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Counterpossibles in Scientific Practice

Three Case Studies in support of Worldly Hyperintensionality

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Abstract

Hyperintensionality – the failure of substitutivity *salva veritate* of intensionally equivalent expressions – is one of the most debated topics in recent philosophy of language. Being a phenomenon that affects a wide variety of different sentential contexts, a question concerning its source arises: is hyperintensionality something that can originate from actual features of the world, or it is simply some kind of representational phenomenon, which entirely depends on our conceptual faculties and preferred semantics? After a brief general introduction to the topic from the semantic standpoint, in the present work I defend a view developed by Daniel Nolan, according to which hyperintensionality can be a worldly, non-representational phenomenon. I first reconstruct Nolan's view and argument for worldly hyperintensionality, understood through the comparison with *de re* intensional modality. Then, I address the main criticism that has been raised against it, developing three objections. Evidence for worldly hyperintensionality is thereby presented, by focusing on a paradigmatic trigger: counterpossible conditionals. I provide two jointly sufficient criteria for a counterpossible to be a genuine instance of worldly hyperintensionality, then illustrate three case studies from scientific practices that all rely on counterpossible reasoning: reducibility in relative computability theory; scientific explanation of certain substances' essential properties; and emergent molecular structure in chemistry. Each of them is evaluated by checking if it satisfies the criteria for worldly hyperintensionality, leading to the detection of patterns of counterpossible dependence occurring in at least some specific instances of the case studies discussed. Finally, I suggest a possible realist reading of such dependence relation, locating its *relata* in the world, independently from our representations.

Keywords: hyperintensionality; counterpossibles; relative computability; scientific explanation; strong emergence; negative facts.

I. Introduction

1.1 Compositionality

Contemporary semantics heavily relies on set-theoretic machinery to account for essential features of natural languages. Among them, *compositionality* is one of the most important. Before beginning my journey through the formal frameworks and their philosophical motivations that will constitute the basis of this work, I will briefly introduce this property, together with a principle that will soon be recognized as fundamental for the upcoming discussion.

In the literature we find different ways of understanding compositionality, but a rough characterization that will be suitable for the current purposes is the following:

(C) The meaning of a complex expression is determined by its structure and the meanings of its constituents (Szabó 2017).

(C) is meant to capture, at least *prima facie*, the huge amount of evidence we have for semantic phenomena like *productivity* – our ability as competent speakers of potentially understanding infinitely many complex expressions we never encountered before, just by understanding the meaning of each of their parts – and *systematicity* – the fact that there are definite and predictable patterns among the expressions we understand. It follows that every compositional language satisfies a rule, commonly known as *substitution principle*, according to which:

(SP) The substitution of a meaningful linguistic item occurring in a complex expression with another item that has the same meaning, preserves the meaning of the whole expression.

An instance of such semantic preservation is the following:

(1) Anna talks

If one substitutes 'talks' in (1) with an expression that has the same meaning, for example 'speaks', the overall meaning of (1) does not change.

But what exactly is *a meaning*? According to standard accounts, the primary aspect of meaning has to do with the *referents* of linguistic items: a meaningful linguistic expression must, obviously, "pick out" something out there in the world. Then, the first step towards a formal account of meaning amounts to find the right sort of entity that fits this role, for each kind of basic linguistic expressions such as names, predicates and sentences.

1.2 Extensionality

A first set-theoretic attempt in this direction that has its roots in Frege 1892, consists in employing the notion of *extension*. Following Nolan 2019,

> the extension of a name is its bearer, the extension of a predicate is the set of things that the predicate applies to, and the extension of a sentence is its truth-value. [...] In the simplest cases of subject–predicate sentences, the extension of the subject term and the extension of the predicate together determine the extension (truth-value) of the sentence: the sentence is true if the extension of the subject term is a member of the extension of the predicate, and false otherwise. (For example, if the extension of 'Peter' is Peter and the extension of 'is a rabbit' is the set of rabbits, then 'Peter is a rabbit' is true if, and only if, Peter is a member of the set of rabbits) (p. 1).

Within this framework the extensions of complex sentences are truth-functions of the extensions of simple sentences (sentences of the subject-predicate form) occurring in them: the truth-value of a whole sentence is fully determined by the truth-values of its constituents, given a set of rules for the logical connectives.

By identifying the primary aspect of the meaning of a sentence with its truth-value, we can easily notice how (SP), so far understood merely as an intuitive principle, can be reinterpreted in a more rigorous way:

> (SVE) Two expressions can always be inter-substituted *salva veritate* if and only if they have the same extension.

Here is an example. On the assumption that I actually have ten fingers and ten toes, the following sentence is true:

(2) The number of my fingers = 10

Since the extension of 'the number of my fingers' is the same of 'the number of my toes' – namely, the number 10 – we can substitute the former with the latter in (2) without changing the truth-value of the resulting sentence:

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(2a) The number of my toes = 10

So far, so good. But is this enough to conclude that 'the number of my fingers' and 'the number of my toes' have the same meaning? Even if, in actuality, they refer to the same object, they seem to express different contents. Is truth preservation in extensional contexts – places in a sentence that allow for substitution according to (SVE) – sufficient for modeling synonymity? Something in this picture of meaning is still missing.

Besides a *prima facie* intuition that two extensionally equivalent expressions can differ in meaning, what can go wrong if we limit ourselves to a purely extensional semantics? Consider the following sentence:

(3) 10 = 10

Since (3) is an identity statement, if it is true, it is necessarily so[1]:

(3a) It is necessary that 10 = 10

Now, suppose to substitute in (3a) the first occurrence of '10' with a co-extensional expression from the previous example, 'the number of my fingers':

(3b) It is necessary that the number of my fingers = 10

(3b) is obviously false[2]: for instance, I could have lost one of my fingers in an incident. Another famous example of (SVE) failure, due to Quine 1951, involves predicates:

(4) It is necessary that all renates are renates

(4) is a straightforward conceptual truth. Now, since in the actual world 'being a renate' has the same extension of 'being a cordate' (every animal with a heart has also a kidney and

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1   The necessity of identity is a popular view in metaphysics that will be assumed here (see Kripke 1980).
2   Informally, the scope of the necessity operator in (3b) is the whole expression, therefore its default reading is *de dicto*. The correspondent *de re* variant is typically written as 'the number of my fingers is necessarily 10'. See section 2.2 for a more detailed discussion.

vice-versa), according to (SVE) the two expressions should always be inter-substitutable *salva veritate.* However, by substituting an instance of 'renates' in (4) with 'cordates', one gets:

(4a) It is necessary that all renates are cordates

But there could have been renates that are not cordates. So, (4a) turns out to be false, despite the truth of (4). It seems that even if the predicates 'renate' and 'cordate' have the same extension, they still express different *properties*.

1.3 Intensionality

I showed how easily one can find counterexamples to (SVE), namely, cases in which substitution *salva veritate* of co-extensional expressions fails. Is the project of modeling compositionality doomed? The answer is pretty obvious: it is doomed, as long as one only focuses on the extensional aspect of meaning. But, as already noticed, there is a lot more to say about it. For instance, intuition suggests that the meaning of an expression should not depend merely on how the world *actually is*, but also on *how it could have been*. What happens in cases like (3b) and (4a) is that the presence of the modal operator 'it is necessary that' creates opaque contexts in which truth preservation requires something stronger than mere extensional equivalence.

The notion of *intension* has been introduced precisely for this purpose. As Nolan points out, «the intension of a piece of language is traditionally thought to be an aspect of the meaning of that language which goes together with how the world is to determine the extension of the piece of language» (p. 1). Intensional accounts try to fully capture the identification of the meaning of a sentence with its truth-conditions, namely, the conditions under which it is true. The seminal idea behind this view is commonly traced back to Wittgenstein's *Tractatus*, in which it is claimed that in order to understand a sentence one must know what is the case in the world if that sentence is true. In the very same passage – proposition 4.024 –, Wittgenstein also points out that one understands a sentence if one understands its constituents; in other words, that language is compositional.

What are intensions then? For basic categories (names, predicates and sentences), intensions are generally defined as functions from possible situations – commonly labeled as *possible worlds* – to extensions: the intension of a predicate, i.e. the property it expresses, can be identified with *the set of all objects that it applies to* (not merely actual ones), and the intension of a sentence with *the set of possible worlds in which it is true*. At least since Kripke 1980, proper names, along with demonstratives, indexicals and natural kinds are typically taken to be *rigid designators*: expressions that refer directly to the same object in every possible world[3].

Possible worlds are *ways in which things could be or have been* and were first introduced to ground the truth of modal statements like (3a). From a purely semantic perspective, the formal device that captures the idea of a possible world is that of a maximally complete and consistent set of sentences: for every world w and every sentence p, either p is true in w or it is false. According to the framework of standard Possible Worlds Semantics (PWS), a sentence is necessarily true if and only if it is true in every possible world, and it is possibly true if and only if it is true in some world.

Such modal operators behave like quantifiers over worlds, and, as already noticed, give rise to peculiar sentential contexts that require more than simple co-extensionality to warrant truth preservation. These are commonly called *intensional contexts* (as opposed to extensional ones), because substitution *salva veritate* within them requires at least identity of intension. Given this picture, a corresponding substitution principle can be expressed:

(SVI) Two expressions can always be inter-substituted *salva veritate* if and only if they have the same intension.

Subscribing to this new framework enables to understand what went wrong in the previous examples: they simply did not satisfy the correct substitution principle, (SVI). Let us see it at work.

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3 Both the rigidity thesis and the notion of possible world have generated a huge amount of literature that encompasses almost every discipline within analytic philosophy, and the debate around them is still going on nowadays. In the present work I will not focus on the epistemic and metaphysical issues concerning them, but I will assume the rigidity thesis and take an ontologically neutral stance towards possible worlds semantics.

Consider two names that actually refer to the same individual: 'Brian Hugh Warner' and 'Marilyn Manson'. Assuming the rigidity of proper names, they refer to the same individual in every possible world.

(5) It is necessary that Marilyn Manson is Marilyn Manson

(5) is both conceptually and metaphysically true (it is true both on a *de dicto* and on a *de re* reading), therefore, by substituting the first instance of 'Marilyn Manson' with the intensionally equivalent expression 'Brian Hugh Warner', one gets:

(5a) It is necessary that Brian Hugh Warner is Marilyn Manson

which is also true. Through the adoption of an intensional account, it seems possible to associate each linguistic expression with the appropriate function that allows for substitution *salva veritate*, also within the scope of modal operators.

Now, one may ask if sameness of meaning is fully captured by sameness of intension: is satisfying (SVI) sufficient for two expressions to be synonyms? If that is the case, (PWS) would provide a full and elegant account of compositionality. But what about cases like the following:

(6) Anna believes that Marilyn Manson is a great singer

Suppose that (6) is true, yet Anna does not know that Marilyn Manson is Brian Hugh Warner. Now, by operating the same substitution saw in the previous example, one gets:

(6a) Anna believes that Brian Hugh Warner is a great singer

which is false. The same arguably happens with any other intentional concept like hope, fear, desire, knowledge, and so on, even without appealing to the rigidity of proper names. Suppose it is true that:

(7) Anna knows that 2+2=4

All mathematical truths, being true in the same possible worlds (all of them), are intensionally equivalent. Consider now a less friendly mathematical truth like 9!=362880. According to (SVI), it is possible to substitute it with '2+2=4' in (7) *salva veritate*:

(7a) Anna knows that 9!=362880

But suppose that Anna does not actually know that 9!=362880. Then, the inference from (7) to (7a) would be illegitimate, even though (SVI) allows it. This generalizes to any other necessary truth: in a standard intensional framework it is sufficient to know a single necessary truth, no matter how trivial, for knowing *every* necessary truth as well[4].

