

Kant's Conception of Logical Extension and Its Implications

By

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DISSERTATION

Submitted in partial satisfaction of the requirements for the degree of

DOCTOR OF PHILOSOPHY

in

Philosophy

in the

OFFICE OF GRADUATE STUDIES

of the

UNIVERSITY OF CALIFORNIA, DAVIS

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2012

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Abstract

It is a received view that Kant's formal logic (or what he calls "pure general logic") is thoroughly intensional. On this view, even the notion of logical extension must be understood solely in terms of the concepts that are subordinate to a given concept. I grant that the subordination relation among concepts is an important theme in Kant's logical doctrine of concepts. But I argue that it is both possible and important to ascribe to Kant an objectual notion of logical extension according to which the extension of a concept is the multitude of objects falling under it. I begin by defending this ascription in response to three reasons that are commonly invoked against it. First, I explain that this ascription is compatible with Kant's philosophical reflections on the nature and boundary of a formal logic. Second, I show that the objectual notion of extension I ascribe to Kant can be traced back to many of the early modern works of logic with which he was more or less familiar. Third, I argue that such a notion of extension makes perfect sense of a pivotal principle in Kant's logic, namely the principle that the quantity of a concept's extension is inversely proportional to that of its intension. In the process, I tease out two important features of the Kantian objectual notion of logical extension in terms of which it markedly differs from the modern one. First, on the modern notion the extension of a concept is the sum of the objects actually falling under it; on the Kantian notion, by contrast, the extension of a concept consists of the multitude of *possible* objects—not in the metaphysical sense of possibility, though—to which a concept applies in virtue of being a general representation. While the quantity of the former extension is finite, that of the latter is infinite—as is reflected in Kant's use of a plane-

geometrical figure (e.g., circle, square), which is *continuum* as opposed to *discretum*, to represent the extension in question. Second, on the modern notion of extension, a concept that signifies exactly one object has a one-member extension; on the Kantian notion, however, such a concept has no extension at all—for a concept is taken to have extension only if it signifies a *multitude* of things. This feature of logical extension is manifested in Kant’s claim that a singular concept (or a concept in its singular use) can, for lack of extension, be figuratively represented only by a point—as opposed to an extended figure like circle, which is reserved for a general concept (or a concept in its general use). Precisely on account of these two features, the Kantian objectual extension proves vital to Kant’s theory of logical quantification (in universal, particular and singular judgments, respectively) and to his view regarding the formal truth of analytic judgments.

In Chapter 1, I give a close reading of Kant’s various characterizations of logic (“pure general logic”) as formal: logic is formal in (a) that it abstracts from all empirical psychological conditions of thinking on the part of the thinking subject, (b) that it abstracts from all (differences among) objects of thinking, and (c) that it abstracts from whether a thought is empirical or *a priori*. By my analysis, (a)-(c) are three complementary aspects of the formality of Kantian logic, each of which is related to a particular concern that Kant has in demarcating the proper boundary of logic as a formal science. I focus on arguing that neither (b) nor (c) requires that thinking be treated in Kantian logic as if it were non-intentional (i.e., not directed at objects). This argument paves the way for my subsequent interpretations of Kant’s notion of logical extension, of his account of logical quantification, and of the formal truth of Kantian analytic judgments.

In Chapter 2, I argue that Kant has a hybrid notion of logical extension that incorporates both [EXT_O] (extension = the multitude of *objects* falling under a given concept) and [EXT_C] (extension = the multitude of *concepts* subordinate to a given concept). My argument partly draws on the accounts of logical extension in various early modern works of logic to which Kant had direct or indirect access. My analysis of these accounts suggests a way to reconcile [EXT_O] and [EXT_C]. But my main interest is to clarify the relevant notion of “object” that is involved in

[EXT_o]. In this regard, I draw special attention to the geometrical analog—an extended figure such as a circle or a line—that is used by Kant, as well as by Leibniz, Lambert and Euler, to represent a concept with respect to its objectual extension ([EXT_o]). I take such a figurative representation to suggest that the relevant notion of objects is that of *possibilia* rather than *actualia*. In that connection, the objectual notion of extension I ascribe to Kant makes perfect sense of his logical principle regarding the inverse proportionality between the quantity of a concept's extension and that of its intension.

In Chapter 3, I apply some of the observations made in Chapters 1 and 2 to address certain issues about Kant's account of singular judgment at A71/B96-7. I concentrate on making formal-logical sense of Kant's claim that (i) a singular judgment stands to a general (universal or particular) one, in respect of quantity, as unity (*Einheit*) to infinity (*Unendlichkeit*), even though (ii) a singular judgment can be treated just like a universal one when used in syllogisms. Contrary to the view that the contrast between (i) and (ii) boils down to that between a transcendental-logical standpoint and a strictly formal-logical one, I argue that it is a contrast between two formal-logical standpoints (Kant's own vs. the previous logicians'). I am mainly interested in showing that Kant has adequate formal-logical resources to establish (i) and can illustrate the *Einheit-Unendlichkeit* contrast by applying his own method of representing a singular concept by a point and a general one by a circle. This interpretation has a further implication for how to approach the controversy over the correlation between the logical moments of quantity (i.e., universal, particular, singular) and the categories of quantity (i.e., unity, plurality, totality).

In Chapter 4, I address some challenges to the view that Kantian analytic judgments are about objects. Opponents of the view argue their case from the thought that it would be implausible for Kant to explain the formal truth of analytic judgments on the conception of truth as agreement with objects. I use the Kantian notion of objectual extension developed in Chapter 2, together with my interpretation of Kant's logical principle regarding the reciprocity between the intensional and the extensional relations of concepts, to show how he can consistently hold that

the formal truth of an analytic judgment consists in the agreement of its predicate with the object(s) signified by its subject-concept and yet can be determined solely on the basis of the intension-containment relation between its two concepts. (The judgment in question is assumed to be a universal affirmative one.)

Acknowledgments

There are many people to thank. For the development both of my thesis and of my passion for Kant study, I wish to thank all five of my dissertation advisors (Henry Allison, Michael Friedman, John Malcolm, G.J. Mattey and Daniel Warren). I thank them especially for their enthusiasm for my project and their probing comments on my work in progress. In addition, I benefited tremendously from Pierre Keller's written comments on an earlier version of the chapter on singular judgments, from auditing Lanier Anderson's seminar "Kant's Criticism of Metaphysics" in the fall of 2008, and from exchanges with Clinton Tolley on several topics touched in this dissertation. I also thank the Adler family for their emotional support. I thank my mother-in-law Nancy in particular, who helped me through many difficult moments with her generous love and unfailing optimism. My greatest gratitude goes to my husband Greg, for his profound love and understanding.

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Introduction

There is no doubt *that* Kant takes formal logic or “pure general logic” (B77-8) to have some significant roles to play in various parts of the *Critique of Pure Reason*. It is fairly clear that such logic is, as Paton puts it, “largely based on the logic of his time” (Paton 1936:188) or, more precisely, based on what Kant takes to be the traditional Aristotelian logic. And, in Tolley’s words, “one of the central pillars of Kant’s Critical project is his second-order *reconceptualization* of the status and significance of this [first-order] traditional logic, within the philosophical architectonic of transcendental idealism.” (Tolley 2007:4) But there is no consensus among commentators about how exactly to construe the nature or the content of the first-order logic as Kant sees it, or about the extent to which his second-order philosophical reflections may have affected how he uses the first-order logic. Controversies over these issues can affect the exegesis of many topics in the *Critique*. I will examine such impacts through a study of two cases.

The first case concerns Kant’s claim that singular and infinite judgments are two special forms of judgment that must be included in his so-called Table of Judgments. Kant recognizes that, in the traditional logic, singular and infinite judgments are treated—correctly in his view—just like universal and affirmative ones, respectively.(A71-3/B96-8) But he also claims that in constructing the Table he has what he takes to be already finished works of the logicians lying before him.(*Prolog.* Ak 4:323) The question is then whether Kant can find adequate resources in the formal logic as he knows it, or whether he would have to resort to his newly introduced transcendental logic instead, in order to establish singular and infinite judgments as two special forms of judgment. Some commentators (e.g., Brandt [1991/1995], Longuenesse [1998]) find it necessary for Kant to invoke his transcendental-logical apparatus in order to *distinguish* singular and infinite judgments from other forms of judgment in the first place. By contrast, Krüger [1968]

and Allison [2004] hold that there are adequate formal-logical resources for Kant to single out singular and infinite judgments as two special forms of judgment and that transcendental logic, if it is to be involved at all, serves only as some sort of higher-order principle. But little has been said about what relevant formal-logical resources are at Kant's disposal.

The second case to be investigated concerns Kant's view on the formal truth of analytic judgments. On Kant's conception of truth, for a (categorical) judgment to be true is for its predicate to agree with the *object*. And, if a judgment is made by means of two concepts, intuitively it is made *about* the object(s) signified by the subject-concept (as opposed to being about this concept). But Kant also takes it that a judgment, if it is analytic, is true in virtue of the intension-containment relation between its two *concepts*, viz., in virtue of the predicate being contained in the subject-concept as part of the latter's intension (assuming the judgment is universal affirmative). This view of analytic truth seems to make all references to objects irrelevant and allows there to be true analytic judgments where the subject-concept has no object—or maybe even cannot have an object—falling it. The question is then whether Kant can consistently hold, on the one hand, that an analytic judgment, *qua* judgment, is about objects and its truth consists in its predicate agreeing with the objects and, on the other hand, that such a judgment, as analytic, is true solely in virtue of the intension-containment relation between its concepts. For some the answer is “no”: Kant's view must be either that analytic truths are formal truths merely in the syntactical sense (Tolley 2007) or that analytic judgments do not have truth values in the first place (Rosenkoetter 2008)—either way, analytic judgments are not about objects at all. Others (e.g., Paton [1936], MacFarlane [2002], Heis [2007]) insist that Kantian analytic judgments are always about objects. But no cogent account has been provided by these commentators as to exactly what it means for Kant to think that analytic judgments are about objects, in a way that is compatible with everything else he has to say about such judgments.

In my view, there are adequate resources in Kant's formal logic for him to treat singular and infinite judgments, respectively, as two special forms of judgment and also to account for the

formal truth of analytic judgments in terms of agreement with objects. By my analysis, the formal-logical notion that plays the key role in these regards is an objectual notion of logical extension, according to which the extension of a concept is the multitude of objects falling under it. But there are three substantive reasons that one might have to resist such an ascription. First, Kant often describes logic as abstracting from all objects, from all differences among the objects and from all relation of thoughts to objects, and as instead treating thoughts merely in respect of their form. One may think that by these remarks Kant means, as Tolley puts it, that “[in formal logic] we must restrict ourselves to a syntactical analysis of thinking *as such*”, leaving to transcendental logic the entire business of semantical analysis, analysis that involves references to objects. (Tolley 2007:19-20) In that case, no objectual notion of extension would belong to the Kantian formal logic properly so called. Second, if one is of the view that the Kantian formal logic is thoroughly intensional, in which even the extension of a concept must be understood only in terms of the *concepts* subordinate to it, one may seek support for such a view in the early modern logics against the backdrop of which Kant’s view of logic was developed. For, as Tolley [2007] has presented it, on the notion of extension that was introduced into early modern logic through the Port-Royal Logic and that found its way into the works of many Wolffian logicians with which Kant was directly acquainted, the extension of a concept consists of all and only the *concepts* that are subordinate to it. Third, one may side with Anderson [2004b] and Tolley [2007] in believing that if Kant were to hold an objectual notion of extension, a pivotal principle in his logic—i.e., the principle that the quantity of a concept’s extension and that of its intension are inversely proportional—would no longer be intelligible.

Obviously, then, what notions we think legitimately belong to Kant’s formal logic are largely constrained by how we interpret Kant’s remarks about the formality of such logic and, in addition, by what early modern logics we take to form the relevant backdrop for the development of Kant’s logic and how we interpret them. So, to justify my ascription of an objectual notion of logical extension to Kant, I must show, in response to the dissenting opinions mentioned above,

that Kant's conception of the formality of logic actually allows such a notion of extension and that, moreover, in the early modern logics with which Kant was acquainted we can indeed find a similar notion. And, finally, if the principle of the inverse proportionality between the quantity of a concept's extension and that of its intension is an indispensable part of Kant's logic, I also have to show that such a principle makes sense on the objectual notion of extension I ascribe to Kant.

Accordingly, I shall begin with a close reading of Kant's various characterizations of "pure general logic" with respect to its formality. On my reading, there are three aspects in view of which this logic is considered formal by Kant: logic is formal in that it treats all thoughts only in respect of their form, (a) regardless of what empirical psychological conditions under which we may think, (b) regardless of what particular objects they may be about, and (c) regardless of how they are related to the objects (empirically or *a priori*). Each of these aspects is presented in close connection with a specific concern on Kant's part. (a) reflects Kant's disagreement with certain early modern logicians or philosophers of logic (e.g., Locke) over the nature and proper subject matter of logic: the latter logicians/philosophers think that logic should study the natural operations of our faculty of thinking; but Kant denies that such a study, as empirical psychology, has any place in logic *qua* proper science. (b), as is initially developed in Kant's logic corpus, has to do with his attempt to introduce logic and establish its status as an independent rational science that must come prior to all special sciences (such as mathematics): if in each of the latter sciences the understanding is used with respect to a particular kind of objects and in accordance with a specific set of rules, logic contains the rules of thinking that govern the universal use of the understanding without regard to the particular nature of its objects. Finally, (c) can be made intelligible only in the context of the *Critique* and through a contrast with transcendental logic, in terms of Kant's critical distinction between pure *a priori* and empirical ways of thinking objects: while transcendental logic presents laws for the use of the understanding only insofar as it is related to objects *a priori*, general logic contains laws of the understanding whether it relates to objects empirically or *a priori*. So construed, none of these three aspects of the formality of pure

general logic suggests that thinking be treated as if it were non-intentional—i.e., as if it were not related to objects at all—in such logic. To that extent, there is indeed room for an objectual notion of logical extension in the Kantian formal logic.

Having thus charted out the theoretical space for ascribing to Kant an objectual notion of logical extension, I shall then examine several early modern works of logic that are directly relevant for us to understand the particular manner in which Kant talks about logical extension. I do not dispute that the subordination relation among concepts is an important theme in Kant's logic as well as in many early modern logics. But my analysis of the relevant texts will yield a more nuanced conception of logical extension than simply viewing it in terms of what is subordinate to a given concept. To begin with, I take the notion of logical extension introduced in the Port-Royal Logic to be a hybrid one, according to which the extension of a concept can be considered both in terms of the concepts subordinate to it and in terms of the individual objects falling under it. Similarly, in Wolff's logics (where "extension" is not explicitly mentioned), a concept is said to "contain under" itself both the concepts subordinate to it and the individuals represented by it. This dual conception of the containment-under relation will then morph into two diverging definitions of extension in the works of some Wolffian logicians: while Meier—whose logic works will serve as the basis for Kant's logic lectures—defines the extension (*Umfang*) of a concept as the sum of all the *concepts* contained under it, Knutzen—who was once Kant's teacher in logic—and Reusch define the extension (*extensio*) of a concept as the multitude of *individual things* to which it applies. The latter, objectual notion of extension is also clearly present in Leibniz', Lambert's and Euler's logics, but with an important extra feature: for each of these logicians, the extension in question can be represented by means of a plane-geometrical figure, such as a line or a circle.

In light of this historical survey, I shall then analyze Kant's own account of logical extension. Recognizing that Kant characterizes the extension of a concept as much in terms of the objects represented by it as he does in terms of the concepts subordinate to it, I take it that he has

a hybrid notion of logical extension that resembles the one introduced in the Port-Royal Logic. I shall argue that whether Kant presents the extension of a concept in one way or the other largely depends on what explanatory role he expects the notion of extension to play in a particular context. In particular, I shall show that the objectual notion of extension prominently figures in his account of the logical forms of (categorical) judgment. In this regard, I shall draw special attention to Kant's use of the Leibnizian-Lambertian-Eulerian method to represent the objectual extension by a geometrical figure (e.g., a circle, a square). Kant's adoption of such a method gives a significant indication regarding the nature of the objectual notion of extension that is permissible in the formal logic as he sees it. Especially, on such a notion, the extension of a concept consists of all the *possible* objects that can be represented by the concept on account of its capacity as a general representation. (Here the possibility of an object is not considered in any metaphysical sense or in terms of Kant's transcendental-logical conception of possibility.) When Tolley [2007] argues, for example, that an objectual notion of extension would falsify Kant's logical principle regarding the inverse proportionality between the quantity of a concept's extension and that of its intension, he has thereby refuted, I shall point out, only the modern notion of extension, according to which the extension of a concept consists of the objects that *actually* fall under it. The principle makes perfect sense on the notion of objectual extension that I have ascribed to Kant. And the principle so construed will indeed serve as part of the logical basis for Kant to hold, consistently, that the formal truth of analytic judgments lies in a certain agreement with objects and yet can be determined solely by the intension-containment relation between its concepts.

I shall stress a second aspect in which Kant's notion of objectual extension markedly differs from the modern one: on the latter notion, a concept that applies to exactly one object has a one-member extension, whereas on the former notion, such a concept has no extension at all (which is different from saying that it has an empty extension). In the early modern logics in which the notion of extension is discussed, the tendency is to ascribe extension explicitly to

general (or universal) concepts—as opposed to singular ones—and to *define* it in terms of a certain *multitude*. Such a practice suggests that the notion of extension may be considered simply as inapplicable to singular concepts: these concepts may not be said to have any extension at all. This contrast between general and singular concepts in respect of extension is clearly articulated in Lambert's explanation as to why he chooses a line to represent the extension of a general concept: the extension in question can be represented by a line (viewed as an extended continuum) precisely because the concept to which it is ascribed is considered as applying or extending to (infinitely) *many* individuals, whereas a discrete individual—which would be signified by a singular concept—can only be represented as a point because it lacks extension. Kant accepts the basic tenet of such a contrast between general and singular concepts in respect of extension, with the caveat that the contrast is, strictly speaking, between general and singular *uses* of concepts (for all concepts are *essentially* general representations). In brief, on Kant's objectual notion of extension, a concept has extension only if it represents a *multitude* of objects. A concept of general use, i.e., when it is used (in a judgment) on account of its capacity as a general representation, is used to signify a multitude of objects and hence is considered to have an extension. By contrast, a concept of singular use—as the subject-concept of a singular judgment—is used to signify exactly one object and hence is considered to have no extension. In either case, as far as formal logic is concerned, the signifying function of a concept (in a judgment) is considered without regard to specific qualities of the signified object(s) and without regard to whether there is in fact—or can be (according to Kant's transcendental logic)—any object falling under the concept. Thus, for Kant, while a concept in its general use can be figuratively represented by means of an extended geometrical figure such as a circle, a concept in its singular use can be figuratively represented by means of an extension-less point. This formal contrast between general and singular concepts constitutes part of the logical basis for Kant to treat singular judgment both as a distinct logical form vis-à-vis general (universal or particular) judgment and as a form of judgment that can be viewed as universal when used in syllogisms.

Chapter 1

The Formality of Logic

At the very start of the Preface to the second edition of the *Critique of Pure Reason* (“B Preface”) Kant devotes two paragraphs to discussing logic as a successful science. (Bviii-ix) He begins the discussion with the observation that, since the time of Aristotle, logic seems to be “completed [vollendet]” (Completeness Claim); he then proceeds to reflect on the proper boundary of logic *qua* science, arguing that logic “exhaustively presents and strictly proves nothing but the formal rules of thinking” (Formality Thesis). There are two noteworthy ways in which this discussion has been approached. The first approach is endorsed especially by Kemp Smith [1918; 1929], who (mis)represents the Completeness Claim as being about Aristotle’s logic as a completed *body of doctrine* and assesses Kant’s references to logic in the rest of the *Critique* in light of that (mis)representation, but gives no serious attention to the Formality Thesis. The second approach has been taken by, among others, Paton [1936], MacFarlane [2000; 2002] and Tolley [2007]. These commentators place the Formality Thesis at the center of Kant’s philosophy of logic, recognizing both that it has important roles to play in the *Critique* and that it represents a unique conception of formality in the history of logic. But they disagree about how exactly to interpret the formality in question. And for Paton and Tolley, such a disagreement goes hand in hand with their divide over how to construe Kant’s account of the logical forms of judgment, over whether Kantian analytic judgments are about objects, and hence over how to interpret the nature of Kantian analytic truth.

This overview suggests that an accurate reading of Kant’s account of logic in the B Preface must precede any serious attempt at understanding the exact role that logic is supposed to play in the *Critique* as a whole. As I will address, in Chapters 3 and 4 respectively, the issues of

whether Kant has adequate logical resources to include singular (and infinite) judgments as distinct logical forms in his Table of Judgments and to account for the formal truth of Kantian analytic judgments in terms of agreement with object, I shall use the present chapter to examine Kant's account of logic in the B Preface and develop a reading of its key points that are presupposed by much of what will be said in subsequent chapters. First, I shall argue, against Kemp Smith, that Kant's Completeness Claim in the B Preface is not meant as a dogmatic, uncritical commitment to the specifics of Aristotle's logic, but as a way to draw out the Formality Thesis. I shall give priority to Kant's philosophy of logic over specific logical doctrines, granting him a critical philosophical perspective from which to assess and freely adopt (or reject) any specific legacies of other logicians. This move will in turn justify my strategy in Chapters 2 and 3 of invoking a whole spectrum of logical legacies to inform Kant's view of logical extension and of its role in accounting for the logical forms of judgment. Second, I shall analyze the Formality Thesis itself, in terms of the three distinct ways in which Kant characterizes the formal nature of logic in the B Preface: logic is formal in that it (a) abstracts from all empirical psychological conditions of thinking on the part of the thinking subject, (b) abstracts from all (differences among) objects of thinking, and (c) abstracts from whether a thought is empirical or *a priori*. I consider (a)-(c) as three complementary aspects of the formality of Kantian logic, each of which is related to a particular concern that Kant has in demarcating the proper boundary of logic as a formal science. I shall explain how aspects (a) and (b) may be fully fleshed out in Kant's logic corpus before being carried over into the *Critique*, whereas (c) makes sense only in the *Critique*, after the idea of a transcendental logic has been introduced and formulated. I shall argue that neither (b) nor (c) requires, contrary to what Tolley [2007] has argued, that thinking be treated in Kantian logic as if it were non-intentional (i.e., not directed at objects). This argument is crucial in paving the way for my subsequent interpretations of Kant's notion of logical extension, of his account of the logical forms of singular and infinite judgments, and of the form and truth of Kantian analytic judgments.

I. The Completeness Claim and Kant's Formality Thesis about logic *qua* science

I shall first examine Kant's Completeness Claim in its context and explain why it is best understood as an entry point to Kant's philosophical reflection on the nature and boundary of logic *qua* science. For that purpose, it is useful to begin with a comparison of two different ways in which the Claim has been translated from German into English.

- (i) Since Aristotle and "to the present day this logic has not been able to advance a single step, and *is* thus to all appearance a closed and completed *body of doctrine*." (Kemp Smith 1929, emphasis added)
- (ii) Since Aristotle and "until now it [i.e., logic] has been unable to take a single step forward, and therefore *seems* to all appearance to be finished and complete [*allem Ansehen nach geschlossen und vollendet zu sein scheint*]." (Guyer and Wood 1998, emphasis added)

Obviously, Guyer and Wood's translation is quite faithful to the German text. By contrast, Kemp Smith's translation has conveyed a significantly stronger claim than the German text can warrant. For one thing, Kemp Smith has translated '*zu sein scheint*' as the factive "is" rather than the much weaker "*seems to be*".¹ For another, he has added the phrase 'body of doctrine', which is not in the German text at all. On Kemp Smith's translation, the Completeness Claim is about the specific *content* of Aristotle's logic as a "body of doctrine". This is actually how Kemp Smith had interpreted the Claim in his earlier book, *A Commentary to Kant's Critique of Pure Reason*, where Kant's "attitude towards [the traditional] formal logic" is characterized as such that, "however many provisos he made and defects he acknowledged, they were to him merely minor matters, and he *accepted its teaching as complete and final*." (Kemp Smith 1918: 184, emphasis added) So construed, the Completeness Claim is seen as the warrant that Kant gives to himself for "rel[ying] upon its prestige and upon the assumed *finality of its results*" to advance his own philosophical projects. (ibid. 184, emphasis added)

Since formal logic is a completed and perfectly *a priori* science, which has stood the test of 2000 years, and remains practically unchanged to the present day, its *results* can be accepted as final, and can be employed without question in all further inquiries. (ibid. 185, emphasis added)

Having thus taken Kant to hold an “invincible belief in the adequacy and finality” of the results of the traditional logic (ibid. 184), Kemp Smith can give no better than a negative interpretation of the cases in which Kant does break away from the specifics of Aristotle’s logic. In particular, he finds it “strangely perverse” (ibid. 184) that Kant, in many parts of the *Critique*, “recasts, extends, or alters, to suit his own purposes, the *actual teaching* of the traditional logic.” (ibid. 185, emphasis added)²

I shall argue, however, that Kemp Smith’s reading of the Completeness Claim as a mere claim about the *de facto* finality of Aristotle’s logic *qua body of doctrine* misses the real gist of Kant’s discussion of logic in the B Preface. In the B Preface, to begin with, the discussion of logic is part of Kant’s reflection on the essential features of a science that explain its success and security *qua* science. He begins the discussion with a criterion by which to judge whether a science has taken a secure path or not.

Whether the treatment of the cognitions that belong to the business of reason travels the secure path of a science or not is easily judged from the *outcome*. (Bvii, emphasis added)³

Kant then gives various characterizations of the outcome that indicates that a study of cognitions has taken the secure path of a science. In particular, if a study, in order to attain its objective (*Zweck*), “must often go back and pursue another path”, then “one may be convinced that such a study is still far from having entered the secure path of a science, but is a mere groping about.” (Bvii) In contrast, logic has traveled the secure path of a science precisely because it has not had the need to retrace its steps and start a new path in order to achieve its purpose.

That logic has already traveled this secure course since the earliest times can be seen from the following: that since the time of Aristotle it has not had to retrace any step, unless one cares to count as improvements the removal of a few dispensable subtleties or a more distinct determination of its presentation, which however belongs more to the elegance than to the security of that science. (Bviii)

So far, Kant has not yet *explained* what it is about *logic* that has made it so successful. Kant will identify the essential element that explains such success in the next paragraph, when he proceeds to observe that “for the advantage that has made it so successful, logic has solely its own

limitedness [*Eingeschränktheit*] to thank.” (Bix) This claim, as I read it, is a thesis about the boundary of logic as a science of the formal rules of all thinking (Formality Thesis). Kant arrives at such a thesis by applying what he would regard as an analytic method, namely, by analyzing a certain *phenomenon* about logic so as to reveal the essential truth about logic that underlies such a phenomenon. And the Completeness Claim can be construed as a statement about the relevant phenomenon, the analysis of which will lead Kant to the Formality Thesis. Let me explain.

Immediately after stating that logic has taken the secure path of a science based on the observation that it has not had to retrace a single step since Aristotle, Kant makes the following *further* observation about logic.

It is further remarkable [about logic] that *until now* it has also been unable to take any step forward and hence *seems to all appearance to be* closed and completed. (Bviii, emphasis added)

This is what I have called Kant’s Completeness Claim about logic. There are two things to be noted about the way in which the claim is stated. First, as the expression ‘seems to all appearance to be’ suggests, the claim is about a *phenomenon* of logic as it has been practiced up till Kant’s time. Second, the statement conveys no commitment on Kant’s part to any specific content of Aristotle’s logic. Indeed, what Kant says next suggests that he rather has in mind what he calls “the peculiar *nature*” of logic *qua* science, a nature that is to be cashed out in terms of the *determined* boundary of such a science.

More specifically, having observed that logic appears to be completed, Kant makes a two-fold remark about the boundary of logic *qua* science. On the one hand, Kant criticizes “some moderns” for attempting to “extend” logic by inserting “*psychological* chapters about our different cognitive powers (about imagination, wit), or *metaphysical* chapters about the origin of cognition or the different kinds of certainty in accordance with the diversity of objects (about idealism, skepticism, etc.) or *anthropological* chapters about our prejudice (about their causes and remedies).” (Bviii) As to which “moderns” Kant might be referring to, some hints might be

gathered from his discussions of the history of logic. Consider, for example, these brief remarks from his logic notes and lectures:

- (1) “Locke: origin of concepts. Wolf and Leibniz: demonstrative and distinct. Modern: not pure, but mixed logic with psychology. Crusius: fanatic logic. Lambert: an organon of mathematics and physics.” (R1629, Ak 16:48)
- (2) Malebranche and Locke have not treated “real logic”, for their putative logic is also concerned with “the content of cognition and with the origin of concepts.” Likewise, Crusius, one of the “modern logicians”, has not “consider[ed] how things stand with logic. For his logic contains metaphysical principles and so to this extent oversteps the limits of this science.” (Log, *Intro*. Ak 9:21)
- (3) “Locke became famous through his *Essay Concerning Human Understanding*—he speaks of the origin of concepts, <but> this really does not belong to logic, but rather to metaphysics.” (V-Lo/Dohna, Ak 24:701)

So Kant has at least Malebranche, Locke and Crusius in mind as representative modern logicians who have tried to expand logic beyond what has been expounded in what they considered traditional Aristotelian logic.⁴ In Kant’s view, all such attempts are vain and have betrayed their makers’ “ignorance of the peculiar nature of this science [of logic].” For “[i]t is not proliferation [*Vermehrung*] but deformation [*Verunstaltung*] of the sciences when their boundaries [*Grenzen*] are allowed to run over into one another.” (Bviii)⁵ On the other hand, Kant suggests that the boundary of logic is indeed strictly determined:

the boundary of logic [...] is determined completely clearly [*ganz genau bestimmt*] by the following: that it is a science that exhaustively presents and strictly proves nothing but the formal rules of all thinking. (Bviii-ix)⁶

In these words, Kant has cashed out his Formality Thesis. Such a thesis presumably *explains* the sense in which Kant takes it that logic, as it has been practiced up till his time, appears to be completed. Roughly put, logic since Aristotle has such an appearance of closure and completion only because it seems to have achieved its sole vocation of presenting and proving nothing but all the formal rules of thinking in general. Thus, strictly speaking, the Formality Thesis a higher-order, *philosophical* reflection on Kant’s part about the special nature and determinate boundary

of logic *qua* science. And the Completeness Claim has served no other purpose in the B Preface than that of drawing out such a thesis. Especially, it does not entail the further claim that Kemp Smith has attributed to Kant, the claim that Aristotle's logic is indeed a finished "body of doctrine" all the specific contents of which are final.

This textual analysis of Kant's discussion of logic in the B Preface has significant exegetical consequences for parts of the *Critique*. By my analysis, in the B Preface Kant is not so much expressing a commitment to the finality of Aristotle's logic in all its specifics as taking a philosophical stand on the nature of logic and defending its status as a science of the formal rules of all thinking. If Kemp Smith has found occasional aberrations in the *Critique* from the teachings of Aristotle's logic, there is nothing "strangely perverse" about such aberrations as long as they occur in principled ways. As Winfried Lenders once wrote about the relation of Kant's logic to the logics of Leibniz, Wolff, Meier, Lambert etc., for a fruitful investigation of such a relation we must consider the possibility that Kant eventually "came to a standpoint of his own", and that there may be an "aspect of the creative thinking of Kant" through which the influences of those logics are filtered." (Lenders 1971:152) A similar, methodological point may be made about the relation of Kant's logic to Aristotle's: in order to understand the role of logic in the *Critique* in a fruitful (and charitable) way, we must consider the possibility that Kant has a *perspective* of his own through which the influence of Aristotle's logic—together with its later developments by Leibniz et al.—may be filtered. Accordingly, whenever there appear to be departures in the *Critique* from what Kant takes to be the Aristotelian logic tradition, we shall investigate the perspective from which Kant may take such aberrant steps with good reasons. As for what such a perspective is exactly, we may gather from the above analysis that at least Kant's Formality Thesis is constitutive of it. The real question now is how to interpret the Formality Thesis itself. In particular, considering that, as I mentioned at the very start of this chapter, how one interprets the Formality Thesis may directly or indirectly affect what sort of logical apparatus one is willing

to grant to Kant in the *Critique*, it is important to gain a precise understanding of the Thesis before we investigate Kant's actual use of logic in the *Critique*.

II. Logic and its formality, from Kant's logic corpus to the *Critique*

In the B Preface, the formal nature of logic is further specified in the following terms: logic presents rules of thinking, "whether it be *a priori* or empirical, whatever origin or object it may have, and whatever contingent or natural obstacles it may encounter in our minds." (Bviii-ix) This characterization of the merely formal nature of logic is intended, as we have seen, partly to counter what Kant sees as a psychologistic approach to logic by such logicians (or philosophers of logic) as Locke: logic properly so called ("Kantian logic" for short) (a) abstracts from any considerations about the contingent, natural conditions under which we actually think.⁷ But the characterization also says something more: Kantian logic, as a science of the merely formal rules of all thinking, must (b) abstract from the object of thought—that is, "from all objects of cognition and all the distinctions between them" (Bix), and (c) abstract from whether a thought is *a priori* or empirical. I shall refer to (a)-(c) as three *aspects* of the formality of Kantian logic. I shall examine how (a) and (b) are developed in the context of Kant's logic corpus and then carried over into the *Critique*. And I shall show that (c) is unique to the *Critique* and makes sense only in terms of Kant's contrast between general and transcendental logics. But I shall do so with an eye to resolving a controversy surrounding (b) that is caused by a *prima facie* tension between two ways in which Kant spells out the idea that logic, *qua* formal science, must abstract from the objects of thinking: (b₁) logic abstracts from all *differences* among the objects of thinking; and (b₂) logic abstracts from *all* objects of thinking.

A certain awareness of the difference or tension between (b₁) and (b₂), together with its consequence for understanding the exact sense in which Kantian logic is formal, is suggested in Paton [1936]. But it is in Tolley [2007] that the tension in question—and its philosophical implication—is taken most seriously. In Tolley's terms, (b₁) represents an *object-neutral* interpretation of the formality of logic, and (b₂) represents a *non-intentional* interpretation. The

former is attributed to Paton [1936], against which interpretation Tolley argues in favor of the latter.⁸ Each side of this controversy has its own textual evidence. According to Paton, the “formal logic” as Kant considers it is concerned with the necessary laws of thought that “hold whatever be the nature of the objects thought about”. It thus still “recognizes that thought has an object”; but it applies to “all objects in general”, abstracting “entirely from the character of the objects, and from all differences between them”; and only to such extent is it “truly *general*”. (Paton 1936:187; 191) This view of logic is “not too clearly expressed” by Kant himself, Paton laments (*ibid.* 191), and Kant may sometimes give commentators the (false) impression that he takes logic to “[treat] thought as if thought had no object.” (*ibid.* 191*n.*1)⁹ Nonetheless, Paton contends, when Kant is “speaking carefully”, his view is really that “it ignores *differences* in objects (see, for example, A52 = B76), and this is true.” More specifically, “though Formal Logic always assumes that there are objects of thought, it is under no obligation to explain what such objects are. It merely supposes that objects are given.” (*ibid.* 191*n.*1) In support of this interpretation, Paton mentions two places in §1 of the Introduction to the *Jäsche-Logik*, one in Ak 9:12, the other in Ak 9:16. (*ibid.* 187*n.*2; 191*n.*1) Although Paton did not quote the specific passages he had in mind, they can be easily located, respectively as follows.

If we [...] merely reflect on the use just of the understanding, we discover those of its rules which are necessary without qualification, for every purpose and without regard to any particular objects of thought, [...] they contain merely the conditions for the use of the understanding in general, *without distinction among its objects*. (Log, *Intro.* Ak 9:12, original emphasis)

Logic is [...] a science *a priori* of the necessary laws of thought, not in regard to particular objects, however, but *to all objects in general*; — hence a science of the correct use of the understanding and of reason in general [...]. (Log, *Intro.* Ak 9:16, emphasis added)

No further textual evidence is invoked by Paton to support the object-neutral interpretation of the formality of Kantian logic.

Paton’s textual evidence for the object-neutral interpretation, as is drawn primarily from the *Jäsche-Logik*, turns out to be rather weak. The authenticity of the *Jäsche-Logik*, as is compiled by Jäsche, has been called into question. Paton’s case for the object-neutral

interpretation is, Tolley observes, thus significantly weakened.¹⁰ In comparison, Tolley believes, the non-intentional interpretation enjoys much stronger textual evidence. Among such evidence Tolley mentions precisely Kant's claim in the B Preface that logic abstracts "from all objects of cognition", and that "in logic, therefore, the understanding has to do with nothing further than itself and its own form", by virtue of which it differs from a science which "does not have to deal merely with itself, but has to deal with objects as well". (Bix) In addition, Tolley also invokes the following passages.

Logic deals with thinking without object [*Denken ohne object*]. (R5665, Ak 18:323)

[L]ogical principles (which abstract completely from everything concerning the possibility of the object), merely concern itself with the formal conditions of judgment. (Ak 8:193)

[The Principle of Contradiction, as a logical principle] is valid for thought in general, without regard to any object. (Ak 8:195; cf.B189)

Universal logic [...] as a mere canon abstracts from all objects. (Log, *Intro*. Ak 9:17)

Apart from these passages explicitly cited by Tolley (Tolley 2007: 134-5, 135n.11), we can find more places in which Kant seems to suggest that logic is formal in the sense of abstracting from all objects. For instance, in two logic lectures Kant reportedly says:

Logic is then a science of the form of understanding and reason in general. Universal logic must *abstract from all objects* [...]; thus logic exhibits the *form*, without being concerning with the matter. (V-Lo/Warschauer, LV 2:506, emphasis added)

The content of a cognition is the object or the matter and is distinguished from the *form*. A universal logic must *abstract from all objects of thought*. (V-Lo/Dohna, Ak 24:693)

At the same time, however, further textual evidence can be found beyond the *Jäsche-Logik* to support Paton's object-neutral interpretation. For example,

Understanding is the faculty of rules. A universal theory of understanding [*viz.*, logic] therefore puts forward only the necessary rules of thinking *without distinguishing the objects*, i.e., the matter, that are thought, hence only the *form* of thinking in general and the rules without which nothing could be thought at all. (R1620, Ak16:40, emphasis added; cf. R1628, Ak 16:44; R1603, Ak 16:33; V-Lo/Busolt, LV 2:609; V-Lo/Wiener, Ak 24:790, 792)

All rational cognition is either material and considers some object, or formal and deals only with the *form* of understanding and of reason itself and with the universal rules of thinking in general *without distinction of objects*. The formal philosophy is called logic[.] (GMS, Ak 4:387, emphasis added)

If these passages are evidence of the object-neutral interpretation, the variety of the sources from which the evidence has been obtained—published and unpublished, Kant’s own notes and students’ recordings of his lectures—should fend off doubts about the strength of the evidence.

Suppose the claim that logic studies the form of thinking in abstraction from all objects and the claim that the study abstracts from differences among objects of thinking indeed correspond, as Tolley has argued, to the non-intentional and the object-neutral interpretation of the formality of logic, respectively. Then, at least insofar as *prima facie* textual evidence is concerned, the two interpretations seem to be on equal footing. This does not mean that it is simply textually underdetermined whether Kant takes the formality of logic to require mere abstraction from differences among objects of thinking or to require treating thinking as non-intentional altogether. Nevertheless, given that there are as many cases of Kant claiming logic to abstract from differences among objects of thinking as there are of him claiming logic to abstract from all objects, it would be rather uncharitable to pit one set of cases against the other and dismiss the unfavorable cases as mere occasions of Kant speaking loosely. We may instead pause and ask: are the two claims in question really incompatible with each other? For both Paton and Tolley, the answer to this question is “Yes”. I shall argue, however, that the notion of object in one claim differs significantly from that in the other. I shall proceed in two steps. First, I shall show that, as it is treated in Kant’s logic corpus,¹¹ the formal nature of logic is best understood in light of Kant’s view of logic as a rational science that serves as the propaedeutic for all other sciences. In that connection, logic is considered, first and foremost, as a science that investigates the necessary rules of thinking in general—necessary both in the sense of normativity and in the sense of universal applicability. This necessity claim gives rise to two corollaries: that logic is pure and *a priori*, and that logic is universal. The universality of logic requires abstraction from all *differences* among objects of thinking, where by ‘object’ is simply meant that at which thinking may be directed in various special sciences. Although Kant occasionally also states that

universal logic must abstract from *all* objects of thought, this statement does not differ in any interesting or meaningful way from the claim that universal logic abstracts from differences among objects. There is no obvious reason to interpret the statement as suggesting Tolley's non-intentional reading of the formality of Kantian logic. Second, I shall show that when Kant takes logic to be concerned merely with forms of thinking in abstraction from *all* objects of cognition or from all relation (*Beziehung*) of cognition to objects, he has in view a philosophical contrast between logic (as pure general logic) and transcendental logic, the latter of which investigates the epistemic conditions under which we may cognize objects of experience *a priori*.

2.1. Logic and its formality in Kant's logic corpus

In Kant's view, there are a few essential characteristics of logic concerning its rules: necessity, formality, universality and apriority. In various places, Kant actually defines logic in terms of one or more of these characteristics. Logic is, more specifically,

- (i) a "science of *universal* rules of the use of understanding in general," (R1620, Ak 16:41, emphasis added) or
- (ii) "a science that contains merely the *formal* rules of thinking" (R1624, Ak 16:42, emphasis added) or "a science that is occupied with the form of the understanding," (V-Lo/Wiener, Ak 24:791) or
- (iii) "the science of the *necessary* laws of the understanding and of reason in general." (Log, *Intro*. Ak 9:13, emphasis added) or
- (iv) "a science (*a priori*) of the pure laws of understanding and reason in general." (R1603, Ak 16:33).

But Kant does not take any one of the mentioned characteristics, considered by itself, to fully determine the nature of logic. One or the other of these characteristics may be emphasized in a particular context. But logic as a science cannot be sharply distinguished from other sciences without keeping in sight all of its essential characteristics. This point is reflected, for instance, in the final definition of logic that concludes the very first section of the *Jäsche-Logik*: logic is "a science *a priori* of the necessary laws of thought, not in regard to particular objects, however, but to all objects in general." (Log, *Intro*. Ak 9:16) For our purpose, we need to investigate the relation among the four characteristics mentioned (necessity, formality, universality and apriority), so as to understand the exact sense in which Kant, in his logic corpus, takes the rules of logic to

be purely formal and to be concerned not with particular objects, but with objects in general. We shall see that there is no definitive textual evidence that the Formality Thesis presented in the B Preface entails Tolley's non-intentional reading of the formality of Kantian logic.

We may begin with the first section of the Introduction to the *Jäsche-Logik*, a section entitled "Concept of logic". In this section one can clearly trace the steps by which the very concept of logic is introduced and gradually fleshed out with all its essential marks. One begins with the observation that "like all our powers, the understanding in particular is bound in its actions to *rules*." (ibid. Ak 9:11, emphasis added; cf. R1628, Ak 9:43) The rules of thinking are then distinguished into necessary and contingent ones.

All rules according to which the understanding operates are either *necessary* or *contingent*. The former are those without which no use of the understanding would be possible at all, the latter those without which a certain determinate use of the understanding would not occur. (Log, *Intro*. Ak 9:12; cf. V-Lo/Pölitiz, Ak 24:502; Log, *Intro*. Ak 9:12; R1620, Ak 16:41)

The contingent rules for the determinate use of the understanding are sometimes considered "conditionally necessary." (V-Lo/Wiener, Ak 24:790) The rules are conditional in that they "depend upon a *determinate object* of cognition, [...] for example, a use of the understanding in mathematics, in metaphysics, morals, etc." (Log, *Intro*. Ak 9:12, emphasis added)¹² There is then a two-pronged move—from the *contingent* rules to the *absolutely necessary* ones and from the rules governing *particular* uses of understanding to those governing the *general* use of understanding—that leads to the introduction of logic as a science of universal and necessary laws of thinking, a science that therefore prepares understanding in its use in all other sciences. In making such a move, one *abstracts* from the conditions under which particular rules of thought apply in particular sciences and thereby ascends to the universal rules of thinking that apply in all possible sciences without qualification. Kant presents such a move in various places in the logic corpus:

Thinking is the occupation of the understanding. But as the objects are different, there must also be different rules of thinking [...]. Each science has its particular rules. There must, however, be one that comes prior to all sciences and contains the rules of thinking in general.

This [science] must abstract from all differences of objects. [...] Logic. It is the propaedeutic of all sciences[.] (R1628, Ak 16:43-4)¹³

There must also be something that [...] contains the rules of thinking in general. This must abstract from all differences of objects. [...] Therefore such a science would bring (merely) the form of thinking under rules. [...] These rules are necessary (without which nothing can be thought at all, hence abstracting from the difference of objects) and essential to thinking in general. Logic. (R1628, Ak 16:44, emphasis added; Log, *Intro.* Ak 9:16)

Our understanding has various objects of cognition and of science, such as history, mathematics—but universal logic abstracts from all this content, from all variety of cognition, and considers in everything only the form of concepts, judgments, and inferences. In short, it is one of the sciences that prepares us for others. (V-Lo/Wiener, Ak 24:791)

The last two of these passages seem to suggest that the logical rules of thinking must, precisely because they are supposed to be necessary and universal, be merely formal, i.e., must concern nothing other than the form of thinking. As Kant puts it, “the universal and necessary rules of thought in general can concern merely its *form* and not in any way its *matter*.” (Log, *Intro.* Ak 9:12) Accordingly, logic is introduced as a “science of the *necessary* laws of the understanding and of reason *in general*, or *what is one and the same*, of the mere *form* of thought as such.” (ibid. Ak 9:13, emphasis added) The form of thought is here presented merely in terms of its contrast with the content or matter of thought.¹⁴ The latter in turn concerns the “objects of cognition and of science”, where the relevant notion of “object” may be rather loosely construed, merely as whatever thinking may be directed at.

