On atomic composition as identity

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Abstract
In this paper I address two important objections to the theory called '(Strong) Composition as Identity' ('CAI'): the 'wall-bricks-and-atoms problem' ('WaBrA problem'), and the claim that CAI entails mereological nihilism. I aim to argue that the best version of CAI capable of addressing both problems is the theory I will call 'Atomic Composition as Identity' ('ACAI') which consists in taking the plural quantifier to range only over proper pluralities of mereological atoms and every non-atomic entity to be identical to the (proper) plurality of atoms it fuses. I will proceed in three main steps. First, I will defend Sider's (2014) idea of weakening the comprehension principle for pluralities and I will show that (*pace* Calosi 2016a) it can ward off both the WaBrA problem and the threat of mereological nihilism. Second, I will argue that CAI-theorists should uphold an 'atomic comprehension principle' which, jointly with CAI, entails that there are only proper pluralities of mereological atoms. Finally, I will present a novel reading of the 'one of' relation that not only avoids the problems presented by Yi (1999a, 2014) and Calosi (2016b, 2018) but can also help ACAI-theorists to make sense of the idea that a composite entity is both one and many.

Keywords Mereology · Composition as Identity · Collapse · Mereological Nihilism

1. Introduction

Taken in its strongest form, Composition as Identity (CAI) is a very radical thesis. It says that, whenever a singular entity is the mereological fusion of a plurality of entities, then the singular entity is—literally!—*identical* to the things it fuses. Despite its radical nature, CAI is not a theory without a certain allure, as it appears to be able to account in a simple and straightforward way for the alleged 'ontological innocence' of mereology. In fact, if a whole is *identical* to the parts it fuses, then there seems to be a clear sense in which, once one is committed to the existence of the parts, one is thereby also committed to the existence of the whole without any 'additional' ontological cost: 'it just *is* them, they just *are* it' (Lewis 1991: 83).[1]

Beyond the incredulous stare by which it is usually met, CAI appears to be saddled with a series of problems concerning the so-called 'Collapse' principle (which CAI appears

[1] '[…] perhaps the major motivation for CAI is that it implies the 'ontological innocence' of classical mereology' (Cotnoir 2014: 7). 'If Lewis's claim were that the fusion is *literally identical* to the cats that compose it, he would clearly be entitled to ontological innocence' (Bennett 2015: 256). '[…] the thought that a fusion is numerically identical to the things that compose it *taken together* […] would vindicate the intuition that such double countenancing is ultimately redundant, hence the innocence thesis' (Varzi 2014: 49). 'But why think that mereology is ontologically innocent? If composition is identity, then ontological innocence is secured' (Hawley 2014: 72).

to entail) and the interaction of the notion of parthood with the notion of 'being one of' (Yi 1999a, 2014; Sider 2007, 2014). These problems give rise to two serious objections to CAI. According to the first objection (which I will here label the 'wall-bricks-and-atoms problem', or 'WaBrA problem' for short), CAI is incompatible with the idea that there are entities that have proper parts that have themselves proper parts (like a wall that has bricks as proper parts which have in turn atoms as proper parts). According to the second objection, CAI actually entails mereological nihilism, that is, the thesis that *nothing* has proper parts. Sider (2014) has suggested a way out for CAI-theorists which consists in taking the plural quantifier to quantify over 'fewer pluralities than one normally expects' (Sider 2014: 213). Recently, however, Calosi (2016a) has argued that Sider's strategy is ineffective against the threat of mereological nihilism.

In this paper I aim to defend CAI and to argue that, at least insofar the WaBrA problem and the threat of mereological nihilism are concerned, CAI-theorists should embrace the specific version of CAI which I will call 'Atomic Composition as Identity' (or 'ACAI'), according to which (i) the plural quantifier only quantifies over proper pluralities of atoms and (ii) every non-atomic entity is identical to the plurality of atoms it fuses. I will proceed as follows. After some stage-setting in section 2, in section 3 I will present both the WaBrA problem and some of the arguments given for the claim that CAI entails mereological nihilism. In section 4 I will defend Sider's (2014) strategy on behalf of CAI-theorists and argue that, *pace* Calosi (2016a), it is effective with respect to both the WaBrA problem and the claim that CAI entails nihilism. In section 5 I will present ACAI and argue that it should be preferred over other ways CAI-theorists have to comply with the gist of Sider's strategy. In section 6 I will present two arguments (adapted from Calosi (2016b, 2018) and Yi (1999a, 2014)) to the effect that ACAI entails nihilism, and propose a novel interpretation of the one-of relation capable of withstanding both of them. Finally, in section 7, I will show how ACAI-theorists can make sense of the idea that a composite entity is both one and many.

2. Stage-setting

I will be working within the framework of plural logic. The following notational conventions are employed throughout: (i) 'x', 'y', …, 'z' are *singular* variables, (ii) 'X', 'Y', ..., 'Z' are *plural* variables,[2] (iii) '$y \prec X$' stands for 'y is one of the Xs', and (iv) '$x \leq y$' stands for 'x is part of y'. The notion of *proper part*, *overlap* and *mereological fusion* are defined as follows:

(Proper Part) $x < y =_{df} x \leq y \land x \neq y$

(Overlap) $Oxy =_{df} \exists z(z \leq x \land z \leq y)$

[2] Notice that, for the ease of expression, I will sometimes use 'plurality' and plural terms like 'X', 'Y' and 'Z' as grammatically singular and say things like 'there is a plurality X such that *it* is such-and-such'.

(Fusion) $xFuY =_{df} \forall z(z \prec Y \rightarrow z \leq x) \wedge \forall z\big(z \leq x \rightarrow \exists w(w \prec X \wedge Ozw)\big)$[3]

In this paper I will work on the background of Classical Mereology understood as the conjunction of the following three principles:[4]

(Transitivity) $\forall x \forall y \forall z\big((x \leq y \wedge y \leq z) \rightarrow x \leq z\big)$

(Weak Supplementation) $\forall x \forall y\big(x < y \rightarrow \exists z(z \leq y \wedge {\sim} Ozx)\big)$

(Universalism) $\forall X \exists y(yFuX)$

Furthermore I will assume the three following singular, plural, and mixed versions of Leibniz's Law[5]:

(LL1) $\forall x \forall y(x = y \rightarrow \big(\varphi(x) \leftrightarrow \varphi(y)\big)$

(LL2) $\forall X \forall Y(X = Y \rightarrow \big(\varphi(X) \leftrightarrow \varphi(Y)\big)$

(LL3) $\forall x \forall Y(x = Y \rightarrow \big(\varphi(x) \leftrightarrow \varphi(Y)\big)$

A notable instance of (LL2) is (LLI), according to which identical pluralities have the same 'members':

(LLI) $\forall X \forall Y\big(X = Y \rightarrow \forall z(z \prec X \leftrightarrow z \prec Y)\big)$

I will also assume that no plurality is empty[6]

(NEP) $\forall X \exists y(y \prec X)$

and the following extensionality principle for pluralities, according to which pluralities having the same members are identical:[7]

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[3] This definition of fusion appears to be the most common in the debate that is relevant to this paper. See, for instance, Yi (1999a: 143; 2014: 183), Sider (2007: 52; 2014: 212), Calosi (2016a: 221; 2016b: 3; 2018: 282), and Loss (2018: 370). For a discussion of alternative definitions of mereological fusion see, *inter alia*, Hovda (2009) and Varzi (2016: section 4).

[4] See Hovda (2009) for alternative axiomatizations of Classical Mereology.

[5] As Sider (2007) puts it: 'Whatever else one thinks about identity, Leibniz's law must play a central role. [..] To deny it would arouse suspicion that their use of 'is identical to' does not really express identity' (Sider, 2007: 56-7).

[6] See Linnebo (2017: section 1.2). Without this assumption the mereological axiom of universalism (see p. 3) would have to be in conditional form: $\forall X(\exists z(z \prec X) \rightarrow \exists y(yFuX))$.

[7] See, *inter alia*, Linnebo (2017: section 1.2) and Hovda (2014: 202).

(EXT) $\forall X \forall Y(\forall z(z \prec X \leftrightarrow z \prec Y) \to X = Y)$

Finally, Composition as Identity (CAI) will be understood as the thesis according to which, whenever an entity *x* fuses a plurality of entities *Y*, then *x* is identical to the *Y*s:

(CAI) $\forall x \forall Y(xFuY \to x = Y)$

3. Comprehension and Collapse

The following Comprehension Principle for pluralities is usually taken to be an axiom of plural logic:[8]

(CMP) $\exists x \phi x \to \exists Y \forall x(z \prec Y \leftrightarrow \phi x)$

(CMP) can be used to prove the principle commonly called Plural Covering, saying that if *x* is part of *y*, then there is a plurality of entities *W* such that *y* fuses the *W*s and *x* is one of the *W*s:[9]

(PC) $\forall x \forall y\big(x \leq y \to \exists W(yFuW \wedge x \prec W)\big)$

In turn, from CAI, (LLI) and (PC) the Collapse Principle can be derived, according to which, if *x* fuses the *Y*s, then for every *z*, *z* is part of *x* if and only if *z* is one of the *Y*s:[10]

(Collapse) $\forall x \forall Y\big(xFuY \to \forall z(z \leq x \leftrightarrow z \prec Y)\big)$

[8] See, for instance, Linnebo (2017: section 1.2).

