Structuralism, Fictionalism, and Applied Mathematics[1]

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ABSTRACT. A natural and attractive view of the applications of mathematics in empirical science is that expressed by *ante rem* structuralist, Stewart Shapiro, according to whom "mathematics is to reality as pattern is to patterned" [6, p. 248]. Viewing mathematics as the science of abstract patterns, *ante rem* structuralists such as Shapiro have the means for an explanation of the applicability of mathematics which proceeds by considering the various ways in which our theories about 'freestanding' mathematical patterns can be used to provide information about the patterns instantiated in physical reality.

But does the view that the world is mathematically structured require a belief in the existence of abstract structures? This paper considers three ways in which mathematical structures are appealed to in the context of our scientific theories, and considers how a fictionalist might view these appeals to mathematical structure. While the first of these three kinds of applications of mathematics does not require us to posit abstract mathematical structures that are related in various ways to nonmathematical objects, the second and third applications do require the supposition of ideal models and/or pure mathematical structures over and above the physical reality our theories aim to describe. In such cases, since a fictionalist cannot accept the existence of these ideal models/mathematical structures, to the extent that such applications are indispensible to our empirical scientific theories, a fictionalist must be anti-realist about such theories, believing not that they are true, but only that they are *good* representations of the physical world, in that "the physical world holds up its end of the "empirical-science bargain"" [1, p. 134].

In this respect, then, it seems that *ante rem* structuralism has some advantages over fictionalism. Through its realism about mathematical structures, *ante rem* structuralism can preserve the assumption that our scientific theories are (at least approximately) true, and can do so without appeal to the less than crystal clear notion of the "physical world" being the way it would have to be for our theories to be true. This paper argues that these advantages are illusory. Given structuralist metaphysics, the mere claim that our scientific theories are true is far too weak to account for the applicability of those theories. In order to account for applicability, the structuralist will also have to hold that "the physical world holds up its end of the "empirical-science bargain"". If it is this claim that is doing the work in accounting for the applicability of our scientific theories, it is unclear what is gained by the additional claim that our empirical theories are true.

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1 Introduction

Our scientific theories are mathematical through and through. It seems that we cannot even state their laws without talking, not just about physical systems and their properties, but about their relations to mathematical objects. Why should this be? Why should a body of truths about abstract, nonspatiotemporal things turn out to be indispensable to our best theories of the physical world? Mathematical structuralism provides the beginnings of an attractive answer to this question. According to structuralism, mathematics is the science of patterns, and, to quote one prominent structuralist, Stewart Shapiro, "mathematics is to reality as pattern is to patterned" [6, p. 248]. The physical world is structured mathematically, so it is no wonder that mathematics, as the general theory of structure, is relevant in understanding that world.

Stewart Shapiro is an *ante rem* structuralist. That is, he holds that "free-standing" abstract mathematical structures exist over and above any physical instantiations. The structuralist view of applications slots in well to this metaphysical outlook, since it allows one to be straightforwardly realist about both sides of the pattern/patterned relation. However, as Shapiro himself notes [6, p. 248], one does not have to adopt the structuralist metaphysics to adopt something like this view of applications. Traditional Platonists, who do not reify mathematical *structures* but do believe in structured systems of mathematical *objects* can deal with applications in much the same way, albeit with a slight difference of emphasis. Rather than holding that freestanding mathematical universals are sometimes instantiated in particular systems of physical objects, a traditional Platonist can hold that the structures exemplified in physical systems are modelled in systems of mathematical objects, expressing this in terms of the existence of isomorphisms between systems of physical objects and systems of mathematical objects.

But does the view that mathematics applies to physical reality because the physical world is mathematically structured require a belief in *any* abstract mathematical reality, whether that be a reality of structures and their positions, or of more traditional mathematical objects? I would like to probe the structuralist account of mathematics a little more carefully in order to get clearer on its commitments, with a view to answering this question. In the process, I will argue that a version of the account is available to mathematical fictionalists, who do not accept the existence of abstract mathematical objects *or* of abstract mathematical structures. Furthermore, while one may think that the *ante rem* structuralist's use of this account has some advantages over this fictionalist account, in that it can be stated more straightforwardly and allows us to remain wholesale realists about our scientific theories, I will argue that these advantages are illusory.