From the necessity and universality of the rules of logic also follows their *apriority*. The rules are *a priori* both in the sense that they are presupposed by all special sciences as their “foundation” (Log, Ak 9:13)—and hence are not derived from any of the sciences—and in the sense that they are not derived from experience.

[Logic] may not borrow any principles either from any science or from any experience; it must contain nothing but laws *a priori*, which are necessary and have to do with the understanding in general. (Log, *Intro.* Ak 9:13-14)

The necessary, universal rules of thought must be *a priori*, moreover, not derived from experience. (V-Lo/Dohna, Ak 24:693; see also V-Lo/Wiener, Ak 24:792; R1603, Ak 16:33; R1607, Ak 16:34)

For its rules to be absolutely necessary and universal, logic must abstract “from all things *accidental*, all things *particular*.” (V-Lo/Busolt, LV 2:609, emphasis added) Kant mentions two accidental and particular elements in this regard, concerning the object and the subject of thinking, respectively. Accordingly, the apriority in the sense of experience-independence may also be considered in two ways, with respect to the objects of thought and to the thinking subjects, respectively. On the one hand, it is accidental what particular objects our thinking may be directed at. Rules that govern the use of thinking in respect of particular objects are merely contingent. But rules of logic, insofar as they are to govern thinking in general, must disregard all particularities of the objects at which thinking may be directed. Hence we must be able to know the rules *a priori* without considering any particular object of thought and hence without borrowing anything from experience.

If now we put aside all cognition that we have to borrow from *objects* [...] we discover those of its rules which are necessary without qualification, for every purpose and without regard to any particular objects of thought, because without them we would not think at all. Thus we can have insight into these rules *a priori*, *i.e.*, *independently of all experience*, because they contain merely the conditions for the use of the understanding in general, *without distinction among its objects*[.] (Log, *Intro*. Ak 9:12, original emphasis)

On the other hand, the rules of logic are *a priori* also in the sense that they must not presuppose any psychological principles or considerations about how we actually think. For logical rules, as “necessary laws of thought”, are conditions under which the understanding “can and ought to” be used. (Log, *Intro*. Ak 9:13) In other words, such rules are “[a]bout the objective¹⁵ and possible use of the understanding, not about the subjective and actual use”, and hence involve “no empirical *principia* [which would belong to] psychology”. (R1063, Ak 16:33)¹⁶

If we were to take principles from psychology [...] we would merely see *how* thinking does take place and *how it is* under various subjective obstacles and condition; this would lead then to cognition of merely *contingent* laws. In logic, however, the question is not about *contingent* but about *necessary* rules; not how we do think, but how we ought to think. (Log, *Intro*. Ak 9:14; see also V-Lo/Dohna, Ak 24:694; V-Lo/Blomberg, Ak 24:25-6)

Logic, considered as an *a priori* doctrine in this sense, is given three descriptions. Firstly, it is a “pure logic”, in contradistinction to the so-called “applied logic”. The latter, in Kant’s view,

“really ought not to be called logic”; for it would be a mere psychology that considers how things “customarily go on” in our thought. (Log, *Intro.* Ak 9:18)¹⁷ Secondly, logic as an *a priori* science is a *logica artificialis*, which investigates the rules of thinking *in abstracto*, as opposed to a putative *logica naturalis*, which considers the workings of the understanding *in concreto* and is, for that very reason, not really a logic. (V-Lo/Wiener, Ak 24:791)¹⁸ Finally, logic is a “canon” to the extent that it “has clear *principia a priori* and not empirical principles, thus it borrows nothing from psychology.” (R1613, Ak 16:36)¹⁹

Now we have the proper context for understanding the sense in which logic is taken, in Kant’s logic corpus, to abstract from objects of thought. A key feature of the context is that Kant’s primary concern is to explain the “possibility of such a science [i.e., logic],” (Log, *Intro.* Ak 9:13) by precisely determining the concept of logic in terms of its relation with other sciences, sciences that are taken as given and understood as rule-governed uses of the understanding with respect to particular objects. Logic is thus introduced as a *scientia propaedeutica*²⁰ that must come prior to all special sciences, as that which contains the absolutely necessary rules for all possible uses of the understanding. It is in that connection that logic is said to bring the mere form of thought under the rules, without regard to any particular or determinate objects of thought. The “object” in question is rather loosely understood as whatever may be thought or simply as the subject matter of a special science. To that extent, the notion of object-neutrality does capture the formal nature of logic *qua* propaedeutic science for all special sciences. Admittedly, in the very same context, we also find a few instances of the claim that logic abstracts from *all* objects of thought. Nevertheless, there is no obvious reason that this claim says any more than the earlier claim that logic is not concerned with the differences among particular objects at which thinking may be directed. In the context of the logic corpus, it rather seems to be treated as interchangeable to say that logic abstracts from all *differences* among objects of thinking and to say that it abstracts from *all* objects of thinking. This point may be illustrated by taking a closer look at the context in which logic is said to abstract from all objects of thought.

In the *Jäsche-Logik*, shortly after logic is introduced as a science that contains the rules of thinking that are “necessary without qualification [...] without regard to any particular objects of thought” and that “contain[s] merely the conditions for the use of the understanding in general, *without distinction among its objects*,” (Log, *Intro*. Ak 9:12, original emphasis) Kant says the following:

As a science that deals with all thought in general, *without regard to objects as the matter of thought*, logic

1. is to be regarded as the foundation for all the other sciences and as the propaedeutic to all use of the understanding. Just because it does *abstract wholly from all objects*, however, it also

2. cannot be an organon of the sciences.

[...] because an organon presupposes exact acquaintance with the sciences, their objects and sources. [...] Logic, on the other hand, as universal propaedeutic to all use of the understanding and of reason in general, may not go into the sciences and anticipate their matter. (*ibid.* Ak 9:13, emphasis added)

In this passage, Kant is apparently drawing further implications from the previously introduced concept of logic *qua* science of the necessary and universal laws of thinking and hence *qua* science of the mere form of thought in general. In that connection, the characterization of logic here as entirely abstracting from all objects is presumably intended as no more than a reiteration of what has already said of the abstraction that must be made in logic *qua* science of the necessary laws of thinking in general, namely, abstraction from the “particular” objects of thought and from any “distinction” among them.²¹ And no stronger reading of the claim that logic abstracts from all objects of thought is required to make sense of the points 1 and 2. More specifically, in order for logic to serve as the “foundation” for all special sciences or as the “propaedeutic” to all use of the understanding, the abstraction *required* of it is, as we have already seen, no more than abstraction from the *particular* features of the objects at which thinking may be directed. It is not further required, contrary to what Tolley has argued, that thinking be treated as if it were non-intentional, i.e., without object. Similarly, in order to understand why logic cannot serve as an organon of the sciences, namely, as a “directive as to how a certain cognition is to be brought about,” (*ibid.*) we only need to see that logic is only the

“universal propaedeutic to all use of the understanding and of reason in general.” Considered as a universal propaedeutic to all special sciences, logic must come prior to all special sciences and thus presuppose nothing about their objects (which would be *determinate* objects). This means that logic cannot “anticipate” what the matter of a special science may be, i.e., *what the object may be* with respect to which the understanding operates in the science. In other words, the universality of logic entails that logical rules are *indeterminate* about the objects of thinking, for which reason it cannot serve as organon for bringing about a specific cognition.

[Logic is] not of the particular and determinate, but rather of the universal, use. Just because it is *determinate about no object*, it is only a principle of the assessment, not of the construction, of cognition, a canon and not organon. (R1612, Ak 16:36, emphasis added; cf. R1627, Ak 16:43)

This passage contains the key terms for precisely articulating the gist of the claim in Kant’s logic corpus that logic abstracts from all objects of thinking: it is not that logic ought to treat thought as if it were not directed at any object whatsoever, but that, whatever object a thought may be about, logic disregards all its particularities and hence its rules remain silent about the conditions under which any particular object of thought may be determined.

In sum, whether the formality of logic is presented in the logic corpus as consisting in its abstraction from all differences among the objects of thinking or from all objects of thinking, the two characterizations converge on the same notion of abstraction in terms of which the formal nature of Kantian logic may be understood.

[Formal_o] Logic is formal in that it treats all thoughts only in respect of their form, regardless of what particular objects they may be about.

This notion of formality will be carried over, as we shall see, into the *Critique*, when logic properly so called (“pure general logic”) is distinguished from putative special logics. And [Formal_o] captures one of the several aspects of the formal nature of (pure general) logic mentioned in the B Preface: logic presents the formal rules of all thinking “whatever [...] object it may have.” (Bix) A second aspect of the formality of logic mentioned in the B preface has also been captured in the account of logic developed in the logic corpus: logic treats the rules of all

thinking “whatever contingent or natural obstacles it [i.e. thinking] may meet with in our minds.”

(ibid.) This aspect of the formal nature of logic may be roughly described as follows.

[Formal_s] Logic is formal in that it treats all thoughts only in respect of their form, regardless of what empirical psychological conditions under which we may think.

If both [Formal_o] and [Formal_s] are developed in Kant’s logic corpus and will be carried over to the *Critique*, it is only in the latter context that a third aspect of the formal nature of pure general logic will emerge. This last aspect of the formal nature of logic consists in logic’s treating the rules of all thinking “whether this thinking be empirical or *a priori*.” (Bix) The terms in which to present and make sense of this aspect are put in place only, as we shall see, by comparing pure general logic with a transcendental logic. And the empirical-vs.-*a priori* distinction will concern the different *ways* in which the understanding and reason are *related* to objects, a unique Kantian distinction that will add a significant extra dimension to the exegetical controversy over the formal nature of logic.

2.2. Logic and its formality in the *Critique*: general logic vs. transcendental logic

In Kant’s logic corpus, as we have seen, logic is presented as a pure science *a priori* of the necessary and universal rules of thinking in general. The nature of such a science has been viewed primarily in light of its relation to all the sciences to which it must serve as the universal propaedeutic. In that connection, no other science can be given the title “logic”. In the *Critique*, however, the term ‘logic’ is used in a noticeably broader sense. Here the discussion of logic occurs in a context in which Kant has a rather different concern than in the logic corpus. After the B Preface, the topic of logic does not resurface until in the Introduction (“The idea of a transcendental logic”) to “The transcendental logic”.(A50-64/B74-88) The general notion of logic or “logic in general” (not to be confused with “general logic”) is introduced in light of the contrast between two faculties of cognition, namely, sensibility and understanding. Even though only from the unification of these two faculties can there arise cognition, Kant cautions against mixing up their roles and suggests that we should instead “separate them carefully from each

other and distinguish them.” (A51-2/B76) As the function of each faculty is governed by certain rules, two sciences are distinguished accordingly: aesthetic, as “the science of the rules of sensibility as such [*überhaupt*]”; logic, as “the science of the rules of thinking as such [*überhaupt*].”²² In the latter case, the following distinctions are made:

- (a) logic of the general use of the understanding (general logic) vs. logic of the particular use of the understanding (special logic), and within the former, pure vs. applied logics;
- (b) pure general logic vs. transcendental logic;
- (c) within pure general logic, analytic vs. dialectic; and
- (d) within transcendental logic, transcendental analytic vs. transcendental dialectic.

These four sets of distinctions are expounded, respectively, in the four sections of “The idea of a transcendental logic”: (I) “On logic in general”, where a “general but pure logic” (referred to simply as “general logic” in subsequent sections) is introduced in contradistinction to special logic and to applied logic, respectively; (II) “On transcendental logic”, where the idea of a transcendental logic is formulated and fleshed out, in comparison with which a new aspect of the formality of the pure general logic is introduced; (III) “On the division of general logic into analytic and dialectic”; (IV) “On the division of transcendental logic into the transcendental analytic and dialectic.” My analysis of the text shall focus on (a) and (b), and hence on sections (I) and (II). My main goal is to tease out the precise sense in which the (pure) general logic is taken, in the *Critique*, to abstract “from all content of cognition, i.e., from all relation [*Beziehung*] of it to the object,” (A55/B79) in terms of which Kant distinguishes such a logic from the transcendental logic.

The notion of a pure general logic in section (I) of the Introduction to “The transcendental logic” is developed in the familiar terms in which Kantian logic has been presented in the logic corpus. The development of the notion takes two steps in the *Critique*, to account for the generality and the purity, respectively, of the logic in question. Logic (broadly construed as the science of the rules of understanding as such) is first divided into general and special logics, in accordance with the fact that

logic in turn can be undertaken with two different aims, either as the logic of the general or of the particular use of the understanding. The former contains the absolutely necessary rules of thinking, without which no use of the understanding takes place, and it therefore concerns these rules without regard to the difference of the objects at which it may be directed [*gerichtet sein mag*]. The logic of the particular use of the understanding contains the rules for correctly thinking about a certain kind of objects. [...] the latter [can be called] the organon of this or that science. (A52/B76)

This distinction echoes a point made in the logic corpus about the logic strictly so called that it is a science of the absolutely necessary rules of thinking in general, without distinction of its objects, and hence cannot serve as an organon for the use of the understanding in this or that science. Such a logic is, in the terminology of the *Critique*, a general one. A putative special logic, by contrast, would contain merely conditionally necessary rules for the use of the understanding with respect to a particular kind of objects of thinking—as, for example, in mathematics, metaphysics, morals, etc.²³ General logic is then divided into applied and pure logics. Applied logic is

a representation of the understanding and the rules of its necessary use *in concreto*, namely under the contingent conditions of the subject, [...] which can all be given only empirically. (A54/B78-9)

By contrast, in pure logic

we abstract from all empirical conditions under which our understanding is exercised, e.g., from the influence of the senses, [etc.]. A *general* but *pure* logic therefore has to do with strictly *a priori* principles, and is a *canon of the understanding* and reason[.] (A53/B77)

This contrast reminds us of the one Kant draws in the logic corpus between the logic strictly so called—*qua* pure science *a priori* of the rules of thinking—and a putative applied logic. In the logic corpus, as we have seen, Kant denies that the latter is logic in the proper sense of the term. In the similar spirit, Kant claims in the *Critique* that only pure logic is “properly science”. (A54/B78) An applied logic, because it contains empirical, psychological principles, is “neither a canon of the understanding in general nor an organon of particular sciences, but merely a cathartic of the common understanding [*Kathartikon des gemeinen Verstandes*].” (A53/B77-8; cf. A54-5/B79)²⁴ Precisely for the reason that an applied though general logic is not a science *a priori*, it will drop out of the picture in Kant’s discussion of transcendental vs. general logics in the subsequent sections. In these later sections, even though Kant refers to “general logic”

without qualification, he obviously has in mind only *pure* general logic. (He omits the qualification “pure” presumably because the same feature of purity is also possessed by transcendental logic.) This restriction to pure logic is evidenced by the division of general logic into analytic and dialectic in section III; for the division in question correlates with the canon-organon division and, as Kant suggests elsewhere, only logic *qua* pure doctrine *a priori* can be analytic. (cf. R1629, Ak 16:49) Accordingly, all references to general logic henceforth will be to *pure* general logic.

The section in the *Critique* where the notion of a transcendental logic is introduced (“On transcendental logic”) begins with the following statement:²⁵

General logic abstracts, *as we have shown*, from all content of cognition, [...] and considers only [...] the form of thinking in general. (A55/B79, emphasis added)

The italicized phrase suggests that Kant is referring back to what has supposedly been established in the previous section where the notion of a pure general logic is introduced and fleshed out. We can indeed find the relevant reference in that section, where the *a priori* principles of a pure general logic are said to concern “only what is formal in their use, be the content what it may (empirical or transcendental).” (A53/B77) And this indifference to whether the content of cognition is empirical or transcendental will certainly be the key feature that Kant uses to separate general and transcendental logics. (A55-6/B80) But no *argument* for the said indifference can be found in the previous section, a lack of argument that is suggested by two observations. On the one hand, the above-cited phrase from A53/B77 that expresses the indifference of pure general logic to the content of cognition—content that can be either empirical or transcendental—is inserted in Kant’s account of the distinction between pure and applied logic. Yet such a distinction, as we have seen, is drawn only in terms of whether the use of understanding is studied *in abstracto* or *in concreto* (i.e., in reference to the empirical psychological conditions of the thinking *subject*). The fact that the *content* of cognition may be either empirical or transcendental is irrelevant to such a distinction. On the other hand, when Kant distinguishes general logic from

special logics, the distinction concerns the kind of objects at which thinking may be directed. In that context the notion of “object” is so construed that each science (e.g., mathematics, history, morals, metaphysics) may be said to have its own object. And general logic is general in that it disregards all differences among such objects. There is no reference to, nor is there any need to invoke, the contrast between empirical and transcendental content of thought in order to form such a notion of general logic vis-à-vis special logics.

The contrast between empirical and transcendental content of cognition that is relevant to Kant’s distinction of general and transcendental logics in the *Critique* is, it is to be noted, actually not cast in terms of the difference in the kind of objects that may be thought. The contrast has rather to do with different ways in which the understanding may be *related* to the object. In the context of the *Critique*, such relation to object (through intuition) is what gives content to thought. (A51/B75) In these terms, the section “On transcendental logic” begins with the characterization of general logic that it “abstracts from all content [*Inhalt*] of cognition, i.e., from all relation [*Beziehung*] of it to the object.” (A55/B79) Kant then distinguishes two ways in which to think an object, in accordance with a previously established distinction between pure and empirical intuition: “since there are pure as well as empirical intuitions (as the transcendental aesthetic proved), a distinction between pure and empirical thinking of objects could also well be found.” In these terms, transcendental logic is introduced as “that logic that contained merely the rules of the pure thinking of an object”, which “would therefore concern the origin of our cognitions of objects insofar as that cannot be ascribed to the objects[.]” (A55-6/B79-80, emphasis added) This putative logic is called “logic” in the broad sense of the term (contrasted with “aesthetic”): it “has to do merely with the laws *of the understanding and reason*.” (A57/B81, emphasis added) But it is “transcendental”, in that it contains only the rules of the understanding and reason by which to cognize that certain representations (e.g., concepts) “are not of empirical origin at all and [to cognize] the possibility that they can nevertheless be related *a priori* to objects of experience.” (A56/B81) And only in the “expectation” that “there can perhaps be concepts that may be related

to objects *a priori*, [...] that are thus concepts of neither empirical nor aesthetic origin”, do we formulate the idea of a “transcendental logic”, as “a science of pure understanding and of the pure cognition of reason, by means of which we think objects completely *a priori*.” (A57/B81) In sum, such a science presents the laws of the understanding and reason, in virtue of which it has the title “logic”; but it treats the understanding only as it is “related to objects *a priori*”, for which reason it is a “transcendental” logic. (A57/B81-2) Otherwise put, in transcendental logic “we isolate the understanding (as we did above with sensibility in the transcendental aesthetic)” (A62/B87), as we do in any *logic* understood in contradistinction to aesthetic and as “the science of the rules of understanding *as such*”; (A52/B76, emphasis added) and we “elevate from our cognition merely the part of our thought that has its origin solely in the understanding”, namely “pure cognition” (A62/B87), a feature that is unique to a *transcendental* logic.

This way of introducing the idea of a transcendental logic does not presuppose any particular conception of a general logic, even though the section “On transcendental logic” begins, as we have seen, with a statement about the nature of general logic. In order to “provisionally formulate the idea of [transcendental logic as] a science of pure understanding” (A57/B81), Kant needs and has used only the following propositions: (a) logic is the science of the rules of understanding as such; (b) understanding is “the faculty for *thinking* of objects of sensible intuition” (A51/B75); (c) in accordance with the distinction between pure and empirical intuitions, there is a distinction between pure and empirical thinking of objects; and (d) thinking is always carried out by means of concepts and in accordance with rules. Hence one can expect that there may be pure thinking of objects by means of concepts that relate to objects *a priori*, and accordingly it is possible to have a logic that specifies nothing but the rules of pure thinking of objects, a logic that would be “transcendental” in Kant’s sense. On the other hand, however, a new aspect of the formality of the pure general logic has now been teased out through the comparison with transcendental logic, in terms of abstraction from the origin (*a priori* or empirical) of cognition.

[G]eneral logic, on the contrary, has nothing to do with this *origin* of cognition, but rather considers representations, *whether they are originally given a priori in ourselves or only empirically*, merely in respect of the laws according to which the understanding brings them into relation to one another when it thinks, and therefore it deals only with the *form* of the understanding, which can be given to the representations *wherever they may have originated*. (A56/B80, emphasis added)

If cognitions, in respect of their origin, may be either empirical or pure *a priori*, then general logic deals with laws of thinking as they pertain “to empirical as well as pure cognitions of reason *without distinction*.” (A57/B82, emphasis added) This captures yet another aspect, in addition to the two aspects already mentioned in 2.1, of the formality of logic presented in the B Preface: logic deals with merely “the formal rules of thinking (whether this thinking be empirical or a priori [...]).” (Bix) This aspect is, like the other two, again presented only in negative terms, namely in terms of what logic abstracts from or is indifferent to:

[Formal_R] Logic is formal in that it treats all thoughts only in respect of their form, regardless of how they are related to the objects (empirically or *a priori*).

I argued in 2.1 that the aspect of the formality of logic expressed through the formula [Formal_O], according to which logic abstracts from all (differences among) objects of thinking, does not license the claim that in logic thinking should be treated as if it were non-intentional, i.e., not directed at objects at all. Now it also seems clear that the aspect of formality captured in [Formal_R] does not support such a claim either. For [Formal_R] suggests only that (pure general) logic is not concerned with *how* thinking may be related to objects. It disregards any differences in the ways in which our cognitions may be related to object. Yet it may very well *take for granted that* that the thoughts being dealt with are related to objects however such relations may come about.²⁶

If this interpretation of [Formal_R] (as well as the earlier interpretation of [Formal_O]) thus seems to leave room, insofar as textual evidence is concerned, for Paton’s object-neutral reading of the formality of pure general logic, it also helps deflate a “clear and substantial ‘doctrinal’ reason” that Tolley brings up against the object-neutral interpretation and in favor of his own non-intentional reading. (Tolley 2007:135) Tolley refers to the famous passage at B105 where Kant claims:

the same understanding [...] indeed by means of the very same actions [*Handlungen*] through which it brings the logical form of a judgment into concepts [...], also brings a transcendental content [*Inhalt*] into its representations [*Vorstellungen*] [...], on account of which they [*sie*]²⁷ are called pure concepts of the understanding that pertain to objects *a priori*. This can never be achieved by general logic. (B105; the “B105 passage” for short)

Tolley takes it that the representations of the understanding into which “transcendental content” is brought are the pure concepts of the understanding. (Tolley 2007:137)²⁸ The crux of his argument from the B105 passage against treating thoughts (including concepts) as object-related in general logic is then this: since identifying the process by which to introduce a transcendental content into the concepts of the understanding, whereby such concepts pertain to objects at all, is “*beyond* the purview of general logic,” such a logic lacks proper resources to consider concepts as object-related. (ibid.) It is certainly correct to say that general logic is incapable of *showing* that the representations it deals with are in fact related to objects. But it would be *non sequitur* to infer that such representations must therefore be treated as non-intentional. If Tolley is correct in saying that transcendental logic is—whereas general logic is not—concerned with “the generic conditions for the intentionality of thinking, as it contains the principles for ‘pure thought *about* objects *in general*’,” (ibid. 138) it follows at most that general logic *does not investigate* the enabling *conditions* for thinking to be object-related. This nevertheless leaves such a logic with the freedom to take the intentionality of thinking for granted without asking how it is possible.

It is also worth pointing out that Tolley’s argument hinges on a problematic reading of the B105 passage, on which reading the main point of the passage concerns bringing content into the *pure concepts*. We get a rather different picture if we take into consideration the entire context of the section in which the passage occurs, “On the pure concepts of the understanding or categories.” Here Kant seeks those concepts which are “the third thing necessary for cognition of an object that comes before us.” (A79/B104) These concepts turn out to be “all original concepts of synthesis that the understanding contains in itself *a priori*,” concepts *by means of* which alone can the understanding “understand something in the manifold of intuition, i.e., think an object for it.” (A80/B106) If by ‘transcendental content’ is meant a relation to an object of “intuition in

general” (A79/B105),²⁹ then the representations into which such content is said to be brought at B105 seem to be the manifold of representations contained in an intuition whereby an object is “given” to us (but not yet “thought” as an object);³⁰ and the “actions” of the understanding through which this is achieved are precisely “the pure concepts of the understanding that pertain *a priori* to objects [of intuition in general].” (B105)³¹ Identifying these pure concepts—*qua* pure actions of the understanding whereby a transcendental content can be brought into a given manifold of representation—certainly cannot be achieved by general logic, but only by a transcendental logic. For, as we have seen, Kant takes the very idea of a transcendental logic to be formulated only “in the expectation [...] that there can perhaps be concepts that may be related to objects *a priori*, [...] merely as actions of pure thinking”, and it is up to such a logic to determine the “origin” of these pure concepts. (A57/B80) But this does not rule out the possibility for general logic to take it *that* the thoughts, including concepts of all sorts, it deals with are object-related. This point will become very important when we address, in Chapter III, the issue of whether Kant has adequate general-logical resources to include singular judgments as a distinct form of judgments in his Table of Judgments.

To summarize, I have explained the three aspects of the formality of (pure general) logic mentioned in the B Preface. Each of these aspects has been characterized in terms of a certain abstraction: logic is formal in that it treats merely the form of thinking (a) in abstraction from the empirical psychological conditions of the thinking subject ([Formal_s]), (b) in abstraction from (all differences among) the objects of thought ([Formal_o]), and (c) in abstraction from the difference between pure and empirical thinking (or between pure and empirical relations of thought to the object) ([Formal_r]). I have presented the proper context in which to understand each of these aspects. More specifically, (a) is set against the backdrop of Kant’s disagreement with certain early modern logicians or philosophers of logic (e.g., Locke) over the nature and proper subject matter of logic: while the latter logicians advocate that logic should investigate the natural

operations of the faculty of thinking, Kant purges logic, as a proper science, of such study as mere empirical psychology. And (b), as it is developed in the logic corpus, is understood in the context of Kant's attempt to introduce logic and establish its status as an independent science that must come prior to all sciences (such as mathematics): while in each of the latter sciences the understanding is used with respect to a particular kind of objects and in accordance with a specific set of rules, the former science contains the rules of thinking that govern the universal use of the understanding regardless of its object. Finally, (c) can be made intelligible only in the context of the *Critique* and through the contrast with transcendental logic, in terms of Kant's critical distinction between pure *a priori* and empirical ways of thinking objects: while transcendental logic presents laws for the use of the understanding only insofar as it is related to objects *a priori*, general logic contains laws of the understanding whether it relates to objects empirically or *a priori*.

So construed, none of these three aspects of the formality of pure general logic suggests that thinking be treated as if it were non-intentional in such logic. Indeed, as far as the texts in which Kant discusses the formality of logic are concerned, we have found no clear evidence that thinking should be treated either as intentional or as non-intentional. For we have not yet seen any *positive* characterization, besides the merely negative presentation in terms of abstraction, of what exactly the "form" of a thought amounts to when Kant claims, say, that logic "considers in everything only the form of concepts, judgments, and inferences" and brings merely such forms under rules. (V-Lo/Wiener, Ak 24:791; R1628, Ak 16:44) To understand what the specific form of a concept or of a judgment is in Kant's view, we will have to wait till next chapter, when we discuss Kant's notion of logical extension. To anticipate, in that chapter I shall explain the sense in which Kant's characterization of the form of a thought may, and does, involve reference to objects. My analysis above served to chart out the conceptual space for such reference.

¹ This mistranslation can also be found in Pluhar 1996.

² Kemp Smith means these remarks to be elaboration of his earlier statement about Kant's Completeness Claim: "For Kant's view of the logic of Aristotle as complete and perfect, cf. below, pp.184-5." (Kemp Smith 1918:21)

³ Translations of the passages from the B Preface in this chapter are mine. But I have also consulted both Kemp Smith's and Guyer and Wood's translations.

⁴ The Cartesian and Lockean logicians in the early modern period were not so much trying to extend the Aristotelian logic as attempting to replace it with what has come to be known as their "facultative logic". (See Buickerood 1985; cf. Passmore 1953; Gaukroger 1989; Easton 1997; Capozzi & Roncaglia 2009)

⁵ Kant is consistently of the view that, for the success of any science, it is necessary to isolate it from other sciences. (see MAN, Ak 4:477-8)

⁶ Explicit reference to "formal rules" can also be found in R1620, Ak 16:40; R1624, Ak 16:42; V-Lo/Wiener, Ak 24:795.

⁷ For a brief but helpful account of the anti-psychologistic element of Kant's conception of logic, see Hanna 2001:71-6.

⁸ Tolley 2007, ch.2. The object-neutral interpretation is also attributed to Hanna 2001:74 and Young 1992a:109, though both use the term 'topic-neutral' instead to describe Kantian formal logic. (see Tolley 2007:131n.5)

⁹ Paton claims that this is a common mis-interpretation of Kantian formal logic. Though he does not single out any particular commentator holding this view, the prime suspects are the Marburg neo-Kantians, especially Hermann Cohen and Walter Kinkel, with whom Paton debates, among other topics, whether Kantian analytic judgments are about objects or only about concepts. (Paton 1936:84; 84n.2) The debate goes hand-in-hand with Paton's disagreement with the two neo-Kantians over the nature of Kantian formal logic. On Cohen's and Kinkel's reading, Kantian formal logic (1) is only about analytic judgments and (2) treats thinking as totally object-less (and hence Kantian analytic judgments are about concepts and not about objects). (Cohen 1871/1885; Kinkel 1904) Paton rejects both (1) and (2) (see Paton 1936:213-15; 187-92, 191n.1). I shall revisit these points in Ch.4.

¹⁰ Tolley observes that "it has been argued by many that Jäsche's text is unreliable in general", and accordingly submits the following "guideline for interpretive procedure": "wherever it [the *Jäsche-Logik*] departs from, or adds to, Kant's published material, its claims must be discounted, or at least flagged as not necessarily Kant's own." (Tolley 2007:133) For the problem of authenticity of the *Jäsche-Logik*, see Boswell 1988 (also see Conrad 1994:62-5). Note, however, that the authenticity problem concerns mainly Jäsche's claim that what is included in the *Jäsche-Logik* has been approved by Kant himself as representative of his final views on matters of logic. It is not as if Jäsche made things up. As a matter of fact, the contents of the text can be corroborated by—and are often almost repetition *verbatim* of—other texts in Kant's entire logic corpus. In that case, Tolley's "guideline" may go too far. Things said in the *Jäsche-Logik* that appear to depart from or add to Kant's published material need not be discounted simply for that reason. Instead, we should take two steps to handle such departures. First, to see whether a substantive amount of corroborative evidence can be found in other texts in Kant's logic corpus. (see Young 1992b:xvii-xix) Second, to evaluate the departures in the larger context of those texts. Meanwhile, we should bear in mind that the *perspective* from which Kant discusses a logical topic may vary from his logic corpus to his published materials. Especially, in the latter case the discussions are likely influenced by certain philosophical issues being addressed. Such will be my own methodological guideline in sorting out Kant's conception of a formal logic.

¹¹ Michael Young mentions three groups of materials from which to gain a "reliable picture of Kant's views on logic": (a) Kant's remarks about logic in his published works, especially in the *Critique*; (b) Kant's *Reflexionen* on logic that are published in Volume 16 of the Academy edition; (c) the texts stemming from Kant's logic lectures. (Young 1992b:xviii-xix; cf. Conrad 1994:43-6; and see Vázquez-Lobeiras 2001 for the further addition of Kant's Inaugural Dissertation) When I refer to Kant's logic corpus while examining his conception of the formality of logic, I shall be referring only to (b) and (c). I exclude (a) from the logic corpus on the basis of the following consideration: Kant's discussion of the nature of logic in the *Critique* is an integral part of the overall philosophical project of his transcendental idealism; so it is very likely—and, as we shall soon see, it is indeed the case—that Kant takes a different perspective in such a discussion than in the texts grouped under (b) and (c).

¹² For a similar interpretation of Kant's contrast between logic and other special sciences in terms of the different modal statuses of their laws, see MacFarlane 2000:52-3.

¹³ The characterization of logic as the propaedeutic to the use of understanding in all other sciences can also be found in R1629, Ak 16:49; R1579, Ak 16:21; Log, *Intro.* Ak 13; V-Lo/Blomberg, Ak 24:26.

¹⁴ Throughout the logic corpus, Kant often speaks of "content", "matter" and "object" of cognition without distinction and contrasts each of them equally with the form of cognition. He says, for instance: "The content of a cognition is the object or the matter and is distinguished from the form. A universal logic must abstract from all objects of thought." (V-Lo/Dohna, Ak 24:693, emphasis added; see also V-Lo/Blomberg, Ak 24:25; V-Lo/Wiener, Ak 24:792; R1698, Ak 16:87)

¹⁵ By 'objective' is meant here the normative nature of the rules of logic: "Logic is a philosophy about the universal laws (rules) of the correct use of our understanding and reason [...]. (objective: how it should be used.)" (R1579, Ak 16:20)

¹⁶ Also see R1612, Ak 16:36; R1627, Ak 16:43; R1628, Ak 16:44; R1579, Ak 16:21.

¹⁷ Also see V-Lo/Dohna, Ak 24:694; R1629, Ak 16:49; R1604, Ak 16:33-4; R3332, Ak 16:783; Ak 21:207. A similar conception of logic as a pure philosophy is presented in the *Groundwork of the Metaphysics of Morals*. There logic is characterized as a *pure* philosophy—"insofar as it sets forth its teachings simply from *a priori* principles"—which is also formal. And only pure logic is logic properly so called, which "can have no empirical part, that is, no part in which the universal and necessary laws of thinking would rest on grounds taken from experience; for in that case it would not be logic, that is, a canon for the understanding or for reason, which holds for all thinking and which must be demonstrated." (GMS, Ak 4:387-8)

¹⁸ Also see V-Lo/Dohna, Ak 24:694, 698; V-Lo/Busolt, LV 2:609; V-Lo/Blomberg, Ak 24:26; R1629, Ak 16:48. Precisely because logic properly so called is artificial as opposed to natural and contains merely *a priori* rules, it possesses a feature that is not shared, say, by grammar, even though Kant often introduces logic via its analogy with the latter. What logic and grammar have in common is their formality and the universality of their rules: just as grammar concerns merely the form of language and contains its universal rules, so does logic concern merely the form of thought and contain its universal rules. (V-Lo/Dohna, Ak 24:693; see also Log, *Intro.* Ak 9:11; V-Lo/Wiener, Ak 24:790-1) But whereas logic "must contain principles *a priori*" and its rules are "universal according to reason", grammatical rules are "empirically universal, only insofar as it is always so in experience," for which reason "logic is a science and grammar is not." (ibid. Ak 24:694; cf. R1587, Ak 16:26)

¹⁹ Also see R1579, Ak 16:21; R1628, Ak 16:44-5.

²⁰ The expression '*scientia propaedeutica*' is used at V-Lo/Blomberg, Ak 24:26.

²¹ The same analysis applies to V-Lo/Dohna, Ak 24:693. There, right after speaking of "the necessary universal rules of thought, *without distinction as to what it deals with*" that they "concern only the form of all rules of thought," Kant contrasts the "content" of a cognition—understood as "the object or the matter" of the cognition and says: "A universal logic must abstract from *all objects of thought*." (emphasis added)

²² *Überhaupt* may be translated as "in general" or "as such". I opt for the latter in the present context, which I think captures more precisely Kant's point that sensibility and understanding are investigated in separation, each being considered all by itself.

²³ Kant gives no example of such a putative special logic. It may very well have to do with what Kant calls the "pure part" of a science (be the science a theoretical or a practical one), a part in which "reason determines its object wholly *a priori*" and which therefore "must be expounded all by itself." (Bx; cf. GMS, Ak 4:387-8)

²⁴ Kant, following Meier's practice, usually contrasts the "common understanding" (*sensus communis*) or common use of the understanding with the "learned use" of understanding. (R1529, Ak 16:18) The contrast clearly lines up with the distinction between natural "logic" and artificial logic (pure general logic) mentioned in 2.1: the common use of understanding is a "natural use of rules", whereas the learned use is an "artificial use of rules"; "the science of the rules in the common use of reason is the *critica sensus communis*", a science that "serves as a *catarticon*", whereas "the science of the rules in the learned use of reason is *logica proprie dicta*." (ibid. Ak 16:18-19; cf. R1600, Ak 16:31)

²⁵ To my knowledge, there are only two places in the logic corpus where Kant explicitly contrasts general (or universal) and transcendental logics: R2162, Ak 16:256 and Log, *Intro.* Ak 9:15. Consider the latter:

Now as propaedeutic to all use of the understanding in general, universal logic [viz. general logic] is distinct also on the other side from *transcendental logic*, in which the object itself is represented as an

object of the mere understanding [*Gegenstand des bloßen Verstandes*]; universal logic, on the contrary, deals with all objects in general [*alle Gegenstände überhaupt*]. (Log, *Intro*. Ak 9:15)

The wording of this passage seems to suggest a contrast between general and transcendental logics that resembles the one between general and special logics: while the former treats thinking with respect to all objects in general without distinction, the latter treats it with respect only to a particular kind of objects. Kant thus seems, as MacFarlane puts it, “to regard the restriction of transcendental logic to objects capable of being given in human sensibility as a *domain restriction*, like the restriction of geometry to spatial objects.” (MacFarlane 2002:48n.35; cf. MacFarlane 2000, ch.4; for the treatment of transcendental logic as a special logic, also see C. H. Beck 2003:120) But the account of transcendental logic and its contrast with general logic in the *Critique* will prove to be significantly more nuanced than that. For a recent criticism of the domain-specific reading of transcendental logic, see Tolley 2012b (cf. Tolley 2012a).

²⁶ Kant sometimes also suggests that the distinction between general and transcendental logics may be understood in terms of the distinction between formal and real uses of the understanding. As he puts it in R1608, while logic properly so called is the “canon of all formal use of the understanding”, transcendental logic (as a “transcendental philosophy”) is the “canon of all real use of the understanding,” where the real use is “undetermined, when it pertains to things in general. But because it therefore pertains to things insofar as they are given through experience, it is predetermining, in that it contains the conditions under which all appearances can be cognized in accordance with a rule.” (Ak 16:34)

²⁷ Here *sie* may be referred either to *Handlungen* or to *Vorstellungen*. As I shall explain next, while Tolley suggests it is *Vorstellungen*, I think it is *Handlungen* that is meant by *sie*. The former rendering is not uncommon. Kemp Smith, for instance, simply translates *sie* as “these representations.” This interpretation is shared by Paton, who is aware of the choice between *Handlungen* and *Vorstellungen*, but dismisses the former alternative as “impossible” without explaining why. (Paton:1936:287, 289-90; see also Pippin 1982:94-6) In case one might find it implausible to call a pure *concept* “action”, it should be pointed out that B105 would not be the only place in which concepts are said to be *Handlungen*. Indeed, at A57/B81 Kant already suggests that, if there were “concepts that may be related to objects *a priori*”, such concepts would be considered “merely as actions [*Handlungen*] of pure thinking.” Likewise, in R4276 categories are defined as “the general actions of reason;” (Ak 17:492) in the *Metaphysical Foundations of Natural Science* the table of categories is said to contain “all formal actions of the understanding in judging.” (Ak 4:475)

²⁸ This point is reiterated in Tolley 2012b.

²⁹ The content in question seems to be “transcendental” in a different sense than the sense in which transcendental logic has been said to be transcendental. The transcendental nature of the content in question, which consists in the *Beziehung* of given representations to object, has to do with the fact that the object is here considered only as an object of intuition in general (*überhaupt*), in abstraction from the conditions of *our* sensible intuition. This interpretation is suggested in Kant’s following statement at A247/B304: “Thinking is the action of relating given intuitions to an object. If the manner of this intuition is not given in any way, then the object is merely transcendental[.]” And, when “abstraction is made from any condition of sensible intuition as the only that is possible for us,” the actions by which the transcendental content is brought into given representations are *pure* concepts of the understanding, through which “only the thought of an object in general is expressed in accordance with different *modi*.” (ibid.; cf. B148)

³⁰ Such representations are somehow given to the understanding before its synthesizing and unifying function takes place. This is actually the sense of representation with which Kant begins §16 of the Transcendental Deduction in the B edition, where it is argued that the *I think* must be able to accompany all representations given to me in sensible intuition, if they are to represent some object to me. (B131-2) This suggests that a manifold of sensible intuition would not have any representational content, unless it is first represented as a synthetic unity through their necessary relation to the single consciousness *I think*, (B132) unity of consciousness that “alone constitutes the relation [*Beziehung*] of representations to an object.” (B137; cf. B139)

³¹ A similar reading is indicated in Allison 2004:154-5. The alternative reading—that a transcendental content is given to categories (as pure concepts) by means of synthetic unity of a manifold of intuition in general—is problematic for a further reason. In Kant’s view, the “content” of (schematized) categories consists in a relation (*Beziehung*) to our sensible objects, in which case “the formal conditions of sensibility” (as specified in the Transcendental Aesthetic) must be taken into account. (A239/B298) By

contrast, *pure* categories can only have “transcendental significance”, transcendental in the sense of abstracting from such formal conditions of sensibility. (A248/B305; cf. A247/B304) In that connection, attributing a *transcendental content* to categories would amount to contradiction. Meanwhile, at B105 the transcendental content is said to be brought into representations through the synthetic unity of the manifold in “intuition in general,” which does indicate an abstraction from the formal conditions of sensibility.

Chapter 2

Logical Extension

I presented the formality of Kantian logic¹ merely in negative terms in Chapter 1, namely in terms of what it abstracts from. It was left unexplained what exactly *is* the form of a thought that is to be brought under the rules of logic. Now, there are three sorts of thought the forms of which may be treated in logic: concept, judgment, inference. In Kant's logic corpus, concepts are always treated prior to judgments (and inferences), presumably for the reason that a judgment, with respect to its logical form, is essentially a relation of given concepts² and, to that extent, the relevant formal features of concept must be expounded before we can correctly account for forms of judgment.³ Just as Kant expounds the formal features of a concept with regard to its intension and extension, so does he account for the form of a judgment in terms of the relation between given concepts with regard to their intension and extension. It is relatively uncontroversial that Kant takes the intension (*Inhalt*)⁴ of a concept to be the multitude of all the conceptual marks contained therein.⁵ And there is clear evidence that Kant takes the *form* of a concept to consist in its being a universal representation—i.e., a representation that contains a multitude under itself—and hence in it having an extension (*Umfang, sphaera*).⁶ But it is far from clear what the multitude that constitutes a concept's extension ("logical extension" henceforth) is in Kant's sense: Should we construe a Kantian extension as

[EXT_O] = the multitude of all the *objects* falling under a given concept, or as
 [EXT_C] = the multitude of all the *concepts* subordinate to a given concept?

Assuming [EXT_O] and [EXT_C] to be two incompatible notions of extension, one might think that only [EXT_C], as opposed to [EXT_O], can belong to Kant's logic, for the following initial reasons. First, one might interpret Kant's conception of the formality of logic in such a way that no reference to objects at all would be allowed in the Kantian formal logic, in which case [EXT_O]

would not be a formal-logical notion for Kant. Second, in the text on which Kant's logic corpus is based, namely Meier's *Auszug aus der Vernunftlehre*, the extension of a concept is explicitly defined as "the sum of all the *concepts* that are contained under" it (*Auszug*, §262, emphasis added). But neither of these alleged reasons would hold water. First, as we have seen in Chapter 1, Kant's view of the formality of (pure general) logic does not banish all reference to objects from logic. It is thus left open that some sense of [EXT_O] may still be admitted in the Kantian formal logic. Second, as I have also pointed out in Chapter 1, Kant has a philosophical standpoint from which to approach critically the legacies of other logicians. A logical doctrine should not, then, be ascribed to Kant simply because it was held by a logician with whose relevant works he was intimately acquainted. Moreover, as we shall soon see, Kant was also familiar with the logics of many other early modern logicians, many of whom explicitly took logical extension in the sense of [EXT_O].

In Kant's logic corpus there is as much *prima facie* evidence for taking extension as [EXT_O] as there is for taking extension as [EXT_C]. In this chapter, I shall argue that Kant has a hybrid notion of extension that incorporates both [EXT_O] and [EXT_C]. The resources for spelling out such a notion will be drawn partly from the accounts of logical extension in various early modern logic works to which Kant had direct or indirect access. My analysis of these accounts will suggest a possibility of reconciling [EXT_O] and [EXT_C]. But, to prepare for some of my arguments in Chapters 3 and 4, my main interest is to clarify the relevant notion of "object" that is involved in [EXT_O]. In this regard, I shall draw special attention to the geometrical analog—an extended figure such as a circle or a line—that was used by Leibniz, Lambert and Euler to represent a concept with respect to its objectual extension ([EXT_O]), an analog that will also be adopted by Kant. I take such a figurative representation to suggest that the relevant notion of objects is that of *possibilia* rather than *actualia*. Most commentators who have taken stand in the controversy over [EXT_C] vs. [EXT_O] have failed, I shall argue, to appreciate or even recognize

the possibility that by [EXT_o] Kant could mean a multitude of possible, as opposed to actual, objects represented by a given concept.

I. Two notions of logical extension in the early modern logics

Kant often begins his logic lectures with an introduction on the nature and history of logic. In doing so he occasionally refers to logicians of his time. For instance, he says in the *Wiener Logik*:

Among the moderns Leibniz and Wolff are to be noted. The logic of Wolffius is the best to be found. It was subsequently condensed by Baumgarten, and the latter was again extended by Meier. After them, Reusch and Knutzen wrote logics. [...] The logic of Crusius is crammed full of things that are drawn from other sciences, and it contains metaphysical and theological principles. Lambert wrote an organon of pure reason. (V-Lo/Wiener, Ak 24:796; cf. V-Lo/Pöhlitz, Ak 24:509; Log, 9:21; V-Lo/Philippi, Ak 24:337-38)

In this passage, Kant is evaluating various logicians' works from the viewpoint of his philosophy of logic. But thereby he has also exhibited acquaintance with the developments of early modern logic in general. The fact that, as we have seen in Chapter 1, he has philosophical disagreements with some of the mentioned logicians over the nature of logic does not prevent him from adopting their views on a specific logical topic. We shall keep this in sight while investigating the accounts of logical extension in the early modern logic that form the backdrop for the development of Kant's own account.