[9] Suppose that, for some *x* and *y*, x is part of y. It follows that that there is something that is identical to either *x* or y. By (CMP) we have, thus, that there is a plurality W of entities such that something is one of the *W*s if and only if it is identical to either *x* or *y*. Therefore, *x* is one of the *W*s. From the definition of fusion it follows that *y* fuses the *W*s. QED

[10] Suppose that *x* fuses the *Y*s. By the definition of fusion we have that if an entity *z* is one of the Ys, then *z* is part of *x*. Conversely, if *z* is part of *x*, it follows from (PC) that there is some plurality W such that *x* fuses the *W*s and *z* is one of the *W*s. By CAI, *x* is identical to both the *W*s and the *Y*s. Therefore, *W* and *Y* are identical. By (LLI), they have the same members, so that *z* is also one of *Y*s. QED

As it has been shown in the literature, Collapse appears to be a highly problematic principle.[11] Consider, for instance, a scenario in which the only mereological atoms in the world are A1, A2, and A3. Suppose, furthermore, that B1 has only A1 and A2 as proper parts, B2 has only A1 and A3 as proper parts, B3 has only A2 and A3 as proper parts, and that A1, A2, A3, B1, B2, and B3 are all the proper parts of C. In this scenario (CMP) predicts that there is both the plurality of the As (the atoms) and the plurality of the Bs (in what follows I will refer to the Bs as 'the bricks' and to C as 'the wall'). By the definition of fusion it follows that the wall fuses *both* the plurality of the atoms *and* the plurality of the bricks. However, according to Collapse, if x is a fusion of a plurality of entities, then each of its parts is a member of that plurality. Therefore, if the wall is the fusion of the bricks, then each of its parts is a brick, and so—given that the atoms are not bricks—it cannot also be the fusion of the atoms. Alternatively, if the wall is the fusion of the atoms,

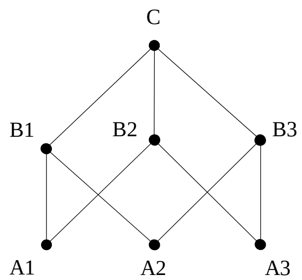

Figure 1-The WaBrA scenario

then it cannot be the fusion of the bricks. It follows, thus, that the wall can*not* be the fusion of both the bricks and the atoms. *Contradiction*! Let's call this the 'wall-bricks-and-atoms problem', or the 'WaBrA problem' for short.

The WaBrA problem is clearly a serious issue for CAI-theorists. However, the bad consequences of Collapse don't seem to stop there. In fact, as it has been argued in the literature, Collapse also appears to entail mereological nihilism (or 'Nihilism' for short), that is the idea that everything is a mereological atom, with no proper parts:[12]

(Nihilism) $\forall x \forall y (y \leq x \rightarrow y = x)$

Calosi (2016) and Loss (2018) have recently offered three arguments to this effect. They all (implicitly or explicitly) rely on (CMP) to argue that a certain kind of pluralities exist, and then argue for Nihilism by employing those pluralities. The pluralities whose existence is presupposed by Calosi's (2016) and Loss's (2018) arguments are pluralities *Y* such that,

[11] See, in particular, Yi (1999, 2014) and Sider (2007, 2014).

[12] See Calosi (2016a, 2016b, 2018), and Loss (2018). Gruszczyński (2015) offers an argument employing sets instead of pluralities.

for some *x*, (i) *x* is one of the *Y*s, (ii) *x* fuses the *Y*s, and (iii) for some *z*, *z* is a proper part of *x* and yet *z* is not one of the *Y*s. I will call them 'incomplete thick pluralities'.[13]

Loss's (2018) first argument can be presented as follows:

> *The argument from improper pluralities*:
> Suppose that *x* is a composite entity. By (CMP) some (incomplete thick) plurality *X* has *x* as its only member (*X* is thus an 'improper plurality'). By the definition of fusion, *x* fuses the *X*s. By Collapse, every part of *x* is one of the *X*s. Therefore, every part of *x* is identical to *x*. There is, thus, no *y* such that *y* is part of *x* and different from *x*. *x* has, thus, no proper parts and is not a composite entity. *Contradiction*! Therefore, there are no composite entities.[14]

Calosi's (2016) argument pivots around the Duplication Principle, according to which if *x* fuses the *Y*s, then each of the *Y*s is a duplicate of *x*—where '*x* and *y* are (singular) duplicates […], if they have the same properties' (Calosi, 2016: 226; in what follows '$y \triangleq x$' stands for 'y is a duplicate of x'):

(Duplication) $\forall x \forall Y \big(xFuY \rightarrow \forall y(y \prec Y \rightarrow y \triangleq x)\big)$

Duplication entails Nihilism.[15] However, as Calosi (2016) shows, Duplication is also entailed by Collapse. In order to prove this Calosi considers first the scenario in which *x* has a property *P* that one of its proper parts *y* lacks, and then the scenario in which *x* lacks a property *P* that is instantiated by one of its proper parts. Let us focus here on the first case, which can be formulated as follows:[16]

> *From Collapse to Duplication*
> Suppose that *y* is a proper part of *x* and assume for *reductio* that *P* is a property that *x* has but *y* has not. By (CMP) some plurality *Y* is the (incomplete thick) plurality of things that are *P*-parts of *x* (that is, the things that are *P* and are also part of *x*). By the definition of fusion, *x* fuses the *Y*s. From Collapse it follows that every part of *x* is one of the *Y*s, and therefore, that every part of *x* is a *P*-part of *x*. However, *y* is a

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[13] I take here a *thick plurality* to be any plurality *Y* of entities such that, for some *x*, (i) *x* is one of the *Y*s and (ii) *x* fuses the *Y*s. A *complete* thick plurality can thus be taken to be any plurality *Y* of entities such that, for some *x*, (i) *x* is one of the *Y*s, (ii) *x* fuses the *Y*s, and (iii) for every *z*, *z* is one of the *Y*s if and only if *z* is a part of *x*.

[14] See Loss (2018: 371). I reformulated the argument in order to highlight the way in which it (implicitly) relies on (CMP). See also Gruszczyński (2015: 536-7) for a similar argument with sets in place of pluralities.

[15] Suppose *y* is a proper part of *x* and consider the property *P*='being a proper part of *x*'. *y* clearly has *P*. However, *x* doesn't have *P* (nothing is a proper part of itself), thus contradicting Duplication.

[16] Notice that for the second case Calosi's argument relies on the existence of different pluralities fused by the same composite entity, which makes it relevantly similar to the WaBrA scenario: '[…] assume that there is a property *P* that at least one of the proper parts of *x*, let's say *y*, has but *x* has not. Consider the plurality W_2 of things that have the following property: "being part of *x* and having a *P*-part". […] Hence $xFuW_2$. […]. Now consider the plurality W_3 of *P*-parts of *x*. Once again, $xFuW_3$ […]' (Calosi 2016: 227). Here W_2 and W_3 are different pluralities, since *x* is one of the W_2s but not of the W_3s, and yet *x* fuses both.

part of *x* that is not a *P*-part of *x*. *Contradiction*! Therefore, every proper part of *x* has every property that *x* has.[17]

Loss's (2018) *second* argument appeals to Weak Company (a weaker principle than Weak Supplementation) and relies on the existence of incomplete thick pluralities having just two members and such that one of their members is a proper part of the other:

(Weak Company) $\forall x \forall y \big(x < y \rightarrow \exists z (z < y \wedge z \neq x)\big)$

The argument from Weak Company:
Suppose that *y* is a proper part of *x*. By (CMP) there is a (thick incomplete) plurality *W* of entities such that each of the *W*s is identical to either *x* or *y*. By the definition of fusion, *x* fuses the *W*s. By Collapse, every part of *x* is identical to one of the *W*s. Therefore, every part of *x* is identical to either *x* or *y*. It follows, thus, that there is no *z*, such that *z* is a proper part of *x* and different from *y*, thus contradicting Weak Company. Therefore, nothing is a proper part of *x*. By generalization, nothing has proper parts, and so Nihilism is true.[18]

It appears, thus, that—assuming the validity of (CMP)—not only CAI is incompatible with the rather minimal pattern of parthood relations featured in the WaBrA scenario, but also with the very existence of composite entities.

4. A weaker Comprehension Principle

The problems of Collapse appear to depend on (CMP) in two ways. On the one hand, (CMP) appears to be necessary to derive Plural Covering.[19] On the other hand, (CMP) is necessary to infer the existence of the pluralities needed for the WaBrA problem and the arguments for Nihilism. For this reason, it is at least *prima facie* plausible to think that CAI-theorists should reject (CMP) and claim that there are 'fewer pluralities than one normally expects' (Sider 2014: 213), in the sense that, for some ϕ, there is actually *no plurality of the ϕ-ers* (that is, no plurality of entities such that something is one of them if and only if it ϕs). For instance, supposing that the weaker comprehension principle chosen by CAI-theorists still validates Collapse,[20] there will be

[17] See Calosi (2016: 226-7).

[18] See Loss (2018: 372)

[19] In fact, without (CMP) there is no guarantee in the proof of Plural Covering that there is a plurality W of entities such that something is one of the *W*s if and only if it is identical to either *x* or *y* (see footnote 9).

[20] Notice that there are two (non-exclusive) ways in which the assumption of a weaker comprehension principle may block the problems of Collapse: (i) by invalidating Collapse; (ii) by blocking the arguments from Collapse. Sider's weak comprehension principle (see below) actually validates Collapse, and yet it blocks the argument from Collapse by excluding the existence of the problematic pluralities.

> no *X*s such that something is one of them if and only if it is a human being. For any *X*s including all humans will also include some non-humans, and thus will not include only humans. If each human is one of the *X*s then the fusion of the *X*s (which must exist given the fusions principle) contains many non-human parts (non-human parts of individual humans, and non-human objects containing multiple humans as parts, for example), and each non-human part of the fusion of the *X*s must be one of the *X*s given Collapse. (Sider 2014: 213).

