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Priority, Platonism, and the
Metaontology of Abstraction

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*To Valentina,
Alma and Tobia*

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Chapter 1

INTRODUCTION: PRIORITY, PLATONISM AND ABSTRACTION

1.1 What NeoFregeanism is about

NeoFregeanism is an highly original and much discussed view on the relation between language and reality.¹ It is widely believed that a certain portion of our language, that constituted by assertions, namely sentences that intuitively can be true or false, stands in a certain relation with reality. It is often said that assertions are *about* reality and reality *makes* some assertions true and some other false. These two claims, though extremely appealing, are quite generic. What does it mean for a sentence to be about reality? What kind of relation is in play here? Moreover: what does it mean for a portion of reality to make a sentence true or false? Reality seems to be so discouragingly complex and many-sided that it's not easy to imagine how a certain portion of it can be assigned as a truthmaker for a sentence, while that same portion can be thought as making true another, very different, sentence or the negation of a sentence. These questions have given rise to a variegated and fruitful research program and they are still matter of curiosity and puzzlement.

An interesting side of this problem is constituted by existential statements, namely those sentences in which an existential quantifier features, like, for example 'there is a key in my pocket', 'there are infinitely many prime numbers', 'there are no dragons in the world'. The sentences whose general form is $\exists x_1, x_2, \dots, x_n \phi(x_1, x_2, \dots, x_n)$ are supposed to be made true by the fact that, among the things that

¹ A general introduction to NeoFregeanism can be found in MacBride (2003), Zalta & Linsky (2006), Hale & Wright (2001). Wright (1983) and Dummett (1956) can still be considered fundamental introductory readings.

furnish the world, there are the objects a_1, a_2, \dots, a_n capable of satisfying the conditions imposed by ϕ . It is widely accepted that the truth of a statement of that form *depends* on the occurrence of facts of the kind just outlined. The consequences of this general thesis are very important. Consider the subfield of our natural language constituted by arithmetical statements. Within this domain of discourse, existential statements are quite common and many of them are demonstrably true. ‘There are infinitely many prime numbers’ is a good example of this kind of statements. It is certainly an existentially quantified statement and it is known to be true, at least since Euclid’s time. The general thesis about the relation between existential statements and reality that we have just outlined entails that the statement at issue cannot be true without there being infinitely many prime numbers among the objects of the world. It is the fact infinitely many prime numbers are part of reality that makes that well known theorem true.

This last point may appear puzzling, at least for two reasons. On the one hand the fact that infinitely many prime numbers furnish the world doesn’t seem to be the reason why we believe that the theorem above is true. No serious mathematician would accept such an answer to the question ‘why is it true that there are infinitely many prime numbers?’ The answer that such a question requires is a mathematical proof, namely an argument in behalf of that theorem that respects the epistemic standards holding in mathematical discourse. One may object ‘you are confusing the issue! There are two different questions: one is the question of what makes a statement true and the other is why we believe that such a statement is true. The former is about truthmaking, the second is about epistemic justification’. This might seem to be a good point, but is it really convincing? I could easily

reply that the question worth asking is ‘why don’t we make the same distinction in all the other cases?’ If the sentence at issue were ‘there are finitely many stars in the universe’ we would never consider what makes it true and why we (common speakers) believe that this is true as two different things. The fact that there are finitely many stars in the universe is what makes that sentence true and it’s the reason why we assert that the sentence ‘there are finitely many stars in the universe’ is true. No speaker, unless she is a professional astronomer, knows exactly how to prove that there are finitely many stars in the universe. Anyway a scientific argument in favour of the truth of this sentence is not part of its meaning, unless we embrace a strong verificationist view (and nobody is keen to do so). There’s no serious reason to introduce the distinction in question when only the *meaning* of a sentence is in play; what makes it true and what makes us assert that it’s true are the same. Getting back to our arithmetical example, we are compelled to notice that the existence of infinitely many prime numbers doesn’t seem to be generally taken as the truthmaker of the theorem in question or, equivalently, as the reason why we assert that such a theorem is true. Indeed some philosophers believe that there are no numbers at all; nevertheless they believe that statements like the theorem of prime numbers are true. It seems that, at least within the subfield of arithmetic language, the truth of existential statements may not depend upon a portion of reality being such and such.