In the early modern logic, the extension of a concept is usually contrasted with its intension: while the intension of a concept (general idea, notion) consists of the characteristic marks that are contained in the concept, its extension consists of what is contained under it. Such a distinction was supposedly introduced into the early modern logic through what came to be known as the Port-Royal Logic.⁷ Although there is no evidence that Kant ever read the book, it can serve as a useful starting point in our examination of the resources in the early modern logic that might have informed, one way or the other, Kant's conception of logical extension. For one thing, the book was widely popular for an extended period. Even if Kant had no first-hand knowledge of its content, it is not unlikely that he was at least indirectly influenced by it—and likely even more so on the topic of logical extension.⁸ For another, the definition of “extension”

given in the book has a certain ambiguity that will resonate in the works of many other logicians. And a hybrid notion of extension suggested thereby, viz., a notion that incorporates both [EXT_O] and [EXT_C], provides us with a helpful model for sorting out the relation between the [EXT_O] and the [EXT_C] that occur in some of the later logic texts. Accordingly, I shall begin with the Port-Royal Logic in my survey of the early modern accounts of logical extension. I shall then examine two distinct definitions of extension in the logics of three Wolffian logicians, namely Knutzen, Reusch and Meier: while Meier defines the extension of a concept in terms of the other concepts subordinate to it ([EXT_C]), both Knutzen and Reusch take the extension to consist of the individual objects to which the concept applies ([EXT_O]). It will be noted, however, that in Meier's *Auszug* the notion of extension defined as [EXT_C] is never explicitly referred to beyond the context of a theory of how to obtain a concept through continued logical abstraction. By contrast, when we finally turn to the objectual notion of extension ([EXT_O]) shared between Leibniz, Lambert and Euler, we shall see that such a notion—together with its geometrical analog (circle, line)—is introduced and formulated explicitly to account for the basic logical forms of judgments.

1.1. Two notions of extension in the Port-Royal Logic

In the Port-Royal Logic (*PRL*), a distinction between comprehension (*comprehension*) and extension (*étendue*) is made with respect to a general idea:⁹ while the comprehension of an idea consists of “the attributes that it contains in itself, and that cannot be removed without destroying the idea”, the extension consists of “the subjects to which this idea applies”. (i.6, pp.39-40) It is not immediately clear what is meant by the “subjects” that make up the extension of an idea. Consider the provided example: “the idea of a triangle in general extends to all the different species of triangles.” (i.6, p.40) One may put stress either on “species” or on “triangle” and get two different results regarding the make-up of the extension of the given idea: the extension consists either of all the species that are “inferior” to the given idea (ibid. p.40) or of all the individual triangular objects to which the idea of triangle applies.¹⁰ These seem to be two

dramatically different notions of extension. For in the first case the species that constitute the extension of the idea of triangle are themselves general or “common” *ideas*, differing from the given idea (as genus) only in the degree of generality: “An idea is called a genus when it is so common that it extends to other ideas that are also universal, [...]. *Common ideas* that fall under a more common and general idea are called *species*.” (i.7, p.40, emphasis added) Thus we have two possible renderings of the Port-Royal definition of extension, which roughly correspond to the [EXT_C] and [EXT_O] mentioned at the outset of this chapter.

[EXT_C] The extension of a general idea consists of all the *general ideas* subordinate to it.

[EXT_O] The extension of a general idea consists of all the individual *objects* falling under it.

Both [EXT_C] and [EXT_O] seem needed to make sense, respectively, of two distinct ways in which we may “restrict [an idea] as to its extension, by applying it to only some of the subjects it can be applied to, without thereby destroying it.” (i.6, p.40) Consider the idea of triangle again. We may add to this idea another “determinate idea”, e.g., *having a right angle*, and thereby restrict it to a particular species of triangle (*right triangle*). (i.6, p.40; cf. i.8, p.45) Or we may restrict the given idea by adding to it an “indeterminate idea”, e.g., *some*, by which restriction the idea “become[s] particular, because it now extends to only a part of the subjects to which it formerly extended, but without the part to which it is restricted being determined.” (i.6, p.40; cf. i.8, p.45) While the first kind of restriction makes sense if “extension” is taken in the sense of [EXT_C], the second kind is intelligible only in terms of [EXT_O]. For restricting an idea by means of the quantifier “some” does not lead to any determinate species subordinate to it. Rather, the idea is so restricted as to apply only to an *indeterminate* part of its original extension. This restriction would make no sense if by “extension” were meant [EXT_C]. To illustrate, consider the case of a “lowest species [*species infima*]”, which is a general idea that has under itself only individuals, not other general ideas: e.g., the idea of circle, which “has under itself only individual circles”. (i.7, p.41) On account of its generality, the idea of circle has an extension and hence can be restricted as to extension: we may refer to *some* circles. As a lowest species, however, the idea

of circle has no other general ideas subordinate to it. Hence, by “some circles” cannot be meant a few sub-species of circles, but only certain individual objects to which the idea of circle applies.¹¹

Charitably interpreted, then, the notion of extension presented in the Port-Royal Logic is a hybrid one, i.e., a combination of [EXT_C] and [EXT_O]. It is not that the extension of an idea is thought to be an entity composed of both the general ideas subordinate to it and the individual objects to which it applies or is applicable.¹² It is rather that the extension of an idea is conceived in two different but compatible ways, either (a) as consisting of the subordinate ideas or (b) as consisting of the applicable individuals. Insofar as the Port-Royal Logic is concerned, these two ways of viewing extension capture two distinct aspects of a general idea. On the one hand, (a) primarily concerns the genus-species relation a general idea has with other general ideas and, to that extent, is the basis for restricting a given genus-idea to certain determinate species under it. On the other hand, (b) has mainly to do with the applicability of an idea to individuals and serves as the basis for the restriction of a general idea by means of the quantifier “some”. Thus, as Capozzi and Roncaglia aptly put it, if the definition of extension in the Port-Royal Logic seems ambiguous between [EXT_C] and [EXT_O], “[t]he ambiguity [...] is somehow *intended*, for it serves [...] to define different properties of the operations [e.g., restriction] that can be performed on ideas, as well as to solve classical problems of quantification [e.g., quantification involving the use of ‘some’] in the doctrine of judgment and reasoning.” (Capozzi & Roncaglia 2009:100, emphasis added) To this remark I add: if only one of the two conceptions of extension sketched above is operative in a certain context, it does not show that the other conception is thereby ruled out *tout court*; for the first notion may merely be *emphasized* in the context in question on account of being more pertinent to the treatment of a particular issue. We shall bear this point in mind when we proceed to examine the accounts of logical extension in later logicians’ works.

1.2. Two notions of extension in the logics of the Wolffians

Kant considers Wolff’s logic as “the best to be found” and takes it that this logic “was subsequently condensed by Baumgarten”, the latter logic in turn to be “extended by Meier”. (V-

Lo/Wiener, Ak 24:796) On the particular topic of extension, however, there is no traceable passage from Wolff via Baumgarten to Meier. The technical terms Meier uses for extension, namely *Umfang* and *sphaera*, have not been used in either Wolff's or Baumgarten's logics. As a matter of fact, Meier's two logic monographs, i.e., *Vernunftlehre* (1752) and *Auszug aus der Vernunftlehre* (1752),¹³ appeared much earlier than Baumgarten's *Acroasis logicae* (1761). And, while Meier defines the *Umfang* of a concept solely in terms of other concepts contained under it ([EXT_C]), Wolff introduced two sorts of containment-under relations: (i) less universal concepts are contained under a relatively more universal one, and (ii) individuals are contained under a universal concept through which they are represented. It is also noticeable that, before the publication of Meier's *Vernunftlehre* and *Auszug*, a definition of extension in terms of (ii)—hence as [EXT_O—was given by two other *Wolffschüler*, Reusch and Knutzen. Since Kant was familiar with both Reusch's and Knutzen's logics,¹⁴ their notion of extension is not to be ignored if we wish to understand, as comprehensively as possible, the resources that might have informed Kant's own notion. Accordingly, before turning to the account of *Umfang* in Meier's *Auszug*, I shall examine (i) and (ii) in Wolff's logics and the [EXT_O] shared by Knutzen and Reusch.¹⁵

In his *German Logic* (1712), Wolff remarks that if we obtain a concept by abstracting from various determinations that we “encounter in individual things”, then such a concept “comprehends the individual things under itself [*eintzele Dinge unter sich begreiffet*]” or has under itself “the entire genera or various kinds of individual things [*ganze Geschlechter oder verschiedene Arten einzeler Dinge*]”. (Wolff 1712, §27) Every universal concept thus seems to contain two sorts of things under it: all the individuals to which it applies, and all the *Arten* into which these individuals may be sorted. A concept is “universal [*allgemein*]” insofar as it “applies to all the things of one kind [*allen Dingen von einer Art zukommet*]”; but it is *more* universal—presumably in comparison with another universal concept—if it applies “to more *kinds* of things [*mehreren Arten der Dinge*]”. (ibid. §28, emphasis added) The degree of a concept's universality in comparison with that of another concept is determined by its relative position in a genus-

species hierarchy. Wolff gives a detailed account of how such a hierarchy may be generated in his *Latin Logic* (1740). In a nutshell, on the basis of certain similarities among individuals or “singular things [*res singulares*]”, we form a species (Wolff 1740, §44)—or, rather, a “lowest species”, which “embraces under itself [*sub se complectitur*] only individuals” (ibid. §47);¹⁶ then, with respect to similarities among certain species, we form a genus “embracing those species under itself [*sub se complexae*]”; (ibid. §45) among certain genera there may be further similarities, in which case an even higher genus can be formed; (ibid. § 46) and so on, and so forth (till we reach the highest genus). (ibid. §47) The further and higher we go in this process, the more universal a concept—in comparison with the relatively lower ones—we may obtain (until we encounter the most universal one, viz., the highest genus).

Like Wolff, Knutzen characterizes a universal concept as that which represents what is “common to many individuals”, and the individuals are taken to be contained under the concept. (Knutzen 1747, § 63) Unlike Wolff, however, Knutzen introduces the notion of extension to capture such a containment-under relation:

the extension [*extensio*, *Weite*]¹⁷ of an idea is the multitude of the subjects or of the individuals [*seu indiuiduorum*]¹⁸ that are somehow contained under a certain universal idea or to which an idea of this kind applies. (ibid. §71)

This definition of the extension of a universal concept in terms of the individuals contained under the concept can also be found in the *Systema logicum* (1734) by Reusch, another *Wolffsschüler*.

Objects [*obiecta*] in which an abstract idea is contained [...] are said to be contained or enveloped [*involvi*] under the idea, i.e., the idea is said to extend to [*extendi ad*] them. From this those objects are called the extension [*extensio*] [...] of the notion. (Reusch 1734, § 21)

Thus defining the extension of a notion solely in terms of the individual objects contained under it, Reusch and Knutzen share [EXT_O].¹⁹ By contrast, Meier—yet another *Wolffsschüler*—will define the extension (*Umfang*) of a universal concept in terms of the other containment-under relation discussed by Wolff—i.e., in terms of the genus-species relation that the concept has to the other universal concepts that are subordinate to it—and hence as [EXT_C]. However, with close attention

to the context in which Meier introduces the notion of *Umfang*, we shall see no obvious reason for Meier to consider Knutzen's objectual notion of extension as incompatible with his own.

Meier introduces the notion of logical extension along with his theory of how the so-called abstract concepts (as opposed to singular ones) are made through "logical abstraction [*logische Absonderung*]". According to Meier, we "make a concept through logical abstraction [...] when we compare congruent concepts of different things and represent distinctly only the marks that they have in common." (*Auszug*, §259) The concepts from which a concept is abstracted are said to be "contained under" it: "[the] abstract concept contains under [*unterenthält*] itself those from which it was abstracted." (ibid. §260) The notion of *Umfang* is introduced precisely to capture such a containment-under relation:

The sum of all the concepts that are contained under an abstracted concept is its extension [*Umfang*] (*sphaera notionis*). (*Auszug*, §262)

Noticeably, after §262 there is no more explicit reference to this notion of *Umfang*.²⁰ Meanwhile, however, Meier continues referring to what is "contained under" a concept without explicitly restricting such reference to the concepts contained under the given concept. This is clear especially in how Meier contrasts universal and particular judgments in terms of quantification over what is contained under a concept: in the former case, the predicate is affirmed or denied of "all that is contained under the subject"; in the latter, the predicate is affirmed or denied of a few of what are contained under the subject. (*Auszug*, §301; cf. §302) It is not explained what is meant here by the things that are contained under the subject-concept of a judgment. If these things may be said to constitute the "extension" of the given concept, such an extension may very well be Knutzen's *extensio* and not Meier's own *Umfang*.²¹

If the *extensio* in Knutzen's sense may be allowed in the *Auszug*, a close connection can be forged between it and the *Umfang* in Meier's sense. We may appreciate such a connection by comparing the seemingly different ways in which Meier and Wolff contrast general (or abstract) and singular concepts (or notions). On Wolff's theory of concept formation, the process of

forming general concepts begins with *individuals*: it is by noting certain similarities among the individuals that we form the first level of general notions. (Wolff 1740, §§ 44-7) But there are also singular notions, which differ from the general ones with respect to the quantity of the individuals to which a notion applies.

A general notion [*notio communis*] is that which presents [*exhibit*] what are common to many; a singular notion is that which represents [*repraesentat*] a singular thing [*rem singularem*] or an individual.

The notion of human being in general is a general notion, since there are many individuals to which it applies, which correspond to that notion. But the notion of this human being, e.g., Peter, is a singular notion, since there is only one individual, to which that notion corresponds.

(Wolff 1740, §113)

Unlike this contrast of general and singular notions in terms of the *quantity* of what is represented by a concept, Meier's distinction between abstract and singular concepts concerns different *modes* of representation.

All concepts that are made through logical abstraction are abstracted [*abgesonderte*] or abstract [*abstracte*] concept (*conceptus abstractus, notio*). Concepts that are not abstracted are called singular [*einzelne*] concepts (*conceptus singularis, idea*). For example: *Leibniz*. All immediate [*unmittelbare*] empirical concepts are singular concepts. (*Auszug*, §260)

If immediacy characterizes a singular concept's mode of representation, what is represented thereby is still an individual.²² For Meier, it is the singular concepts as *representations*, not the individuals per se, that are the basis for making abstract concepts through logical abstraction. Nevertheless, if an abstract concept is formed ultimately on the basis of certain singular concepts through logical abstraction, it presumably also represents whatever individuals may be represented by the latter concepts, albeit *in a different way*: if the singular concepts represent the individuals immediately, the abstracted concept does so only in a *mediate* way, i.e., through the mediation of all the concepts—and ultimately through the mediation of the singular ones—from which it has been abstracted.

Reading this characterization of an abstract concept as a *mediate* representation of individuals together with Wolff's view of a general notion as representing *a multitude* of individuals, we shall see a two-faceted profile of every general concept. On the one hand, a

general concept may be considered as a product of logical abstraction from other concepts in which it is contained as a mark. In this respect, it contains the less general concepts under itself and thus has an *Umfang* in Meier's sense ([EXT_C]). On the other hand, a general concept may also be considered in terms of its applicability to a multitude of individuals, on account of which it is a general representation and hence has an *extensio* in Knutzen's sense ([EXT_O]). These two aspects of a general concept are connected in the following way. If a general concept, A, contains another general concept, B, under itself so that the latter is part of the former's *Umfang*, then A is also *more general* than B. In other words, A has a greater *extensio* than B does in virtue of being a higher concept relative to B. Or, as Crusius puts it, insofar as B stands to A as species to genus in a conceptual hierarchy, B is narrower than A in respect of *extensio*: B is included, "with regard to the multitude of individuals, [as] a part of" A, to the extent that it "comprehends under itself only a part of those individuals" that are comprehended under the latter. (Crusius 1747, §136)²³ This relation between the *Umfang* (= [EXT_C]) of a concept and its *extensio* (= [EXT_O]) will become more salient in Kant's account of logical extension.

1.3. Leibniz, Lambert, and Euler: [EXT_O] and its geometrical analog

I have ascribed *extensio* ([EXT_O]) to such *Wolffschüler* as Knutzen and explained how it may be connected with Meier's *Umfang* ([EXT_C]). And I have shown that in Wolff's logics a concept is considered both to contain individual objects and to contain less general concepts under it. But it remains unclear whether the objects that are contained under a given concept and constitute its *extensio* ([EXT_O]) are (a) *actualia*, namely the objects to which the given concept actually applies or are (b) merely *possibilia*, namely the possible objects that the concept is capable of representing. If one asks whether either Wolff or any of the *Wolffschüler* mentioned so far is committed to (a) or (b), one may find it difficult to pin down a precise answer to the question. For the answer may vary with whether one addresses the question in regard to one or the other of the two aspects of a general concept sketched above. The answer may be (a), if we view the *extensio* in light of how a general concept is generated in the first place: to the extent that a general

concept is a result of comparing certain representations of *given individuals*, its *extensio* is composed of those individuals, which are the *actualia* falling under the concept. But the answer can also be (b), if a general concept is considered merely in virtue of its capacity as a general representation, regardless of how it is generated in the first place. We shall find clear indications of (b), though, in the logics of Leibniz, Lambert and Euler.²⁴ These logicians' accounts of logical extension share two important features. First, the extension of a concept is explicitly taken to be the multitude of individual objects falling under the concept. Second, the logical extension so construed is represented by such a geometrical figure as a line or a circle, a method of representation that strongly suggests that the objects constituting the extension of a concept are the *possibilia*, not the *actualia*, falling under the concept.

1.3.a. Leibniz on *extensio* and its representation by a line or a circle

Leibniz briefly discusses a distinction between extension and intension in the *New Essays*.²⁵ The distinction was introduced to address whether these two syllogisms are of a single mood: (i) Every A is B; every B is C; therefore, every A is C; (ii) Every B is C; every A is B; therefore, every A is C. In Leibniz's view, (i) and (ii) merely reflect two different perspectives of demonstrating the same syllogism (Barbara), depending on whether the judgments involved are interpreted with respect to the *extension* of the terms A, B and C or with respect to their *intension*. The judgment "Every A is B", for instance, can be read in two ways: either it "concerns individuals" (extension) and says that all *individual* As are "included among" all individual Bs, or it "refers rather to ideas or universals" (intension) and says that the *idea* of B is "included in" the *idea* of A. (*NE*, Bk.IV, Ch.xvii, §8) Accordingly, there are two ways of proving Barbara as valid, in accordance with the principle of "the containing and the contained". (ibid. §8) First, with "Every A is B" as its first premise, Barbara is valid by the transitivity of *extensional inclusion*: if all the individual As are included among all the individual Bs, and all of the latter are in turn included among all the individual C's; then all the individual As are included among all the individual C's. Second, with "Every B is C" as its first premise, Barbara is also valid by the

transitivity of *intensional inclusion*: if the idea of C is contained in that of B, and the latter is contained in the idea of A, then the idea of C is contained in the idea of A.

The view that the same judgment can be construed both in terms of the extensional inclusion and in terms of the intensional inclusion between its terms goes hand in hand with Leibniz's conception of a dual relation between genus and species. As Kauppi has noted, Leibniz takes the concepts of genus and species to stand in two kinds of part-whole containment relations. On the one hand, the concept of a genus is contained as a part (together with *differentia*) in that of its species, an intensional containment relation that pertains essentially to Leibniz's logical calculus. On the other hand, the genus-concept is broader than the species-concept with respect to extension, so that the individuals falling under the latter are contained as a proper part of those falling under the former. One containment relation is the inverse of the other, to the extent that if the concept A (of genus) is contained in the concept B (of species), then the extension of B (= the multitude of all the individuals falling under B) is contained in the extension of A, and vice versa.²⁶ This inverse relation between intensional containment and extensional containment presupposes that the extension of a concept consists of possible individuals, not actual ones.²⁷

That, for Leibniz, the extension of a concept is the multitude of *possibilia* falling under the concept becomes more evident when we consider the figurative means he uses to represent the extension. In the essay "General inquiries about the analysis of concepts and of truths" (1686), Leibniz uses two parallel lines to exhibit how any two given concepts may be formally related in a judgment with respect to extension, i.e., on the grounds that "a species is a part of (or at least are included in) the individuals of a genus." (C 385)²⁸ The form of "Every A is B", for instance, is shown as follows:

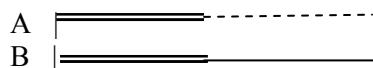


Fig. 1 (C 385)²⁹

This figure shows that the concepts A and B are so related that all the *individuals* falling under A are at least a proper part (indicated by the double-line segment) of all the *individuals* falling under

B. (ibid.) In another essay, “On the proof of logical form by the drawing of lines” (1686),³⁰ Leibniz also introduces a circle symbolism, alongside a line symbolism, to exhibit the same logical relation between two given concepts with respect to extension, as well as their relations in the other three basic forms of judgments. (Fig.2)³¹

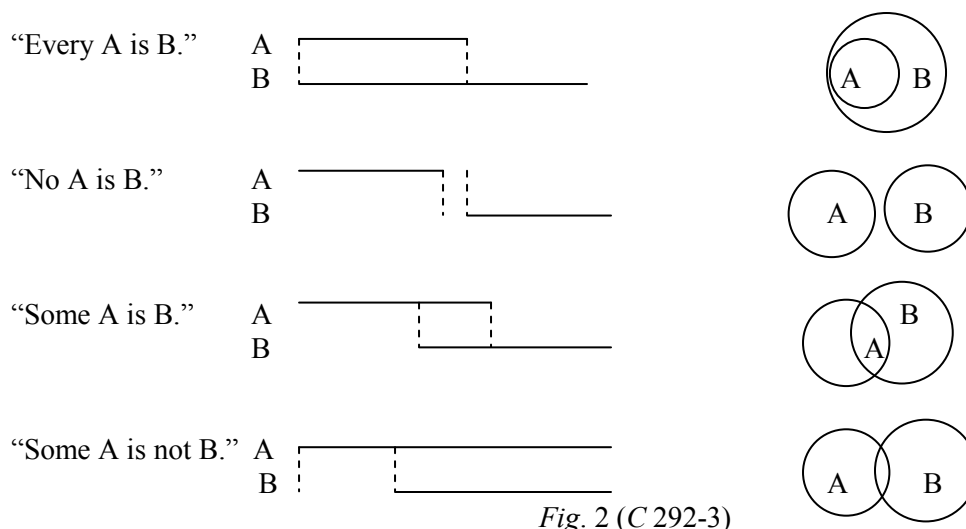


Fig. 2 (C 292-3)

Leibniz gives no explanation of how exactly to understand these figures. But it is at least safe to speculate that, if the lines or circles are introduced to represent concepts with respect to their logical extension, then the extension in question must consist of all the possible individuals falling under each concept. The justification for such a speculation will be spelled out by Lambert when he introduces a similar line symbolism into his logic.

1.3.b. Lambert on *Ausdehnung* and its representation by a line

In Lambert’s *Neues Organon* (1764), *Umfang* and *Ausdehnung* are the two notions that correspond to Leibniz’s *intension* and *extension*, respectively. Unlike Leibniz or any of the logicians mentioned so far, Lambert discusses these two notions in two separate chapters, without making any explicit connection between them. The notion of *Umfang* appears in the first chapter of *Dianoilogie* (volume 1 of the *Neues Organon*), entitled “*Von den Begriffen und Erklärungen*”. Here Lambert echoes Wolff’s and Meier’s view that concepts are formed through the process of “abstracting”, whereby one separates the common marks from the proper ones and designates the former by a general or abstract concept. (*NO*, §17) By repeating the same process we obtain a

hierarchy: “The similarity of individual things [*einzelne Dinge*] constitutes the species, the similarity of species constitutes the genus, and the similarity of genera constitutes the higher genus. And so it goes on and on, until one arrives at what is common to all things in general.” (*NO*, §15; cf. §10) Every species-concept has an *Umfang* (= Leibniz’s *intension*) composed of two sorts of marks: the common marks, which make up the concept of a relevant genus; and the proper marks, by which the species in question is differentiated from other species under the same genus. (*NO*, §14; cf. §§ 29, 36, 51, 56)

The notion of extension (*Ausdehnung*) is introduced in the third chapter of *Dianoilogie*, entitled “*Von den Urtheilen und Fragen*”. Noticeably, in this chapter there is no more discussion of the genus-species relations among concepts. Nor is there any reference to the *Umfang* of a concept introduced in the first chapter. By contrast, *Ausdehnung* now serves as *the* central notion in an account of the basic logical forms of judgments, forms that in turn determine valid patterns of syllogisms. An *Ausdehnung* is ascribed to a “universal concept” (*NO*, §17) and is characterized explicitly in terms of all the individuals to which the concept applies.³²

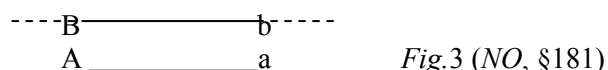
Every universal [*allgemein*] concept extends [*erstreckt*] to all the individuals to which it applies. Therefore it has a certain *extension* [*Ausdehnung*]. (*NO*, §174; cf. §176)

Lambert then introduces a line symbolism to represent such an *Ausdehnung*: “If one represents all these individuals in a *sequel* [*Reihe*] or *line*, then the length of this line will represent figuratively the *extension* of the universal concept.” (*NO*, §174) A line that represents a concept is a *continuum*—which contains infinitely many points—as opposed to a discrete row of finite points. This is meant to show that the individuals falling under a concept are also numerically infinite.

[T]he individuals that belong under a universal concept [...] are numerically just as infinite [*unendlich*] as the points of a line. Thus the universal concept, or its *extension*, cannot be represented through a certain number of points, but must be represented through a line. (*NO*, §177)

Obviously, the individuals making up the extension of a concept are considered as *possibilia* as opposed to *actualia*. For only as *possibilia* can the individuals falling under a concept be infinite. This view of what constitutes the extension of a concept has to do with the universality of the

concept: every universal concept, in virtue of its universality, is *capable* of representing many individuals, whether or not there is any individual actually falling under it. This conception of a universal concept, together with a line symbolism used to represent its extension, is central to Lambert’s account of the logical forms of judgment. To the extent that a judgment is a relation of two universal concepts, its form concerns “the relation of the extension of both concepts”. (§179) Such a relation can then be exhibited through a certain arrangement of two parallel lines that represent, respectively, the subject- and predicate-concepts with respect to their extension. In this manner, every judgment can be, “with respect to its form”, “*figuratively* represented and *designated* [gezeichnet]”. (NO, §173) E.g., the form of “Every A is B” can be represented as such:



The concepts A and B are so related in the judgment “Every A is B” that *every individual A belongs also under the concept B*. In Fig.3, such a relation is expressed by line A-a (which represents the concept A in respect of its extension) being placed directly under line B-b (representing the concept B), when line B-b is at least as long as line A-a. All the other basic forms of judgments are represented in the same fashion. (§§181-5)

1.3.c. Euler on extension and its representation by a circle

Euler’s account of logical extension is contained in several of the famous letters that he wrote to a German princess.³³ On Euler’s account, every “notion”, as a general idea, can “extend to” (*Lettres*, c, p.332) and be “realized in an infinite number of objects.” (ibid. p.333; cf. p.337)³⁴ In virtue of such generality, every notion has an extension, which can be represented by a closed, extended geometrical figure, such as a circle.³⁵

As a general notion contains an infinite number of individual objects, we may consider it as a space in which they are all contained. Thus, for the notion of *man* we form a space [...] in which we conceive all men to be comprehended. For the notion of *mortal* we form another [...] in which we conceive every thing mortal to be comprehended. (*Lettres*, cii, p. 339)



The extension of a concept, as is represented by a circle,³⁶ consists of *infinitely many* individual objects—and hence of *possibilia* as opposed to *actualia*. It is in this capacity of representing infinitely many possible objects that a concept figures in Euler’s account of the logical forms of judgment. A judgment, in Euler’s view, is a relation between two (general) notions. (cii, p.338)

A judgment is nothing else but affirmation or negation, that a notion is applicable or not applicable; [...]. All men are mortal, is a proposition [judgment]³⁷ which contains two notions; the first, that of men in general—and the second, that of mortality, which comprehends whatever is mortal. The judgment consists in pronouncing and affirming that the notion of mortality is applicable to all men. (cii, p.338)

Generally speaking, any two notions, A and B, can be related in four ways: either B is affirmed, or denied, of *all* of A; or B is affirmed, or denied, of *only some* of A.³⁸ These correspond to the four basic forms of judgments in the Aristotelian logic: universal affirmative, universal negative, particular affirmative, and particular negative. (cii, pp.338-39) With A and B being represented by two distinct circles, these forms of judgments can be exhibited through four possible ways in which the two circles may be related: either circle A is “entirely inside” or is “entirely outside” circle B, two relations that represent, respectively, the universal affirmative and universal negative judgments; or circle A is “partly inside” or has “at least a part outside” circle B, two relations that represent, respectively, the particular affirmative and particular negative judgments. (ciii, p.341; cf. cii, p.340)

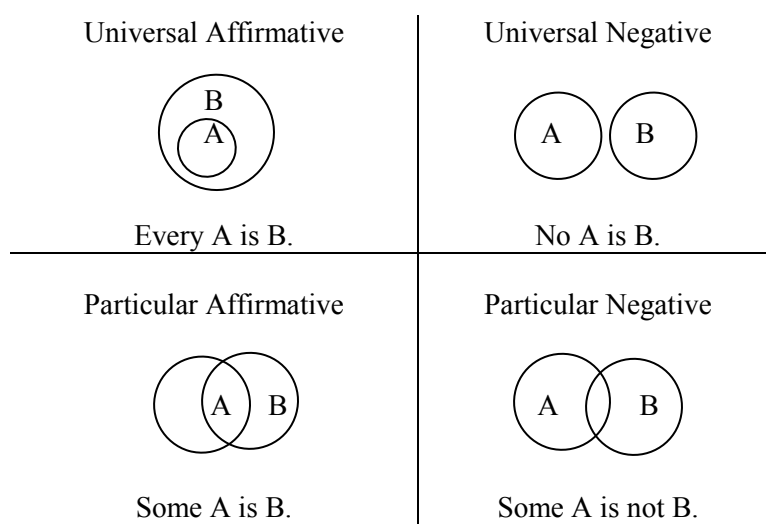


Fig.4

For Euler, as for Leibniz, the point of representing judgments by means of the circle symbolism is to provide “a great assistance towards understanding more distinctly wherein the accuracy of a chain of reasoning consists”. (cii, p.339) By the “chain of reasoning” is meant syllogism, “of which the objective is to draw a just conclusion from certain given propositions”. (ciii, p.342) Equipped with the circle symbolism, Euler seeks to demonstrate formally every valid syllogism the major premise of which has one of the four forms presented in Fig.4 and do so systematically, on the basis of two Leibnizian principles concerning “the nature of the *containing* and the *contained*”: whatever is inside the contained must likewise be inside the containing, and whatever is outside the containing must likewise be outside the contained. (civ, p.350)³⁹

In sum, to get a clear view of the historical backdrop against which to understand Kant’s notion of logical extension, we have investigated the accounts of logical extension given by many of the logicians mentioned in Kant’s logic corpus, as well as the account contained in the Port-Royal Logic, which supposedly introduced the notion of extension into the discourse of early modern logic. Here are the results of this investigation in a nutshell.

- (1) The definition of logical extension given in the Port-Royal Logic seems ambiguous between [EXT_C], by which the extension of a general idea consists of other (general) ideas it has under itself, and [EXT_O], by which the extension is the sum of the individual objects that the idea applies to. [EXT_C] and [EXT_O] are compatible in that both are needed, respectively, to make sense of two ways of restricting an idea with respect to extension: [EXT_C] is the basis for us to restrict the idea by adding to it a determinate idea, while [EXT_O] is the basis on which we restrict the idea by adding to it the indeterminate quantifier *some*. Thus, the notion of logical extension introduced in the Port-Royal Logic may be intended as a hybrid one that incorporates both [EXT_C] and [EXT_O].
- (2) Wolff presents two containment-under relations that pertain to two aspects of a general concept: in virtue of its generality, a concept contains a multitude of individual objects under itself; on account of having been obtained from other concepts through abstraction, it contains the latter concepts under itself. Among *Wolffschüler*, Reusch and Knutzen define the extension (*extensio*) of a concept as the multitude of individuals to which it applies ([EXT_O]), whereas Meier defines the extension (*Umfang*) of a concept as the sum of the concepts contained under it ([EXT_C]). Meier’s *Umfang* is noticeably tied up, however, with a theory of concepts according to which a general concept arises through logical abstraction from the concepts in which it is contained and, to that extent, contains the latter concepts under itself. Meanwhile, a notion of extension like Knutzen’s seems to be implicated in

Meier's account of the distinction between universal and particular judgments in terms of quantification over what is contained under the subject-concept.

- (3) Leibniz, Lambert and Euler share a distinct conception of [EXT_o], according to which the extension of a concept consists of all the *possible* objects falling under the concept and, to that extent, can be represented figuratively by a geometrical figure such as a line or a circle. This notion of logical extension, together with its geometrical analog, serves as the basis for exhibiting the logical forms of propositions (judgments) and for systematically demonstrating all valid syllogisms.

For our purpose, one important thing to take away from these results is that Meier's notion of *Umfang* ([EXT_c]) should not be taken as the default sense in which Kant must have conceived logical extension. Nor should this notion of *Umfang* be seen as obviously incompatible with the objectual notion of extension presented either by Knutzen or by Leibniz et al. After all, we have seen plausible ways not only to reconcile but also to connect these two notions. Keeping these lessons in mind, we shall be able to read Kant's varying characterizations of logical extension—at times suggesting [EXT_c], at times suggesting [EXT_o]⁴⁰—in a charitable spirit: not as signs of either equivocation or inconsistency, but as indicating a nuanced, hybrid notion of logical extension that may be intended to capture different aspects of a concept.

II. Kant on logical extension

Commentators are divided with regard to Kant's view of logical extension. Schulthess [1981], to begin with, thinks that Kant distinguishes between the “extensional intension” and the “intensional extension” of a concept:⁴⁰ the former is the multitude of the things thought under the concept, whereas the latter consists of all the concepts that are subordinate to the concept in a species-genus hierarchy. (Schulthess 1981:103-6) But Schulthess also thinks that Kant changes his mind from time to time about which of these two notions to hold. (ibid. 103-17) Other commentators, though they all think that Kant is of one mind on the issue, are divided into various camps. We may sample one representative view of each camp.

- (1) Prien [2006] argues that “by extension [*Umfang*] Kant means what Schulthess has called extensional extension”, according to which “the extension of a concept consists of objects [*Gegenstände*] and not of concepts.” (Prien 2006:76; cf.83) And “in the formal logic by extension is always meant a multitude of *possible* things.” (ibid. 76)⁴¹

- (2) Anderson [2004b] contends that Kant's conception of logical extension is "thoroughly 'intensional' in our modern sense" (Anderson 2004b:508): "for Kant, even the *extension* of a concept (in the logical sense) is understood to be the group of *intensional concepts* contained under it, rather than the individual objects it applies to." (ibid. 512; cf. 507; 508n.17; also see Anderson 2004a:186-92)⁴²
- (3) According to Stuhlmann-Laeisz [1976], the Kantian extension of a given concept is composed of two sorts of *representations*, i.e., intuitions and all the subordinate concepts. Although objects are said to be represented thereby (immediately by intuitions, but mediately by the subordinate concepts), they are not considered as part of the extension of the given concept. (Stuhlmann-Laeisz 1976:87-8)⁴³
- (4) Hanna [2001] takes it that for Kant every concept has *both* a "notional comprehension" ([EXT_C]), which consists of "every concept more specific than that concept", *and* an "objectual comprehension" ([EXT_O]), which is "the set of all actual or possible things satisfying the intension's descriptive criteria". (Hanna 2001:130; cf.136-7) So construed, a Kantian extension is a "hybrid intensional entity". (ibid. 136n.48)⁴⁴ But Hanna is especially interested in the objectual extension, which is "the result of treating the concept's intension as a function mapping from possible worlds into corresponding indefinitely large sets of possible objects". (ibid. 137)

On my reading, Kant construes the extension of a concept both in terms of the concepts subordinate to it and in terms of the objects thought under it. My reading differs from Hanna's in two significant respects. First, my view is not that for Kant the extension of a concept is, as Hanna sees it, a hybrid "entity", but that Kant holds two distinct but compatible notions of extension, a conceptual and an objectual one. Second, I take the objects that constitute a Kantian logical extension to be *possibilia*, but not *possibilia* in Hanna's sense. I see no need to appeal, as Hanna does, to the possible-world semantics in order to spell out the objectual notion of extension on Kant's behalf. Rather, I shall explain the relevant sense of *possibilia* in connection with Kant's use of circle symbolism to represent a concept as regards its (objectual) extension.

My reading will be a result of applying two methodological principles to interpret the observation that throughout Kant's logic corpus there is as much *prima facie* evidence for [EXT_O] as there is for [EXT_C]. First, the principle of charitable interpretation commends that, if there is room for a better, alternative explanation, one should not take the observation simply to show that Kant is confused or careless. The second principle is a context principle, according to which Kant's use of a particular notion of logical extension should be interpreted in light of the context

for such a use. In the following, I shall sample Kant's discussions in his logic corpus of the *Umfang* (*sphaera*, *Sphäre*) of concepts. The samples are taken from three groups of texts: the *Logik Blomberg* and the *Logik Philippi*, both of which are based on Kant's logic lectures from early 1770's; the *Logik Pölitz*, the *Wiener Logik* and a few other student-notes based on Kant's logic lectures during and after 1780's; and the *Jäsche-Logik*, supplemented by certain materials from Kant's logic *Reflexionen*. In each group of texts, there appears to be as much evidence that Kant holds an objectual notion of logical extension ([EXT_O]) as there appears to be evidence that Kant holds a conceptual notion ([EXT_C]) instead. But we shall notice the difference in the contexts in which these two notions operate.

2.1. Logical extension in Kant's logic corpus

2.1.a. In the *Logik Blomberg* and the *Logik Philippi*, the extension or "sphere" of a general notion is defined as the "multitude of things" thought under it.

The sphere of a notion [*sphaera notionis*] means actually the multitude of things [*Menge der Dinge*] that are comprehended under a concept as a general notion [*nota communis*]. (V-Lo/Blomberg, Ak 24:257)

The multitude of things that is contained under a concept as a general notion constitutes the sphere of the general notion. (V-Lo/Philippi, Ak 24:454)

Note that the extension so defined is attributed to a concept *as* general notion. A general notion is contrasted with a so-called singular concept (*conceptus singularis*): by means of a singular concept I "think only one thing"; but a general notion always concerns "that which is common to many things", and hence by means of it I think "a complex of many individual things". (V-Lo/Blomberg, Ak 24:257) The question is whether by '*Ding*' here Kant means "object", so that the definition given above unequivocally amounts to an objectual notion of extension ([EXT_O]).

Tolley [2007] warns against reading Kant's reference to *Dinge* in such contexts straightforwardly as reference to objects. He suggests that many present-day readers of Kant's logic texts take the "things" which belong in the extension of a concept to be individual objects because they are "predisposed" to do so—"given the now-standard use of the term"—without

independent reason. (Tolley 2007:357) To the contrary, Tolley argues, once one recognizes that “Kant moves effortlessly between claiming that the ‘Umfang’ of a concept is concerned with a ‘multitude of *things* [Menge der Dinge] that are contained under a concept’ (24:911), and claiming that ‘every concept contains more *possible concepts* under itself [unter sich]’ (24:910)”, one shall have an “initial reason” to think that the *Dinge* that constitute the extension of a concept include the *concepts* contained under it. (ibid.)⁴⁵ To my knowledge, however, Kant never explicitly *equates* a concept—which is a representation of things—with a *Ding*. By contrast, he often uses the term ‘*Ding*’ interchangeably with the term ‘*Gegenstand*’. In the *Logik Blomberg*, for instance, immediately after saying that “a concept that contains much under itself arises through the fact that what is common to many things [*viele Dinge*] is abstracted”, Kant says: “a general concept arises only through the fact that what is common to many objects [*viele Gegenstände*] is considered.”⁴⁶ (V-Lo/Blomberg, Ak 24:258) Likewise, in the *Logik Philippi*, having said that the more general a concept is “the more things [*Dinge*] it is common to”, Kant characterizes a concept of “maximal sphere” as one that “contains under itself all objects [*omnia objecta*]”. (V-Lo/Philippi, Ak 24:454) Thus, Kant’s reference to *Dinge* in the definition above of logical extension is a reference to objects, in which case the definition amounts to [EXT_O].

In the same texts, however, there are also hints of a conceptual notion of extension ([EXT_C]). Besides taking a multitude of things or objects to be contained under a general concept, Kant also speaks of other *concepts* being contained under it. The latter containment-under relation has to do with Kant’s theory of how a concept may be obtained through logical abstraction.

In logical abstraction we compare many concepts with one another, we see what these contain in common, [...]. If many concepts have a mark in common, then this mark is contained in them. (V-Lo/Blomberg, Ak 24:255; cf. V-Lo/Philippi, Ak 24:453)

Insofar as a concept is obtained through logical abstraction from other concepts, it contains these concepts under itself. (V-Lo/Blomberg, Ak 24:257-60; V-Lo/Philippi, Ak 24:453-4) This concept, as a higher one, admits “logical division”, a division with respect to its extension. Such a division is said to be “complete” just in case “the members of division (those are just the lower concepts)

taken together exactly constitute [the extension of] the universal concept.” (V-Lo/Blomberg, Ak 24:273; V-Lo/Philippi, Ak 24:461) Here, logical extension appears to be considered as [EXT_C].

The notion of logical extension also figures in Kant’s theory of judgment, especially in his account of the distinction between universal and particular judgments. But here it becomes less clear whether the notion should be understood as [EXT_O] or [EXT_C]. In the *Logik Blomberg* the distinction is presented as follows:

All judgments are either *universal* or *particular*. It is universal if the notion of the subject is either completely contained in the sphere of the predicate [*in der Sphaera des Praedicats enthalten*] or not at all, [...]. A particular judgment, however, is one where the notion of the subject is only partly contained, or not contained, under the sphere of the predicate [*nur dem Theile nach unter der Sphaera des Praedicats enthalten*]. (V-Lo/Blomberg, Ak 24:275)

By that the *notion* of the subject is wholly or only partly contained or not contained *in* or *unter*^{A7} the sphere of the predicate, what is meant is really that the *extension* or sphere of such a notion is entirely or only partly contained, or not contained, in that of the predicate. This can be seen from the way in which universal and particular judgments are characterized in the *Logik Philippi*:

If the sphere of the subject is entirely contained in the notion of the predicate [*ganz in der Notion des Prädicats enthalten*] or is considered entirely outside it, then it is a universal affirmative judgment in the first case and a universal negative judgment in the last case. [...]

If the sphere of the subject partly falls in that of the predicate [*zum Theil in diejenige des Prädikats fällt*], and partly does not, then it is particular judgment. [...]

With all judgments I compare nothing other than the spheres of concepts and see whether one is contained in the other or not. (V-Lo/Philippi, Ak 24:463)

If we read the three sentences together, it is clear that different forms of particular and universal judgments are viewed in terms of different ways in which the subject and the predicate of a judgment are related with respect to both of their *extensions*. What is not immediately clear, however, is whether the extension in question is to be considered in terms of [EXT_C] or [EXT_O].

2.1.b. In the logic lectures during the critical period, Kant often emphasizes the *essential* generality of every concept and denies that there is such a thing as a singular *concept*. Logic, Kant says, “deals only with concept as concept”. (V-Lo/Pölit, Ak 24:567) That is, it deals only with “the logical form of concept”, which lies in its being “common to many [*vielen gemein*]”.

(ibid. Ak 24:568) What a concept is common to includes both things (*Dinge*) and their representations (concepts). Accordingly, both things and their representations are said to be contained under the concept. As a concept “contains that which is common to various representations of several things [*Vorstellungen mehrer Dinge*]”, “these things are contained *under* it” *and* the representations—or “possible concepts”—of the things are also contained under it. (V-Lo/Wiener, Ak 24:910; V-Lo/Pölitz, Ak 24:568) With respect to the first containment-under relation, the concept is considered “a ground of cognition for many things”; with respect to the second, the concept is a “higher representation” of what are represented by the subordinate concepts. (ibid.) In these terms, the extension (*Umfang, sphaera*) of a concept is introduced and defined in contrast with its intension (*Inhalt*): the extension is “the multitude of *things* [*Dinge*] that are contained under the concept”, and the intension is “the multitude of the *representations* that are contained in the concept itself.” (V-Lo/Wiener, Ak 24:911, emphasis added)⁴⁸

The two containment-under relations—that a concept contains many *Dinge* under itself and that it contains the lower representations (concepts) of those things under itself—seem to be closely connected. On the one hand, the generality or “universality” of every concept lies in it representing what is common to a “multitude of things”, and hence in it having an extension.