Once (CMP) is rejected for a weaker principle it is, thus, very important to be 'very careful with the locution 'the ϕs'' (Sider 2014: 213). For instance, supposing again that the weaker comprehension principle in question validates Collapse, it follows that

> the plural term '[the] subatomic particles [of that person]' denotes nothing. It is intended to denote *X*s such that something is one of them iff it is a subatomic particle that is part of the person in question; but any *X*s of which each such part of a person is one will also include further things—anything (such as the person's head) that contains multiple subatomic particles from the person will also be one of such *X*s. (Sider 2014: 213)

On behalf of CAI-theorists Sider (2014) proposes a specific weak version of the comprehension principle for pluralities. It uses the notion of *schematic fusion* (or 'S-fusion' for short):

(S-Fusion) $xSFu\phi =_{df} \forall z(\phi z \rightarrow z \leq x) \wedge \forall z\big(z \leq x \rightarrow \exists w(\phi w \wedge Ozw)\big)$

For reasons that will be clear in a moment it is very important to keep in mind that 'ϕ' in the definition of S-Fusion is *not* a plural variable. In fact, speaking of 'the ϕs' may lead one to think that when '$xSFu\phi$' is true we are thereby guaranteed that there is the *plurality* of 'the ϕs' (which *x* schematically fuses). However, as we will see below, this is *not* what the notion of schematic fusion entails. In order to avoid any kind of talk that might mislead one into assuming that when 'ϕ' is concerned we are speaking of a plurality, '$xSFu\phi$ can be read as '*x* is the S-fusion of every *z* such that ϕz' or, more simply, as '*x* is the S-fusion of everything that ϕs'.

Sider (2014: 214-5) then assumes the principle of Unrestricted Schematic Fusion

(USF) $\exists y\phi y \rightarrow \exists y(ySFu\phi)$

and states his Weakened Comprehension Principle as follows:

(WCP) $\exists x\phi x \rightarrow \exists Y\exists z\big(zSFu\phi \wedge \forall x(x \prec Y \leftrightarrow x \leq z)\big)$

(WCP) says that if something is a ϕ-er, then there is a plurality *Y* and some entity *z*, such that *z* is the S-fusion of everything that ϕs and something is one of the *Y*s if and only if it is part of *z*. As Sider (2014) shows, (WCP) appears to offer a way out of the WaBrA problem. In order to better appreciate his point, it may be helpful to introduce some new terminology to make it easier to talk about the kind of pluralities whose existence is ruled out by the choice of a certain plural comprehension principle.

Let a plurality X *correspond* to a certain set s ('$X \rhd s$') if and only if something is one of the Xs if and only if it is a member of s:

(COR) $X \rhd s =_{df} \forall y(y \prec X \leftrightarrow y \in s)$

In addition, let the plural quantifier be Σ-*weak* just in case there is some non-empty set *s* such that there is *no* plurality corresponding to it

(Σ-Weak) $\exists s\big(\exists x(x \in s) \wedge {\sim}\exists Y(Y \rhd s)\big)$

With this new terminology in play, Sider's main insight can be reformulated as the idea that, if CAI is true, then the plural quantifier must be Σ-weak. In fact, what the WaBrA problem appears to show is that, if CAI is true, there can*not* be, for instance, *both* a plurality corresponding to {A1,A2,A3} *and* a plurality corresponding to {B1,B2,B3}. Taken together, (WCP) and CAI entail not only that the plural quantifier is Σ-weak, but also that the only sets that have a corresponding plurality are the sets that contain all the (proper and improper) parts of a certain given entity:

(SR1) $\forall s\big(\exists X(X \rhd s) \rightarrow \exists y \forall z(z \in s \leftrightarrow z \leq y)\big)$

Proof. Consider an arbitrary set *s* having a corresponding plurality *X*. Let ϕ be 'v is one of the *X*s'. By (WCP) there is a plurality *W* and an entity *y* such that (i) *y* is the S-fusion of everything that is one of the *X*s, and (ii) something is part of *y* if and only if it is one of the *W*s. Since *y* has all and only the *W*s as parts it follows by the definition of fusion that *y* fuses the *W*s. By CAI *y* is identical to the *W*s. On the other hand, by the definition of S-fusion, to say that *y* is the S-fusion of everything that is one of the *X*s is to say that (i) each one of the *X*s is part of *y* and (ii) every part of *y* overlaps some of the *X*s. Therefore, by the definition of fusion, *y* fuses the *X*s. By CAI, *y* is identical to the *X*s. Since *y* is identical to both the *X*s and the *W*s it follows that the *X*s are identical to the *W*s. By (LLI) the *Xs* and the *W*s have the same members. Therefore, since something is one of the *W*s if and only if it is part of *y*, it follows that something is one of the *X*s if and only if it is part of *y*. *X* corresponds to *s*, so that something is a member of *s* if and only if it is one of the *X*s. It follows, thus, that *y* is such that something is a member of *s* if and only if it is part of *y*. QED

It follows from (SR1) that in the WaBrA scenario (WCP)+CAI allow *only* the following sets to have corresponding pluralities: {A1}, {A2}, {A3}, {A1, A2, B1}, {A1, A3, B2}, {A2, A3, B3}, {A1, A2, A3, B1, B2,B3, C}. In fact, in the WaBrA scenario these are the only sets such that there is some *x* having as parts all and only their members. Most importantly, (WCP) and CAI jointly *exclude* the existence of pluralities corresponding to either {A1,A2,A3} and {B1,B2,B3}. In fact, neither of these sets is such that there is some entity having as parts all and only its members, as no entity in this scenario has *only* either A1, A2, and A3, or B1, B2, and B3 as parts. More in general, (WCP) and CAI entail (UF), that is, the claim that every entity is the fusion of just *one* plurality of entities: the plurality of all of its proper and improper parts:[21]

(UF) $\forall x \exists Y \big(xFuY \wedge \forall z(z \prec Y \leftrightarrow z \leq x) \wedge \forall W(xFuW \rightarrow W = Y)\big)$

Therefore, by embracing (WCP) CAI-theorists can adequately address the WaBrA problem.

Calosi (2016) has claimed that CAI entails Nihilism even if (WCP) is assumed. His argument appears to rely on taking (WCP) to entail that whenever there is at least a ϕ-er, then there is the *plurality* of things that ϕ, as passages like the following seem to suggest:

> [(WCP)] states that, provided there is at least a ϕ-er, there is a *plurality* of things such that *it* has a schematic fusion and something is among them only if it is part of their S-fusion. (Calosi 2016: 231; my italics)

However, (WCP) *doesn't* say that if there is at least a ϕ-er, then there is a plurality *Y* such that *it*—*the plurality*—has a schematic fusion. (WCP) says that if there is at least a ϕ-er, then there is a plurality Y such that some entity *z* is such that *z* S-fuses everything that ϕs and every x is one of the Ys if and only if it is part of z. In order to better appreciate this point, notice that (WCP) is equivalent to (WCP*), in which '$zSFu\phi$' *doesn't* lie in the scope of the plural quantifier:

(WCP*) $\exists x \phi x \rightarrow \exists z\big(zSFu\phi \wedge \exists Y \forall x(x \prec Y \leftrightarrow x \leq z)\big)$

(WCP*) states even more clearly that, supposing there is indeed at least a ϕ-er, then the plurality of entities that we are guaranteed to have isn't the plurality of 'the ϕs' but a plurality of entities such that something is one of them if and only if it is part of the S-fusion of everything that ϕs. Suppose, for instance, that ϕ is 'v is a red atom' and that *a* and *b* are the only red atoms in the world. It follows then from (WCP)/(WCP*) that there is an entity *z* such that

(i) both *a* and *b* are parts of *z* and each part of *z* overlaps either *a* and *b* (so that *z* S-fuses everything that is a red atom), and
(ii) there is a plurality *Y* such that something is part of *z* if and only if it is one of the *Y*s (that is, the plurality of all the proper *and* improper parts of *z*).

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[21] *Proof*. Consider an arbitrary entity *x*. Let ϕ be 'v is part of *x*'. By (WCP) there is a plurality *Y* such that the *Y*s are all the parts of *x*. By the definition of fusion, *x* fuses the *Y*s. By CAI, *x* identical to the *Y*s. Suppose that, for some plurality *W*, *x* fuses the *W*s. By CAI *x* is identical to the *W*s. Hence, the *Y*s are identical to the *W*s. QED

However, on the one hand, since not every part of *z* is a red atom (*z* is part of *z* and is not an atom) and yet every part of *z* is one of the *Y*s, it *doesn't* follow from (i) and (ii) that there is the plurality of the red atoms.[22] On the other hand, it is easy to see that (SR1)—which, as we just proved, is entailed by (WCP) and CAI taken together—actually *excludes* that there is the plurality of the red atoms. In fact, if there was such a plurality, then, by (COR), it would correspond to the set of red atoms. In turn, this would entail, by (SR1), that there is some *y* such that for every *z*, *z* is part of *y* if and only if it is a member of the set of the red atoms, or in other words, that there is some *y* that has the red atoms as its all and only (proper *and* improper) parts. But since *y* is part of itself, it would follow that it itself is a red atom. But no red atom can have *all* the red atoms (that is, both *a* and *b*) as its parts: an entity with two different parts is not an atom. Therefore, there is no plurality corresponding to the set of red atoms and there is, thus, no plurality of the red atoms.

Calosi's (2016) and Loss's (2018) arguments for Nihilism make use of incomplete thick pluralities which are pluralities corresponding to what we may call 'incomplete thick sets', that is, sets *s* such that, for some *x*, (i) *x* is a member of *s*, (ii) every member of *s* is part of *x*, (iii) some proper part of *x* is not a member of *s*.[23,24] However, incomplete thick sets are among those sets that, according to CAI+(SR1), can*not* have a corresponding plurality ('$IT(s)$' stands for 's is an incomplete thick set').