Apparently, substitution *salva veritate* of co-intensional expressions within representational contexts – places in a sentence created by the presence of verbs for intentional mental states – can fail. Now, either propositional attitude reports like (6) are non-compositional, or (SVI)-based semantics like (PWS) lack expressive power. But why should one consider sentences like (6) non-compositional? Is the failure of (SVI) in such cases sufficient for rejecting compositionality altogether? Epistemically, they behave exactly like the others discussed so far. Suppose that I never encountered (6) before. I would still be able to understand it as long as I could grasp the meaning of each of its constituents (i.e. 'Anna', 'believing', 'Marilyn Manson', 'being a good singer'). In other words, sentences like (6) seem to comply with the productivity of natural language, and productivity entails compositionality. Denying this would require to provide a non-*ad hoc* explanation of alternative mechanisms by means of which one understands expressions like (6). Furthermore, as will be explored in detail later, (SVI) seems to fail also with sentences that do not contain any kind of propositional attitude verb. In other words, it can fail also with sentences that have as constituents only pieces of language that typically behave compositionally.

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4   This phenomenon is an instance of the more general *problem of logical omniscience* (see Rendsvig and Rasmus 2021).

Given these remarks, it seems more reasonable to conclude that (SVI) is not sufficient as a basic principle to account for compositionality, instead of rejecting compositionality altogether in problematic cases.

1.4 Representational hyperintensionality

At a first glance, the problem of finding an alternative to (SVI) and to standard intensional semantics is the problem of finding a formal device for representing meanings that do not collapse the distinction between necessary equivalents. In other words, we need a principled way of delivering the appropriate semantic *fineness of grain* for blocking inferences like that from (6) to (6a), while retaining all of the substitutional properties associated with compositional languages. This constitutes the core of the issues concerning what is nowadays called *hyperintensionality*.

The most common way of framing the problem in the literature focuses on *hyperintensional contexts*, of which those created by propositional attitudes, as previously seen, are a prominent example. Hyperintensional contexts are places in a sentence in which (SVI) fails. To put it in a more rigorous way, a place in a sentence is said to be hyperintensional if and only if it does not always allow substitution *salva veritate* of co-intensional expressions.

What about the entities that should warrant substitution *salva veritate* in hyperintensional contexts? As Nolan 2019 notices,

> it is less common to talk about an expression's 'hyperintension' than it is to talk about its extension or intension. Presumably the hyperintension of an expression would determine how it behaved in hyperintensional contexts as well as extensional and merely intensional ones, and how it contributed to the truth-conditions of any sentence in which it appeared (p. 3).

Needless to say, a corresponding substitution principle that should capture the appropriate fineness of grain for drawing hyperintensional distinctions would sound like this:

(SVH) Two expressions can always be inter-substituted *salva veritate* if and only if they have the same hyperintension.

In order to comply with (SVH) and assess its suitability, one should first find a precise definition of hyperintension: something far from being an easy task. Furthermore, the problem of hyperintensionality appears to be at least twofold: it is not simply that of finding a way of making distinctions between semantic contents that are not too coarse-grained, but also that of finding a way of making distinctions that are *not too fine-grained.* This is known as *the granularity problem*, or the problem of "how hyper" hyperintensions should be. Borrowing Berto and Nolan 2021 words,

> a hyperintensional account of this or that notion shouldn't make it as fine-grained as the syntax of the language one is working with—on pain of giving away the very point of having a semantics for it. Propositions may be more fine-grained than sets of possible worlds, but they had better not be mappable 1:1 to the sentences expressing them.

To better understand this point, look at one of the most ambitious historical attempts to model semantic distinctions: Lewisian semantic trees.

Lewis 1970 claims that meanings are to be identified with peculiar structures built out of intensions, which are able to cut more fine-grained distinctions between co-intensional expressions. Such devices are semantic trees, namely functions that impose a hierarchy on sets that mirrors the internal structure of meaningful expressions. Formally, «a partial order with a unique maximum element, x_0, such that for any element, x_n, there is a unique finite chain of elements $x_n \leq x_{n-1} \leq ... \leq x_1 \leq x_0$» (Priest 2012, p. 6). Hence, according to this picture meanings are *semantically interpreted trees*, having at each of their nodes an ordered pair in which the first member is a category – entities like names, predicates and sentences – and the second one is an appropriate intension for that category.

Within this framework, it is possible to account for the difference between sentences like:

(8) If the sky is blue then the sky is blue

(9) Grass is green or grass is not green

(8) and (9) are both tautologies, therefore, assuming classical truth conditions for the disjunction and the conditional, they are true in exactly the same circumstances: all possible ones. In other words, they are co-intensional. But if meanings are semantic trees, then a difference between any node of the trees representing (8) and (9) amounts to a difference in their meaning, regardless of their intensions (where the intension of a whole tree is identical with the intension of its root, i.e. the top node representing the whole expression). Sameness of meaning entails sameness of intension, but sameness of intension is compatible with distinction in meaning. So, Lewis' trees seem to be able to draw this specific kind of hyperintensional distinctions. But what about sentences like:

(10) Grass is green.
(11) It is not the case that grass is not green.

One can easily notice that (11) is the double negation of (10). Thus, they have the same truth conditions[5]. Do they have the same meaning? Intuitively, the answer is yes. (10) and (11) do not simply share their intensions, but also their *subject matter*: they are both about 'the color of the grass'. Nevertheless, according to Lewis' account (10) and (11) would differ in meaning: two different trees are needed to represent a sentence and to represent its double negation, so there cannot be identity between their meanings. Then, as Lewis himself recognizes, semantic trees seem to cut meanings *too finely*, but he quickly dismiss the issue by saying:

> this difficulty does not worry me: we will have both intensions and what I call meanings, and sometimes one and sometimes the other will be preferable as an explication of our ordinary discourse about meanings. Perhaps some entities of intermediate fineness can also be found, but I doubt that there is any uniquely natural way to do so (p. 32).

Was Lewis' pessimism justified? Nowadays hyperintensional talk is widespread across several areas of philosophical inquiry, so the need for rigorous ways of dealing with it resulted in the development of a variety of formal frameworks that account for more or less specific instances of it.

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5   Unless one subscribes to some form of intuitionism, which rejects the classical rule of double negation elimination (see Moschovakis 2018).

So far I have provided a general introduction of the topic, which could not have been approached without making reference to some intuitive semantic principles. As we have just seen, the task of finding an appropriate formal framework to account for them is currently one of the most difficult and debated in philosophy. But regardless of one's semantic preferences, in the present work I want to focus on a side-issue that has yet received relatively little attention: the problem of *the source of hyperintensionality*. The hyperintensionality of the cases discussed in section 1.3 seems to depend entirely on the way in which some agent represent certain contents. We can label them as cases of *representational hyperintensionality*. But what if hyperintensionality could depend also on how the world is like, independently of our ways of representing it? In the following chapters I will explore this hypothesis by focusing on a peculiar class of hyperintensional contexts, chosen for their significance that goes beyond the scope of philosophy and formal semantics.

II. Non-representational Hyperintensionality

2.1 Hyperintensional contexts without propositional attitudes

There are at least two distinct ways of understanding non-representational hyperintensionality. The first and weaker sense is simply a label for the set of those hyperintensional contexts triggered by something different than a sentential operator standing for a propositional attitude like belief, desire, hope, fear and so on. The second and stronger sense is what Nolan 2014 calls *worldly hyperintensionality,* which is based on the idea that «we need hyperintensional resources to adequately represent features of the world independent of us, and not merely to understand and explain our *representing* of that world» (p. 2). I will try to clarify Nolan's position in section 2.2; in the current section, I will illustrate some of the most discussed cases of non-representational hyperintensionality in the weaker sense.

Recent years have witnessed a constantly growing amount of evidence for hyperintensional contexts that apparently arise independently from any direct representational constraint. The property of being knowable *a priori* displays hyperintensional features even without explicit ascriptions of knowledge to a particular agent. Consider this classic example:

(12) It is *a priori* that water is water

'water is water', arguably just like any other expressible tautology, is knowable *a priori*, therefore (12) is true. We know that water is H_2O and, following Kripke, that it is necessarily so[6]. This means that 'water' and 'H_2O' are intensionally equivalent. But if one substitutes an instance of the former with the latter in (12), the resulting sentence is:

(12a) It is *a priori* that water is H_2O

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6   It is important to notice that one can resist the inference by rejecting the kripkean notion of *a posteriori* necessity (see Kripke 1980).

which seems false: the identity between water and H_2O is something that must be discovered through experience, therefore it is not possible to know it *a priori.*

The property of being informative seems to display the same feature. To see this, consider a false statement like:

(13) It is informative that water is water

By applying the same substitution of the previous example, one gets:

(13a) It is informative that water is H_2O

which is arguably true.

Yet, one might still be tempted to treat this kind of hyperintensionality as a representational phenomenon, since the property of being *a priori* has to do with the blatantly epistemic notion of knowability, and knowledge is a propositional attitude. Likewise, one might argue that the property of being informative cannot be ascribed to a proposition without indexing it to a particular agent or group of agents: something cannot be informative or uninformative *simpliciter,* but only with respect to some agent.

However, there are cases of hyperintensionality in which no implicit epistemic notion seems to be involved. Several hyperintensional distinctions manifest at a superficial semantic level, even without the need of substitutions that are not truth-preserving. This has already been partially shown by discussing Lewis in the previous chapter, but it is worth to notice that something analogous happens with distinctions between impossibilities (necessary falsehoods):

(14) The grass is green and it is not the case that the grass is green
(15) The sky is blue and it is not the case that the sky is blue

The intuitive difference in meaning between (14) and (15), due to a difference in their subject matter – the former is about *the color of the grass*, while the latter is about *the color of the sky* – cannot be accounted for in intensional terms[7]. Intensionally, they are true in

7   See Yablo 2014 for a hyperintensional theory of subject matter.

exactly the same worlds (none of them), so they are both identified with the null set, just like any other contradiction.

Finally, conditionals seem to display hyperintensional features as well. Consider the following sentences:

(17) If Hesperus is not Phosphorus, then Hesperus is not Phosphorus

(17a) If Hesperus is not Phosphorus, then Hesperus is not Hesperus

Since 'Hesperus' and 'Phosphorus' are intensionally equivalent, (17) and (17a) should be both true. However, one might want to maintain that the former is true, while the latter is false[8]. Something analogous happens with the infamous paradoxes of the material conditional and of the strict conditional, in which the supposed logical relation between the antecedent and the consequent seems to be irrelevant for the intuitive truth-value of the whole conditional[9]. One of the most interesting and discussed cases, that of counterpossible conditionals, will be explored in detail in chapter 3.

The source of hyperintensionality in these cases is less clear. Even if they do not involve neither explicit representational attitudes nor implicit epistemic notions, they ultimately seem to depend on our semantic intuitions. This lead authors like Berto and Nolan to locate them in the representational realm[10]. In the next section, I will introduce the concept of worldly hyperintensionality, namely hyperintensionality triggered by wholly non-representational phenomena.

2.2 Worldly hyperintensionality

Nolan 2014 acknowledges the conspicuous amount of data for non-representational hyperintensionality and moves a step forward. The author's original insight is that we should not consider hyperintensionality simply as a semantic or representational phenomenon which manifests at the level of language (regardless of whether the subject

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8   See Jago 2014 for a detailed discussion.
9   See Berto and Nolan 2021.
10   *Ibidem.*

matter of a sentence is non-representational), but also as a real, worldly phenomenon largely independent from our conceptual faculties.

He begins by extending the applicability of hyperintensional talk to phenomena outside of the field of language: in his view, it is not enough to say that a phenomenon is hyperintensional on the basis of the fact that the words we use for talking of it occupy a hyperintensional position. A better way of classifying phenomena consists in detecting a principled need for a certain kind of expressive resources to *explain* them. For instance, *de re* modal properties systematically require intensional language to be accounted for, while propositional attitudes like belief need something more fine-grained. The shift is then from the linguistic phenomena to the phenomena that the language captures, and a rough picture of their classification is the following:

> Running might then count as an extensional phenomena [sic], if we think an account of running can be adequate if it only mentions actual arrangements of legs and bodies. Necessity would then naturally be classified as an intensional phenomena [sic], needing expressions like "necessarily", that create intensional positions, to capture it. Belief might naturally be classified as a hyperintensional phenomena [sic], needing an expression like "believes that..." to adequately capture it. Most theories of phenomena will consist of a variety of sentences, so if we want to assign each phenomenon one label from "extensional", "intensional" and "hyperintensional", it might be best to assign "hyperintensional" if hyperintensional language is needed to capture a phenomenon, "intensional" if intensional but not hyperintensional language is needed, and "extensional" only if no intensional nor hyperintensional language is needed (p. 4).

