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Realism and the Infinite

"Not empiricism and yet realism in philosophy, that is the hardest thing." -Wittgenstein

"A human is that being which prefers to represent itself within finitude, whose sign is death, rather than knowing itself to be entirely traversed and encircled by the omnipresence of infinity." -Badiou

I

In his 1951 Gibbs lecture, "Some basic theorems on the foundations of mathematics and their philosophical implications," drawing out some of the "philosophical consequences" of his two incompleteness theorems and related results, Kurt Gödel outlines an alternative which, as I shall try to show, captures in a precise way the maximal disjunctive horizon of the varieties of realism available today:

> Either mathematics is incompletable in [the] sense that its evident axioms can never be comprised in a finite rule, i.e. to say the human mind (even within the realm of pure mathematics) infinitely surpasses the powers of any finite machine, or else there exist absolutely unsolvable Diophantine problems of the type specified (where the case that both terms of the disjunction are true is not excluded so that there are, strictly speaking, three alternatives).[1]

As I shall try to show, the disjunction captures the contemporary situation of reflective thought in the essentially ambiguous relationship of formalism to the real, a relationship engendered by the most significant results of the twentieth century's sustained inquiry into the problem of the infinite and its givenness to finite thought. A consequence of this aporeatic contemporary situation, as I shall try to show, is that the longstanding philosophical debate over the relative priority of thought and being that finds expression in contemporary discusssions of "realism" and "anti-realism" (whether of idealist, positivist, or conventionalist forms) can only be assayed from the position of what I shall call a *meta-formal* reflection on the relationship of the *forms* of thought to the *real* of being, exactly the kind of reflection exemplified by Gödel's argument in the Gibbs lecture. Moreover, if Gödel's argument is correct, and if it bears (as I shall try to show it does) not only on the question of "mathematical reality" narrowly conceived but, more generally, on the very "relationship" of thought and being that is at issue

[1] Gödel (1951), p. 134.

in these discussions, it is also not neutral on this question of priority, but rather rigorously establishes a (necessarily disjunctive) *realism* that, though singularly appropriate to the widely ranging consequences of the projects of formalism and formalization in our time, nevertheless differs significantly both from traditional kinds of "metaphysical realism" and the newer varieties of "speculative realism" on offer today.

The type of realism I shall defend here is not primarily a realism *about* any particular class or type of objects or entities. Thus it is not, a fortiori, an *empirical* realism or a naturalism (although I also do not think it is inconsistent with these positions).[2] In particular, its primary source is not any empirical experience but rather the experience of formalization, both insofar as this experience points to the real-impossible point of the *actual relation* of thinkable forms to being and insofar as it schematizes, in results such as Gödel's, the intrinsic capacity of formalization problematically to capture and decompose its *own* limits. In *The Politics of Logic*, I systematically interrogated the consequences of formalism and formalization in this sense for contemporary political, social, and intersubjective life according to the various orientations that appear to be possible today for thought in its total relation to being, seeking to locate, in each case, the actual point and limits of the effective formal capture of the real in thought. In particular, I suggested there, and will argue more fully here, that both of the orientations I presented there as "post-Cantorian" demand a *realist* attitude grounded, in different ways, in this experience of the transit of forms, and capable of acknowledging their inherent difference from anything simply created or produced by finite human thought. Accordingly, I believe the metaformal realism common to the two post-Cantorian orientations might be formulated precisely, referring in passing to the Lacanian motto according to which "the Real is the impasse of formalism," as a realism *of* the "Real," in Lacan's sense, according to which the Real represents both an inherent limit-point and obscurely constitutive underside for both of the other two "registers" of the Imaginary and the Symbolic.[3] As the name "post-Cantorian" is meant to index, what is most decisive in producing the possibility of this distinctive kind of realism is the chain of consequences following from the Cantorian event and the problematic accessibility of the infinite to mathematical thought, up to and including Gödel's incompleteness results, as these consequences offer to challenge and reconfigure the traditional conception of the human as an essentially finite agent of thought.

To arrive at his disjunctive conclusion, Gödel draws centrally on a concept that is central to twentieth-century inquiry into the foundations of mathematics, that of a "finite procedure." Such a procedure is one that can be carried out in finite number of steps by a system governed by well-defined and finitely stateable rules, a so-called "formal system." As Gödel points out, there are several rigorous ways to define such a system, but they have all been shown to be equivalent to the definition given by Turing of

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[2] I return to this issue in section IV, below.

[3] Of course, Lacan's concept of the "Real" is complex and undergoes many changes of specification and inflection over the course of his career. I do not take a view here about how precisely to define it or which formulation is most important, but seek only to preserve the link that is constitutive for Lacan between the Real and formalization at the latter's point of inherent impasse. For a very exhaustive and illuminating treatment of Lacan's concept, see Eyers (2012). I also discuss Lacan's motto and Badiou's reversal of it into his own claim for a "theory of the pass of the real, in the breach opened up by formalization…" in Livingston (2012), pp. 188-192.

a certain specifiable type of machine (what has come to be called a "Turing machine").[4] The significance of the investigation of formal systems for research into the structure of mathematical cognition and reality lies in the possibility it presents of rigorously posing various general questions about the capacities of such systems to solve mathematical problems or prove mathematical truths; for instance, one can pose as rigorous questions i) the question whether such a system is capable of proving *all* arithmetic truths about whole numbers; and ii) whether such a system is capable of proving a statement of its own consistency. Notoriously, Gödel's first and second incompleteness theorems, respectively, answer these two questions, for *any* consistent formal system capable of formulating the truths of arithmetic, in the negative: given any such system, it is possible to formulate an arithmetic sentence which can (intuitively) be seen to be true but cannot be proven by the system, and it is impossible for the system to prove a statement of its own consistency (unless it is in fact *inconsistent*).

In its outlines, Gödel's argument from these results to his "disjunctive conclusion" is relatively straightforward. The first incompleteness theorem shows that, for any formal system of the specified sort, it is possible to generate a particular sentence which we can "see" to be true (on the assumption of the system's consistency) but which the system itself cannot prove.[5] Mathematics is thus, from the perspective of any specific formal system, "inexhaustible" in the sense that no such formal system will ever capture *all* the actual mathematical truths. Of course, given any such system and its unprovable truth, it is possible to specify a *new* system in which that truth is provable; but then the new system will have its own unprovable Gödel sentence, and so on. The question now arises whether or not there is some formal system which can prove *all* the statements that *we* can see to be true in this intuitional way. If not, then human mathematical cognition, in perceiving the truth of the successive Gödel sentences, essentially *exceeds* the capacities of all formal systems; this is the first alternative of Gödel's disjunction (mechanism is false). If *so*, however, then there is some formal system that captures the capacities of human mathematical thought. It remains, however, that there will be statements that are undecidable for this system, including the statement of *its* consistency. Thus it is impossible, on this alternative, to claim simultaneously that actual mathematical cognition is based wholly on principles that are consistent and that it decides all mathematical problems. In this case there are thus problems that cannot be solved by any formal system we can show to be consistent *or* by any application of our powers of mathematical cognition themselves; there are well-defined problems which will remain unsolvable, now and for all time.

We can understand the issues, as Gödel himself does, in terms of a distinction between "subjective" and "objective" mathematics. By "objective" mathematics, Gödel means the totality of arithmetic statements that are true in an "absolute" sense; by "subjective" mathematics he means the totality of statements that are demonstrable, or *knowably* (with mathematical certainty) true.[6] From the first

[4] This is a formulation of the "Church-Turing" thesis, which holds that the structure of a Turing machine (or any of several provably equivalent formulations) captures the 'intuitive' notion of solvability or effective computability.
[5] I here state the first theorem, roughly and intuitively, appealing to a notion of "truth" that is in some ways problematic. For discussion of the issues involved in the difference between this and other, less potentially problematic statements, see Livingston (2012), chapter 6.
[6] This distinction, as Gödel uses it, does not, however, imply or involve a *metaphysical* distinction between subject and object.

incompleteness theorem, it already follows that "objective" mathematics is "inexhaustible" in the sense that any consistent formal system will fail to exhaust all its truths. We can now pose Gödel's question as the question whether "subjective" mathematics and "objective" mathematics (defined this way) coincide. If so (the first disjunct) then human cognition includes the capacity to demonstrate all mathematical truths, but this capacity cannot ever be captured or modeled by any (finitely specifiable) formal system, and the powers of human cognition essentially exceed those of any such system. If not (the second disjunct) then it may indeed be possible for human cognition to be captured by some formal system, but there will be truths (including the truth of the statement of the *consistency* of that system itself) which cannot be demonstrated by that formal system and hence, a fortiori, by any humanly accessible means whatsoever; hence there will again be absolutely unsolvable problems.

The two options left open by Gödel's disjunctive conclusion correspond directly to the two post-Cantorian orientations of thought, or positions on the relation between thought and Being, that I called in *The Politics of Logic* the "generic" and "paradoxico-critical" orientations.[7] On the first of Gödel's disjunctive options, the power of the human mind to grasp or otherwise comprehend truths beyond the power of any finite system effectively to demonstrate witnesses an essential *incompleteness* of any finitely determined cognition and a correlative capacity on the part of human thought, rigorously following out the consequences of the mandate of consistency, to traverse by means of a "generic" procedure the infinite consequences of truths essentially beyond the reach of any such finite determination. On the second of the options, the essential indeterminacy of any such system witnesses, rather, the necessary *indemonstrability* of the *consistency* of any procedural means available to the human subject in its pursuit of truth, and thereby to the necessary existence of mathematical problems that are absolutely unsolvable by any specifiable epistemic powers of this subject, no matter how great. Both orientations, as I argued in the book, as well as the necessity of the (possibly non-exclusive) decision between them, result directly from working through the consequences of the systematic availability of the *infinite* to mathematical thought, as accomplished most directly through Cantor's set theory and its conception of the hierarchy of transfinite cardinals. More broadly, as I argued in the book, what is most decisive for the question of the orientations available to thought today is the consequences of the interlinked sequence of metamathematical and metalogical reflection running from Cantor, through Gödel's incompleteness theorems, up to Cohen's demonstration of the independence of the Continuum Hypothesis from the axioms of ZF set theory; it is thus not surprising that Gödel's own "philosophical remarks" about the implications of his own results should replicate the general disjunction in a clear and specific form.

We can further specify the underlying issue, and move closer to discerning its deep philosophical significance, by noting that, by Gödel's second theorem, the undecidable Gödel sentence for each system is equivalent (even within the system) to a statement, within that system, of its own consistency. As Gödel emphasizes, it is (given classical assumptions) an implication of the *correctness* of any system of axioms that we might adopt for the purposes of arithmetic demonstration that the system be consistent; but then it is an implication of the second incompleteness theorem that if we are in fact *using* a specific (and consistent) formal system to derive all the mathematical truths (that we know) we

[7] For the four orientations, see Livingston (2012), pp. 51-60.

could not know that we are. For if we could know this, i.e. if we could know the truth of the assertion of the consistency of the system, we would thereby know a mathematical truth that cannot be derived from that system. Accordingly, as Gödel says, it is

> …impossible that someone should set up a certain well defined system of axioms and rules and consistently make the following assertion about it: All of the axioms and rules I perceive (with mathematical certitude) to be correct and moreover I believe they contain all of mathematics.[8]

Thus if a system is (knowably) consistent it is, by that token, and *demonstrably* incomplete; if it is complete, we cannot know it to be consistent (and hence we cannot know it to be correct). Accordingly, on the assumption that we are in fact using a finite procedure to demonstrate mathematical truths, the assumption of the consistency of the system we are actually using is shown to be *essentially* unsecurable in any way that is itself consistent with our (in fact) using (only) that system at all.

Again, by considering the question of the axiomatization of mathematics, we can see how the issue is connected to the problem of the accessibility of the infinite, and the higher levels of infinity. Specifically, in order to axiomatize arithmetic set-theoretically without contradiction, it is necessary to introduce axioms in a step-by-step manner, and in fact, as Gödel suggests, this process can be continued infinitely: thus

> Instead of ending up with a finite number of axioms, as in geometry, one is faced with an infinite series of axioms, which can be extended further and further, without any end being visible and, apparently, without any possibility of comprising all these axioms in a finite rule producing them.[9]

The successive introduction of the various levels of axioms corresponds to the axiomatization of sets of various order types; in each case the introduction of a new level of axioms corresponds to the assumption of the existence of a set formed as the *limit* of the iteration of a well-defined operation.

