From McGee's puzzle to the Lottery Paradox

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Abstract. Vann McGee has presented a putative counterexample to modus ponens. I show that (a slightly modified version of) McGee's election scenario has the same structure as a famous lottery scenario by Kyburg. More specifically, McGee's election story can be taken to show that, if the Lockean Thesis holds, rational belief is not closed under classical logic, including classical-logic modus ponens. This conclusion defies the existing accounts of McGee's puzzle.

Keywords. McGee's counterexample to modus ponens; Lottery Paradox; Belief Closure; Lockean Thesis

1. The election scenario

In a well-known article, McGee (1985, p. 462) has proposed the following scenario:

Opinion polls taken just before the 1980 election showed the Republican Ronald Reagan decisively ahead of the Democrat Jimmy Carter, with the other Republican in the race, John Anderson, a distant third. Those apprised of the poll results believed, with good reason:

[1] If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.

[2] A Republican will win the election.

Yet they did not have reason to believe

[3] If it's not Reagan who wins, it will be Anderson.

McGee (1985) speaks of a "counterexample to modus ponens". In fact, the question whether, and in which sense, (1)-(3) deserves such a label, remains, as of today, highly controversial. Still, there is at least one claim on which students of McGee's example seem to agree, i.e., the claim that the puzzle is dissolved if we assume a material interpretation of the natural language conditional "if … then …". Indeed, if we assume the material conditional, we should interpret (3) as the disjunction "either Reagan wins or Anderson wins", which is very plausible, for the simple reason that Reagan is hot favourite.

That is, as McGee himself specifies, if we interpret (1)-(3) according to the material conditional, we believe both the premises and the conclusion (McGee 1985, p. 464).

Interestingly, starting from McGee's scenario it is also possible to generate what looks like a counterexample to modus tollens (see Gauker 1994, but also Kolodny and MacFarlane 2010):

(1) If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.

(4) If it's not Reagan who wins, it's not the case that Anderson will win.

(5) The winner won't be a Republican.

(5) does not seem to follow from (1) and (4). Indeed, "a Republican will win" is very plausible, as long as the winning Republican is Reagan.

Of course, we can only apply modus tollens to (1) and (4) if (4) and the nested consequent of (1) contradict each other. Assuming that this is the case (i.e., that (4) and the nested consequent of (1) contradict each other) involves what is often called, in the literature on conditionals, "conditional non-contradiction", i.e., $P \rightarrow \sim Q$ if and only if $\sim(P \rightarrow Q)$ (where "→" is the non-material, indicative conditional and "~" is the negation symbol). As far as indicative conditionals with a possible antecedent are concerned, conditional non-contradiction is one of the most uncontroversial principles of conditional logic: Bennett (2003, p. 84) goes as far as to say that it is "almost indisputably true" that it holds.

Exactly as in (1)-(3), in (1)-(5) as well, however, it seems that if we give the conditionals a material interpretation the puzzle disappears: on a material reading of the conditional, (4) and the nested consequent of (1) cannot be seen as contradictory.

In this paper I will challenge the idea that McGee's puzzle is dissolved if we give a material interpretation of the conditionals in (1)-(3) and (1)-(5). Indeed, I will show that (a slightly modified version of) McGee's election scenario has the same structure as a famous lottery scenario by Kyburg (1961). More precisely, McGee's election story can be taken to show that, given a specific assumption on the link between probability and rational belief, the latter is not closed under classical logic, including classical-logic modus ponens. This conclusion defies the existing accounts of McGee's puzzle. Before that, however, I will have to provide some clarifications about the nature of McGee's example.

2. McGee on his "counterexample"

Truth-preserving modus ponens may be defined as the principle according to which if P is true and $P \rightarrow Q$ is also true, then Q is true as well. (In what follows, $P \rightarrow Q$ will denote the indicative conditional, and $P \supset Q$ will denote the material conditional. I will also assume that the indicative conditional is not given a material interpretation.)

For an argument to be a counterexample to this principle, it must be the case that P and $P \rightarrow Q$ are true and Q is false. The title of McGee's paper ("A counterexample to modus ponens") may at first suggest that McGee regards (1)-(3) as a counterexample in this sense. However, in the body of the article, McGee describes (1)-(3) (and the other, structurally similar, examples he provides) as cases in which "one has good grounds for believing the premises of an application of modus ponens but yet one is not justified in accepting the conclusion" (p. 462). Moreover, McGee's example revolves around what "those apprised of the poll results" *had reason to believe*, and not about truth.