A first matter of potential controversy with this method of classification is addressed by the author. On the one hand, one might believe that an alleged extensional phenomenon like running is actually intensional, or even hyperintensional, on a more careful reading. On the other hand, one might argue that necessity can be accomodated in a purely extensional language, for instance by means of quantification over worlds. However, as remarked later in the paper, even if a fully extensional language for intensional or hyperintensional phenomena is actually available, this would not threaten the idea that *the phenomena themselves* are intensional or hyperintensional, just like the fact that we can offer a theory of possible worlds in an extensional language does not undermine the idea that possible worlds themselves are intensional entities. Nevertheless, as we will see in section 2.3, one could reject the very idea of worldly hyperintensional phenomena even while accepting that

a language contains hyperintensional positions occupied by non-representational subject matters.

Now, the idea that some non-representational properties can be explained away by treating them as properties of pieces of language instead of properties of the objects denoted by them is not new in the history of philosophy. Nolan mentions the extensionalist reaction to the so-called *intensional revolution* that took place in analytic philosophy between the 1960s and 1980s: in this flourishing era of new powerful formal devices like possible worlds and the wide range of modal notions employed across almost every field of philosophical inquiry, many were those unwilling to take intensionality seriously outside the merely linguistic domain. For instance, a common strategy adopted by prominent extensionalists like Davidson and Quine consisted in paraphrasing intensional concepts by mentioning them in quotation, in order to reinterpret modality as a predicate of sentences, or propositional attitudes as relations between people and sentences. This allowed them to read intensional phenomena as if they were all about representations. According to such extensionalists, when we say that I contingently have ten fingers, we are not stating a fact about a non-representational object. What we are really doing is expressing a property of a sentence: "'I have ten fingers' is contingent". In other words, there was no place for *de re* modality in the extensionalist picture. Nowadays, this would constitute a rather unorthodox view, as Nolan points out: «while extensionalism still has some adherents, I think it is fair to say that most philosophers working on necessity, belief etc. find these extensionalist maneuvers clumsy, inadequate to the data, and philosophically perverse» (p. 7).

A similar kind of resistance, this time on intensionalist grounds, exists today towards certain kinds of hyperintensional contexts, as will be explored in detail later for the case of counterpossible conditionals. Nevertheless, the need for expressive resources able to draw fine-grained distinctions between intensionally equivalent contents is widely recognized. What is more controversial is the view that hyperintensionality can be due also to the world rather than just to our representations of the world. According to Nolan, the source of the difficulty in accepting this idea is to be found in the common characteristics of the most famous cases of hyperintensionality. As already underlined, even the cases of weaker non-representational hyperintensionality can be indirectly traced back to our representations or conceptual faculties. Besides the obvious cases like intentional mental

states denoted by propositional attitude reports, it is always possible to develop arguments for the conceptual nature of synonymity, truth according to fictions or even according to semantics and epistemology, as we saw with the properties of being *a priori* or being informative. If such contexts exhausted the whole range of hyperintensional phenomena, this would be enough to conclude that hyperintensionality is essentially a representational matter. The fact that we need hyperintensional positions in our languages could be explained with our inability to have perfect access to all of the intensions associated with each piece of language and its corresponding representational content in our minds. Another reason why one might be tempted to consider hyperintensionality as merely representational is that one's favorite theory for analyzing hyperintensionality makes use of representational entities only. This is similar to the strategy adopted by Byrne and Thompson in opposing Nolan's view, as we shall see in the next section.

But the crucial passage is that, according to Nolan, the above mentioned examples are only a subset of the whole range of hyperintensional phenomena.

> I want to suggest here that hyperintensionality is needed for an adequate account of the non-representational world as well. In suggesting this, I do not mean to be disputing the truism that, in one sense, we need representations for hyperintensionality. Hyperintensional positions were defined in terms of substitution instances and truth-preservation under substitutions, and so is a criterion that applies only to representations. Nor do I mean the relatively innocuous claim that sentences that contain hyperintensional positions are sometimes about the non-representational world, and are sometimes not synonymous with non-hyperintensional sentences about the same subject matter (p. 10).

Hence, Nolan's intuition seems to be that hyperintensionality can be involved with non-representational content also at a deeper level than that of linguistic subject matter: hyperintensionality can be a worldly phenomenon independent from our representations. But what evidence do we have for this? According to Nolan, a first indicator is the fact that we often require hyperintensional *explanations* for worldly phenomena that do not depend at all on our representations of them: «if we find ourselves appealing to hyperintensional distinctions in our theory of non-representational matters, that would be good evidence that hyperintensional phenomena are not just representational» (p. 10).

The author then proceeds to enumerate some of the main examples of hyperintensional explanations within the field of metaphysics[11]. Since metaphysics is supposed to be the inquiry on the ultimate structure and nature of reality, and the subject matters of metaphysical theories are typically non-representational, to show that some fundamental features of reality require hyperintensional explanations amounts to provide genuine evidence for the claim that hyperintensionality can be due to the world.

The first example mentioned by Nolan is counterpossibles – counterfactual conditionals with a necessarily false antecedent –, that I will explore in detail in the next chapter. Next, we have explanation itself: as long as we take it as a relation between states of affairs (which are worldly entities), as many do, in claims like:

(E) It is true that grass is green because grass is green

the *explanans* and the *explanandum* are necessarily equivalent (necessarily, it is true that grass is green if and only if grass is green), but the relation captured by 'because' goes only in one direction[12]. Of course, one might raise a worry concerning explanation similar to that discussed in section 2.1 concerning other epistemic notions. However, according to Nolan it is sufficient to point out that those who already accepted that explanation can be a worldly matter have reasons to posit worldly hyperintensionality here.

Regardless of one's opinion on this specific issue, there seems to be cases in which the absence of representational mediation is less controversial. Nolan mentions the debate on *essences*, in which it has been argued that a merely modal (intensional) account is insufficient to draw the appropriate distinctions[13], and this might also generalize to *properties* as a whole (triangularity and trilaterality necessarily apply to the same objects, but they seem to be different properties). This includes the ways of classifying the various kinds of properties, like *intrinsicality*, in which the traditional modal accounts have been contrasted with a number of counterexamples[14] and even positive arguments for its hyperintensionality[15]. Furthermore, one of the most flourishing topics in metaphysics of

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11   See Nolan 1997 for a discussion on the necessary nature of metaphysical claims.
12   See Schneider 2011.
13   See Fine 1994.
14   See Dunn 1990.
15   See Eddon 2011.

recent years is *metaphysical grounding*, i.e. a non-causal relation of ontological dependence between objects, properties or states of affairs. Despite its being a matter of huge disagreement, there is a widespread consensus regarding its hyperintensionality. A classical example is the following:

(G) The existence of Socrates grounds the existence of {Socrates}

{Socrates} is Socrates' singleton, i.e. the set containing Socrates as its only member. Apparently, {Socrates} and Socrates are co-intensional, but if we switch the former with the latter in (G), we get a false statement: it seems that {Socrates} exists in virtue of Socrates' existence and not the other way around.

Finally, Nolan himself developed arguments for the view that some disposition ascriptions are hyperintensional: objects can have non-trivial *impossible dispositional properties*, i.e. properties that would manifest only in impossible circumstances[16]. Consider the following:

(ID) Anna is disposed to be surprised if she sees a round square
(IDa) Anna is not disposed to be surprised if she sees a round square

Since the stimulus condition – the sight of a round square – for the manifestation of the disposition is metaphysically impossible, (ID) and (IDa) are supposed to express the same proposition (the null set), hence they should have the same truth value. However, we have a strong intuition that the former would be true while the latter would be false. In sum, as Berto and Nolan 2021 remark: «almost any area of metaphysics where necessity or counterfactuals have played a role is a candidate for hyperintensional approaches».

However, the shift from the language to the world might still raise some perplexity. All of the just sketched cases have in common the fact of being about non-representational matters and of requiring hyperintensional resources to account for them, but this does not suffice for concluding that hyperintensionality itself is something arising from the world rather than from our representations of it. After all, hyperintensionality has been defined in linguistic terms, theories are ultimately linguistic items and Nolan writes that allowing for

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16 See Jenkins and Nolan 2012.

hyperintensional contexts with non-representational subject matters is relatively innocuous. How does this justify to locate the source of hyperintensionality outside the field of language? On the face of it, I believe that it is simply the most natural way to read certain instances of hyperintensionality. The author's comparison with intensionality is appropriate: just like the natural way of reading intensional modality is often the *de re* interpretation (otherwise, we would need to resort to dubious paraphrases), the same is true for certain hyperintensional statements. To better understand this point, look at how we ordinarily treat ambiguous intensional cases. Consider the following:

(P) Every poor is necessarily poor[17]

We have two different ways of reading (P). On a *de re* interpretation, it says:

(Pr) $\forall x(poor(x) \to \Box poor(x))$

namely, that every poor is such that he or she is necessarily so (he or she possesses the property of being poor essentially), making (P) obviously false. Hence, the natural and correct reading seems to be *de dicto*, according to which (P) says:

(Pd) $\Box \forall x(poor(x) \to poor(x))$

Here, the scope of the modal operator includes the whole expression contained within the scope of the universal quantifier, therefore (Pd) ascribes a modal property to the expression 'every poor is poor', which is a necessary conceptual truth. On the other hand, take a sentence like:

(F) The number of my fingers is necessarily even

On a *de dicto* interpretation, (F) would say:

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17 The example is from Casalegno 1997.

(Fd) □∃x(the number of my fingers(x)∧∀y(the number of my fingers(y)→y=x)∧even(x))

namely, that I necessarily have the property of having an even number of fingers, which is clearly false. So, in this case the natural reading (on the assumption that I actually have ten fingers) should be *de re*:

(Fr) ∃x(the number of my fingers(x)∧∀y(the number of my fingers(y)→y=x)∧□even(x))

according to which the object denoted by 'the number of my fingers' in the actual world – i.e. the number 10 – is necessarily even. Apparently, some statements are intensional because they capture features that the world possess intrinsically, not because we represent them intensionally.

Likewise, some statements are hyperintensional because they capture features that the world possesses intrinsically, not because we represent them hyperintensionally. Just like it would be implausible to say that it is true that I contingently have ten fingers because the sentence 'I have ten fingers' has the property of being contingent, it would be equally implausible to say that {Socrates}'s existence is grounded on Socrates' existence because the grounding relation behave *semantically* like that, or because we are unable to conceptualize it otherwise. Notice that in this last case the subject matter is non-representational and the two *relata* behave hyperintensionally, but the relation seems to hold regardless of our representational faculties or preferred semantics.

Now, despite the strong intuitions towards this reading within metaphysics, Nolan does not make a clear statement of what he believes to be the sufficient and necessary conditions for worldly hyperintensionality. However, I think that it is possible to follow its guiding principle of looking for hyperintensional explanations (especially those that rely less on intuition than metaphysical ones, like scientific explanations) in order to reconstruct some criteria for making the idea of at least one kind of worldly hyperintensionality more clear, as I will discuss in chapter 3.

In the following section I will explore the main criticism that has been put forward against the idea that hyperintensionality can be something that originates from the world, and in the next chapter I will illustrate three case studies for defending its plausibility from outside the field of metaphysics.

2.3 Against worldly hyperintensionality

Byrne and Thompson 2019 criticize Nolan's idea of worldly hyperintensionality on conceptualist grounds. More precisely, even if they accept the view that hyperintensional language is needed to accurately describe the world, they reject the idea that hyperintensionality can be a genuine non-representational phenomenon: «hyperintensionality derives from features of representations» (p. 153).