2 Kinds of Applications of Mathematical Structure

The slogan that mathematics is to reality as pattern is to patterned is an attractive one, but needs some filling out. Shapiro points to two closely related senses in which mathematical patterns can relate to physical systems, by direct exemplification and by modelling:

> At least some applications consist of incorporating mathematical structures into physical theories, so that physical systems exemplify mathematical structures. What is almost the same thing, in some theories, the structures of physical systems are modeled or described in terms of mathematical structures. [6, p. 243]

The Platonist account mentioned above aims to incorporate all applications into the modelling account, avoiding talk of exemplification altogether. On the other hand, as this quote from Shapiro makes clear, even though the structuralist account can make some use of the notion of a mathematical structure being exemplified in physical systems, at least some applications will require us to talk of the *relations* between abstract mathematical structures and physical systems. As we will see, this aspect of the structuralist account of applications causes some difficulty for fictionalists, as it makes it hard for a fictionalist to be a scientific realist. But before we turn to this point, it is worth looking a bit more closely at kinds of applications by instantiation and by modelling, and consider the question of just what, according to the structuralist account, is being modelled.

2.1 Type I: Direct and Approximate Exemplification

The most basic type of application is via direct exemplification. Shapiro takes the example of the Kariera system, a class structure at work in a tribe discovered by Lévi-Strauss:

> A tribe is divided into four classes, and there is a certain function for determining the class of a child from the classes of his or her parents. There is an "identity class" in the sense that when a member of this class mates with any member of the tribe, the offspring are members of the other's class. If two members of the same class mate, then their offspring are of the identity class. [6, p. 249]

It turns out that, taking the four classes as objects together with the rules for determining the class of a child as a function on those four classes, the structure exemplified is that of the Klein group. Theorems about the Klein group thus receive an interpretation in terms of the class system, and can allow us to infer properties of that system (for example, as Shapiro points out, the fact that the Klein group is Abelian means that if a child's parents are from class A and class B, we do not need to know which parent is from which class in order to determine the class of the child).

Now this kind of application is relatively easy for a fictionalist, who does not believe in abstract mathematical objects or structures, to make sense of, since it does not require us to presuppose that there are any such things. When a mathematical theory is directly exemplified, its nonlogical terminology can be interpreted in such a way that its axioms assert ordinary truths about nonmathematical objects and their relations. One view of mathematical axioms suggests that we think of axioms such as the axioms of group theory as *implicit definitions* of their primitive terms: a group $\langle G, +, 0\rangle$ is any system of objects together with a binary relation satisfying certain properties and a distinguished identity element. A Klein group is any such system which satisfies some further restrictions. In the case of the Kariera system, the claim that the Klein group structure is instantiated in this system is just the claim that, when we interpret classes as objects, the rules for determining the class of a child as a function on those classes, and a particular class as the identity, the defining features of the Klein group are true of the Kariera system. And since, as a matter of logical consequence, theorems about Klein groups hold true of any system satisfying the definition, such theorems will also be true, suitably interpreted, about the Kariera class system. This application requires no appeal to abstract mathematical objects or structures. All that is required is for us to find interpretations of the primitives G, $+$ and 0 as ordinary nonmathematical objects and relations, such that the axioms for group theory are true in that interpretation.

Unfortunately, such neat applications are not always readily available. Indeed, more often than not, even in applications by instantiation, the axioms of the mathematical theories we apply are not strictly speaking true of their nonmathematical interpretation. Take, for example, Euclidean geometry: we apply this theory to physical space with a high degree of success, interpreting the theory's assumptions about points and straight lines as truths about physical points and straight lines. But in fact these assumptions are not true of physical points and straight lines: according to our current best science, ordinary physical space is not Euclidean. Or consider our mathematical theory of fluid dynamics. The 'fluids' of this theory are continuous substances, while fluids in the real world are not continuous. Yet the mathematical theory is highly successful when we interpret its technical term 'fluid' as applying to fluids in the usual sense.