But each axiom "entails the solution of certain Diophantine problems, which had been undecidable on the basis of the preceding axioms;" in particular, according to a result that Gödel had achieved in the 1930s, the consistency statement for any given system of axioms can be shown to be equivalent to a statement asserting the existence of integral solutions for a particular polynomial.[10] Since consistency is undecidable within the system itself, so is the problem of the truth-value of the statement concerned, but it becomes decided in a stronger system which adds, as a new axiom, a statement of the former system's consistency (or something equivalent to this).[11] But since the problem of the truth of the

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[8] Gödel (1951), p. 133.
[9] Gödel (1951), p. 130.
[10] Gödel (1951), p. 132.
[11] The result that Gödel refers to in 1951 is that the consistency statement is equivalent to some statement of the form:
$\forall x_1 . . . x_n \exists y_1 … y_m [p(x_1,. . ., x_n, y_1, …y_m) = 0]$ where p is a polynomial with integer coefficients and the variables range over natural numbers; later the work of Davis, Putnam, Robinson and Matiyasevich showed that one can replace the statement with something of the form:
$\forall x_1 . . . x_n [p(x_1,. . ., x_n) \neq 0]$

statement about the solutions to a polynomial is itself simply a number-theoretical problem, it follows that each particular system, if it *is* consistent, cannot solve some mathematical problem; and that *if* human cognition is equivalent to some *particular* system then there is some problem of this form (equivalent to the statement of its own consistency) that *it* cannot solve either. This is then an "absolutely undecidable" problem. If, however, there is no formal system to which human cognition is equivalent, then for *any* specified machine the mind can prove a statement which that machine cannot, and accordingly "the human mind … infinitely surpasses the powers of any finite machine."[12]

The issue can, again, be connected to that of the status of the most famous unsolved (and, as we now know, unsolvable) problem of set theory, Cantor's problem of the size of the continuum. From the work of Gödel himself in 1939 and Cohen in 1962-63, we now know that the continuum hypothesis (CH), which holds that the size of the continuum is the same as that of the first non-countable ordinal, cannot be demonstrated or refuted on the basis of the standard ZF axioms of set theory. Gödel himself thought, for a time at least, that the status of the continuum hypothesis might be resolved by the addition of one or more *new* axioms, in particular new axioms affirming the existence of certain "large" cardinals. If we were able intuitively to establish or otherwise have insight into the truth of some such axiom capable of resolving the status of the CH, this might provide evidence for the first horn of Gödel's disjunction, on which the power of the human mind to have insight into evident axioms true of mathematical reality essentially exceeds the capacities of axiom systems such as ZF. However, although the program of investigating the implications of such additional axioms continues actively today, none of the axioms that have so far been considered actually suffice to establish the truth of the CH, and none of them appear in any direct way "intuitively" motivated. Thus, the results of the inquiry so far might rather reasonably be taken to support the second horn of Gödel's disjunction, on which there are simply unsolvable problems; indeed, it might well be thought that the problem of the CH is one such, and that its unsolvability bears witness to an essential ontological feature of indeterminacy or undecidability characteristic of the universe of sets itself.[13]

For discussion, see Feferman (2006), p. 6.

[12] I follow here the trenchant and careful exegesis of Gödel's conclusion in the Gibbs lecture given by Feferman (2006), pp. 1-7. As Feferman notes (p. 7), there are a few auxiliary premises that are needed to assure the validity of Gödel's argument for the "disjunctive conclusion": first, that the human mind, in demonstrating truths, "only makes use of evidently true axioms and evidently truth-preserving rules of inference"; second, that these axioms include those of Peano Arithmetic; and third that a "finite machine", in the relevant sense, proves only theorems that are also provable by the human mind (or in other words that the power of a formal system is in any case no *greater* than that of the human mind).

[13] In *Being and Event* (Badiou 1988), Alain Badiou considers the "ontological" implications of Gödel and Cohen's research on the continuum hypothesis under the assumption of an identity between mathematics, as axiomatized by the ZFC axioms, and the ontological theory of "being qua being". Under this assumption, and the further auxiliary identifications of sets with "situations" and their power sets with (what Badiou calls) their "states", the issue of the CH and the means used to show that both it and its negation are consistent with the axioms of ZFC bear important implications for the question of the "power" of the state over a situation that it captures, and hence for the possibility of transformative action that undermines or challenges this state power. Drawing in detail on Cohen's method of forcing, which was used to establish the consistency of the *negation* of the continuum hypothesis with the axioms, Badiou argues that the large degree of arbitrariness that this introduces into the actual size of the continuum motivates a positive argument for the possibility of such action, at a distance from the

Most of the discussion in the philosophical literature over the broader implications of Gödel's theorems so far has been directed toward the question of the truth or falsity of *mechanism*. This is the question whether the mathematical thought of an individual subject, or perhaps of the whole community of mathematicians, can "in fact" be captured by some formal system. Gödel himself, particularly in his later years, was, as is well known, a dedicated anti-mechanist, and sometimes referred to his incompleteness theorems as providing evidence against mechanism; more recently, philosophers such as Lucas and Penrose have followed Gödel in arguing for this conclusion. Gödel also sometimes suggested that the truth of the first disjunct of his disjunctive conclusion in the Gibbs lecture, on which mechanism is false, might be established by means of independent (perhaps partly empirical) considerations. Nevertheless, the recent literature witnesses a consensus that (as Gödel himself seems to affirm in the lecture) the only conclusion relevant to the mechanism debate that can really legitimately be drawn from the incompleteness results themselves is the disjunctive one: *either* mechanism is false, and the human mind (or the community of mathematicians) has access to mathematical truths that cannot be proven by any formal system *or* mechanism is true and there are well-specified problems that cannot be solved by any means whatsoever.

Additionally, there are some good reasons to think that the "hypothesis" of mechanism cannot in fact be specified clearly or uniquely enough to use the incompleteness theorems to establish anything about its truth or falsity at all. Thus, for instance, in a recent very comprehensive review of discussion about Gödel and mechanism, Stuart Shapiro concludes that "there is no plausible mechanist thesis on offer that is sufficiently precise to be undermined by the incompleteness theorems."[14] One reason for this is that any proposal to treat the cognition of a subject, or human mathematical cognition overall, as embodying a specific formal system will clearly involve a significant degree of idealization with respect to actual practice; actual mathematicians make mistakes, and any determination of which formal procedure they are "actually following" would thus require a motivated distinction between what counts as mistaken performance and what does not. Similarly, any determination of *what* class of performance is to count as evidencing the postulated formal system is bound to be somewhat arbitrary; do we consider, for example, the behavior of just the *best* mathematicians, or all who are formally trained in (some kind of) mathematics at all, or perhaps of everyone who is even (minimally) competent in mathematics at all? Finally, even if these worries about the idealization of performance can be overcome, one might wonder whether there is any "well-defined" way to consider questions involving the totality of *all* formal systems, as we must in fact do if we are to consider the truth-value of *either* term of Gödel's disjunctive result.[15]

For all of these kinds of reasons, it seems that it is not possible to draw any unequivocal conclusions directly from Gödel's incompleteness theorems about the hypothesis of mechanism with respect to

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state. However, as I argued in *The Politics of Logic* (chapter 9), even given Badiou's interpretive assumptions and identifications, it is much more trenchant in light of the current development of set theory to suppose that the truth-value of the CH, and hence the "ontological" possibility of radical change in Badiou's sense, is *itself* simply undecidable, and that a more completely grounded interpretation of formalism might have to take this fact of undecidability as a basic, positive result.

[14] Shapiro (1998), p. 275.

[15] I am indebted to discussion with Reuben Hersh for this point.

human mathematical capacities. Nevertheless, despite these worries relevant to mechanism and idealization, it may still be possible to see the upshot of Gödel's "disjunctive conclusion" as bearing relevance to somewhat different philosophical issues.[16] In particular, I shall argue that it points to a distinctive and non-standard, but comprehensive position of *realism*, what I shall call *meta-formal* realism. The decisive issue here is not, primarily, that of the reality of "mathematical objects" or the possibility of understanding them as determinate independently of the routes of access to them (epistemic or otherwise) involved in the exercise of our human capacities. It is, rather, that both terms of Gödel's disjunction capture, in different ways, the structural point of contact *between* these capacities and what must, on *either* horn of the distinction, be understood as an infinite *thinkable* structure determined quite independently of anything that is, in itself, finite. Thus, each term of Gödel's disjunction reflects the necessity, given Gödel's theorems, that any specification of our relevant capacities involve their relation to a structural infinity about which we must be realist, i.e. which it is not possible to see as a mere production or creation of these capacities.

On the first alternative, this is obvious. If human mathematical thought can know the truth of statements about numbers which are beyond the capacity of *any* formal system to prove, then the epistemic objects of this knowledge are "realities" (i.e. truths) that also exceed any finitely determinable capacity of knowledge. (It does not appear possible to take these truths as "creations" of the mind unless the mind is not only credited with *infinite* creative *capacities*, but understood as having actually already *created* all of a vastly infinite and in principle unlimitable domain). But on the second alternative, it is equally so. If there are well-specified mathematical problems that are not solvable by any means whatsoever, neither by any specifiable formal system nor by human cognition itself, then the reality of *these problems* must be thought of as a fact determined quite independently of our capacities to know it (or, indeed, to solve them).[17] On this alternative, we must thus acknowledge the existence of a reality of forever irremediable problems whose very issue is the inherent undecidability that results from the impossibility of founding thought by means of an internal assurance of its consistency. In this way the implications of the mathematical availability of the infinite, on either horn of the disjunction, decompose the exhaustiveness of the situation underlying the question of realism vs. idealism in its usual sense: that is, the question of the relationship of a presumptively finite thought to its presumptively finite object.

The actual underlying reason for the realism which appears forced upon us on either alternative is the phenomenon Gödel describes as that of the *inexhaustibility* of mathematics, which results, as we have seen, from the possibility of considering, given any well-defined ordinal process, its infinite limit (or

[16] I thus follow Feferman (2006), p. 11 in considering that, even if there are problems with applying Gödel's reasoning directly to the question of mechanism, "…at an informal, non-mathematical, more every-day level, there is nevertheless something to the ideas involved [in his argument for the "disjunctive conclusion"] and something to the argument that we can and should take seriously."

[17] Gödel says this about the second term of the disjunction: "… the second alternative, where there exist absolutely undecidable mathematical propositions, seems to disprove the view, that mathematics (in any sense) is only our own creation…So this alternative seems to imply that mathematical objects and facts or at least *something* in them exist objectively and independently of our mental acts and decisions, i.e. to say some form or other of Platonism or "Realism" as to the mathematical objects." (pp. 135-36). I return to the question of the implications of the second disjunct in section IV below.

totality). On the first alternative, this inexhaustibility yields a structurally necessary *incompleteness* whereby each finite system by itself points toward a truth that it cannot prove but which is nonetheless, by this very token, accessible to human thought. On the second, it yields an equally necessary *undecidability* which leaves well-specified mathematical problems unsolveable by any means (finitely specified or not) by any means whatsoever. The form of the relevant realism is, in each case, somewhat different: the orientation underlying the first disjunct corresponds, as I argued in *The Politics of Logic*, to a realism of *truth beyond sense*, a position that affirms the infinite existence of truths and the infinite genericity of our dynamic insight into them beyond any finitely specifiable language or its powers, while the realism of the second consists is a realism of sense beyond truth, affirming the existence of linguistically well-defined *problems* whose truth-value remains undecidable under the force of any powers of insight whatsoever. But in either case, reflective thought about human capacities must reckon with the consequences of their structurally necessary contact with an infinite and inexhaustible reality essentially lying beyond the finitist determination of the capacities of the human subject or the finitely specifiable powers of its thought. In this way, the consequences of Gödel's theorem, however we interpret them, engender a structurally necessary realism about the objects of these powers that is the strict consequence of the entry of the infinite into mathematical thought.

It would probably not be difficult to show that each of the controversies between varieties of "realism" and "idealism", signed by prominent names in the history of philosophy, unfolds in direct and demonstrable connection with varying conceptions of the infinite and its availability to thought; one could consider, for instance, the difference between Plato's late conception of the Idea as owing its genesis to the ongoing struggle between the principle of the One and that of the *apeiron dyas*, or the unlimited dyad, and Aristotles merely *potential* infinity; or the difference between Leibniz's harmoniously ordered infinite continuity of monadic powers, up to the divine itself, and Kant's determination of the infinite as thinkable only in the form of the infinitely deferred, regulative idea). Nevertheless, wherever the actual infinite *has* been thought philosophically prior to the twentieth century, it has been thought simply as a theological (or, more broadly, onto-theological) Absolute. The singular significance of the event of Cantor thus lies, as Badiou has emphasized, in its rendering a de-absolutized infinite accessible to non-theological thought, in making mathematics as the "science of the infinite" the possible site for a renewed rigorously formal thinking of the powers and limits of thought.

As Gödel immediately goes on to point out, the only position from which it appears possible (while accepting Gödel's assumptions about mathematical reasoning and the incompleteness theorems themselves) to resist the "disjunctive conclusion" is a *strictly finitist* one according to which "only particular propositions of the type 2+2=4 belong to mathematics proper…"[18] and no *general* judgments applying to an infinite number of cases are ever possible. This kind of position would indeed avoid the disjunctive conclusion, since there is no way to apply the incompleteness theorems themselves consistently with it. However, as Gödel points out, the strict finitist view is very implausible as a view of mathematical reasoning, since it ignores that "it is by exactly the same kind of evidence that we judge that 2+2=4 and that a+b=b+a for any two integers a,b"; and it would moreover appear to disallow the use of even such simple "concepts" as "+" (which "applies" to all integers). Outside these very severely

[18] Gödel (1951), p. 135.

limited finitistic point of view, on the other hand, it appears inevitable that the disjunctive conclusion will apply, and thus we will be forced to acknowledge the validity of one or both of its disjuncts.

It is thus that the inherent character of reasoning in mathematics invokes the infinite, and marks the consequences of its availability to thought. Because it bears witness in a rigorous way to the consequences of thought's inherent drive to formalization, and thereby also witnesses the consequences of the transit of forms and limits through, and beyond, the limit of finitude itself, I propose to call the kind of realism exhibited here, on either horn of Gödel's disjunction, "metaformal realism."[19] More broadly, we can extend the label to *any* position that takes a realist attitude with respect to the actual and intra-temporal basis and implications of the development of formalization. As is evident in Gödel's interpretation of the implications of his own metaformal results, this kind of realism draws on the rigorous consequences of the formal thought of the infinite, and thus cannot be sustained solely within a position of finitism.