The second reason to be puzzled about the thesis that, in general, existential statements are made true by the fact that certain objects furnish the world, is about the enormous ontological inflation that it seems to elicit. We have already cited the case of arithmetical discourse. The thesis at issue poses us in a predicament: either we accept the existence of numbers and so we save the truth of arithmetic

(existential) statements or we favour a sober ontology giving up with the claim that arithmetical (existential) statements are true. It's worth noticing that, in arithmetic, existential statements are not the only ones to be problematic. Every atomic sentence in which number terms occur is subject to this dilemma. For example, '5 is bigger than 3' requires us to choose between its truth and our predilection for a parsimonious ontology. Analogous problems arise in many other domains of discourse outside arithmetic.

NeoFregeanism constitutes an alternative view of the relation between language and reality. As we have already said, the traditional view holds that the truth of existential and atomic statements depend on the existence of some objects. NeoFregeanism reverses this order of explanation, maintaining that the existence of some objects depends on the truth of some existential or atomic statements. Let's try to give a loose formulation of this thesis:

PRIORITY: the following two facts:

- a) the singular term 'a' in the atomic sentence 'Fa' refers to an existing object
- b) the existence of objects satisfying the condition ϕ that features in the sentence ' $\exists x\phi(x)$ '

are *grounded* respectively in the following two facts:

- a') 'Fa' is a true sentence
- b') ' $\exists x\phi(x)$ ' is a true sentence

I purposely use the term general 'ground' to indicate the relation that links the existence of certain objects with the truth of certain sentences. The term, at least according to the most authoritative

authors,² indicates a relation of metaphysical explanation. It's not merely a matter of modal dependence. For sure if ϕ grounds ψ , then if it happens to be the case that ϕ , it is necessarily the case that ψ . Nevertheless this is not a sufficient condition for a grounding relation. Grounding is a relation of metaphysical explanation: if ϕ grounds ψ then, in some sense the definition/essence of ϕ determine the definition/essence of ψ . In the case of Priority, what we mean is that, for example, in reality there are objects that satisfy condition ϕ if the sentence ' $\exists x\phi(x)$ ' is true, because of the very nature of truth and reality. So defined, Priority is a very general thesis; as we are going to see there are various ways to articulate and defend it.

I believe that one of the essential characteristic of NeoFregeanism, intended as a thesis about the relation between language and reality, is the endorsement of Priority. Certainly there are some philosophers, inspired by Fregean philosophy of mathematics, who try to revive the Logicist Program in philosophy of mathematics. Although some of them would apply the label 'NeoFregean' to their own views, I prefer to reserve this term for the philosophical attempts to justify and develop Priority and some other related theses. For the broadly Frege-inspired theories that defend the idea of an epistemic access to mathematical truths by means of sole logic, I would prefer to reserve the term NeoLogicism.

The present doctoral dissertation is exclusively about what I call NeoFregeanism. Its first aim is to examine Priority showing the different ways in which it can be justified. Its second aim requires the introduction of a second key-player in the theoretical landscape that we have just outlined.

² See, for example Fine (2012), Audi (2012) or Schaffer (2009).

There's a natural, though not necessary, link between Priority and an ontological view known as Platonism. When it comes to entities, whose existence is controversial because of their being "immaterial" or "abstract", e.g. numbers, propositions, geometrical shapes, universals, Platonists are those who claim that these entities really exist and their existence is independent from the existence of other things.³ Usually a philosopher is not Platonist without further specification, but rather Platonist about certain entities. Indeed the endorsement of Platonism about, say, numbers doesn't involve to be Platonist about everything else. A Platonist not only believes that there really are the objects she is committed to; she also believes that these objects don't depend upon other objects (like, for example, properties depends on their bearers). As a matter of fact, supporters of NeoFregeanism are Platonist about abstract entities. In particular they believe that Priority offers a robust ground for such a position. In the course of this introduction we are going to see in greater details why they believe so; for the moment it's sufficient to observe that Priority, plus the claim that arithmetical existential theorems are true, entails the existence of natural numbers. Applications of analogous arguments to sentences talking about other abstract entities would lead to analogous results. Hence we can certainly say that Priority is closely related to the following thesis:

PLATONISM : there are self-subsistent abstract objects

Frege was certainly a Platonist about numbers and about other abstract entities. Some passages of his *Grundlagen der Arithmetik* suggest that he believed that the existence of abstract entities is not as demanding

³ There are various notions of independence. Here we adopt a rather generic one; when we speak of 'independence of abstract objects' we mean that their existence is not grounded in our thought and practices. For an introduction to Platonism in Philosophy of Mathematics see Linnebo (2013).