There are two ways in which such applications can be understood. One way is to continue with the 'application by instantiation' idea, holding in this case not that the axioms of the theory are *true* of the physical system in question, but that they are nevertheless *approximately true*. In this case, the account of applications is not substantially different from the account given for group theory above: we do not need to suppose that there are any abstract mathematical objects our structures, only that certain statements are approximately true when interpreted as talking about concrete things. Such an account is fairly plausible for the application of Euclidean geometry, where locally we know that the Euclidean axioms are close enough to the truth that the approximation makes no detectable difference. But the example of fluid dynamics is somewhat more complex: do we really wish to say that it is even *approximately* true that fluids are continuous?

In the case of fluid dynamics, a more natural understanding of such an application holds that real fluids behave in important respects *like* the continuous fluids of our mathematical models. Thus, in a fairly standard fluid dynamics textbook we are asked to assume the following 'continuum hypothesis' (!):

> ... that the macroscopic behaviour of fluids is the same *as if* they were perfectly continuous in structure [2, p. 4, my italics].

One way of understanding this *as if* claim is as a direct comparison between actual fluids and the ideal fluids of some ideal, abstract model of the pure mathematical theory of fluid dynamics. And in such a case, in order to make such a comparison it seems that we have to suppose, not that our mathematical theory is true when interpreted about some specific concrete system, but rather, that that theory is true of a system of abstract objects, some ideal model. While such applications appear relatively straightforward for *ante rem* structuralists (who hold that mathematical theories describe abstract structures and are automatically true of the positions in those structures, considered as objects), fictionalists will have more difficulty in making sense of these cases while still refusing to accept the existence of *abstracta*.

2.2 Type II: Structural Embeddings

In Type I applications, even when our mathematical theories are not perfectly instantiated in a physical system, we do at least have a resemblance between all parts of our ideal mathematical models and the concrete systems they are used to describe. But in many applications of mathematics, there are parts of our abstract mathematical theories that correspond to nothing in the 'concrete' physical realm. Shapiro points to applications of complex analysis and set theory in this regard:

> A case in point is complex analysis. The very name "imaginary number" suggests that no straightforward physical interpretation of these mathematical structure positions is forthcoming. Yet complex analysis is useful in physics and engineering.[2] A related phenomenon is the use of a physically uninterpreted theory, such as set theory, to solve problems that are undecidable in a weaker theory, such as arithmetic. ... [S]ome of these results have applications in recursive function theory and thus in computability. One would be hard put to find a physical exemplification of the set-theoretic hierarchy, let alone an exemplification that relates to computability. [6, pp. 249–50]

Here we have mathematical structures being applied even when the systems to which they are applied do not even approximately exemplify those structures. What can the structuralist view of applications say about these?

In fact, when we think in terms of structure, such applications should not be too problematic, as can be seen if we think first about applications of mathematics to mathematics itself. Why is set theory helpful in solving problems about arithmetic? Because the structure of the natural numbers can be modelled in the sets. Why is complex analysis helpful in learning about real-valued functions? Because the real numbers are embedded in the complex plane. If one structure is embedded in a larger one, then we may find ourselves appealing to the larger structure to discover truths about the embedded structure. And in such cases, if the embedded structure also receives a concrete exemplification, we may find ourselves learning truths about the concrete exemplification by appeal to mathematical structures that are not themselves exemplified.

So such applications of unexemplified mathematical theories to concrete physical reality are not surprising, so long as their applicability is due to their ability to tell us about theories that are (at least approximately) physically exemplified. This of course leaves some applications potentially mysterious (as Shapiro points out [6, p. 247], we cannot neatly explain all uses of mathematics in quantum mechanics by means of this kind of account, simply because we have so little idea

[2] Shapiro is rather too quick in assuming that there could be no straightforward applications of complex numbers via direct exemplification. In fact, the use of complex numbers in quantum theory to measure wave phase can be given a straightforward 'modelling' interpretation. (I am grateful to McLarty, Colin for stressing this point.) However, so long as complex numbers are used in *some* contexts where they do not directly represent any physical phenomenon, Shapiro's point, that not all applications can be via direct exemplification, stands.

of what the underlying 'physical reality' we are describing is like). But insofar as we do have an understanding of what it is that we are applying our mathematics *to*, we should be able to make use of the notion of structural embeddings in order to explain the applicability of wider mathematical structures.

Again, though, for a fictionalist who doesn't believe that there are 'wider mathematical structures' over and above those structures that are physically instantiated, something more will need to be said about such applications.