The attitude I am calling "metaformal realism" might certainly be developed as a position within the philosophy of mathematics itself. Developed in this way, it would bear a resemblance to a "methodological" realism about mathematics, for example of the kind suggested by Maddy (2005), that characteristically looks to mathematical practice itself as the source for its "ontological" claims and assumptions. This kind of realism has the advantage that it does not entertain, or attempt to solve, "metaphysical" problems about the "existence" of mathematical objects, except insofar as these problems are formulable and resolvable, in a motivated way, within mathematical practice itself (here, including the kind of "metamathematics" or "metalogic" that Gödel uses to produce his incompleteness theorems).

In fact, it is very important to distinguish this kind of attitude from "Platonism" as it is traditionally construed. In particular, as Badiou (1998) has argued, there is no need to invoke, even in service of a realist attitude that here takes the event of the infinite and the consequences of mathematical practice seriously, the "Platonistic" claim of the "real existence" of mathematical objects. As Badiou suggests, the "Platonist" attitude of object-invoking realism is in fact quite alien to Plato's own concerns; in particular, it relies upon a "distinction between internal and external, knowing subject and known 'object'" which is, as Badiou says, "utterly foreign" to Plato's own thought about thought and forms.[20] Plato's fundamental concern is not, as Badiou argues, at all with the question of the 'independent existence' of mathematical objects, but rather with the 'Idea' as the name for something that is, for Plato, "always already there and would remain unthinkable were one not able to 'activate' it in thought."[21] Similarly, this is, as Badiou emphasizes, not an attitude of accepting or *believing in* the existence of sets or classes corresponding to well-defined monadic predicates, but rather one of maintaining, quite to the contrary, that what correlates to a well-defined concept may well be "empty or

[19] In The Politics of Logic (p. 291), I called this position simply "formal realism". I add the prefix 'meta-', here, to reflect that what is concerned is not primarily an attitude (e.g. a Platonist one) about the "reality" or "actual existence" of forms, but rather the implications of the *transit* of forms in relation to what is thinkable of the real, the transit that can, in view of Cantor's framework, be carried out beyond the finite.
[20] Badiou (1998), p. 49.
[21] Badiou (1998), p. 49.

inconsistent"; it is thus a *metalogical* inquiry into the structure of forms for which, as Badiou emphasizes, "the undecidable constitutes a crucial category" and in fact becomes the central "reason behind the aporetic style of the [Platonic] dialogues," wherein thought constantly proceeds through forms to their own inherent points of dissolution or impasse. Whether or not we follow Badiou in his desire to redeem for this attitude the name of "Platonism," against its standard, ontological mis-approriation, what is most important to note is that what is involved here is thus not any direct attitude of realism toward objects of any kind but rather only a philosophical reflection of the internal consequences of the meta-formal inquiry into forms and their limits, including the open dialectic of finite and infinite thought.

Because this attitude, along with Plato himself, accords mathematical experience a certain privilege as, precisely, a non-empirical experience of forms, the realism suggested by it can be worked out, as I have said, as a position within the "philosophy of mathematics" itself. But it seems to me that the kind of realism exhibited here can also find fruitful application more broadly, to domains other than simply that of mathematics. For as I argued in *The Politics of Logic*, the consequences of formalism and formalization in their contemporary practical and theoretical development are by no means limited to mathematics, but extend to a broad range of phenomena and many aspects of contemporary social and political life. As a leading example of this (though there are certainly others) one might consider the pervasiveness of informational and computational technologies and the forms of abstract social organization they make possible, themselves grounded in the technology of the computing machine which was directly made possible by the development of the implications of the concept of a formal system in thinkers such as Hilbert, von Neumann and Turing. If this and many other developments of twentieth century praxis and organization are indeed, as I argued there, intimately linked to the project of formalization in its various dimensions, then a realism that is, as I have suggested, itself directly linked to the aporeatic result of this project's development may be singularly appropriate to contemporary critical and reflective thought.

Here, as I argued in the book, the relevance of leading developments in mathematics and metamathematics is not limited to the "philosophy of mathematics" narrowly construed, but extends to the broader impications of the ongoing project of formalization itself. If, accordingly, the *metaformal realism* I am recommending here arises in an intrinsic way from the structure of forms in their capture of life, then a rigorous understanding of the relationship of thought to being may today require such a position, which takes account of the implications of the dimensions of the infinite as they occur at the horizon of our contemporary understanding of ourselves and the world. The specific relevance of mathematics and metamathematics, in this connection, does not lie in the identification of a particular realm or region of entities, but rather in the way that mathematics, *as* the "science of the infinite", possesses the ability to capture and schematize the constitutively "infinite" dimension of form itself. As I argued in the book, this infinite dimension of forms is a constitutive part of the thinking of form, even when it is dissimulated or foreclosed, ever since Plato, and is inherently involved, as well, in every contemporary project of the analysis of logical form or the discernment of the formal determinants of contemporary life and practices. This twentieth-century inquiry into formalism has, as I argued in the book, many interacting dimensions, including (but not limited to) the philosophical inquiries, both "analytic" and "continental," which in the twentieth century interrogate the structures of language as essential guidelines to their inquiry into forms of life. As such, its results capture the most important implications of contemporary reflective inquiry for the constitutive idea of the rational human subject or agent of capacities and thought.

In particular, as is clear in relation to Gödel's development of his results, this metaformal realism, with its constitutive conception of the powers of thought in relation to a real determined as infinite, marks the unavailability of any traditional opposition between the finitude of the human subject and a transcendent matter thought under the heading of the absolute. If, on the contrary, thought is capable, in its capacity for formalization, of rigorously conceiving an infinite-real to which it is immediately adequate (whether this capacity be thought as itself infinite, or as grounded in the finite systematics that comprise a formal system), then it is no longer possible to oppose an attitude of realism (in the traditional sense) to one of idealism according to the different positions taken on thought's capacity to know its object in itself.

The metaformal realism thus indicated has several further distinctive features, which I briefly adumbrate:

1. Metaformal realism is not a "metaphysical realism" or an "empirical realism." In particular, because it is grounded solely in an *internal* experience of the progress of forms to the infinite, it avoids any need to posit an empirical or transcendent referent beyond the effectiveness of forms and formalization and does not ground its realism in any such referent. Because of the way it turns on the entry of the infinite into mathematical thought, it does not require that one assure oneself of the existence of a world "in itself" and independent of thought. It is thus completely distinct from any realism of a "mind-independence" variety, which always requires a problematic doctrine of the bounding of thought in relation to its empirical objects. It also does not require, and does not encourage, the possibility of a "view from nowhere" or a "single unique description of reality." Rather, we have here a rigorous internal development of the limitology of thought from within thought itself, a development of "thought thinking itself" which is nevertheless not 'dialectical' and does not attest, either, to the power of thought consistently to appropriate *everything* within itself. For all of these reasons, metaformal realism does not involve the difficult metaphysical and epistemological questions (how is it possible to know or have access to a "thing in itself"? What is the status of the "world independent of the mind"?) which recurrently appear to make forms of "metaphysical realism" untenable and have often been taken to motivate a contrasting position of idealism (or pragmatism, or 'internal realism,' etc.)
2. Metaformal realism is a reflective, not a 'speculative' idealism. It develops all of its consequences internally, from internal reflection on the limitology of thought and its inherent formal features. It thus has no need to posit an object of *speculation* simply external to this limitology or to engage in the uncertain investigation of the features of such an object. If it is, as

I shall try to show, engaged in an inherent dialectic of thought with being, this dialectic is thus not a *speculative* dialectic of "determinate negation."[22]

3. Metaformal realism de-absolutizes the world as a transcendent object of thought. As I argued in *The Politics of Logic*, the twentieth-century inquiry into forms pursued in its narrower aspect as the inquiry of "metamathematics" or "metalogic" has the consequence of consigning formal thought about the totality of the world (indeed, thought about totality in general) to an unavoidable disjunction, what I called there the "metalogical duality" between consistent incompleteness and inconsistent completeness, essentially the same alternatives involved in Gödel's 'disjunctive' conclusion. This means, as well, the fundamental diremption of any figure of thought that countenances a (complete *and* consistent) Absolute, and forces a choice between acknowledging the essential incompleteness of consistent thought or countenancing the existence of the totality of the world only under the heading of the reality of the inconsistent. But the consequence of this is that there is, then, no world that is *both* whole in itself and immune from structural inconsistency. If this is correct, then it is henceforth obligatory to think through the rigorous consequences of a "realist" attitude toward a world inherently incomplete in itself, *or* marked by the structural presence of real inconsistency. This means the deposition of every absolutism of thought in relation to its real matter.

II

In contemporary philosophical discourse, no project has done more to illuminate the issue of realism and its underlying formal determinants than Michael Dummett's. Familiarly, in a series of articles and books beginning in 1963 with the article "Realism," Dummett has suggested that the dispute between realism and anti-realism with respect to a particular class of statements may be put as a dispute about whether or not to accept the principle of *bivalence* (i.e., the principle that each statement is determinately true or false) for statements in the class concerned.[23] Though this issue yields differing consequences in each domain considered, the acceptance of bivalence generally means the acceptance of the view that all statements in the relevant class have truth values determined in a way in principle independent of the means and methods used to verify them (or to recognize that their truth-conditions actually obtain when they, in fact, do so); the anti-realist, by contrast, generally rejects this view with respect to the relevant class.

Dummett did not envisage that this comprehensive framework would or should support a single, *global* position of metaphysical "realism" or "anti-realism" with respect to all domains or the totality of the

[22] I refer here, in passing, to the distinction between "reflection" and "speculation" drawn by Hegel in the "Preface" to the *Phenomenology of Spirit*, para. 59. That I thus distinguish the post-Cantorian orientations of metaformal realism from Hegel's pre-Cantorian speculative dialectic should not exclude that metaformal realism, particularly in its paradoxico-critical variant, nevertheless exhibits a number of important parallels to aspects of Hegel's system, particularly in its treatment of the nature of contradiction *prior* to its dialectical sublation or resolution; for discussion of these relationships to Hegel, see *The Politics of Logic*, pp. 253-54.

[23] Dummett (1963); for some later reflections on the development of the framework and issues related to it, see Dummett (1978).

world; rather, his aim was to illuminate the different kinds of issues emerging from the traditional disputes of "realism" and "idealism" in differing domains by submitting them to a common, formal framework.[24] From the current perspective, however, it is just this aspect of formal illumination which is the most salutary feature of Dummett's approach. For by formally determining the issue of realism with respect to a given domain as one turning on the acceptance or nonacceptance of the (meta-)formal principle of bivalence with respect to *statements*, Dummett points toward a way of conceiving the issue that is, in principle, quite independent of any *ontological* conception of the "reality" or "ideality" of *objects* of the relevant sort. In particular, it is in this way that Dummett avoids the necessity to construe realism and anti-realism in *any* domain as involving simply differing attitudes toward the ontological status of its objects (for instance that they are "mind-independent" or that, by contrast, they are "constituted by the mind"). What this witnesses, *along with* what I have called meta-formal realism, is the possibility of a purely formal and reflective determination of the issue of realism that connects its stakes directly to those of the truth of claims, thereby instantly short-circuiting the laborious and endlessly renewable dialectic of the "actual relationship" of mind to world.

Dummett's framework is sometimes glossed in terms that suggest that, for him, the adoption of realism or anti-realism in any particular case turns primarily on our judgment about the (primarily epistemological) issue of whether a certain type of entities can be considered to be real in themselves, independently of our access to them or ability to possess evidence for their existence. But that this kind of formulation is, at best, highly misleading, both with respect to Dummett's own motivations and the actual merits of the framework he recommends, can be seen from the introductory formulation of the issue of realism and anti-realism in the original article "Realism" itself:

> For these reasons, I shall take as my preferred characterisation of a dispute between realists and anti-realists one which represents it as relating, not to a class of entities or a class of terms, but to a class of *statements*, which may be, e.g., statements about the physical world, statements about mental events, processes or states, mathematical statements, statements in the past tense, statements in the future tense, etc…[T]he realist holds that the meanings of statements of the disputed class are not directly tied to the kind of evidence for them that we can have, but consist in the manner of their determination as true or false by states of affairs whose existence is not dependent on our possession of evidence for them. The anti-realist insists, on the contrary, that the meanings of these statements are tied directly to what we count as evidence for them, in such a way that a statement of the disputed class, if true at all, can be true only in virtue of something of which we could know and which we should count as evidence for its truth. The dispute thus concerns the notion of truth appropriate for statements of the disputed class; and this means that it is a dispute concerning the kind of *meaning* which these statements have.[25]

There are two points here that bear important implications for the issue of how best to characterize realism and anti-realism. The first is that, on Dummett's formulation, it is an issue, not of the reference

[24] Dummett (1978), pp. xxx-xxxii.
[25] Dummett (1963), p. 146.

of terms or the existence of objects, but of the way in which the truth-values of statements are determined. The second, following from the first, is that the question of realism within a given domain is not directly an epistemological question about our knowledge of (or 'access to') entities, but rather a semantic question about the basis of the *meaning* of statements. As Dummett points out, both points are helpful in characterizing the real underlying issue and separating it from other issues that have become confused with it in the history of discussion of realist and idealist (or nominalist and universalist, etc.) positions. For example, in the traditional debate between phenomenalists and realists about material objects, which has sometimes been put as a debate about their "existence", Dummett argues that his framework allows the actual question of realism to be separated from what is in fact a conceptually different one, the question of *reductionism* (i.e. of whether 'material objects' can in fact be reduced to something like sense-data). Somewhat similarly, with respect to mathematics, concentrating on the question of the reference of terms tends, Dummett suggests, to "deflect the dispute from what it is really concerned with"; in particular, "the issue concerning platonism relates, not to the existence of mathematical objects, but to the objectivity of mathematical statements."[26] Here again, a framework primarily directed toward the question of the meaning of statements is more useful than one concerned primarily with questions of the existence of objects. This is, at least in part, because in mathematics (as opposed to some other cases) it is generally implausible to suppose we can have "access" to the relevant "objects" independently of a recognized procedure (i.e. a calculation or a proof) for establishing the truth of statements about them; and on the other, that such a procedure is also generally taken to be *sufficient* for whatever access to mathematical objectivity we can enjoy.