A later paper by the author contains some important elucidations. In McGee 1989, he explicitly admits that his examples concern the preservation of acceptability, versus truth preservation: "Such examples show that *modus ponens* fails in English [...] More precisely, the examples show that *modus* *ponens* does not preserve warranted acceptability. As I [McGee] pointed out (1985, p. 463) and as Sinnot-Armstrong, Moor, and Fogelin (1986) have emphasized, the examples have no direct bearing on the question whether *modus ponens* is truth-preserving" (McGee 1989, p. 512 and fn. 20). That is, McGee seems to target a principle that may be formulated along these lines (note that even though there may be subtle differences between the concept of (rational) acceptance and that of (rational) belief, these are not relevant for my purposes; as a result, in this paper, I will take the two terms to be synonyms):

Epistemic modus ponens. If $Bel(P \rightarrow Q)$, and $Bel(P)$, then $Bel(Q)$, where *Bel* is a rational belief operator.

If we now turn to (1)-(5), it seems that, by McGee's own criteria, we should regard it as a failure of the following schema:

Epistemic modus tollens. If $Bel(P \rightarrow Q)$, and $Bel(not\ Q)$, then $Bel(not\ P)$.

As for rational belief (or acceptability) itself, McGee does not provide many details about the way it should be defined in order for his examples to go through. However, in McGee 1985, he mentions high probability as a reason for believing the premises of his examples, and low probability as a reason for disbelieving their conclusions. That is, he speaks of such reasons in terms of likelihood ("[i]t is more likely that […]"; "[…] it is virtually certain that […]"; "[…] it is entirely certain that […]"; McGee 1985, p. 463). Although he does not endorse it explicitly, he seems to adopt a principle called "Lockean Thesis":

Lockean Thesis. One should believe P if and only if, given one's evidence, P is very probable (where "very probable" means "probable to a degree equal to or higher than a specified threshold value t").[1] Or equivalently: it is rational to believe P if and only if, given one's evidence, P is very probable (where "very probable" means "probable to a degree equal to or higher than a specified threshold value t").[2]

In what follows I argue that McGee's scenario gives us reasons to believe that, if the Lockean Thesis holds, epistemic modus ponens and modus tollens fail, even if natural language conditionals are given a material interpretation (i.e., even if $P \rightarrow Q$ and $P \supset Q$ are taken to be equivalent). More precisely,

[1]In what follows I will apply the Lockean Thesis to (among others) indicative conditionals. That is, I will make use of the principle that one should believe, or accept, an indicative conditional $P \rightarrow Q$ if and only if $P \rightarrow Q$ is very probable. Now, in the wake of Lewis' famous triviality results (Lewis 1976), a number of philosophers have argued that indicative conditionals are not propositions. So I avoid committing myself to the controversial claim that indicative conditionals have propositional content. In other words, when I will talk about the probability of $P \rightarrow Q$ I will not be talking about the probability of $P \rightarrow Q$ *being true*. Following Adams (1975), I will take such a probability to be the probability of Q conditional on P. That is, in what follows, the Lockean Thesis (when applied to indicative conditionals) will read: one should believe, or accept, $P \rightarrow Q$ if and only if the probability of Q conditional on P is high (provided that $P \rightarrow Q$ is a simple conditional and that $p(P) \neq 0$, see section 3 below). If you think that taking indicative conditionals to be non-propositional is not enough to escape Lewisian triviality, note that I can propose a stronger line of defence: to the best of my knowledge, all Lewis-style triviality results assume a classical Bayesian framework. This means that we can avoid triviality by abandoning (or suitably modifying) such a classical setting. For instance, Lassiter (2020) has shown how to avoid triviality by replacing the classical bivalent semantics for probability with a three-valued semantics. Goldstein and Santorio (2021) and Santorio (2021) also show that we can avoid triviality by adopting a nonstandard account of credence and credal update. In particular, Goldstein and Santorio (2021) suggest that we reject Bayesian conditionalization in favour of an imaging-based account of conditionalization. Note that I am not claiming that we should embrace either of these proposals (Lassiter's or Goldstein and Santorio's); in fact, discussing the possible ways out of triviality, as well as their consequences, would deserve a separate paper. What I want to stress here is simply that renouncing classical Bayesianism may offer some interesting ways out of triviality, which may be especially worth exploring if you dislike the non-propositional view, or are unconvinced that it suffices to avoid triviality. (For more on this point, see fn. 3 below.)