2.3 Type III: Applications to Non-Physical Systems

In our discussion of applications so far, we have assumed that the ultimate 'targets' of our mathematical models are systems of concrete physical objects. However, many applications of mathematics do not have such obvious targets. Many of our physical theories, including classical Hamiltonian mechanics and quantum mechanics, are so-called 'phase space' theories. Such theories use vectors to represent, not actual systems of physical objects (as when vectors are used to represent space-time points), but *possible states* of a physical system. In these cases, mathematics may still be to reality as patterned is to patterned, but the reality that is patterned is not a reality of actual, physical objects. If we want to look for a system that exemplified the mathematical structure in such cases, it looks like it would have to be a system of *possible* objects, ways that one and the same physical system could have been.

Do structuralists who wish to adopt the pattern/patterned account of applications have to be realists about *possibilia* in order to account for these cases? Shapiro is unclear on this. On the one hand, he emphasizes that in such applications

> the theorist describes a class of mathematical objects or structures and claims that this class represents the structures of all possible systems of a certain sort. *Relations among the objects or structures represent relations among the possible objects.* [6, p. 250, my italics.]

Talk of relations among possible objects certainly suggests a commitment to the existence of possibilia which are modelled by our mathematical theories. On the other hand, though, in what follows he seems to suggest that the move to structure allows us to avoid realism about possibilia themselves:

> Classical mechanics entails that there is at least a continuum of possible configurations of physical objects. There is even a continuum of possible pairs of point masses. *We do not have to reify the "possibilities"; we speak of their structure instead.* [6, p. 250]

This is rather confusing: Shapiro claims that mathematics is to reality as pattern is to patterned, but when the patterned system appears to be a system of *possibilia*, he seems to want to retreat and say we need only accept the existence of the pattern itself. But perhaps an explanation of this attitude can be gleaned from another comment he makes about such applications. When a mathematical theory applies to a class of 'possible systems', Shapiro tells us that

> theorems about the class of structures will correspond to facts about the possible systems—about what is and what is not possible. [6, p. 250]

Now, while talk of 'facts about possible systems' suggests the reification of such systems and their objects (possibilia), the alternative talk of facts 'about what is and what is not possible' invites another reading. If one wishes to be realist about modality without being realist about possible worlds and their contents, one will accept that there are (modal) facts about what is and is not possible, without holding these to be reducible to facts about what is true in systems of really existing *possibilia*. One might in such a case think that talk of possible systems of possible objects is a helpful way of representing facts about what is and is not possible, even if one does not think that there are any such possible objects. The reason that speaking *as if* there are systems of such objects is helpful is that the structure one supposes such systems to have respects the structure of the modal facts one is aiming to model.

This, of course, is extremely speculative – I do not pretend to know what Shapiro had in mind in his own tantalizing comments on possibilities in this discussion, but rather suggest this as a reading consistent with some of his comments. There is certainly no need for a structuralist such as Shapiro to take this option: there is nothing in structuralism that would speak against adopting a standard reductive account of modality. On the other hand, fictionalists who do not accept abstract objects are likely to be equally squeamish about possibilia (while still accepting primitive, unreduced modal facts: as far as I know, no fictionalist has tried to do without modality). So if it is possible for the structuralist to bypass commitment to possibilia while making use of mathematical theories that appear to describe the structure of possible systems of objects, then this will be a welcome option for fictionalists too.

3 Fictionalism and Type I-III Applications

Of the types of applications of mathematics considered here, only the most basic type I applications (via the interpretation of our mathematical axioms as truths or approximate truths about ordinary systems of concrete nonmathematical objects) are easily dealt with by fictionalists. At least on the face of it, all other cases seem to require that we suppose that our mathematical theories correctly describe systems of problematic objects (whether they be abstract 'ideal' counterparts of physical objects, such as ideal fluids (Type I); abstract purely mathematical objects (such as transfinite sets or complex numbers) (Type II); or systems of possible objects (Type III, setting aside for now the proposed anti-reductionist dodge). How one deals with these cases as a fictionalist will depend on whether one wishes to remain a realist about our scientific theories, in the sense that one holds those theories to be true or approximately true.