In the 1973 article "The Philosophical Basis of Intuitionistic Logic," Dummett considers the question of what rationale might reasonably serve as a basis for replacing classical logic with intuitionistic logic in mathematical reasoning (hence, in his framework, for replacing realism with anti-realism).[27] As in the earlier article, Dummett here emphasizes that the primary issue is not epistemic but semantic: thus, "Any justification for adopting one logic rather than another as the logic for mathematics must turn on questions of *meaning*"; and again, "it would be impossible to construe such a justification [i.e. for adopting classical or intuitionistic logic] which took meaning for granted, and represented the question as turning on knowledge or certainty."[28] In fact, Dummett suggests, there are just two lines of argument that could plausibly be used to support the replacement. The first turns on the idea that "the meaning of a mathematical statement determines and is exclusively determined by its *use*"; beginning from this assumption, it is plausible to hold that any difference between two individuals in their understanding of mathematical symbolism would have to be manifest in observable differences of behavior or capacities. The second turns on considerations about learning, and in particular on the thought that what it is to learn mathematical reasoning is to learn how to *use* mathematical statements (i.e. when they are established, how to carry out procedures with respect to them, how to apply them in non-mathematical contexts, etc.).[29] On either assumption, it is then reasonable, Dummett suggests, to hold that *since* meaning is exhausted by use (in one way or the other) we cannot claim that a notion of truth,

[26] Dummett (1963), p. 146.
[27] Dummett (1973).
[28] Dummett (1973), p. 215.
[29] Dummett (1973), pp. 216-217.

understood classically as imposing bivalence on all mathematical statements independently of the use we actually make of them, can any longer serve as the “central notion” for a characterization of the meanings of mathematical statements. In place of the classical notion of truth, Dummett suggests, we must substitute a notion grounded in the practices of which we have actually gained a mastery; in particular, we must replace the classical notion of truth with the claim that “a grasp of the meaning of a statement consists in a capacity to recognize a proof of it when one is presented to us.”[30] (p. 225). This, in turn, allows the recognition that certain classical arguments and proof-procedures are unjustified from this perspective, and should accordingly be replaced with intuitionistic ones.

Dummett thus presents the best route to the adoption of intuitionistic logic in mathematics as motivated by considerations very different from those that motivated arguments to the same conclusions for classical intuitionist thinkers such as Brouwer and Heyting; in particular, as Dummett points out, whereas intuitionism was motivated for those thinkers primarily by the requirement that mathematical objects be present or given in subjective, private experience, Dummett’s arguments turn on what is in some ways the exactly opposite idea, namely that of the mastery of a socially learned and publically evident intersubjective *practice*. In fact, Dummett suggests against the views of the early intuitionists, there is *no* plausible route from the view that mathematical entities such as natural numbers are “creations of human thought” to the application of intuitionistic logic, unless we are prepared to adopt a very severely restricted (and implausible) view of mathematical practice (including rejecting unbounded quantification over all numbers, etc.)[31] For this reason, Dummett suggests as well that there is no good reason to think that any successful argument for anti-realism in mathematics can turn on considerations bearing simply on the supposed ontological peculiarities of the mathematical domain; both of the reasonable arguments that one might make turn, instead, on considerations about the link between meaning and use which have nothing special to do with mathematics and would seem to be applicable much more broadly, to any number of classes of sentences “about” widely differing kinds of things.

Thus, according to Dummett, it is ultimately general issues about the capacities or practices that we learn in learning a language and deploy in speaking one that determine, given his framework, equally general issues about whether realism or anti-realism is better justified in any given domain. Nevertheless, although this kind of consideration finds application quite generally, it is certainly no accident from the present perspective that the historical dispute which forms the basic model for Dummett’s formal framework itself is, in fact, the dispute between formalists and intuitionists about the foundations of mathematics in the 1920s and early 1930s. This dispute was, of course, both prompted and driven by the question of the role of the infinite in mathematical reasoning, particularly in the form of the mathematical *theory* of sets as infinite and transfinite wholes that had recently been given by Cantor. One of the most central issues of dispute, in particular, was that between Hilbert’s formalist picture which allows, in accordance with classical logic, the admissibility of reasoning about arbitrary infinite quantities as long as such reasoning is (demonstrably) consistent within a specific axiom system, and the various versions of intuitionism, which place more limiting strictures on infinitary reasoning,

[30] Dummett (1973), p. 225.
[31] Dummett (1973), p. 247.

including disallowing the general application of the law of the excluded middle beyond those cases in which it is actually possible to construct a particular mathematical entity or verify it in experience. Partisans of the two positions thus reach deeply opposed conclusions about the nature of reasoning about the infinite, but for both positions the idea of a *finite (i.e., finitely specifiable) procedure* or process of demonstration plays an absolutely central role. In particular, whereas the formalist positiion allows the axioms and rules of a formal system to extended classically, by means of such a procedure, to arbitrarily extended reasoning about the infinite *provided* that the system can be shown to be consistent, intutionism generally restricts the positive results of mathematics about the infinite to what can be shown by means of a finite, constructivist procedure of proof.

By posing the issue of realism vs. anti-realism, not only in the mathematical case but more generally, as turning on the question of the provision of sense in accordance with the learning and practice of such a procedure, Dummett shows that the question of realism in a particular domain is most intimately related, not to the question of the ontological status of, or our epistemological access to, its objects, but rather to the question of the coherence and range of the procedures by means of which the meanings of statements about the domain are learned and manifested. But this is none other than, again, the question of the way that the infinite becomes available on the basis of a finite procedure. For the intuitionist (and by analogy, the anti-realist more generally), it is possible to establish the existence of an object only if it can be shown to result from its actual construction in a finite number of steps or from a finite, constructivist proof (i.e. one that does not involve reasoning over arbitrarily complex infinite totalities); by contrast, for the formalist (realist), all that is needed is to show that it is possible to refer to the object without contradiction within a specified formal system.

Though Dummett's framework is itself not meant, by itself, to decide this issue between the anti-realist and the realist, at several points in his consideration of the possible motivations of the anti-realist argument for adopting intuitionist logic in mathematics, Dummett appeals to what he takes to be the "Wittgensteinian" view that the meaning of a class of statements must be exhausted by their intersubjectively verifiable and learnable "use". It is this identification of meaning as use (in this sense), in particular, that is supposed to make the requisite link between processes of verification (i.e. calculation and proof) and anything that we could reasonably take to be a way of supplying *meaning* for the statements in question. As I have argued elsewhere, it is not actually the late Wittgenstein's view that meaning is *identical* to use or necessarily to be reduced to it in all cases.[32] But even on a view that does hold this, there is a further step that has to be taken in order for the anti-realist view to follow. In order to draw the conclusion that bivalence does not generally hold, it is necessary for the anti-realist to argue *not only* that meaning cannot go beyond use, but *also* that "use" in the relevant sense does not go beyond what is involved in finitary and constructivist procedures.

It is just here, with regard to the specific question of what is involved in the learning and pursuit of a finite procedure, that the possibility of meta-formal reflection of the sort that I have portrayed Gödel as engaging in proves to be decisive. For Gödel's own incompleteness theorems, of course, result directly from a rigorous meta-formal consideration of the range and capacities of formal systems (in Hilbert's

[32] Livingston (2004).

sense and related ones). In particular, Gödel's first theorem shows that for any such system, there will be a number-theoretical sentence that is beyond its capacity to prove or refute, and the second theorem shows that no such system can prove its own consistency (assuming that it *is* consistent). In this way Gödel's results render the formalist conception of finite procedures unsuitable for anyone who wishes to assert the realist position that the statements of number theory have determinate truth-values, quite independently of our ways of verifying them; but on the other hand, as I have argued, in invoking under the heading of the "inexhaustibility" of mathematics an essential reference to a reality that marks the point of impasse of any given finite procedure (whether this be a reality of further mathematical truths, platonistically construed, or only of unsolvable problems), Gödel's argument shows the intuitionist strictures to be untenable as well.

In this way, just as Gödel's theorems themselves might be thought to overcome the debate between intuitionism and formalism, narrowly construed, by conceptually fixing and reflecting upon the contours of a central concept (that of a finite procedure) commonly appealed to by both, the meta-formal realism I have discussed as suggested by Gödel's argument thus provides a new basis for critically interrogating the central concept of a *rule of use*, as it figures in both "realist" and "anti-realist" conceptions of the structure of language. In particular, as I argued in more detail in *The Politics of Logic*, we may take considerations analogous in some ways to those which establish Gödel's second incompleteness theorem, in particular, to demonstrate, incapacity of a finitely specifiable system of such rules to establish its *own* consistency. It is then apparently possible to draw, with respect to our actual practices and institutions of linguistic use, a conclusion directly analogous to that drawn by Gödel with respect to mathematical reasoning specifically: namely that *either* the consistency of our regular practices can only be known, and assured, by a deliverance of an essentially irregular insight that essentially cannot be subsumed within them or determined by them insofar as they can be captured by rules; *or* it cannot be known at all and thus can *only* be treated as a perpetually deferred problem. On either assumption, the claim of consistency is shown to be, from the perspective of the regular provision of sense, the point of an impossible-Real that always escapes, drawing along with it any possibility of an internal systematic confirmation of the infinite noncontradictory extensibility of the rule to ever-new cases. In relation to this, the metaformal realism I have discussed does not consist simply in affirming this consistency; nor, like Dummett's framework, does it by itself make a decision as to the attitude we should take toward the scope of rules of use or their actual relation to processes of verification in any particular case.

But as I argued in the book, its effect is, among other things, to expose the *assumption* of the consistency and extensibility of rules to immanent and meta-formal critique; indeed, as I argued in the book, we might see a major strand of the late Wittgenstein's critical "rule-following considerations" as doing just this. It is always possible, as well, to take the other horn of Gödel's disjunct, assuming rather that it *is* possible to have access to the truth of consistency by means essentially outstripping those of any regular system. But in either case, what is made possible by the metaformal reflection is a rigorous deconstruction and displacement of the concept of the effective rule as it figures in the idea of the human subject of knowledge as the operator of effective, finite procedures; it shows, in other words, that either there is something in knowledge that forever escapes this idea of effectivity *or* that to secure its coherence would require something that it itself cannot provide. It is in this way, as I have argued, that the phenomenon that Gödel calls the "inexhaustibility of mathematics" points toward a metaformally justified realism of the impossible-Real correlative to what we may describe as our essential openness toward the infinite and based in rigorous metaformal reflection about the limits and transit of forms. In so doing, it radically unhinges any possible claim of the humanistically conceived "finite" subject finally to ground itself, or to secure by its own means the ultimate sense of its language and life.

III

For the thinkers and positions that have characterized themselves, over the last few years, as "speculative realist", the work of Quentin Meillassoux has been seen as both an inspiration and a leading example (though he himself does not apply this term directly to his own work, preferring to call it instead "speculative materialist").[33] In particular, in *After Finitude*, Meillassoux develops a bold original position on the basis of his critique of what he calls "correlationism", arguing for the renewed possibility (against what he supposes to be Kantian and other critical forms of 'correlationism') of a realist attitude toward the *mathematically* describable properties and relations of the objects of natural science.[34] There is much in Meillassoux's argumentation that I am not certain I have been able to understand, and it seems to me that there are significant gaps and failures in Meillassoux's argumentation at several important points. But since his argument, like the present one, seeks to establish a form of realism on the basis of considerations about mathematics and in particular about the significance of the infinite therein, I believe it might be helpful to consider briefly the similarities and differences between Meillassoux's argument – and the position it yields – and the meta-formal realism I have argued for here. In particular, since Meillassoux's argument is predicated upon excluding, under the heading of "correlationism," any fundamentally *critical* position with respect to the relationship of thought and being, I believe it may be helpful to show that, as I argued in *The Politics of Logic*, such a position is not only possible in today's "post-Cantorian" context but even positively motivated internally by the very availability of the infinite to which Meillassoux decisively appeals. But to see this it is essential to consider, as well, the most important results of twentieth-century philosophical inquiry into language and the nature of linguistic signification; in the following, therefore, I shall briefly discuss not only *After Finitude*, but also a more recent article by Meillassoux entitled "Iteration, Reiteration, Repetition: A Speculative Analysis of the Meaningless Sign," which develops and supplements some of the considerations in his book, extending them to involve an analysis of the nature and structure of signs.

Much of the influence of Meillassoux's work has derived from the force of his critique of what he calls "correlationism," and it is primarily on the basis of this critique of what he holds to be "the central notion of modern philosophy since Kant" that he is able to present his own position as substantially new in this history.[35] What, then, is correlationism? It is, Meillassoux says in the opening pages of *After*

[33] See, e.g., Bryant et al. (2011), pp. 3-4.
[34] Meillassoux (2006).
[35] Meillassoux (2006), p. 5.