(NOC) $\forall s\big(IT(s) \rightarrow {\sim}\exists Y(Y \rhd s)\big)$

Proof. Suppose that *s* is an incomplete thick set having a corresponding plurality *Y*. Since *s* is an incomplete thick set, there is some entity *x* such that, (i) *x* is a member of *s*, (ii) every member of *s* is part of *x*, and yet (iii) some proper part of *x* is not a member of *s*. Since *Y* corresponds to *s*, it follows that (i) *x* is one of the *Y*s (and, therefore, every part of *x* overlaps one of the *Y*s), (ii) each one of the *Y*s is part of *x*, and yet (iii) some proper part of *x* is not one of the *Y*s. From the definition of fusion it follows, thus, that *x* fuses the *Y*s. By CAI, *x* is identical to the *Y*s. From (SR1) we have that, since *Y* corresponds to *s*, there is some entity *z* such that something is a member of *s* if and only if it is part of *z*. *z* fuses the *Y*s (each of the *Y*s is a member of *s* and is, thus, part of *z*; each part of *z* is a member of *s* and is, thus, one of the *Y*s, so that each part of *z* overlaps at least one of the *Y*s, namely *z* itself). Therefore, *z* is identical to the *Y*s. It follows, thus, that *z* is identical to *x*. However, while all the

[22] To be clear, what does *not* follow from (i) and (ii) is the claim (where 'R' stands for 'is a red atom'):

(*) $\exists X\forall y(y \prec X \leftrightarrow Ry)$

[23] *Proof*. Suppose that *Y* is an incomplete thick plurality. This means that, for some *x*, (*a*) *x* is one of the *Y*s, (*b*) *x* fuses the *Y*s, and for some *z*, (*c*) *z* is a proper part of *x* and yet *z* is not one of the *Y*s (see section 3). Consider, now the set *s* of things that are one of the *Ys*. *Y* clearly corresponds to *s*. Since *x* is one of the *Y*s, (i) *x* is a member of *s*. From the fact that *x* fuses the *Y*s it follows that each of the *Y*s is part of *x*. Therefore, (ii) every member of *s* is part of *x*. Some proper part of *x* is not one of the *Y*s, so (iii) it is also not a member of *s*. *s* is, thus, an incomplete thick set. QED

[24] Notice, that if *s* is an incomplete thick set and *x* complies with (i)-(iii), then *x* 'set-fuses' *s*:

(Set-Fusion) $xSetFs =_{df} \forall y(y \in s \rightarrow y \leq x) \wedge \forall y\big(y \leq x \rightarrow \exists z(z \in s \wedge Ozy)\big)$

parts of z are members of s, some part of x is *not* a member of s. *Contradiction* (by LL1)! Therefore, incomplete thick sets don't have corresponding pluralities. QED

Therefore, if (CMP) is rejected and (WCP) is assumed in its place, not only CAI-theorists can successfully deal with the WaBrA problem, but neither Calosi's (2016) nor Loss's (2018) arguments can be used to prove that CAI entails Nihilism as they all rely on a kind of pluralities (incomplete thick pluralities) that don't exist according to CAI+(WCP).

5. On *Atomic Composition as Identity*

Let's recap. Sider's strategy—equivalent to a qualified Σ-weakening of the plural quantifier—seems to be able to deal with both the WaBrA problem and the threat of mereological nihilism. All this appears to confirm that (CMP) is the real culprit behind these problems and that the strategy of Σ-weakening the plural quantifier is the right one.

Beyond (WCP) there appear to be at least two other ways to Σ-weaken the plural quantifier to address both the WaBrA problem and the threat of mereological nihilism. The first corresponds to the following principle of 'Properly Weak Comprehension'

(PWCP) $\exists x\phi x \to \exists Y\exists z\big(zSFu\phi \wedge \forall x(x \prec Y \leftrightarrow x < z)\big)$

which, if conjoined with CAI, entails that the only pluralities corresponding to a set are pluralities of all the *proper* parts of a certain entity:

(SR2) $\forall s\big(\exists X(X \rhd s) \to \exists y\forall z(z \in s \leftrightarrow z < y)\big)$[25]

Alternatively, one could Σ-weaken the plural quantifier to a certain *kind* of proper parts of an entity. Let *atomism* be the claim that every entity is the fusion of a plurality of atoms (in what follows 'Ax' stands for 'x is an atom' and '$\mathbb{A}X$' for '$\forall y(Xy \to Ay)$', that is 'each one of the Xs is an atom'):

(AT) $\forall x\big(Ax \vee \exists Y(\mathbb{A}Y \wedge xFuY)\big)$

If we assume atomism, the most plausible alternative to (PWCP) appears to be the following 'Atomic Comprehension Principle' according to which, if there is at least a ϕ-er, then (i) there is some entity z and some plurality of entities Y such that that (i) z is the S-fusion of everything that ϕs and (ii) Y is the plurality of the *atomic proper parts* of z:[26]

(ACP) $\exists x\phi x \to \exists Y\exists z\big(zSFu\phi \wedge \forall x(x \prec Y \leftrightarrow (x < z \wedge Ax)\big)$

[25] The proof is very similar to the proof that (WCP)+CAI entail (SR1) and is, thus, left to the reader.

[26] Alternatively, one may Σ-weaken the plural quantifier so that it quantifies only to pluralities corresponding to the atomic proper *and improper* parts of a certain entity, thus admitting also improper pluralities of atoms in one's ontology. The reason I am not pursuing this strategy will be clear in section 6 (footnote 37).

(ACP) and CAI entail not only that the only pluralities corresponding to some set are pluralities comprising all and only the atomic proper parts of a certain entity

(SR3) $\forall s\big(\exists X(X \rhd s) \rightarrow \exists y \forall z(z \in s \leftrightarrow (z < y \wedge Az)\big)$[27]

but also that every non-atomic entity is identical to a plurality of atoms

(ATI) $\forall x(\sim Ax \rightarrow \exists Y(\mathbb{A}Y \wedge x = Y))$

and, therefore, that that *every non-atomic entity is identical to a unique plurality of atoms*:

(ATI-2) $\forall x(\sim Ax \rightarrow \exists! Y(\mathbb{A}Y \wedge x = Y))$

In what follows I will call the theory comprising both CAI and (ACP) 'Atomic Composition as identity' or 'ACAI' for short.

It is easy to see that, as CAI+(WCP), both CAI+(PWCP) and ACAI can successfully deal with the WaBrA problem and the threat of Nihilism. In fact, as we saw in the previous sections, the WaBrA problem and the threat of Nihilism seem to require either (i) the existence of different pluralities fused by the same composite entity, or (ii) the existence of incomplete thick pluralities, and it is easy to check that CAI+(PWCP) and ACAI exclude both.[28] The question is, thus, whether there is some reason to prefer one of the three versions of CAI over the others. In what follows I will argue that this is in fact the case and that CAI-theorists should prefer ACAI over both CAI+(WCP) and CAI+(PWCP).

Let a set s be *crowded* just in case it has at least two members (so that s is neither the empty set nor a singleton):

(Crowded set) $C(s) =_{df} \exists x \exists y(x \neq y \wedge x \in s \wedge y \in s)$

Consider, then, following four principles (where '$\cup$', '$\cap$', and '$-$' stand for set-union, set-intersection, and set-difference):

(PQ1) $\forall X \forall Y \forall s \forall r\Big(\big(X \rhd s \wedge Y \rhd r \wedge C(s \cup r)\big) \rightarrow \exists W(W \rhd s \cup r)\Big)$

(PQ2) $\forall X \forall Y \forall s \forall r\Big(\big(X \rhd s \wedge Y \rhd r \wedge C(s - r)\big) \rightarrow \exists W(W \rhd s - r)\Big)$

[27] The proof is very similar to the proof that (WCP)+CAI entail (SR1) and is, thus, left to the reader.

[28] (ATI-2) immediately excludes the possibility of different pluralities fused by the same entity, while from CAI+(PWCP) it follows that every composite entity is the fusion of only the plurality of its proper parts. From (SR2) it follows that there are only pluralities containing all the proper parts of certain entities, thus ruling out the existence of incomplete thick pluralities. Instead, (SR3) excludes the existence of incomplete thick pluralities by entailing that no composite entity can be a member of a plurality.

(PQ3) $\forall X \forall Y \forall s \forall r \left(\left(X \rhd s \wedge Y \rhd r \wedge C(s \cap r) \right) \rightarrow \exists W (W \rhd s \cap r) \right)$

(PQ4) $\forall X \forall s \left(X \rhd s \rightarrow \forall r \left((C(r) \wedge r \subset s) \rightarrow \exists Y (Y \rhd r) \right) \right)$

Suppose that s and r are sets that have a corresponding plurality. (PQ1-3) say that their union, intersection and difference also have a corresponding plurality, provided that they are crowded. Instead, (PQ4) says that if a set has a corresponding plurality, then each of its crowded proper subsets has one.

(PQ1)-(PQ3) strike one as highly plausible. They entail that pluralities corresponding to sets have 'plural unions', 'plural intersections' and 'plural differences', so to speak (provided that the union, intersection and difference of their corresponding sets is crowded). (PQ4) also has the ring of intuitiveness to it. How could the plurality consisting of the Earth, Mars, and Venus exist without there being any plurality containing only Mars and Venus? However, it is easy to check that (PQ1), (PQ2) and (PQ4) are invalidated both by CAI+(WCP) and by CAI+(PWCP). For instance, in the WaBrA scenario there is, according to CAI+(WCP), a plurality corresponding to the set {A1,A2,A3,B1,B2,B3,C}, and according to CAI+(PWCP), a plurality corresponding to the set {A1,A2,A3,B1,B2,B3}. However, in both cases there is no plurality corresponding to the crowded set {A1,A2,A3}, *contra* (PQ4). Similarly, according to both CAI+(PWCP) and CAI+(WCP) B1 and B2 are identical to two pluralities corresponding to two sets ({B1,A1,A2} and {B2,A1,A3} for CAI+(WCP); {A1,A2} and {A1,A3} for CAI+(PWCP)) such that their union ({B1,B2,A1,A2,A3} and {A1,A2,A3}, respectively) is a crowded set without no corresponding plurality, thus contradicting (PQ1). As for (PQ2), consider {A1,A2,A3,B1,B2,B3,C} and {A1,A2,B1} for CAI+(WCP), and {A1,A2,A3,B1,B2,B3} and {A1,A2} for CAI+(PWCP).[29]

Instead, (PQ1)-(PQ4) are all validated by ACAI. The crowded union of two sets of atoms r and s (if any) has as corresponding plurality the plurality of atoms contained by either r or s. Their crowded intersection (if any) has as corresponding plurality the plurality of atoms that are members of both r and s. Their crowded difference (if any) is just the plurality of atoms belonging to r but not to s. Finally, since every crowded proper subset of a set of atoms is a crowded set of atoms (SR3) guarantees also the validity of (PQ4).