3.1 Realist Fictionalism

Hartry Field is best known as having attempted a defense of realist fictionalism. As a scientific realist, Field wants to hold that our best scientific theories are at least approximately true. But if we take the applications we have considered at face-value, as representative of ordinary scientific theorizing, it looks like our scientific theorizing involves us in (a) comparing physical systems to mathematically-described systems of ideal objects; (b) embedding mathematically-described physical systems in more complex mathematical structures and learning about those systems by consideration of their relation to those more complex structures; and (c) providing mathematical models of systems of possible objects. If one does not believe in any of the systems of ideal, mathematical, or possible objects posited in the context of our ordinary scientific theorizing, how can one claim to be a realist about the scientific theories that make use of these systems in helping to understand the physical world?

Field's answer in *Science without Numbers* is to agree that he cannot be a realist about *those* theories, but to hold that these are not our ultimate theories of spatiotemporal reality. Rather, he argues, we can express our theories of the underlying physical systems that our mathematically-stated theories are normally used to describe directly, in non-mathematical terms, and it is these theories whose truth we are committed to in doing science. Furthermore, given these descriptions, he argues that we can *prove* that the physical systems we believe in are correctly represented by the systems of mathematical objects we use to model them. That is, if Field is correct in his assumption that his nominalization project can by carried out, then *given the suppositions our mathematical theories make about the structured systems of mathematical objects they concern*, it will be possible to show that, if there *were* such objects, they would indeed be related to the physical objects of our nonmathematical theories in the way our mathematically-stated empirical theories describe.

Field's nominalization project has, therefore, two aims. First, to provide attractive nonmathematical versions of our usual scientific theories, that a scientific realist can believe to be (approximately) true without thereby becoming committed to the existence of mathematical objects. And second, to use those theories to *explain* why the mathematically-stated versions of our scientific theories are so useful. Such an explanation would hold that, since we have shown that, *if there were* mathematical objects satisfying the hypotheses of our mathematical theories, then they *would be* related to nonmathematical objects in the ways our mathematically-stated theories describe, then if we indulge in the pretense that there *are* such objects, and work out the consequences of that assumption, we will be able to uncover truths about the nonmathematical objects that we really believe to exist.[3]

Field's project is most plausible for Type I and II applications. In such applications, we learn about systems of nonmathematical objects by relating them to the objects of mathematical models, and inquire into the nature of these mathematical models by describing their essential structure and perhaps embedding that structure into wider models. So long as we can describe the original systems of nonmathematical objects in nonmathematical terms (itself no mean feat, since many of our ordinary concepts are infused with mathematics), then the prospects for an explanation of the applicability of the mathematical versions of our scientific theories in terms of these nonmathematical descriptions are reasonable. But Type III applications are more problematic for Field, as critics of Field's nominalization programme have been quick to point out. If the system

[3] I have presented Field's project in non-technical (and arguably somewhat loaded) terms for my own rhetorical purposes, as will become clear. His own presentation, in terms of representation theorems and the semantic conservativeness of mathematics over nonmathematical theories, is presented in [3], and is well known, so I make no apologies for applying my own slant on the essence of the project here.

being 'modelled' by the mathematics is not itself a system of nominalistically acceptable objects, then there should be no solace in an explanation of the applicability of the mathematical 'pattern' that proceeds by means of a detailed nonmathematical description of the equally problematic 'patterned' reality. As Malament, David puts it, at best in applying Field's strategy to a phase-space theory we will be able to

> reformulate the theory so that its subject matter is the set of "possible dynamical states" (of particular physical systems) and various relations into which they enter. But this is no victory at all! Even a generous nominalist like Field cannot feel entitled to quantify over *possible dynamical states.* [4, p. 533]