[2]It could be objected that it is not so obvious that "One should believe P" and "It is rational to believe P" are equivalent. And indeed, a whole debate has arisen, in recent years, on the nature of the relations between epistemic obligations and rational belief. However, we can ignore this debate for present purposes, since it is not relevant to the argument to be presented in the main text.

under the assumption that the Lockean Thesis holds, McGee can be taken to show that the two following principles of the logic of belief are falsified (where, as we already know, "⊃" is the material conditional and "~" is the negation symbol):

*Epistemic modus ponens**. If $Bel(P \supset Q)$, and $Bel(P)$, then $Bel(Q)$.

*Epistemic modus tollens**. If $Bel(P \supset Q)$, and $Bel({\sim}Q)$, then $Bel({\sim}P)$.

Of course, the failure of epistemic modus ponens* (or modus tollens*) entails the failure of a more general principle, often called "Belief Closure":

Belief Closure. Rational belief is closed under classical logic.

In the literature on rational belief and rational degrees of belief it is commonly held that the Lockean Thesis and Belief Closure cannot be jointly satisfied. Indeed, joint acceptance of Belief Closure and the Lockean Thesis gives rise to the Lottery Paradox (Kyburg 1961). I will show that the latter is intimately linked to McGee's election scenario. More specifically, here is how I now proceed: in section 3 I propose my interpretation of McGee's scenario. In section 4 I present what I call "the restaurant scenario". As I show in section 5, the importance of this scenario lies in that, first, it preserves the relevant features of McGee's election story, and that, second, it is just a lottery scenario. Section 6 spells out the conclusion that the same kind of treatment should be provided for both the restaurant scenario and Kyburg's lottery scenario. Section 7 clarifies the consequences the discovery of the restaurant variant of McGee's scenario has on the way we should handle McGee's original argument.

3. The Argument Schema

I will make the reasonable assumption that if we are justified in believing (2) above it is because of its high probability (for recent papers that make a similar assumption, see Stern and Hartmann 2018 as well as Neth 2019; as specified above, evidence for this assumption can also be found in McGee's original paper). Essentially, I will assume that McGee endorses the principle I called "Lockean Thesis". However, note that, in spite of my hypothesis being reasonable, it is actually not necessary for my purposes to rely on the claim that McGee indeed made this assumption. What only needs to be the case for this article's purposes is that there is a plausible interpretation of McGee's puzzle in which the Lockean Thesis is assumed. The same holds for the assumption that epistemic modus ponens is the principle involved in (1)-(3): no matter what McGee really had in mind, there is a reasonable and easily accessible interpretation of his puzzle that involves epistemic modus ponens: this is all I need for my aims here.

So let us go on and assume that the reason why we should believe (2) is that it is highly probable. It follows that if (1)-(3) is to be taken as a potential counterexample to (epistemic) modus ponens, the reason why we should believe (1) must be the same (that is, its high probability); and the reason why we should not believe (3) must be that its probability is not high enough.

One popular way of interpreting the conditionals in McGee's example is compatible with the author assuming the Lockean Thesis. According to this interpretation, (1) has a probability of 1 because, supposing that a Republican wins, the conditional probability that Anderson will win given that Reagan doesn't win is 1. In this view, (2) is also likely, because the unconditional probability that a Republican will win is high. However, the conditional probability that Anderson wins, given that Reagan doesn't win, is low, that is, (3) is unlikely.

This interpretation of the premises can be made more precise by adopting what is often called, in the literature on conditionals, "Adams' Thesis"; that is, by assuming that the acceptability of an indicative conditional is equal to the probability of its consequent given its antecedent (see Adams 1975). In the literature on conditionals, many versions of the Thesis can be found, involving subtle differences; however, the one below should be enough for my purposes. Note that it only holds for simple conditionals, $P \to Q$, such that $\mathrm{p}(P) \neq 0$:

Adams' Thesis. The acceptability of $P \to Q$ is equal to the probability of Q given P (i.e., of Q conditional on P).

Stern and Hartmann (2018) also adopt an account of the conditionals in (1)-(3) based on Adams' Thesis. However, as they observe, the latter does not provide us with an analysis of (1), as Adams' Thesis only applies to simple conditionals and (1) is an embedded conditional. Indeed, consider an indicative conditional of the form $P \to (Q \to R)$: "If we were to apply [Adams' Thesis] to this conditional, it would seem that $Acc(P \to (Q \to R)) = \mathrm{p}((R|Q)|P)$, but there is no such probability expression as $\mathrm{p}((R|Q)|P)$" (Stern and Hartmann 2018, p. 608; here and below, I modified the authors' notation to make it coherent with mine). However, there is such a probability expression as $\mathrm{p}(R|P \wedge Q)$ (where "∧" is the conjunction symbol). Here, I will follow Stern and Hartmann (2018) in assuming that choosing to analyse $Acc(P \to (Q \to R))$ as $\mathrm{p}(R|P \wedge Q)$ is safe. This step can be motivated by a plausible principle of conditional logic, i.e., import-export, according to which $P \to (Q \to R)$ is equivalent to $(P \wedge Q) \to R$. An acceptability version of the principle can be formulated as below (see, again, Stern and Hartmann 2018).