[29] Instead, (PQ3) appears to be validated by CAI+(WCP) but not by CAI+(PWCP). *Proof*. Suppose that $X \rhd s$, $Y \rhd r$, and $C(s \cap r)$. Letting ϕ be '$v \in s \cap r$', it follows by (USF) that some entity *k* S-fuses everything that is a member of $s \cap r$. It can be proved that, for every *z*, *z* is part of *k* if and only if it is a member of $s \cap r$ (The proof is left to the reader [*Hint*: if $s = r$, the proof is trivial; if $s \neq r$, then both (SR1) and (SR2) entail that *k* is the mereological product—see below—of the fusion of the *X*s with the fusion of the *Y*s]). Assuming (WCP), there is a plurality *W* such that *W* is the plurality of the *proper and improper* parts of *k*. By (COR), *W* corresponds, thus, to $s \cap r$. *Instead*, from (SR2) we have that there is a plurality corresponding to $s \cap r$ *only if* there is some entity *z* such that $s \cap r$ is the set of all the *proper* parts of *z*. Suppose such an entity exists. *z* is not a proper part of *z* and is, thus, not a member of $s \cap r$. It follows, therefore, that either *z* is not a part of *x*, or *z* is not a part of *y*. However, every part of *z* overlaps both *x* and *y* so that, by Strong Supplementation (see the Appendix), *z* is part of both *x* and *y*. Hence, *z* is a member of $s \cap r$. *Contradiction*! Therefore, if (PWCP) is assumed, there is no plurality corresponding to $s \cap r$. QED

Furthermore, let us define the notion of mereological sum, mereological difference and mereological product as follows (‘$\iota z.\phi$’ stands for the definite description ‘the z such that ϕ’):[30]

(Sum) $x + y =_{df} \iota z.\big(x \leq z \wedge y \leq z \wedge \forall w(w \leq z \rightarrow (Owx \vee Owy))\big)$

(Difference) $x - y =_{df} \iota z.\forall w(w \leq z \leftrightarrow (w \leq x \wedge {\sim}Owy))$

(Product) $x \times y =_{df} \iota z.\forall w(w \leq z \leftrightarrow (w \leq x \wedge w \leq y))$

In addition, let (i) ‘$\mathbb{P}^x$’ stand for ‘the plurality *Y* such that x is identical to the *Y*s’, (ii) ‘$\cup$’, ‘$\cap$’, and ‘$-$’ stand (for simplicity’s sake) also for the notions of plural union, plural intersection, and plural difference, and (iii) ‘$IY.\phi$’ stand for the definite description ‘the plurality *Y* such that ϕ’:

($\mathbb{P}^x$) $\mathbb{P}^x =_{df} IY.(x = Y)$[31]

($\cup$) $X \cup Y =_{df} IW.\forall z(z \prec W \leftrightarrow (z \prec X \vee z \prec Y))$

($-$) $X - Y =_{df} IW.\forall z(z \prec W \leftrightarrow (z \prec X \wedge {\sim}z \prec Y))$

($\cap$) $X \cap Y =_{df} IW.\forall z(z \prec W \leftrightarrow (z \prec X \wedge z \prec Y))$

It is then easy to prove that ACAI allows for an interesting correspondence between the notions of mereological sum, difference, and product and the notions of plural union, plural difference, and plural intersection, respectively. In fact, ACAI entails that (MP1)-(MP3) hold *for every non-atomic x, y and z*: [32]

(MP1) $x = y + z \leftrightarrow \mathbb{P}^x = \mathbb{P}^y \cup \mathbb{P}^z$

(MP2) $x = y - z \leftrightarrow \mathbb{P}^x = \mathbb{P}^y - \mathbb{P}^z$

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[30] I assume in what follows that both singular and plural (see below) definite descriptions are eliminable using Russell’s theory of descriptions:

(RT1) $F(\iota x.\phi x) =_{df} \exists x(\phi x \wedge \forall y(\phi y \rightarrow y = x) \wedge Fx)$

(RT2) $P(IX.\phi X) =_{df} \exists X(\phi X \wedge \forall Y(\phi Y \rightarrow Y = X) \wedge P(X))$

[31] Notice that it follows directly from ACAI that, for every non-atomic *x*, something is one of the $\mathbb{P}^x$s if *and only if* it is an atomic part of *x* or, in other words, that $\mathbb{P}^x$ is the plurality of the *atomic parts* of *x*:

($\mathbb{P}$A) $\forall x\big({\sim}Ax \rightarrow \forall z(\mathbb{P}^x z \leftrightarrow (Az \wedge z \leq x))\big)$

[32] *Proof.* See the Appendix.

(MP3) $x = y \times z \leftrightarrow \mathbb{P}^x = \mathbb{P}^y \cap \mathbb{P}^z$

Notice, furthermore, that it follows from ACAI that every non-atomic entity x S-fuses everything that ϕs if and only if the plurality to which it is identical contains all and only the atomic parts of all the things that ϕ, or in other words, that (PQ-SF) holds *for every non-atomic x*:[33]

(PQ-SF) $xSFu\phi \leftrightarrow \forall z\left(\mathbb{P}^x z \leftrightarrow \left((Az \wedge \phi z) \vee \exists w(\phi w \wedge \mathbb{P}^w z)\right)\right)$

(MP1)-(MP3) and (PQ-SF) are principles that appear to be highly plausible within a CAI framework. However, CAI+(WCP) and CAI+(PWCP) fail to validate both (MP1) and (PQ-SF).[34] In the WaBrA scenario, for instance, the following counterexamples to (MP1) can be inferred from both CAI+(WCP) and CAI+(PWCP), as according to both there is no plural union of $\mathbb{P}^{\mathrm{B1}}$ and $\mathbb{P}^{\mathrm{B3}}$:

(CE1) $\mathrm{C} = \mathrm{B1} + \mathrm{B3} \wedge \mathbb{P}^{\mathrm{C}} \neq \mathbb{P}^{\mathrm{B1}} \cup \mathbb{P}^{\mathrm{B3}}$

Similarly, in the WaBrA scenario the wall C S-fuses everything that is identical to either B1 or B3. However, according to both CAI+(WCP) and CAI+(PWCP), (i) B2 is one of the $\mathbb{P}^{\mathrm{C}}$s and yet (ii) B2 (which is not an atom) is neither one of the $\mathbb{P}^{\mathrm{B1}}$s nor one of the $\mathbb{P}^{\mathrm{B3}}$s. It is, thus, possible to conclude that—at least insofar as the WaBrA problem and the threat of Nihilism are concerned—CAI-theorists have good reasons to prefer ACAI as the best version of their theory.

Before moving further, notice that if the world is at least as mereologically structured as in the WaBrA scenario, both Collapse and Plural Covering are *in*validated by ACAI: (i) the wall fuses the plurality of the atoms without being one of the atoms (*contra* Collapse); (ii) the brick B1 is part of the wall, but there is no plurality *Y*, such that the wall fuses the *Y*s and B1 is one of the *Y*s (*contra* Plural Covering): for instance, ACAI rules out that there is a plurality of entities that is the plurality of *the bricks*. This situation has admittedly something odd to it. In fact, even if every brick is part of the wall, and each part of the wall overlaps at least one brick, we cannot say that the wall fuses *the bricks*, if by 'the bricks' we purport to refer to the plurality of entities such that something is one of them if and only if it is a brick. Notice, however, that there are other notions of fusion that may help us in this case. For instance, one may adopt the notion of 'set-fusion'

(Set-Fusion) $xSetFs =_{df} \forall y(y \in s \rightarrow y \leq x) \wedge \forall y\left(y \leq x \rightarrow \exists z(z \in s \wedge Ozy)\right)$

[33] *Proof.* See the Appendix.

[34] (MP2) and (MP3) are validated by CAI+(WCP) but not by CAI+(PWCP). The proof is left to the reader (see, however, footnote 29 for (MP3): k is the product of x and y, yet only according to CAI+(WCP) the plural intersection of $\mathbb{P}^x$ and $\mathbb{P}^y$ is the plurality to which k is identical).

and claim that the wall set-fuses the set $\{B1,B2,B3\}$. Or, alternatively, one may simply say that the wall schematically fuses (see section 4) 'v is a brick'. Furthermore, ACAI validates the following 'atomic' counterparts of Collapse and Plural Covering:[35]

(Atomic Collapse) $\forall x \forall Y \big(xFuY \rightarrow \forall z (z \prec Y \leftrightarrow (z < x \wedge Az)\big)$

(Atomic Covering) $\forall x \forall y \Big(x < y \rightarrow \exists W \Big(yFuW \wedge \big((Ax \wedge x \prec W) \vee \mathbb{P}^x \subseteq W\big)\Big)\Big)$

According to Atomic Collapse, if *x* fuses the *Y*s, then something is one of the *Y*s if and only if it is an atomic proper part of *x*. According to Atomic Covering, if *x* is part of *y*, then there is a plurality *W* of entities that *y* fuses and is such that, either *x* is an atom and *x* is one of the *W*s, or the plurality *Y* that is identical to *x* (which, given ACAI, must be a plurality of atoms) is such that the *Y*s are among the *W*s. Therefore, even if, strictly speaking, the wall *doesn't* fuse 'the bricks', it still fuses what we may call the 'atomic footprint' of the bricks, that is, the plural union of all the pluralities that are identical to a brick.

6. Other paths to Nihilism?

There appear to be two final arguments potentially threatening ACAI of collapsing into Nihilism that need to be discussed:

> *First argument* [adapted from Calosi (2018: 287-8)]
> Suppose *y* is a composite entity. By (AT), there is a proper plurality *W* of atoms such that *y* fuses the *W*s. Consider an arbitrary entity *x* such that *x* is one of the *W*s. By ACAI *y* is identical to the *W*s. By (LL3) *x* is, thus, one of *y*.[36] However, under the 'most natural reading' (Calosi 2018: 287) of '*x* is one of *y*' it follows that *x* is identical to *y*. *Contradiction*! Therefore, there are no composite entities.
>
> *Second argument* [adapted from Yi (1999b: 146)]
> Suppose *y* is a composite entity. By (AT), there is a proper plurality *W* of atoms such that *y* fuses the *W*s. Everything is one of itself. Therefore, *y* is one of *y*. By ACAI, *y* is identical to the *W*s. By (LL3) *y* is, thus, one of the *W*s. But *y* is not an atom and so it is not one of the *W*s. *Contradiction*! Therefore, there are no composite entities.