as it could appear. In particular he seems to claim that the existence of some abstract entities amounts to the fact that some absolutely not controversial concrete entities stand in a certain reciprocal relation. Abstraction principles are exactly about this. They are statements of the following form:

$$A(a) = A(b) \leftrightarrow R_{eq}(a,b)$$

The symbol ‘A’ stands for a function assigning abstract objects to the entities designed by the terms ‘a’ and ‘b’, ‘R_{eq}’ is an equivalence relation.⁴ An abstraction principle in general asserts that two objects stands in an equivalence relation if and only if they are associated to the same abstract item. Abstraction principles are the other key-players I was referring to. As we are going to see they play an important theoretical role in the theoretical framework that NeoFregeanism consist in. The first, very intuitive, example of abstraction principle is provided by Frege’s *Grundlagen* and it is the following one:

$$\text{DIRECTION ABSTRACTION: } D(r) = D(s) \leftrightarrow P(r,s)$$

In words: the direction of the straight line r is identical with the direction of the straight line s if and only if r and s are parallel. This is an example of *objectual* abstraction principle, since the domain on which the equivalence relation holds includes only objects. There are also *conceptual* abstraction principles; they differ from the objectual ones for the fact that the equivalence relation applies to a domain whose members are (also) concepts. An example provided by Frege is the following one:

⁴ A relation R is an equivalence relation if and only if i) it’s reflexive, i.e. for every a, R(a,a); ii) it’s symmetric, i.e. for every a,b, if R(a,b) then R(b,a); iii) it’s transitive, i.e. for every a,b,c, if R(a,b) and R(b,c), then R(a,c). Examples of equivalence relations are: identity, parallelism among straight lines, 1-1 correspondence among sets, simultaneity, and many others.

HUME PRINCIPLE: $N(F) = N(G) \leftrightarrow F \equiv G$

In words: the numbers of Fs is identical with the number of Gs if and only if the Fs and the Gs stand in 1-1 relation. Conceptual abstraction principles are notoriously more powerful than objectual ones and potentially unstable.⁵ Some of them play a significant role in some theories of foundation of arithmetic and set theory.⁶

From a purely ontological point of view, abstraction principles are interesting because of the equivalence that they establish between two different states of affairs. The logical symbol employed to signify this equivalence is a normal biconditional. I will use it, because it's quite common in the subject's literature to write abstraction principles in such a way. Nevertheless we clearly attach to the symbol ' \leftrightarrow ' a stronger meaning. Consider, for example, Direction Abstraction. The reason why such a statement is considered so interesting (and philosophically controversial) lies in the fact that, in Frege's intention, its two sides are internally related in a stronger sense than mere identity of truth values signified by the biconditional. They have, in some sense to be made precise, the same meaning. In Hale's words, 'anyone who understands both of them can tell, without determining their truth values individually, that they have the same truth value' (Hale 2001b, p. 13). Certainly this view enjoys an intuitive support, since, presumably, anyone who is ready to accept that two lines are parallel is also ready to accept that they have the same direction and

⁵ Some principles like, for example, the infamous Basic Law V of Frege's *Grundgesetze der Arithmetik*, require the existence of more objects than the domain they apply to contains. They are defined unstable (see Hale & Wright 2001) essentially because, whatever the cardinality of the domain is, they require a larger domain.

⁶ Frege's Theorem asserts that a second order theory enriched with impredicative Hume Principle is equi-interpretable with full second-order Peano Arithmetic. This stunning mathematical result was correctly proven by Frege in its *Grundgesetze*, as shown by Heck (1993). Conceptual abstraction principles has proven to be powerful axioms also for set theory (see, for example, Shapiro 2003).

vice versa (granted that she masters the concepts of <line> and <direction>). Nevertheless it's easy to see that the ontological commitments associated with the two sides of Direction Abstraction differ heavily: the right hand side requires the existence of lines, while the left hand side cannot be true without being the case that there are directions. How can two sentences with so different ontological carry-on have the same meaning or, alternatively, describe the same state of affair?

As we are going to see in the course of this dissertation, there's no easy answer to this question. Robust theorizing is needed in order to face this challenge. What we can say without doubts is that, if a theorist want to employ abstraction principles for her theoretical purposes, she need to find some compelling arguments in behalf of the following thesis:

ABSTRACTION EFFECTIVENESS: some abstraction principles are effective stipulations, i.e. they are such that their two sides share the same content.

Another distinctive feature of abstraction principles is a particular kind of asymmetry between its two sides. The entities mentioned in its right hand side are less problematic than those mentioned in its left hand side. Loosely speaking, the right hand side is the place of concrete entities, while the left hand side is the place of the (relatively) abstract ones. This asymmetry is explanatorily meaningful: for example, the fact that two lines are parallel explains why their direction is the same, while the converse does not hold. The "concreteness" of the right hand side, as we are going to see soon, plays an important role in the theoretical framework we are outlining.