Perhaps Field could try to deal with such applications in a similar manner to our proposal for the structuralist who does not believe in a realm of possible objects. That is, Field could take the basic realm to be described by our mathematical theory to be a realm of irreducible modal facts, which are nevertheless modelled well if we suppose that there is a realm of possible dynamical states that are related to these modal facts. Field's scientific realism requires him to do more than the structuralist does at this point, though: he is required to give a theory which yields the modal facts he accepts as consequences, and does so without quantifying over either possible dynamical states *or* mathematical objects. But arguably our main access to those facts is through thinking in terms of possible dynamical states. It is, then, unclear whether we would be able to come up with an attractive theory which yielded the modal facts we wanted without detouring via consideration of a realm of possibilia. In fact, this problem is quite general, arising even for attempted nominalizations of more straightforward physical subject matters. Can we be confident that, for any of the more complex uses of mathematics to describe a nonmathematical subject matter, we will be able to come up with attractive theories of the subject matter in question that do not make use of mathematical concepts? Field's sample nominalization replaces talk of real number quantities applied to measure properties of physical objects (for example, the lengths of physical line segments) in favour of comparison predicates: 'is congruent (e.g., in respect of length) to' and 'is greater (e.g., in respect of length) than'. But not all applications of mathematics involve the use of quantities as measurements in this way. It is just a hope that we can find appropriate non-mathematical substitutes for all of the mathematical concepts we find ourselves applying in describing the nonmathematical world.

3.2 Instrumentalist Fictionalism

We said that Field's project had two aims: first, to preserve his scientific realism while remaining a nominalist, by providing attractive nonmathematical versions of our usual scientific theories, and second, to use those theories to explain why we should expect our ordinary mathematically stated theories to be useful instruments in finding out about the nonmathematical realm, even if we don't believe those theories to be true. Instrumentalist fictionalists[4] focus on Field's second aim. In his (sketched) nominalization of Newtonian gravitational theory, Field

[4] Amongst whose number I count myself and, in some incarnations, Balaguer, Mark [1, Part II] and Joseph Melia [5].

has a plausible explanation of why one mathematically-stated scientific theory manages to be good without being true – i.e., that it is correct in the way it represents the nonmathematical objects it talks about. Instrumentalist fictionalists can seize on this explanation in providing an (inductive) argument for the instrumental success of other mathematically-stated theories. We can explain why it is good to indulge in the pretense that there are real numbers related to nonmathematical objects in the way our mathematical version of gravitational theory describes, on the grounds that that theory is correct in the picture it paints of the nonmathematical objects it concerns (something we can *prove* once we have our nominalistic theory together with appropriate representation theorems in hand). Why not, then, think that other mathematically-stated scientific theories are successful for the same reason: that is, even in cases where we do not have a nominalistic version of the 'nonmathematical' content of our theory to hand, we can suppose that the reason such a theory is predictively successful is not that it is *true* in all of its parts, but that it provides a good representation of the systems of nonmathematical objects it is ultimately concerned with describing. The appropriate attitude to such theories would not, then, be to *believe* that they are literally true, but rather, to indulge in the pretense that they are true for the representative advantages they bring.

Fictionalists of this sort cannot be straightforwardly realist about those scientific theories that they cannot nominalize. Insofar as their use of such theories involves them in an element of belief, the belief must be in the truth of the *nominalistic content* of our theories, i.e., in the correctness of the picture they paint of the *nonmathematical* realm. Balaguer expresses the attitude as follows:

> Now, of course, the actual mixed sentences of empirical science would not be true, but the point is that in moving from empirical science to its nominalistic content—that is, to the claim that the physical world holds up its end of the "empirical-science bargain"—we do not lose any important part of our picture of the physical world. [1, p. 134]

Unfortunately, in the absence of nominalized versions of our scientific theories, this picture is something we may only be able to access by means of the 'pretense' that there are mathematical objects. We have to indulge in what Joseph Melia calls the "way of the weasel", using the framework of mathematics to paint a picture of the nonmathematical world, but having done so, withholding belief from the mathematical parts of the framework. Indeed, Melia thinks that this is quite a plausible interpretation of the attitude taken by many ordinary scientists:

> Whilst almost all scientists will admit that they must quantify over numbers in order to formulate their scientific theories, almost all will go on to deny that there are such things as mathematical objects. Philosophers typically represent these scientists as engaging in double-think—denying by night what they believe by day. But it is surely uncharitable to regard so many scientists as hypocrites! Surely it is more charitable to think that we must have misinterpreted them. But look at the kind of things they say: "The force between two massive objects is *proportional* to the *product* of the masses *divided* by the square of the distance"; "There is a one-to-one differentiable *function* from the points of space-time onto *quadruples of real numbers*" — how can we have misinterpreted them? By thinking that any theorist who presents a theory of the world must do so by asserting a set of sentences, each one believed by the theorist. This is our mistake. As soon as we allow theorists to take away details that were added before, to subtract parts of their earlier discourse, the theorists no longer appear to believe contradictory things. The mathematics is the necessary scaffolding upon which the bridge must be built. But once the bridge has been built, the scaffolding can be removed. It is surely more charitable to take scientists to be weasels rather than inconsistent hypocrites. [5, p. 469]