These arguments rely on the following principles concerning the one-of relation:

(OF1) $\forall x \big((x \prec y) \rightarrow x = y\big)$

(OF2) $\forall x (x \prec x)$

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[35] The proof is straightforward and is, thus, left to the reader.

[36] See below on the (admittedly awkward sounding) phrase '*x* is one of *y*'.

(OF1) and (OF2) can be seen as following from the principle, according to which x is one of y if and only if it is identical to y:

(OF3) $\forall x \forall y(x \prec y \leftrightarrow x = y)$

Calosi (2018) explicitly acknowledges that 'there may be other readings [of 'x is one of y'] such that the desired conclusion does *not* follow' (Calosi 2018: 290, fn 34) so that the task of the friends of CAI is that 'to come up with a reading of the problematic formula that does not entail the more problematic conclusion' (*Ibid*.). ACAI-theorists appear to be perfectly in position to meet this challenge. In fact, ACAI-theorists can claim that the one-of relation should be understood as a notion of being *properly* one-of, so to speak. In other words, using 'α' as a 'generic' variable that can take both singular entities and pluralities as values ACAI-theorists can claim that for any (either singular or plural) α, an entity x is one of α only if it is *properly* included in α and is, thus, *different* from α. According to this picture, nothing can be one of itself:

(OF4) $\forall x \forall \alpha(x \prec \alpha \rightarrow x \neq \alpha)$

(OF5) $\sim \exists x(x \prec x)$

(OF4) and (OF5) block both arguments for Nihilism just reviewed. However, the first argument still shows that the atom x is one of y (a composite entity). To this ACAI-theorists can reply that an entity x is one of a certain α if and only if α is identical to a plurality W of entities (which within ACAI is guaranteed to be a proper plurality of atoms) such that x is one of the Ws:

(OF6) $\forall x \forall \alpha\big(x \prec \alpha \leftrightarrow \exists W(W = \alpha \wedge x \prec W)\big)$

Therefore, ACAI-theorists can accept that the atom x is not only part of y¸ but also *one of* y. This, however, shouldn't come as a surprise, given that y is *identical* to the Ws and x is one of the Ws. In the same vein, ACAI-theorists can continue, since being one of α entails that α is identical to a plurality, and thus, that α is identical to a plurality of atoms, it follows that only *atoms* can be one of something:

(OF7) $\forall x \forall \alpha(x \prec \alpha \rightarrow Ax)$

Therefore, ACAI-theorists can conclude, an entity x is one of another entity y just in case x is an *atomic proper part* of y: [37]

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[37] Since, according to ACAI, there are no improper pluralities it follows that atoms are not fusions of any plurality. Notice that an atomic Σ-weakening allowing also the existence of improper pluralities of atoms would be incompatible with (OF4-5) given that—quite independently from CAI—it is highly plausible to think that

(OF8) $\forall x \forall y(x \prec y \leftrightarrow (x < y \wedge Ax)$

Some may retort that to say things like 'the atom is one of the wall' is ungrammatical and that the one-of predicate should be thought of as admitting only of plural terms on its right-hand side. Notice, however, that—as noticed by Calosi (2016b: 12, fn 30, 2018: 290, fn 34)—the idea that the second place of 'is one of' can admit of singular terms is pretty common in the literature on mereology. What is controversial is the idea that an entity x may be one of a *different* entity y. However, once it is assumed that (i) y is identical to a plurality of entities W and that (ii) x is one of the Ws, the idea that x is one of y shouldn't sound too outlandish. In fact, ACAI-theorists can actually claim that this is exactly the result one should expect: if x is one of the Ws and y is *identical* to the Ws, then surely x is also one of y.

Notice, finally, that it is possible to define on the basis of the primitive notion of 'being (properly) one of' a corresponding *improper* notion as follows:

($\preccurlyeq$) $x \preccurlyeq \alpha =_{df} x \prec \alpha \vee x = \alpha$

Everything is improperly one of itself in this sense. This can help ACAI-theorists explain why some may have thought that (OF2) is a valid principle: it is just because they were confusing the notion of 'being properly one of' and the notion of 'being improperly one of'. At the same time, whenever x is *not* an atom, it follows that x is improperly one of y if and only if x is identical to y. The same is true also whenever y is *not* identical to any plurality of atoms. In turn, this can help ACAI-theorists explain the intuitions of those who thought (OF1) and (OF3) to be true: they were simply either thinking of cases in which the entity that is one of another entity is not an atom or implicitly assuming the falsity of ACAI. Therefore, by employing a primitive notion of 'being properly one of' and by defining in terms of it a notion of 'being improperly one of' ACAI-theorists appear to be in position to not only ward off the objections represented by the two arguments opening this section, but also to explain why some may have thought that the problematic principles (OF1-3) had the ring of plausibility to them.

(OF4)-(OF8) express a notion of being one-of that appears to be both plausible and legitimate within the framework of ACAI. It appears, thus, possible to conclude that—at

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an improper plurality is identical to its only member. In this case, one may reformulate (OF4-5) as follows and allow atoms to be one of themselves:

(OF4*) $\forall x \forall \alpha(x \prec \alpha \rightarrow (x \neq \alpha \vee A\alpha))$

(OF5*) $\forall x(x \prec x \rightarrow Ax)$

Although this route seems to be viable (at least *prima facie*) the asymmetry it introduces between entities that can be one of themselves (atoms) and entities that cannot be one of themselves (composite entities) strikes me as unnatural. It appears to be more natural to think of the notion of being properly one-of by analogy to other familiar 'proper' notions, like that of proper part and proper subset, which are irreflexive across the board, so to speak (see below on the possibility to define an improper one-of notion on the basis of the proper one).

least absent other considerations—ACAI-theorists needn't be too worried by Calosi's and Yi's objections.[38]

7. One and many

A final objection to ACAI addresses what is not only the first and main concern many have with respect to CAI but also the very reason Lewis famously came only close to CAI without ever fully embracing it:

> […] even though the many and the one are the same portion of Reality […] still we do not really have a generalized principle of indiscernibility of identicals. It does matter how you slice it—not to the character of what's described, of course, but to the form of the description. What's true of the many is not exactly what's true of the one. After all *they are many while it is one*. (Lewis 1991: 87; italics mine)

Clearly, in order to articulate this argument, we must first clarify what 'being one' and 'being many' amount to. Yi (2014) defines 'being one' and 'being many' as follows:

(O1) $\mathbf{One_1}(\alpha) =_{df} \exists y \forall z(z \prec \alpha \leftrightarrow z = y)$
'They are *one* ≡: something is such that something is one of them if and only if the latter thing is the former thing' (Yi 2014: 175)

(M1) $\mathbf{Many_1}(\alpha) =_{df} \exists y \exists z(y \neq z \wedge y \prec \alpha \wedge z \prec \alpha)$
'They are *many* ≡: there is something that is one of them and something else that is one of them.' (Yi 2014: 175)

According to these definitions, being one and being many are incompatible characteristics. Therefore, the argument goes, supposing that a is the fusion of the *W*s and the *W*s are the atoms *b* and *c*, a cannot be identical to the *W*s, given that *a* is one but not many and the *W*s are many but not one. With (OF4)-(OF8) in place this argument is blocked.[39] Yet, given (O1) and (M1), Yi's argument appears to be blocked for the wrong reason, since (OF4)-(OF8), (O1), and (M1) jointly predict that *both a and* the *W*s are *many* without being *one*.[40] Fortunately, ACAI-theorists can give the following simple, alternative account of the notions of 'being one' and 'being many':

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[38] Yi (1999b: 147; 2014: 178) advances also a version of the second argument presented in this section based on a relation '$\mathbf{H}'$' (Yi 2014: 178) defined on the basis of the 'is one of' relation as follows ('$[x_1, \dots, x_n]$' stands for 'the plurality of entities that are identical to either x_1,…, or x_n'):

($\mathbf{H}'$-df) $x\mathbf{H}'\alpha =_{df} \forall y(x \prec [y, \alpha])$

However, since for ACAI no plurality contains a composite entity, there is no composite entity *x* that for ACAI is such that $x\mathbf{H}'x$, so that Yi's second version of the second argument presented in this section is also blocked.

[39] According to (OF5), nothing can be one of itself. Therefore, a cannot be *one* in the sense of (O1).

[40] In fact, $\exists y \forall z(z \prec a \leftrightarrow z = y)$ contradicts (OF4), while $\exists y \forall z(z \prec W \leftrightarrow z = y)$ would entail that W is an improper plurality, *contra* ACAI.

(O2) $\mathbf{One_2}(\alpha) =_{df} \exists x(\alpha = x)$

(M2) $\mathbf{Many_2}(\alpha) =_{df} \exists x(x \prec \alpha)$

According to (O2) and (M2) α is *one* if and only if it is identical to a *singular* entity, while it is *many* if and only if something is *one of* α. Notice that, given (OF6) and (ACP), (M2) is equivalent to (M3), which says that to be many is to be identical to a proper plurality of entities:

(M3) $\mathbf{Many_2}(\alpha) =_{df} \exists X\big(\alpha = X \land \exists y \exists z(y \neq z \land y \prec X \land y \prec X)\big)$

On the background of ACAI and (OF4-8), (O2) and (M2)/(M3) are not incompatible. If, in fact, a thing *x* can be identical to a proper plurality of entities *Y*, then both *x* and the *Y*s are *both* one *and* many: *x* is identical to *x*, and thus, it is one, but is also such that some atom is indeed one of *x*, so that it is many; *Y* is such that for some *z*, *z* is one of the *Y*s, so that the *Y*s are many, but the *Y*s are also identical to some singular entity (that is, x), so that they are one. Therefore, both *x* and the *Y*s *can* both be both one and many, which is precisely as it should be, given that x and the Ys are identical.