For a concept is always valid for what is contained under it[;] consequently the universality of cognition always rests on the sphere. (V-Lo/Wiener, Ak 24: 912; cf. V-Lo/Hechsel, LV:400)

On the other hand, a concept may have a larger logical extension than does another concept depending on their relative positions in a conceptual hierarchy. If two concepts relate to each other as genus to species, “the *genus_L* [i.e., the relatively higher concept]⁴⁹ is always a broader concept [*conceptus latior*], the *species_L* [i.e., the relatively lower concept] a narrower concept [*conceptus augustior*].” (V-Lo/Wiener, Ak 24:912; cf. V-Lo/Hechsel, LV:400) And, as one “go[es] to the heights” along a hierarchy of concepts, “[t]he logical sphere always grows”; in the process, one thinks more and more things with less and less *Inhalt* in one’s concept—just “as a leaf of gold stretches when it loses in thickness”—until one reaches the highest concept which

represents “only what is common to *all* things”. (ibid.) The metaphor of a stretching leaf of gold captures what Kant takes to be an inverse relation between the intension and the extension of a concept: for any concept, the less is in its intension, the greater is its extension; and vice versa. (V-Lo/Wiener, Ak 24:911; cf. V-Lo/Dohna, Ak 24:755) Commentators who think that by logical extension Kant *always* means the concepts, as opposed to objects, contained under a concept sometimes use this inverse relation to support their reading, arguing that the relation would not hold if ‘extension’ were taken in the objectual sense. I shall address such an argument in 2.2, showing that the inverse relation fails on an objectual notion of extension only if the objects constituting the extension are *actualia*, whereas on my reading the objects are *possibilia*.

In the *Logik Wiener* etc., however, the notion of extension involved in Kant’s account of logical division again appears to be [EXT_C]. To logically divide a “universal concept”—insofar as it “has a sphere, and has lower concepts under itself”—is said to “divide the sphere, the lower concepts, as far as they are contained under the universal”; and the lower concepts or *membra dividantia* “taken together, are equal to the sphere of the whole concept.” (V-Lo/Wiener, Ak 24:925; cf. V-Lo/Pöhlitz, Ak 24:576) The claim that the lower *concepts*, as members of the logical division of a given concept, taken together are “equal to the sphere” of the divided concept is repeated in many other logic lectures from the same period.

Logical division is actually when I enumerate the particular concepts that [...] stand under a universal concept. [...] the concepts [...] taken together constitute the sphere of the divided concept; [...]. (V-Lo/Bauch, LV:169)

Division is in general the representation of the manifold of concepts that are opposed to one another and that fill up the entire sphere of the concept [...]. (V-Lo/Hechsel, LV:421)

By logical division I look to the sphere [...] of a concept. Logical division is therefore division of sphere. [...] The other concepts called members of division must be equal to the sphere of the divided concept. (V-Lo/Busolt, Ak 24:660-1)

By logical division is understood the taking apart of [...] sphere. [...] A division expresses, then, that lower concepts subordinated to higher ones are together equal to the whole sphere of the concept. (V-Lo/Dohna, Ak 24:760-1)

All these passages, literally read, suggest that [EXT_C] is the operative notion of extension in Kant's account of logical division: the extension of a concept consists of the concepts that are contained under it as members of its logical division.

Finally, the notion of extension involved in Kant's account of forms of judgment again seems ambiguous between [EXT_C] and [EXT_O]. Kant's characterization of the form of an infinite judgment, for instance, is given in terms of a certain relation between the subject-*concept* and the extension (sphere) of the predicate:

I do not merely exclude one concept from the sphere of another concept, but also think the concept under the whole remaining sphere which does not belong under the concept that is excluded. [...] I say that the soul can be counted among all the concepts in general that may be thought outside the concept of mortality. (V-Lo/Wiener, Ak 24:930; cf. V-Lo/Hechsel, LV:424-5)

By the judgment “the soul is non-mortal”, the *concept* of the soul is said to be placed in the sphere of the non-mortal, a sphere that consists of all the *concepts* that are not contained under the concept of mortality. The notion of extension operative here seems to be [EXT_C], then. Nevertheless, when we turn to the account of infinite judgment given in, say, the *Logik Pölitz*, we no longer see any clear sign of [EXT_C]. Here, an infinite judgment is said to be one in which “the subject is contained in a sphere other than in [that of] the predicate.” For example, when I judge that the soul is non-mortal, “I think that the soul does not belong to the mortal things [*zu den Sterblichen gehöre*]; but I think still more, [...] I think that it [*sie*] is contained in a sphere other than in [that of] the concept [of mortal].” (V-Lo/Pölitz, Ak 24: 578; cf. V-Lo/Warschauer, LV:624-5) In this case, the soul (which may very well be understood in the objectual sense), not its concept, is said to be contained in a sphere that lies outside the sphere of mortal things. It seems more natural to understand the sphere in question in the sense of [EXT_O] as opposed to [EXT_C]. Or, to say the least, there is no obvious reason why it should not be [EXT_O].

2.1.c. In the *Jäsche-Logik* the notion of *Umfang* is first introduced in §7, as one of the two central aspects of a concept. First, a concept is a “partial concept” that is “contained in the

representation of things [*in der Vorstellung der Dinge enthalten*]; in this respect, a concept is said to have intension (*Inhalt*). Second, “these things [*diese Dinge*] are contained *under*” the concept *qua* “ground of cognition”; in this respect, the concept is said to have extension (*Umfang*). (Log, §7, Ak 9:96; cf. R2902, Ak 16:567; R2881, Ak 16:557-8) As Prien [2006] puts it, here we are presented with two distinct relations that a concept stands in and that correspond, respectively, to the *Inhalt* and the *Umfang* of the concept. On account of being contained in other representations as part of their intension, a concept relates to those representations as genus to species. But a concept, in virtue of being a “representation” or “ground of cognition”, also relates “to the objects [*Gegenständen*] that fall under it”. (Prien 2006:82-3) To the extent that the extension of a concept has strictly to do with the latter relation, it consists only of the *objects* falling under the concept. (ibid. 77) And so the “extension” presented in §7 of the *Jäsche-Logik* is a notion of [EXT_o].

This conception of *Umfang* as [EXT_o] is further suggested in a remark about the “quantity of the extension of concepts” in the subsequent section: “The more things [*Dinge*] stand under a concept and can be thought through it, the greater is its extension or sphere.” (Log, §8, Ak 9:96) Note, however, that the quantity of the extension of a concept is not determined by how many objects *actually* fall under the concept. Nor does it make sense to say that the extension of a concept is large or small *tout court*. For, as Prien has pointed out, the multitude of *Dinge* that constitutes the extension of a concept is “a multitude of possible things” and, to that extent, the extension of a concept, considered by itself, “is always infinitely large [*unendlich groß*]”. (Prien 2006:76) Rather, the extension of a concept is only *greater* or *smaller* than that of another concept, when the two concepts stand in a logical relation of subordination, i.e., when one concept is higher or lower than the other. More specifically, when one concept (A) is said to be “broader”—i.e., have a greater extension—than another (B), it is “*not* [...] *because* it contains *more* under itself—for one cannot know that—but rather insofar as it contains under itself the *other concept* and *besides this still more*.” (Log, §13, Ak 9:98, the first emphasis added) In other words, A’s

having a greater extension is *not determined on the basis* that more things fall under it than under B, but by A's being *higher* than B: a "higher concept is also called a broader concept". (Log, §12, Ak 9:98) This echoes our observation in 2.1.b of a close connection between [EXT_C] (= all the concepts subordinate to a given concept) and [EXT_O] (= the multitude of possible objects falling under a concept): the higher we go in the series of subordination, the more general a concept we reach, which represents more things in comparison with any of the subordinate concepts.

Kant occasionally uses two distinct kinds of figurative means—a plane-geometrical figure and a tree-like structure—to represent, respectively, the extension of a concept in the objectual sense and the hierarchical relation that the concept has with other concepts. To begin with, Kant suggests that a circle—or, more precisely, the space enclosed therein—may be used to represent the extension of a concept that is the multitude of objects falling under the concept.

The multitude of things [*Dinge*] that are contained under the concept is called the logical sphere of the concept. [...] One understands by that [...] the circle of application, a line that has in itself no breadth but nonetheless comprehends a great space. (V-Lo/Dohna, Ak 24:755)

This circle symbolism for representing a concept with respect to its logical extension (in the objectual sense) is used, on Kant's part, primarily to exhibit the logical relations of concepts in judgments and syllogisms. If, for instance, two concepts are related in a judgment with respect to extension, then here is a crude way of representing the basic schema of such a relation:



Fig.5 (R3216, Ak 16: 716)

The two circles represent, respectively, the subject- and predicate-concepts of a judgment.⁵⁰ The four basic forms of judgment in the Aristotelian logic are expressed accordingly:

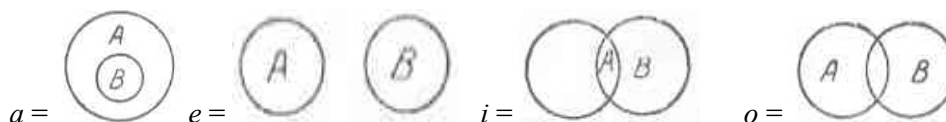


Fig.6 (R3215, Ak 16:715; cf.R3036, Ak 16:627; R3063, Ak 16:637)⁵¹

And a syllogism, insofar as it is a relation of such forms of judgments, can be represented by means of the same circle symbolism. For instance, *Fig.7* represents “all C is B; all A is C; therefore, Every A is B” or “all A is C; all C is B; therefore, some B is A.” (R3215, Ak 16:716) And *Fig.8* represents “all C is B; some A is C; therefore, some A is B.” (ibid.)

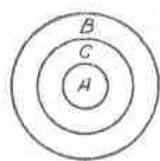


Fig.7

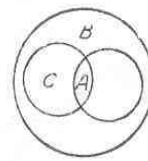


Fig.8

In general, the form of any judgment (and hence of any logical inference), so long as it consists in a relation of two or more *concepts* with respect to extension, can be presented in the same fashion as in *Fig.6-Fig.8*. For that purpose, it does not matter what figure is used to represent a concept.⁵² Kant sometimes uses a square-symbolism to represent, for example, a disjunctive judgment:

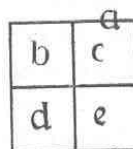


Fig.9 “(x) which is *a*, is either *b*, or *c*, or *d*, or *e*.”⁵³ (R3096, Ak 16:658)

A disjunctive judgment, in Kant’s view, is based on the division of the extension of a given concept into mutually exclusive but jointly exhaustive parts.⁵⁴ As such, the judgment says that “[w]hat is contained under the sphere of a concept is also contained under one of the parts of this sphere.” (Log, §29*n*, Ak 9:107) In *Fig.9*, the extension of the concept *a* is divided into four parts corresponding, respectively, to the concepts *b*, *c*, *d* and *e*. To the extent that these parts are mutually exclusive, but jointly exhaustive of the extension of *a*, anything *x*, which falls in the extension of *a*, must fall in exactly one of these parts.

This figurative representation of disjunctive judgment gives us a new perspective to view the notion of extension involved in Kant’s account of logical division. The account given in the *Jäsche-Logik* is no different from what we have seen in 2.1.a and 2.1.b: by a logical division, we go from a given concept (the divided concept) to certain concepts (members of division) that are

subordinate to it, in such a way that “taken together they [i.e., the subordinate *concepts*] constitute the sphere of the divided concept or are equal to it.” (Log, §112, Ak 9:146) The terminology changes, however, when Kant presents the result of such a division in a disjunctive judgment.

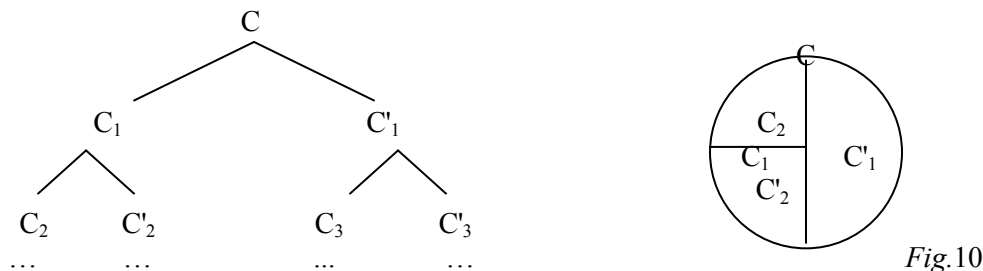
If, for example, I make the disjunctive judgment, A learned man is learned either historically or in matters of reason, I thereby determine that these concepts are, *as to sphere*, parts of the sphere of the learned man, [...] and taken together they are all complete. (Log, §29n., Ak 9:107, emphasis added)

Saying that the lower *concepts* revealed through a logical division constitute the extension (sphere) of the divided concept is not the same as saying that these concepts, in respect of *extension*, constitute the extension of the divided concept. If, as we have seen in light of Fig.9, the extension in the latter case is taken in the objectual sense ([EXT_o]), the extension in the former case appears to be taken in the conceptual sense ([EXT_c]). And yet logical division and disjunctive judgment are considered, on Kant’s part, to be so connected that the latter is based on the former. What, then, is the precise nature of the connection between logically dividing a given concept (e.g., *learned*) into the lower concepts that are members of the division (e.g., *learned historically* and *learned in matters of reason*) and making the disjunctive judgment “A learned man is learned either historically or in matters of reason”? It is clear that, if a disjunctive judgment says that “[w]hat is contained under the sphere of a concept is also contained under one of the parts of this sphere”, then “the sphere must *first* be divided.” (Log, §29n., Ak 9:107, emphasis added) But if the “sphere” involved in the disjunctive judgment is taken in the *objectual* sense, how is *its* division—as is exhibited in Fig.9—related to the division of the given concept into lower *concepts*?

The answer to this question is partly suggested, I submit, by the expression ‘*Glieder der Eintheilung*’ used for the concepts that are subordinate to a divided concept.⁵⁵ This expression has a figurative connotation: it literally means “*branches of division*”. This suggests that the result of logically dividing and sub-dividing a given concept can be figuratively represented by means of a tree-like structure, in which every relatively higher node stands for a divided concept and the

branches extending from this node lead to the concepts that are immediately subordinate to it.⁵⁶

We can use *Fig. 10* to illustrate this kind of division—or, more aptly put, branching—and how it may be intimately connected with the division of logical extension in the sense of [EXT_O].



The tree on the left shows that a given concept, *C*, branches into two concepts, *C*₁ and *C*'₁, each of which in turn branches into even lower concepts; and so on. The plane or area enclosed within the circle on the right designates the multitude of possible objects represented by *C*. The tree presents a step-by-step manual, as it were, for how to divide the plane: first into *C*₁ and *C*'₁; the former in turn into *C*₂ and *C*'₂; and so on. The logical relations of concepts shown in the tree are thus mirrored, so to speak, in the divisions of the plane. In particular, as the tree shows that *C*₁ is a higher concept than *C*₂ and relates to the latter as genus to species, the plane must be so divided that the part corresponding to *C*₂ is properly contained in the part corresponding to *C*₁. This vividly reflects a principle explained earlier: a higher concept is also a broader one.

2.2. [EXT_C] and [EXT_O]: Kant's hybrid notion of logical extension

In 2.1 we examined how Kant presents the *Umfang* or *sphaera* of a concept in his logic corpus. What we found in the logic lectures (2.1.a and 2.1.b) should disappoint any one who wishes to pin down either [EXT_C] or [EXT_O] as *the* sense in which Kant construes the extension of a concept. For, these logic lectures taught us only the following. First, there is *prima facie* evidence, especially in Kant's definition of *Umfang*, that the extension of a concept is taken to be the multitude of objects represented by it ([EXT_O]). Second, there is also *prima facie* evidence, especially in Kant's account of logical division, that the extension of a concept is taken to consist of all the concepts subordinate to it ([EXT_C]). Finally, the notion of extension involved in Kant's

account of the logical forms of judgment appears simply to be ambiguous between [EXT_C] and [EXT_O]. Thus, as far as the logic lectures are concerned, there is no decisive textual evidence for the reading that by logical extension Kant always means [EXT_C] *as opposed to* [EXT_O], or vice versa. If anything, the data seemed more amenable to a compatibilist interpretation: it may very well be that by logical extension Kant means *both* [EXT_C] *and* [EXT_O].

A firmer textual basis for such a compatibilist reading, together with the proper vocabulary for spelling it out, was gradually teased out in our analysis of the account of logical extension in the *Jäsche-Logik* (2.1.c). A concept is taken to have an objectual notion of extension ([EXT_O]), to begin with, on account of being an essentially general representation *qua* “ground of cognition”. By contrast, if a concept is also taken to have a conceptual notion of extension ([EXT_C]), as implicated in Kant’s account of logical division, such a notion would serve only to capture how the concept relates to those concepts that are subordinate to it. And, if Kant uses an enclosed geometrical plane to represent [EXT_O] and a tree-like structure to represent [EXT_C], it is remarkable that only the former figurative means is used to present the logical relations of concepts in judgments and syllogisms. We may interpret these observations in light of some findings of section I. Let’s recall: in Meier’s logic the notion of *Umfang*, understood as [EXT_C], is never explicitly referred to beyond the context of a theory of how a concept may be formed through logical abstraction from lower concepts; by contrast, the logical extension in Leibniz’s, Lambert’s and Euler’s logics—explicitly construed as [EXT_O] and represented by means of a circle or line symbolism—underscores those logicians’ account of the formal relations of concepts in judgments and inferences. In terms of such a contrast, we may regard Kant as having presented a hybrid account of logical extension: a concept has a logical extension both in the sense of [EXT_C] and in the sense of [EXT_O]; [EXT_C] is intended primarily to capture how concepts relate to one another in a series of subordination, whereas [EXT_O] reflects the essential generality of a concept and plays a pivotal role in accounting for certain forms of judgment.

If one still wishes to show that by logical extension Kant must mean either [EXT_C] or [EXT_O], but not both, one would then have to argue to the following effect: with respect to any *prima facie* textual evidence for ascribing one of the two notions, there is a strong reason for not taking that evidence at its face value and for thinking, instead, that what Kant *really means* is the other notion. This roughly captures the strategy adopted by some proponents (e.g., Anderson 2004b; Tolley 2007) of the reading that by logical extension Kant always means [EXT_C] and definitely not [EXT_O]. Since, to prepare for Chapters 3 and 4, I am mainly interested in defending the ascription of [EXT_O] to Kant, I shall conclude the present chapter by addressing the main argument made by Anderson [2004b] and Tolley [2007] against such an ascription. The argument builds on Kant's claim that the extension of a concept is inversely proportional to its intension. In a nutshell, the argument runs as follows: Kant is committed to the logical law that the intension and extension of a concept are inversely proportional ("law of inverse proportionality" for short);⁵⁷ but this law holds only if extension is taken in the conceptual sense; therefore, Kant cannot mean the extension of a concept in the objectual sense (even if his wording may occasionally suggest otherwise).⁵⁸ For details, let's consider Tolley's version of the argument. Tolley refers to the formulation of the law of inverse proportionality in §7 of the *Jäsche-Logik*:

The intension [*Inhalt*] and extension [*Umfang*] of a concept stand in an inverse relation [*in umgekehrtem Verhältnisse*] to each other. That is, the more a concept contains under itself, the less it contains in itself; and vice versa [*umgekehrt*]. (Log, §7, Ak 9:96)

In Tolley's view, this law of inverse proportionality "cannot possibly be thought to hold if we are supposed to take an 'Umfang' to consist in the *objects* to which the concept is applicable". For

it might be the case that concepts of differing intensional-'quantities' could still apply to the same 'quantity' of objects. Consider concepts like 'president of the United States' and 'male president of the United States': these have *different* intensional-quantities, insofar as the idea of 'president' does not contain within itself the idea of 'male', and yet both concepts have the *same* quantity of objects (indeed, the very same objects) falling under them. Inverse proportionality would thus be compromised. (Tolley 2007:362-3)

Obviously, the notion of objectual extension targeted in this argument is our modern notion, according to which the extension of a concept is the set of objects that *actually* fall under the

concept.⁵⁹ But the notion of objectual extension I have ascribed to Kant is an early modern one, especially as it was developed in Leibniz's, Lambert's and Euler's logics. According to this notion, the extension of a concept is the multitude of *possible* objects represented by the concept, regardless how many, if any, objects actually fall under it.⁶⁰ This objectual extension is, as we have observed, intimately connected with the hierarchical relation among concepts, to the extent that the relative quantity of the extension of a given concept can be determined only in reference to the concept's relative position in a conceptual hierarchy. This observation, together with the logical principle that a concept contains another concept under itself only in virtue of being contained in the latter as part of its intension, serves as the basis for understanding Kant's law of inverse proportionality and for showing that the law necessarily holds for the objectual extension when by objects is meant *possibilia*. Or so I shall argue.

To begin with, we have seen that, in Kant's view, if one concept is contained in another (as part of its intension), then the latter is subordinate to or contained under the former; to that extent, the first concept relates to the second as genus to species and as a higher to a lower concept. There are two cases in which the containment-in and the reverse containment-under relations arise. First, we obtain higher and higher concepts by means of logical abstraction, in which case a relatively higher concept is always contained in those from which it is abstracted, and the latter concepts are contained under and subordinate to the former. Second, we generate lower and lower concepts "by continued logical determination", (Log, §15, Ak 9:99) viz., by adding to a given concept certain predicates that are not already contained in it,⁶¹ in which case the given concept is necessarily contained in all the concepts generated from it, and the latter concepts are necessarily subordinate to the former as species to genus. In either case, a relatively higher concept always contains less in itself—and hence has a smaller intension—than any concept subordinate to it does; and vice versa. But, as we have seen in 2.1.c, for Kant a relatively higher concept is, on that account, also a relatively *broader* one—i.e., represents a greater multitude of possible objects—than any concept subordinate to it does; and vice versa. Therefore,

necessarily, the smaller an intension a given concept has in comparison with another concept, the greater is its objectual extension (provided that the objects constituting the extension are *possibilia*); and vice versa.

To illustrate, take Tolley's example. Let $C = \textit{president of the United States}$, $A = \textit{male}$ and, in addition, $B = \textit{female}$. Insofar as the concept C does not contain either A or B within itself, A and B may be added, respectively, to C . Two lower concepts are generated thereby: $C_1 = \textit{male president of the United States}$, and $C_2 = \textit{female president of the United States}$. Assuming that *male* and *female* are two contradictorily opposed concepts for Kant, we have a logical division or "branching" of C into two subordinate concepts, C_1 and C_2 . Both the containment-in and the containment-under relations between C and C_1/C_2 can be exhibited through Fig.11.

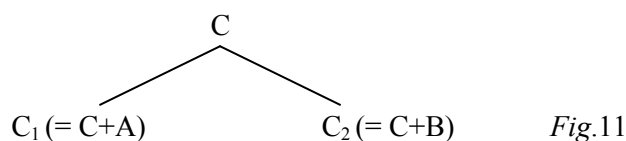


Fig.11

With C thus branching into C_1 and C_2 , the objectual extension of C —i.e., the multitude of possible objects falling under it—must be divided accordingly. Insofar as the concepts C_1 and C_2 are contradictorily opposed, such an objectual extension is to be equally divided into the multitude of possible objects falling under C_1 and the multitude of those falling under C_2 (Fig.12).

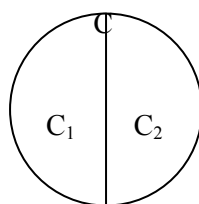


Fig.12

With respect to quantity, then, the objectual extension of C_1 (*male president of the United States*) is necessarily smaller than that of C (*president of the United States*)—despite the fact that, as Tolley has pointed out, exactly the same amount of actual objects fall under C and C_1 . Such a difference in the quantity of objectual extension follows simply from the fact that we have obtained C_1 by *adding* a predicate to the intension of C , an addition that in turn determines the subordination of C_1 to C . If we add yet another predicate (say, "black") to C_1 , we will get a

concept which has an *even greater intension* (thanks to the addition), but which also has an *even smaller objectual extension* (on account of being subordinate to C_1); and so on, and so forth. In this way, Kant's law of inverse proportionality holds necessarily of an objectual notion of logical extension: the greater the intension of a concept is, the smaller is its objectual extension, provided that the objects constituting the extension are *possibilia* as opposed to *actualia*.⁶²

¹ In the present chapter by 'logic' I always mean the pure general logic discussed in Chapter 1.

² V-Lo/Dohna, Ak 24: 764; cf. V-Lo/Blomberg, Ak 24:274; V-Lo/Busolt, Ak 24:664; V-Lo/Philippi, Ak 24:461; V-Lo/Wiener, Ak 24: 928. Unless otherwise noted, by 'judgment' in the present chapter I mean categorical judgment, in which two concepts are related as subject to predicate.

³ In the *Critique*, judgments are said to have priority over concepts in the sense that the understanding has no other use of its concepts than judging by means of them (A68/B93), to which extent every concept is "the predicate for a possible judgment". (A69/B94; cf. *Prol.* Ak 4:304, 323) Such priority has to do with the discursive nature of concepts: "a concept is never immediately related to an object, but is always related to some other representation of it", and a judgment, accordingly, is "the mediate cognition of an object, hence the representation of a representation of it". (A68/B93; for discussion, see Paton 1951:252; Allison 1973: 63-4) Taking judgments to have priority over concepts in this sense is compatible with regarding concepts as being prior to judgments in the sense that they are the *given* "matter" out of which to make judgments. (see V-Lo/Hechsel, LV:423; V-Lo/Wiener, Ak 24:928; V-Lo/Dohna, Ak 24:764; V-Lo/Blomberg, Ak 24:273-4; Log, §18, Ak 9:101)

⁴ Kant never uses the term '*intensio*', but uses only '*Inhalt*', in his logic corpus to mean what most early modern logicians took to be intension. The term '*Inhalt*' is usually translated as "content". But there is noticeable difference between the *Inhalt* of cognition in the *Critique*, by which is meant the cognition's *Beziehung* to the object, and the *Inhalt* of a concept in the logic corpus, by which is meant the marks contained in the concept. In light of such difference, I have reserved "content" for the former *Inhalt* (discussed in Chapter 1) and will use "intension" for the latter *Inhalt*.

⁵ See V-Lo/Hechsel, LV:399; V-Lo/Busolt, Ak 24:655; V-Lo/Warschauer, LV: 613; V-Lo/Pölitz, Ak 24: 569; V-Lo/Philippi, Ak 24: 454. I am setting aside the difficult task of deciding, for any particular concept, what marks are contained in it or of understanding the relevant sense of containment in the first place.

⁶ See Log, §2, Ak 9:91; V-Lo/Pölitz, Ak 24:568; V-Lo/Warschauer, LV:609; V-Lo/Hechsel, LV:391; V-Lo/Blomberg, Ak 24: 253.

⁷ The original title of the book is "*La logique ou l'art de penser*" ("Logic or the Art of Thinking"). The better known title "Port-Royal Logic" has to do with the fact that the authors Arnauld and Nicole were associated with the Port-Royal Abbey. The work was first published in 1662. After four major revisions, its final edition appeared in 1683. The English translation I am using is the *Logic or the Art of Thinking* by Buroker [1996]. The translation is based on the 1683 French edition. My reference is made to: part and chapter (e.g., i.6), and the page number in Buroker's translation. Necessary modifications are made based on French edition.

Admittedly, the extension-intension distinction was already suggested by Aristotle's theory of genus and species developed in the *Posterior Analytics*. (See Frisch 1969:115-21) But it was first through the Port-Royal Logic that the distinction received its modern characterization. On this point, see Peirce 1867, §1; Frisch 1969:5; Baynes 1865:xxxiii; Buroker 1996:xxv; McCosh 1891:27.

⁸ According to Schulthess [1981], Knutzen, Kant's logic teacher, developed the extension-intension distinction "partly in reference to the Port-Royal". (Schulthess 1981:96)

⁹ All ideas are first divided into singular and general ones. Singular ideas "represent only a single thing". The things represented are called "individuals". Such ideas are marked by "proper names", such as

‘Socrates’ and ‘Rome’. Ideas that “represent several things” are called “universal”, “common”, or “general”. They are marked by “general terms”, such as ‘man’ and ‘horse’. (i.6, p.39)

¹⁰ This ambiguity has also been noted in Kneale and Kneale 1962:318-19.

¹¹ [EXT_o] is also suggested in the Port-Royal Logic by the characterization of “singular ideas”. Singular ideas are said to be “always taken in their entire *extension*”. (i.8, p.45, emphasis added) And a judgment with a singular idea as its subject is treated as a universal one precisely because its subject “is necessarily taken through its entire *extension*”. (ii.3, p.84, emphasis added) If a singular idea is thus taken to have an extension, what constitutes such an extension can only be the individual object represented by the idea. In Chapter 3 we will see that Kant also thinks that a singular concept characteristically represents exactly one object. Unlike the Port-Royal Logic, however, Kant will treat a singular concept to *have no extension*—precisely because it represents only one object.

¹² There is no clear indication in the Port-Royal Logic as to whether the individuals that constitute the extension of an idea are the ones that the idea *does* apply to (i.e., *actualia*) or the ones that it *can* apply to (i.e., *possibilia*)

¹³ The *Auszug* is the abridged version of *Vernunftlehre*. The two texts are alike in all essential respects. Kant based his logic lectures mostly if not exclusively on the *Auszug*. (see “Kant in the Classroom”, <http://www.manchester.edu/kant/Home/index.htm>; see also Pozzo 2005) Hence, in my discussion of Meier’s account of logical extension, I will focus on the materials from the *Auszug*.

¹⁴ Especially, Knutzen was a professor of logic and metaphysics at the University of Königsberg (spring semester 1734 through winter semester 1745/46), with whom Kant studied logic among other topics. In a letter written in April 1756, Kant praised Knutzen for having led “an extraordinary profession of logic and metaphysics”. (Ak 10:3) For the logic courses, Knutzen probably used Wolff’s German Logic until 1746, when he used his own *Elementa philosophiae rationalis seu logicae*. (see “Kant in the Classroom”, <http://www.manchester.edu/kant/Home/index.htm>) For an informative discussion of Knutzen’s important influence on Kant during his student years at Königsberg, see Kuehn 2001:78-84.

¹⁵ As for Baumgarten, there is no clear commitment to either [EXT_o] or [EXT_c] in his *Acroasis logicae* (1761), a commentary on Wolff’s German Logic. Like Wolff, Baumgarten speaks of a concept or notion containing some other concepts under itself: “A notion contains [...] under itself the concepts in which it is contained.” (Baumgarten 1761, §45) And he also speaks of individuals being contained under a concept: “A notion that applies to [all] the individuals [*singulis*] contained under a concept is called universal in that respect, i.e., objectively.” (ibid. §49) However, although Baumgarten does refer to *extensio* (*Erweiterung*, *Verbreitung*, *Ausdehnung*, *Reichthum*), it is ascribed to cognition, but not to concepts specifically, and is characterized as “the perfection of the cognitive faculty that somehow cognizes many”. (ibid. §51) Whatever is meant by such a statement, it suggests very little as to what Baumgarten would take the *extensio* of a concept to be—except, perhaps, that it would involve a multitude (the “many”).

¹⁶ Wolff notes that the term ‘species’ may be used in two senses, one sense being narrower than the other. In the narrower sense, a species is nothing other than a lowest species, which has under itself only individuals. In the broader sense, any genus that is lower with respect to the higher genus under which it is contained is a species in that respect. (Wolff 1740, §47)

¹⁷ The German terms are given on the margin of the relevant Latin text.

¹⁸ I am taking ‘or’ here to express equivalence.

¹⁹ The same notion of extension is also shared by Crusius (not a *Wolffsschüler*), another person that Kant mentions as a logician of his time. In Crusius’ view, to consider a concept with respect to its extension (*Weite*) is to consider the concept “individually [*individualiter*], i.e., with respect to the multitude of individuals [*Menge der Individuorum*] to which it applies”. (Crusius 1747, §135)

²⁰ Immediately after giving the above definition of *Umfang*, Meier adds: “Therefore, the more abstract and the higher a concept is, that is, the more often logical abstraction is repeated toward it, the greater is its extension (*Umfang*).” (*Auszug*, §262) But there is no further use of this notion of *Umfang*. The expression ‘*Umfang*’ does appear in several other places (e.g., §§48-9, in the phrase ‘*Umfange des Horizonts der menschlichen gelehrten Erkenntniss*’), but not in the technical sense defined in §262.

²¹ As we shall also see in Chapter 3, most early modern logicians explained the quantity of a judgment in terms of quantification over the individual things to which the subject applies, i.e., over its *extensio* in Knutzen’s sense. Wolff and many other *Wolffsschüler* are no exception in this regard.

²² For the definition of a concept as a certain representation (*Vorstellung*), see *Auszug*, §249.

²³ As I mentioned earlier, Kant is also familiar with Crusius' logic, although he is critical of how Crusius construes the nature and boundary of logic.

²⁴ Although, to my knowledge, Kant never explicitly mentioned Euler while lecturing on the history of logic, he certainly exhibited acquaintance with Euler's logic. The key component of Euler's logic is the use of geometrical figures (circles) to represent concepts in judgments and inferences. Kant explicitly refers to such figures at least once, in the *Logik Philippi*, Ak 24: 454. To the extent that the use of figures in Euler's logic is, as we shall see, tied to his conception of logical extension, and to the extent that the figures share certain important features with the ones used by Leibniz and Lambert, it is appropriate to include Euler's account of logical extension, together with its figurative representation, in our discussion.

²⁵ According to Schulthess [1981] (p.100), Leibniz was the first to use the term '*intension*' (in French) to express what was meant by '*comprehension*' in the Port-Royal Logic. Leibniz's *New Essays* was posthumously published in 1765, which Kant would read in 1769. The term '*intension*' is used in Bk.IV, Ch.xvii, §8.

²⁶ See Kauppi 1960: 43-4, in reference to Leibniz's *Elementa calculi* (April, 1679).

²⁷ See Capozzi & Roncaglia 2009:118-19 and Lenzen 2003.

²⁸ The page number refers to Couturat 1903 (*C* = Couturat).

²⁹ A similar line symbolism is also used to represent a judgment with respect to intension, i.e., "on the grounds that the concept of a genus is a part of (or at least is included in) the concept of a species". (*C* 384)

³⁰ *De Formae Logicae Comprobatione per Linearum Ductus*. In this essay, judgments are again treated with respect to extension (*C* 292-9) as well as with respect to intension (*C* 300-11). The intensional treatment is the same as in the previous essay.

³¹ To my knowledge, Leibniz never used circles to represent a judgment with respect to intension. Also, although both lines and circles are used to validate Aristotle's syllogisms in extensional terms (*C* 294-8), neither was used to show how the same syllogisms may be validated with respect to intension. When Leibniz attempts to validate syllogisms from the intensional point of view, he applies his logical calculus instead. (*C* 301-11) Leibniz might be aware of the difficulty of using the geometrical figures to validate syllogisms on the intensional basis. On this point, see Parkinson 1966:xli-xlii.

³² Interestingly, Lambert takes both individuals *and* species-concepts to be "under" a genus-concept. (*NO*, §§14, 16, 28) Besides his own notion of *Ausdehnung* ([EXT_O]), Lambert might also allow Meier's notion of extension ([EXT_C]), but would have confined the latter to the first chapter of *Neues Organon* and used it merely to capture the relation that a genus-concept has with the species-concepts.

³³ These letters were written during 1760-63. They cover a wide range of topics, logic being one of them. Written in French, the letters were published first in 1768 by St. Petersburg, as *Lettres à une Princesse d'Allemagne sur divers sujets de physique & de philosophie*. In 1969, they were published in German by Leipzig, as *Briefe an eine deutsche Prinzessin über verschiedene Gegenstände aus der Physik und Philosophie*. I use the English translation by Henry Hunter, *Letters of Euler on Different Subjects in Natural Philosophy: addressed to a German princess* (1858), with necessary modifications on the basis of the French edition. Page numbers cited are of the English edition.

³⁴ Euler takes "notions", or "general ideas", to be "acquired by abstraction" and to differ from "sensible ideas, which represent to us objects really existing". (*Lettres*, c, p.332) Among notions, a distinction is made between *genus* and *species*: "genus is the more general and contains in it [*renferme en lui*] several species." (*Lettres*, c, p.333) For instance, "the notion of a tree may be considered as a genus, as it contains the notions of pear trees, apple trees, oaks, firs, and so on, which are species". Each of these species-notions in turn "comprehends [*comprehend*] existing individuals—all the pear trees, oaks, etc. that exist". (ibid.) These remarks have led Schulthess to claim that in the *Eulersche extensionale Logik* there are two kinds of extension for every general idea or concept: to the extent that a concept "extends to" a multitude of *objects*, it has an "extensional extension"; to the extent that a concept (genus) contains under itself other, relatively less general *concepts* (species), the concept has an "intensional extension". (Schulthess 1981:100-1) If Schulthess is right in this claim, however, the notion of extensional extension alone will be operative in Euler's account of the logical relations of concepts in judgments and syllogisms with respect to extension, and only for this notion of extension will Euler introduce the circle symbolism.

³⁵ For Euler, the shape of the figure is not important. Indeed we can use "space formed at pleasure to represent every general notion", so long as the figures formed in this way eventually to render the whole subject of syllogism "sensible to the eye". (*Lettres*, ciii, p.340-1)

³⁶ More precisely, the extension of a general notion is represented by the *space* enclosed within a circle. For the sake of simplicity, I shall follow Euler's practice and treat *circle* and the *space enclosed within a circle* as interchangeable notions.

³⁷ In the present context, I treat 'judgment' and 'proposition' as interchangeable.

³⁸ Euler's view on the logical quantification involved in particular judgments is rather complicated. According to the Kneales, Euler regards the "some" in all particular judgments as "some but only some". (Kneale & Kneale 1984:349, in reference to letter ciii) This interpretation is backed by the following "essential" distinction between a particular judgment and a universal one that Euler draws in Letter cvii: while the latter "speaks of all [of the beings comprehended in the notion of the subject] without exception", the former "does not speak of all", but instead "turn[s] only on a part", of those beings. (*Letters*, cvii, p.362) In letters ciii-cv, however, Euler seems to treat "some" in *affirmative* judgments differently from that in the *negative* ones: it means "only some" in the former, but "at least some" in the latter. Such a contrast can be gathered from the different figurative representations that Euler gives to the two forms of particular judgments throughout the three letters.

On the one hand, the inclusion relation between two circles that shows the logical relation between two concepts in an *affirmative* particular judgment is *merely* partial ("some but only some"). This is suggested by Euler's remark that the same figure which represents "Some A is B" can also be used to represent these forms of judgments: "Some B is A", "Some A is not B" and "Some B is not A." (*Letters*, p.342) Also, on pp.345, 346-47 and 351-53, where a figurative representation is given of "Some A is B" (used as a premise of certain syllogisms), the circle representing A and the one representing B are always presented as merely partially intersecting. On the other hand, the exclusion relation that shows the relation between two concepts in a *negative* particular judgment is *at least* partial, but *may* be total ("some but may be all"). This can be seen in the three possible ways of arranging two circles—or, more precisely, the spaces enclosed therein—in relation to each other whereby the logical form of "Some C is not B" is exhibited: either (i) circle C is wholly outside circle B, or (ii) the space enclosed within circle C includes the one enclosed within circle B as a proper part, or (iii) the two circles only partially overlap. (*Letters*, p.345)

³⁹ This echoes Leibniz's claim in the *New Essays* that "the whole theory of syllogism could be demonstrated from the theory *de continente et contento*, of container and contained." (*NE*, Bk.IV, Ch.xvii, §8) But Leibniz had in mind intension- as well as extension-containment.

⁴⁰ "Kant distinguishes extensional and intensional extension [*extensionale und intensionale Extension*]." (Schulthess 1981:103) Earlier Schulthess defined the two notions of extension as follows: "The *extensional extension* is the multitude of all the things [*Menge aller Dinge*] that fall under a concept, that is, are contained under it. The *intensional extension* is the multitude of all the concepts that are contained under a concept." (Schulthess 1981:16)

⁴¹ This "formal-logical extension" is contrasted with "empirical extension", the latter "consist[ing] of the actual objects [*wirkliche Gegenstände*] that exhibit a certain determination [*eine gewisse Bestimmung aufweisen*]" (Prien 2006:76)

⁴² A similar claim has also been made by Friedman [1992]: "Kant's notion [of extension] involves a relation between a concept and other concepts—its species, subspecies, and so on—rather than a relation between a concept and the objects falling under it." (Friedman 1992:68; cf. 67) Friedman refers to this conceptual notion of extension—in direct contrast with the modern notion of extension (= the set of objects falling under a concept)—to make sense of Kant's reference to infinity at B40, where every concept is said to be "a representation that is contained in an infinite multitude of different possible representations (as their common mark), and it therefore contains these *under itself*". (ibid.67-8; cf. 64) I agree that the claim at B40 is best understood in light of Kant's view that the series of species, subspecies, and so on, that are contained under any given concept is infinite. But it does not follow that by the extension of a concept Kant always means the series of concepts contained under it "rather than" the objects falling under it.

⁴³ For what I take to be a correct criticism of the view that Kant takes the extension of a concept to include intuitions (or singular representations), see Codato 2008:142-4.

⁴⁴ Longuenesse also—generally, but not always unequivocally—attributes a hybrid notion of extension to Kant. She observes that Kant sometimes "speaks indifferently of *things* or *concepts* being subordinated to a 'higher concept'", an indifference that she claims to be common in eighteenth-century logics. (Longuenesse 1998:92n.23; cf.88n.16) Although she sometimes mentions only objects in reference to the extension of a concept (ibid. 87; 132), she does not thereby exclude concepts from being comprised in the extension as

well. (ibid. 93-4) When she does talk about the objectual extension of a concept, by objects she usually means “the objects of sensible intuition”. (ibid. 87) At one point she reconciles the two notions of extension by connecting them with two different standpoints: on the one hand, “from a strictly logical standpoint, Kant’s notion of extension is essentially inherited from Port-Royal” (understood as [EXT_C]); on the other hand, “when general logic is considered in relation to transcendental logic, [...] then ‘extension’ tends to be identified with ‘manifold of *individual things*’”, which is however a notion of extension that “[has] no strictly logical status”. (ibid. 383n.97)

⁴⁵ This interpretation can also be found in de Jong 1995. Referring to Kant’s definition of the extension of a concept, A, as “the set of ‘things contained under’ A”, de Jong asks: “which are these ‘things’?” He answers the question by appealing to Kant’s account of the genus-species relation among concepts: “To make this clear, the *Logic* introduces the (relative) distinction between lower and higher concepts; this distinction turns out to be nothing else than that between species and genus. A genus is a higher concept with regard to its (possible remote) species and a species is a lower concept in connection to its proximate genus and (possible) remote genera (L 96-97). Moreover, lower concepts are contained under its higher concepts (XVI 565; L 96). And thus, the extension of a concept includes in each case all the (remote) species contained under it.” (de Jong 1995:626, emphasis added)

⁴⁶ So construed, a general concept is contrasted with a singular one, the latter containing under itself “only one object [*Gegenstand*]”. (V-Lo/Blomberg, Ak 24:258)

⁴⁷ The German ‘*unter*’ can mean both “under” and “amongst”. In the cited passage it seems to mean the latter, considering that it is being used interchangeably with ‘*in*’.

⁴⁸ V-Lo/Hechsel, LV:399; V-Lo/Warschauer, LV:613; V-Lo/Dohna, Ak 24:755; V-Lo/Bauch, LV:152. These texts are all based on Kant’s logic lectures during or after 1780’s.

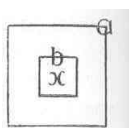
⁴⁹ The subscript *L* indicates that the relevant term is a Latin one. (see Young 1992b:xxix)

⁵⁰ Fig.5, as it appears in Kant’s note, does not show clearly what the two marks written inside the two circles, respectively, are. Nevertheless, if we interpret the figure in connection with other similar figures used by Kant (e.g., R3096, Ak 16:658), it seems safe to think that they are two schematic letters used to represent the subject and the predicate, respectively, in a typical categorical judgment.

⁵¹ For the sake of consistency, in the first of these figures, the letters of A and B should have their positions switched, so that the four judgments represented, respectively, by the four figures are: “Every A is B.” “No A is B.” “Some A is B.” “Some A is not B.”

⁵² But the figure has to be an *extended* one, which alone is fit for representing the *multitude* of things that constitute the extension of the concept. We shall see the significance of this point in Chapter 3.

⁵³ This figure is used to contrast a disjunctive judgment with a categorical one. The form of the latter (“*x*, which is *b*, is *a*.”) is presented as follows:



(R3096, Ak 16:658)

⁵⁴ Log, §111, Ak 9:146; cf. V-Lo/Bauch, LV: 169-70; V-Lo/Warschauer, LV: 622; V-Lo/Philippi, Ak 24: 461; V-Lo/Hechsel, LV: 419; V-Lo/Pöhlitz, Ak 24: 576; V-Lo/Busolt, Ak 24: 660-61; V-Lo/Dohna, Ak 24: 761-62; V-Lo/Wiener, Ak 24: 926. Also see Meier *Auszug*, §285.

⁵⁵ Log, §110n.1, Ak 9:146; cf. R3010, Ak 16:612; R3013, Ak 16:614; R3031, Ak 16:623. The same expression is used by Meier (*Auszug*, §285).

⁵⁶ This model of a tree-like structure is also noted in Tolley 2007 (Ch.IV, §33). As Tolley see it, this model captures “the standard view for the logical articulation of the nature of conceptuality through much of the history of philosophy, from Aristotle up to Kant’s time, and especially among his Wolffian predecessors”, and hence gives us the “historically pertinent, philosophical resources” by which to interpret Kant’s doctrine of concepts. (Tolley 2007:349) In particular, such a model allegedly “gives us resources to portray the generality of concepts *without* making the assumption that they relate to, or are being predicated of, individual objects.” (ibid. 349-50)

⁵⁷ The expression ‘inverse proportionality’ is used in Peirce 1868 (§4). Peirce also refers to the law of the inverse proportionality between intension (comprehension) and extension as “Kant’s law”. (ibid.)

⁵⁸ Anderson 2004b:507-8; Tolley 2007:361-5. See also de Jong 1995:626-7.