Finitude, the position that holds that "we only ever have access to the correlation between thinking and being, and never to either term considered apart from the other" and furthermore that the "correlation so defined" is "unsurpassable".[36] Though Meillassoux does not specify the kind of "correlation" figuring in this position as any one type of relation, he suggests that "the subject-object correlation," "the noetico-noematic correlation," and the "language-referent correlation" may *all* be treated as examples of the kind of relation with which he is critically concerned.[37] Again, a few pages later, Meillassoux suggests that the kind of relation between "being and man" specified, at a great explicit distance from any "subject-object" picture, by the late Heidegger as their "co-propriation" and as "Ereignis", may *also* be treated as an instance of "correlationism" in the relevant sense.[38] To all of these varieties of correlationism, Meillassoux raises a single objection, that of what he calls the "ancestral." In particular, correlationism in any of its forms, he suggests, cannot account for the existence of a "reality anterior to the emergence of the human species;" Meillassoux calls any material evidence for the existence or obtaining of such a reality an "arche-fossil".[39] According to Meillassoux, the correlationist cannot account for the arche-fossil because, in considering it, he must insist upon a "retrojection of the past on the basis of the present" whereby "it is necessary to proceed from the present to the past, following a logical order, rather than from the past to the present, following a chronological order;" this "enjoins [him] to subordinate the apparent sense of the ancestral statement to *a more profound counter-sense*" according to which "it is not ancestrality which precedes givenness, but that which is given in the present which retrojects a *seemingly* ancestral past."[40] To this apparent doubling of meaning in the correlationist's treatment of the arche-fossil, Meillassoux opposes the maxim of what he calls an "irremediable realism": that an ancestral statement "either …has a realist sense, and *only* a realist sense, or it has no sense at all."[41] Such a conclusion, Meillassoux suggests, is in fact the only possible one to reach if we are indeed to affirm that such statements are not *completely* "illusory".

Meillassoux's argument against correlationism has been aptly criticized elsewhere for the apparently "straw" character of the figure of the "correlationist" which it invokes; for example, as Hallward points out, "almost no-one…apart except from a few fossilized idealists" actually accepts the argument presented by Meillassoux as the correlationist's for the literal falsehood of claims about the ancestral past, and even as characteristic an idealist as Husserl, as Hallward correctly remarks, only considers claims about the "correlation" of thought or consciousness and objects within an attitude of *bracketing* claims about their existence (rather than attempting to explain or derive them).[42] This and similar considerations about what is involved in actual idealist positions, including those of Kant himself, may lead us to conclude, along with Hallward, that Meillassoux has, in constructing his critique of correlationism, essentially committed an equivocation of epistemological considerations with ontological ones, mistaking (reasonable) arguments about the conditions for our *knowledge* of the ancestral for (unreasonable) ones about its ontology or existence. On the other hand, Meillassoux at

[36] Meillassoux (2006), p. 5.
[37] Meillassoux (2006), p. 6.
[38] Meillassoux (2006), p. 8.
[39] Meillassoux (2006), p. 10.
[40] Meillassoux (2006), p. 16.
[41] Meillassoux (2006), p. 17.
[42] Hallward (2011), p. 137.

least sometimes suggests that what is decisive for the correlationist position as he is portraying it is an order of precedence that is not either epistemological or ontological, but rather logical or semantic: thus, for instance, in describing the temporal "retrojection" that the correlationist must perform, he describes it as substituting a "logical" for a "chronological" order, and at least at one point he specifies the problem which the correlationist must answer as the problem of the possibility of the *meaningfulness* of scientific statements about the past.[43] `

If we take this last suggestion seriously, it might be possible to see the main concern of Meillassoux's argument as turning not on the ontological issue of the existence of objects, or the epistemological one of the conditions for our knowledge of them, but rather on the question of the basis of the provision of sense for sentences about the ancestral past. In this way, Meillassoux could be construed as avoiding the equivocation between epistemology and ontology of which Hallward accuses him; and if construed this way, Meillassoux's argument would approach more closely both Dummett's framework for discussion of realism and anti-realism and the position of meta-formal realism I am recommending here. For as we have seen, both of the latter are best understood as turning most decisively, not on ontological issues or epistemological ones, but on questions about the nature and provision of sense. Nevertheless, even following this suggestion, it is not at all evident how to interpret Meillassoux's "correlationist" as an anti-realist in Dummett's sense. For example, though Dummett has discussed within his framework the question of the reality of the past, even the anti-realist position has reason to reject the application of bivalence *only to* statements about the past *for whose truth or falsity there is presently no available conclusive evidence*; for this sort of anti-realist, there is no problem at all in admitting the straightforward truth or falsity of sentences of the sort that Meillassoux considers (for instance statements about the age of the earth established on the evidentiary basis of radio-carbon dating). Similarly, it is not at all clear how to think about the issue of "anteriority" that forms the linchpin of Meillassoux's argument against the "correlationist" within Dummett's framework or the meta-formal one; in particular, if the underlying issue is indeed that of the possibility of a "logical" order of anteriority on the basis of which the position opposed to realism (whether it be called "correlationism" or anti-realism or whatever) seeks to establish logical conditions for the sense or meaningfulness of a class of statements, it is not clear why this "anteriority" should pose any deeper problem than that posed by the "anteriority" of premises to a conclusion in a rational argument, or of a smaller number to a larger one in the sequence of natural numbers. In particular, the claim that a premise or item is "anterior" to another in this kind of sense is in no evident competition with any directly chronological sense of anteriority; but such a conflict is needed if Meillassoux's argument is to pose any problem for the relevantly "anti-realist" position.

At any rate, whether Meillassoux intends this kind of consideration about the provision of sense to be the primary strand in the argument, or whether he, again, simply equivocates among epistemological, ontological and logical/semantic conclusions, the least that can be said is that his identification of the correlation problem as it applies, on the one hand, to classical forms of idealism specified in terms of the

[43] Meillassoux (2006), p. 9. In his critical response to Hallward's critique of Meillassoux, Nathan Brown (2011, p. 143) makes this a central point of contention, charging that Hallward "…ignores Meillassoux's critique of logical retrojection altogether."

subject/object distinction, and on the other to contemporary positions (both "continental" and "analytic") within the scope of the "linguistic turn" is much too quick. In particular, while it may be that it is possible to direct some of the question which are, for Meillassoux, embarassments for the "correlationist" to pre-20th century (typically idealist) positions that presuppose such a distinction as one between two spheres or realms of existence, and thus raise the problem of the relations between them, the effect of the interrelated set of conceptions of "logical form", propositional structure, and the structure of language that characterize 20th century philosophy in the linguistic mode is to displace these questions from that setting and render largely innocuous the specific problems that Meillassoux raises about chronological precedence. This is not to exclude that there may be additional problems about the ontology or epistemology of logic or linguistic structure which would have to be dealt with by anyone who takes the implications of these types of formal structure seriously; only that these problems are not the same as the ones that Meillassoux raises and that they do not fit well with the sketch of the "correlationist" and his realist opponent that Meillassoux provides. From this perspective, then, Meillassoux's position (at least insofar as it is determined by the "disqualification" of correlationism) is not only (as he happily admits) pre-Kantian, but in a certain (and this time unacknowledged) sense importantly pre-Fregean and pre-Saussurian as well. In particular, by failing to take into consideration the implications for the proper form of the realist/anti-realist debate of the massive project of the formalization of language begun, in different contexts, by these respective "forefathers" of the analytic and structuralist traditions, Meillassoux effectively ignores the relevance to this ongoing debate of some of the deepest philosophical results and most useful methods of our time.

Nevertheless, at other moments in Meillassoux's argument, he appeals directly and decisively to what may seem to be the direct implications of mathematical formalism, and specifically to the implications of the availability of the infinite and transfinite to mathematical thought. In particular, after "disqualifying" the correlationist position on which objects (or our knowledge of them or perhaps their sense) are essentially conditioned by *finite* forms of human thought, Meillassoux appeals, following Badiou, to Cantor's discovery of the transfinite hierarchy to motivate an anti-"frequentialist" position according to which it is no longer possible to hold natural or physical laws to be (even in a relative sense) necessary and, accordingly, that Hume's problem about the necessity of causal and other laws must be re-instated against Kantian and other critical attempts to resolve it.[44] Meillassoux's basic argument for this conclusion is that since all reasoning about probabilities "presupposes the notion of [a] numerical toality" of possibilities, Cantor's demonstration of the essentially open and non-totalizable hierarchy of infinite sets, if taken as applicable to the question of the conceivability of a total space of possibility, can "provide us with the resources for thinking that the possible is untotalizable" and hence for at least questioning the "necessitarian" assumption that reasoning about the relative probability of laws and events must be possible.[45]

Drawing as it does upon the implications of Cantor's hierarchy of transfinite sets, this argument resembles in some ways the appeal made to formal structures of the infinite in motivating what I have called meta-formal realism. However, there are several problems with the appeal as Meillassoux makes

[44] Meillassoux (2006), pp. 100-108.
[45] Meillassoux (2006), p. 105.

it. To begin with, there is in fact no evident direct way to connect Cantor's open hierarchy of the transfinite with *any* kind of reasoning about probabilities and necessity. As Meillassoux in fact recognizes, it is perfectly possible to determine relative probabilities over domains that admit of infinite or even uncountably infinite ranges of possible values; thus, even if Cantor's results are taken to show that there may be infinitely or even uncountably many "possible worlds," this by itself has no tendency to show that probability measures over the totality of them are not well-defined.[46] Meillassoux's attempt to show that the "possible is untotalizable" (or at least that we can think so) apparently rests, instead, on the thought that the possible should be identified, not with any particular finite or infinite quantity, but with the "universe" of *all* of them, and it is true that, on Cantor's own assumptions at least, this "universe" cannot exist as a totality, and thus that, if we can understand the space of possibilities as equivalent to this universe, it is indeed untotalizable. But the requisite link between probability and the universe of *all* sets and quantities is obscure, and Meillassoux does not clarify how we are to understand it. All he says, in fact, is that "…although we have not positively demonstrated that the possible is untotalizable, we have identified an alternative between two options – viz., the possible either does or does not constitute a totality – with regard to which we have every reason to opt for the second…" (p. 107). But it cannot be said that the untotalizability of Cantor's hierarchy provides an alternative to a totalizable (or total) possibility space unless we know how to identify the space of possibilities with *all* of Cantor's hierarchy, and Meillassoux has given us no suggestion as to how to do so; indeed, if we do actually take the "universe" of sets to be untotalizable, this identification (since it calls for identifying *all* of the possibility space with *all* of the "universe", which is exactly what does not, on this telling, exist as a whole) is in fact not only unmotivated but in a certain sense impossible.

Meillassoux's appeal to Cantor thus does not give any positive reason or even any positive motivation for believing that the probability of events or the necessity of laws cannot be well-defined relative to the (possibly infinite or transfinite) domains in which they take place, since there is no evident link between the untotalizability of the Cantorian universe and any relevant considerations about probability at all. However, there is a second, deeper issue here which would additionally problematize Meillassoux's strategy of drawing conclusions from the structure of Cantor's hierarchy of transfinite sets, even if the connection between it and probability could be independently motivated. For as I argued in *The Politics of Logic*, the availability of the transfinite to thought does not in fact demand the conclusion that Meillassoux follows his teacher, Badiou, in drawing: that of the in-existence of the All of the universe, or the untotalizability of the universe of sets and situations.[47] Rather – and this is the key to what I describe there as *another* possible post-Cantorian orientation of thought, distinct from and formally opposite to Badiou's own "generic" orientation – it forces a decision on the level of totality and its thinkability; the decision is the one between, on the one hand, the combination of consistency with incompleteness (the alternative Badiou takes and in which Meillassoux apparently follows him) and, on the other, the combination of completeness (or totality) with inconsistency. That is, the implication of Cantor's transfinite and the formal paradoxes and aporias associated with it is not simply to demonstrate or show the incompleteness or inexistence of the Whole, but rather to force the

[46] Meillassoux (2006), p. 102.
[47] See especially chapters 1 and 9.

metalogical decision between the *two* orientations of the generic and the paradoxico-critical, the two orientations that correspond directly, as I have argued above, to the two alternatives of Gödel's disjunctive conclusion.

If Meillassoux had taken the paradoxico-critical alternative, or even considered it seriously as a possibility for thought, he could by no means have drawn the conclusions that he does about the "necessity of contingency" and the consequent need to assume, outside the "correlationist circle", the absolute existence of an ultimate power of "chaos" by means of which "nothing is or would seem to be, impossible."[48] Rather, on the paradoxico-critical side, he would have had to be driven to consider the inherent and structural aporias involved in conceiving of a force of laws and rules that is, within its own sphere, always certainly capable of being complete, but nevertheless always constitutively imbricated with the paradoxes of its own foundation and recurrently involved in the quixotic attempt to prohibit or foreclose its own inherent point of contradiction. On this kind of position, there is no special problem with the coherence of judgments of relative probability or probabilistic causal laws, so long as the *general* structure of the law as such, as a *consistent* repetition of the same, can be uncritically assumed; but this structure itself always rests on the ultimately aporeatic foundation of a consistency that can never be ultimately guaranteed. Pursuing or even considering this other option does not have the effect of reinstating the unproblematic availability of the (actually pre-Cantorian) Kantian orientation, but it does make available a reflective and *critical* position from which it is possible to consider the force of laws (including natural laws) and the ultimately paradoxical support of their "unlimited" possibility of application. Since the key point here is not the fixation or absolutization of an unlimited principle of contingency according to which "nothing is…impossible" but rather the acknowledgment of the structurally constitutive possibility of real inconsistency which corresponds to the ultimate unavailability (in accordance with Gödel's second incompleteness theorem) of any intra-systematic guarantees of consistency, this provides another, more critical and less "absolutist," way of considering the nature of scientific (and other laws) and their determination as necessary, one which removes none of the critical force of Hume's problem, but rather situates it within a more radical interrogation of the ultimate basis of the rationally thinkable force of laws as such.