Acceptability Import-Export. $Acc(P \to (Q \to R)) = Acc((P \wedge Q) \to R)$[3]

[3]As specified in fn. 1 above, my approach eludes the so-called triviality results, for we are only interested here in the acceptability conditions for indicative conditionals, and not in the question whether indicative conditionals are propositions (see Stern and Hartmann 2018). However, as also made clear in fn. 1, the reasons why triviality results do not affect my argument are deeper: triviality results crucially assume a classical Bayesian framework. Indeed, as Lassiter (2020),

By Acceptability Import-Export and Adams' Thesis, we obtain that $Acc(P \rightarrow (Q \rightarrow R)) = \mathrm{p}(R|P \wedge Q)$.[4] That is, our attitudes towards (1), (2), and (3) are represented as indicated in (a), (b), and (c) respectively:

(a) $\mathrm{p}(R|P \wedge Q)$

(b) $\mathrm{p}(P)$

(c) $\mathrm{p}(R|Q)$

I will call this way of representing our attitudes towards McGee's argument's premises and conclusion "the Argument Schema". According to it, both (1) and (2) have a high degree of acceptability (as (a) and (b) are both high), whereas (3) is only acceptable to a low degree (because (c) is low). The Argument Schema is clearly compatible with McGee assuming the Lockean Thesis: by the latter, we should (fully) accept both (1) and (2), while we should (fully) reject (3).

Note that I do not mean to claim that (a)-(c) is the only representation of our attitudes towards (1)-(3) compatible with McGee assuming the Lockean Thesis. However, (a)-(c) are certainly a very natural way of representing them, which I will thus take as the main reference here.

I will now show that, if my interpretation of McGee's argument (according to which it involves both the Lockean Thesis and epistemic modus ponens) is granted, we can provide a slightly modified

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Goldstein and Santorio (2021), and Santorio (2021) show, these results can be avoided precisely by modifying such a setting (see fn. 1 above). An in-depth discussion of the different ways out of triviality falls beyond the scope of this paper, so the only point I want to make here is that the mere assumption of Adams' Thesis and Acceptability Import-Export cannot lead to triviality: further assumptions, among which a classical account of credence and credal update, are needed.

[4] As the authors specify, "this follows only when [Acceptability Import-Export] is restricted to settings where $\mathrm{p}(P \wedge Q) > 0$ (since [Adams' Thesis] applies only in these settings)" (Stern and Hartmann 2018, fn. 15).

version of McGee's election scenario, in which (i) both epistemic modus ponens* and epistemic modus tollens* fail, and (ii) the relevant features of the scenario are preserved.

4. The restaurant scenario

I am sitting in a restaurant with my Italian friend Pasquale. I know that Pasquale always orders one of the day's specials. Today's specials are pizza, pasta and roast beef. I know that Pasquale loves both pizza and pasta, and that he does not like roast beef very much. I estimate that there is a 0.4 probability that Pasquale will have pizza, a 0.4 probability that he will have pasta and a 0.2 probability that he will have roast beef.

Assume the material conditional and set $t = 0.6$. In this context, I should believe both (6) and (7):

(6) If Pasquale doesn't have pizza, then he will have pasta.

(7) Pasquale won't have pizza.

Indeed, they both have a probability of at least 0.6. Now, from (6) and (7), using epistemic modus ponens*, I should infer (8):

(8) Pasquale will have pasta. (!)

But (8) only has a probability of 0.4; so I should not believe (8), that is, epistemic modus ponens* fails.

Let us now turn to epistemic modus tollens*. By the Lockean Thesis, I should believe (9), which has a probability of 0.6:

(9) Pasquale won't have pasta.

Now, from (9) and (6) I should draw, by epistemic modus tollens*, the following conclusion:

(10) Pasquale will have pizza. (!)

But (10) only has a probability of 0.4; therefore, I should not believe (10), i.e., epistemic modus tollens* fails.

So we have both a failure of epistemic modus ponens* and a failure of epistemic modus tollens*. Or, more precisely, given $t = 0.6$, in the above examples rational belief is not closed under modus ponens* and modus tollens*, respectively.