They begin by sketching a Fregean picture of representational hyperintensionality analogous to one that, they argue, is possible to be employed for analyzing intensional notions. Fregean accounts maintain that linguistic expressions – at least those embedded in intensional contexts – do not refer directly to worldly entities such as objects, properties and states of affairs. Instead, reference is mediated by the *senses*, or *modes of presentation* of the objects denoted by the linguistic expressions[18]. Hence, what speakers actually express is not the *denotation* of a linguistic expression – a worldly object independent of our representational faculties –, but a way in which we conceptualize such denotation: a kind of representation. In other words, meanings are identified with senses. It is important to notice that in the paper we do not find neither a full Fregean semantics, nor even a principled way for building a rough formal framework. Instead, the authors choose to provide some examples to defend their suggestion.

To see their strategy at work, consider a pair of sentences from the previous chapter:

(3a) It is necessary that 10=10

(3b) It is necessary that the number of my fingers = 10

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18 Explaining what sort of entity Fregean senses are supposed to be, metaphysically speaking, is one of the toughest issues that a defender of such view must face; the authors avoid this specific point.

The difference in meaning (and the corresponding difference in truth value) between (3a) and (3b) does not depend on the denotation of '10' and that of 'the number of my fingers' – i.e. the number 10 –, which is contingently the same. Rather, it depends on the fact that they express different *senses*, and obviously, the typical Fregean view is that substitution *salva veritate* of expressions with different senses can fail.

According to Byrne and Thompson, the same happens with regular cases of representational hyperintensionality, such as those triggered by propositional attitudes. From a Fregean perspective, propositional attitudes are relations that agents bear with senses. Therefore, pairs of sentences like:

(6) Anna believes that Marilyn Manson is a great singer

(6a) Anna believes that Brian Hugh Warner is a great singer

once again differ in truth value because Anna bears a relation with two different senses, even if the objects to which the senses refer are necessarily the same: for her, 'Marilyn Manson' and 'Brian Hugh Warner' are associated with different descriptions, and this is enough to explain their semantic difference at the hyperintensional level.

With this in mind, the authors proceed to summarize Nolan's core argument in the form of a conditional:

(N) If a subject-matter is not representational, then we cannot explain the fact that we need hyperintensional idioms to describe and explain it in conceptualist terms.

Their second step is to compare (N) with its intensional counterpart:

(M) If a subject-matter is not representational, then we cannot explain the fact that we need intensional idioms to describe and explain it in conceptualist terms.

Finally, they claim that if (M) can be successfully rejected, there is no reason to think that the same cannot be done with (N). Since «a Fregean analysis of the non-representational intensional operator *per excellence* [i.e. necessity] is well-known» (p. 156), they take a case involving necessity like (3a) vs (3b), read in a conceptualist fashion, as evidence for the possibility of rejecting (M). More precisely, they argue that what a sentence like (3a) says is that *a certain sense has the property of being necessary*, therefore it cannot guarantee truth preservation if one of its components is substituted with another having a different sense. They conclude that since senses are features of representation, intensionality arises from it, even if the subject matter of a sentence like (3a) is not representational. By analogy, the same should happen with hyperintensionality:

> if there's a plausible conceptualist account of an intensional locution about a non-representational phenomenon, and if plausible conceptualist accounts of intensional locutions can be extended to deliver plausible conceptualist accounts of hyperintensional ones, why should we expect these extensions to break down when the locutions are about non-representational phenomena? (p. 157).

In the previous section we saw some intuitive and historical reasons for rejecting the view that *de re* modality can be systematically paraphrased into *de dicto* modality, like the authors seem to suggest. But regardless of this, I think that Byrne and Thompson's argument from analogy is problematic for a number of independent reasons. First of all, the analogy itself. Even if a full Fregean account of intensional content is available, how is that supposed to constitute evidence for the possibility of an extended Fregean semantics that also accounts for all kinds of worldly hyperintensional content? They declare themselves «confident that such semantics could be formulated for most of them at least» (p. 155), but in the paper they merely present alleged counterexamples to (M) and claim that it is possible to reject (N) on the same basis, without directly addressing the issue of how the supposed extended Fregean semantics might look like. Of course, they do suggest an analogy at the level of ontological commitment between the two semantics. But I believe that the real value in such a proposal would have been a preliminary understanding of the *differences* between a hyperintensional Fregean semantics and its intensional version, rather than simply remarking the obvious similarities. Furthermore, the idea of a hyperintensional Fregean semantics able to account for all kinds of hyperintensional distinctions may sound

too optimistic at the current time: many different semantics have so far been developed for dealing with more or less broad families of hyperintensional contents, but «hardly any of them has been put forth as a systematic account of all hyperintensional notions» (Berto and Nolan 2021). Hope is the last to die, but for the time being, I think that the burden of proof is still on Byrne and Thompson.

Second, the choice of a semantics for settling a philosophical matter might look, in a sense, question-begging. If we try to prove that an entity, or a phenomenon (like worldly hyperintensional content or, in Nolan's jargon, hyperintensional phenomena) is explainable in representational terms by adopting a semantics that literally *assumes* that the entity at stake is representational, we are probably just walking in circles. Independent arguments for preferring that particular semantics over others with equal expressive power are needed, otherwise we are simply showing something rather trivial: that some worldly hyperintensional contents might be read also in Fregean terms. But *why* should we read them in that way? One might argue on the basis of a principle of ontological economy (a theory with a lighter ontological commitment is to be preferred to a theory with a heavier one), but this would simply move the problem from the object of our semantics to that of the trade-off between ontology and ideology[19].

In my opinion, the crucial point to establish is not whether it is possible to have a conceptualist reading of worldly hyperintensional content, but rather *how* and *from where* hyperintensionality arises, regardless of our semantic preferences. If I am right, the conditional reconstruction of Nolan's idea misses the point. Even if every semantics comes with its own ontological commitment – most obviously to the entities that are chosen to represent meanings –, appealing to the mere fact that a semantics might work at the formal level cannot be used to justify the acceptability of its ontology. One might reject a semantics by judging its ontological commitment as problematic, but it is not clear if one can assess a metaphysical claim (like that on the source or nature of hyperintensionality) by simply choosing a certain semantics. As noticed by Berto and Nolan 2021, «one might think that a theory of meaning should be silent on substantive questions about what the world is like. [...] Likewise, it might be good for a theory of meaning to be neutral about how to understand essences, or intrinsicness of properties, or causation».

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19 See Berto and Plebani 2015 for a detailed discussion.

Finally, remember Byrne and Thompson's opening claim: «hyperintensionality derives from features of representations» (p. 153). Suppose that the mere adoption of a certain semantics suffices for establishing its truth, as I believe they suggest. If that is the case, one might develop the same kind of pseudo-argument put forward by the authors for proving the opposite of their claim. All one needs to do is choosing a semantics for intensional content with non-representational ontological commitments and saying that, by analogy, the same can be done for hyperintensional content. For instance, suppose that for dealing with intensionality one adopts a version of Possible Worlds Semantics with a heavy ontology, such as Lewis' modal realism. Within this framework, possible worlds are concrete entities, by no means less real than the actual one. More precisely, possible worlds are defined as maximally interrelated – causally and spatiotemporally – objects, causally isolated from one another. This allows Lewis to provide a fully extensional analysis of modal notions like necessity and possibility, drawing a straightforward realist (Lewisian extensions are uncontroversially worldly entities) and reductionist picture of intensionality. In other words, it is a semantics that locates the source of intensionality out there in reality, by assuming the concrete nature of possible worlds.

Now, it is pretty obvious how one might replicate Byrne and Thompson's move for extending the worldly nature of intensional content to hyperintensional one: it would be sufficient to extend Lewisian realist PWS in a way that accomodates hyperintensional distinctions. Funnily enough, in this case we do not even need to *imagine* such a semantics: Yagisawa's extended modal realism does precisely this job. Just like Lewis with possible worlds, Yagisawa 1988 describes an ontology of concrete *impossible* worlds that represent impossibilities by instantiating them directly. If we set aside the extremely controversial notion of *real* impossibilities from the metaphysical standpoint, and focus on the purely semantic aspect of this framework – just like Byrne and Thompson do with Fregean senses –, we can easily recognize that, by assuming it, we would have a full non-representational, worldly notion of hyperintensional content. For instance, the distinction between necessary equivalents would depend entirely on the fact that, in some worlds, they do not denote the same objects, regardless of our ways of representing them. By replicating Byrne and Thompson's reasoning, we could then conclude that hyperintensionality *does not* derive

from features of representations: after all, we do have a non-representational account for hyperintensional content.

To rephrase them once again, if there is a plausible *worldly* account of an intensional locution about a non-representational phenomenon, and if plausible *worldly* accounts of intensional locutions can be extended to deliver plausible *worldly* accounts of hyperintensional ones, why should we expect these extensions to break down when the locutions are about non-representational phenomena? If such an argument would not be acceptable, as I believe, then the same is true of Byrne and Thompson's argument.

III. Counterpossibles and Worldly Hyperintensionality

3.1 The orthodoxy on counterpossibles and two criteria for worldly hyperintensionality

In the previous chapter I introduced the idea of worldly hyperintensionality and discussed an attempt to explain it away by appealing to our conceptual faculties. As we saw, one of the most straightforward ways for understanding worldly hyperintensionality (regardless of whether one is skeptical about it) passes through the comparison with its more known relative: worldly intensionality. The same kind of comparison might be illuminating for a particular trigger for hyperintensionality: that of counterfactual conditionals with an impossible antecedent, commonly known as *counterpossibles*. I already mentioned some hyperintensional features of conditionals in sec. 2.1, but in the present chapter I want to focus on a peculiar class of counterpossible conditionals, for their possession of unique properties that, I believe, constitutes the most convincing evidence for accepting hyperintensionality as a real, non-representational matter. In particular, I want to highlight two features that some counterpossibles can possess:

(I) *Indispensability:* if they are required in some of our best scientific practices.

(II) *Reality*: if they have a metaphysically impossible antecedent with a non-representational subject matter.

These will also constitute my two jointly sufficient criteria for identifying cases of genuine worldly hyperintensionality. But first, a brief introduction to ordinary counterfactuals and the standard approach to account for them will be useful.

Counterfactuals are conditionals with a *contrary-to-the-facts* antecedent, expressing what would have been the case if the antecedent was true. A typical counterfactual is a sentence of the form "*if it was the case that p, then it would be the case that q*", and it is supposed to capture a peculiar relation between the facts denoted by 'p' and 'q': *counterfactual dependence*. Something counterfactually depends on something else if and only if the former obtains in every relevant situation (i.e. holding fixed certain facts) in

which the latter obtains. Standard approaches, mainly due to Lewis and Stalnaker, employ the apparatus of standard possible worlds semantics for evaluating the truth of counterfactuals by means of a *similarity* relation across worlds. The technical details will not be explored here, but an intuitive characterization of similarity is that of a way for ordering worlds by their degree of resemblance to the actual one: the more a possible world is similar to our world, the closer it is. This idea of proximity between worlds is often represented as a structure of concentric spheres, in which the actual world is located at the center, and worlds in the outermost spheres are progressively less similar to the actual world than those in the innermost spheres.

With this hierarchy of worlds in mind, the truth conditions of a counterfactual are given by checking what happens in the closest worlds in which its antecedent is true: a sentence of the form "*if it was the case that p, then it would be the case that q*" is true if and only if all of the closest *p-worlds* – the worlds in which the antecedent is true – are also *q-worlds*. To see a classical example due to Lewis 1973:

(K) If kangaroos had no tails, they would topple over

(K) is true if and only if in every closest world to the actual one in which kangaroos have no tails, they topple over. As one can easily notice, within this framework the conditional behaves like a restricted necessity operator, quantifying only over those worlds with the appropriate degree of similarity to the actual one.

What about counterfactuals with an impossible antecedent? The standard account holds that every counterfactual with a necessarily false antecedent, having no possible world that verifies it, turns out as vacuously true. The main motivations suggested by Lewis for such *vacuism*, that soon became the orthodoxy on counterpossibles, primarily concern the fact that, within classical frameworks, impossibilities imply anything (then any kind of conditional with an impossible antecedent will be true, regardless of the consequent). Secondly, vacuism is based on the view that *nothing can counterfactually depend on non-contingent matters*: in other words, that counterpossibles are uninformative. According to Lewis, nothing can counterfactually depend on, for instance, 2+2=5, because we are not able to say how our opinions would be different given such a circumstance.