Like Badiou, Meillassoux does not generally recognize the possibility of an alternative, post-Cantorian orientation in thought resulting from a decision to preserve completeness along with the recognition that consistency cannot be guaranteed, rather than the "generic" orientation of consistency and incompleteness.[49] Nevertheless, Meillassoux appears to recognize that his own conclusions about the bearing of the infinite on the question of chance and law will only be possible if an interpretation of the infinite in terms of inconsistency is first disqualified; thus he argues that, if we are to accept his

[48] Meillassoux (2006), p. 64.

[49] This is not to say that, in terms of the fourfold schema of orientations of thought that I developed in *The Politics of Logic*, Meillassoux simply replicates Badiou's "generic" orientation; rather, Meillassoux's position appears to mirror the generic one in certain respects (e.g. its insistence upon incompleteness) while at other points, particularly with its 'absolutization' of the principle of 'hyper-Chaos', falling back into the essentially pre-Cantorian position of onto-theology. For some trenchant critical remarks on the implications of this absolutization of a new principle of the divine and its problematically ontotheological structure, see Adrian Johnston's argument in Johnston (2011).

overarching principle of the "necessity of contingency" we must also hold that "the principle of non-contradiction is absolutely true."[50]

Meillassoux's argument for this is not easy to follow. It begins by considering the relationship between an affirmation of the "sheer becoming of all things", understood as a "sovereign flux," and the assertion of the "reality of contradiction"; such an affirmation of flux and becoming should not in fact, Meillassoux avers, be identified with that of the existence of any "contradictory entity" because the latter would in fact be an "utterly Immutable instance against which even the omnipotence of contingency would come to grief."[51] In particular, such an entity would:

> …prove incapable of undergoing any sort of actual becoming – it could never become other than it is, since it already *is* this other. As contradictory, this entity is always-already whatever it is not. Thus, the introduction of a contradictory entity into being would result in the implosion of the very idea of determination – of being such and such, of being this rather than that. Such an entity would be tantamount to a 'black hole of differences', into which all alterity would be irremediably swallowed up, since the being-other of this entity would be obliged, simply by virtue of being other than it, *not* to be other than it.[52]

The reasoning here appears to be (at least roughly) that if an entity were possible which is "contradictory" in Meillassoux's sense, it would in some way simultaneously bear *all* properties, including both existence and non-existence, and so could not ever take on any property that it initially lacks. However, even if it is logically coherent to entertain the possibility of such an entity (it is not clear that it is), this possibility has little to do with any possibility that is relevant to establishing the necessity of the law of non-contradiction. In particular, even if Meillassoux's argument does establish the impossibility of a "contradictory entity" in this sense, this has *no* tendency to show that it is not possible for there to be any *true contradictions*, i.e. *true sentences* of the form P & ~P. The truth of such a sentence, though sufficient to establish that the law of noncontradiction does not hold in general, does not in any way establish or even invite the existence of a "contradictory" entity in Meillassoux's sense (the sense in which such an entity apparently bears *all* properties). But the *possible* truth of *some* such sentences (not the claim that all truths are contradictory or that contradictions are ubiquitous) is all that the paradoxico-critical orientation must countenance in order to accommodate the implications of the infinite within the assumption of a fundamental completeness of thought in relation to the totality of what it can consider.

Apparently realizing that the argument against the possibility of a "contradictory entity" (in his sense) by itself does not succeed in establishing the necessity of his absolutist position with respect to noncontradiction, Meillassoux in fact considers, a few pages later, the position formalized by paraconsistent logics, according to which "it is perfectly possible for a logical system to *be* contradictory without thereby being inconsistent…"[53] Such a position could be developed, Meillassoux admits, to

[50] Meillassoux (2006), p. 71.
[51] Meillassoux (2006), p. 69.
[52] Meillassoux (2006), pp. 69-70.
[53] Meillassoux (2006), p. 77. For paraconsistency, see, e.g., Priest (1987).

maintain, first, that contradiction is not, after all, "unthinkable" and, second, that a world containing a particular contradiction would not thereby, by any means, have to be a necessary world. Meillassoux says that both objections "provide an opportunity to flesh…out" his speculative position, but he does not in fact do so; rather, he says only that in order to answer the first point "we would need to correct our thesis by reformulating it" to "verify that the reasoning through which we arrived at the impossibility of contradiction also manages to establish the claim 'nothing can be *inconsistent* because nothing can *be* necessary'" and that, with respect to the second point, "…We would …have to try to demonstrate that dialectics and paraconsistent logics are only ever dealing with contradictions inherent in *statements* about the world, never with *real* contradictions in the world…" But Meillassoux does not provide even so much as a sketch of either of these suggested demonstrations; rather, he simply abruptly ends the discussion a few lines later.[54]

Meillassoux does not, then, seem to have done anything to establish the impossibility or incoherence of an alternative interpretation of the availability of Cantor's hierarchy to mathematical thought, one which would emphasize the limit-paradoxes and contradictions involved in it rather than its incompleteness or non-totalizability. Moreover, even the very limited considerations he does give in this direction are *subsequent* to his argument for the "necessity of contingency" and so can seemingly do nothing to support that argument. Accordingly, he does not appear to succeed in motivating the claim that it is necessary to interpret Cantor's hierarchy in terms of any kind of essential incompleteness or non-totalizability; and the further claim that it can be treated as motivating any claims about *probabilities* or *contingency* at all is, as we have seen, also problematic. But since Cantor's transfinite hierarchy is the *only* actual mathematical or formal structure to which Meillassoux appeals anywhere in his argument, all of this tends to suggest that Meillassoux has not really provided any convincing argument for the central claim of the book, namely that it is possible to draw from a consideration of the implications of the infinite some motivation for adopting a realist position with respect to the quantifiable "primary" properties of natural objects. Nevertheless, since the position that I am here calling "metaformal realism" in fact concurs, in broad terms at least, with the claim that considerations about the mathematical infinite *can* be appealed to in successfully motivating a (somewhat different kind of) realist position, it is worth examining in some more detail how this kind of argumentation succeeds or fails.

In the recent article, "Iteration, Reiteration, Repetition: A Speculative Analysis of the Meaningless Sign," after rehearsing the argument of *After Finitude*, Meillassoux develops in more detail the basis of his claim for the capability of "mathematics and mathematized physics" to give us "the means to identify the properties of a world that is radically independent of thought."[55] His concern, in particular, is to support a realist position with respect to the "modern sciences of nature" which are "characterized," he says, by mathematization. In order to do so, Meillassoux considers what he calls a "minimal condition" of formal (logical and mathematical) languages. In particular, our ability to "think a meaningless sign" of the sort employed in such languages is, Meillassoux holds, an essential condition for the possibility of understanding laws and regularities of nature, when mathematizable, as capable of holding in a way

[54] "We will not pursue this investigation any further here…" (p. 79)
[55] Meillassoux (2012), p. 18.

that is "radically independent of thought."[56] Thus, according to Meillassoux, the structure of the "meaningless sign" provides a "specific criterion of logicity and of mathematics" as opposed to natural languages, capable of underlying the claim that it is possible through a formal language in this sense – though not through natural language – to make reference to (or gain access to) a reality independent of thought. He finds this structure represented in "formal languages" in the sense of Hilbert's formalist project, developing specifically the example of ZF set theory as such a system.[57] Particularly important to Meillassoux's development of the theme of the potential "meaninglessness" of signs in such formal systems is their use of the device of "implicit definition", whereby signs are defined contextually by their roles in stipulated axioms, without any necessity for them first to be coordinated to determinate referents external to the system.[58] This difference, according to Meillassoux, yields a "criterion" for the difference between natural and formal languages:

> From what we have said so far, we can draw a precise principle of distinction between a natural language and a formal language: for we can decide *to differentiate them according to the role that meaningless signs play within them*. We shall therefore say that a formal language, unlike a natural language, accords a structural role to the meaningless sign – at least on a *syntactical* level. For alphabetical natural languages do indeed make use of letters and syllables that, in themselves, have no meaning – but they do so on the morphological level of the constitution of words, and not at the syntactic level of the constitution of phrases. On the syntactical level, a natural language can certainly also use meaningless words – for example Mallarme's 'ptyx', if we agree that this word means nothing – but there is no rule that *imposes* this type of word upon natural languages. Their propensity is, on the contrary, to avoid them so as to fulfill their ordinary function of communication. Within a natural language, at the level of syntax, the meaningless sign plays a contingent (and, in general, marginal) role; whereas in a formal language, at the same syntactical level, it plays an essential, structural role.[59]

Based on the distinction thereby draws, Meillassoux undertakes to give an "ontology of the empty sign" which is to provide an account of the possibility of thinking a meaningless sign in this sense.[60] It is essential to such a sign, Meillassoux suggests, that one draw a type/token distinction between its instances and its general type, and furthermore that such a sign is one whose function "can be fulfilled by *any sensible mark*." For Meillassoux, the identity (as type) of a sign that is "meaningless" in his

[56] Meillassoux (2012), p. 18.
[57] Meillassoux (2012, p. 20); he also mentions category theory, in passing.
[58] Meillassoux sketches the difference between this method of definition, pioneered by Hilbert in his axiomatization of geometry, and the older method of explicit definition exhibited by Euclid; however, somewhat misleadingly, he characterizes Hilbert's method as one which "does not begin with any initial definition." (p. 21). But this is accurate only if we restrict the meaning of "definition" to the older, constructive sense; if we consider the signs to be implicitly defined, then it is exactly such an "initial definition" that the axioms in fact yield. There are other minor infelicities in Meillassoux's presentation of the vocabulary and logical language of formal systems; for example, signs for the logical truth-functions are described by him as designating "operations that will be able to be carried out on the base-signs" (p. 21) and the set-theoretical axiom of extensionality is characterized as "a rule of substitution of one set for another" (this appears to be, rather, a description of the axiom of replacement).
[59] Meillassoux (2012), pp. 22-23.
[60] Meillassoux (2012), p. 24.

privileged sense depends on the structural feature of *iteration*: this he defines as “a recurrence that is non-differential and therefore unlimited because it produces a pure identity of marks.”[61] That is, in recognizing a “meaningless” sign in his sense, it is essential that “I end up being able to recognize as perfectly identical in type, occurrences of marks that I can *for this reason alone* think as iterable at will.”[62] This structural “iterability” is, in turn, Meillassoux says, the basis for the possibility of a structure of “reiteration” by which the iteration of the same “operator-sign”, such as ‘+’, allows for the production of a “differential” effect, for instance when the iteration of the operation of adding one allows for the “differentiated series:” 1, 2, 3, 4…(etc.); reiteration thereby makes possible mathematical reasoning, according to Meillassoux, and gives rise to the idea of the ‘potential infinite’ in mathematics.[63] (p. 34). In the final pages of the essay, Meillassoux seeks to connect the iteration and the reiteration of the “meaningless sign”, thus defined, back to the “necessity of contingency” for which he argued independently in After Finitude, holding here that “*it is because I can intuit in every entity its eternal contingency, that I can intuit a meaningless sign*.”[64] In particular, according to Meillassoux, the iterability of the (“meaningless”) sign is rooted in a prior iterability of the “contingency of a thing” which is “in itself always identical” and in a “speculative contingency” or “arbitrariness” that shows it to be “eternally the same, since contingency is eternal.”[65] In this way, the structural iterability of the “meaningless” sign and the regular operative reiterability it makes possible are claimed to be dependent upon a conception of a “pure gratuitousness” resting in the recognition that “anything whatsoever” could fulfill the function of the sign “just as well as it does.” This iterability is thus said to be rooted in our access to a “pure semiotic world – where nothing has a reason to be, where nothing has meaning – and where everything, in consequence, breathes eternity.”[66]

As Meillassoux in fact admits in the last lines of the article, his consideration of the structure of the “meaningless” sign in fact has done little or nothing to support the possibility of a realist conception of the world (or perhaps what is mathematizable within it) as “independent of thought.”[67] For it has not solved or really even addressed the question of how a sign that is “meaningless” in Meillassoux’s sense gains application to such a reality. Moreover, it is not even clear how this connection *could* be motivated, since there is no scientific theory of the world that is axiomatic in the sense of Hilbert’s formal systems, and the aspects of “mathematization” that do occur in particular empirical theories are, of course, heavily dependent on interpretation in terms of particular properties and (ultimately

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[61] Meillassoux (2012), p. 32.
[62] Meillassoux (2012), p. 33.
[63] Meillassoux (2012), p. 34.
[64] Meillassoux (2012), p. 35.
[65] Meillassoux (2012), pp. 36-37.
[66] Meillassoux (2012), p. 37. I should say that I am not certain I understand the argument Meillassoux is offering in this section; in particular, like other arguments given by Meillassoux for the “necessity of contingency” it appears to be circular in essentially assuming what it is attempting to prove. Moreover, it is not clear why we must understand the “contingency” involved in the fact that any lexicographic sign could serve the same role as any other as having anything to do with the kind of contingency Meillassoux wishes to assert of events in the actual, mind-independent universe.
[67] “…we have not at all shown that the empty sign allows…the description of a world independent of thought…The new puzzle that appears before us is the following: how can a meaningless sign allow us to describe the world, without becoming once again a meaning*ful* sign, and thereby capable of referring to a world outside of it?” (p. 37)

observable) phenomena. Similarly, even if it were possible to motivate, by means of this kind of argument, a general claim of realism for the *mathematized* parts of empirical theory or their objects, this would leave aside most of our actual scientific understanding of the world. For, of course, only some highly restricted portions of physics and perhaps physical chemistry can really be considered to be "mathematized" in this sense; vast tracts of our empirical understanding of the world (comprising, e.g., most of biology, psychology, and anthropology among others) would have to be left aside by this approach. Thus, the links between Meillassoux's claims here about the structure of signs and formalisms and *any* realism worth holding (including the empiricist kind he purports to defend or "absolutely" found)[68] are tenuous or nonexistent. Nevertheless, in the present context it is worth considering these claims on their own, since they resemble in some respects the kind of appeal to formalism that, I have argued, is at the basis of meta-formal realism, whether of the generic or paradoxico-critical variety. If, in particular, the link suggested here between the specific features of formal systems such as Hilbert's system of geometry and some compelling motivation for a variety of realism can indeed be forged, then despite Meillassoux's failure to motivate the specific ("Gallilean") kind of realism he prefers, the meta-formal realist can evidently make some common cause with Meillassoux's project here nevertheless.