5. From McGee's puzzle to the Lottery Paradox

It can be noted that epistemic modus ponens* and modus tollens* are not the only logical principles that fail in the restaurant scenario. Indeed, epistemic conjunction introduction[5] does not hold either: given $t = 0.6$, we should believe:

(7) Pasquale won't have pizza.

(9) Pasquale won't have pasta.

(11) Pasquale won't have roast beef.

[5]Epistemic conjunction introduction can be defined as the principle according to which if $Bel(P)$, and $Bel(Q)$, then $Bel(P \wedge Q)$.

However, we should not believe the conjunction of these three propositions (actually, we should believe its negation). That is, the failure of epistemic modus ponens* and modus tollens* is not the only relevant feature of the restaurant scenario. Indeed, as anticipated above, it can be shown that the restaurant example has the same structure as Kyburg's Lottery Paradox.

Famously, the Lottery Paradox goes as follows: suppose that I participate in a fair 1000-ticket lottery with exactly one winner. In this context, I have very good reasons to believe that my ticket will lose. Indeed, the probability that it will win is 0.001. I believe the same about the ticket of the person next to me, and about all the other tickets. Nonetheless, if I apply this reasoning to every ticket from n°1 to n°1000 I reach the conclusion that all tickets will lose, which is false.

In both Kyburg's scenario and mine, a disjunction must be satisfied: in the lottery scenario, one ticket must win; in the restaurant example, it is assumed that Pasquale will pick one of the day's specials. However, at the same time, in both scenarios we should not believe any of the disjuncts: in the lottery scenario each of the 1000 tickets is unlikely to win; in the restaurant scenario, none of the day's specials is likely to be Pasquale's choice. That is, the restaurant scenario is a lottery scenario, at least if we adopt the standard definition of a lottery scenario as a scenario where, given t higher than 0.5 (and lower than 1), the Lockean Thesis and epistemic conjunction introduction come into conflict, i.e., one should end up believing a contradiction. This definition clearly applies to the restaurant scenario, as in it a probability of 0.6 is assumed as a threshold for rational belief.

So if we assume the above definition of a lottery scenario, then a slight modification of McGee's original example leads to a version of the Lottery Paradox, namely one with three tickets and a probability threshold for rational belief of 0.6. All one has to do in order to obtain such a scenario is to decrease the probability of "Reagan will win"; more specifically, one has to assign to "Reagan will win" a probability lower than the threshold: this is enough to generate a probability distribution where the probability of each of the three disjuncts is below t. That is, this is enough to generate a lottery scenario (provided, of course, that some specific proportions are respected between the probabilities of the propositions; the restaurant scenario exemplifies such proportions).

I would like to stress that such a modification (i.e., the one that takes us from McGee's original scenario to the restaurant story) is an innocent one: the fact that in the original scenario "Reagan will win" is very likely can be regarded as a contingent feature of the scenario itself. That is, transforming McGee's original story into the restaurant story by no means betrays the original scenario. Suppose that X = "Carter loses the election" (i.e., "a Republican wins"), Y = "Reagan loses", and Z = "Anderson loses". Given these interpretations of X, Y and Z, (1)-(3) has the following form:

$X \rightarrow (Y \rightarrow \sim Z)$

X

$\therefore Y \rightarrow \sim Z$

Consider now the restaurant scenario and assume that X = "Pasquale doesn't have pizza", Y = "Pasquale doesn't have pasta" and Z = "Pasquale doesn't have roast beef": we obtain (1')-(3'), which is structurally identical to (1)-(3):

(1') If Pasquale doesn't have pizza, then if he doesn't have pasta, he will have roast beef.

(7) Pasquale won't have pizza.

(3') If Pasquale doesn't have pasta, he will have roast beef. (!)

Clearly, the Argument Schema can be applied to (1')-(3') as well: the probability that Pasquale will have roast beef, given that he does not have pizza or pasta (i.e., p($R|P \wedge Q$)) is 1, the probability that he will not have pizza (i.e., p(P)) is 0.6, while the probability that he will have roast beef given that he does not have pasta (i.e., p($R|Q$)) is very low (much lower than 0.6). So assuming t = 0.6, we should believe the argument's premises, and should disbelieve its conclusion.

What is very interesting here is that in the restaurant scenario the relevant properties of McGee's example are preserved: this happens because even if in the restaurant scenario the probability of ~*Y* ("Pasquale will have pasta", corresponding to "Reagan will win" in the election story) is low, the probability of *X* ("Pasquale won't have pizza"/"A Republican will win") is high. That is, even if in the election scenario the probability of "Reagan will win" were to be lower than it is, McGee should still have to regard (1)-(3) and (1)-(5) as failures of modus ponens and modus tollens respectively. Removing the contingent fact that one of the disjuncts in the original scenario has a probability higher than the threshold simply allows us to gain further insight into the puzzle.