Counterpossibles are not apt to say anything informative, since any impossible counterfactual fails to correspond to any way that the world could be: as we just saw, the modal space of counterfactuals is that of standard possible worlds semantics, therefore there is simply no information for them to capture or convey at all.

Now, one might object that this kind of vacuism depends entirely on the semantics that has been chosen to deal with the phenomenon at stake (counterfactual dependence), just like I did in section 2.3 with Byrne and Thompson's extended Fregeanism for hyperintensionality. This might be true for Lewis' vacuism, but in recent years we saw the development of new arguments in defense of the orthodoxy that do not necessarily depend on any particular semantics. Among them, the most influential are certainly those of Timothy Williamson, in which the vacuity of counterpossibles is inherited from the vacuity of strict implication and the substitution principles to which counterfactuals seem to comply[20]. I will not explore the details of such defense here.

What is more interesting for the current purposes is that, according to Williamson, hyperintensionality occurs only in constructions that are about representational features: «if the substitution of coreferential names in propositional attitude ascriptions does not preserve truth value, the reason is that such ascriptions are about representational features» (p. 175). Thus, counterpossibles are not hyperintensional, because counterfactuals in general do not have the representational character of propositional attitudes that makes (SVI) fail, so it must be always possible to substitute intensionally equivalent expression within a counterfactual without changing its truth value[21]. In other words, Williamson seems to suggest that one reason to accept vacuism is the fact that there is no such thing as non-representational hyperintensionality[22]. So, if we are able to provide good reasons for taking worldly hyperintensionality seriously, that might constitute a threat to vacuism. But this means also that, if Williamson is right in claiming that counterfactuals are generally non-representational, the mere acceptance of some non-vacuous counterpossible would amount to accept cases of non-representational hyperintensionality. However, as we are about to see, I think that establishing the nature of a hyperintensional content does not

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20 See Williamson 2007.

21 According to Williamson, if we substitute the antecedent of a counterpossible with some intensionally equivalent expression, namely any other impossibility, the whole conditional will still be true.

22 This is also explicitly stated in Williamson 2013: «Hyperintensionality arises at the level of thought and linguistic meaning, and should be explained at that level, not at the level of anything like a general theory of properties and relations» (p. 266).

depend merely on its subject-matter. In what follows I will not engage directly with Williamson's arguments, but I will focus on some counterexamples that have been offered for challenging the orthodoxy – cases of alleged non-vacuous, informative counterpossibles – and check if they can be regarded as genuinely worldly hyperintensional, according to the criteria sketched above.

The first piece of evidence against vacuism comes from, once again, intuition. We have a *prima facie* insight that there are plenty of non-vacuously *false* counterpossibles. Consider this example from Nolan 2014:

> (R) If there was a piece of steel in the shape of a 36-sided platonic solid, it would have fewer sides than a dodecahedron.

Since the maximum amount of sides that a solid in three-dimensional space can have is 20, the antecedent in (R) is necessarily false (it is a metaphysical impossibility). 36 sides are definitely more than 20, nevertheless, if one subscribes to the standard account, (R) must be taken as true. We can draw countless examples of this kind, but according to the orthodoxy they would turn out to be all vacuously true. Quite obviously, the distinction between different impossible antecedents and their semantic interconnections cannot be accounted for in purely intensional terms: they are all verified in the same worlds, namely none of them. Hence, vacuists cannot account for the apparent differences in meaning between different impossibilities and the apparent difference in truth value between a sentence like (R) and:

> (Ra) If there was a piece of steel in the shape of a 36-sided platonic solid, it would not have fewer sides than a dodecahedron.

However, even if we are willing to accept the falsity of a sentence like (R) and the truth of (Ra), they can hardly be judged as epistemically fruitful in any non-trivial way.

So, the next question is: can counterpossibles be genuinely informative? Let us consider a quote from Field 1989:

It is doubtless true that nothing sensible can be said about how things would be different if there were no number 17; that is largely because the antecedent of this counterfactual gives us no hints as to what alternative mathematics is to be regarded as true in the counterfactual situation in question. If one changes the example to "Nothing sensible can be said about how things would be different if the axiom of choice were false", it seems wrong […]: if the axiom of choice were false, the cardinals wouldn't be linearly ordered, the Banach-Tarski theorem would fail and so forth. (pp. 237-8).

It seems that it is possible to formulate at least some conditional statements with impossible antecedents that, if true, would have huge consequences on our understanding of mathematics (as we shall see in section 3.2, there is an entire field that systematically adopts this strategy for inferring truths about certain *actual* features of sets). Once again, merely intensional resources such as those of the standard analysis of counterfactuals look inadequate: we face a whole class of non-vacuous, informative counterfactuals that are sensitive to hyperintensional distinctions.

But how do these relate to worldly hyperintensionality? I believe that, if we are willing to accept the idea of a worldly intensional phenomenon – counterfactual dependence – expressed by ordinary counterfactuals like (K), in which nothing "representational" seems to be involved, we have no reason to reject the idea of a corresponding worldly hyperintensional phenomenon – *counterpossible dependence* – expressed by counterpossibles like (R). As Berto and Nolan 2021 remark, just like the behavior of a tailless kangaroo is independent of our representations of it, «which blocks of steel have which shapes is not just about us and our representations. It would not be even if steel could take shapes that it in fact cannot».

Now, cases in which the alleged truth or falsity of a counterpossible that rely on intuitions like (R) are surely interesting, but are they really compelling? The orthodoxy defender might still stress out that the theoretical virtues of the standard approach cannot be undermined on merely intuitive grounds. Sure, counterpossible thinking might be useful in metaphysics, and we can formulate interesting hypothetical scenarios in counterpossible terms, as seen with Field's example. But the first remark would not impress someone who takes a skeptical or deflationary stance towards metaphysics, and the second one would not constitute good evidence for allowing a *de re* reading of counterpossible modality. A false mathematical theory speaks about non-representational objects, but in formulating an

hypothesis like that of Field we are simply ascribing the property of being true or false to certain statements of the theory, which is ultimately a representational object[23].

But what if counterpossibles are actually *indispensable* in some of our best scientific practices, in accounting for real features of the actual world? That would be enough for rejecting vacuism, since its alleged theoretical virtues would force us to consider scientifically significant statements as pretty much nonsensical. Furthermore, that would satisfy Nolan's guiding principle: the explanatory role of hyperintensional talk for certain features of reality. However, if we want to take this as evidence for worldly hyperintensionality in Nolan's sense, we will need something stronger: not simply that some counterpossibles used in science are non-vacuous, but also that at least some of them have *metaphysically* impossible antecedents (as opposed to, for instance, logically or physically impossible antecedents) with a non-representational subject matter. Mere physical impossibility – the violation of laws of physics – can be accomodated within a standard possible worlds framework, therefore a non-vacuous counterpossible with a physically impossible antecedent would not be enough to constitute evidence for worldly hyperintensionality (not even for hyperintensionality in general). Logical impossibility, on the other hand, does not seem to entirely depend on features of reality regardless of our conceptual faculties: the variety of logical systems which validates different types of inference might not be a good guide to worldly matters. Of course, these three kinds of (im)possibility do not exhaust the whole field of modality, and the relations among them is the object of a debate that will not be explored further here[24]. For the current purposes, sticking to metaphysical impossibility will suffice.

This leads us back to our two criteria, that are meant to capture the indispensability of the right sort of counterpossibles within scientific explanations, as opposed to the mere *possibility of describing* some phenomena in counterpossible terms. If such indispensability is paired with metaphysically impossible antecedents and non-representational subject matter, we have a more rigorous way than Nolan's for identifying at least one kind of worldly hyperintensionality. The first case study in this spirit that I shall discuss is that of *relative computability*, as presented in Jenny 2018.

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23 Furthermore, a counterpossible with a mathematical falsity as antecedent might be considered an epistemic impossibility rather than a metaphysical one, so it might not constitute good evidence for worldly hyperintensionality.
24 See Kment 2021.

3.2 Relative computability

Jenny's strategy consists in showing how counterpossibles play an ineliminable role in relative computability theory. An intuitive characterization of this discipline is often put forward in counterfactual terms: for instance, given an unsolved problem *P*, we may ask what else we could solve if we could solve *P*. More precisely, relative computability theory is the study of what sets of natural numbers are *algorithmically decidable*.

A set is algorithmically decidable if and only if it is possible for a computing agent, by following an algorithm, to establish for any natural number whether or not it is a member of the set, in finite time and after finitely many steps. Among non-decidable sets we find:

- the *validity problem*, the set that encodes the logically valid sentences of first order logic;
- the *halting problem*, the set that encodes the problem of deciding whether a computer will eventually halt when it is given a certain input;
- *arithmetical truth*, the set that encodes the true sentences of the language of arithmetic.

According to relative computability theory, it is possible to show that a problem is *reducible* to another one, namely, it is possible to show that if we have an algorithm for deciding membership in a certain set, then we could also retrieve an algorithm for deciding membership in another. For instance, it has been proved that the halting problem is reducible to the validity problem, while arithmetical truth is not, by employing peculiar theoretical devices called *oracle Turing machines*.

Oracle Turing machines are ordinary Turing machines[25] that are supplied with an oracle, namely

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25 A Turing machine is an abstract computational device «capable of a finite set of configurations $q_1,…,q_n$ [...] supplied with a one-way infinite and one-dimensional tape divided into squares each capable of carrying exactly one symbol. At any moment, the machine is scanning the content of one square r which is either blank (symbolized by S_0) or contains a symbol $S_1,…,S_m$ with $S_1=0$ and $S_2=1$. The machine is an automatic machine which means that at any given moment, the behavior of the machine is completely determined by the current state and symbol being scanned» (De Mol 2019).

an external storage device that deliver the correct answer to any "yes" or "no" question about a particular decision problem we may ask them. For example, an oracle for the validity problem contains, for arbitrary sentences of the predicate calculus, the answer to the question whether they are logically valid or not. […] An oracle Turing machine with an oracle for the validity problem can algorithmically transform the answers it gets about the validity problem into answers about the halting problem. That's how the halting problem is Turing reducible to the validity problem. However, even if the oracle Turing machine can ask the oracle questions about the validity problem, it won't be able to transform these answers into answers about arithmetical truth. That's how arithmetical truth isn't Turing reducible to the validity problem (pp. 6-7).

These results are proved in a purely mathematical way, but the reducibility or irreducibility relation between sets that is proven can be (and, according to the author, *must be*) expressed in counterfactual terms:

(valid>halt) If the validity problem were algorithmically decidable, then the halting problem would also be algorithmically decidable.

On the other hand, arithmetical truth is not reducible to the validity problem, therefore the following claim is false:

(valid>arith) If the validity problem were algorithmically decidable, then arithmetical truth would also be algorithmically decidable.

Since we know that the validity problem is not algorithmically decidable, (valid>halt) and (valid>arith) both have an impossible antecedent, therefore we can take them as *prima facie* non-vacuous counterpossibles. The next step for our current purposes is to understand whether or not they are actually indispensable for relative computability theory and which kind of impossibility is at stake in the antecedent, provided that its subject matter is non-representational.

According to the author, in cases like (valid>halt) and (valid>arith) we are dealing with metaphysical impossibility, for two reasons. First, because decidability in the sense sketched above is grounded in a metaphysically necessary principle: the *Church-Turing thesis*, which says that «the sets that are algorithmically decidable in the informal sense, i.e.

the sets that are decidable by any algorithmic means, are just the sets that are decidable by a Turing machine» (p. 6). Since it has been arithmetically proved that, for instance, the validity problem cannot be decided by a Turing machine, the validity problem cannot be algorithmically decidable at all. But this kind of Turing computability has nothing to do with our technological or physical capability. When Church and Turing say that the validity problem is not decidable, they mean that it would impossible to find an algorithm for deciding it, *even if we had no technological limitations or constraints from the actual laws of physics*. The tape of a Turing machine – and hence its memory – is infinite, so for a function to be Turing computable it is sufficient that a set of instructions that will result in a Turing machine computing the function exists, no matter how much time will it take. But non-decidable sets cannot be computed even in an infinite amount of time. In Jenny's words:

> Before anyone had built anything resembling a modern computer, Church and Turing had already identified computational problems that no computer could ever decide. And since the invention of the first computer, all technological innovations in computing, including innovations involving quantum computers that are yet to be realized, have merely lead to an increase in computing speed and efficiency; they never have and never will lead to an improvement in what can be algorithmically decided (p. 8).