In this context, however, it must first be pointed out that virtually every aspect of Meillassoux's discussion of the iterable structure of the "meaningless" or "empty" sign and is prefigured in existing twentieth-century structuralist and post-structuralist discussions of the character of signs and their syntax, and *pre-eminently* in Derrida's project of deconstruction (which Meillassoux does not address even once, either in *After Finitude* or in the more recent article). Thus, for example, for Derrida in "Signature, Event, Context:"

> My "written communication" must, if you will, remain legible despite the absolute disappearance of every determined addressee in general for it to function as writing, that is, for it to be legible. It must be repeatable—iterable—in the absolute absence of the addressee or of the empirically determinable set of addressees. This iterability…structures the mark of writing itself, and does so moreover for no matter what type of writing… A writing that was not structurally legible – iterable – beyond the death of the addressee would not be writing.[69]

Again, for Derrida, "…the possibility of repeating, and therefore of identifying, marks is implied in every code, making of it a communicable, transmittable, decipherable grid that is iterable for a third party, and thus for any possible user in general" (p. 315). Thus Derrida, like Meillassoux, certainly recognizes and accords a primary theoretical role to the possibility of a sign's being recognized as a type and to the underlying ideal self-identity of the mark so conceived. In particular:

> …let us say that a certain self-identity of [the] element (mark, sign, etc.) must permit its recognition and repetition. Across empirical variations of tone of voice, etc., eventually of a certain accent, for example, one must be able to recognize the identity, shall we say, of a

[68] "My absolutism is so far from being hostile to empiricism, that in fact it aims to found the absolute necessity of the latter." (p. 12).
[69] Derrida (1971), p. 315.

> signifying form. Why is this identity paradoxically the division or dissociation from itself which will make of this phonic sign a grapheme? It is because the unity of the signifying form is constituted only by its iterability, by the possibility of being repeated in the absence not only of its referent, which goes without saying, but of a determined signified or current intention of signfification, as of every present intention of signification.[70]

Here, as in many other places, Derrida determines the structure of the sign, just as Meillassoux does, in terms of its iterability and self-identity as type quite independently of its referent or signified, and in such a way as to make it capable of an infinite recognizable repetition.

It would take us too far afield to point out exhaustively the many other texts developing from diverse schools and methods that have pointed out something similar. But one could doubtless locate substantially the same conception, for example, in Lacan's discussions of the "materiality" of the signifier; and Derrida himself points to the roots of the conception of the "purely significative" meaning of the sign without referent in Husserl's *Logical Investigations*. What is important in the present context, however, is less the startling temerity and apparent ingenuousness with which Meillassoux replicates the central claims and theoretical categories of the twentieth-century philosophical discourse on signs as if they were his own original innovations, than the way that doing this permits him to beg central questions that also bear on the very prospects for a critical continuation of the twentieth-century linguistic turn today. In particular, despite all the similarities I have pointed out, there is one obvious and significant difference between the way that Meillassoux characterizes the object of his remarks about iterability and the way that Derrida does: while what is characterized by Meillassoux as structurally iterable and essentially capable of being recognized as a type in an infinite number of cases of iteration is restricted to what he calls the *meaningless* or "*empty*" sign, Derrida explicitly characterizes this precise structure as a much more general one, the structure of the written *sign as such*. For Meillassoux, recall, the "meaninglessness" of a sign in this sense stems from its being defined implicitly or in terms of axioms or rules of use for the language as a whole, whereas signs that are "meaningful" are defined by the presentation of some intuitive referent; it is this distinction which, in turn, licenses for Meillassoux the distinction he draws between formal systems and "natural" languages, which are, he says, characterized by the "(negative) property of ordinary meaning" which is "the *absence of the rule-governed use of syntactical units devoid of meaning*."[71] By contrast with this, Derrida and other post-structuralist thinkers will, familiarly, consider the structure of natural languages and natural-language texts to be determined in central ways by regular structures of repetition and difference, determined and determinable quite independently of the empirical provision of intuitive reference, and thus will treat these structurally determinable aspects of natural language as analogous or identical to the parallel aspects of "formal systems" narrowly considered.

Is there, then, any good reason to consider natural languages to be characterized by a complete "absence of the rule-governed use of syntactical units devoid of meaning"? In fact there is *no* existent natural language whose structure could even remotely be thus characterized, and to simply stipulate the

[70] Derrida (1971), p. 318.
[71] Meillassoux (2012), p. 23.

difference between formal and natural languages in these terms, as Meillassoux does, is to do nothing more than commit a gross equivocation on “devoid of meaning.” The equivocation would only be tempting if we were to suppose not only (as Meillassoux perhaps does) that the only way to equip any natural-linguistic term with “meaning” in a full or proper sense is to equip it with a direct empirical reference but, furthermore, that there are *no* structurally determinative terms that are essential to the functioning of natural languages whose meaning cannot be provided in some such way. But to assume the latter would obviously be to invoke what can only be considered, in the wake of what is now more than a century of concerted philosophical investigation into the regular structure of natural languages, to be a caricature of the baldest and most misleading sort. Anyone who is tempted by such a caricature can quickly be cured of the temptation by considering the conditions for the functioning in natural language of, among other things, adjectives, adverbs, conjunctions, and interjections, none of which have a direct, intuitive meaning; not to mention natural-language terms for larger numbers, abstract relations, and other *abstracta*, none of which depend for their functioning on being provided with direct intuitively accessible referents. Again, one should certainly not forget the essentially “meaningless” signs of ordinary written language, such as spaces and punctuation marks, which play an essential role in the syntactic and structural articulation of writing as such, as Derrida (among others) has often emphasized. In fact, it is not obvious that there are *any* terms of natural language, beyond perhaps proper names and those directly standing for sensory qualities or sense-data (if any there be) which require for their functioning to be supplied with a directly present intuitive referent, as Meillassoux supposes all “meaningful” terms (in his sense) must be.

It is thus very difficult to grant Meillassoux’s attempt to distinguish in principle between “natural” and formal languages any credibility, once we have attended even in a cursory way to the actual details about how meaning is determined in natural languages; nevertheless, this does not imperil the possibility of pursuing, following Derrida and in abeyance of any principled distinction between “natural” and “formal” languages, the consequences of the formal and structural determinants of linguistic function and meaning as these articulate, and play at the boundaries of, natural languages and texts as well as “artificial” ones.[72] For this approach, formal and regular iterability, far from being a characteristic only of the “meaningless” signs of specifiable formal and axiomatic systems, is in fact one of the most essential underlying structural features of natural languages, and can thereby be investigated and analyzed on the level of the structure of language as such. An obvious advantage of following Derrida in this approach (relative to certain ends, at least) is, as he points out, that it makes possible not only the deconstruction of the classical concept of the “meaningfulness” of the sign as determined by the provision of intuitive meaning (essentially the conception that Meillassoux presupposes) but ultimately, as well, “the disruption, in the last analysis, of the authority of the code as a finite system of rules…”[73] In other words, it is the structural iterability of the sign as such that for

[72] The underlying thought here is that the distinction between “natural” (or “historical”) and “formal” (or “artificial”) languages is bound to lose any ultimately determining significance once it is removed from the context of an assumed distinction between “natural” and “artificial” processes (essentially, between “nature” and “culture”), a distinction which appears – though I do not have time to argue this here – to be less capable of assurance today than ever before. See Livingston (2012), p. xii.
[73] Derrida (1971), p. 316.

Derrida, far from confirming the necessity of present "meaning" or the cogency of a "hermeneutics" devoted to its elicitation, ultimately provides the positive basis for decomposing and deconstructing the "finitude" of language as it is figured in the classical "formalist" conception of a language (one accepted as fully by, e.g., Carnap as by Hilbert) as a determinate totality of rule-bound signs. Here, in other words, we can witness a singular application of formalism to its own deconstruction, a formal tracing of the very boundary-complications which must henceforth be seen to characterize the "real" meaning and internal auto-decomposition of any finitely specifiable system of regular language as such.

More generally, although it is often simply assumed that a deconstructive project thus formally determined must devolve into a limited "textualism" or linguistic idealism, the ultimately formal determination of this deconstructive project is not, as I shall argue, in competition with a substantive realism about its results. As I have argued elsewhere, the formal and syntactic determination of features of natural languages such as structural iterability and undecidability in the context of deconstruction can be directly supported by reference to the analogous features of formal systems, including, for example, the essential undecidability of formal systems of mathematics demonstrated by Gödel's incompleteness results; here, since the underlying formal structure is analogous or homologous, one can in fact, as I have suggested, see Derrida's application of deconstructive methods to particular texts and contexts as having a formal provenance in these metamathematical or metalogical results themselves. Moreover, although such structures as iterability, trace, and differance are most directly evident in connection with the determinate structure of natural languages, it is not only possible but reasonable to see these formal features of language as such, as Hägglund has recently argued, as having a deeper basis in the "real" structure of the world, for example in the structure of space and time themselves. Thus, for instance, Hägglund (2012) argues, *contra* Meillassoux, that the structure that Derrida describes as that of the "trace" rests on a more basic *temporal* structure of necessary "survival" which characterizes the possible endurance of both living and non-living entities, and thereby (in answer to Meillassoux's problem of the "arche-fossil") also ultimately underlies any possibility of referring to distal and ancestral existents. If either of these connections is convincing, the formally determined features of language as treated by deconstruction, and the radical internal critique they make possible, have direct application to a reality ("mathematical reality" on one hand, and the reality of space and time on the other) that is not limited to that of the "linguistic" or "textual"; rather, this formal register *itself* provides the basis, along the lines of the "metaformal realism" I have been sketching, for the rigorous thinking as well as the deconstructive critique of the reality of the provenance and transit of forms as they determine and decompose the world itself, beyond and before the thought of the finite subject of rules and signs.

If deconstruction's recourse to formalism can indeed be interpreted as embodying, as I have suggested here and have argued at more length in *The Politics of Logic*, one of a series of examples of "metaformally realist" substantive and critical positions in the history of twentieth century thought that are grounded, in various ways, in the linguistic turn taken in both "analytic" and "continental" thought, then it is certainly neither necessary nor adequate to base a contemporary argument for realism, as Meillassoux does, on the presumed identity of these positions with earlier idealist or subjectivist positions in the history of philosophy, or with the (largely "straw," as we have seen) position of "correlationism". Nevertheless, I have dwelled on the details of Meillassoux's argument, as noted, not simply in order to point out the often breathtaking leaps in his argumentation or his recurrent tendency to rely on what are mere gestures toward arguments not given and whose credibility is supported only by the provocativeness and heterodox quality of the claims they are said (but seldom shown) to support, but rather, primarily and more positively, in order to consider more closely the actual prospects for a "meta-formal" argument for realism based upon the demonstrable features of forms and their internal capacity to trace within themselves the very limits of formalization as such. With this in mind, we may finally return to Meillassoux's direct appeal to Hilbert's formalist position as underlying the conception of the "meaningless sign" which he presents as essential to all mathematization and to the structure of a realist conception of the objects and properties treated within the mathematized discourses of the natural sciences.