⁵⁹ Tolley presents his argument from the perspective of a “good Quinean”. (Tolley 2007:362) Anderson likewise has the specter of Quinean worries in mind when he stresses that Kant’s logic is thoroughly intensional and that “even the *extension* of a concept (in the logical sense) is understood to be the group of *intensional concepts* contained under it, rather than the individual objects it applies to”. (Anderson 2004b:512)

⁶⁰ Actually, the same objection that Tolley has raised against attributing an objectual notion of extension to Kant in reference to the law of inverse proportionality has also been raised against Leibniz’s view that the intension and the (objectual) extension of a concept are inversely proportional. (Swoyer 2005; Tolley refers to this objection on p.362) Like Tolley, Swoyer assumes that the objects in question are *actualia*, an assumption that has been rebutted in Lenzen 2003 and Capozzi & Roncaglia 2009 (pp.118-19).

⁶¹ On Kant’s account of logical determination, a concept can be determined only by what is not contained in it: “One can determine only through synthesis, not through analysis, for in view of that which lies within it, it is not determined. For I can determine the concept only when I add something.” (V-Met/Mrongovius, Ak 29:819) For instance, it would be incorrect to say that “the concept of body is determined by extension”; for the predicate “extension” already “lies in” the concept of body. Weight, by contrast, can be a determination of body, since it is not already contained in the concept of body. (ibid.)

⁶² We shall see further implications of this rendering of Kant’s law in Chapter 4, when we discuss Kant’s account of the formal truth of an analytic judgment.

Chapter 3

The Logical Form of Singular Judgments

In the *Critique*, Kant claims that the basic forms of judgment can be classified and ordered in the following 4 (titles) × 3 (moments) table.

Quantity (universal, particular, singular)	
Quality (affirmative, negative, infinite)	Relation (categorical, hypothetical, disjunctive)
Modality (problematic, assertoric, apodictic)	

Such is the Table of Judgments (A70/B85), on the basis of which is to be constructed a Table of Categories that allegedly contains all the pure concepts of the understanding (A80/B106). While the derivation of the second Table directly involves the perspective of transcendental logic, Kant suggests that the moments of judgment included in the first Table can all be identified within the pure general logic that we discussed in Chapter 1. (I shall refer to the latter logic as “general logic” in the present chapter.) As he puts it in the *Prolegomena*, in putting together the Table of Judgments he has “the [already finished] work of the logicians” lying before him. (*Prolog.* Ak 4:323) If, however, by “the logicians” Kant is referring to the ones I mentioned in Chapter 2, there is a noticeable departure on his part from those logicians in treating the forms of judgment with respect to both quantity and quality. As we saw in Chapter 2, for the logicians who were interested in presenting all the basic forms of judgment (in which two concepts are related as subject to predicate), there are only four such forms: universal affirmative, universal negative, particular affirmative, particular negative. And, as we shall see, when they did mention singular and infinite judgments, they granted neither the status of an irreducibly basic logical form of judgment. Rather, singular judgments were treated as universal, and infinite ones as affirmative.¹

Kant is well aware of such a common practice in the early modern logics. He readily admits that the inclusion of “singular” under Quantity, and “infinite” under Quality, in the above Table of Judgments has deviated from “the customary technique of the logicians”, and finds it necessary to make certain “caveats [*Verwahrungen*] against a worrisome misunderstanding”. (A70-1/B96)

Whatever “worrisome misunderstanding” Kant is trying to forestall,² the reasons that he gives for including the singular and the infinite in his Table seem to raise more concerns than they might resolve. The most pressing concern of all has to do with the standpoint from which such reasons are given. Kant spends two full paragraphs (A71-3/B96-8) explaining why the singular and the infinite constitute two “special member[s] of the classification” (A72/B97) of the forms of judgment regarding quantity and quality, respectively. The explanations involve a certain appeal to transcendental logic—explicitly so in the case of infinite judgments (A71-2/B97), though only implicitly so in the case of the singular ones.³ Brandt [1991/1995] takes it that this appeal to transcendental logic raises a specter of circularity. He proposes to remove the specter by distinguishing two *aspects* of transcendental logic, a “transcendental-philosophical” and a “purely formal” one: if Kant needs to employ transcendental logic in order to justify the inclusion of the singular and the infinite in his Table of Judgments, he needs only to invoke “the purely formal, not yet transcendental-philosophical aspect” of transcendental logic. (Brandt 1991/1995:73-5) Brandt assumes here that general logic does not contain suitable resources for Kant to *distinguish* the singular and the infinite from the universal and the affirmative. By contrast, Allison [2004] suggests that there is indeed room for Kant to make the singular-vs.-universal and infinite-vs.-affirmative distinctions within his general logic. (Allison 2004:141-2) And this suggestion may be supplemented by Krüger’s thesis that, if Kant has to invoke any transcendental-logical considerations at A71-3/B96-8, such considerations serve only to make “relevant” those general-logical distinctions that Allison has alluded to. (Krüger 1968:348) But neither Allison nor Krüger has identified the exact *general-logical* materials on the basis of which the distinctions in question may be made.⁴

I am sympathetic to Brandt's intent to remove whatever specter of circularity he perceives there to be in Kant's justification for including the singular and the infinite in the Table of Judgments. But I am not convinced about the need to invoke a distinction between purely formal and transcendental-philosophical aspects of transcendental logic. If the circularity issue comes down to whether Kant has adequate general-logical resources for *distinguishing* the singular from the universal, and the infinite from the affirmative, in the first place, a simpler way to handle the issue is suggested by Allison's and Krüger's insights that there is room for Kant to draw the needed distinctions within general logic, even if afterwards he may have to appeal to certain transcendental-logical considerations in order to make such distinctions relevant to his project in the *Critique*. To make concrete such a suggestion, we need to answer the following questions. First, if there indeed are adequate general-logical resources for Kant to distinguish the singular and the infinite as special forms of judgment from the universal and the affirmative, respectively, what are the relevant resources? And how could such distinctions be reconciled with Kant's acknowledgement that the logicians have nonetheless rightly treated the singular as universal and the infinite as affirmative? Second, presumably not all appeal to transcendental logic would render Kant's arguments for treating the singular and the infinite as special forms of judgment viciously circular. What kind of transcendental-logical appeal, then, would give rise to vicious circularity and hence must be prevented, and what kind would not? Especially, if Kant's reference—be it explicit or implicit—to transcendental logic at A71-3/B96-8 cannot be simply brushed aside, in what way might the perspective of transcendental logic still be involved in his justification for including the singular and the infinite in the Table of Judgments, without the justification being made viciously circular thereby?

I shall address these questions in the present chapter through a case study of Kant's account of singular judgments at A71/B96-7. My analysis of the account will draw crucially on two things developed in Chapters 1 and 2: first, my reading of Kant's view regarding the formality of general logic (especially with respect to its contrast with transcendental logic);

second, my interpretation of Kant's and many of his logician-predecessors' objectual notion of logical extension ([EXT_o]) together with the circle symbolism Kant used to represent it. In section I, I shall articulate the particular circularity threat that faces Kant's account of singular judgments at A71/B96-7 due to his explicit reference to the notion of unity (*Einheit*). In section II, I shall develop a reading of Kant's account of singular judgments by comparing it with the account held by some of his logician-predecessors. We shall see that Kant shares with those logicians a similar view regarding the distinctions among universal, particular and singular judgments, but that he differs from them in the unique perspective he adopts in the *Critique* to assess such distinctions. And I shall explain that, when Kant ascribes *Einheit* to the singular judgment at A71/B96, he need not assume any substantive connection of such a judgment with the category of *Einheit* to be introduced latter. By thus weakening the relation between the two occurrences of *Einheit*, I will have resolved the circularity threat presented in section I. In section III, I shall discuss a further implication of the reading developed in section II, namely an implication for how to approach the controversy over the correlation between the logical moments of quantity (i.e., universal, particular, singular) and the categories of quantity (i.e., unity, plurality, totality). Finally, in section IV, I shall sketch and briefly address a potential objection to my reading of Kant's account of singular judgment. I shall argue that Kant's account in the *Critique* regarding what is characteristic of a singular judgment, as I have interpreted it, is a general-logical as opposed to a transcendental-logical one.

I. The problem of singular judgment: an introduction

Kant gives a two-fold account of singular judgments at A71/B96-7, so as to show why its inclusion in the Table of Judgments is justified regardless of how it has been treated by the logicians. First, it is granted that the logicians are right to treat singular judgments like universal ones, for this reason: "just because they [i.e., singular judgments] have no extension at all, their predicate [...] holds of that concept [i.e., the subject-concept] without exception, just as if the latter were a general concept with an extension, with the predicate applying to the whole of what

is signified.” (A71/B96) Second, it is argued that a singular judgment nevertheless differs from “a general one” to the extent that, considered “merely as cognition”, the former relates to the latter “as unity relates to infinity” in respect of quantity, for which reason it must be treated as a special moment of judgment. (ibid.) Kant thus recognizes the apparent tension between treating singular and universal judgments as alike and treating the former judgment as special. But he tries to handle the tension by affixing a distinct perspective to each treatment, so as to make the treatments mutually compatible. Suppose the first treatment conforms to a logicians’ perspective, while the latter conforms to a Kantian one. The question is how exactly to characterize the two perspectives and their relation.

The contrast of the two likely perspectives is sometimes thought to be that between a general-logical and a transcendental-logical perspective. As Longuenesse [1998] puts it, the above-sketched two-fold account of singular judgments suggests that “in Kant’s view, [...] from a strictly [general-]logical standpoint, the singular is not distinguished from the universal judgment because in both alike the predicate is attributed to the totality of what is thought under the subject.” (Longuenesse 1998:139) If singular judgments are to be distinguished from the universal ones, Longuenesse suggests, then Kant must appeal to a transcendental-logical conception of the former, whereby they are “considered in their relation to sensibility”. (ibid.) In Kant’s *stated* reason for treating the singular as a distinct form of judgment, of course, there is no explicit reference to transcendental logic. For all Kant says is that a singular judgment relates to a general one, in respect of quantity, as *Einheit* to *Unendlichkeit*, for which reason the former is treated as a special moment of judgment. But one may read an implicit transcendental-logical appeal into Kant’s stated reason, on the basis that he begins the next paragraph, where he explains why infinite judgments should be regarded as a special moment of judgment with the following claim: “*Likewise*, in a transcendental logic infinite judgments must also be distinguished from affirmative ones, even though in general logic they are rightly included with the latter[.]” (A71-2/B97, emphasis added) If so, one might then ask, what transcendental-logical consideration

could have been implicated in Kant's contrast of singular and general judgments as *Einheit* vs. *Unendlichkeit*? Some commentators have indeed questioned the very intelligibility of such a contrast. Allison, for instance, is baffled by the reference to infinity: "why not totality?" (Allison 2004:141) And Kemp Smith suggests that it is inconsistent for Kant to ascribe "unity" to the singular instead of the universal judgment:

the statement that the singular judgment stands to the universal as unity to infinity (*Unendlichkeit*) is decidedly open to question. The universal is itself a form of unity, as Kant virtually admits in deriving, as he does, the category of unity from the universal judgment. (Kemp Smith 1923:192)

In expressing such misgivings about the *Einheit-Unendlichkeit* contrast, Allison and Kemp Smith seem to share two related assumptions. First, it is assumed that Kant means such a contrast to characterize the essential difference between a singular and a universal (*allgemein*) judgment.⁵ Second, it is assumed that the *Einheit* mentioned at A71/B96 is the same as the category of *Einheit* to be introduced later (A80/B106). Thus, for Allison, who supposes that the universal judgment correlates with the category of totality, Kant's reference to *Unendlichkeit* at A71/B96 is puzzling. (Moreover, there is no such thing as the category of infinity for Kant.) And for Kemp Smith, who takes universal judgment to correlate with the category of unity, there is inconsistency in Kant's ascription of *Einheit* to singular judgment at A71/B96.⁶

However we may understand the correlation between the logical moments of quantity (i.e., singular, etc.) and the categorial ones (i.e., unity, etc.), the assumption that Kant's reference to *Einheit* at A71/B96 is just a reference to the category of unity gives rise to a serious exegetical challenge. To put the challenge crudely, the assumption in question feeds into the suspicion that Kant's construction of the Table of Judgments is somewhat "artificial", probably driven by the need to coordinate it with an already worked-out Table of Categories. (Kemp Smith 1923:192)⁷ As Tonelli [1966] has observed, the two Tables in their final shapes actually occurred around the same time. (Tonelli 1966:134)⁸ This observation suggests to Tonelli that the Table of Judgments, as it appears in the *Critique*, may very well be built on the finalized Table of Categories. (ibid.

150)⁹ It would be a *non sequitur*, of course, to infer from the mere observation that the two Tables matured around the same time to the claim that, in the *Critique*, Kant needs to *justify* the specific constitution of his Table of Judgments by appealing to that of the Table of Categories. Nonetheless, Tonelli's observation and the hypothesis he has submitted to explain it—together with the common assumption that Kant's reference to *Einheit* at A71/B96 is but a reference to the category of unity—gesture toward a vicious circularity threat regarding Kant's ground for treating the singular as a distinct form of judgment vis-à-vis the universal. Roughly put, the threat is this: provided that Kant intends to derive the categorial moments of quantity from the relevant logical moments, there would be a vicious circle if the *only* ground on which Kant can distinguish the singular from the universal in the first place presupposes the categories of quantity. For anyone who wishes to interpret Kant's account of singular judgments at A71/B96-7 charitably, then, there is a burden to prove that Kant has *other* intelligible grounds on which to make the needed distinction.

Most commentators who have taken on such a burden have sought the alternative grounds directly within transcendental logic. Brandt's distinction between two aspects—i.e., between the “purely formal” and “transcendental-philosophical” aspects—of transcendental logic mentioned earlier can, for instance, be seen as one such attempt: if the categories pertain to the transcendental-philosophical aspect of transcendental logic, Kant's account of the singular as a special moment of judgment may be considered to involve only the purely formal aspect of the same transcendental logic. Longuenesse, by contrast, appeals to Kant's transcendental-logical distinction between sensible intuition and discursive thought and suggests that a singular judgment essentially differs from a universal one in virtue of having a special “relation to sensibility”; for it “refer[s] concepts to what is beyond the discursive capacity [namely, to singular intuition]”. (Longuenesse 1998:139)¹⁰ For our purpose, we need not assess any specific weaknesses or strengths of such views. But we may question their shared assumption that the singular cannot be *distinguished* from the universal judgment within Kant's general logic, so that

a transcendental-logical perspective must be invoked to establish such a distinction. On my reading, the gist of Kant's concession to the logicians is not that the distinction in question cannot be made within the general logic strictly speaking. On the contrary, the distinction can be *made* on the basis of the same logical materials that Kant shares with his logician-predecessors, although, for those logicians, it is made only to be deemed *inconsequential* from their syllogism-centered perspective.

I shall flesh out such a reading in section II. I shall analyze Kant's account of singular judgments at A71/B96-7 in light of the relevant materials from early modern logics as well as from his own logic corpus, especially those involving the objectual notion of logical extension discussed in Chapter 2. In brief, I take it that Kant's explanation for including the singular as a special form of judgment in his Table of Judgments boils down to the following triad of theses.

- (1) A distinction can be *made* between singular and general/universal judgments within general logic, both according to Kant and according to many of his logician-predecessors.
- (2) But such a distinction can be *assessed* from two different perspectives. From the logicians' perspective, the distinction is made only to be set aside as *inconsequential*; for singular judgments play no unique inference role vis-à-vis the universal ones in syllogisms.¹¹ To that extent, the perceived distinction does not establish the former as a fundamental form of judgment over and above the latter. From Kant's perspective in the *Critique*, by contrast, the distinction does establish the singular as a separate basic form of judgment insofar as it is considered merely as "cognition as such" (A71/B96). And, to the extent that the Table of Judgments contains all the basic forms of judgment considered as such, the singular must be included therein.
- (3) A further, *second-order* justification may be needed as to why considering a judgment as "cognition as such" is *the* appropriate perspective for Kant to adopt in the *Critique* in order to determine which forms of judgment are irreducibly basic. This is where certain transcendental-logical considerations may be involved, if at all.

My main focus in section II will be (1) and (2). I shall begin by showing that Kant shares with the previous logicians a two-tiered distinction among the forms of judgment concerning quantity: first, the singular vs. the general; second, concerning the latter, the universal vs. the particular. Especially, a singular judgment is distinguished from a general one with respect to the quantity of what is signified by the subject-concept: in the first case, the subject-concept is used to signify exactly *one* object, whereas in the second case the subject-concept is used to signify a *multitude*

of possible objects (i.e., a logical extension). Using the symbolic means available to him, Kant can present such a distinction as a formal-logical one through the contrast of a point with an extended plane-geometrical figure. Such a distinction will be shown to be the basis for both parts of Kant's two-fold account of singular judgments at A71/B96-7. On the one hand, given such a distinction (together with the fact that, on the early modern notion of logical extension, a concept that signifies exactly one object has no extension), Kant has good reason to believe that a singular judgment, precisely because its subject-concept has no extension, necessarily exhibits the same subject-predicate relation concerning quantity as a universal one does and hence can be treated like the latter in logical inferences. On the other hand, the same distinction also serves to make sense of Kant's claim that a singular judgment, considered as cognition as such and with respect to quantity, relates to a general one as *Einheit* to *Unendlichkeit*, a relation that can be illustrated by how a point relates to an extended plane figure in terms of quantity.

II. The form of singular judgments and its logical status: Kant vs. "the logicians"

The following two statements at A71/B96-7 convey the gist of Kant's reason for including the singular in the Table of Judgments without thereby discrediting "the customary technique of the logicians" from which such inclusion deviates.

The logicians rightly say that *in the use of judgments in syllogisms* singular judgments can be treated like universal ones [*gleich den allgemeinen behandeln könne*]. (emphasis added)

If, *on the contrary*, we compare a singular judgment with a general one [*mit einem gemeingültigen*],¹² *merely as cognition*, with respect to quantity, then the former stands to the latter as unity to infinity [...] (emphasis added)

The two italicized clauses indicate a contrast of two *perspectives*. Such a contrast becomes even clearer as we read on: "Therefore, if I assess [*schätze*] a singular judgment not only with respect to its inner validity, but also as cognition as such [*überhaupt*], with respect to the quantity it has in comparison with other cognitions, then [...]" In these terms, the two perspectives being contrasted can be roughly expressed as follows.

- [Perspective-s] Consider a singular judgment with respect to its “inner validity [*Gültigkeit*]”, i.e., with respect to how the predicate applies (*gilt*) to the subject,¹³ a predicate-to-subject relation that determines its use in syllogisms.¹⁴
- [Perspective-c] Consider a singular judgment in terms of its quantity *qua* cognition as such.¹⁵

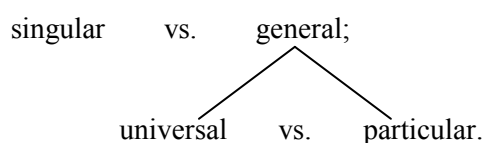
The contrast between Perspective-s and Perspective-c may be considered in light of Kant’s suggestion elsewhere that a certain “logical distinction” among singular, particular, and universal judgments must be made “at the beginning”, although one can say “afterward” that the singular can be treated as universal in use.¹⁶ Kant’s stated reason at A71/B96-7 for including the singular judgment in his Table of Judgments may then be reconstructed as follows. First, a singular judgment, considered as “cognition as such”, can be logically distinguished from a general one (be the latter universal or particular). Second, although, having made such a distinction, we can proceed to treat singular judgments like universal ones from Perspective-s, this is a further step which *need not* be taken as far as the *Critique* is concerned; rather, a judgment can be treated as cognition as such (i.e., from Perspective-c), in a way that abstracts from all considerations about its possible use in syllogisms. Hence, whatever logical distinction has been drawn between the singular and the general *remains* as is and can in turn serve as the basis for including the former in the Table of Judgments (provided that this Table includes all the basic forms of judgment *qua* cognition as such).¹⁷

The question then becomes this: what logical distinction between singular and general judgments—considered as “cognition as such”—is Kant in the position to draw in the first place? To answer this question, I will begin with a two-tiered distinction shared by most of the early modern logicians mentioned in Chapter 2: first, singular vs. general judgments; second, within general ones, universal vs. particular judgments. (I shall consider especially those versions in which the distinction is presented in terms of the quantity of what is signified by the subject-concept of each judgment.) There are clear indications in Kant’s logic corpus that he shares the basics of such a two-tiered distinction. I will then enter Kant’s circle symbolism, which was examined in Chapter 2, and conjoin it with the metaphor of point he uses for the individual

signified by a singular term. By means of the resulting circle-*plus*-point symbolism, we shall be able to give a formal articulation, on Kant's behalf, both of his recognition that a singular judgment behaves like a universal one in syllogisms and of his view that the singular judgment stands to a general one, with respect to quantity, as *Einheit* to *Unendlichkeit*.

2.1. The conception of singular judgments shared by early modern logicians comprises theses [I] and [II], respectively.

[I] There is a two-tiered distinction among the forms of judgment¹⁸ with respect to quantity:



More specifically, the singular-vs.-general distinction is drawn in terms of whether the subject of a judgment is a singular or a general concept. The universal-vs.-particular distinction has to do with whether the predicate applies to all or to only a part of what is contained under the subject-concept.

[II] A singular judgment can be treated as universal to the extent that, like the latter, its predicate applies to what is contained under its subject-concept without exception.

Both theses are clearly present in the Port-Royal Logic, which, with the notion of logical extension it introduced into the early modern logical discourse, also initiated the basic conceptual framework within which later logicians would present their versions of [I] and [II].

In the Port-Royal Logic there is first a distinction between singular and general (or common, universal) ideas: the former “represent only a single thing” or an “individual”; the latter “represent several things”, things that constitute its extension. (*PRL*, i.6, pp.39-40) The distinction among universal, particular and singular judgments, which “arises from the subject”, is characterized in these terms. First, a singular judgment is one “whose subject is singular”, on account of which it differs from a judgment whose subject is a “common term”. Second, the latter judgment is in turn either universal or particular, depending on whether its subject is “taken in its entire extension” or “taken only through an indeterminate part of its extension”. (*ibid.* ii.3, p.83) A reason is then given as to why singular judgments “take the place of universals in arguments”:

[singular judgments] have a singular subject which is necessarily taken through its entire extension. This is the essence of universal propositions and distinguishes them from particulars. For it makes no difference to the universality of a proposition whether the subject's extension is large or small, provided that whatever it is, it is taken completely throughout. (ibid. ii.3, p.84)

Note that this is given as the reason for treating singular judgments as universal “in arguments”, i.e., in formal inferences, primarily syllogistic ones. In the traditional Aristotelian logic, a basic syllogism is a relation of terms, so that its formal validity is determined by how the predicate *relates* to the subject—more specifically, by whether the predicate is *affirmed* or *denied* of the subject through the *entirety* or through only a *part* of the latter's extension. It is presumably with an eye to this conception of syllogistic validity that the authors of the Port-Royal Logic, having distinguished singular from both universal and particular judgments, proceed to stress the subject-predicate *relation* that is shared by both singular and universal judgments.¹⁹

The basics of this account of singular judgment, viz., the theses [I] and [II] sketched above, are shared by most of Kant's immediate logician-predecessors. For the sake of argument, I shall focus on the versions that contain clues to the sense in which Kant thinks that the essential difference between singular and general judgments concerns their “quantity” and that the former stands to the latter as *Einheit* to *Unendlichkeit*. Here are three samples to consider.

- (1) In Baumgarten's *Acroasis logica in Christianum L.B. de Wolff* (1761), whether a judgment is singular, universal or particular is said to concern its quantity (*quantitas*) or extent (*Ausdehnung, Weite*), which is the character of the judgment “with respect to the number [*numero*]” of things cognized. (§§142-3)²⁰ A judgment is either singular or general (*communis, gemein*). In the latter case it is either universal (*universalis, allgemein*) or particular, depending on whether the predicate is affirmed/negated of every one or only some of what is contained under the general notion of the subject. (§§ 135-41)²¹ As Wolff puts it, the subject of a singular judgment is a singular term that signifies (*significat*) an individual or a singular thing (*rem singularem*); that of a universal or a particular judgment is a general term, which designates (*designat*) a general notion that represents a multitude of individuals. (Wolff 1740, §§113-14, 240-1)
- (2) In Knutzen's *Elementa philosophiae rationalis seu logicae* (1747), a judgment is said to be either singular, if its subject is a singular concept that represents an individual, or universal or particular, if its subject is a universal concept that contains under itself a multitude of individuals.²² In a universal judgment, the predicate is affirmed/negated of each one of the sub-contained multitude, but in a particular one the predicate is affirmed/negated of only a few of this multitude. (§141)

- (3) In Crusius' *Weg zur Gewissheit und Zuverlässigkeit* (1747), the distinction among universal, particular and singular judgments is said to concern the extent (*Weite*) of each judgment, extent being the quantity (*Menge*)²³ of the individuals that are grasped under the subject. A judgment is either singular (*singularis*) or not: in the former case something is said of a given individual, whereas in the latter case something is said of a logical whole (*totum logicum*). The second type of judgment is either universal or particular, depending on whether something is affirmed/negated of the logical whole “without restriction [*Einschränkung*] of its extent” or of an indeterminate part thereof. (§230) The *totum logicum* of a general concept (*Generalbegriff*) is, moreover, said to be “of infinite extent [*unendlicher Weite*]”. (§231)

From these accounts we can extract the following version of the two-tiered distinction among universal, particular and singular judgments sketched earlier. First, there is a distinction between singular and general judgments that concerns the *quantity* or *number* of individuals signified by the subject of each judgment. To the extent that the subject of a singular judgment signifies exactly one individual, whereas that of a general one signifies a *totum logicum*, and to the extent that, as Crusius puts it, the *totum logicum* of a general concept is of infinite extent, the contrast between singular and general judgments regarding quantity is that between *one* and an *infinite multitude*. Second, when general judgments are divided into universal and particular ones, such a division concerns the restriction—or lack thereof—of the *totum logicum* signified by the subject. This version of the two-tiered distinction among universal, particular and singular judgments is, as we shall see next, roughly the logical distinction that Kant thinks must be drawn “at the beginning” before any further assessment can be made about singular judgments.

2.2. Kant adopts the two-tiered distinction sketched above with an important caveat. Strictly speaking, for Kant, the distinction between singular and general judgments comes down to that between singular and general *uses* of a concept, not that between singular and general *concepts*.²⁴ For, in his view, every concept is in essence a general representation: “The form of a concept consists in universal applicability [*Gültigkeit*].” (V-Lo/Hechsel, LV: 395) The use of a concept can nevertheless be general or singular. More specifically,

the use of a concept can be singular, since what applies [*gilt*] to many things can also be applied to a singular instance. I think a man individually [*in individuo*], that is, I use the concept of man for one singular being [*ens singulare*]. I can use a concept insofar as it is

applied to several objects [*Gegenstände*]; then the concept is used as general representation[.] (V-Lo/Hechsel, LV:395-6)²⁵

The most fitting example Kant uses to illustrate the distinction between general and singular uses of one and the same concept is recorded in the *Wiener Logik*: the concept *house* may be used in its capacity *qua* general representation, e.g., in “All houses must have a roof” and “Some houses must have a gate”; but the same concept may also be used “only for an individual thing: e.g. this house is plastered this way or that.” (V-Lo/Wiener, Ak 24:909) For the sake of argument, I shall treat “This A is B” as the paradigmatic schema of singular judgment for Kant.²⁶

With this schema of singular judgment in mind, let’s turn to Kant’s claim at A71/B96 that singular and general judgments differ in quantity when considered “as cognition as such”. A hint as to what is meant by treating a judgment as “cognition as such” can be gathered from the notion of judgment introduced in a previous section titled “On the logical use of the understanding in general” (A67-9/B92-4). The following statements found in that section are especially pertinent.

[Judgment is] the mediate cognition of an object, hence the representation of a representation of it. (A68/B93)

[To judge is to judge by means of concepts, and all concepts,] “as predicates of possible judgments, are related to some representation of a still undetermined object. (A69/B94)

We can read these statements in conjunction with what may be called an *x-a-b* conception of judgment presented in some of Kant’s *Reflexionen*. On this conception, a judgment is cognition of an object, a “something *x*”, by means of two predicates: the first predicate (*a*, as the “logical subject”) is the given conceptual representation of the object, while the second predicate (*b*, as the “logical predicate”) is compared with the first. (R4634, Ak 17:616-17)²⁷ The judgment thus comprises two relations: the relation of *a* to *x* and the relation of *b* to *a*. To borrow Kant’s terminology at A71/B96, we may regard the first as a signifying relation: *a* is used in the judgment to signify (*bedeuten*) an object = *x*, and the object is what is signified or the *Bedeutung* of *a* in the judgment. To say that *b* is predicated of *a* is then to say that *b* applies (*gilt*) to the *Bedeutung* of *a*.²⁸ Accordingly, Kant’s above-quoted two statements about judgment may be

taken to suggest that to consider a judgment *as cognition as such* is to consider it as a mediate cognition of an object = x by two predicates: the first predicate is the logical subject by which the object is signified, the second is the logical predicate that applies to the object so signified.

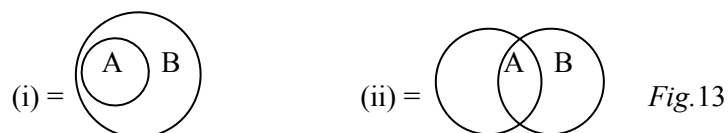
In these terms, Kant's version of the distinction among universal, particular and singular judgments can be sketched as follows. First, a judgment is either singular or general, depending on whether its logical subject is used *in individuo*, signifying exactly *one* object, or used *as a general representation*, signifying a *multitude* of objects. Second, a general judgment is either particular or universal, depending on whether the multitude of objects signified by the logical subject *qua* general representation is restricted or not. As far as Kant's view of singular judgment in the *Critique* is concerned, it is especially important to understand the precise nature of the singular-vs.-general distinction. After a brief analysis of this distinction, I will present it using Kant circle-*plus*-point symbolism, so as to accentuate its formal character. I shall then explain why the same distinction gives Kant both the logical basis to treat the singular as a special form of judgment in respect of quantity (*Grösse*) and (part of) the logical basis to treat it like the universal judgment in its use in syllogisms.

In a general judgment, to begin with, the concept used as the logical subject is used in its capacity *qua* general representation. In Kant's view, the generality (or universality) of a concept lies in it representing a multitude of things and hence in it having an extension.²⁹ In Chapter 2, I argued that the relevant notion of extension is an objectual one ([EXT_o]), so that the multitude that constitutes the extension of a concept is that of all the possible objects to which the concept applies. And we saw that logical extension in this sense can be represented by Kant's circle symbolism. Now, just as a circle can be used to represent a concept in respect of the logical extension that it has *qua* general representation, so can a point be used to represent a concept in respect of its singular *Bedeutung*. As Kant puts it, a concept, when used to signify exactly one individual, "e.g., that of the individual Julius Caesar, is equal to a point." (V-Lo/Dohna, Ak 24:755)³⁰ Kant is thus equipped with a circle-*plus*-point symbolism to represent a concept in its

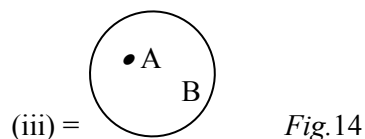
different signifying functions: a circle to represent the concept in its general use, and a point to represent it in its singular use.³¹

In Chapter 2, the circle- or line-symbolism introduced by Leibniz, Lambert and Euler was shown to have a significant role to play in their logics: it serves to make precise the logical form of a judgment, which consists in a certain relation between two given concepts with respect to extension. The relation of concepts represented by means of such a symbolism is *formal*, to the extent that it is represented thereby in abstraction from what features each concept may have other than that it has an extension (in virtue of being a general representation) and from what—or whether any—objects actually fall under the concept. As Euler puts it, if a circle is used to represent the logical extension of a concept, it represents the multitude of objects to which the concept applies simply on account of its generality, “whatever” the objects may be. (*Lettres*, cii, p.338) In the same spirit, Kant’s circle-*plus*-point symbolism serves as the means by which to represent the relation one concept stands to another in a judgment with respect to *Bedeutung*. The relation, so represented, is formal in that it is being considered in abstraction from any other feature a concept may have than that it is used in a judgment to signify a multitude of objects—or to signify exactly one object—and in abstraction from what (if any) object actually falls under the concept. So construed, the circle-*plus*-point symbolism can be used to articulate the formal *distinction* between singular and general judgments—as well as that between universal and particular ones—regarding the quantity of what is signified by the logical subject of each judgment, without reference to any particular (actual) objects.

To illustrate, take any two concepts, A and B. Using A as the logical subject, B as the logical predicate, we can make three judgments that differ from each other only in respect of quantity: (i) “All A is B.” (ii) “Some A is B.” (iii) “This A is B.” In Chapter 2, we saw Kant using his circle symbolism and representing the forms of (i) and (ii) as follows.



Now, with his circle-*plus*-point symbolism, Kant can also represent the form of (iii) as this:



In Kant's terms, (iii) differs from both (i) and (ii) with respect to the quantity of what is signified by the subject-concept A. In (i) and (ii), A is used in its capacity as a general representation and hence used to signify a *multitude* of objects. In (iii), by contrast, A is used to signify *exactly one* object. The restriction of A to singular signification in this case is a necessary function of *this*, a quantifier that renders irrelevant the fact that A, *qua* concept, is capable of representing more than one individual.³² Such a distinction between (iii) and (i)/(ii) holds for any concepts that may take the places of A and B, and holds regardless what (if any) actual objects may fall under each concept. To that extent, the distinction is a formal one. It can be shown by the contrast between the two ways in which A is represented in Fig.13 and Fig.14. In Fig.13, A is represented by a *circle*, in view of the multitude of objects—whatever they may be—that it is used to signify in the judgments (i) and (ii). In Fig.14, by contrast, A is represented by a *point*, in view of the sole individual—whatever it may be—that it is used to signify in the judgment (iii). Now, for Kant, as for Euler et al., a concept *qua* general representation is applicable to infinitely many possible objects. Accordingly, the multitude of objects that A is used to signify in (i) and (ii) is infinite.³³ To that extent, (iii) stands to both (i) and (ii), regarding quantity, as *Einheit* to *Unendlichkeit*. More generally put, as regards the quantity of what is signified by its logical subject, a singular judgment stands to a general one as *one* to *infinity*.³⁴ This contrast of quantity is reflected in the point-vs.-circle representation of A in Fig.14 vs. Fig.13: the circle representing A in Fig.13 encloses infinitely many points.

2.3. On the reading given above, Kant takes it that the unique formal character of a singular judgment consists in its logical subject signifying exactly one object, on account of which it

differs, in form, from a general judgment (be this judgment universal or particular). In a nutshell, to the extent that the form of a judgment lies in the relation of two given concepts with respect to *Bedeutung*, the form of a singular judgment (“This A is B.”) is such that the object signified by the logical subject (A) falls within the extension of the logical predicate (B). Now, as mentioned in 2.1, the subject-predicate relation of a judgment with respect to extension determines how the judgment may be used in a syllogism (insofar as its formal validity is concerned). So, when Kant agrees with “the logicians” in treating a singular judgment like a universal one in syllogisms, we expect him to justify such a treatment precisely in terms of the account of singular judgments I attributed to him in 2.2. But it is not immediately clear that he has met such an expectation. Here is how he states the relevant justification:

just because they [i.e., singular judgments] have no extension at all, their predicate is not merely related to some of what is contained under the concept of the subject while being excluded from another part of it. Thus it applies to that concept without exception[.] (A71/B96)

What would strike a modern reader as puzzling is the claim that a singular judgment or, more precisely, its subject-concept, *has no extension*.³⁵ Codato [2008] even goes as far as to declare that the notion of *concept without extension* is “oxymoron” and “border[s] on nonsense”.(Codato 2008:144) Codato’s reaction is understandable. After all, as we have seen earlier, for Kant every concept is an essentially general representation and, as such, necessarily has an extension; hence it would seem to be an obvious contradiction for Kant to talk about a *concept that has no extension*. Anderson [2004b], by contrast, suggests that Kant’s reference to “concept without extension” is perfectly meaningful—but *only* if by extension is meant the multitude of all the *concepts* subordinate to the given concept ([EXT_C]). Indeed, as Anderson sees it, that Kant takes logical extension to be [EXT_C] *as opposed to* [EXT_O] is implied by the “otherwise puzzling” doctrine “that the subject concept of a singular judgment has no extension (A71/B96), i.e., no general concepts under it”. (Anderson 2004b:512n.28) Here, then, is the only way that Anderson thinks we can make sense of Kant’s reference to concept without extension:

- (a) the extension of a concept consists of all and only the concepts that are subordinate to it, and a concept *has extension only if* it has at least one subordinate concept;³⁶
- (b) the subject-concept of a singular judgment is a concept with no subordinate concepts;
- (c) therefore, the subject-concept of a singular judgment is a concept without extension.

If, Anderson might press on, Kant had taken the extension of a concept to consist of all the objects falling under it, he would have said that the logical subject of a singular judgment, on account of signifying exactly one object, is a concept with a one-member extension rather than a concept with no extension at all. Otherwise put, interpreting Kantian logical extension as [EXT_O] would make nonsense of Kant's claim that the logical subject of a singular judgment is a concept without extension.

There are two problems, however, with this line of argument. First, it does not square with what is usually said in Kant's logic corpus about a concept with no subordinate concepts, on the one hand, and about a singular concept, on the other. The concept often characterized as (putatively) having no subordinate concepts is a *species infima*, which is a *general* concept³⁷ and which is never said to have no extension for containing no other general concepts under it. Meanwhile, a singular concept is mostly characterized as a concept used to signify exactly one individual,³⁸ but rarely *as* having no subordinate concepts.³⁹ Second, as I pointed out in Chapter 2, the version of [EXT_O] that Anderson has in mind is only our modern notion of extension, according to which a concept with exactly one object falling under it has a one-member extension. The fact that this notion of extension makes nonsense of Kant's claim that a singular concept has no extension—when such a concept is considered in terms of its singular signification—leaves it open that the claim may be perfectly intelligible on an idiosyncratically early modern notion of [EXT_O]. In fact, as noted in Chapter 2, in early modern logics the notion of logical extension was always introduced for *general* concepts, *as opposed to* the singular ones; and, for the logicians who explicitly construed logical extension as [EXT_O], a concept has an extension precisely in virtue of its generality, i.e., in virtue of its capacity to represent a *multitude* of objects. With such a notion of [EXT_O], then, comes an implicit restriction on when a concept

may be said to *have extension* in the first place: it has extension *only if* it applies to a *multitude* of objects.⁴⁰ It follows that a singular concept, to the extent that it signifies one and only one object, has no extension. Thus, contrary to Anderson's contention, in terms of [EXT_o] we can make the following sense of Kant's claim that the subject-concept of a singular judgment has no extension:

- (a') the extension of a concept consists of the multitude of objects to which it applies, and a concept *has extension only if* it is applied to more than one object;
- (b') the subject-concept of a singular judgment is applied to only one object;
- (c') therefore, the subject-concept of a singular judgment has no extension.

This, as it turns out, is roughly how Kant himself explains in his logic corpus why the subject-concept of a singular judgment has no extension (or sphere), for which reason such a judgment can be treated like a universal one in syllogisms. In the *Wiener Logik*, for instance, Kant says the following about the singular judgment "Caesar is mortal":

no exception can occur here, because the concept Caesar [...] does not comprehend a multitude under itself, but is only an individual thing [*nur ein einzelnes Ding ist*]⁴¹ [...] that is to say, it *does not have a sphere at all* from which something could be excluded. [...] consequently a singular judgment is like the universal one in use. (V-Lo/Wiener, Ak 24:931, emphasis added)⁴²

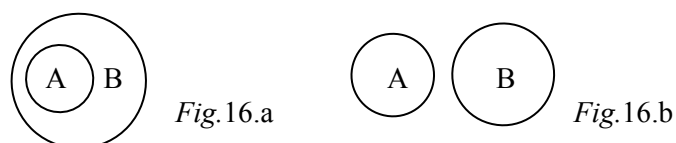
An individual, precisely because it is not a multitude, is not an extension. It cannot be restricted (*beschränkt*) in the way that an extension can. Hence it must be taken in its entirety when related to something else. In that sense, the predicate of a singular judgment always applies to the logical subject—more precisely, to its *Bedeutung*—"without exception [*Ausnahme*]". (A71/B96)

A singular judgment thus resembles a universal one in how the predicate is *related* to the logical subject in respect of the later concept's *Bedeutung*. Such a resemblance in relation can again be formally demonstrated by means of the circle-*plus*-point symbolism introduced above. Suppose we make a singular judgment out of two given concepts, A and B, with A as the logical subject, B as the logical predicate. In the fashion of *Fig. 14*, we may use a point to represent A in respect of its singular *Bedeutung*, and a circle to represent B in respect of its logical extension. There are only two possible ways for the point to be related to the circle: it is either entirely inside

or entirely outside the circle. These two possibilities correspond, respectively, to two basic forms of singular judgments: “This A is B” (*Fig. 15.a*) and “This A is not B.” (*Fig. 15.b*)



These two ways in which the subject and predicate of a singular judgment are *related* resemble the subject-predicate relations in the universal affirmative (“All A is B.”) and the universal negative (“No A is B.”), which can be represented by *Fig. 16.a* and *Fig. 16.b*, respectively. (see *Fig. 4* and *Fig. 6* in Chapter 2)



These four figures show that a singular judgment has the same *innere Gültigkeit*—i.e., the same way in which the predicate applies to the subject—as a relevant universal one does. (A71/B96) This is the basis for Kant to grant that the former is not “special” in comparison with the latter “in that logic which is limited only to the use of judgments with respect to each other”, i.e., limited to the syllogistic use of judgments.⁴³(A71/B97) More specifically, a singular judgment does not count as an irreducibly basic form of judgment in that logic where syllogisms have primacy in the *assessment* of the logical status of a form of judgment.⁴⁴ In this syllogism-centered logic, whether a judgment has a special form vis-à-vis another is determined by whether it plays a unique inference-role in syllogisms, which is in turn determined by whether it exhibits a special subject-predicate relation. Since a singular judgment does not have a unique subject-predicate *relation* in comparison with a universal one, it can be treated like the latter in syllogisms and so does not count as a basic form of judgment over and above the latter.

If Kant grants that one *can* take this syllogism-oriented perspective to evaluate the logical status of a singular judgment, however, he also makes it clear that his assessment of singular judgment in the *Critique* need not be limited to such a perspective. Rather, for the purpose of

deciding which forms of judgments should be included in the Table of Judgments, he suggests that we consider a judgment as cognition as such—viz., as mediate cognition of object by means of two concepts ($x-a-b$)—and in abstraction from its potential use in syllogisms. Accordingly, the formal distinction that has been drawn between a singular judgment and a general one, both being considered as cognition as such, suffices for the former to be deemed a distinct form of judgment with respect to quantity. In sum,

[if, contrary to considering all judgments in view of their use in syllogisms] we compare a singular judgment with a general one, *merely as cognition*, with respect to quantity, then the former stands to the latter as unity to infinity, and is thus in itself essentially different from the latter. Therefore, [... the singular] deserves a special place in a complete table of the moments of thinking in general (though obviously not in that logic that is limited only to the use of judgments with respect to each other). (A71/B96-7, emphasis added)

In thus treating the singular as a special moment of judgment, Kant is, as C. H. Beck [2003] puts it, “not anticipating the transcendental logic within the formal logic. On the contrary he remains in the formal logic, though not in the logic of inference, but rather in the logic of judgment.” (Beck 2003:126) More precisely, as far as the formal-logical resources are concerned, Kant has been faithful to his claim in the *Prolegomena* that, in his search for the various moments of judgment, he has lying before him “already finished though not yet wholly free of defects, the work of the logicians”. (*Prolog.* Ak 4:323) For, as we learned in 2.1, the basics of the logical distinction that Kant draws among universal, particular and singular judgments were already clearly present in the early modern logics with which he was familiar. If the work of the logicians is nevertheless defective in this regard, it is not, contrary to what Beck has thought, that it has “overlook[ed] a few distinctions [e.g., that between the singular and the general] that are important for the logic of judgment”. (C.H. Beck 2003:126) It is rather that in their “syllogism-fixated logic” (*ibid.*) the logicians can find no room for giving a special logical status to the singular vis-à-vis the universal, even though they recognize that the singular differs from both the universal and the particular judgment with respect to quantity. As for Kant, it is by *shifting the perspective* from what I have called Perspective-*s* to Perspective-*c* that he finds himself being

“put in the position” (*Prol.* Ak 4:323) to secure a special logical status for the singular judgment. In doing so, Kant has not added any new *materials* to the traditional logic as he knew it. If, on the other hand, Kant is pressed on the further question as to why Perspective-*c*—whereby a judgment is considered as cognition as such—is *the relevant perspective* from which to sort out all the moments of judgment, he may very well have to invoke certain transcendental-logical considerations. What such considerations might be is an issue we need not address here. But it is worth pointing out that, if some transcendental-logical considerations are to be involved in Kant’s justification for including the singular *in the Table of Judgments*, they need to enter only at a higher order. In other words, they need to be invoked only to explain why Perspective-*c*, as opposed to any other perspective, should be assumed for Kant to determine which forms of judgment to include in his Table of Judgments; once such a perspective has been fixed, however, whatever *distinction* needs to be drawn among moments of judgment can be drawn on the basis of none other than the relevant formal-logical materials.