The way in which we have just reformulated Yi's definition of 'being many' suggests also the following reformulations of Yi's (1999a) definitions of 'being two', 'being three', *et cetera*:

(Two) $\mathbf{Two}(\alpha) =_{df} \exists x \exists y \Big(x \neq y \land \forall z\big(z \prec \alpha \leftrightarrow (z = x \lor z = y)\big)\Big)$

(Three) $\mathbf{Three}(\alpha) =_{df} \exists x \exists y \exists z \Big(x \neq y \land x \neq z \land y \neq z \land \forall w\big(w \prec \alpha \leftrightarrow (w = x \lor w = y \lor w = z)\big)\Big)$

(…) ...

(*n*) $\boldsymbol{n}(\alpha) =_{df} \exists x_1 \dots \exists x_n \Big(x_1 \neq x_2 \land \dots \land x_1 \neq x_n \land x_2 \neq x_3 \land \dots \land x_2 \neq x_n \land \dots \dots \land x_{n-1} \neq x_n \land \forall y\big(y \prec \alpha \leftrightarrow (y = x_1 \lor \dots \lor y = x_n)\big)\Big)$

According to these definitions, to be two is to 'contain' (in the sense of the one-of relation) exactly two things, to be three is to contain exactly three things, …*et cetera*. Therefore, not only composite entities can be *both* one *and* many, but since according to ACAI every composite entity is identical to the plurality of atoms it fuses, it follows that every composite entity can be associated with *two* numbers, so to speak: *one*, and *the number of atoms it fuses*. So, in our toy example, *a* is both one entity (that is, a) and two entities (that is, *b* and *c*). Similarly, *b* and *c* (taken together) are both two entities (namely, *b* and *c*) and one (namely, a). This, however, shouldn't come as a surprise, given that a is identical to b and c taken together.

8. Conclusion

Let's sum up. ACAI is the theory that endorses CAI and for which the only pluralities that are quantified over by the plural quantifier are proper pluralities of atoms. As I have argued, ACAI

(i) is immune from the WaBrA problem;
(ii) doesn't entail mereological nihilism;
(iii) validates the plausible principles (PQ1-4), (MP1-3) and (PQ-SF);
(iv) can assume a reading of the one-of relation that successfully wards off the objections raised by Calosi (2016b) and Yi (1999a, 2014);
(v) can clearly explain, drawing on Yi (1999b), how something can consistently be both one and many, and (drawing on Yi 1999a) also how composite objects can be associated with two numbers (one, and the number of atoms they fuse).

(i)-(v) strike one as very important theoretical virtues showing at least that ACAI is a stable theory that has the potential of becoming a serious contender in the current debate on mereology.

One may worry that ACAI's commitment to atomism is costly. I have two replies. First, as every strong version of CAI, ACAI's overall cost must be measured against what appears to be the main motivation behind the theory: giving a clear and straightforward account of the innocence of mereology. If, as I have been assuming here, ACAI is indeed successful in this task, then atomism may after all be a bullet well worth biting. Second, at least some of the rival theories on the mereological market have been presented in combination with atomism, like Cotnoir's (2013) version of CAI[41] and Sider's (2013) version of mereological nihilism. Therefore, the dialectical disadvantage of assuming atomism in this debate may be less severe than it may seem at first sight.

Needless to say, more discussion is clearly needed to provide a full defence of ACAI as the true theory of parthood. Be that as it may, however, if what has been argued in this paper is on the right track, it appears possible to conclude that ACAI is a theory that deserves careful attention in the contemporary debate on parthood, identity and composition.

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[41] To be fair, Cotnoir leaves it actually open whether the ultimate entities in his theory are atoms: 'You may think of them as atoms, but one *needn't* think of them as atoms. […] Another possible interpretation—to be explored in future work—is to think of them merely as propertied spacetime points. I largely leave the underlying metaphysics open, since the semantic approach endorsed here is compatible with a number of metaphysical views.' (Cotnoir 2013: 302).

Appendix

I. ACAI entails (MP1-3)
ACAI entails (L1):

(L1) $\forall x\left(\left(\sim Ax \wedge \forall z(z \leq x \leftrightarrow \phi_v z)\right) \rightarrow \mathbb{P}^x = IW.\forall z\left(Wz \leftrightarrow (Az \wedge \phi_v z)\right)\right)$

Proof. Suppose that *x* is not an atom and that something is a ϕ-er if and only if it is a part of *x*. By (AT), every ϕ-er has some atomic part. By (ACP), there is, thus, a plurality *W* such that something is one of the *W*s if and only if it is an atomic ϕ-er. Each of the *W*s is a ϕ-er and is, thus, part of *x*. Therefore, every atomic ϕ-er is part of *x*. By (AT), every part of *x* has atomic parts. By transitivity, every atomic part of a part of *x* is an atomic part of *x* and is, thus, an atomic ϕ-er. Therefore, every part of *x* overlaps an atomic ϕ-er. By the definition of fusion, *x* fuses the *W*s. By ACAI, *x* is identical to the *W*s. By (ATI-2), $\mathbb{P}^x$ is identical to *W*. QED

(T1) is a theorem of classical mereology that follows from the definition of mereological sum and Strong Supplementation (which is itself a theorem of classical mereology):

(Strong Supplementation) $\forall x \forall y\left(x \nleq y \rightarrow \exists z(z \leq x \wedge \sim Ozy)\right)$

(T1) $\forall x \forall y \forall z\left(x = y + z \rightarrow \forall w\left(w \leq x \leftrightarrow \forall k\left(k \leq w \rightarrow (Oky \vee Okz)\right)\right)\right)$

Proof. Suppose that *x* is the sum of *y* and *z*. (*a*) Let *w* be an arbitrary part of *x*. By the definition of sum, every part of *x* overlaps either *y* or *z*. By the transitivity of parthood, every part of *w* overlaps either *y* or *z*. (*b*) Let *w* be an entity such that every part of *w* overlaps either *y* or *z* and suppose that *w* is not part of *x*. Since *y* and *z* are parts of *x* it follows that every part of *w* overlaps *x*. By Strong Supplementation, *w* is part of *x*. *Contradiction*! Therefore, something is part of *x* if and only if all of its parts overlap either *y* or *z*. QED

The left-to-right directions of (MP1-3) can be, thus, proved as follows (*x*, *y* and *z* are thought of as ranging over *non-atomic* entities):

(MP1-lr) $x = y + z \rightarrow \mathbb{P}^x = \mathbb{P}^y \cup \mathbb{P}^z$

Proof. Suppose that *x* is the sum of *y* and *z*. By (T1), something is part of *x* if and only if all of its parts overlap either *y* or *z*. Letting ϕ be '$\forall w\left(w \leq v \rightarrow (Owy \vee Owz)\right)$' it follows from (L1) and the definition of mereological sum that $\mathbb{P}^x$ is the plurality of atoms overlapping either *y* or *z*, and thus the plurality of the atomic parts of either *y* or *z*. Therefore, by the definitions of '$\mathbb{P}^x$' and '$\cup$', it follows that $\mathbb{P}^x = \mathbb{P}^y \cup \mathbb{P}^z$. QED

(MP2-lr) $x = y - z \rightarrow \mathbb{P}^x = \mathbb{P}^y - \mathbb{P}^z$

Proof. Suppose that *x* is identical to $y - z$ and let ϕ be '$v \leq y \wedge {\sim}Ovz$'. It follows from (L1) and the definition of mereological difference that $\mathbb{P}^x$ is the plurality of the atomic parts of *y* that don't overlap *z*. Therefore, $\mathbb{P}^x$ is the plurality of the atomic parts of *y* that are not part of *z*. By the definitions of '$\mathbb{P}^x$' and '$-$', it follows, thus, that $\mathbb{P}^x = \mathbb{P}^y - \mathbb{P}^z$. QED

(MP3-lr) $x = y \times z \rightarrow \mathbb{P}^x = \mathbb{P}^y \cap \mathbb{P}^z$

Proof. Suppose that *x* is identical to $y \times z$ and let ϕ be '$v \leq y \wedge v \leq z$'. It follows from (L1) and the definition of mereological product that $\mathbb{P}^x$ is the plurality of atoms that are parts of both *y* and *z*. Therefore, by the definitions of '$\mathbb{P}^x$' and '$\cap$', it follows that $\mathbb{P}^x = \mathbb{P}^y \cap \mathbb{P}^z$. QED

From the definition of '$\mathbb{P}^x$' and ($\mathbb{P}$A)[42]

($\mathbb{P}$A) $\forall x \left({\sim}Ax \rightarrow \forall z\big(\mathbb{P}^x z \leftrightarrow (Az \wedge z \leq x)\big)\right)$

it is possible to derive (L2) which says that, for every *non-atomic* *x* and *y*, *x* is part of *y* if and only if the $\mathbb{P}^x$s are among ('$\subseteq$') the $\mathbb{P}^y$s:

($\subseteq$) $X \subseteq Y =_{\text{df}} \forall z(Xz \rightarrow Yz)$

(L2) $\forall x \forall y\big(({\sim}Ax \wedge {\sim}Ay) \rightarrow (y \leq x \leftrightarrow \mathbb{P}^y \subseteq \mathbb{P}^x)\big)$

Proof. *Left-to-right*. Suppose that (i) *x* and *y* are not atomic, (ii) *y* is part of *x*, and (iii) *w* is one of the $\mathbb{P}^y$s. By ($\mathbb{P}$A), *w* is an atomic part of *y*. Therefore, since *y* is part of *x*, *w* is also an atomic part of *x*. By generalisation, the $\mathbb{P}^y$s are among the $\mathbb{P}^x$s. *Right-to-left*. Suppose that (i) *x* and *y* are both not atomic, (ii) $\mathbb{P}^y \subseteq \mathbb{P}^x$, and (iii) *y* is not part of *x*. By Strong Supplementation, there is a part *w* of *y* that doesn't overlap *x*. Therefore, since, by (AT), *w* is either an atom or has atomic parts, there must be an atomic part *v* of *y* that doesn't overlap *x* and is, thus, not part of *x*. Since *v* is an atomic part of *y*, it is one of the $\mathbb{P}^y$s. But we are assuming that each of the $\mathbb{P}^y$s is also one of the $\mathbb{P}^x$s. *Contradiction*! Therefore, *y* is part of *x*. QED

The right-to-left directions of (MP1-3) can be proved by means of (L2) (*x*, *y* and *z* are thought of as ranging over *non-atomic* entities):

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[42] See footnote 31.