The second reason, more direct yet more controversial, has to do with the nature of abstract objects. According to the author, claiming that a set is not algorithmically decidable is equivalent to claiming that there is no algorithm that could decide it. Since algorithms are mind-independent abstract objects, if an algorithm does not exist in the actual world, it cannot exist in any other metaphysically possible world. Therefore, any counterfactual in which the antecedent expresses the existence of a non-actual algorithm, like (valid>halt) and (valid>arith), must be a counterpossible in the stronger metaphysical sense[26].

If Jenny's arguments on the nature and non-vacuity of such counterpossibles are compelling, the only thing left to establish is whether they are actually indispensable for relative computability theory. The author's general defense for this view consists in the appeal to a certain *philosophical humility* against the orthodoxy on counterpossibles: «whenever an established mathematical or scientific discipline purports to study a certain

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26 The necessary existence of abstract objects like numbers has been disputed (see Yablo 2014).

phenomenon, we shouldn't give in to philosophical considerations that suggest that there is no such phenomenon to be studied» (p. 11). In other words, the property of a set of being reducible to another is something that we cannot deny or explain away by appealing to the fact that our only way of understanding it is in counterpossible terms. Skepticism towards a phenomenon on the basis of mere semantic preferences is highly problematic, as extensively shown in section 2.3.

In light of this, the last strategy available to the die-hard vacuist consists in arguing that when relative computability theorists asserts sentences like (valid>halt) they are not actually employing counterpossibles. For instance, (valid>halt) might simply be a useful evocative idiom for illustrating reducibility, but it is not meant to be taken literally. However, as Jenny notices, sentences like (valid>halt) do not semantically behave like idioms: they are compositional, while idioms typically are not[27].

In sum, if Jenny is right in taking counterpossible talk in relative computability theory seriously, since counterpossibles like those discussed above sit at the very core of the theory, it seems that the indispensability criterion is satisfied as well. A *de re* reading of reducibility or irreducibility relations between sets, which are relations of *counterpossible dependence* – i.e. a special case of counterfactual dependence in which metaphysical impossibility is involved –, might then constitute our first candidate for worldly hyperintensionality.

3.3 Explanation and counterpossible reasoning in natural sciences

Tan 2019 adopts a similar strategy for arguing against the orthodoxy on counterpossibles, providing a set of examples from natural sciences. In the current section I will illustrate them and assess their eligibility as genuine cases of worldly hyperintensionality.

The author identifies three kinds of counterpossible indispensability in science: scientific explanation, model-based reasoning and reasoning about superseded theories. He begins by noticing how counterfactual dependence is a key element for our understanding

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27 A sentence like 'I'm keeping an eye out for you' sounds linguistically appropriate, while 'it's an eye that I'm keeping out for you' sounds odd. This is because the meaning of the former is not derived compositionally from the meanings of its parts, but it is directly encoded by the whole expression. In contrast, counterfactuals about relative computability interact with other sentence constructions just like ordinary counterfactuals do. See Jenny 2018 for the replies to other objections.

of the relations that backs explanation in natural sciences, such as causality (not surprisingly, one of the most popular theories of causality is Lewis' counterfactual account). Furthermore,

> the role of the counterfactual conditional in scientific explanations is not limited merely to specifying that some state of affairs physically depends on another. Counterfactuals also help identify what is *relevant*, explanatorily, for one state of affairs to bring about another (pp. 38-39).

For instance, the fact that a glass breaks if exposed to a sound of the right pitch and amplitude does not depend on whether that sound was meaningful – like that of a song –, precisely because it would still break even if the sound expressed a different content, or no content at all, while it would not break if the pitch or the amplitude were different: we express all of this in the form of a counterfactual. As remarked by the author, explanation is *a matter of exhibiting systematic relations of counterfactual dependence*.

In light of this ubiquitous explanatory character of counterfactual dependence, the idea of rejecting the appropriateness of a counterfactual reading whenever we face something that looks like a counterpossible conditional within a scientific context begins to appear problematic. Consider the following:

> (D) If diamond had not been covalently bonded, then it would have been a better electrical conductor.

(D) is meant to capture the dependence of diamond's properties at the macroscopic level on its structural properties at the microscopic level, an essential part of the explanation of its poor electric conductivity. That is so because, following Woodward 2003, «the provision of any explanation of some fact is incomplete until one specifies, counterfactually, how that very fact would have been different under various circumstances» (p. 43). If this is the case, we can safely conclude that a sentence like (D) satisfies our indispensability criterion.

Now, it is an essential property of diamond of being covalently bonded – it is what distinguishes it from other allotropes of carbon such as graphene, which is an exceptional conductor –, therefore it is necessarily so: it is covalently bonded in every possible world. But this means that the antecedent in (D) is a metaphysical impossibility: (D) is a

counterpossible in the stronger metaphysical sense sketched in the previous sections, so it satisfies our reality criterion. Indeed, according to the author,

> the fact that (D) features in an explanation of a particular substance's properties, and describes the relationship between two of that substance's properties, suggests that a *de re* reading of its apparent referring terms is its correct interpretation (p. 41).

And precisely by adopting a *de re* reading of the antecedent we can conclude that this kind of counterfactual dependence is a genuine instance of worldly hyperintensionality: a fact grounded in the properties of a real actual object, regardless of our representations or the epistemic roles that we associate with its descriptions.

The second set of examples is that of idealized models. A typical strategy for testing hypotheses in science consists in building highly idealized models in which contingent factors that would be extremely difficult to compute are minimized. Among classical examples of such idealizations we find blatant impossibilities such as frictionless planes, pendulums with massless strings and so on. According to the author, some of them are *metaphysical* impossibilities, yet they are extremely useful for understanding a wide class of phenomena. This is, again, expressed in the counterfactual form. Take the case of models that are commonly used for wave propagation in water like the Navier-Stokes equations for fluid flow, which represent it as a continuous and incompressible medium. Since water is necessarily H_2O – namely, a substance composed of discrete molecules –, any counterfactual with an antecedent that represents water as a continuous substance is a counterpossible. Nevertheless, an idealization like:

(W) If water were a continuous, incompressible medium, then it would behave
as the Navier-Stokes equations describe

is successful, since actual water behaves approximately like how it would have behaved if it was a continuous substance. In light of this, it becomes difficult for the defender of the orthodoxy to maintain that (W) is vacuously true. For what concerns our notion of worldly hyperintensionality, here the point is less obvious. Sure, the antecedent is about a metaphysically impossible non-representational object, but the counterfactual dependence relation is ascribed with respect to a set of equations employed in a theoretical framework,

which one might be not willing to consider totally non-representational, in contrast to an object's properties. Nevertheless, (W) seems to satisfy both of our criteria for worldly hyperintensionality.

Last but not least, reasoning about superseded theories. Tan's point here is that, just like in the case of idealized models, there is a widespread use of counterpossibles in science when it comes to explain why a certain theory fails to account for a certain phenomenon. The main example that he chooses to illustrate this is the following:

(B1) If Bohr's theory of the atom had been true, then an electron's angular momentum L in the ground state would have been observed at $L=\hbar$ (that is, the reduced Planck constant).

(B2) It is not the case that an electron's angular momentum L in the ground state is observed at $L=\hbar$.

(B3) Therefore, Bohr's theory of the atom is false.

Suppose that (B1) is vacuously true. If that is the case, it would not manage to convey any actual information about Bohr's theory, therefore one would not be able to employ an empirical observation like (B2) to reject it. Tan then concludes that «in order for this commonplace pattern of reasoning to be epistemically fruitful, theory-evaluating conditionals must describe genuine relations of counterfactual dependence and implication» (p. 49). In other words, a conditional like (B1) must be be non-vacuously true. But is (B1) a counterpossible? Since Bohr's theory of the atom is a paradigmatic case of an inconsistent theory – it rests on assumptions from both classical and quantum physics for representing electrons –, a sentence that claims its truth would be a contradiction. Therefore, (B1) is clearly a logical impossibility. According to the author, since metaphysical possibility is a subset of logical possibility, (B1) is *a fortiori* metaphysically impossible.

Regardless of one's opinion on modal matters like the relation between logical and metaphysical possibility, I believe that, despite its being an interesting case, this would not constitute a genuine case of worldly hyperintensionality. Since it involves counterpossible reasoning, it can obviously be accounted for only within a hyperintensional framework, but this does not suffice for instantiating our stronger notion of worldly hyperintensionality.

Even on a *de re* reading of the antecedent, a counterpossible like (B1) is neither about an object independent from our conceptual faculties (theories are a product of human thinking), nor suggests any other reason for taking it as describing some non-representational phenomenon. The difference between this case and those discussed above is that here we use counterpossibles to describe a methodological process, so they are ultimately sentences that *speak about a theory*, while in the previous cases the counterpossibles are *part of a theory* and speak about certain non-representational phenomena. Nonetheless, it is still perfectly reasonable that some counterpossibles implied in Bohr's theory which are directly about the nature of the atom might be genuine cases of worldly hyperintensionality.

Despite the relative controversy of the last two examples, I believe that what is most valuable for the present discussion is that we identified at least another kind of non-vacuous counterpossibles that admits a non-representational *de re* reading (the most natural one), which is also to a large extent commonplace in our best scientific practices. Something along these lines emerges also from Tan's concluding remarks against the orthodoxy on counterpossibles. As already noticed, the standard approach to counterfactuals is meant to capture a certain *phenomenon*, that of counterfactual dependence, which is widely regarded to be at the core of our best theories of explanatory reasoning, causality, microphysical and mechanistic realization, and so on. We saw that science makes an extensive, more or less explicit use of counterpossibles, that the orthodoxy treats as vacuous. But this is equivalent not simply to saying that many meaningful pieces of science are nonsensical, but also to considering the very *use* of counterfactuals as a mistaken choice:

> The orthodoxy's incompatibility with the way that science uses counterfactuals means that it fails to accurately capture the phenomenon it was meant to capture, namely, the counterfactual conditional. [...] The counterfactual is not merely a toy of philosophers and linguists. Hence, if one's philosophical or linguistic theory of how the counterfactual works is incompatible with the way it is used in those core contexts, that is a sufficient reason for the theory to be rejected. For that would mean that that theory fails to accurately capture the very phenomenon for which it was built to account. It is the phenomena that must be saved, not the theories built to account for them. A theory of the counterfactual fails to save the phenomenon if it is inconsistent with the way the counterfactual is used in those contexts, or worse, if it declares that those uses constitute an incorrect way of using the counterfactual (pp. 58-59).

With this in mind, we might finally try to figure out the hyperintensional phenomenon (in Nolan's sense) at stake in the case studies discussed so far. My modest suggestion is that we might consider the relation of counterpossible dependence as the true hyperintensional phenomenon involved in our examples. Such relation, as seen in cases like (D), might hold between higher-level and lower-lever properties of real objects, just like ordinary counterfactual dependence might hold between facts, chains of events or objects and their modal properties. Since we ordinarily admit a *de re* reading of counterfactual dependence, I believe that the widespread use of counterpossibles in our best scientific practices – in which the commitment to the non-representational character of their contents is not easily disputable – constitutes sufficient support for such reading also in some cases of counterpossible dependence.

3.4 Strong emergence in chemistry

In the previous section we saw that one of the most compelling examples of worldly hyperintensionality comes from a special kind of counterfactual dependence fundamental to the explanation of certain chemical phenomena, such as the poor electrical conductivity of diamond. Chemistry is also the environment to which belongs my final case study: *strong emergence,* a thesis on the nature of the relationship between certain chemical and physical properties. In a series of papers on the topic, Robin Hendry extensively describes how to characterize this phenomenon in a rigorous way and defends his plausibility against reductionist perspectives. In what follows, I will explore this idea and assess whether it is possible to understand it as another instance of worldly hyperintensionality.