III. Correlation between logical and categorial moments of quantity: settling a controversy

We have clarified the main points of Kant’s account of singular judgments at A71/B96-7. In sum, Kant’s agreement with the logicians is not that the singular is not a distinct form of judgment from a strictly logical point of view, but that it plays no special role in comparison with the universal judgment in syllogisms. For Kant, this treatment of singular judgments in syllogisms indeed presupposes a certain logical distinction between these judgments and the general ones. By Kant’s account of the distinction in question, a singular judgment stands to a general one as *Einheit* to *Unendlichkeit*, in virtue of which the former—considered as cognition as such—counts as a special form of judgment with respect to quantity. In this explanation of why a singular judgment should be included in the Table of Judgments regardless of how it has been commonly treated by the logicians, Kant does not require resources beyond the formal logic as he knew it, except that he may need a non-formal-logical higher-order justification for his choice of

Perspective-*c*—as opposed to the logicians’ Perspective-*s*—from which to assess the logical status of singular judgment. In this account of singular judgments there is also no need for Kant to presuppose any knowledge about the specific content of the Table of Categories. Especially, even if the *Einheit* ascribed to the singular judgment at A71/B96, in which lies its essential difference from the general one, may turn out to be intimately connected with the category of unity, no such connection is required in order for Kant to explain the distinction between singular and general judgments in the way he has. Otherwise put, in deciding which moments of judgment to place under the title “Quantity” in the Table of Judgments, Kant can set aside any view about the specific ways in which these moments may in turn be connected with the categories of quantity. This has an immediate consequence for how to approach a controversy over the correlation between the logical moments of quantity in the Table of Judgments and the categorial moments in the Table of Categories.

The controversy in question begins with the observation that Kant presents the three logical moments of quantity in these two different orders in different texts:

[Order I]	universal, particular, singular;
[Order II]	singular, particular, universal.

[Order I] can be found in the *Critique*, in the *Prolegomena* (§21, Ak 4:302-3) and throughout Kant’s logic corpus whenever the three forms are listed together.⁴⁵ And [Order II] can be found in many of Kant’s lectures on metaphysics.⁴⁶ But the categories of quantity are always presented in the order of <unity, plurality, totality>. This contrast has prompted discussions surrounding two questions. First, which of [Order I] and [Order II] is the right order? Second, how should the logical and the categorial moments of quantity correlate? Especially, should the correlation be universal-unity and singular-totality, or singular-unity and universal-totality?⁴⁷ The reading I presented in section II of Kant’s stated justification at A71/B96-7 for treating the singular as a special logical moment of judgment undermines two sorts of strategies that have been used in addressing these questions. First, the reading thwarts any attempt at using the explicit connection

drawn between “singular” and “*Einheit*” at A71/B96 to pit one correlation against the other. Such an attempt is exemplified in Thomas Swing’s argument that Kant’s comparison of singular judgments to *Einheit* at A71/B96 is “evidence” that he already has in mind the more “natural” correlation, namely, singular-unity, particular-plurality, universal-totality. Swing says:

We find evidence indeed to show that Kant had not forgotten this more natural line of thought at the time of the Metaphysical Deduction. He observes that “the singular stands to the universal as unity to infinity” (A71/B96). (Swing 1969:20)

Apart from mistaking Kant to be contrasting a singular with a universal judgment, this argument hinges on the assumption that by comparing the singular to *Einheit* at A71/B96 Kant is already explicitly relating it to the category of unity. My reading of the *Einheit-Unendlichkeit* contrast in 2.2 has hollowed out such an assumption. On my reading, ‘*Einheit*’ is meant simply to capture the distinctive logical feature of a singular judgment, namely that its subject-concept is used to signify exactly *one* object. However this notion of *Einheit* may, in the end, be connected with the category of unity, there is no need for Kant to assume any such connection in order to ascribe *Einheit* to the singular judgment at A71/B96. Hence, simply because a notion of *Einheit* is involved in Kant’s reason for including the singular in the Table of Judgments, it does not follow that Kant must have assumed that the singular correlates with the category of unity.

Second, my reading of Kant’s account of singular judgments at A71/B96-7 also undermines any effort to how the logical moments of quantity *should* be ordered in the Table of Judgments in light of how the categorial ones are ordered. It is not unusual for commentators to think that the way the three logical moments are to be ordered in the Table of Judgments must reflect how they are to be correlated with the three categories of quantity. Bennett [1966], for instance, has argued that in the *Critique* Kant *should have* arranged the logical moments of quantity as a trio of <singular, particular, universal> and that it is a “slip” on Kant’s part to have reversed the order. For “[t]he associated trio of concepts is given as ‘unity, plurality and totality.’” (Bennett 1966:77, 77n.2) A more substantive argument to the same effect comes from Longuenesse [1998], which draws on a footnote in the *Prolegomena* where Kant suggests that, “if

I start from unity (in singular judgment) and proceed to totality, [...] I think only a plurality without totality”, and that this progression from unity to totality is “necessary, if the logical moments are to be placed under” the categories. (*Prol.* Ak 4:302) Taking such a suggestion to mean that “to understand the categories of quantity one must consider their genesis as parallel to the progression from singular to particular, then to universal judgment”, Longuenesse thinks that the footnote in question has “answered conclusively” the question as to whether “one of the tables [should] be reversed (and then, which one?), the correspondence then being singular judgment/unity, universal judgment/totality”. The alleged conclusive answer is that (i) the logical and categorial moments of quantity should be correlated as singular-unity, particular-plurality, universal-totality and that, given the necessity to progress from unity to totality, (ii) the moments of quantity in the Table of Judgments *should* be arranged in the order of <singular, particular, universal>, i.e., in [Order II] as opposed to [Order I], even though the latter is the order in which they are actually presented in the *Critique*. (Longuenesse 1998:249)

Given what we saw in section II, however, the move from (i) to (ii) is not warranted.⁴⁸ Even if, in order to understand how the categorial moments of quantity may be derived from the logical ones, the latter moments must be taken in the progression from the singular to the particular and then to the universal ([Order II]), it does not follow that [Order II] is the way in which these moments should be arranged in the Table of Judgments initially. For there are two standpoints from which the logical moments of quantity may be *interpreted*. On the one hand, they may be deemed as the moments of judgment from which specific categories are to be generated in the order, say, of <unity, plurality, totality>.⁴⁹ On the other hand, they may be viewed merely as three basic logical moments of judgment, regardless how specific categories may be derived from them. The contrast between these two interpretations is actually hinted in the very footnote that Longuenesse has built her argument on. In that footnote, Kant gives the following explanation as to why ‘*judicia plurativa*’ is a more suitable term than ‘*judicia*

particularia’ for particular judgments when they are used in bringing a given *intuition* under the category of plurality.

For the latter [i.e., ‘*particularia*’] already contains the thought that they [the judgments in question] are not universal. If, however, I start from unity (in singular judgments) and proceed to totality, [...] I think only a plurality without totality, not the exception of the latter. This is necessary, *if* the logical moments are to be placed under the pure concepts of the understanding; *in logical usage things can remain as they were*. (*Prol.* §20n., Ak 4:302, emphasis added)

Here Kant is making a qualified claim: the progression from singular to particular and then to universal must be assumed, *if* these logical moments “are to be placed under the pure concepts of the understanding”. The *if*-clause involves an *interpretation* of the judgments of quantity according to which they are, roughly put, the judgments through which a given intuition is to be subsumed under the pure concepts of quantity. (see *Prol.* §20, Ak 4:300-2) Such an interpretation, however, goes beyond what Kant needs to assume in order to introduce the singular as a distinct logical moment of judgment. That may be why Kant *adds* at the end of the quoted footnote: “in logical usage things can remain as they were”. And that may also be why, immediately after the footnote, Kant proceeds to present a Table of Judgments in which is included the trio of <universal, particular, singular> ([Order I]) instead. (*Prol.* §21, Ak 4:302)

Indeed, the wording of the quoted footnote also indicates that [Order I] may be more suitable to capture the two-tiered distinction among universal, particular and singular judgments that Kant shares with his logician-predecessors. In the footnote, Kant suggests that there are two ways of construing the relation between particular and universal judgments, which are captured, respectively, by [Order II] and [Order I]. In a nutshell, if we go from the singular to the particular and then to the universal ([Order II]), then, by a particular judgment, we are thinking of “only a plurality without totality”. By contrast, if we follow [Order I], in which the universal comes prior to the particular, then, by a particular judgment, we are thinking of a plurality that is the “exception of [*Ausnahme von*]” the totality. To recast this contrast in terms of what was shown in section II, in the first case, we think of the *Bedeutung* of the subject-concept of a particular

judgment as a discrete collection of (actual) individuals, to which extent the particular is dependent on the singular, but independent of the universal. In the second case, however, we consider the relation among the universal, the particular and the singular in light of Kant's two-tiered distinction: first, the general vs. the singular; second, the universal vs. the particular. Accordingly, the *Bedeutung* of the subject-concept of a particular judgment is viewed not as a discrete collection of individuals, but as the result of a certain restriction of what is signified by the subject-concept of a relevant universal judgment,⁵⁰ namely the concept's entire logical extension (= a continuum of infinitely many possible individuals thought under the concept *qua* general representation). (see *Fig. 13*) To that extent, the universal—as opposed to the singular—is prior to the particular. Accordingly, when the logical moments of quantity are first included in the Table of Judgments on account of being three distinct forms of judgment as to quantity, they may be most naturally arranged in the order of <universal, particular, singular>. This initial order of the logical moments reflects the sense in which Kant takes them to be *logically distinct* forms of judgment with respect to quantity and, as such, it is neither determined by nor determines the order of the categorial moments of quantity.

Given the above distinction between two ways of considering the logical moments of quantity, the fundamental problem with an argument like Longuenesse's for favoring the order <singular, particular, universal> seems to be that of conflating two questions. It is one thing to ask (a): how should the logical and the categorial moments of quantity be correlated, and hence in what order should we consider the logical moments themselves so that they may serve as the basis for generating the relevant categorial moments in the right order? But it is another to ask (b): how should the logical moments be arranged as they are first introduced and included in the Table of Judgments? My reading in section II of Kant's account of singular judgments helps disentangle these two questions, so that each can be addressed independently of the other—and hence more effectively. On the one hand, an answer to (b) is adequate so long as it reflects the logical grounds on which certain forms of judgments are included in the Table of Judgments. If I

am right in suggesting that the two-tiered distinction among universal, particular and singular judgments—first between general and singular ones, and then between universal and particular ones—is, viewed from Perspective-*c*, the basis for Kant to include all three of them in the Table of Judgments, then it is of little importance whether these forms of judgments are arranged in [Order I] or in [Order II], except that the former order would be a more *natural* way to capture the relevant logical grounds on which these forms of judgment have been distinguished. To that extent, the fact that Kant adopts both [Order I] and [Order II] from time to time does not simply signal his recurring “slips”, *à la* Bennett, inadvertently to reverse the should-be order. On the other hand, to address question (a) more effectively, one should not dwell on a particular order in which the logical moments of quantity appear in the initial Table of Judgments. Handling (a) takes a different viewpoint than what has been assumed in introducing those logical moments in the first place. Especially, it involves the issue about the most meaningful or plausible way in which the categories of quantity may be *derived* from the logical moments of quantity, which is an issue that need not be treated at the time when the Table of Judgments was initially built. To that extent, whichever way the logical moments are ordered in the latter Table has no direct consequence for any particular order in which to consider the categories of quantity.

IV. Singular judgments in general logic vs. in transcendental logic

I have given a reading of Kant’s account of singular judgments at A71/B96-7 mainly to address a specific circularity worry, raised in section I, about the ground on which for Kant to include the singular judgment in his Table of Judgments. To take stock, it was shown in section II that a formal distinction may be drawn, on Kant’s part, between singular and general judgments regarding the quantity of the objects that the subject-concept of each judgment is used to signify: the subject-concept of a singular judgment is used to signify exactly *one* object, whereas that of a general one signifies an *infinite multitude* of possible objects. I argued that the *materials* based on which for Kant to draw such a distinction were already present in the early modern logics familiar to him, and that the departure of his treatment of singular judgment in the *Critique* from that of

those logicians boils down to a difference in the *perspectives* from which the logical status of such judgment is evaluated. I also pointed out that, insofar as Kant may have to invoke certain transcendental-logical considerations to justify his choice of the relevant perspective, these considerations are only part of a higher-order justification for treating the singular as a special form of judgment, without being directly involved in the initial making of the *distinction* between the singular and the general. Finally, in section III, I distinguished two ways of interpreting the logical moments of quantity regarding their relation to the categorial ones. I granted, for the sake of argument, that the logical moments may be thought of as the basis for the derivation of the specific categorial moments, in which case they are viewed, *à la* Longuenesse, in relation to sensibility and how the categorial moments are understood (e.g., concerning their order) may in turn constrain how the logical moments are to be understood. But I contended that the initial ground on which each logical moment is included in the Table of Judgments does not have to involve considerations about its connection with a particular category via relation to sensibility. I take it that I have thus resolved the circularity worry that, if Kant's justification for treating the singular as a special moment of judgment consists precisely in ascribing to it the quantity of *Einheit*, then his only ground for such a treatment involves the perspective of categories of quantity (in particular, the category of unity), in which case the justification in question would be viciously circular. My answer to such a worry is, in a nutshell, as follows: however the *Einheit* ascribed to the singular judgment at A71/B96 may eventually be connected to the category of unity (by being considered in relation to sensibility), there is no need to interpret this *Einheit* as more than a primitive notion of *one* (in abstraction from all relation to sensibility), a quantity that is ascribed to the singular judgment simply to capture its unique logical character (i.e., that its subject-concept is used to signify exactly one object).

In thus presenting Kant's account of singular judgment, I am assuming that he can speak about the object-signifying function of the subject-concept of a singular judgment without having to consider the judgment in relation to sensibility. But one might doubt that this is a *bona fide*

Kantian way to interpret his account of singular judgments in the *Critique*. Especially, one might doubt that there is really any conceptual room for Kant to say that, in a singular judgment, the subject-concept is used to signify exactly one object without any reference to sensible intuition. After all, one might argue, given that for Kant our conceptual cognition is essentially “discursive” (A68/B93), that an (individual) object can be given to us only in sensible intuition and that there is no thoroughly determinate cognition of an individual merely by means of concepts, a concept cannot, in Kant’s view, possibly be used to refer—in its own right—to exactly one individual. Rather, if a concept may be used in relation (*Beziehung*) to any objects at all, such relation must *always* be mediated by sensible intuition. Thus, as Longuenesse puts it, a singular judgment—insofar as it refers to a determinate individual—necessarily refers its concepts “to what is beyond discursive capacity: the singular intuition”. (Longuenesse 1998:139) To the extent that this allegedly necessary involvement of singular intuition is also an involvement of transcendental logic, Kant does not seem to be in the position, contrary to what I have thought, to draw a *general-logical*, as opposed to transcendental-logical, distinction between singular and general judgments in terms of the former judgment’s characteristically singular signification.⁵¹

There is a crucial assumption behind this potential objection to my reading of Kant’s account of singular judgments in the *Critique*. Roughly put, what is assumed is the view that both singularity and immediacy are the essential features of intuition (whereby it differs from concept), so much so that all singular reference necessarily requires intuition. For our purpose, we need not assess the merits—or the lack thereof—of such a view.⁵² I only wish to point out an important distinction between two ways—(i) and (ii) below—in which Kant characterizes singular concept, in light of which distinction we may appreciate the sense in which he allows the immediacy of a cognition and the singularity of its reference (or signification) to come apart, so that the latter feature can be attributed independently of the former.

- (i) Kant often speaks freely of a “singular concept”, as a concept “which does not grasp a multitude under itself, but is only a single thing”. (V-Lo/Hechsel, LV: 426)⁵³

- (ii) Kant sometimes explicitly denies there to be any singular concept, as a “thoroughly determinate concept” of an individual. (Log, §15, Ak 9:99) For such a concept would have to contain a unique combination of conceptual determinations by which an object is individuated, a requirement that is inconsistent with the essence of every concept as a representation only of what is *common* to a multitude of things. (V-Lo/Wiener, Ak 24:912)

It would no doubt be uncharitable to see Kant as simply contradicting himself, or speaking loosely here and there, in these two sets of remarks about singular concept. Rather, by (i) and (ii) Kant may be stressing, respectively, two distinct aspects of a concept, so that it is in principle permissible to speak of singular concept regarding one aspect but not the other. To spell out this suggestion, we may consult the following distinction in philosophy of language: semantics of a singular term (i.e., its contribution to the meaning of the sentence in which it occurs) vs. theory of reference (e.g., about how to secure the referential link between the singular term and its referent).⁵⁴ In that connection, we may also recall the distinction made in Chapter 1 between the proposition *that* a thought is related to an object and a theory as to *how*—or, more specifically, the possibility conditions under which—such a relation (*Beziehung*) may come about. (I argued that the former proposition can be assumed in Kantian general/formal logic, but transcendental logic has a unique claim to the latter theory.) In these terms, Kant’s remarks about singular concept in (i) and (ii) may be recast as follows. In (i), a singular concept is considered with respect to its *semantic* function in singular judgment: by means of it is always signified exactly *one* object. In (ii), a putative singular concept is viewed in terms of its putative *epistemic* function of *thoroughly determining* an object solely and directly by the conceptual marks or determinations contained in it. Kant denies singular concept in the second sense due to his theory of *Beziehung*, according to which a concept can be related to an individual object only through the mediation of sensible intuition and hence cannot individuate the object all by itself. Kant can nevertheless speak meaningfully of singular concept in the first sense, independently of his or any other view about the epistemic conditions under which to secure the singular reference or signification of such concept. In other words, singularity can be ascribed to a concept as a mere

semantic feature of its signifying function in judgment, even though the concept is essentially incapable of representing the signified object in an immediate and thoroughly determinate way.

In these terms I shall briefly respond to the above-sketched objection to my view that the distinction between singular and general judgments at A71/B96 can be drawn within Kant's general logic. Take Kant's paradigmatic form of singular judgment, "This A is B." On my reading, the affixation of *this* to the subject-concept A determines its semantic property, i.e., that A's *Bedeutung* in the judgment in question is necessarily one and only one object. In ascribing this semantic property to A *qua* subject-concept of a singular judgment, we make two abstractions in a characteristically Kantian manner. First, the object being signified is not assumed to be actual.⁵⁵ Nor do we consider any particular qualities of the object other than that it is what is signified by the concept A in the judgment in question. (That is why, just as the logical extension of a concept in its general use can be represented by a mere circle, so can the object signified by a concept in its singular use be represented by a mere point.) Second, we are not concerned with any epistemic conditions under which a concept can be related to an individual object. By taking the concept A, as it is used in a singular judgment, to have the semantic property of signifying exactly one object, we are not committed to any view regarding the epistemic ground on which A's *Beziehung* to the signified object is to be secured. (It is left to transcendental logic to explain any such epistemic ground.) Accordingly, if Kant (at A71/B96) takes the special feature of a singular judgment to consist in the singularity of its subject-concept's *Bedeutung*, he can give an account of this semantic feature that is general-/formal-logical in two senses explained in Chapter 1—both in that it abstracts from all specific features of the signified object and in that it abstracts from any considerations about the epistemic ground on which the concept may be related to the object. In virtue of the latter abstraction, especially, the account is a general-logical *as opposed to* transcendental-logical one.

¹ I will not discuss the details of Kant's or other early modern logicians' treatment of infinite judgments. For explicit treatment of infinite judgments as affirmative, see Wolff 1740, §§208-9; Baumgarten 1761, §217; Meier's *Auszug*, §294 and his *Vernunftlehre*, §327.

² For various ways to spell out the likely "misunderstanding", see Greenberg 1994.

³ Transcendental logic is not mentioned at all in the paragraph in which Kant discusses singular judgment. But the next paragraph, in which infinite judgment is discussed, begins with this claim: "*Likewise*, in a *transcendental logic* infinite judgments must also be distinguished from affirmative ones, even though in general logic they are rightly included with the latter[.]" (A71-2/B97, emphasis added) The wording of this claim suggests that the perspective of transcendental logic was somehow involved in the previously given account of singular judgment.

⁴ The distinctions Krüger does mention are only linguistic or grammatical (*sprachlich*) ones. (Krüger 1968: 347-51)

⁵ Both commentators take it that at A71/B96 Kant is only trying to distinguish singular judgments from *universal* ones. But I shall show in section II that Kant is contrasting singular with *general* judgments instead, a fact that is of great exegetical consequence. Until then, however, I shall follow the common practice and speak loosely of the distinction between singular and universal judgments.

⁶ Allison and Kemp Smith have in mind the following two different correlations, respectively, between the moments of quantity in the two tables.

Correlation I: singular-unity, particular-plurality, universal-totality

Correlation II: universal-unity, particular-plurality, singular-totality

The difference of these two correlations results from two ways in which the three moments of the quantity of judgment are ordered in different texts: <universal, particular, singular> vs. <singular, particular, universal>. As I shall argue in section III, however, there is no reason to assume that the way these moments are ordered in the Table of Judgments should have any direct consequence for how they are to be correlated with the categories of quantity.

⁷ The same dismissive remark can be found in Lovejoy 1907 (especially p.591).

⁸ In this essay, Tonelli gives a detailed survey of the classification of judgments by the logicians, both German and non-German ones, of the 18th century (up till 1777). On the basis of such a comprehensive survey, Tonelli observes that Kant's Table of Judgments in its final shape (as it is presented in the *Critique*) was not simply taken over from any previous logicians. (Tonelli 1966:150)

⁹ This suggestion has been challenged by Krüger [1968]. Krüger refutes Tonelli's interpretation that the Table of Judgments depends on the Table of Categories by refuting all three reasons that Tonelli cited to support the interpretation. (Krüger 1968:344-6) Krüger's own view is that Kant has independent grounds for the construction of the Table of Judgments in the *Critique*. (ibid. 344-53)

¹⁰ Another noteworthy, though extremely controversial, attempt at finding a transcendental-logical perspective from which for Kant to derive the Table of Judgments can be found in Reich 1986/1992 (first published in 1932, then reissued in 1948 and 1986). On Reich's reading, the entire Table can be derived step by step from a single principle, namely the principle of objective unity of apperception (which is not clearly articulated until §19 of the Transcendental Deduction in the B edition). This reading became a main target of objection in Krüger 1968, Brandt 1991/1995 and Wolff 1995.

¹¹ Treating a distinction between singular and universal judgments as "inconsequential" is not the same as treating one form of judgments as reducible to the other. The logicians can consistently treat singular judgments as irreducibly different from the universal ones in form and yet hold that, as far as syllogisms are concerned, exactly the same rules governing the use of universal judgments can be used to determine the use of the singular ones. It is in the latter sense that the perceived distinction between the two forms of judgments may be deemed as inconsequential.

¹² At A71/B96 Kant uses different terms for "general" and "universal" judgments. A general judgment is *gemeingültig* (in Latin: *judicium commune*), whereas a universal one is *allgemein*. In Kemp Smith's translation—and in many others—of the passage at A71/B96, '*gemeingültig*' is rendered as "universal". Guyer and Wood translate the word literally, as "generally valid", but explain in a footnote that "it clearly refers to the universal (*allgemein*) judgment", indicating that the apparent difference between *gemein* and *allgemein* is merely verbal and hence negligible. (Kant 1998:207a) As we shall see more clearly later, however, the difference is rather substantial, with important exegetical consequences.

¹³ I have kept the customary translation of ‘*innere Gültigkeit*’ as “inner validity”. But, for our purpose, it is important to caution that one should not be misled into thinking that by ‘*innere Gültigkeit*’ is meant none other than inferential validity, although it is certainly related to the latter. Rather, since ‘*Gültigkeit*’ comes from ‘*gelten*’ (to apply), by ‘*innere Gültigkeit*’ is more precisely meant the relation of the predicate being somehow applied to the subject. As we shall see more clearly later, Kant’s main reason for likening a singular judgment to a universal one is that, in both cases, the predicate applies (*gilt*) to the subject in the same way, namely, “without exception” (A71/B96).

¹⁴ As in Chapter 2, we are considering only categorical judgments and the syllogisms formed from them.

¹⁵ It is not immediately clear whether Perspective-*c* is exclusive or inclusive of considering a judgment in terms of its inference role. Kant’s characterization of the perspective seems ambiguous between the exclusive and the inclusive reading. The perspective is first presented as follows: “*on the contrary*, we compare a singular judgment with a general one, *merely as cognition*, with respect to quantity[.]” (A71/B96, emphasis added) The italicized phrases suggest the exclusion of considering a singular judgment also with respect to its use in syllogisms. But Kant goes on to say: “if I consider a singular judgment (*judicium singulare*) *not only* with respect to its inner validity, *but also, as cognition as such*, with respect to the quantity it has in comparison with other cognitions, then it is surely different from general judgments (*judicia communia*)[.]” (ibid., emphasis added) The italicized phrases in this statement seem to favor the inclusive reading instead. Note, however, Kant means this latter statement to be a recap of what has been said previously, for which reason he starts this statement with the expression ‘therefore’. It is then plausible to say that the “not only ... but also ...” clause is used to summarize two *alternative* perspectives: that of considering a singular judgment *merely* in terms of its inference role vs. that of considering it *merely* with respect to its quantity *qua* cognition as such.

¹⁶ V-Lo/Warschauer, LV:624; V-Lo/Pöhlitz, Ak 24:577; V-Lo/Wiener, Ak 24:929, 931; V-Lo/Hechsel, LV:426-7. I am setting aside the question regarding the extent to which a singular judgment can be treated like a universal one in syllogisms. There are at least two significantly different ways in which such a treatment may be construed: (i) in any syllogism in which a singular judgment is in fact used, it can be treated as universal; or (ii) whatever role can be played by a universal judgment in syllogisms can likewise be played by a singular one. For a discussion, see Czezowski 1955.

¹⁷ This interpretation makes good of Kant’s claimed deference to the previous logicians (*Prol.* Ak 4:323) that I cited earlier. I mentioned that, among commentators on Kant’s Table of Judgments, Krüger takes such a claim very seriously, reading it as suggesting that Kant takes the logical forms already discovered by previous logicians to require “only a critical revision under a uniform perspective”—the perspective being, by Krüger’s account, the conception of the understanding as unity-producing spontaneity. (Krüger 1968:344) Krüger makes a useful distinction between two points: that the logical distinctions under discussion were *not made* by the previous logicians, and that some logical distinctions were already made, but were somehow *neglected*. (ibid. 348) With regard to singular judgment, Krüger’s interpretation is similar to mine in spirit: the universal-singular distinction can—for Kant as well as for the logicians—already be made within formal/general logic, which was however neglected by the logicians; what Kant does is to make the distinction “relevant”, from the above-mentioned perspective.

¹⁸ ‘Proposition’ was more often used in place of ‘judgment’ in the early modern logics. As in Chapter 2, however, I treat ‘proposition’ and ‘judgment’ as interchangeable in the present context.

¹⁹ Among later logicians, Euler is most articulate about the exact reason for treating singular judgments as universal in syllogisms. To the extent that, in a singular judgment just as in a universal one, the subject (an individual) is taken “in all its extent” while being related to the predicate, the “same rules” that apply to universal judgments apply equally to singular ones. (Euler, *Lettres*, cvii) By “rules” Euler is referring to inference rules. The point is that, insofar as singular judgments behave in exactly the same way as the universal ones do, there is no need to treat them separately or specify separate inference rules for them. To illustrate this point, Euler uses the example of a syllogism with two singular judgments as premises: “Voltaire is a philosopher; Voltaire is a poet; therefore, some poets are philosophers.” He argues that the validity of this inference can be proven by treating the two singular premises as universal judgments, so that the inference takes the following form: “Every A is B; every A is C; therefore, some C is B.” (ibid.)

²⁰ The body of Baumgarten’s text is in Latin. But German equivalents are often provided in the footnotes for key terms.

²¹ The same two-tiered distinction, presented in exactly the same terms, can be found in §301 of Meier’s *Auszug*.

²² For the distinction between universal and singular concepts or ideas (*allgemeine und einzelne Begriffe oder Ideen*), see Knutzen 1747, §63, where a universal concept/idea is said to represent what is common among a plurality of individuals.

²³ ‘*Menge*’ can be translated also as “multitude”, and ‘*Weite*’ as “extension”, in which case ‘*Weite*’ would mean a collection of multiple individuals falling under a concept. But a close reading of §230 as a whole suggests that Crusius is really talking about the quantity of the individuals falling under the subject of a judgment, which can be one or many. (also see Crusius 1747, §136)

²⁴ That being said, for the purpose of understanding Kant’s view of the *logical* feature of a singular judgment, whatever otherwise significant difference there is between “singular concept” and “concept of singular use” is negligible. For Kant, as well as for the other logicians, the characteristic logical feature of a singular judgment is that by its subject-concept is signified exactly *one* individual.

²⁵ Also see V-Lo/Warschauer, LV:609; V-Log/Wiener, Ak 24:908-9; V-Lo/Pölitz, Ak 24:567; Log, §1, Ak 9: 91.

²⁶ Like many of the previously mentioned logicians (Knutzen 1747, §142; Wolff 1740, §241), Kant uses both proper names and demonstrative pronouns—and the former more often than the latter—as indicators of the singularity of a judgment. And he often calls a proper name a “singular concept”. This need not be read as Kant contradicting himself. When Kant denies that there are singular *concepts*, he is really denying the possibility of determining an individual object by means of concepts *alone*. But when he calls a proper name a singular concept, his focus is on the singularity of what is being signified thereby: “the concept Caius is a singular concept, which does not grasp a multitude under itself, but is only a single thing[.] (V-Lo/Hechsel, LV: 426; cf. V-Lo/Wiener, Ak 24:931; V-Lo/Warschauer, LV: 625; V-Lo/Pölitz, Ak 24: 578; V-Lo/Busolt, Ak 24: 665; R3095, Ak 16:657) Understood in this sense, Kant can speak freely of singular concepts without contradicting his deeply held conviction that no individual can be determined by concepts alone.

As for in what sense Kant might regard a proper name as a *concept* of singular use, I am open to taking Thompson’s quasi-Russellian/Quinean approach: “A proper name represents an empirical concept used with an existence and a uniqueness claim and is hence eliminable in favor of a predicate expression. ‘Caius is mortal’ has the force of ‘The man who ... is mortal’ just as ‘This is F’ has the force of ‘The ... that is here is F’.” (Thompson 1972:334-5; cf. 334-5n.15) But it is important to point out a reservation that Kant would have about this appeal to Russellian definite description: it could be meant only to capture the logical feature of a singular judgment (i.e., *that* it always has a singular reference), but not as an account of what secures the singular reference.

I shall revisit these points in section IV.

²⁷ Also see R4676, Ak 17:653-6; R3920, Ak 17:344-5; R3921, Ak 17:345-6, R3933, Ak 17:353.

²⁸ The relevant notion of *Bedeutung* is contained in Kant’s following remark about a singular judgment at A71/B96: “it [i.e., the predicate of a singular judgment] applies to that concept [i.e., the logical subject of the judgment] without exception, just as if the latter were a general concept [...] to whose entire signification [*Bedeutung*] the predicate applied.” I share Guyer and Wood’s interpretation of ‘*Bedeutung*’ in this context: “here Kant uses *Bedeutung*, as Frege was later to use it, to mean the reference or denotation of a concept.” (Kant 1998: 207 n.b) Translating ‘*Bedeutung*’ as “signification” is appropriate, considering that, as we saw in 2.1, such logicians as Wolff already think that a singular term “signifies” (*significat*) an individual.

²⁹ V-Lo/Blomberg, Ak 24:257; V-Lo/Philippi, Ak 24:454; V-Lo/Wiener, Ak 24:911, 912; V-Lo/Hechsel, LV:399, 400; V-Lo/Warschauer, LV:613; V-Lo/Dohna, Ak 24:755; V-Lo/Bauch, LV:152; Log §7, Ak 9:96; R2902, Ak 16:567; R2881, Ak 16:557-8.

³⁰ In R3095 Kant suggests that we can think of a general or universal concept as a “plane [*Fläche*]”, and a singular one as a “point”, in respect of what is represented by each concept. (Ak 16:657) By a “plane” in such a context I take Kant to mean a bounded plane figure. (In Chapter 2, I explained that, when Kant suggests that the logical extension of a concept can be represented by a *circle*, he is really referring to the *space* or geometrical plane bounded within the circle.)

³¹ The point symbolism used in this sense differs significantly from the metaphor of “point”—which really means “standpoint”—that Kant uses in the *Critique* to characterize *every* concept (considered as an essentially general representation). There Kant says: “One can regard every concept as a point [*Punkt*], which, as the standpoint [*Standpunkt*] of an observer, has its horizon, i.e., a multiplicity of things [*Menge von Dingen*] that can be represented and surveyed, as it were, from it. Within this horizon a multiplicity of

points must be able to be given to infinity, each of which in turn has its narrower field of view; i.e., every species contains subspecies.” (A658/B686) These remarks may be read in light of the tree-like structure I used in Chapter 2 to exhibit the genus-species hierarchy of concepts. In that connection, each point or standpoint used to describe a concept (*qua* general representation) can be thought of as, more precisely, a node on the tree-like structure. (This image actually fits well with Kant’s use, a few paragraphs later, of the metaphor of various “branches” (*Zweige*) extending from the same “root” (*Stamm*). (A660/B688))

³² For a discussion of the use of ‘this’ as logical quantifier (as opposed to determiner), see Gumański 1960.

³³ This can be understood in connection with Crusius’ claim that the *totum logicum* of a general concept—which is equivalent to the Kantian logical extension—is of *unendliche Weite*, where *Weite* is “the quantity of the individuals that are grasped under the subject”. (see 2.1) In other words, the *totum logicum* of a general concept is of infinite quantity, because such a concept comprehends infinitely many individuals. The individuals are here understood as *possibilia* as opposed to *actualia*, a contrast that is suggested by the way Crusius compares a universal judgment that has an infinite *Weite* with one that has a finite *Weite*. As he puts it, a universal judgment has an infinite *Weite* if it is about possible individuals: e.g., “All bodies are composite.” By contrast, a universal judgment has a finite *Weite* if it is only about actual individuals: e.g., “All the planets in our heavens are smaller than the sun.” (Crusius 1747, §231)

³⁴ In *Metaphysik K₂*, Kant explicitly connects *Einheit* with *conceptus singularis* for the reason that it applies “to only one thing” (Ak 28:714-15).

³⁵ The same claim is repeated in the *Jäsche-Logik*: in the singular judgment “a concept that has no sphere at all is enclosed [...] under the sphere of another.” (Log, §21, Ak 9:102)

³⁶ Note that Kant is *not* claiming that the subject-concept of a singular judgment has an *empty* extension. That is why the second clause is needed, which specifies the necessary condition for a concept to *have extension* at all.

³⁷ See Log, §11n., Ak 9:97. Most early modern logicians took there to be *species infima*, as a *general* concept that represents the *immediate* similarity among individuals and that has under itself no other general concepts. (see, for instance, *PRL*, i.7, p.41; Wolff 1740, §§44-7; Knutzen 1747, §65; *Auszug*, §261) Kant denies *species infima* in this sense out of the consideration that, for any given concept, it is always theoretically *possible* to find concepts that are contained under it. As it is put in the *Jäsche-Logik*, “in the series of species and genera there is no lowest concept or lowest species, under which no other would be contained, [...]. For even if we have a concept that we apply immediately to individuals, there *can* still be specific differences in regard to it, which we either do not note, or which we disregard.” (Log, §11n., Ak 9:97, emphasis added; see also V-Lo/Warschauer, LV:612-13; V-Lo/Hechsel, LV:399; V-Lo/Pöhlitz, Ak 24:569; V-Lo/Dohna, Ak 24:755)

³⁸ In section IV, I shall discuss a significant difference between two ways in which Kant characterizes “singular concept”, so much so that he can deny there to be singular concept under one characterization, but freely admit it under the other.

³⁹ In Kant’s two early lectures, the *Logik Philippi* and the *Logik Blomberg*, a singular concept is sometimes presented as a lowest concept (*conceptus infimus*) and is explicitly distinguished from a *species infima*. (V-Lo/Blomberg, Ak 24:259; V-Lo/Philippi, Ak 24:454) In the *Jäsche-Logik*, however, when the expression ‘*conceptus infimus*’ is used, it is used interchangeably with ‘*species infima*’. (see Log, §11n., Ak 9:97)

⁴⁰ It is not clear, admittedly, that all early modern logicians were committed to such a restriction. In the Port-Royal Logic, although the definition of extension is given explicitly for general ideas (after they have been distinguished from singular ones), singular ideas are also said to be “always taken in their entire *extension*”, (*PRL*, i.8, p.45, emphasis added) and a proposition with a singular idea as its subject is treated as a universal one precisely because its subject “is necessarily taken through its entire *extension*.” (ibid. ii.3, p.84, emphasis added; see note 11 of my Chapter 2) In comparison, Lambert explicitly contrasts the extension attributed to a *universal concept* with an individual as without extension, and suggests that, figuratively, just as the extension of a concept can be represented by a *line*, so can an individual be represented by a *point*. So he says:

Every universal concept extends to all the individuals to which it applies. Therefore it has a certain *extension*. If one represents all these individuals in a *sequel* or *line*, then the length of this line will represent figuratively the *extension* of the universal concept. (*NO*, §174)

As we give an extension to the universal concept because it extends to many individuals, so *an individual necessarily has no extension*, because it [...] cannot extend any further. Therefore it must be represented by means of a *point*, because a point likewise has no extension. (*NO*, §176, emphasis added)

In that connection, Kant's view that a singular concept—in respect of what it signifies—has no extension may very well have to do with his Lambert-style representation of the extension of a general concept by means of an *extended* plane-geometrical figure. As we have seen, Kant also uses a *point*—on account of its lack of extension—to represent an individual (as it is signified by a singular concept in judgment).

⁴¹ The use of '*ist*' should not be taken to show that Kant simply equates a concept with a thing. For Kant, as we have seen in Chapter 2, a concept is always a *representation* of a thing.

⁴² See also V-Lo/Hechsel, LV:426-7; V-Lo/Warschauer, LV:625; V-Lo/Pölitz, Ak 24:578; V-Lo/Busolt, Ak 24:665; Log, §21*n.1*, Ak 9:102. Noticeably, the passage quoted from the *Wiener Logik* ends with this qualifying statement: "There is a distinction here nevertheless, which one must look up, although in the formal use the two can go together."

⁴³ Generally speaking, judgments are used with respect to each other in "inferences". Kant distinguishes two kinds of such inferences: inferences of understanding and inferences of reason (syllogisms). The former are immediate inferences from one judgment to another. By contrast, syllogisms are mediate inferences, in which a third judgment must be added in order to deduce one judgment from the other. (see Log, §44, Ak 9:115) At A71/B96-7 Kant apparently has only the syllogisms in mind, which is clear from the very first sentence of the paragraph: "Logicians are justified in saying that, *in the use of judgments in syllogisms*, singular judgments can be treated like the universal ones." (emphasis added)

⁴⁴ It is in this sense of primacy that I understand the sense in which Kant takes the logic in question to be "limited" to the use of a judgment in syllogisms. Kant himself sometimes discusses the primacy of syllogism in determining the basic forms of judgments. For instance, in *Logik Hechsel* he explains that, as far as the use of a judgment in syllogisms is concerned, one need consider it only with respect to three questions: *quae* (relation—whether it is categorical, hypothetical or disjunctive); *qualis* (quality—whether it is affirmative or negative); and *quanta* (quantity—whether it is universal or particular). And, if that is where "the quality and quantity originated", one would certainly take universal and particular judgments as the only two *basic* forms of quantity. (V-Lo/Hechsel, LV:427)

⁴⁵ Also see V-Met/Wrongovius, Ak 29:801-2 and V-Met/L₂, Ak 28:547.

⁴⁶ See V-Met/Vockmann, Ak 28:396; V-Met/Dohna, Ak 28:626; V-Met/Vigilantius, Ak 29:985-7; V-Met/Schön, Ak 28:480.

⁴⁷ See Frede and Krüger 1970 for a useful overview of the discussions.

⁴⁸ To be fair, Longuenesse need not make such a move, even though it is clearly suggested in her discussion. Longuenesse's main thesis is that <singular-unity, particular-plurality, universal-totality> is the correlation "we must consider *in order to understand the genesis and meaning* of the schemata and categories of quantity". (Longuenesse 1998:249, emphasis added) My argument against the move from (i) to (ii) leaves the integrity of such a thesis intact.

⁴⁹ I am assuming this order of categories only for the sake of argument.

⁵⁰ Here we may recall what we learned, in Chapter 2, from the Port-Royal Logic about how to *restrict* a general term in respect of its extension by means of the quantifier "some".

⁵¹ This sketch of a potential objection to my interpretation also captures the sense in which Hanna [1990] thinks that Kant's account of singular judgments at A71/B96-7 shows "the essentially paradoxical nature of the conceptual aspect of judgment" (Hanna 1990:347). The paradox, as Hanna sees it, is as follows. On the one hand, in Kant's view a concept that could pick out an existent individual would have to contain a complete description or thorough determination of that individual; but there are no such concepts; for only intuitions can supply us with thorough determination of actual individuals. On the other hand, Kant's account of singular judgments at A71/B96-7 suggests (on Hanna's reading) that "that in the singular judgment which is supposed to take us directly and necessarily to the individual is not an intuition, but rather only an empty concept." (ibid.) In addition, Hanna thinks that Kant's account also contradicts the supposition that the singular subject of a judgment acts as a "strictly proper name (that is, as a singular term that directly picks out an individual thing" in the semantic structure of the judgment. (ibid. 344) For, Hanna explains, if the subject-concept is an "empty concept", or a concept with "null extension", then "obviously it cannot act as a strictly proper name." (ibid. 347)

No doubt, these misgivings that Hanna has about Kant's account are partly to be blamed on his misreading of Kant's notion of "concept without extension" as our modern notion of "empty concept" (= a concept with *no object* falling under it). But, as we shall see more clearly soon, such misgivings have also betrayed a failure on Hanna's part to distinguish Kant's view about the semantic property of the subject-concept of a singular judgment from his view about the epistemic conditions for the possibility of referring—in a thoroughly determinate way—to an actual individual.

⁵² For a representative account and defense of such a view, see Hintikka 1969. For challenge to this view, see Thompson 1972.

⁵³ Cf. V-Lo/Wiener, Ak 24:931; V-Lo/Warschauer, LV:625; V-Lo/Pöhlitz, Ak 24:578; V-Lo/Busolt, Ak 24:665; R3095, Ak 16:657.

⁵⁴ Lycan 2008:52-3. Lycan complains that philosophers of language, Russell being one of them, often fail to note such an important distinction.

⁵⁵ To appreciate this separation of singular signification and existential import, we may consider Kant's view about the use of the concept *God* in judgments. (A598-9/B626-7)

Chapter 4

The Object and the Formal Truth of Kantian Analytic Judgments

A Kantian analytic judgment is one in which the predicate is contained in the subject-concept, e.g., “All bodies are extended.” (A6-7/B10-11)¹ The Marburg neo-Kantians interpret such judgments to be merely about concepts, against which interpretation Paton [1936] counters:

[AO] Kantian analytic judgments are about objects.

This proposition is, in my view, at least *prima facie* correct. Its contrary, namely the view that Kantian analytic judgments are only about concepts, would be a non-starter. On that view, an analytic judgment would be a second-order claim about a given concept regarding, say, its intensional content: e.g., “All bodies are extended” would *mean* “The concept *extension* is contained in the concept *body*.” No doubt, the intension-containment relation between the two given concepts can, as we shall see, serve as the ground of one’s knowing, say, the truth of an analytic judgment. But the judgment is not therefore *about* the given concepts. To borrow Geach’s perspective, although a Kantian analytic judgment, in view of its subject-predicate structure, is composed of two concepts, the predicate is not *said of* the subject-concept; rather, the predicate is attached to the subject-concept, but is predicated of what is signified by the latter (or what it stands for). (Geach 1950:462) In short, a Kantian analytic judgment, *qua* categorical judgment, is a first-order judgment *about* objects made *by means of* concepts.²

It is not immediately clear, however, that [AO] squares with other related views held by Kant. Paton thinks that [AO] is entailed by what he takes to be two commitments on Kant’s part:³

- (i) analytic judgments are true; and
- (ii) the truth of all judgments consists in agreement with objects.⁴

But not all are convinced by this argument in favor of [AO]. Rosenkoetter [2008], for instance, counters that [AO] is incompatible with (iii), the latter being a view that Kant seems also to hold.

(iii) Some analytic judgments (e.g., “A two-sided polygon is two-sided.”) have no objects with which they *could* agree.