Plausible though this interpretation may be, instrumentalist fictionalists must admit that it is a disadvantage that they have to rely on unanalyzed notions such as that of "the physical world holding up its end of the "empirical-science bargain"", or of mathematical "scaffolding", in stating what it is to believe the nominalistic content of our scientific theories. Surely, in this respect, standard realist structuralism has an advantage over fictionalism?

4 Structuralism and Applied Mathematics

Let us suppose that Field's nominalization project cannot be satisfactorily completed: that we cannot give attractive nonmathematical versions of our ordinary mathematically-stated scientific theories, and must instead adopt the hypothesis that there are mathematical objects in presenting our best scientific theories. In that case, the only option for fictionalists is to reject scientific realism and adopt instrumentalist fictionalism, together with its belief in the correctness of the elusive 'nominalistic content' of our ordinary scientific theories. *Ante rem* structuralists, on the other hand, appear to do better: they believe in freestanding mathematical structures, and hence in mathematical objects interpreted as pure positions in those structures. So (setting worries about the realms of *possibilia* presupposed by some Type III applications aside), they can account for those applications of mathematics that are not via direct instantiation in terms of relations between nonmathematical objects and the positions in mathematical structures. So they can be realist about our scientific theories, holding simply that they are true, and thus avoiding the instrumentalist fictionalist's elusive claim that they are correct in their nominalistic content.

Actually, things are not so simple, as a look at structuralist metaphysics will make clear. For *ante rem* structuralists, the truth of a mathematically stated theory comes cheap: Shapiro hypothesizes that any coherent theory correctly describes some structure.[5] Insofar as it is interpreted as talking about the positions in that structure, then, any coherent theory is automatically true. Take, for example, a simple scientific theory which describes an isomorphism between the objects in some physical system and the positions in some freestanding mathematical structure. This theory itself can be viewed as a mathematical theory (standardly, in the language of set theory with urelements, the isomorphism the theory asserts to exist will just be some set of ordered pairs whose first members are urelements and whose second members are pure sets). If it is any good at all,

[5]Shapiro's 'coherence' is a primitive (unreducible) modal analogue of semantic consistency.

this theory will be coherent, and hence will describe some mathematical structure, about whose positions it asserts the truth. But content of the scientific realist's claim that our best scientific theories are true or approximately true cannot be that they are true in this sense, since *any* coherent scientific theory so-interpreted will be equally true. What, then, is the claim that structuralists who wish to be scientific realists will have to make about our ordinary, mathematically-stated scientific theories?

Clearly, truth of a 'freestanding' mathematical structure is not enough. Rather, the theory will also have to remain true when its urelements are interpreted as the nonmathematical objects the theory intends to model, with relations the theory induces on these urelements being mirrored in relations that do hold between those objects. In other words, the theory won't just be true of some positions in some abstract mathematical structure: *the nonmathematical world will also have to be as the theory describes it as being.* Might we express this account by saying that the nonmathematical world holds up its side of the "empirical-science bargain"? It is not clear to me what else the structuralist can be saying here. If I am right, then although *ante rem* structuralists can be scientific realists, they must express this realism as holding not only that our scientific theories are true, but also that they are true in their nonmathematical content. And if so, it is not clear (to me at least) what would be wrong with simply holding on to the latter claim as one's account of the applicability of empirical science while ditching the former.

5 Conclusion

Of course, structuralists may well have more to say about the sense in which they take our mathematically-stated scientific theories to be not just true, but true in their picture of the nonmathematical objects they concern. It remains to be seen whether a more detailed account is available, and indeed whether such an account is adaptable to instrumentalist-fictionalist purposes. But for now at least, I think that the jury should remain out on whether *ante rem* structuralism offers any advantage over instrumentalist fictionalism. For if the notion of a scientific theory's being correct in its picture of the nonmathematical realm is required by both accounts, it is unclear what is added by the additional claim that that theory is also true.

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