(MP1-rl) $\mathbb{P}^x = \mathbb{P}^y \cup \mathbb{P}^z \rightarrow x = y + z$

Proof. Suppose that $\mathbb{P}^x = \mathbb{P}^y \cup \mathbb{P}^z$. (*a*) Clearly, $\mathbb{P}^y \subseteq \mathbb{P}^y \cup \mathbb{P}^z$ and $\mathbb{P}^z \subseteq \mathbb{P}^y \cup \mathbb{P}^z$. By (L2), both *y* and *z* are parts of *x*. (*b*) Suppose that *w* is part of *x*. If *w* is an atom, then *w* is one of the $\mathbb{P}^y \cup \mathbb{P}^z$ and so it clearly overlaps either *y* or *z*. If *w* is not atomic, we have by (L2) that $\mathbb{P}^w \subseteq \mathbb{P}^y \cup \mathbb{P}^z$, so that all the atomic parts of *w* are atomic parts of either *y* or *z*. Therefore, *w* overlaps either *y* or *z*. Either way, *w* and thus, by generalization, every part of *x* overlaps either *y* or *z*. Therefore, *x* is the sum of *y* and *z*. QED

(MP2-rl) $\mathbb{P}^x = \mathbb{P}^y - \mathbb{P}^z \rightarrow x = y - z$

Proof. Suppose that $\mathbb{P}^x = \mathbb{P}^y - \mathbb{P}^z$.
Part I. Let *w* be part of *x*. (*a*) Suppose that *w* is not atomic. Then, by (L2), $\mathbb{P}^w \subseteq \mathbb{P}^x$ and, so $\mathbb{P}^w \subseteq \mathbb{P}^y - \mathbb{P}^z$. Clearly, $\mathbb{P}^y - \mathbb{P}^z \subseteq \mathbb{P}^y$. Therefore, $\mathbb{P}^w \subseteq \mathbb{P}^y$, and by (L2), *w* is part of *y*. Suppose *w* overlaps *z*. Then some *v* is part of both *w* and *z*. By (L2), we have that $\mathbb{P}^v \subseteq \mathbb{P}^w$ and $\mathbb{P}^v \subseteq \mathbb{P}^z$. Therefore, each of the $\mathbb{P}^v$s is both one of the $\mathbb{P}^w$s and one of the $\mathbb{P}^z$s, thus contradicting $\mathbb{P}^w \subseteq \mathbb{P}^y - \mathbb{P}^z$ (by the definition of '$-$'). Therefore, every non-atomic part of *x* is a part of *y* that doesn't overlap *z*. (*b*) Suppose that *w* is an atom. Then *w* is one of the $\mathbb{P}^x$s and, by the definition of plural difference, also one of the $\mathbb{P}^y$s without being one of the $\mathbb{P}^z$s. Suppose *w* overlaps *z*. Since *w* is an atom, *w* is an atomic part of *z* and, thus, one of the $\mathbb{P}^z$s. *Contradiction*! Therefore, also every atomic part of *x* is a part of *y* that doesn't overlap *z*.
Part II. Let *w* be a part of *y* that doesn't overlap *z*. (*a*) Suppose that *w* is not an atom. By (L2), we have, thus, that $\mathbb{P}^w \subseteq \mathbb{P}^y$ and that none of the $\mathbb{P}^w$s is one of the $\mathbb{P}^z$s. Therefore, $\mathbb{P}^w \subseteq \mathbb{P}^y - \mathbb{P}^z$ and thus $\mathbb{P}^w \subseteq \mathbb{P}^x$. By (L2), *w* is part of *x*. (*b*) Suppose that *w* is an atom. *w* is, thus, an atomic part of *y* and, thus, one of the $\mathbb{P}^y$s. Since *w* is an atom that doesn't overlap *z*, *w* is not an atomic part of *z* and is, thus, not one of the $\mathbb{P}^z$s. Therefore, *w* is a member of $\mathbb{P}^y - \mathbb{P}^z$ and, thus, one of the $\mathbb{P}^x$s. By (L2), *w* is part of *x*. Every part of *y* that doesn't overlap *z* is, thus, part of *x*.
Therefore, *x* is the mereological difference between *y* and *z*. QED

(MP3-rl) $\mathbb{P}^x = \mathbb{P}^y \cap \mathbb{P}^z \rightarrow x = y \times z$

Proof. Suppose that $\mathbb{P}^x = \mathbb{P}^y \cap \mathbb{P}^z$.
Part I. Let *w* be part of *x*. (*a*) Suppose that *w* is not an atom. By (L2), $\mathbb{P}^w \subseteq \mathbb{P}^y \cap \mathbb{P}^z$. By the definition of '$\cap$', we have both $\mathbb{P}^w \subseteq \mathbb{P}^y$ and $\mathbb{P}^w \subseteq \mathbb{P}^z$. By (L2), *w* is, thus, part of both *y* and *z*. (*b*) Suppose that *w* is an atom. *w* is, thus, one of the $\mathbb{P}^x$s. By the definition of '$\cap$', *w* is both one of the $\mathbb{P}^y$s and one of the $\mathbb{P}^z$s. Therefore, *w* is part of both *y* and *z*.
Part II. Let *w* be part of both *y* and *z*. (*a*) Suppose that *w* is not an atom. By (L2), we have both $\mathbb{P}^w \subseteq \mathbb{P}^y$ and $\mathbb{P}^w \subseteq \mathbb{P}^z$ and, thus, that $\mathbb{P}^w \subseteq \mathbb{P}^y \cap \mathbb{P}^z$. By (L2), *w* is part

of *x*. (*b*) Suppose that *w* is an atom. *w* is, thus, both one of the $\mathbb{P}^y$s and one of the $\mathbb{P}^z$s. By the definition of '∩', *w* is also one of the $\mathbb{P}^x$s. *w* is, thus, part of *x*.
Therefore, *x* is the product of *y* and *z*. QED

II. ACAI entails (PQ-SF)

In what follows *x* is thought of as ranging over *non-atomic* entities:

(PQ-SF-lr) $$xSFu\phi \rightarrow \forall z\left(\mathbb{P}^x z \leftrightarrow \left((Az \wedge \phi_v z) \vee \exists w(\phi_v w \wedge \mathbb{P}^w z)\right)\right)$$

Proof. Suppose that *x* S-fuses everything that ϕs.
Left-to-right. Let *z* be one of the $\mathbb{P}^x$s. *z* is, thus, an atomic part of *x*. Since *x* S-fuses everything that ϕs, it follows there is some *w* such that *w* is a ϕ-er and *z* overlaps *w*. *z* is, thus, an atomic part of *w*. Therefore, if *w* is a composite entity, *z* is one of the $\mathbb{P}^w$s. If, instead, *w* is an atom, then *z* is identical to *w* and is, thus, a ϕ-er. In either case, it follows that *z*, and thus, by generalization, each of the $\mathbb{P}^x$s is either an atomic ϕ-er or it is one of the $\mathbb{P}^w$s for some *w* that ϕs.
Right-to-left. (*a*) Suppose that *w* is a ϕ-er and that *z* is one of the $\mathbb{P}^w$s. Since *x* S-fuses everything that ϕs, *w* is part of *x*. By (L2), $\mathbb{P}^w \subseteq \mathbb{P}^x$, so that *z* is also one of the $\mathbb{P}^x$s. (*b*) Suppose instead that *z* is an atomic ϕ-er. Then, since *x* S-fuses everything that ϕs, it follows that *z* is an atomic part of *x* and, thus, that *z* is one of the $\mathbb{P}^x$s. QED

(PQ-SF-rl) $$\forall z\left(\mathbb{P}^x z \leftrightarrow \left((Az \wedge \phi_v z) \vee \exists w(\phi_v w \wedge \mathbb{P}^w z)\right)\right) \rightarrow xSFu\phi$$

Proof. Suppose that, for every *z*, *z* is one of the $\mathbb{P}^x$s if and only if, either *z* is an atomic ϕ-er or there is some *w* such that *w* is a ϕ-er and *z* is one of the $\mathbb{P}^w$s.
Part I. Let *w* be a ϕ-er. Suppose that *w* is a composite entity. Then each of the $\mathbb{P}^w$s is an atomic ϕ-er and, thus, one of the $\mathbb{P}^x$s. Suppose that *w* is, instead, an atom. Therefore, *w* is an atomic ϕ-er and, thus, one of the $\mathbb{P}^x$s. It follows that $\mathbb{P}^w \subseteq \mathbb{P}^x$ and, by (L2), that *w* is part of *x*. Therefore, every ϕ-er is part of *x*.
Part II. Let *w* be part of *x*. Suppose that *w* is a composite entity. By (L2), $\mathbb{P}^w \subseteq \mathbb{P}^x$. Suppose that *z* is one of the $\mathbb{P}^w$s. *z* is, thus, also one of the $\mathbb{P}^x$s and, therefore, either an atomic ϕ-er or one of the atomic parts of a ϕ-er. Either way, *z* is also part of a ϕ-er. Therefore, *w* overlaps a ϕ-er. Suppose, instead, that *w* is an atom. Then, *w* is one of the $\mathbb{P}^x$s, and so either an atomic ϕ-er or one of the atomic parts of a ϕ-er. In both cases, *w* overlaps a ϕ-er. Therefore, every part of *w* overlaps a ϕ-er.
We have, thus, proved that (i) every ϕ-er is part of *x*, and that (ii) every part of *w* overlaps a ϕ-er, and so that *x* S-fuses everything that ϕs. QED

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