Rosenkoetter recognizes that (i) and (ii) together entail [AO] but doubts that Kant is committed to both (i) and (ii) in the first place. He argues that Kant has no need for—though he sometimes appears to endorse—(i) as far as his critical projects are concerned. Thus Rosenkoetter recommends that we relieve Kant of (i) and, with it, [AO]. Tolley [2007], by contrast, grants (i) to Kant, but contends that [AO] would contradict Kant’s insight—and an important one for that matter—that analytic judgments tell us nothing positive about objects in the world. And so Tolley suggests that Kantian analytic judgments must be true in a different sense of truth than is specified in (ii): if (ii) captures Kant’s conception of *material* truth, analytic truths are merely logical or *formal* truths for Kant; accordingly, one does not have to assume [AO] in order to account for the truth of a Kantian analytic judgment. It especially seems more natural, given that Kant characterizes an analytic judgment primarily in terms of the intension-containment relation between two given concepts, to construe its truth in exactly the same terms, without having to invoke any reference to object whatsoever. Thus, Tolley takes Kantian analytic truths to be formal truths that consist in nothing other than the agreement between given concepts, on account of their intension-containment relation governed by what in his opinion are merely syntactic rules (specified in Kant’s formal logic).⁵

Rosenkoetter and Tolley have thus presented three main challenges to any commentator who is of the view that Kantian analytic judgments are both true and about objects. First, suppose Rosenkoetter’s observation about (iii) is at least *prima facie* correct, is there a cogent Kantian way to formulate [AO] so as to accommodate such cases as “A two-sided polygon is two-sided” to which no object possibly corresponds? Second, how could the conception that Kantian analytic judgments are true in terms of agreement with an object be reconciled with Kant’s view that such

judgments express no material truths about objects in the world? Third, given that Kant does characterize analytic judgments predominantly in intensional terms, why should he account for its truth in terms of agreement with an object rather than being content with the merely syntactic treatment of analytic truth that Tolley has ascribed to him? And, if Kant does need an account of analytic truth in terms of agreement with an object, how is the intension-containment relation still relevant (given that such a relation is characteristic of Kantian analyticity)? I shall address these challenges in the present chapter. In section I, I shall briefly examine the accounts of [AO] that can be found in Paton [1936] and in the more recent MacFarlane [2002], Allison [2004] and Heis [2007]. In doing so, I will borrow Rosenkoetter's critical perspective. Although, as I shall show, Rosenkoetter's criticisms can be easily answered, they will help bring to light the lack of clarity—or sometimes even lack of consistency—in those accounts as to what it means to say that Kantian analytic judgments are “about objects”. In section II, to close this exegetical gap regarding the sense in which Kantian analytic judgments are “about objects”, I shall submit two distinct theses: first, the “objectual purport thesis” that a Kantian analytic judgment, *qua* judgment, is about an object, albeit only an object in the sense of an object in general = x ; second, the “contentfulness thesis” that an analytic judgment has no Kantian content (*Inhalt*) unless the object represented by its subject-concept is a real possibility in Kant's sense. In light of the view I developed in Chapter 1 that Kantian formal logic treats thought as related to an object without assuming that the object can be given in intuition, I shall argue that the objectual purport thesis can be asserted on Kant's part independently of the contentfulness thesis. Thus we can handle the case of (iii) mentioned by Rosenkoetter as follows: insofar as Kant holds [AO] in the sense of the objectual purport thesis but independently of the contentfulness thesis, he can coherently say that a judgment like “A two-sided polygon is two-sided” has objectual purport and yet corresponds to no object that can be given in our sensible intuition (i.e., is devoid of Kantian content). And, in response to Tolley, we can say that taking Kantian analytic judgments to have objectual purport is compatible with denying that they convey any material truths about objects in the world. Finally,

I shall argue in section III that the truth (= agreement with an object) of a Kantian analytic judgment presupposes no more and no less than the objectual purport in the above sense. In doing so, I shall use the geometrical figures introduced in Chapter 2 by means of which to exhibit the logical relation between concepts in respect of extension. I shall also use the model developed in that chapter by which was illustrated a certain reciprocity—or inverse proportionality in respect of quantity—between intensional and extensional relations of concepts to show that knowing the truth (= agreement with an object) of a Kantian analytic judgment nevertheless requires no more than knowing the intension-containment relation between two given concepts. For this reason, I shall argue, analytic truths are indeed formal—not, however, in Tolley’s modern sense of formality (in syntactical terms), but in the sense of Kantian formality that I explained in Chapter 1 (especially what was called Formal_R).⁶ In these terms, I shall sharply distinguish what makes analytic judgments true from what may render them contentful, a distinction that will help capture an important role that such judgments can be expected to play in Kant’s critique of a certain Leibnizian illusion.

I. Rosenkoetter vs. Paton et al.: what does it mean to say that Kantian analytic judgments are “about objects”?

Against the Marburg neo-Kantians’ interpretation that Kantian analytic judgments are about concepts alone,⁷ Paton argues:

The analytic judgment, although it takes place by means of analysis of the subject-concept, is not a judgment about the concept, but about the objects which are supposed to fall under the concept. (Paton 1936:84)

This thesis—that Kantian analytic judgments are about objects—plainly follows, Paton believes, from the fact “that an analytic judgment can be true [... and that] for Kant truth is always correspondence with an object.” (ibid. 214*n*.3) For Paton, this means that the truth of an analytic judgment “depends on the supposition that there is an object corresponding to the subject-concept.” (ibid.) Note that the reference to an object is merely “supposed”. Accordingly, the thesis that analytic judgments are about objects can, as Paton construes it, be specified as follows.

[AO₁] Analytic judgments are about objects, in that their subject-concept is supposed to refer to objects.

The qualification “supposed” is not trivial for Paton. It is meant to accommodate cases in which no object corresponds to the subject-concept. (ibid. 84) But what does it mean to say that an analytic judgment has “supposed” reference to an object in the first place? Paton describes this supposed reference in terms of the judgment-makers’ intention: “Some analytic judgments of metaphysics may have no object, but their authors *intend* them to refer to an object.” (ibid, emphasis added) Unfortunately, he says little about how to interpret ‘intend’ in such a context: is it to be taken normatively (say, as a necessary condition for making genuine judgments), or merely descriptively (say, as a mere psychological fact)? Nor is it clear what the purported “reference” to objects amounts to: does it mean that an analytic judgment has existential import? If not, what else does it mean?

These are the questions that Rosenkoetter wonders about in his assessment of Paton’s version of [AO]. By Rosenkoetter’s analysis, Paton’s account of the sense in which analytic judgments are “about objects” is problematic for two reasons.

[1] Yet it is beside the point what people *commonly* intend when they are judging. [...] On the other hand, [2] insofar as a judgment’s author is judging [...] that it has an object, this judgment will be false if there is no such object, since in this case no object *corresponds* to the judgment. (Rosenkoetter 2008:198)

These criticisms hinge, of course, on Rosenkoetter’s own reading of what Paton takes to be analytic judgments’ supposed reference to objects. On this reading, every analytic judgment has “supposed” reference to objects insofar as the judgment-makers *commonly intend*, as a psychological fact, the subject-concept to refer to objects (hence criticism [1]);⁸ and the supposed “reference” boils down to the existential import of each judgment (hence criticism [2]).⁹ This is a rather uncharitable interpretation of Paton’s [AO₁], however. An informed and charitable-spirited commentator like Paton would not seriously attribute [AO₁], as interpreted, to Kant himself. For one thing, given the anti-psychologistic tendency of his critical theory of judgment,¹⁰ Kant certainly would not appeal to what people do or do not intend—psychologically speaking—when

making judgments in order to settle the question of whether or in what sense analytic judgments are about objects. For another, as we shall see more clearly in sections II and III, when Kant talks about analytic judgments, he never treats them as having existential import in the sense of implying that there are actual objects falling under the subject-concept. That being said, however, Paton does owe us an explanation of the properly Kantian sense in which analytic judgments have “supposed reference to” objects.

In MacFarlane 2002 we can see the outline of a clearly non-psychologistic account of what it means for a Kantian analytic judgment to have supposed reference to an object or “objectual purport”.¹¹ By MacFarlane’s account, objectual purport is ascribed to a Kantian analytic judgment *qua judgment*. In a nutshell, a judgment is by its very nature about an object (A68-9/B93-4); an analytic judgment is a judgment first and foremost; therefore,

analytic judgments are still judgments about objects, not concepts (cf. Paton 1936, 214 n. 3).
Without “relation to an object” they would not be judgments at all. (MacFarlane 2002:51n.38)

So it is simply in virtue of being judgment that an analytic judgment has objectual purport, i.e., that its concepts have “relation to an object”. (ibid. 51) But in what sense of “object”? After all, we can find at least these four conceptions of “object” in the *Critique*:

[Object₁] “the concept of an object in general (taken problematically, leaving undecided whether it is something or nothing).” (A290/B346)

[Object₂] the concept of a “transcendental object = X”, which is “nothing other than the formal unity of the consciousness in the synthesis of the manifold of the representations.” (A105; A109)

[Object₃] “the concept of pure, merely intelligible objects [... with respect to which] one cannot think up any way in which they could be given.” (A258/B314)

[Object₄] objects of experience, which can only be jointly determined by our faculties of understanding (concepts) and sensibility (intuition). (A258/B314)

However these four conceptions of “object” may or may not be related to one another, a curious reader would press on to ask: which of these is the relevant sense in which Kantian analytic judgments must have objectual purport in order to qualify as judgments?

Different commentators have answered this question differently. According to Allison [2004], to begin with, [Object₁] is the relevant sense of “object” in which to understand the objectual purport of Kantian analytic judgments. Evidence for this interpretation can, in Allison’s view, be found in Kant’s following statement in the *Logik*.

An example of an *analytic* proposition is, To everything *x*, to which the concept of body (*a* + *b*) belongs, belongs also *extension* (*b*). (Log, §36n.1, Ak 9:111)

Articulated here is the “basic schema”, as Allison calls it, shared by all Kantian analytic judgments. (Allison 2004:91) The presence of *x* in the schema suggests to Allison that Kant takes analytic judgments to be about objects generally construed, including non-existent or even impossible ones:

It [i.e., the schema] shows that in such judgments the predicate (*b*) is related to the object *x* by virtue of the fact that it is already contained (as a mark) in the concept of the subject. Analytic judgments are, therefore, “about” an object, [...] it is possible to form analytic judgments about nonexistent, even impossible objects[.] (ibid. 91-2; see also Allison 1985:34)

In this characterization of the objectual purport of analytic judgments, “object” seems to be taken in the sense of [Object₁] introduced above: “an object in general (taken problematically, leaving undecided whether it is something or nothing).” Accordingly, Allison’s version of the thesis that Kantian analytic judgments are about objects may be articulated as follows:

[AO₂] analytic judgments are about objects, in that their subject-concept is supposed to refer to objects in general (be they something or nothing, existent or non-existent, possible or impossible).

But the reference to objects in general, including non-existent or even impossible ones, carries a risk, as is about to be exposed in Rosenkoetter’s discussion of [AO₂].¹²

Rosenkoetter reads [AO₂] as an attempt, on the part of those committed both to the truth of Kantian analytic judgments and to the conception of truth as agreement with an object, to accommodate such cases as “A two-sided polygon is two-sided.” In Kant’s view, a two-sided polygon is a logical possibility, since there is no logical contradiction in its concept; but it is a real impossibility, in that the concept *two-sided polygon* can never be related to any object of experience.¹³ According to [AO₂], “A two-sided polygon is two-sided” is still about an object,

albeit a logically-possible but really-*impossible* one. As Rosenkoetter sees it, however, this way of handling the case amounts to “imputing to Kant the view that there *are* objects such as the two-sided polygon” (Rosenkoetter 2008:192) and thus committing him to an ontology that admits really-*impossible* objects as much as it does really-possible ones (such as triangles). Rosenkoetter sees this as an inflation of Kant’s ontology that compromises “his own hard-won notion of objectivity”, according to which for something to be an object “presupposes our forms of sensibility [which underlie the real possibility of an object]”. (ibid. 193; 194) This compromise would certainly be an undesirable consequence for [AO₂] to have. But, again, Rosenkoetter’s way of reading [AO₂] is quite uncharitable in the first place: one need not interpret the reference to “objects in general” as a naïve ontological commitment when one can, as we shall see, find ample Kantian conceptual resources to construe it otherwise.

Now let’s consider a second way of spelling out the relevant notion of “object” involved in [AO], which can be found in Heis 2007 as well as in MacFarlane 2002. Like Paton, Heis pits [AO] against the Marburg neo-Kantians’ view that Kantian analytic judgments are only about concepts. Unlike Paton, however, he characterizes the objectual purport of Kantian analytic judgments explicitly in terms of *objects of experience*:

[AO₃] analytic judgments are about objects, in that their subject-concept is supposed to refer to objects of experience.

The opposite view that Kantian analytic judgments “do not concern objects of experience”, Heis argues, “does not sit well with Kant’s characterization of analytic judgment at [*Logik*, §36, note 1,]¹⁴ nor with Kant’s deeply-held view that all cognitions must be related to objects if they are to be contentful at all.” (Heis 2007:191-2n.67) Kant is no doubt of the view that all cognitions—concepts and judgments included—must be somehow related to *objects of experience* in order to have content (*Inhalt*).¹⁵ It is probably this view that has convinced Heis of [AO₃], since Kant’s characterization of analytic judgment at *Logik* (§36n.1), which Allison has cited in favor of [AO₂] instead, is neutral about how the object that an analytic judgment is about should be construed—

much less recommending that it be considered as object of experience. Similarly, MacFarlane's remark that a Kantian analytic judgment, *qua* judgment, must be about an object as opposed to concept is premised on what he takes to be Kant's view on the essence of a judgment *as cognition* properly so called:

judgment is *essentially* "the mediate cognition of an object, hence the representation of a representation of it" (A68/B93). The subject concept in every judgment must relate finally to a representation that is "related immediately to the object" (A68/B93)—that is, to a singular representation, or intuition. Otherwise, there would be nothing—*no thing(s)*—for the putative judgment to be *about*, and it would not be a cognition at all [. . .]¹⁶ On Kant's view, then, there can be no such thing as a judgment about concepts themselves: the objective purport of judgment gets spelled out in terms of the *relation of concepts to an object or objects*. (MacFarlane 2002:50-1)

In precisely these terms MacFarlane submits that Kantian analytic judgments, *qua* judgments, "are no exception" in being about objects as opposed to concepts. (MacFarlane 2002:51n.38) It is clear, then, MacFarlane shares Heis' [AO₃]: to say that a Kantian analytic judgment is about an object or has objectual purport is to say that its subject-concept has "relation [. . .] to an object"; and insofar as such a relation is, as MacFarlane sees it, none other than a relation "to a singular representation, or intuition", the objectual purport in question ultimately consists in the relation of the subject-concept to an *object of experience*.

For Heis and MacFarlane, [AO₃] presumably indicates a constraint on how to *interpret* the schema of an analytic judgment presented in the *Logik*: only concepts that have relation to objects of experience can substitute the letters *a*, *b*, etc. in the schema; for otherwise we would not have an analytic *judgment*. On this reading, "A two-sided polygon is two-sided" is not really a Kantian analytic judgment, to the extent that its subject-concept is not related to an object of experience. But this restrictive reading of the schema in question is not obviously called for. Granted: Kant is of the view that a judgment has content only if it has relation to an object of experience. Nevertheless, such a view does not license [AO₃], but only the proposition that, for any Kantian analytic judgment, *if it is to be contentful*, its subject-concept must be somehow related to an object of experience or the object signified by such a concept must at least be a real

possibility in Kant's sense, namely something that can be given or exhibited in our sensible intuition. Meanwhile, as it turns out, the characterization of analytic judgments in the *Logik* shows not only the sort of relation between two given concepts that makes a judgment analytic, but also the basic schema of a *judgment* shared by synthetic *and* analytic judgments alike: x , which is thought through the subject-concept a , is also thought through the predicate b . With the presence of x , this schema remains a prime indicator of the objectual purport of Kantian analytic judgments. I shall expound on these points and their connection in section II.

Before we move on, let's take stock. We have examined, borrowing Rosenkoetter's critical perspective, various attempts to spell out the view that Kantian analytic judgments have objectual purport ([AO]). Although Rosenkoetter's criticism of these attempts has been shown to be flawed,¹⁷ it has succeeded in revealing both the difficulty of fully articulating [AO] in a coherent Kantian way and the failure on the part of each of the surveyed commentators to make full use of Kant's resources. All has not been lost, however, for a believer in [AO]. Our critical analysis of the accounts [AO₁] through [AO₃], as well as of the reason(s) cited for each account, has brought to light certain undesirable consequences of some readings of [AO]. At the very least, we have learned that the objectual purport of a Kantian analytic judgment had better not be understood in terms of the existential import of its subject-concept, nor in terms of what people commonly intend, psychologically speaking, when making the judgment. More importantly, we have teased out—and seen the possibility of separating—two sources that appear to be directly relevant to addressing the issue regarding the sense in which Kantian analytic judgments are (or are not) about objects: one is the schema of analytic judgment presented in the *Logik*, the x of which at least *prima facie* signals a certain objectual purport; the other is Kant's view that a judgment has content only if its subject-concept has relation to an object (through our sensible intuition). And, finally, we have singled out two relevant notions of object, namely *object in general* and *object of experience*, in terms of which to understand references to “object” in those

two sources. Our next task is to utilize these findings and develop a cogent account of the thesis that Kantian analytic judgments are “about objects”.

II. Objectual purport vs. contentfulness of Kantian analytic judgments

My reading of the proposition that Kantian analytic judgments are about objects includes two sub-theses. First is the “objectual purport thesis” that an analytic judgment, *qua* judgment, is about an object, albeit only object in the sense of “object in general = x ”. Second is the “contentfulness thesis” that an analytic judgment is not contentful unless the object represented by its subject-concept is a real possibility in the Kantian sense (i.e., is something that can be given or exhibited in our sensible intuition). The basics of these two theses and their distinction are already suggested in Beck’s characterization of a Kantian judgment as that in which two concepts are “referred to the same object” (and “this common object is called by Kant X ”),¹⁸ but for which an object “may not be given at all”. (Beck 1956b:169) For Beck, this general conception of judgment applies to analytic as well as synthetic judgments, the former being the ones in which “‘ X is A ’ implies logically ‘ X is B ’”. (ibid. 170) But no explanation is given on Beck’s part about how exactly to understand the object = X involved in the analytic judgments—either as that which endows them with objectual purport or as that which may make them contentful.¹⁹ To fill in the gap, I shall flesh out the objectual purport and the contentfulness theses by explaining three key Kantian notions involved therein: an object in general = x , real as opposed to logical possibility of an object, the contentfulness of a concept.

Let’s begin by recalling the schema of analytic judgment in the *Logik*: “To every x , to which belongs the concept c ($a + b$), belongs also the concept a .” This schema, it is to be noted, is compared with that of synthetic judgment: “To every x , to which belongs the concept d ($a + b$), belongs also the concept c .”²⁰ These schemata show how analytic and synthetic judgments differ in respect of the intensional relation between two given concepts. But they also show that the two sorts of judgments share the same logical form *qua* (universal affirmative) judgments: to every x , to which belongs a , belongs also b . The natural way to interpret the x involved in this common

form of judgment is that it stands for an object. This is so especially if we read the two schemata in light of the following passage that we already encountered in Chapter 3:

an object is only a something in general [*ein Etwas überhaupt*] [*x*] that we think through certain predicates that constitute its concept. In every judgment, accordingly, there are two predicates that we compare with one another, of which one [*a*], which comprises the given cognition of the object, is the logical subject, and the other [*b*], which is to be compared with the first, is called the logical predicate. [...] Now *a* as well as *b* belongs to *x*. Only in a different way: either *b* already lies in that which constitutes the concept *a*, [...] or *b* belongs to *x* without being contained and comprised in *a*. (R4634, Ak 17:616)²¹

This passage begins with an explanation of the basic schema of a judgment as such: to *x*, to which belongs *a*, belongs *b* (“*x-a-b* schema” for short).²² Kant uses “A body is divisible”, which turns out to be analytic, to illustrate such a schema: *x*, which is thought under the concept of body, is also thought under the concept of divisibility. (ibid.) This suggests that the feature of an analytic judgment *qua* judgment has priority over, or can at least be treated independently of, its analyticity: an analytic judgment is first and foremost viewed as a judgment and, as such, as possessing the *x-a-b* schema, regardless of how *a* and *b* relate to each other in respect of intension.²³ And it is presumably on account of having the basic *x-a-b* schema *qua* judgment that an analytic judgment is considered to have objectual purport. In that case, for a judgment to have objectual purport is but for it to be about an object in the extremely thin sense, as Kant himself puts it, of “a something in general”. For the reason that, as we shall see next, Kant uses the term ‘something’ also in a more restricted sense (to signify that which is contrary to “nothing”), I shall refer to the thinnest sense of object simply as “object in general = *x*”.²⁴

This concept of an object in general = *x* is, according to Kant, the highest and most general concept of thinking.

The highest concept of the whole human cognition is the concept of an object in general, not of a thing and non-thing, or of something possible and impossible, for these are opposites. Each concept that has an opposite always requires a yet higher concept that contains this division. (V-Met/L₂, Ak 28:543)²⁵

So construed, object is “[w]hat is possible or impossible [...] for object can be thought through impossible predicates.” (V-Met/Mrongovius, Ak 29:811) Accordingly, an object can be “nothing”

or “something”.²⁶ This echoes Kant’s view in the *Critique*: “since every division presupposes a concept that is to be divided, a still higher one must be given, and this is the concept of an object in general (taken problematically, leaving undecided whether it is something or nothing).” (A290/B346) In light of Kant’s account of “problematic” as a modality of judgment, to take a concept problematically is presumably to treat it as “a merely arbitrary admission [*willkürliche Aufnahme*] [...] in the understanding.” (A75/B100-1) Just as taking a judgment problematically leaves it open whether it is true or false, (ibid.) so does taking the concept of an object in general problematically leave it open whether the object is something or nothing, possible or impossible.²⁷ This conception of an object in general = x and of its relation to the possible/something vs. the impossible/nothing is not, contrary to what Rosenkoetter has thought, a naïve ontological division of a given domain of objects into the possible/things and the impossible/non-things. Rather, the notions of possible vs. impossible and something vs. nothing pick out various *modalities* of the object represented by a concept and, as such, “express only the relation to the faculty of cognition” (A219/B266) and concern merely different ways of positing an object.²⁸ Here, instead of starting with objects existing independently of our thinking, Kant takes our thinking to begin with a bare notion of an object = x —which we think through various concepts—and asks about the conditions for it to be represented as a something.²⁹

The relevant conditions come in various sorts. To prepare for our subsequent discussion of the objectual purport vs. the contentfulness of Kantian analytic judgments, let’s briefly consider two sorts of such conditions, the conditions of logical possibility and real possibility, respectively. Whether these conditions are satisfied or failed determines whether the object represented by a certain concept is something or nothing and whether its concept has or lacks content in the Kantian sense.

(I) There are necessary laws for the use of understanding and reason in general. Among these laws is the Principle of Contradiction, according to which no two contradictory concepts can be predicated of the same thing. By this principle is determined the logical possibility or impossibility of an object represented through a concept. Insofar as a concept is free of inner contradiction (that is, it contains no contradicting components), the object

represented thereby is logically possible, hence is a *logical something*. Otherwise, if the concept is self-contradictory, the object thought thereby is logically impossible, hence a *logical nothing*: e.g., a four-cornered circle.³⁰ A concept free of inner contradiction has “logical signification [*logische Bedeutung*]” on that account.(A219/B267)

(II) There are conditions that determine the *real possibility* of an object, conditions that are spelled out in the “postulate of possibility”.(A219-24/B266-72) According to the postulate, “the [real] possibility of things [...] requires that their concept agree with the formal conditions of an experience in general.”(A220/B267)³¹ If a concept fails these conditions (without being self-contradictory), the object represented thereby is a *real impossibility*. A real impossibility in this sense is also called a “nothing”: “the object of a concept to which no givable intuition [*anzugebende Anschauung*] corresponds is = nothing.” (A290/B347)³² A concept would have no content (*Inhalt*) or sense (*Sinn*) without “the possibility of giving it an object [through empirical intuition] to which it may be related.” (A239/B298; A240/B299) A concept has content if it is indeed related to givable intuition.³³

Note that the logical and real possibilities of an object can come apart. Accordingly, a concept may have logical *Bedeutung* without being contentful. Kant illustrates this point by the example of the concept of a figure that is enclosed between two straight lines. The object represented by such a concept is a logical something, since the concept contains no contradiction: “the concepts of two straight lines and their intersection contain no negation of a figure.” Yet such an object is a real impossibility, in that its concept cannot be constructed in our pure intuition of space, intuition that in turn “[contains] *a priori* the form of experience in general.” (A220-1/B268)³⁴

Having thus explained the three relevant Kantian notions (i.e., the notions of an object in general = *x*, of real as opposed to logical possibility of an object and of a concept’s contentfulness), we can now fill out the two theses about Kantian analytic judgments sketched at the outset of this section. On the objectual purport thesis, a Kantian analytic judgment is about an object to the extent that, in respect of its logical form *qua* judgment (as is presented in the *Logik*), it is always a thought of an object in general = *x* by two concepts. On the contentfulness thesis, however, a Kantian analytic judgment has no content unless the purported object is also a real possibility—namely, unless such an object can be given or exhibited in our intuition. Although a judgment must have objectual purport in order to be contentful, it may have objectual purport without being contentful. In these terms, we can answer some of the challenges that have been raised above on the part of Rosenkoetter and Tolley to the thesis that Kantian analytic judgments

are about objects. First, if Rosenkoetter is correct in saying that there are cases of Kantian analytic judgments the subject-concept of which corresponds to no object, he is correct only in the sense that the subject-concept of such judgments does not represent a *really possible* object in Kant's sense, from which it does not follow that the judgments are not about objects *whatsoever*. At most, "A two-sided polygon is two-sided" is an analytic judgment without Kantian content (to the extent that the object represented as "two-sided polygon" is not really possible),³⁵ which nevertheless has as much objectual purport as, say, "A body is extended" does. Second, as regards Tolley's observation that, for Kant, by analytic judgments no claims are made about objects in world, it is important to clarify the sense in which Kant holds such a view: it is not, contrary to what Tolley has suggested, that analytic judgments are not claims about objects *at all*; it is rather that the objects that analytic judgments are about need only be taken in the sense of bare object in general = x , regardless of whether they can also be given or exhibited in intuition.

But some of the challenges mentioned remain unanswered by my account of objectual purport. To begin with, recall that Paton takes the view that Kantian analytic judgments are about objects to be entailed by Kant's alleged commitments (i) that analytic judgments are true and (ii) that the truth of all judgments consists in agreement with an object. And Rosenkoetter means to use the case of "A two-sided polygon is two-sided" to draw out the suspicion that Kant may not be truly committed to both (i) and (ii) after all. Assuming that "A two-sided polygon is two-sided" is a clear instance of Kantian analytic judgment which nevertheless has no object to correspond to, Rosenkoetter thinks that one of (i) and (ii) must go, since their combination entails that the judgment in question does correspond to some object. I have tried to break down this line of argument by separating issues about the objectual purport and the contentfulness, respectively, of any given judgment, to the extent that the conjunction of (i) and (ii) entails that a Kantian analytic judgment has objectual purport only in the sense of being about an object in general = x , allowing that the judgment may turn out not to be contentful in the Kantian sense (for failing to relate to any object of experience). Rosenkoetter might press on, however, asking whether this

notion of object = x is good enough for Kant to explain the *truth* of an analytic judgment as agreement with an object. And Tolley, assuming that Kant can account for analytic truth simply in terms of the intension-containment relation between concepts together with certain syntactical rules of logic, would press for explanations as to why Kant should need an account of analytic truth in terms of agreement with an object in the first place. I shall address these issues in section III. To anticipate, I shall show that the truth of a Kantian analytic judgment is formal but still consists in agreement with an object, by using my reading in Chapter 2 of Kant's notion of logical extension and of the reciprocity between the intensional and extensional relations of concepts. And I shall argue that this account of the formal truth of Kantian analytic judgments—together with my distinction above between the objectual purport and the contentfulness of a judgment—better captures the role they are expected to play in Kant's critique of Leibnizian metaphysics.

III. Objectual purport and formal truth of Kantian analytic judgments

Kant grants the “nominal definition” of truth as “the agreement of cognition with its object”. (A58/B82) This definition is nominal presumably in the sense that, as Vanzo [2010] puts it, it explains what it *means* to say that a cognition “is true”: to say, for instance, that a categorical judgment is true is to say that its predicate agrees with the object it is about. (Vanzo 2010:161-3)³⁶ So construed, this definition is silent about what sort of object the predicate of a specific judgment must agree with—or, for that matter, about the nature of the agreement—in order for the judgment to be true. In Kant's words, the definition does not provide the “criterion of truth” that allows us to determine, for any given judgment, whether it is in fact true or not. If by “criterion” is meant a set of necessary *and* sufficient conditions (a negative *and* a positive criterion), Kant denies there to be one that applies to all judgments. (A58-9/B83) For analytic and synthetic judgments have “entirely different” principles as their positive criteria of truth. (V-Met/Wrongvius, Ak 29:788-9) Especially, while the Principle of Contradiction can serve as the positive (sufficient) criterion of truth for analytic judgments, it is merely the negative (necessary) criterion of truth for all judgments (in that whatever violates the principle is false). (ibid. Ak

29:789-91) Now we are interested to see whether the truth of a Kantian analytic judgment, as it is supposed to be sufficiently determined by the Principle of Contradiction, presupposes no more and no less than the objectual purport in the sense expounded in section II.

To begin with, the Principle of Contradiction (as Kant construes it) says that no predicate that contradicts the concept of a thing can be ascribed to the thing.³⁷ In other words, the principle says: for anything x , which is thought by means of a given concept, any predicate that contradicts the given concept must be denied of x . This formula fits well with the $x-a-b$ schema of judgment mentioned in section II, where a and b are two concepts by which an object in general = x is thought. A judgment, so construed, is true just in case the predicate b is correctly ascribed to the object represented through the subject-concept a . For Kant, of course, there are essentially different *epistemic grounds* on which to determine such ascription as correct for an analytic judgment versus for a synthetic one: in the former case, b is correctly ascribed to the object simply on account of its analytic relation with the concept a and in accordance with the Principle of Contradiction; in the latter case, the ascription must be somehow grounded in the object itself.³⁸ But each judgment, be it analytic or synthetic, is still *true* in the same sense of truth, i.e., in that its predicate somehow agrees with the object signified by the subject-concept. Otherwise put, we must distinguish the sense in which an analytic judgment, as *judgment*, is true from the epistemic ground on which to determine its truth as *analytic* judgment. This distinction is suggested in Kant's following account of how the Principle of Contradiction serves as the sufficient principle by which we can "cognize" the "truth" of an analytic judgment:

one can also make a positive use of [the Principle of Contradiction ...] to *cognize* truth. For, *if the judgment is analytic*, [...] its truth must always be able to be *cognized sufficiently* in accordance with the Principle of Contradiction. For the contrary of that which as a concept already lies and is thought in the cognition of the *object* is always correctly denied, while the concept itself must necessarily be affirmed, of it [viz., of the *object*],³⁹ since the opposite would *contradict the object*.. (A151/B190-1, emphasis added)

Two distinct points can be extracted from this statement. The first is a point about the sense in which a judgment is true: an affirmative judgment is true just in case the predicate agrees with (or

does not contradict) the object thought under the subject-concept. The second is a point about the ground on which we can *know* such truth when the judgment is *analytic*: the truth (in the above sense) of an analytic judgment can be known solely on the basis of the intension-containment relation between the relevant concepts in accordance with the Principle of Contradiction.⁴⁰

Unless there is a strong reason not to take Kant at his word in the above statement, it seems to be the *default* way of presenting his view to say that he takes the truth of an analytic judgment—just as he takes the truth of any other judgment that has the *x-a-b* schema—to lie in the agreement of its predicate with the object it is about. But Kant has also suggested that to know analytic truths we need not go beyond our *concepts* (together with the Principle of Contradiction). How, then, is the notion of object still relevant in accounting for the truth of any given analytic judgment? In what sense is analytic truth still to be understood in terms of agreement with an object? And how would our understanding represent to itself the object—without having to go beyond its concepts and logical principles—with which the predicate of an (affirmative) analytic judgment must agree in order for the judgment to be true? To answer these questions, we may recall two things noted in Chapter 2. First, we saw that Kant uses a geometrical figure to represent a concept in respect of its logical signification—e.g., its logical extension, which consists of all the possible objects the concept represents simply in virtue of being a universal representation, regardless of whether there is indeed any object actually falling under it. Second, I developed a model to illustrate Kant’s logical principle regarding the reciprocity between the intensional quantity and the extensional quantity that one concept has relative to another, and to show how the latter quantity may be entirely determined by the former.

To elaborate, let’s begin with the following figure that Kant uses to represent judgments of the form “Every *x*, which is *b*, is *a*.”

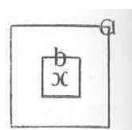


Fig. 17 (R3096, Ak 16:658; Log. §29n, Ak 9:108)⁴¹

Here, both a and b are represented with regard of their form *qua* general representation or “representation common to several objects (*conceptus communis*)”. (Log, §5n.1, Ak 9:94) In that regard, each concept has a logical extension which consists of the multitude of possible objects represented by it and which can be represented by a square (just as it was represented by a circle in Chapter 2). Based on what we learned in Chapter 2, then, logical relations between a and b can—insofar as each is used in its capacity as general representation—be exhibited through certain ways of arranging two squares in relation to each other. In this fashion, *Fig.17* represents the following logical relation between a and b : every x , which falls in the extension of the subject-concept b , falls also in the extension of the predicate a . Such a logical relation holds for every universal affirmative judgment, be it analytic (e.g., “Every body is extended.”) or synthetic (e.g., “Every body is heavy.”). And the variable x , as it figures in the logical form shared by all universal affirmative judgments, stands simply for *that which can be thought by means of concepts*, regardless of whether it is also an object that can be given in intuition.

In light of *Fig.17*, we may recast the conception of truth as agreement with an object, roughly as follows: for any given judgment of the form “Every x , which is b , is a ”, it is true just in case whatever object falls in the extension of the subject-concept is *correctly placed* in the extension of the predicate (which means that the predicate is correctly affirmed of and hence agrees with the object). Now, if a judgment of such a form is also analytic in Kant’s sense, we can see that placing any object that falls in the extension of the subject-concept in that of the predicate is necessarily correct, simply on the basis that the predicate is analytically contained in the subject-concept as part of its intension. To illustrate, let’s recall the figures presented in Chapter 2:

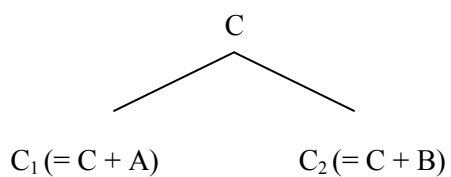


Fig.18

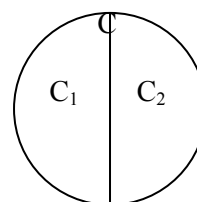


Fig.19

In Chapter 2, I used these figures to show how the hierarchical relation between concepts (*Fig.18*), which is determined by their intension-containment relation, in turn determines their extension-containment relation (*Fig.19*). More specifically, insofar as *C* is contained in *C*₁ as one of the many conceptual marks that constitute *C*₁'s intension, *C* is a *higher* concept relative to *C*₁, containing the latter under itself (*Fig.18*); and, given Kant's logical principle that of any two given concepts the higher concept is also broader, *C* is a *broader* concept—precisely on account of being higher—than *C*₁, to the extent that the extension of the latter concept is properly contained in that of the former (*Fig.19*). I pointed out that such a relation between two concepts with respect to the quantity of extension is a logical relation that holds no matter what—or whether any—objects actually fall under each concept. Now, this logical relation can be articulated precisely by a universal affirmative judgment: to every *x*, to which belongs *C*₁, belongs also *C*. Or, in terms of extension: every *x*, which falls in the extension of *C*₁, falls also in the extension of *C*.⁴² And such a judgment is true, provided that *C* is analytically contained in *C*₁: any *x* that falls in the extension of *C*₁ in fact falls in the extension of *C*. The judgment is indeed necessarily true by the Principle of Contradiction: it would “contradict the object”, in Kant's terms, to deny any *x* that is represented through *C*₁ a place in the extension of *C*; for that would amount to the impossible act of, with respect to *Fig.19*, placing something both inside and outside the plane that represents the extension of *C*₁.

By the above analysis, we can say that the analytic judgment “Every *x*, which is *C*₁, is also *C*” (i) *is true in that* the predicate *C* agrees with whatever object (= *x*) may be represented by *C*₁, and yet (ii) *its truth is determined solely on the basis* of the analytic intension-containment relation between *C* and *C*₁, regardless of what particular objects may be represented by each of these concepts.⁴³ If (i) characterizes the sense in which an analytic judgment is *true*, (ii) captures the *epistemic ground on which to determine* its truth. If these may be considered two aspects of Kantian analytic truths, it is important to distinguish them for our purpose: even though an analytic judgment may be said to be true *in virtue of* intension-containment relation between its

concepts (in the sense of (ii)), this does not affect the application of Kant's above-mentioned nominal definition of truth to analytic judgments (much the same as it applies to the synthetic ones). For the truth of an analytic judgment still *consists in* the agreement of its predicate with the object signified by its subject-concept. The same can be said of "A two-sided polygon is two-sided". In a nutshell, this judgment is true insofar as whatever object falls in the extension of the subject-concept, *two-side polygon* indeed also falls in the extension of the predicate, *two-sided*, in which sense the predicate "agrees with" the object signified by the subject-concept. This "agreement with an object" can nevertheless be determined solely on the basis of the intension-containment relation between the two concepts, in the same manner in which we have shown how the truth (= agreement with an object) of a judgment of the form "to every x , to which belongs C_1 ($= C + A$), belongs also C " can be determined on the basis of the intension-containment relation between C_1 and C .

In this process of determining the truth (= agreement with an object) of an analytic judgment, the involved concepts are treated only in respect of their *form* as general representations. It is on this account that a concept is said to have an extension that consists of the multitude of objects represented through it. And the relevant notion of "object" is that of an object in general = x , understood as whatever may be thought by means of concepts. It does not matter whether the object represented by a particular concept (e.g., *two-sided polygon*), as the concept figures in an analytic judgment, can be given or exhibited in our sensible intuition. To that extent, the domain of objects with respect to which for us to cognize an analytic truth (= agreement with an object) may very well be considered as posited—with no ontological strings attached, though—on the part of our understanding by means of the concept *object in general* = x .⁴⁴ Such a posit serves as a mere device by which for the understanding to represent, say, certain formal properties and formal relations of its concepts (as they were expounded in Chapter 2). In that sense, even though the truth of analytic judgments is in a sense determined with respect to a domain of objects, such truth is, given my account of Kantian formality in Chapter 1, merely

formal—in the sense that it is determined independently of any considerations as to whether or how the concepts involved may be related to objects of experience.

Since there is no presupposition about objects of experience (which would have to be given through intuition), it is “easy”, in Kant’s words, for the understanding to make analytic judgments and cognize their truth. Here the understanding need not go beyond its concepts, posits and logical principles, but “only go through [its] concepts and see what lies therein”, in order to see that a predicate agrees with the object represented by a concept in which the predicate is contained. True judgments made in this way should not, Kant cautions, be “falsely presented as propositions of experience.” (V-Met/Mrongovius, Ak 29:789) Such a warning would not be necessary if analytic judgments did not have the form of judgments about objects (i.e., the *x-a-b* schema) to begin with. Precisely because, as we saw in section II, analytic judgments have the *x-a-b* schema *qua* judgments, one may have the illusion that judgments that are analytically *true*—i.e., agree with *objects*—are *on that account* material truths about the world of experience. Thus Longuenesse, in reference to Kant’s use of the *x-a-b* schema for both analytic and synthetic judgments, observes:

In fact, Kant’s introducing the term ‘x’ in his explanation of the logical form of judgments is precisely due to his awareness that contrary to Leibnizian illusions, not all true judgments can be reduced to analytic judgments or judgments that are true by analysis of the subject-concept. (Longuenesse 1998:86n.10)

Longuenesse has in mind analytic judgments that are true of objects of experience (e.g., “Bodies are extended.”): not all true judgments of experience are true by analysis of concepts. (ibid. 87) But Kant’s warning above against mistaking analytic judgments for judgments of experience indicates a more radical move. Even if the Leibnizians are granted as many analytic judgments as they would like, since the truth of such judgments—assuming they are genuinely analytic—can be easily established without any reference to objects of experience, they are not contentful in the Kantian sense, which would presuppose relation to objects of experience, simply *in virtue of being true*. More generally speaking, an analytic judgment, though true, may very well have no

Kantian content. Even in the case of “Bodies are extended”, though its truth (= agreement with an object) can be cognized through mere conceptual analysis, bodies *as objects of experience* are not conceptually determined; and we cannot give Kantian content to such a judgment without the *additional* move of descending to the sensible intuition through which alone can those objects be given to us.⁴⁵ To borrow the terms in which Kant warns against any dialectical misuse of logical laws, by means of analytic judgments “nobody can dare to judge of objects [i.e., objects that are to be given to us only through our intuition ...] without having drawn on antecedently well-founded information about them from outside” of the understanding. (A60/B85) Yet there is something so “seductive” (ibid.) in the truth of analytic judgments as agreement with objects that one may get the illusion that such judgments, merely on account of being true, teach us something about the world. To such an illusion a sharp separation of the truth and the contentfulness (in the Kantian sense) of analytic judgments is an effective antidote. Or so Kant would say.

¹ It is moot whether this intension-containment relation between the predicate and the subject-concept is all that there is to Kantian *analyticity*. And commentators disagree as to which of Kant’s various characterizations of analytic judgment should be treated as having *definitional* priority over others. For samples of the debate, see Beck 1956b, Garver 1969, Allison 1973, Hanna 2001 (Ch.3), Proops 2005, Anderson 2005 and Hogan 2012. But the main points to be made in this chapter will not hang on any results of the debate.

² Gibson [1995] discusses a similar debate regarding judgments in general: the view that for Kant judgments are *about representations* alone vs. the view that for Kant judgments are *about objects* (as what can be given only in sensible intuition). But analytic judgments are not mentioned at all in Gibson’s discussion.

³ In Paton’s own words, the thesis that Kantian analytic judgments are about objects plainly follows from the fact “that an analytic judgment can be true [... and that] for Kant truth is always correspondence with an object.” (Paton 1936:214n.3)

⁴ The formulation of (i) and (ii), together with that of (iii) (to be introduced below), is borrowed from Rosenkoetter 2008:192. (I have made slight changes in the wording.) One may find (i) strange, since Kant also takes there to be analytically *false* judgments, e.g., “A body is simple.” (V-Met/Mrongovius, Ak 29:789) But the same problem that Rosenkoetter has raised will also arise in the case of analytically false judgments: for Kant, the falsity of such judgments consists in a certain *disagreement* with the *object* (not in lacking an object for the subject-concept to refer), and so seems to presuppose [AO] as well.

⁵ Tolley 2007, Ch.5, §41. Like Rosenkoetter, Tolley explicitly targets Paton’s claim that [AO] follows from Kant’s commitment both to the truth of analytic judgments and to the conception of truth as agreement with an object. Tolley characterizes the basic rules of Kant’s formal logic as syntactic (as opposed to semantic) rules of thought in his Chapter 2 (especially §§14-15).

⁶ Formal_R was presented in Chapter 1 as follows: “Logic is formal in that it treats all thoughts only in respect of their form, regardless of how they are related to the objects (empirically or *a priori*).”

⁷ Paton mentions two of these neo-Kantians, Hermann Cohen and Walter Kinkel. (Paton 1936: 84n.2) The targeted interpretation is tied up with a construal shared by Cohen and Kinkel of Kantian formal logic: (1) Kantian formal logic is only about analytic judgments, (2) formal logic treats thinking as totally object-less,

and hence (3) analytic judgments are about concepts and not about objects. (Cohen 1871/1885; Kinkel 1904) Paton gives separate reasons for rejecting (1) (Paton 1936:213-15) and (2) (Paton 1936:187-92, 191*n.*1) as well as (3).

⁸ Rosenkoetter's proposal at this point is that "it is *possible* to judge analytically without intending or otherwise implicitly claiming to refer to an object"—and so it is "beside the point" whether people *in fact* generally intend to make reference to objects. (Rosenkoetter 2008: 198) Such a proposal is compatible with saying that *some* analytic judgments are about objects. Eventually, however, Rosenkoetter will argue for a stronger claim: Kantian analytic judgments are "not about objects at all." (ibid. 199)

⁹ To illustrate, consider "A unicorn has one horn." Suppose this is a clear case of an analytic judgment, the predicate being contained in the concept of the subject. On Paton's account (as Rosenkoetter construes it), to say that this judgment has a purported reference to an object is to say that the judgment *means* roughly this: (a) anything that is a unicorn has one horn *and* (b) there *is* something that is a unicorn. Rosenkoetter's criticism [2] then amounts to showing how this conflicts with Paton's commitment to the thesis that all analytic judgments are true (by virtue of the predicate being contained in the concept of the subject): *qua* analytic, the judgment "A unicorn has one horn" is necessarily true; however, due to the failure of (b), the same judgment is false.

¹⁰ See Hanna 2001:75-6 for a relevant discussion of Kant's anti-psychologism.

¹¹ MacFarlane himself uses the expression 'objective purport'. (MacFarlane 2002:51)

¹² Rosenkoetter introduces and then refutes a version of [AO₂] without reference to any particular commentator who supports it. (Rosenkoetter 2008:192-5)

¹³ The reason is that it is impossible to construct such a concept in our pure intuition of space. (A220-1/B268) I will explain this point in section II, using Kant's own example of the concept of a figure that is enclosed between two straight lines.

¹⁴ In his original text, Heis mistakenly refers to §16, note 2 instead, where analytic judgments are not mentioned at all. §36 is the only place in *Logik* where a characterization of analytic judgments is given. In a recent private communication to me, Heis confirmed my suspicion that §36*n.*1 is really what he meant.

¹⁵ This is the notion of *Inhalt* that we discussed in Chapter 1 and that was distinguished from the logical notion of *Inhalt* (translated as "intension") in Chapter 2. Kant sometimes also uses the terms 'Sinn' and 'Bedeutung' to express *Inhalt* in the former sense. (see A239-40/B298-9) To avoid confusion with the much thinner, logical notion of *Bedeutung* used in Chapter 3, I shall treat the claim about a cognition's having *Bedeutung* in the sense of being related to an object of experience as a claim about it having *Inhalt* in the same sense.

¹⁶ Here MacFarlane inserts the following quote from B146, in which Kant presents his view that every cognition consists of both concepts *and* intuition: "two components belong to cognition: first, the concept, through which a object is thought at all [...], and second, the intuition, through which it is given; for if an intuition corresponding to the concept could not be given at all, then it would be a thought as far as its form is concerned, but without any object, and by its means no cognition of anything at all would be possible, since, as far as I would know, nothing would be given nor could be given to which my thought could be applied."

¹⁷ Besides trying to show that there is no good Kantian way to spell out the view that analytic judgments are "about objects", Rosenkoetter also invokes Kant's view of the formality of logic as direct evidence to show that "analytic judgments are not about objects at all." (Rosenkoetter 2008:199) According to Kant's conception of formal logic, as is articulated in the *Groundwork of the Metaphysics of Morals*, logic as a "formal" rational cognition is "occupied only with the form of the understanding and of reason itself and with the universal rules of thinking in general, without distinction of objects"; it is thus essentially different from a "material" rational cognition, which "has to do with determinate objects." (GMS, Ak 4:387; Rosenkoetter 2008:199*n.*14) Rosenkoetter does not explain how this claim about the formality of logic is supposed to show that analytic judgments are *not about objects at all*. In referring to the passage from the *Groundwork*, he has glossed over the difference between "not about objects *at all*" and "not about *determinate* objects" (note that in the cited passage formal logic is only said to proceed "without *distinction* of objects"). The burden is on Rosenkoetter to show that such a difference is negligible.