Now, we will leave this problem aside for a moment and focus exclusively on the link between Priority and Abstraction Principles.

As we have already said, Priority is a thesis that reverses what appears to be the natural order of explanation between the truth of certain sentences and the existence of (reference to) certain objects: instead of going from existence of (reference to) certain object to the truth of certain sentences, Priority claims that the truth of certain sentences grounds the existence of certain objects and the fact that certain singular terms really refer. Now, it is widely assumed among the supporters of NeoFregeanism (and not only among them) that each domain of discourse has its own acceptability criteria for sentences.⁷ For example the community of professional mathematicians is inclined to accept a statement only if there is a proof of it that meets certain standards. Mathematical conditions of acceptability are certainly different from the acceptability conditions that rule our discourses about so-called middle size dry goods. In this domain of discourse we are inclined to accept statements that exhibit some kind of correspondence with a fact. A statement is true when it satisfies the ordinary acceptability conditions that hold within the domain of discourse it belongs to. If we couple this widely accepted claim with Priority, we get a thesis according to which, for example, if an existential statement like $\exists x\phi(x)$ meets the acceptability standards ruling the domain of discourse it belongs to, then there really are objects satisfying condition ϕ .

Suppose that a certain abstraction principle, say Direction Abstraction, is effective in the sense specified above and one of its possible instances is such that its right hand side is a true statement. The effectiveness of such a principle assures us that the left hand side (that involved with the existence of directions) is a true statement too.

⁷ Wright (1992) and Lynch (2009) offer sustained arguments in favour of a moderate pluralism about the nature of truth.

Indeed, how can they differ in their truth value if they share the same content? Now, if we endorse Priority we are compelled to admit that there really are directions, since the left hand side of the principle at issue is an atomic sentence in which direction-terms occur. Directions are abstract entities, or, at least, they are abstract with respect to lines. The upshot is that we have just vindicated Platonism, since we have given an example of abstract self-subsistent entities. This line of argument, if correct, provides us with a cheap way to be Platonist about abstract entities.

We can say that Priority, as various authors have noticed,⁸ sets the bar of existence very low. In the example above, the existence of abstract entities, like directions, is justified on the basis of two facts: i) a statement about certain unproblematic entities (lines) is acceptable according to the acceptability standards of the domain of discourse it belongs to; ii) Direction Abstraction is effective. We could say that Priority entails a form of Metaontological Minimalism, according to which, there are object whose existence doesn't impose very demanding requirements to reality. Philosophers committed with Metaontological Minimalism cannot but allow for luxurious ontologies, since, presumably, the argument that we have just outlined can be adapted to many other cases.

In the example that we have just given an essential role is played by an abstraction principle. The couple Priority + Abstraction Effectiveness seems to open an interesting road to Platonism. A further thesis that can be proposed is about the essentiality of abstraction principles for the theoretical framework that we have outlined in these pages. The example given above is such that Direction Abstraction plays an important role in the argument for the

⁸ See, for example, Linnebo (2012b) or Eklund (2006).

existence of directions. Since many abstract entities are susceptible to be defined by means of abstraction principles, one might suppose that our justification for the introduction of abstract object into our ontology necessarily relies on the availability of effective abstraction principles. Nevertheless there seems to be no sufficient reason to be sure of this. Priority seems to be strong enough to support the existence of abstract entities even in absence of sentences presenting all the peculiar feature of abstraction principles. Certainly the atomic sentence ‘the direction of r is F ’, if true according to certain acknowledged standards, entails, in virtue of Priority, the existence of a referent for the singular term ‘the direction of r ’. One may object that, in absence of an abstraction principle, we lack an important norm of correctness for the use of an expression like ‘the direction of r ’. But, is this shortage so dangerous? After all, abstraction principles are not the only way to rule the use of a term referring to an abstract entity. Philosophers who believe that abstraction principles play an essential role in the best arguments aiming at justifying Platonism about abstract entities are committed to this principle.

ABSTRACTION ESSENTIALITY: the best arguments in favour of Platonism about a certain kind of abstract entities require effective abstraction principles.

Many supporters of NeoFregeanism endorse Abstraction Essentiality. Moreover this thesis seems to be a barrier against a possible trivialization of NeoFregeanism. Indeed Abstraction Essentiality seems to restrict the potentially unmanageable power of Priority; only if we provide precise norm of application for abstract terms we are entitled to consider them as really referring.