Hilbert's claim for the possibility of founding mathematics on the basis of axiomatic formal systems whose basic signs are construed as "meaningless" has, of course, been criticized from numerous directions. For example, according to a classical objection from the intuitionist direction, such "meaningless" signs as occur in a Hilbert-style formal system are not signs at all, since a sign essentially must stand for *something*, and simply specifying a syntactic rule for the use or intercombination of a symbol does not suffice to equip it with a meaning in this sense; again, from a more logicist direction, even the development of formal, axiomatic systems constantly requires and presupposes the application of *logical* principles and rules which cannot themselves ultimately be treated as simply stipulated within the formal systems they govern. More significant than either of these types of objections for the present argument, however, are the actual and direct implications of Gödel's metalogical results about the very capacities of formal systems themselves. In appealing to Hilbert's conception of the "meaningless" of signs, Meillassoux passes over these consequences with barely a comment; but it is clear, as I have argued above, that they have a direct and immediate bearing on the very idea of a formal system and the associated idea of the "meaninglessness" of a sign to which Meillassoux appeals. In particular, as we have seen above and as Gödel himself argued, the results that, first, every formal system (of a sufficient degree of complexity) must, if consistent, contain sentences that are strictly undecidable by means of its own procedures, and second, that a sentence asserting the consistency of a particular formal system is itself just such an undecidable one, jointly motivate a disjunctive conclusion that is "realist" in apparently demanding, on either disjunct, the inexhaustible resistance of a reality other than, and constitutively independent of, the capacities or forms of finite thought. It is not necessary, here, to construe this reality, in "Platonist" fashion, simply as a positive reality of existent entities or objects; we may also take it simply as the index of the formal and infinite "inexhaustibility" of actual mathematics with respect to any finitely specifiable and effective procedures of human thought. Even when this thought is figured as the *infinite* application of finitely specifiable rules, as we have seen, the metaformal realism that this exemplifies demonstrates critically the incapacity of any such figuration ever to exhaust, even in principle, the reality of its formal/mathematical correlate. In this way, whether it takes the form of the Gödelian results, Derridian deconstruction, or Wittgensteinian "criticism" of the constitutive idea and formal structure of the rule, metaformal realism provides a direct basis for the critical interrogation, and ultimate decomposition, of the figure of the rule as the potentially infinite repetition of identity; in so doing, it provides a basic realist horizon for the critical interrogation and ultimate decomposition of the constitutive idea of human finitude itself.

IV

In the previous sections, I have articulated and defended the attitude of meta-formal realism as one that captures the most important results of twentieth-century inquiry into the consequences of forms and formalism. In order to take due measure of these results and in particular of the consequences of the twentieth-century thought of the infinite, I have argued, it is important to motivate a realism that is not simply grounded in empirical experience or summarized as an attitude toward objects, but rather takes account of the intrinsic implications of the reflective inquiry into the transit of forms in relation to the Real of being itself; I have, furthermore, argued that the requisite kind of realism cannot be based simply in strawman arguments against idealism or in any simple response to traditional anti-realist positions. My main aim has not been to assay the ever-growing field of varieties and types of realist and anti-realist positions; I have said nothing, for example, about such widely varied positions as "internal realism," "quasi-realism", etc., nor about contemporary positions and projects friendly to "speculative realism" such as "object-oriented ontology" or "guerilla metaphysics". In this last section, I shall say just a bit more about the consequences of metaformal realism, should it be consistently adopted.

As I have argued, metaformal realism is an essentially *disjunctive* position, split between affirming the consequences of two quite distinct and mutually incommensurable orientations of post-Cantorian thought, the generic and the paradoxico-critical. The essential reason for this disjunctive form is, as I argued in The Politics of Logic, the choice with which reflective thought about totality is inherently forced as soon as it takes full account of the implications of Cantor's discovery of the mathematical actual-infinite: this is a choice between the combination of incompleteness and consistency (the generic orientation) and inconsistency with completeness (the paradoxico-critical orientation). As we have seen, Gödel's own disjunctive result witnesses exactly this disjunction with respect to the powers of human thought in relation to a mathematical reality which the constitutive thought of the infinite determines as the inexhaustible-real: this is, in Gödel's terms, the essential distinction between, on one hand, the assumption of an inherent and transcendent power of human thought to bear witness to consistency by exceeding, in relation to this inexhaustibility, the powers of any finitely specifiable system of rules, and on the other, an inherent and inexhaustible inscription of the undecidable as such, including the undecidability of consistency itself, in the very structure of mathematical reality.

Gödel himself sometimes argued that *both* terms of the disjunctive conclusion, beyond their realist implications, also have implications bearing, in various ways, against materialism. However, his primary reason for thinking so with respect to the *first* disjunct was that it appeared to him to show the untenability of mechanism; but besides being (as we have seen) problematic in itself, it is not clear, even if an anti-mechanist position is adopted, that such a position cannot be reconciled with at least some forms of materialism. Gödel himself, for instance, mentions the possibility of a "vitalist" conception of the transcendent powers of human thought whereby its non-mechanistic powers are grounded in a materiality of life, and more recently some have argued that non-classical physical phenomena, for instance those of quantum mechanics, might provide a materialist basis for the capabilities that the human mind must have in relation to mathematical objects on an anti-mechanist position. With respect to the second disjunct, the anti-materialist conclusion is even less certain, since on this disjunct it is certainly possible, and in fact required insofar as we can model human mathematical thinking at all, to understand its capacities as modeled by a particular (mechanistic) formal system. In particular, although, as Gödel argues in the Gibbs lecture and elsewhere, the second disjunct is certainly incompatible with an (anti-realist) *conventionalism* about mathematics according to which mathematical truths are human creations or products of human convention, it is *not* opposed to a thoroughgoingly "materialist" and mechanist position that recognizes these truths as inherent to an (infinite) reality thought as containing nothing other than matter and machines.[74]

Because he was a committed anti-mechanist, Gödel favored the first disjunct (on which the human mind is non-mechanical) and sometimes argued against the tenability of the second on independent grounds, holding both that it ignores the inherent capacity of the human mind to innovate with respect to its guiding axioms and principles and that the existence of absolutely unsolvable problems is untenable since it would imply that "it would mean that human reason is utterly irrational by asking questions it cannot answer, while asserting emphatically that only reason can answer them."[75]

However, as I have suggested in this paper, once we have acknowledged the implications of the availability of the infinite to mathematical thought and made the general decision for meta-formal realism at all, there are some important senses in which the second disjunct, corresponding to the orientation of paradoxico-criticism, is not only not excluded but also enjoys advantages over the choice for the first disjunct (which Gödel himself preferred). In particular, besides being more obviously compatible with materialism because not in any way at odds with mechanism, the paradoxico-critical outlook makes it possible to preserve an outlook and practice that continues the classical orientation of *criticism* with respect to the capacities and practices of the human subject, in the altered conditions post-Cantorian thought. To gain a sense of these ongoing critical implications, one might juxtapose Gödel's remark about reason posing problems that it cannot solve with the infamous opening lines of Kant's first *Critique*:

> Human reason has this peculiar fate that in one species of its knowledge it is burdened by questions which, as prescribed by the very nature of reason itself, it is not able to ignore, but which, as transcending all its powers, it is also not able to answer.

Kant, of course, was a transcendental idealist; and within the fourfold framework of orientations of thought I developed in *The Politics of Logic*, Kant's thought remains a paradigm of the pre-Cantorian constructivist (or criteriological) orientation, which is defined by its attempt to assay the boundaries of knowledge from the exterior position of a limit-drawing project committed to saving jointly the ideas of completeness and consistency. In the post-Cantorian context, it is no longer possible to save these ideas

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[74] For the anti-conventionalist point, see especially Gödel (1953).

[75] Wang (1974), p. 324, discussed in Feferman (2006), p. 12. The former point, about the non-static nature of the mind, is made against Turing's own position in Gödel (1972), remark 3, p. 306.

jointly, and so the constructivist orientation and its associated kind of idealism are both rendered untenable. But by making the paradoxico-critical decision for the combination of a rigorous inquiry into totality with the implication of irreducible paradox at the boundaries, it is possible to maintain the properly critical register of Kant's thought of reflective reason in its ongoing dialectic with itself, and to situate this thought within, as I have argued, a rigorously *realist* position with respect to the relation of thought and being itself. To do so is to transpose the ultimate ground for the development of such a (now thought more in a properly Platonic rather than a Kantian or Hegelian sense) decisively away from the (pre-Cantorian) Kantian oppositional figure of opposition between the finitude of sensory affection and the absolute-infinite divine intellect capable of intellectual intuition, and to reinvent the possiblities of critique on the ontological real ground of the objective undecidability of problems that are problems for (finite or infinite) thought *in itself*, given to it at the point of its very contact with the real of Being as such.

What, finally, are some of the concrete effects of this transposition for contemporary reflective and critical thought? As I argued in *The Politics of Logic,* most generally, the necessity, in a post-Cantorian context, of the forced choice between inconsistent completeness and incomplete consistency indicates, as is confirmed by Gödel's development of the philosophical consequences of his own results, that it is impossible *by* finite, procedural means to confirm rigorously the *consistency* of the finitely specifiable procedures of our social-political, practical, and technological worlds. This suggests, as I argued at more length in the book, that it is impossible by finite means to ensure the *effectivity* of our practices, or procedurally to found whatever faith we may maintain in their ongoing extensibility and capability of continuation. This faith, if it is to be founded at all, must be founded in an essentially infinite capacity of insight and fidelity, bordering on the mystical, to a Real matter of consistency with respect to our own practices that can itself never be guaranteed by any replicable or mechanical procedure; *or* it must be ceaselessly decomposed and deconstructed at the point of the inherent realism of the problematic and undecidable that is necessarily introduced if this faith cannot be assured at all. Such are the consequences, as I have argued in *The Politics of Logic*, of the transformative event of the development of formalization in the light of the accessibility of the mathematical infinite that characterizes our time; and such are the stakes, as I have tried to confirm here, of the metaformal realism that this event rigorously motivates and demands.

Works Cited

Badiou, A. [1988] 2005. *Being and Event*. Transl. Oliver Feltham. London: Continuum.

Badiou, A. 1998. "Platonism and mathematical ontology." Translated and reprinted in *Theoretical Writings*, ed. by Ray Brassier and Alberto Toscano. London: Continuum, pp. 49-58.

Brown, N. 2011. "The speculative and the specific: On Hallward and Meillassoux," reprinted in Levi Bryant, Nick Srnicek and Graham Harman, eds., *The Speculative Turn: Continental Materialism and Realism*. Melbourne: re.press, pp. 142-163.

Bryant, L., N. Srnicek and G. Harman, 2011. "Towards a speculative philosophy" in Levi Bryant, Nick Srnicek and Graham Harman, eds., *The Speculative Turn: Continental Materialism and Realism*. Melbourne: re.press, pp. 1-18.

Derrida, J. [1971] 1982. "Signature, Event, Context," reprinted in *Margins of Philosophy*, transl. and edited by Alan Bass. Chicago: U. of Chicago Press, pp. 307-330.

Dummett, M. [1963] 1978. "Realism" Reprinted in *Truth and Other Enigmas*. London: Duckworth, pp. 145-65.

Dummett, M. [1973] 1978. "The philosophical basis of intuitionistic logic." Reprinted in *Truth and Other Enigmas*. London: Duckworth, pp. 215-247.

Dummett, M. 1978. "Preface" to *Truth and Other Enigmas*. London: Duckworth, pp. ix-li.

Eyers, T. 2012. *Lacan and the Concept of the 'Real'*. Palgrave MacMillan.

Feferman, S. 2006. "Are there absolutely unsolvable problems? Gödel's dichotomy." Philosophia Mathematica 14 (2): 134-152 (page # reference to on-line version at: http://math.stanford.edu/~feferman/papers/dichotomy.pdf).

Gödel, K. [1951] 1995. "Some basic theorems on the foundations of mathematics and their philosophical implications." Reprinted in F.A. Rodriguez-Consiguera, ed., *Kurt Gödel: Unpublished Philosophical Essays*. Springer.

Gödel, K. 1953. "Is mathematics syntax of language?" in Solomon Feferman, et al., (Eds.), *Collected Works, Volume III: Unpublished Essays and Lectures*, pp. 334-353 (Oxford U. Press).

Gödel, K. [1972] 1990. "Some remarks on the undecidability results" in Solomon Feferman, et al., (eds.), *Collected Works, Volume II: Publications 1938-1974*, pp. 305-306.

Hägglund, M. 2011. "Radical atheist materialism: A critique of Meillassoux", reprinted in Levi Bryant, Nick Srnicek and Graham Harman, eds., *The Speculative Turn: Continental Materialism and Realism*. Melbourne: re.press, pp. 114-129.

Hallward, P. 2011. “Anything is possible: A reading of Quentin Meillassoux’s After Finitude”, reprinted in Levi Bryant, Nick Srnicek and Graham Harman, eds., *The Speculative Turn: Continental Materialism and Realism*. Melbourne: re.press, pp. 130-141.

Johnston, A. 2011 “Hume’s revenge: A Dieu, Meillassoux?” reprinted in Levi Bryant, Nick Srnicek and Graham Harman, eds., *The Speculative Turn: Continental Materialism and Realism*. Melbourne: re.press, pp. 92-113.

Livingston, P. 2004. “’Meaning is Use’ in the *Tractatus*”. *Philosophical Investigations* 27 (1):34–67 (2004).

Livingston, P. 2012. *The Politics of Logic: Badiou, Wittgenstein, and the Consequences of Formalism*. New York: Routledge.

Maddy, P. 2005. “Mathematical existence,” *The Bulletin of Symbolic Logic*, Vol. 11, No. 3 (Sep., 2005), pp. 351-376

Meillassoux, Q. [2006] 2008. *After Finitude: An Essay on the Necessity of Contingency*. Transl. by Ray Brassier. London: Continuum.

Meillassoux, Q. 2012. “Iteration, reiteration, repetition: A speculative analysis of the meaningless sign.” Available at: http://oursecretblog.com/txt/QMpaperApr12.pdf

Shapiro, S. 1998. “Incompleteness, mechanism, and optimism.” *The Bulletin of Symbolic Logic*, vol. 4, no. 3 (Sep., 1998), pp. 273-302.