Importantly, the above (namely, the fact that *X* can be likely, even if ~*Y* is not) undermines those accounts of McGee's puzzle according to which (1)-(3) is not a modus ponens argument, but rather contains a fallacy of equivocation of certain kinds. Paoli (2005), for instance, argues that (1)-(3) is not an instance of modus ponens because "A Republican wins" should be given a different interpretation in (1) and (2). An essential role in distinguishing the two interpretations is played by the fact that, according to the author, in (2) "A Republican will win" simply stands for "Reagan will win". Fulda (2010) also claims that (1)-(3) is not a modus ponens, but rather an enthymeme, in which "Regan will win" is the suppressed premise. We now see that both attempts to dismiss McGee's argument are misguided, as they focus on a very contingent feature of the argument: the fact that in McGee's original scenario one of the disjuncts ("Reagan will win") seems rationally acceptable to begin with.

6. The need for a unified solution

My main conclusion will be that it is impossible to solve McGee's puzzle without thereby solving the Lottery Paradox, and the other way around. In this section, I will address one potential objection to this conclusion. The objection is based on a difference between the restaurant story and McGee's original story, i.e., on the fact that in the restaurant scenario both "kinds" of modus ponens (epistemic modus ponens and epistemic modus ponens*) can only fail if *t* is relatively low, namely, if it is equal to 0.6. It goes as follows: what makes (1)-(3) interesting is that each of its two premises seems to have a very high probability (higher than 0.6), and still epistemic modus ponens seems to fail, although not for the material conditional, but rather for the indicative conditional (i.e., epistemic modus ponens seems to fail, unlike epistemic modus ponens*).

In other words, the complaint is this: if we assume that a probability of 0.6 is not sufficient for rational belief (whereas a greater probability does suffice) it is no longer clear whether epistemic modus ponens* would still fail. And if this principle were unscathed for *t* higher than 0.6, then we would still be allowed to regard McGee's original puzzle as a genuine puzzle as far as *indicative* conditionals are concerned, but there would be no puzzle about the restaurant scenario.

A first reply to these remarks is, quite simply, that it is very rarely the case that defenders of the Lockean Thesis commit to a specific value for *t*. Actually, there seem to be only very few authors who have a strong preference for a specific threshold.[6] This notwithstanding, let us go on and assume that the objection can be thoroughly articulated, so that in the restaurant scenario epistemic modus ponens and modus ponens* do not really fail, because a higher probability is needed for rational belief. After all, it does not seem unreasonable to think that a probability of 0.6 is not (at least not always) sufficient for rational belief (think of the defenders of a contextualist version of the Lockean Thesis, who may argue that in the restaurant context there are specific reasons to reject a 0.6 threshold). However, in fact, both epistemic modus ponens and modus ponens* do fail for *t* higher than 0.6.

As far as epistemic modus ponens is concerned, McGee's original scenario itself provides intuitive support for the claim that epistemic modus ponens can fail for *t* higher than 0.6. Now, it turns out that this intuition can be given formal support: Stern and Hartmann (2018) have proved that it is always

[6]One of them is Achinstein (2001), who claims that a probability greater than 0.5 is both necessary and sufficient for rational belief. More recently, Shear and Fitelson (2019) have argued that the inverse of the golden ratio ($\phi^{-1} \approx 0.618$) should be regarded as a non-arbitrary bound on the belief threshold.

possible to find p($R|P \wedge Q$) and p(P) such that they are both high, while at the same time p($R|Q$) is low. In their paper, Stern and Hartmann (2018) consider what I have called above (in section 3) "the Argument Schema":

(a) p($R|P \wedge Q$)

(b) p(P)

(c) p($R|Q$)

Crucially, they arrive at the following expansion of (c):

$$p(R|Q) = p(R|P \wedge Q)\, p(P|Q) + p(R|{\sim}P \wedge Q)\, p({\sim}P|Q).$$

Though the first probability in the expansion corresponds to (a), none of the other probabilities appear in the premises. This means that you can coherently assign (c) a probability as low as 0 (or as high as 1) even when you regard (a) and (b) as highly acceptable. For example, if you assign .99 to (a) and .99 to (b), you can coherently judge (c) to be utterly unacceptable when your estimate for p($R|{\sim}P \wedge Q$) is low. (Stern and Hartmann 2018, p. 610).