In the philosophy of chemistry, reductionism is the view according to which, at least in principle, the whole field of chemistry is reducible to physics. On an epistemic reading, this amounts to say that every phenomenon described by chemistry could in principle be translated in the language of physics; while on an ontological reading it says that every chemical property is nothing more than a physical property: physics is causally closed and exhausts the whole range of phenomena that we observe at the higher level of chemistry. Many accounts of this dependence have been developed (identity, supervenience, and so on), but such technicalities are not important for the current purposes. Instead, the focus

here will be on the emergentist thesis that is offered as an alternative to reductionism, since, according to Hendry 2017, «the scientific evidence for the existence of strong emergence in chemistry is at least as good as the evidence for reductionist positions that rule it out» (p. 2).

An important preliminary distinction to be made is that between *weak* and *strong* emergence. A property is weakly emergent from physics if the current theoretical resources provided by physics are not sufficient for predicting or explaining the fact that a system displays that property. On the other hand, a property strongly emerges from physics if such failure of prediction or explanation is there *in principle*: in other words, if it would be impossible to explain that property by means of the resources of physics only, even with an ideally perfect theory. Hendry adopts a causal criterion for the reality of a property, according to which existing requires the possession of causal powers. Following this principle, one can identify the presence of an emergent property by recognizing the presence of causal powers *in addition* to those explained by physics. Now, one might think that this would conflict with the laws of physics, but according to the author «the possession of novel causal powers does not require the violation of more fundamental laws. Strong emergence requires not that these laws be broken, but only that they fail to determine what happens» (p. 2). In other words, Hendry's view does not conflict with the laws of physics themselves, but only with the causal closure of physics.

So, which kind of causal powers strong emergence involves, if they are not of the sort that is captured by pure physical descriptions? Hendry's opinion is that the distinctive feature of strongly emergent properties is their manifesting *downward causation*: emergent systems exhibit higher-level properties that determine a difference at the lower level from which they emerge, that would not be present if the system complied only with its lower-level basic laws. In Hendry's words: «the subsystems of an emergent supersystem sometimes do something different to what they would do if the causal structure of the world were as imagined by the reductionist» (p. 2). As one can easily notice, this corresponds to a counterfactual claim, that the author calls *counternomic,* due to its reference to certain non-actual laws (those endorsed by the reductionist). In other words, emergence is semantically captured by counterfactuals with a physically impossible antecedent.

Despite the apparent intuitiveness of the reductionist perspective, the choice between emergentism and reductionism should be guided by scientific evidence. So, what evidence do we have for the existence of strongly emergent properties in chemistry? Throughout his works Hendry illustrates a variety of cases, but I believe that the most interesting for the current purposes is that of molecular structure. On this matter, the reductionist stance is that molecules are nothing more than systems of charged particles, interacting according to the laws of quantum mechanics. This is captured by means of a *Schrödinger equation*: a mathematical expression that describes an isolated molecule purely in terms of its electrons and nuclei and that, once solved, explains the peculiar structure of the molecule, which in turn explains the behavior of a substance at the macroscopic chemical level. However, to solve the Schrödinger equations for atoms more complex than hydrogen (the only atom with a Schrödinger equation that is analytically solvable) or for molecules, quantum chemists require to calibrate the physical models with a set of assumptions drawn from chemistry. In other words, quantum chemists need to introduce well-understood approximations at the fundamental level – known falsehoods – that will affect the calculations. As remarked in Hendry 2009,

> quantum chemistry turns out not to meet the strict demands of classical reductionism, because its models bear only a loose relationship to exact atomic and molecular Schrödinger equations, and its explanations seem to rely on just the sort of chemical information that, in a classical reduction, ought to be derived (p. 183).

One of the cases in which such move is most evident is that of isomers. Isomers are distinct molecules sharing the same molecular formula, namely, molecules composed of exactly the same atoms in the same number. For instance, ethanol (CH_3CH_2OH) and dimethyl ether (CH_3OCH_3) are two molecules in which the same number of atoms of the same kind are arranged in different structures, that in turn give rise to very different properties. In order to build the complete Schrödinger equation of a molecule, one needs its *resultant Hamiltonian*, an operator corresponding to the total energy of the system which takes into account all the intra-molecular interactions, using only fundamental physical interactions as input. But since isomers differ only in structure, they will share the same resultant Hamiltonian: in other words, it is impossible to distinguish them at the pure

physical level[28]. In order to account for their distinction, quantum chemists employ the *configurational Hamiltonian,* an operator that allows to specify the nuclear positions within a molecule in order to account for its structure. But this structural insight comes from chemistry's empirical observations: it is not something that could be deduced, even in principle, from our best physical descriptions of the molecules. In other words, configurational Hamiltonians assume higher-level information that impose on the Schrödinger equation the molecular structure that is supposed to be derived. Furthermore, «since a molecule's causal powers depend on its structure, its behaviour would be different were it determined by more basic (quantum-mechanical) laws governing the particles of which the molecule is made» (p. 189).

According to Hendry, this instantiates the counternomic criterion for downward causation, making molecular structure a strongly emergent property.

> In an emergent complex system, the behaviour of the basic stuff of which it is made is governed by a configurational Hamiltonian, which is different from what it would be were its behaviour governed by the resultant Hamiltonian. Since the Hamiltonian of a system determines the precise nature of the physical law that governs its behaviour, to say that some system exhibits downward causation is to make a counternomic claim about it – that its behaviour would be different were it determined only by the more basic laws governing the stuff of which it is made (Hendry 2009 p. 185).

In the author's view, this amounts to say that it is impossible to specify all the properties of an emergent system at the basic physical level. The reason why this happens is not because the mathematics to compute certain properties would be too complex, but because those properties are simply not there: they do not exist at the scale of quantum mechanics. A resultant Hamiltonian cannot specify the structure of a molecule at the quantum scale, precisely because *there is no structure at that scale*. Molecular structure is possessed only at the chemical scale, and it partially determines the entities found at the physical level.

From our semantic perspective, instances of the counternomic criterion has been read so far as counterfactuals with a physically impossible antecedent. But I believe that a more careful reading of it would reveal some curious features. We saw that isomers like ethanol and dimethyl ether are distinguishable only in virtue of their different structures,

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28 Besides their inadequacy for distinguishing molecular structure, resultant Hamiltonians are never used primarily for practical reasons: they would be too complex to calculate (see Hendry 2009).

that, if we agree with Hendry, are emergent properties that can be accounted for only at the chemical level. This is captured by means of a counterfactual claim like the following:

(IS) If the properties of a molecule were determined at the quantum level only, then ethanol and dimethyl ether would be the same molecule.

If Hendry is right, the antecedent in (IS) is physically impossible. But since the structure of a molecule cannot even in principle be retrieved at the quantum level, the antecedent in (IS) would also imply that the structure of a molecule is irrelevant for its identity. However, it seems reasonable to maintain that molecular structure is an essential property: it is metaphysically impossible that a molecule, being a natural kind, has a different structure than his own, otherwise it would be a different molecule, as in the case of isomers.

But if that is the case, a sentence like (IS) would be a counterpossible similar to those saw in the previous sections. Thus, the question for the current purposes is: would (IS) satisfy our criteria for worldly hyperintensionality? As far as non-representational subject matter is concerned, my take is that cases like (IS) satisfy the reality criterion: molecular structure and the properties that it determines are uncontroversially real, regardless of our descriptions of them[29]. Is the use of counterpossibles like (IS) and, more generally, Hendry's counternomic criterion indispensable for science? They can be regarded indispensable at least insofar they feature in the explanation of the philosophical thesis of strong emergentism, which is a way of interpreting ontological relationships between real entities on the one hand, but also epistemic relations between scientific theories on the other. If Hendry is right in claiming that quantum chemists implicitly adopt an emergentist stance when accounting for molecular structure, we might conclude that the counterpossibles used for explaining strong emergence are an essential part of the theory. Therefore, they might be considered indispensable in an indirect way, as long as we choose to subscribe to Hendry's position.

An interesting question that might be raised at this point is whether such hyperintensional reading of the counternomic treatment of emergence can be extended to

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29 Unless one subscribes to some form of hard materialism holding that the things that exists are only those described by physics, which might have some trouble in accounting for molecular structure, as we just saw.

all kinds of alleged emergent properties, even outside the field of chemistry[30]. I will not explore this hypothesis in the present work, but it is worthy to suggest that, if it turns out tenable, then the general notion of *emergence* itself, being ultimately grounded on a relation of counterpossible dependence, might be considered as a worldly hyperintensional phenomenon.

3.5 Making sense of counterpossible dependence

We saw that a worldly hyperintensional phenomenon can be defined as a phenomenon that satisfies some criteria that allow for a *de re* reading of its hyperintensional features. The common feature that all of the cases discussed above share is the relation of dependence captured by the counterpossibles used to explain them. Thus, we might say that some instances of reducibility in relative computability theory, some properties of substances and strong emergence in chemistry are all hyperintensional phenomena in a derivative sense. The hyperintensional phenomenon *stricto sensu* involved in all of them – that invoked for explaining certain aspects of the world – is counterpossible dependence. Making sense of this relation is far from obvious: for the time being, I will limit myself to suggest a possible pathway.

From now on, I will no longer speak about the truth of certain propositions, but about the obtaining states of affairs – i.e. facts – that ground it. We can summarize the relations of dependence between states of affairs identified in our case studies as follows:

- The fact that the halting problem cannot be decided partly depends on the fact that the validity problem cannot be decided.

- The fact that diamond is a poor electrical conductor (despite its being a carbon allotrope) partly depends on the fact that it is covalently bonded.

- The fact that isomers are not the same molecule partly depends on the fact that their properties are not fully determined at the microphysical level.

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30 Emergent properties have been adopted for explaining a wide variety of phenomena in philosophy of mind, physics and biology, among others. See Gibb, Hendry and Lancaster 2019 for an in-depth review.

We can notice that in the first case the dependence relation seems to hold between two states of affairs that obtain in the actual world only in virtue of an absence (there is no algorithm that can decide the validity problem). The second case is less weird, since even if it is captured in counterpossible terms, the dependence relation here is between two positive facts. In the third case, the dependence relation is between two states of affairs that obtain in the actual world in virtue of something that does not (and cannot) happen. At a first glance, this might look quite confusing. But we can try to make sense of it by appealing to a theory of states of affairs, especially one that allows the existence of *negative facts*.

Negative facts are obtaining states of affairs picked out by talk of absences, lacks and ways things are not. For instance, a negative fact can be picked out by a sentence of the form '*a's not being F*', where *a* is a particular, like a substance or an individual, and *F* is a property. Some authors have argued that, besides the free use that we make of them in our ordinary speech, we need negative facts in our ontology in order to explain causal relations, material constitution, or to ground the truth of certain propositions. Despite a widespread suspicion on their metaphysical status, according to authors like Barker and Jago, they come at no additional ontological cost, as long as we already accept positive facts in our ontology. In their 2011's paper, they develop a theory of negative facts that is built upon Armstrong's theory of facts as substantial metaphysical entities. According to this account, facts are irreducible non-mereological wholes having individuals, properties and relations as constituents (even if in Armstrong's picture individuals, properties and relations are only abstractions from the state of affairs, which is the fundamental entity). Without digging deep in the technical details, we can follow Barker and Jago 2011 for a sketch of the difference between positive and negative facts:

> if *the lake's being frozen* is the state of affairs that results when the (thin particular) lake is tied to the property of frozenness in one way, then *the lake's not being frozen* is the state of affairs that results when the (thin particular) lake is tied to the property of frozenness in another way. If the first way is instantiation, then the second is anti-instantiation. The first way gives one kind of non-mereological whole, the second gives another. Neither kind of tie nor corresponding kind of whole is reducible to the other kind of tie or whole (p. 3).

Both positive and negative facts are spatiotemporally located where their constituents are. Hence, a negative fact like *Anna's not being hungry at t* is spatially located where Anna is at *t* and temporally located at *t*. But what about absences captured by negative existential facts? Do they entail the contradiction that some non-existent objects exist? According to Barker and Jago, claiming this is the result of confusing that which is absent (which can be a non-existent object, like an algorithm for deciding the validity problem) with that which exists (the absence itself): within this framework, negative states can exist just like positive ones. The only difference between them is the kind of non-mereological composition involved. So, when we say that there is no algorithm that can decide the validity problem, it is *the algorithm* that does not exist, but *the fact that there is no such algorithm* is an existing state of affairs that belongs to the actual world. The fact is not the algorithm, so no contradiction is involved.