Presumably, Rosenkoetter has the following line of argument in mind: (i) analytic judgments are such that we must be able to make them on the grounds of formal logic alone; (ii) formal logic is not about objects at all; and so (iii) analytic judgments are not about objects at all. (Hints of this argument can also be found in Tolley 2007:429-39.) Given what we saw in Chapter 1, however, such an argument against the

thesis that Kantian analytic judgments are about objects hinges on a controversial interpretation of Kant's conception of formal logic, namely (ii). I argued in Chapter 1 that, simply because Kant takes formal logic to abstract from all relation (*Beziehung*) to objects, it does not follow that all talk of objects must be banished from a Kantian formal-logical discourse.

¹⁸ As I shall explain later (in note 24), there may be a significant distinction to be drawn on Kant's part between an object = x and an object = X , the former alone being the relevant sense in which a Kantian analytic judgment—or, for that matter, any judgment in Kant's sense—is said to have objectual purport. I take it that the X that Beck is referring to is the same as the x in my use.

¹⁹ Beck does specify three senses in which the x in a synthetic judgment may be understood: either as “a schema of an object in general (of a thing, cause, etc.)”, or as “a determinate intuition of space or time or both, which A and B both refer to and make determinate”, or as “a datum or *concretum* of experience”. (Beck 1956b:173) But it is not clear whether any of these is relevant to the interpretation of either the objectual purport or the contentfulness of an analytic judgment.

²⁰ Log, §36n.1, Ak 9:111: “An example of a synthetic proposition is, To everything x , to which the concept of body ($a + b$) belongs, belongs also attraction (c).”

²¹ Also see R4674, Ak 17:645; R4676, Ak 17:657. Kant's account of the logical subject and logical predicate of a judgment reflects what Geach summarizes as the correct way to interpret the subject- and predicate-terms in a sentence of the traditional Aristotelian logic: “A predicate is thus *attached to* a subject, but *predicated of* what the subject stands for.” (Geach 1950:462)

²² As I noted in Chapter 3, this is strictly speaking a mere schema or bare skeleton of a judgment, and not a logical form properly so called. The logical form of a (categorical) judgment would have to specify its quantity (universal or particular) and its quality (affirmative or negative). In the present context, Kant is merely concerned with articulating the basic relata of a judgment and how they are related: x (object), a (logical subject), b (logical predicate).

²³ The analyticity of a judgment has only epistemic relevance. In the *Logik*, the presentation of the x - a - b schemata of analytic vs. synthetic judgments is followed by an epistemic distinction between analytic and synthetic judgments: the latter “increase cognition *materialiter*, analytic ones merely *formaliter*.” (Log, §36n.1, Ak 9:111; for discussion, see Allison 1985:32-4) The view that analytic judgments increase our cognition of objects merely formally is spelled out in the *Critique*: “Analytic judgments do not really teach us anything more *about the object* than what the concept that we have of it already contains in itself, since they do not expand cognition beyond the concept of the subject.” (A736/B764; emphasis added)

²⁴ I take this notion of an object in general = x to differ from the “something in general = X ” (or “transcendental object = X ”) discussed in the A edition of the Transcendental Deduction. (A104; A109) The latter X is introduced in connection with Kant's account of how the understanding, by means of its pure concepts, “brings a transcendental content [*Inhalt*]” into a given manifold of intuition (A79/B105) or “understand[s] something [*etwas*] in the manifold of intuition, i.e., think[s] an object for it.” (A80/B106) The “object” in question is then spelled out in the A edition “only as something in general = X ” (A105) and in the B edition as “that in the concept of which the manifold of a given intuition is *united*.” (B137)

²⁵ Cf. A290/B346; V-Met/Mrongovius, Ak 29:811; V-Met/Vigilantius, Ak 29:960-1.

²⁶ Kant gives the following reason for placing *nothing* under the concept of an object in general: although it “seems striking to think of an object that comprises a nothing”, “a *nothing* also *presupposes only a thought [of object] which then cancels itself*, {i.e., which contradicts itself}.” (V-Met/Vigilantius, Ak 29:960-1)

²⁷ I am of the view that the object in general, as is signified by the subject-concept of a judgment, must at least be a *logical* possibility. What is left open is whether the object is a something/possible or nothing/impossible in the *real* sense. The relevant notions will be explained on next page.

²⁸ This is part of Kant's remark about the peculiar nature of the categories of modality in “The Postulates of Empirical Thinking in General”. To quote the relevant passage in full: “as a determination of the object they [the categories of modality] do not augment the concept to which they are ascribed in the least, but rather express only the relation to the faculty of cognition. If the concept of a thing is already entirely complete, I can still ask about this object whether it is merely possible, or also actual [...]” (A219B266) This characterization can also be extended to the modal notions of possible/impossible and something/nothing we are presently interested in. Here one asks about the object = x , represented through certain concepts (e.g., *body*, *triangle*, *two-side polygon*), whether it is possible, whether it is a something. Such questions can be answered, in Kant's view, only by referring to our faculty of cognition—in this case

sensibility—and examining the epistemic conditions under which the object so represented may be given or exhibited in our intuition.

²⁹ This point is inspired by Allison’s following remark about the “Copernican turn” in Kant’s theory of object: “[Kant replaces] first-order talk about objects [...] by second-order talk about the *concept* of an object and the conditions of the representation of an object.” (Allison 2004: 173)

³⁰ V-Met/Mrongovius, Ak 29:812; V-Met/Vigilantius, Ak 29:961; A291-2/B348. The term ‘logical nothing’ is used in the *Metaphysik Vigilantius* and is treated as semantically equivalent to ‘*nihil negativum*’. The latter term is used at A291/B348.

³¹ By “formal conditions of an experience in general” Kant is referring to space and time as forms of appearances or formal conditions of our sensibility. (see A240-1/B299-300)

³² Also see V-Met/Vigilantius, Ak 29:961. Kant’s other examples include *noumena* (A290/B347), the concept of spirit (V-Met/Vigilantius, Ak 29:961), fairy tales (V-Met/Mrongovius, Ak 29:812) and imaginary being (V-Met/L₂, Ak 28:543).

³³ This sense of contentfulness is what Kant sometimes calls “objective reality” or “objective validity” of a concept, namely, “the possibility of [the] object as is thought through the concept”, possibility that can be determined only in relation to empirical intuition and its conditions. (A220/B268; A239/B298.; A155/B194) Note that the contentfulness requires only the *possibility* of giving or exhibiting an object in intuition: “it does not necessarily matter that the object is sensibly exhibited, but rather only that its exhibition is possible.” (V-Met/Vigilantius, Ak 29:967)

³⁴ For a helpful reading of Kant’s view of the construction of mathematical concepts, see Friedman [1990].

³⁵ For the concept *two-sided polygon* cannot be constructed in our pure intuition of space, in the same sense in which the concept of a figure enclosed between two straight lines has been said not to be constructible. In Kant’s view, the real possibility of an object represented by a mathematical concept is determined by what he calls the “real definition”—as opposed to the “nominal definition”—of the concept, definition through which the object itself is “originally given” *a priori*. (A729-31/B757-9; cf. Log. §106, Ak 9:143-4; V-Lo/Wiener, Ak 24:919-20; R2995, Ak 16:607) Figuratively speaking, the real definition of a geometrical concept gives the instructions for drawing the corresponding figure in our pure intuition. For Kant, there is no such thing as a real but false mathematical definition. A mathematical concept either has a real definition (which is by its very nature true) or has none. (A731/B759) To see whether a judgment that has a mathematical concept as its logical subject has real meaning is to see whether a real definition can be given to that concept. Now, suppose the real definition of “polygon” is as follows: “A polygon is a plane figure bounded by straight lines.” (Wentworth 1891:66) It is impossible to give a real definition to “two-sided polygon”; for the putative definition would issue instructions for us to draw—in our pure intuition of space—a figure with only two straight lines.

On the other hand, a mathematical concept may have a nominal definition. Given the literal meaning of “polygon” (figure with many angles), *two-sided polygon* may be nominally defined as a figure with two sides and many angles. This nominal definition is what is relevant to determining whether the object represented as a two-sided polygon is a *logical* something, and whether “A two-sided polygon is two-sided” is a judgment with logical *Bedeutung*. For a helpful discussion of the relation between Kant’s theory of definition and his account of analyticity, see Beck 1956a.

³⁶ In light of Kant’s own distinction between nominal and real definitions, Vanzo argues that Kant’s nominal definition of truth is meant to capture what is *ordinarily meant*—as Kant sees it—by the use of the predicate “is true”, which is different from giving a full blown *theory* about the truth *maker* of any given cognition. In these terms, Vanzo argues against taking Kant’s nominal definition of truth exactly to express the correspondence *theory* of truth. (Vanzo 2010:164-6) But Vanzo does not profess to show that Kant does not also hold a correspondence theory of truth.

Considering the historical backdrop against which Kant talks about truth, it is safe to assume that Kant holds the following minimalist version of an object-based correspondence theory of truth: a judgment (of the subject-predicate structure) is true just in case the predicate corresponds to the object represented by the subject-concept. This theory is minimalist in that it does not give a full-blown account of what sort of object to which the predicate must correspond, and in what manner of correspondence, in order for a given judgment to be true. (On further details about the object-based correspondence theory of truth and its comparison with the so-called fact-based theory, see Küne 2003 (ch.3) and David 2009.)

³⁷ This formula is stated variously in different texts:

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- (a) “to no subject does there belong a predicate opposed to it.” (V-Met/L₂, Ak 28:544; V-Met/Mrongovius, Ak 29:789, 790; V-Met/Vigilantius, Ak 29:963)
 - (b) “a predicate that contradicts a thing cannot be attributed to it.” (V-Lo/Hechsel, Ak 24:97; A151/B190)
 - (c) “everything must be denied of the thing that contradicts the concept of the thing.” (V-Met/Mrongovius, Ak 29:789; A151/B190-1)

I take these statements to have the same meaning. A predicate contradicts a thing only by contradicting the subject-concept by which the thing is thought. This is suggested in Kant’s following remark: “*b* is always compared with the object *x* by means of *a*.” (R4674, Ak 17:647)

³⁸ R4676, Ak 17:653-5. In the case of synthetic judgments, the object *x* that grounds the predication must be given or exhibited in intuition—for instance, by constructing the subject-concept in pure intuition (in the case of mathematical judgments). (R4674, Ak 17:645-6; R4676, Ak 17:655)

³⁹ This interpolation has its basis in the German text. “Of it”: *von ihm*; “cognition of object”: *Erkenntnis des Objects* (*ihm* can only refer to *Object* in this case).

⁴⁰ Kant sometimes discusses both the Principle of Contradiction and the Principle of Identity as principles for cognizing the truth value of an analytic judgment. But he then argues that the first principle is the “general” one. (V-Met/Mrongovius, Ak 29:789-90)

⁴¹ This figure is originally used to illustrate the relation between the subject and the predicate in all categorical judgments: “In categorical judgments, *x*, which is contained under *b*, is also under *a*.” (Log, §29*n*, Ak 9:108) But, as we saw in Chapter 2 (Fig.6), Kant actually uses four different figures to represent the four basic forms of categorical judgment. The figure resembling Fig.17 was used to exhibit the logical form of a universal affirmative judgment.

⁴² When a concept belongs to a thing, the thing falls under the concept or, which amounts to the same, falls in its extension. See Log, §7, Ak 9:96; cf. R2902, Ak 16:567; R2881, Ak 16:557-8.

⁴³ For our purpose we can, as I have, set aside the difficult issue of determining, for any two particular concepts (especially empirical ones), whether they indeed stand in an *analytic* intension-containment relation. (To appreciate the challenge of such an issue and the importance for Kant to settle it in order to have a sharp analytic-synthetic distinction, see Beck 1956b.) I have only tried to show that, *if* two concepts stand in such a relation, that is, *if* a judgment is a Kantian analytic one, the truth (= agreement with an object) can be determined solely on that basis (in accordance with the logical principle of reciprocity between extension-containment and intension-containment relations, as exhibited in Fig.18 and Fig.19).

My account of Kantian analytic truths in terms of objectual extension thus essentially differs from the one proposed in Hanna [2001]. Hanna introduces an extensional account of Kantian analyticity as an *alternative* to the standard intensional one: “*quite apart from* the analytical decomposition of a concept, there is *another* Kantian route to analyticity—one that relies *exclusively* on containment-under.” (Hanna 2001:137, emphasis added) On my account, by contrast, Kantian *analyticity* is always to be understood in terms of the intension-containment relation between concepts. Although I take it that the *truth* of a Kantian analytic judgment is to be understood in extensional terms, I also think that the relevant extensional relation—insofar as it is to be a necessary one—must be *grounded* on the given analytic intension-containment between the involved concepts. (For a most recent criticism of Hanna’s account from the viewpoint of Kant’s formal extensional logic, see Hauswald 2010.)

⁴⁴ This notion of self-positing objects should not be conflated with that of *noumena* as “beings of the understanding (*Verstandeswesen*)” thought by means of the pure categories. (B306) In the latter case, the beings of the understanding are construed relative to phenomena: “the understanding, when it calls an object in a relation mere phenomenon, simultaneously makes for itself, beyond this relation, another representation of an object in itself [...], a representation that is an “entirely undetermined concept of a being of understanding, *as a something in general outside of our sensibility*.” (B306-7; emphasis added) Such a concept is a mere result of “abstract[ing] from the manner of our intuition” of objects, i.e., the manner in which objects are given to us in our sensible intuition. (*ibid.*)

⁴⁵ ‘Given’ is, and has been, used as a technical Kantian term. In Kant’s view no object can be given to us except through our sensibility, which is “the receptivity of our mind to *receive* representations insofar as it is *affected* in some way.” (A51/B75, emphasis added) An object being “given” through sensible intuition is thus contrary to an object being *posited* on the part of the understanding.

Conclusion

The central notion of this dissertation is that of the logical extension of a concept. I developed an account on which Kant takes such an extension both to consist of all the possible objects to which a given concept is applicable ([EXT_O]) and to include whatever concepts may be subordinate to it in a conceptual hierarchy ([EXT_C]). I focused on articulating the precise sense in which [EXT_O] must be understood and on defending the view that this notion of extension properly belongs to the Kantian formal logic (or what Kant calls the “pure general logic”). My account was presented as an alternative to three readings of Kant’s notion of logical extension championed by other commentators, according to which the Kantian logical extension either (i) is nothing but [EXT_C], or (ii) consists of the intuitions as well as the concepts—but not the objects—that stand under a given concept, or (iii) is a hybrid entity composed both of the subordinate concepts and of the objects falling under a given concept, where objects are understood as those designated by the concept in an actual or a possible world. I especially addressed two reasons invoked by Clinton Tolley and Lanier Anderson, among others, against treating [EXT_O] as a notion of the Kantian formal logic. The first alleged reason has to do with Kant’s conception of the formality of the pure general logic: given Kant’s view that such logic must, on account of its formality, abstract from all objects and from all relations of the cognition to them, it would be against the spirit of a Kantian formal logic to construe the logical extension as [EXT_O]. The second reason concerns Kant’s logical principle that the extension of a concept is inversely proportional to its intension in respect of quantity: such a principle would fail if the extension were viewed as [EXT_O].

In an effort to refute these two alleged reasons against the ascription of [EXT_O] to the Kantian formal logic, I was guided by two methodological principles. First, I considered it crucial to make explicit the *perspective* from which Kant discusses issues related to logic in a particular context. Second, I found it most fruitful, for the purpose of gaining a precise understanding of

both Kant's philosophy of logic and his key logical notions, to investigate the full spectrum of relevant views held by the early modern logicians with whose works he was familiar. Following the first principle, I examined Kant's discussion of the nature of formal logic in his logic corpus in isolation from the discussion in any of his works—in the *Critique* above all—where he clearly has a certain philosophical agenda in view. Such a procedure allowed me to sort out three distinct aspects in which the Kantian formal logic is formal, each aspect being tied with a specific concern on Kant's part in a particular context—either to distinguish the formal logic from an empirical psychology, or to establish such logic as an independent science that is normatively prior to all other sciences, or to contrast it with the transcendental logic. By my analysis, none of these aspects entails that no objectual notion of extension is allowed in the Kantian formal logic. Following the second principle, I went—contrary to the practice common among commentators on Kant's logic—beyond studying Meier's *Auszug der Vernunftlehre* and explored various implications of the perceived similarities between Kant's conception of logical extension and those found in the works of such logicians as Leibniz, Knutzen, Lambert and Euler. In the process, I gathered the relevant conceptual apparatus to articulate the version of [EXT₀] that I took to belong properly to the Kantian formal logic, according to which the logical extension of a concept consists of all the possible objects to which the concept applies simply in virtue of its form as a general representation.

A pivotal element in my account of Kant's conception of logical extension concerns his use of plane-geometrical figures to represent certain logical relations between concepts as regards their extension. Even though such use is rather sparse in Kant's logic corpus and has received little attention from commentators, I teased out its significance by connecting it both with the systematic use of similar figures by Leibniz, Lambert and Euler, respectively, and with my reading of Kant's view on the formality of logic. I deemed the figures in question as the symbolic means for Kant to represent the objects that a concept signifies in a judgment (or in an inference) when the judgment is treated merely from the formal-logical point of view. To that extent, they

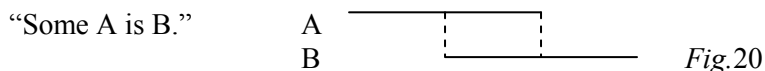
represent the signified objects in a way that abstracts from whatever particular characteristics the objects may have and from how they may be cognized by us (in the strict Kantian sense of cognition). This interpretation of Kant's use of the figures in turn gave me a helpful angle to address certain exegetical issues regarding his views on the logical form of singular judgments and on the formal truth of analytic judgments. Apropos singular judgments, I argued, contrary to a received view, that Kant had enough formal-logical resources to establish them as a distinct form of judgments vis-à-vis the universal and particular ones. Especially, using a point to represent the individual object signified by the subject-concept of a singular judgment, and a circle to represent the multitude of objects signified by the subject-concept of a universal or a particular judgment, I illustrated how to make sense, in the Kantian formal-logical terms, of the following claim of Kant's, which had baffled many commentators: singular judgments count as a special form of judgments vis-à-vis the general (i.e., universal or particular) ones insofar as they stand to the latter "as unity to infinity" in respect of quantity, even though they can also be treated just like the universal ones in syllogisms. As regards the formal truth of analytic judgments, I again used Kant's figurative representation of the logical relation between concepts with respect to extension—together with his logical principle about the reciprocity between the relation of concepts as regards intension and their relation as regards extension—to show, despite what have been argued recently by several commentators, that he can consistently take the truth of an analytic judgment to lie in a certain "agreement with the object" and yet be determinable formally, solely on the basis of the given intension-containment relation between the involved concepts.

In presenting my account of Kantian logical extension as an alternative to various accounts endorsed by other commentators, I mentioned a view according to which the extension of a concept consists of the *intuitions* (as well as the concepts) that stand under it. I did not directly examine or assess such a view. Nor did I rule out the possibility that Kant may, outside of his formal logic, consider the extension of a concept in a way that does involve intuition. Now, I shall conclude my discussion by sketching the sense in which Kant may allow such a non-formal-

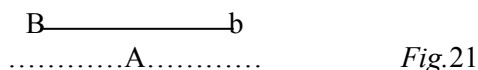
logical conception of extension, in light of a perceived contrast between his two notions of quantification—i.e., the logical vs. the real quantification. In Chapter 3, I considered how Kant and many logicians of his time treated the *logical* quantification involved in judgments, with reference to which universal, particular and singular judgments are distinguished from one another. Generally, such treatments of logical quantification abstract from what actual objects may be signified by the subject-concept of a judgment or whether any of them falls under such a concept at all. (That is why, I argued, it was appropriate for Kant to represent the signified objects by means of geometrical figures.) And the quantification involved in one form of judgment is not, contrary to what we might think, distinguished from the quantification involved in another in terms of how the former determines the truth-condition of a judgment, with reference to a domain of actual truth-makers, in a different way than the latter does. The logical form of “Some bodies are heavy”, for instance, would not, as far as Kant is concerned, be presented as “There is at least one (actual) thing which falls under the concept *body* and which also falls under the concept *heavy*.” Rather, particular and universal judgments are understood as two opposing species of general judgments, the subject-concept of which is always treated as signifying a *multitude* of possible objects (i.e., a logical extension). Accordingly, the logical quantification indicated by “some” in a particular judgment is presented in terms of a certain *restriction* on the entire given logical extension of its subject-concept. Thus Kant, as we saw in Chapter 3, takes such quantification to *exclude* the universal quantification. To that extent, the logical form of the above-mentioned particular judgment is roughly this: “For all bodies, some and only some of them are heavy.” (This form and its contrast with the form of, say, “All bodies are extended” can, as we saw in Chapter 3, be easily illustrated by means of Kant’s circle symbolism.) But we also saw that, when universal, particular and singular judgments are considered in relation to intuition, Kant suggests that we proceed from unity to plurality—the latter corresponding to a particular judgment—in such a way that we think of plurality without excluding totality. If the totality in question corresponds to a universal judgment, this suggests that the judgment “Some bodies are

heavy”, when considered in relation to intuition, roughly means “At least one body is, but possibly all bodies are, heavy.” The quantification involved here may be deemed as a *real* quantification, i.e., quantification over things that are really possible in the Kantian sense. If the extension of a concept relevant to the logical quantification is a logical extension in the strict formal-logical sense, the extension that is pertinent to the real quantification may be called “real extension” by contrast. (The latter extension consists of the objects that can be given or exhibited through sensible intuitions, though not of the intuitions themselves.)

Interestingly, Kant’s contrast between logical and real quantifications in particular judgments echoes the difference in the ways that Leibniz and Lambert, respectively, present the form of a particular judgment by means of a line symbolism. As we saw in Chapter 2, for Leibniz the formal relation between two concepts—as regards extension—in a particular judgment can be exhibited by means of *Fig.20*.



This figure suggests an understanding of the quantification involved in a particular judgment that lines up with Kant’s view of logical quantification sketched above: to say that some A is B is to say that, regarding the multitude of all the individuals that can be signified by A (i.e., regarding the entire logical extension of A), only a part of it falls under the concept B. For Lambert, by contrast, three possibilities are expressed by the same judgment: either exactly one A, or all As, or a few but not all As fall under B. To indicate the “indeterminacy” about which of these possibilities obtains for any given particular judgment, Lambert used *Fig.21* to express “Some A is B.” (Lambert, *NO*, §184)



For Lambert, *Fig.21* exhibits the logical form of an affirmative particular judgment. For Kant, the figure articulates the relation between the two given concepts in respect of real quantification. Given the various aspects of Kantian formality that were presented in Chapter 1, Kant would also

regard the expressed relation as formal, but only in a limited sense: it is formal in the sense of Formal_S and Formal_O , but not in the sense of Formal_R . In other words, the relation is formal insofar as it is considered in abstraction from all empirical psychological conditions of the thinking subject and from all qualitative differences among the objects of thought. But the representation of the relation does not abstract from general considerations about the epistemic conditions under which we cognize objects, conditions that determine the nature of the domain of objects over which we interpret the real quantification involved in a judgment. (As mentioned, the real quantification takes place with reference to the real as opposed to the logical extension of a concept.) This observation gives rise to several issues that are worth further study. Is there a relatively clear divide among early modern logicians in general over the nature of quantification that resembles the above-sketched contrast between Leibniz's and Lambert's views on the topic? If there is such a divide, does it reflect profound differences in conceptions of judgment and, more generally, in philosophical views about the nature of logic (as we might suspect there to be given what we learned in Chapter 1)? If the answers to both of these questions are "yes", does Kant's distinction between treating judgments in purely formal-logical terms and treating them from a perspective involving certain transcendental-logical considerations provide an angle for sorting out the various views of quantification in a meaningful way?

Bibliography

Kant's Works

The *Critique of Pure Reason* is cited by the standard A and B pagination of the first (1781) and second (1787) editions respectively. References to the logic lectures collected in *Logik-Vorlesung* are indicated as follows: abbreviation of the title of the work, followed by LV, and page. All other cited works are from the *Akademie* edition of *Kants gesammelte Schriften* ("Ak" for short), Berlin: Walter de Gruyter, 1902-83. References to them are indicated as follows: title of the work or abbreviation of the title, followed by Ak, volume, and page. For works with existing English translations, I use the translations with necessary but minimal modifications. For other works, translations are my own. The following is a list of abbreviations of Kant's works. The English translations being used are also indicated.

- A/B *Kritik der reinen Vernunft* (Ak 3-4)
 Critique of Pure Reason (Critique), trans. & ed. Paul Guyer & Allen Wood, Cambridge University Press, 1998; other translations of the *Critique* that will be referred to for the sake of comparison include:
 Kemp Smith, Norman. 1929. (trans.) *Critique of Pure Reason*, Macmillan & Co.
 Pluhar, Werner S. 1996. (trans.) *Critique of Pure Reason*, Hackett Publishing Co.
- Diss *De mundi sensibilis atque intelligibilis forma et principiis* (Ak 2)
 Concerning the Form and Principles of the Sensible and Intelligible World (The Inaugural Dissertation), in *Theoretical Philosophy, 1755-1770*, trans. and ed. David Walford and Ralf Meerbote, pp.377-416
- GMS *Grundlegung zur Metaphysik der Sitten* (Ak 4)
 Groundwork of the Metaphysics of Morals, trans. Mary Gregor, in *Practical Philosophy*, Cambridge University Press, 2002, pp.37-108
- LV *Logik-Vorlesung: Unveröffentlichte Nachschriften*, Bd.1 and Bd.2, ed. Tillmann Pinder, Hamburg: Felix Meiner, 1998
- MAN *Metaphysische Anfangsgründe der Naturwissenschaften* (Ak 4)
 Metaphysical Foundations of Natural Science, trans. Michael Friedman, in *Theoretical Philosophy after 1781*, Cambridge University Press, 2002, pp.171-270
- Prol *Prolegomena zu einer künftigen Metaphysik die als Wissenschaft wird auftreten können* (Ak 4)
 Prolegomena to Any Future Metaphysics That Will Be Able to Come Forward as Science, trans. Gary Hatfield, in *Theoretical Philosophy after 1781*, pp.29-170
- R *Reflexionen* (Ak 15-19)
 Notes and Fragments, ed. Paul Guyer, trans. Curtis Bowman, Paul Guyer & Frederick Rauscher, Cambridge University Press, 2005
- UE *Über eine Entdeckung nach der alle Kritik der reinen Vernunft durch eine ältere behrlich gemacht werden soll* (Ak 4)
 On a Discovery whereby any New Critique of Pure Reason Is to Be Made Superfluous by an Older One, trans. Henry Allison, in *Theoretical Philosophy after 1781*, Cambridge University Press, 2002, pp.271-336
- V-Lo/Bauch *Logik Bauch*, LV, Bd. 1
 V-Lo/Hechsel *Logik Hechsel*, LV, Bd.2
 V-Lo/Warschauer *Warschauer Logik*, LV, Bd.2
 V-Lo/Blomberg *Logik Blomberg*, Ak 24, translated in *Lectures on Logic* (LL), trans. & ed. Michael Young, Cambridge University Press, 1992, pp.1-246

V-Lo/Philippi	<i>Logik Philippi</i> , Ak 24
V-Lo/Busolt	<i>Logik Busolt</i> , Ak 24
V-Lo/Pölitz	<i>Logik Pölitz</i> , Ak 24
V-Lo/Dohna	<i>Logik Dohna-Wundlacken</i> , Ak 24, translated in LL, pp.425-516
V-Lo/Wiener	<i>Wiener Logik</i> , Ak 24, translated in LL, pp.247-377
Log	<i>Logik (Jäsche-Logik)</i> , Ak 24, translated in LL, pp.517-640
V-Met/Dohna	<i>Metaphysik Dohna</i> , Ak 28
V-Met/Mrongovius	<i>Metaphysik Mrongovius</i> , Ak 29, translated in <i>Lectures on Metaphysics (LM)</i> , trans. & ed. Karl Ameriks & Steve Naragon, 1997. Cambridge University Press, pp.107-283
V-Met/L ₂	<i>Metaphysik L₂</i> , Ak 28, translated in LM, pp.297-354
V-Met/Schön	<i>Metaphysik von Schön</i> , Ak 28
V-Met/Vigilantius	<i>Metaphysik Vigilantius (K₃)</i> , Ak 29, translated in LM, pp.415-506
V-Met/Vockmann	<i>Metaphysik Vockmann</i> , Ak 28

Other Primary References

- Arnauld, A. & Nicole, P. 1996. *Logic or the Art of Thinking (Port-Royal Logic)*, trans. & ed. Jill V. Buroker, Cambridge University Press
- Baumgarten, Alexander Gottlieb. 1761. *Acroasis logicae in Christianum L.B. de Wolff*, Leipzig, in Wolff 1983, Abl. III, Bd.5
- Crusius, Christian August. 1747. *Weg zur Gewißheit und Zuverlässigkeit der menschlichen Erkenntnis*, Leipzig
- Euler, Leonhard. 1768. *Lettres à une Princesse d'Allemagne sur divers sujets de physique & de philosophie, (Lettres)* St. Petersburg. Translation used: *Letters of Euler on Different Subjects in Natural Philosophy: addressed to a German princess*, trans. Henry Hunter, ed. with notes by David Brewster, Vol.I, Harper & Brother, 1858
- Knutzen, Martin. 1747. *Elementa philosophiae rationalis seu logicae*, Leipzig
- Lambert, Johann Heinrich. 1764. *Neues Organon*, Bd.1, Leipzig
- Leibniz, G.W. 1961. *Die philosophischen Schriften*, G. Olms
- 1686a. “General inquiries about the analysis of concepts and of truths”, in Parkinson 1966: 47-87
- 1686b. *De formae logicae comprobatione per linearum ductus*, in *Opuscles et Fragments Inédits de Leibniz: Extraits des manuscrits de la Bibliothèque royale de Hanovre*, ed. Louis Couturat, Paris, 1903, pp.292-8
- 1765/1996. *New Essays on Human Understanding (New Essays)*, trans. & ed. Peter Remnant & Jonathan Bennett, Cambridge University Press, 1996
- Locke, John. 1690/1975. *An Essay Concerning the Human Understanding*, edited with an introduction, by Peter H. Nidditch, Oxford : Clarendon Press, 1975.
- 1706/1993. *Of the Conduct of the Understanding*, with a new introduction by John Yolton, Bristol, England: Thoemmes Press, 1993
- Meier, Georg Friedrich. 1752a. *Vernunftlehre*, Halle: Gebauer
- 1752b. *Auszug aus der Vernunftlehre (Auszug)*, in volume 16 of the *Akademie* edition of *Kants gesammelte Schriften*
- Reusch, Joanne Peter. 1734. *Systema logicum: antiquiorum atque recentiorum*, Jena
- Wolff, Christian. 1983. *Gesammelte Werke*, G. Olms
- 1712. *Vernünfftige Gedanken von den Kräften des menschlichen Verstandes und ihrer richtigen Gebrache in Erkenntnis der Wahrheit (German Logic)*, in Wolff 1983, Abl.I, Bd.1
- 1740. *Philosophia rationalis sive logica (Latin Logic)*, in Wolff 1983, Abl.II, Bd. 1.2

Secondary References

- Allison, Henry. 1973. *The Kant-Eberhard Controversy*, Johns Hopkins University Press
- 1985. “The originality of Kant’s distinction between analytic and synthetic judgments”, in *The Philosophy of Immanuel Kant*, ed. Richard Kennington, The Catholic University of America Press
- 2004. *Kant’s Transcendental Idealism*, Yale University Press
- Anderson, Lanier. 2004a. “Containment analyticity and Kant’s problem of synthetic judgment”, *Graduate Faculty Philosophy Journal* 25 (2):161-204
- 2004b. “It adds up after all: Kant’s philosophy of arithmetic in light of the traditional logic”, *Philosophy and Phenomenological Research* 69 (3):501–40.
- 2005. “The Wolffian paradigm and its discontents: Kant’s containment definition of analyticity in historical context”, in *Archiv für Geschichte der Philosophie* 87:22-74
- 2010. “The Introduction to the Critique: Framing the Question.” in *The Cambridge Companion to Kant’s Critique of Pure Reason*, ed. Paul Guyer, Cambridge University Press, pp.75-92
- Ashworth, E.J. 1974. *Language and Logic in the Post-Medieval Period*, D. Reidel Publishing Company
- Baynes, Thomas S. 1865. (trans.) *The Port Royal Logic*, Edinburgh
- Beck, C.H. 2003. *Kants Kritik der reinen Vernunft: die Grundlegung der modernen Philosophie*, München
- Beck, Lewis White. 1956a. “Kant’s theory of definition”, *Philosophical Review* 65 (2):179-91
- 1956b. “Can Kant’s synthetic judgments be made analytic?” *Kant-Studien* 47 (2):168—81
- 1957. “On the meta-semantics of the problem of the synthetic a priori”, *Mind*, New Series, 66 (262):228-32
- Bennett, Jonathan. 1966. *Kant’s Analytic*, Cambridge University Press
- Boehner, Philotheus. 1952. *Medieval Logic: an Outline of its Development from 1250 to c. 1400*, Manchester University Press
- Bocheński, I.M. 1961. *A History of Formal Logic*, trans. I. Thomas, Notre Dame
- Boswell, Terry. 1988. “On the textual authenticity of Kant’s *Logic*”, *History and Philosophy of Logic* 9 (2): 193-203.
- Brandt, Reinhardt. 1991/1995. *The Table of Judgment: Critique of Pure Reason A67-76; B92-101*, trans. Eric Watkins. Atascadero, California: Ridgeview Publishing; based on Brandt’s *Die Urteilstafel. Kritik der reinen Vernunft A67-76; B92-101*, Kant-Forschungen 4, Hamburg, 1991
- Buickerood, James G. 1985. “Locke and the rise of facultative logic”, *History and Philosophy of Logic* 6 (1): 157-90
- Buroker, Jill V. 1996. (trans.) *Logic or the Art of Thinking*, Cambridge University Press
- Capozzi, Mirella. 1987, “Kant on logic, language and thought”, in *Speculative Grammar, Universal Grammar, and Philosophical Analysis*, eds. Dino Buzzetti and Maurizio Ferriani, Amsterdam-Philadelphia: John Benjamins, 97-147
- Capozzi, Mirella & Gino Roncaglia, 2009. “Logic and philosophy of logic from Humanism to Kant”, in *The Development of Modern Logic*, ed. Leila Haaparanta, Oxford University Press
- Caygill, Howard. 1995. *A Kant Dictionary*, Blackwell Publishing
- Czezowski, Tadeusz. 1955. “On certain peculiarities of singular propositions”, *Mind*, New Series, 64 (255):392-5
- Codato, Luciano L. 2008. “Judgment, extension, logical form”, *Recht und Frieden in der Philosophie Kants* 5:139-52
- Cohen, Hermann. 1871/1885. *Kants Theorie der Erfahrung*, Berlin: Dümmler

- Conrad, Elfriede. 1994. *Kants Logikvorlesungen als neuer Schlüssel zur Architektonik der Kritik der reinen Vernunft*, Stuttgart-Bad Cannstatt: frommann-holzboog
- David, Marian. 2009. "The correspondence theory of truth", *Stanford Encyclopedia of Philosophy*
- de Jong, Willem R. 1995. "Kant's analytic judgments and the traditional theory of concepts", *Journal of the History of Philosophy* 33 (4):613-41
- Easton, Patricia A. 1997. (ed.) *Logic and the Workings of the Mind: The Logic of Ideas and Faculty Psychology in Early Modern Philosophy*, Atascadero, California: Ridgeview
- Frede, Michael & Krüger, Lorenz. 1970. "Über die Zuordnung der Quantitäten des Urteils und der Kategorien der Größe bei Kant", *Kant-Studien* 61 (1):28-49
- Friedman, Michael. 1990. "Kant on concepts and intuition in the mathematical sciences", *Synthese* 84:213-57
- 1992. *Kant and the Exact Sciences*, Harvard University Press
- Frisch, Joseph C. 1969. *Extension and Comprehension in Logic*. New York: Philosophical Library
- Garver, Newton. 1969. "Analyticity and grammar", in *Kant Studies Today*, ed. Lewis W. Beck, pp.245-73
- Gaukroger, Stephen. 1989. *Cartesian Logic*, Oxford University Press
- Geach, Peter T. 1950. "Subject and predicate", *Mind*, New Series, 59 (236): 461-82
- Gibson, Martha I. 1995. "Reference and unity in Kant's theory of judgment", *Canadian Journal of Philosophy*, 25 (2):229-56
- Greenberg, Robert. 1994. "The content of Kant's logical functions of judgment", *History of Philosophy Quarterly* 11 (4):375-92
- Gumański, Leon. 1960. "Singular propositions, and 'this' as a quantifier", *Mind*, New Series, 69 (276):534-43
- Hanna, Robert. 1990. "Kant's theory of empirical judgment and modern semantics", *History of Philosophy Quarterly* 7 (3):335-51
- 2001. *Kant and the Foundations of Analytic Philosophy*, Oxford
- Hauswald, Rico. 2010. "Umfangslogik und analytisches urteil bei Kant", *Kant-Studien* 101 (3):283-308
- Heis, Jeremy. 2007. *The Fact of Modern Mathematics: Geometry, Logic, and Concept Formation in Kant and Cassirer*, Ph.D Dissertation, University of Pittsburgh
- Hintikka, Jaakko. 1969. "On Kant's notion of intuition (*Anschauung*)", in *Kant's First Critique*, ed. T. Penelhum & J. J. MacIntosh, Wadsworth, pp.38-53
- Hogan, Desmond. 2012. "Metaphysical motives of Kant's analytic-synthetic distinction", *Journal of the History of Philosophy* (forthcoming)
- Kauppi, Raili. 1960. *Über die leibnizsche Logik: mit besonderer Berücksichtigung des Problems der Intension und der Extension*, *Acta philosophica Fennica* 12, Helsinki
- Kemp Smith, Norman. 1923. *A Commentary to Kant's 'Critique of Pure Reason'*, New York: The Humanities Press
- Kinkel, Walter. 1904. "Einleitung" to *Immanuel Kants Logik: Ein Handbuch zu Vorlesungen*, *Philosophische Bibliothek* 43, Dürr
- Kneale, William & Kneale, Martha. 1962. *The Development of Logic*, Oxford University Press
- Krüger, von Lorenz. 1968. "Wollte Kant die Vollständigkeit seiner Unteilstafel beweisen?" *Kant-Studien* 59 (3):333-56
- Kuehn, Manfred. 2001. *Kant: A Biography*, Cambridge University Press
- Künne, Wolfgang. 2003. *Conceptions of Truth*, Oxford: Clarendon Press
- Lenders, Winfried. 1971. "The analytic logic of G.W. Leibniz and Chr. Wolff: A Problem in Kant research", *Synthese* 23:147-153
- Longuenesse, Béatrice. 1998. *Kant and the Capacity to Judge*, Princeton University Press

- Lenzen, Wolfgang. 1983/2004. "Zur 'extensionalen' und 'intensionalen' Interpretation der Leibnizschen Logik", *Studia Leibnitiana* 15:129-48; reprinted in revised form in Lenzen 2004: 23-46.
- . 1989. "Concepts vs. predicates: Leibniz's challenge to modern logic", in *The Leibniz Renaissance: International workshop, Firenze, 2-5 giugno 1986*, Firenze: Olschki, pp. 153-172
- . 2003. "Grundfragen des logischen Kalküls—Eine Art Rezension von F. Schupp (Hrg.), G. W. Leibniz, *Die Grundlagen des logischen Kalküls*", *History and Philosophy of Logic* 24:141–62.
- . 2004. *Calculus Universalis: Studien zur Logik von G. W. Leibniz*, Paderborn: Mentis Verlag
- Lovejoy, Arthur O. 1907. "Kant's classification of the forms of judgments", *Philosophical Review* 16(6):588-603
- MacFarlane, John. 2000. *What Does It Mean to Say That Logic Is Formal?* Ph.D Dissertation, University of Pittsburgh
- . 2002. "Frege, Kant, and the logic in logicism", *The Philosophical Review*, 111 (1): 25-65
- Marquardt, Anton. 1885. *Kant und Crusius: ein Beitrag zum richtigen Verständniss der crusanischen Philosophie*, Kiel: Lipsius & Tischer
- McCosh, James. 1891. *The Laws of Discursive Thought*, New York: C. Scribner's Sons
- Mosser, Kurt. 2008. *Necessity and Possibility: The Logical Strategy of Kant's Critique of Pure Reason*, Cambridge University Press
- Nagel, Ernest. 1945. "Logic without ontology", *Journal of Symbolic Logic* 10 (1):16–18
- Nussbaum, Charles. 1992. "Critical and pre-critical phases in Kant's philosophy of logic", *Kant-Studien* 83 (3): 280-93
- Parkinson, G.H.R. 1966. (trans. & ed.) *Leibniz: Logical Papers*, Oxford: Clarendon Press
- Passmore, John A. 1953. "Descartes, the British empiricists, and formal logic", *Philosophical Review* 62 (4): 545-53
- Paton, H. J. 1936. *Kant's Metaphysic of Experience*, Vol.1, London: George Allen & Unwin Ltd.
- Peirce, Charles S. 1867. "Upon logical comprehension and extension", *Proceedings of the American Academy of Arts and Sciences* 7 (1865-1868):416-32
- Pippin, Robert. 1982. *Kant's Theory of Form*, New Haven and London: Yale University Press
- Pozzo, Riccardo. 1998. "Kant within the tradition of modern logic: the role of the 'Introduction: Idea of a Transcendental Logic'", *The Review of Metaphysics* 52 (2): 295-310
- . 2005. "Prejudices and horizons: G. F. Meier's *Vernunftlehre* and its relation to Kant", *Journal of the History of Philosophy* 43 (2):185-202
- Prien, Bernd. 2006. *Kants Logik der Begriffe: Die Begriffslehre der formalen und transzendentalen Logik Kants*, Berlin: Walter de Gruyter
- Proops, Ian. 2005. "Kant's conception of analytic judgment", *Philosophy and Phenomenological Research*, 70 (3): 588-612
- Reich, Klaus. 1932/1986/1992. *The Completeness of Kant's Table of Judgments*, trans. J. Kneller & M. Losonsky, Stanford, California: Stanford University Press, 1992; from K. Reich's *Die Vollständigkeit der Kantischen Urteilstafel*, Hamburg: Meiner, 1986 (first published in 1932)
- Rosenkoetter, Timothy. 2008. "Are Kantian analytic judgments about objects?" *Recht und Frieden in der Philosophie Kants* 5:191-202
- Schulthess, Peter. 1981. *Relation und Funktion*, Berlin; New York: De Gruyter
- Smart, Harold R. 1955. "Two views on Kant and formal logic", *Philosophy and Phenomenological Research* 16 (2): 155-71
- Stuhlmann-Laeisz, Rainer. 1976. *Kants Logik*, Walter de Gruyter
- Swing, Thomas K. 1969. *Kant's Transcendental Logic*, Yale University Press
- Swoyer, Chris. 1995. "Leibniz on intension and extension", *Nous* 29 (1): 96-114

- Thompson, Manley. 1972. "Singular terms and intuitions in Kant's epistemology". *The Review of Metaphysics* 26 (2): 314-43
- Tolley, Clinton. 2007. *Kant's Conception of Logic*, Ph.D Dissertation, University of Chicago
- 2012a. "Kant and Bolzano on the nature of logic", *History and Philosophy of Logic* (forthcoming)
- 2012b. "The generality of Kant's transcendental logic", *Journal of History of Philosophy* (forthcoming)
- Tonelli, Giorgio. 1959. "Der Streit über die mathematische Methode in der Philosophie in der ersten Hälfte des 18. Jahrhunderts und die Entstehung von Kants Schrift über die 'Deutlichkeit'". *Archiv für Begriffsgeschichte* 9: 37-66
- 1966. "Die Voraussetzungen zur Kantischen Urteilstafel in der Logik des 18. Jahrhunderts", *Kritik und Metaphysik Studien*, Berlin: Walter de Gruyter
- 1994. *Kant's Critique of Pure Reason within the Tradition of Modern Logic: A Commentary on its History*. G. Olms
- Vanzo, Alberto. 2010. "Kant on the nominal definition of truth", *Kant-Studien* 101(2):147-66
- Vásquez-Lobeiras, María Jesús. 2001. "Kants Logik zwischen Tradition und Innovation", in *Vernunftkritik und Aufklärung: Studien zur Philosophie Kants und seines Jahrhunderts*, herausgegeben von Michael Oberhausen unter Mitwirkung von Heinrich P. Delfosse und Riccardo Pozzo, Stuttgart-Bad Cannstatt: Frommann-Holzboog, 365–82
- Walther-Klaus, Ellen. 1987. *Inhalt und Umfang: Untersuchungen zur Geltung und zur Geschichte der Reziprozität von Extension und Intension*, Hildesheim u.a.
- Wentworth, George A. 1891. *New Plane and Solid Geometry*, Boston: Ginn & Company
- Wolff, Michael. 1995. *Die Vollständigkeit der kantischen Urteilstafel: mit einem Essay über Freges Begriffsschrift*, Frankfurt am Main: Vittorio Klostermann
- Young, Michael. 1992a. "Functions of thought and the synthesis of intuitions", in *Cambridge Companion to Kant*, ed. Paul Guyer, Cambridge, pp.101-22
- 1992b. "Translator's Introduction", *Lectures on Logic*, Cambridge University Press, xv-xxxii