The only exceptions are cases in which p($R|Q$) = p($R|P \wedge Q$), as of course, if p($R|Q$) = p($R|P \wedge Q$), we cannot have that both (a) and (b) are high and (c) is low.[7]

Turning to epistemic modus ponens*, it is actually a well-known fact that failure of Belief Closure is not limited to cases in which t is equal to 0.6. As long as an appropriate number of tickets is chosen, failure of Belief Closure can always be observed, no matter the specific threshold t (the only condition is that t must be strictly between 0.5 and 1). Kyburg's original scenario is a case in point: in it t is much

[7]Actually, in pointing out the very minor probabilistic constraints that (a) and (b) impose on (c) the authors focus on the case in which p(P) = 1 (Stern and Hartmann 2018, fn. 18), but actually p(P) = 1 is only a special case of that in which p($R|Q$) = p($R|P \wedge Q$).

greater than 0.6, but Belief Closure still fails. As a result, no matter the kind of lottery scenario we are considering (with 3 tickets, 1000 tickets, or with a still different number of tickets), accusing the specific threshold value adopted (0.6, 0.999, etc.) of being responsible for the failure of Belief Closure seems hopeless. As I will spell out below, this point extends to epistemic modus ponens* specifically.

Note that none of the above entails that a solution to both Kyburg's original Paradox and its restaurant version cannot be provided in contextualist terms. My only point here is that the same kind of treatment should be provided in both cases: this entails nothing regarding the specific treatment that we should provide. A clarification, though: the most famous contextualist proposal (i.e., Leitgeb's) actually belongs to the category of those accounts involving a *modification* of the Lockean Thesis, whose original definition is rejected (see Staffel 2021).[8] Indeed, according to Leitgeb (2014; 2015; 2017), we should supplement the Lockean Thesis with the condition that the probability of P should remain higher than 0.5 when the agent learns new information compatible with P. So adopting

[8]Genin (2019, p. 478) distinguishes a strong, context-independent version of the Lockean Thesis from a weaker, context-dependent version. The version of the Lockean Thesis I am assuming in this paper (see section 2 above) closely matches what Genin calls "The Strong Lockean Thesis", whose definition goes as follows. (Note that Genin is taking the domain of the quantifier to be the set of all belief states a *particular* agent may find herself in, or as the set of all belief states whatsoever (Genin 2019, p. 478).)

Strong Lockean Thesis. There is a threshold $0.5 < t < 1$ such that all rational belief states satisfy $Bel(P)$ if and only if $p(P) \geq t$.

The Strong Lockean Thesis can be contrasted with the Weak Lockean Thesis:

Weak Lockean Thesis. For every rational belief state, there is a threshold $0.5 < t < 1$ such that $Bel(P)$ if and only if $p(P) \geq t$.

As Genin himself (2019, p. 478) points out, the Strong Lockean Thesis is usually regarded as the standard definition of the Lockean Thesis. It is also the one that allows us to generate the Lottery Paradox. By contrast, Leitgeb (2014; 2015; 2017) endorses a precise specification of the Weak Lockean Thesis, which does not give rise to the Lottery Paradox (for the details of his proposal see Leitgeb 2014, 2015 and 2017).

Leitgeb's proposal would still boil down to embracing one of the two "classical" options concerning the Lottery Paradox: rejecting the Lockean Thesis or rejecting Belief Closure (more on these two options below).

I conclude that the sensible ways to deal with McGee's scenario are the same as the sensible ways to deal with the lottery scenario. In both cases, we seem to have two main options: giving up the Lockean Thesis or giving up Belief Closure. (Coherently with what is standard in the literature, and as just suggested concerning Leitgeb's proposal, I regard those authors who propose to modify the Lockean Thesis as belonging to the group of the Lockean Thesis deniers.)

Note that, if we decided to reject Belief Closure, we would be forced to deny at least three principles: epistemic modus ponens*, epistemic modus tollens* and epistemic conjunction introduction. Indeed, I showed that in the restaurant scenario the three of them fail. In fact, this also holds for Kyburg's scenario: even though the Lottery Paradox is generally presented as involving (epistemic) conjunction introduction, we can generate lottery-like paradoxes by using other principles (see Douven 2016). It is instructive to see briefly how.