Putting this idea to work, we might conclude that in the case of relative computability theory (valid>halt) would capture the dependence relation of a negative existential fact – there is no algorithm that can decide the halting problem – on another negative existential fact – there is no algorithm that can decide the validity problem –. Both of them would be real actual facts, in compliance with our picture of a hyperintensional phenomenon that has its source in the world. In the case of emergence, we would have a negative fact – dimethyl ether is not ethanol – that depends on another – their properties' not being fully determined at the microphysical level –, allowing for a *de re* reading of their relation captured by (IS).

IV. Final Remarks

Hyperintensionality, when taken seriously, is a complex theoretical challenge that is still relatively little explored and comes in many shapes and flavors. So, the fact that it might be difficult to appreciate all of them is not a surprise. Nolan believes that we are currently on the verge of a new revolution in philosophy, as already happened with the development of intensional machinery before, and that within a few decades the consensus on hyperintensionality will be the same that we see nowadays on intensionality. But from the radical skepticism of the die-hard intensionalists to the full-fledged realism on hyperintensional phenomena advocated by Nolan, the step is definitely not short. Regardless of one's opinion on the prospects for an "hyperintensional revolution", in the present work I tried to offer a feasible reading of Nolan's suggestion – hyperintensionality can be a worldly phenomenon independent from our representations – by focusing on a paradigmatic case of trigger for hyperintensionality, namely counterfactuals with an impossible antecedent; articulating two jointly sufficient criteria for a counterpossible to capture an instance of worldly hyperintensionality; building an analogy with intensional worldly phenomena and gathering evidence from scientific practices and their philosophical implications. This resulted in the detection of patterns of counterpossible dependence occurring in all of the case studies discussed.

My strategy consisted first in differentiating between a weaker and a stronger sense of non-representational hyperintensionality. In the former case, the alleged non-representational character depends merely on the absence of propositional attitudes, but the contexts at stake can still be indirectly traced back to representations or some pre-theoretic intuitive semantics (see section 2.1). In the latter case, the non-representational character depends entirely on the fact that a certain worldly phenomenon requires hyperintensional language for being *explained*. Following Nolan, we saw that such requirement is the first clue for locating the source of at least some cases of hyperintensionality in the world rather than in our representations of it: this lead to the label of "worldly hyperintensionality". Metaphysics provides the typical theoretical background from which worldly hyperintensionality in this sense can be recognized. After discussing some of the most relevant cases, at least three important points have emerged:

1) Being a hyperintensional phenomenon is not sufficient for being worldly hyperintensional. Apparently, it seems reasonable to assume that every instance of worldly hyperintensionality is a hyperintensional phenomenon, but surely not every hyperintensional phenomenon is a case of worldly hyperintensionality. If we subscribe to Nolan's jargon, even intentional states like belief (which are clearly representational) are hyperintensional phenomena, since they require hyperintensional resources to be accounted for. Hence, we need some further criteria for distinguishing cases of representational hyperintensional phenomena from cases of worldly hyperintensional phenomena.

2) A non-representational content might be *describable* by means of hyperintensional statements, but this does not legitimize to infer the source of its hyperintensionality in features of the objects that it denotes: it might still be the case that it derives from our representations. The distinctive feature of worldly hyperintensionality as understood in the present work is that a non-representational phenomenon cannot be explained without hyperintensional language, for instance by means of paraphrases. However, non-representational subject matter retains a key role in the understanding of worldly hyperintensionality: even epistemically fruitful hypotheses like that of Field concerning different – thus, impossible – mathematical truths expressed in counterpossible terms might not be suitable candidates for worldly hyperintensionality (see section 3.1).

3) The semantics that we choose to account for hyperintensionality should not be taken as a guide for establishing its source or nature. The problem of the source (representations vs. world) of hyperintensionality should be distinguished from the problem of finding a suitable hyperintensional semantics. Collapsing the two epistemic tasks leads to a number of counterintuitive and unpalatable consequences, as shown in section 2.3 with the discussion of Byrne and Thompson's conceptualism, which I rejected.

In light of this, we can attempt to provide some definitions. The first, which is simply Nolan's guiding principle in a more rigorous form, concerns what it takes for a phenomenon to be hyperintensional:

(HP) A phenomenon is hyperintensional if and only if it requires hyperintensional language to be accounted for.

The second definition concerns what it takes for a phenomenon to be worldly hyperintensional:

(WP) A phenomenon is worldly hyperintensional if and only if it satisfies some criteria that support a *de re* reading of its hyperintensional features.

(WP) does not specify the criteria for the *de re* reading, because they may vary across different kinds of hyperintensional phenomena. In section 3.1 I put forward two specific criteria for counterpossibles, that might also serve as a starting point for understanding how general criteria for any case of worldly hyperintensionality should look like.

This brings us to the evidence. As far as metaphysics is concerned, Nolan suggests that we have plenty of examples for taking the idea of worldly hyperintensionality seriously. Among them, counterpossible conditionals turn out to be of particular interest even outside of the field of metaphysics. In particular, the widespread use of them in science already stimulated a number of philosophers to argue against vacuism, the thesis according to which all counterpossibles are vacuously true. On the assumption that they are right, we face potential cases of hyperintensionality from disciplines that uncontroversially speak about real features of the actual world (the subject matters of scientific theories is typically non-representational, at least in the case of natural sciences). Furthermore, the epistemic credit given to science is typically stronger than that given to metaphysics. So, the need for hyperintensional resources (in the form of counterpossibles) in science would constitute the best evidence available for locating some instances of hyperintensionality in the world. However, the mere *possibility* of counterpossible talk within scientific practice would not be enough, just like not every kind of *impossibility* appealed in some explanation would be appropriate.

These remarks lead to the formulation of two jointly sufficient criteria for a *de re* reading of hyperintensionality that are supposed to strengthen Nolan's guiding principle,

which is based on the view that phenomena can be classified by detecting a principled need for a certain kind of expressive resources to explain them. In order for a counterpossible to be considered worldly hyperintensional – i.e. capturing a non-representational phenomenon in the world –, it must at least satisfy the *indispensability* criterion, according to which it must be required in some scientific explanation; and the *reality* criterion, according to which it must have a metaphysically impossible antecedent with a non-representational subject matter.

My first case study (section 3.2) is that of relative computability theory. Following Jenny, we saw how the property of certain sets of being reducible or irreducible to others – e.g. if we had an algorithm to decide the validity problem, we would have also an algorithm to decide the halting problem – is explainable only in counterpossible terms. But we want to maintain that such property is something possessed by the sets themselves, not simply by the sentences that we use to describe them. Hence, it seems reasonable to conclude that we identified a real relation of counterpossible dependence between sets, by means of which we understand real and informative properties of them.

The second case study (section 3.3) is that of counterpossibles within natural sciences. Following Tan, we recognized that at least one kind of counterpossibles, those required to fully explain the behavior of certain chemical substances, satisfies both of our criteria. This, again, legitimizes a *de re* reading of the counterpossible's referring terms, suggesting a real relation of counterpossible dependence between the properties of a substance.

Lastly, I discussed a case study at the crossroads of science and metaphysics: that of strong emergence in chemistry (section 3.4). Here we saw how the relation of downward causation, that Hendry argues to be the distinctive feature of strong emergence, is best understood by means of counterpossibles. The specific example discussed – that of the molecular structure of isomers – seems to satisfy at least our reality criterion, since it involves the relation of counterpossible dependence between non-representational contents like molecules and their structural properties. The indispensability criterion might be considered indirectly satisfied as long as quantum chemists implicitly assume strong emergence in their explanations, as Hendry suggests.

The analogy with intensionality provides us the best way to understand the difference between representational and worldly hyperintensionality, and how the role of counterpossibles in scientific explanation underlies a real pattern of relations of counterpossible dependence between different entities. An intensional ascription can be often read according to a *de dicto* interpretation or to a *de re* interpretation. Now, it is not always clear which one is the best reading, which can depend on the context in which the sentence is asserted. But for the current purposes, it is enough to recognize that intensional ascriptions can sometimes be *de re* (see section 2.2.), as it is orthodox nowadays: intensionality, in the form of modal properties possessed by objects, can be something due to the world, regardless of our representations.

The hope is that what has been explored in the present work could help to establish the same for hyperintensionality. When faced with some hyperintensional content we can ask: is its hyperintensionality due to the world or to our representations? This amounts to say: the hyperintensional distinctions that a semantics would need to retain in order to account for such content's meaning are *de re* – properties of the objects denoted by it – or *de dicto* – properties of the descriptions that we associate to it? In the case of a counterpossible, one promising way to answer the question consists in understanding whether it figures in the explanation of some non-representational phenomenon. If that is the case, provided that it cannot be paraphrased away or substituted with a non-hyperintensional statement with equal epistemic worth, the most natural way of reading it is *de re*. And this is precisely what happens with the case studies discussed above. If ordinary counterfactual modality can be a *de re* feature of the world, the same is true of counterpossible modality, provided that certain conditions (which are ultimately epistemic in nature) are satisfied.

In the present work I primarily focused on scientific practice for its strong degree of commitment to an accurate account of reality, which is typically more acceptable than that of metaphysical theories. But this does not entail that examples from metaphysics should be ruled out from a rigorous understanding of worldly hyperintensionality. This is why my two criteria have been presented only as *sufficient* conditions. However, one might want to extend this notion to the metaphysical cases too (as Nolan assumes), therefore it should be possible to loosen the indispensability criterion for accommodating them. We might sketch

the new *general* criteria for counterpossibles to be worldly hyperintensional (i.e. capturing a relation of counterpossible dependence between objects), this time jointly *necessary and sufficient*, as follows:

(I) *Indispensability:* if they are required in order to explain a phenomenon.

(II) *Reality*: if they have a metaphysically impossible antecedent with a non-representational subject matter.

This version of the indispensability criterion allows also for counterpossibles that feature in metaphysical explanations to be worldly hyperintensional. For instance, it allows to consider an intuitive case of worldly hyperintensionality like that of impossible dispositions (see section 2.2) as a genuine worldly hyperintensional phenomenon, since impossible dispositions are typically explained by means of counterpossibles that satisfy the reality criterion[31].

Finally, in section 3.5 I suggested a possible reading of the counterpossible dependence relation by appealing to a theory states of affairs, in order to account for the non-representational nature of the *relata*. In particular, I adopted Barker and Jago's theory of negative facts for the cases in which the relation is ascribed between absences (as in the case of relative computability) or ways things are not (as in the case of the molecular structure of isomers).

In conclusion, what has been discussed in the present work, especially for what concerns the brief incursion in metaphysics of the very last part, leaves room to a number of further issues that are worthy to be explored. With respect to the problem of the source of hyperintensionality, one can ask whether it is possible to find some general criteria for identifying any case of worldly hyperintensionality, even if it is not captured by a counterpossible, and in turn whether the appeal to a fact-based picture would be appropriate in any such circumstance. On the semantic side, we might find some approaches for accounting cases of worldly hyperintensionality more appealing than others (for instance a truthmaker-based approach instead of one that relies on impossible worlds or structured propositions[32]). These and other related questions shall be discussed elsewhere. For the time

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31 See Jenkins and Nolan 2012.
32 See Fine 2017.

being, I want to conclude my case in support of worldly hyperintensionality with the words of one of its most distinguished opponents, Timothy Williamson, that in assessing the limits of the conceptual turn in philosophy writes:

> The practitioners of any discipline have thoughts and communicate them, but they are rarely studying those very thoughts: rather, they are studying what their thoughts are about. Most thoughts are not about thoughts. To make philosophy the study of thought is to insist that philosophers' thoughts should be about thoughts. It is not obvious why philosophers should accept that restriction [...] Much contemporary metaphysics is not primarily concerned with thought or language at all. Its goal is to discover what fundamental kinds of things there are and what properties and relations they have, not to study the structure of our thought about them. (pp. 17-19).

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