Let us consider Kyburg's original scenario. In it, we should believe "Ticket n°1 wins ∨ ticket n°2 wins… ∨ ticket n°1000 wins" (where "∨" is the disjunction symbol), which is equivalent to "Ticket n°1 loses ⊃ (ticket n°2 loses… ⊃ (ticket n°999 loses ⊃ ticket n°1000 wins))". We should also believe, about each of the tickets between n°1 and n°999, that it will lose. However, we should not believe that ticket n°1000 will win (in fact, we should believe that it will lose). That is, epistemic modus ponens* fails.

In this scenario, epistemic modus tollens* does not hold either. Indeed, we should accept

Ticket n°1 loses ⊃ (ticket n°2 loses… ⊃ (ticket n°999 loses ⊃ ticket n°1000 wins)).

Ticket n°1000 loses.

Ticket n°999 loses.

…

Ticket n°2 loses.

Nevertheless, we should reject

Ticket n°1 wins

and accept its negation. That is, in Kyburg's original scenario, exactly as in the restaurant scenario, there are at least three ways to generate an unacceptable conclusion: using epistemic modus ponens*, epistemic modus tollens*, or (as in the original version of Kyburg's puzzle) epistemic conjunction introduction.

7. Back to McGee's original argument

What I just said concerns the way we should deal with the restaurant scenario and the lottery scenario in general. But what can we say concerning specifically the original version of McGee's argument?

In fact, the dilemma raised by the Lottery Paradox applies in a straightforward manner to (1)-(3). That is, if we reject either the Lockean Thesis or Belief Closure, McGee's original argument is blocked.

Suppose that we deny the Lockean Thesis: we would no longer be compelled to accept (1) and (2), which would block the derivation of (3). (This is a straightforward consequence of the fact that the Lockean Thesis is an implicit assumption in McGee's puzzle, or at least in the version of McGee's puzzle I am considering here (see my understanding of (1)-(3) in section 3 above); therefore, if we renounce the Lockean Thesis, the puzzle vanishes.)

If, instead, we denied Belief Closure (i.e., as we have seen, at least epistemic modus ponens*, epistemic modus tollens* and epistemic conjunction introduction), this would also solve the puzzle. The reason is the following: suppose that we reject epistemic modus ponens*; it seems that, *a fortiori*, we should reject epistemic modus ponens. This is because it is natural to regard the indicative conditional as stronger than the material conditional; i.e., it is generally assumed that if we should believe an indicative conditional, we should also believe the corresponding material conditional. One main argument to this conclusion goes as follows: suppose that we rationally believe the negation of $P \supset Q$, i.e., $P \wedge {\sim}Q$; it seems natural to infer that we rationally believe the negation of the corresponding indicative conditional. This reasonable assumption entails that if epistemic modus ponens* (modus tollens*) turned out to fail, epistemic modus ponens (modus tollens) would also fail. That is, if we rejected modus ponens*, McGee's original puzzle would also be solved, as (1)-(3) would not be an instance of a valid logical schema anymore, whether we assume the material conditional or a stronger conditional.

Of course, the same holds for (1)-(5): suppose that we reject epistemic modus tollens*: (1)-(5) would not instantiate a valid principle anymore, whether, again, we assume the material conditional or a stronger conditional.

So I showed that the two puzzles (McGee's and the Lottery) have the same structure; i.e., that a slight modification of McGee's election scenario is a lottery scenario. This entails that the two scenarios put us before the same dilemma: should we deny the Lockean Thesis or Belief Closure? I then noted that, no matter which of these two principles we choose to deny, McGee's original argument is blocked. In other words, exactly as the Lottery Paradox, McGee's 1985 paper can be taken to show that under the assumption that the Lockean Thesis holds, Belief Closure fails. Of course, this conclusion only follows if one condition is satisfied: the hypothesis that in McGee's original example both epistemic modus ponens and the Lockean Thesis are involved must be true. However, as already specified, even though other interpretations of (1)-(3) are perhaps possible, any author tackling McGee's problem should account at least for this very plausible and easily accessible understanding of the argument. A consequence of this fact is that any student of McGee's puzzle should give up either the Lockean Thesis or Belief Closure. In other words, any account of McGee's puzzle that does not involve either giving up the Lockean Thesis or Belief Closure is unsatisfactory. The interesting and important point here is that the vast majority of the existing accounts of McGee's problem do not address the rejection of either principle.[9]

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[9]Among others, this includes the accounts of the puzzle by Appiah (1987), Lowe (1987), Piller (1996), Katz (1999), Bennett (2003), Gillies (2004), Paoli (2005), Cantwell (2008), Fulda (2010), Kolodny and MacFarlane (2010), Moss (2015), Stojnić (2017), Schulz (2018), Stern and Hartmann (2018), Neth (2019), Edgington (2020), and Williamson (2020).

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