1.2 What NeoFregeanism is

I take NeoFregeanism to be characterized by the four general theses I already presented and that I restate here to ease the reader:

1) PRIORITY: the following two facts:

a) the singular term 'a' in the atomic sentence 'Fa' refers to an existing object

b) there are objects satisfying the condition ϕ that features in the sentence ' $\exists x\phi(x)$ '

are *grounded* respectively in the following two facts:

a') 'Fa' is a true sentence

b') ' $\exists x\phi(x)$ ' is a true sentence

2) PLATONISM : there are self-subsistent abstract objects

3) ABSTRACTION EFFECTIVENESS: some abstraction principles are effective stipulations, i.e. they are such that their two sides share the same content.

4) ABSTRACTION ESSENTIALITY: an argument in behalf of Platonism about a certain kind of abstract entities require effective abstraction principles.

This is not supposed to amount to a definition. It's very likely that a supporter of NeoFregeanism is committed at least with some of these theses, and almost surely with Priority and Platonism. Nevertheless, the purpose of this characterization is not to give an image of NeoFregeanism such that every NeoFregean philosopher would acknowledge it as the core of her theory. I have simply isolated four theses that are widely maintained among NeoFregean theorist and, more importantly, that are related in such a way that they can constitute the frame for an argument whose conclusion is Platonism.

In the preceding section I have outlined an argument for the existence of directions that, using Priority and Abstraction Effectiveness, proves Platonism about directions. That argument was such that the principle that we have called Abstraction Direction plays an essential role. Therefore such an argument verifies Abstraction Essentiality. Are there different (and shorter) paths to Platonism? Is there a way to prove it such that one is not compelled to commit with Priority? Or with Abstraction Effectiveness?

As we have previously stated, the first aim of this dissertation is to examine Priority and see how it can be justified. Its second aim is to understand how it is related with abstraction principles and, in particular, to try to answer questions like these: is Priority sufficient to prove Platonism or something like Abstraction Effectiveness is necessarily required? Can Abstraction Effectiveness alone prove Platonism? Is there a sound and convincing argument in favour of Platonism relying on all the other three theses ?

I'll try to carry out this complex task by means of a detailed analysis of three alternative theoretical approaches to NeoFregeanism. Each one will be examined in a dedicated chapter. I'll show that each of these approaches performs a successful defence of Platonism, but with significant differences. These differences will be explained in virtue of which of the four theses each approach is able to maintain and adequately justify. The final achievement is going to be, hopefully, a deeper understanding of NeoFregean view.

1.3 Three different approaches

In this section I'm going to show how the three approaches to NeoFregeanism, which are the subject matter of this dissertation, have originated. Contrary to reasonable expectations, this is not going to be

an accurate historical *excursus* on the progressive development of that variegated collection of ideas that constitute contemporary NeoFregeanism. It is going to be rather a systematic analysis of the problems that these three approaches are aimed at solving. I'm going to present two theories that have failed to be a satisfactory defence of a Frege-inspired Platonism and I'll show that one of them can overcome its difficulties in two ways, while the other can do the same only with a radical change of perspective. In this section these three new ways of interpreting NeoFregeanism will be placed into the theoretical context that has made them necessary.

1.3.1 Priority, syntax and ontology

The first philosopher to attribute to Frege a view very close to what we have called “Priority” was Michael Dummett. In his Dummett (1956) he states that the root of the idea of a priority of truth over reference lies in a famous Fregean statement, known as Context Principle. This statement says “Nur im Zusammenhange eines Satzes bedeutet ein Wort etwas,” i.e. ‘only in the context of a sentence does a word have meaning’. This claim occurs in Frege's *Grundlagen der Arithmetik* (§§ 60, 62 and Introduction, p. x) and in no other of his writings. Its true meaning is still matter of a complex discussion that we are not going to touch now. What can be said with absolute certainty is that, from Context Principle, Frege deduces that one must “never ask for the meaning of a word in isolation” (*Grundlagen*, Introduction). In Dummett's view, asking for the meaning of a word in isolation is the mistake made by those who take a statement, split it into its components and, focusing on the singular terms that have been extracted, ask themselves whether these terms *really* refer to something or not. Dummett casts doubt on the legitimacy of this

operation: why do they ask whether a term 'really' refer? Is there a meaningful distinction between 'real' reference and 'apparent' or even 'spurious' reference? If a term acts as a singular term in the context of a sentence, then it must be regarded as a referring term. The distinction between 'really referring' and 'apparently referring' calls for a *philosophical sense of existence* that plays no role in our common linguistic practice.

One of the consequences of A [= the Context Principle] is the repudiation of this philosophical existence. If a word functions as a proper name, then it is a proper name. If we have fixed the sense of sentences in which it occurs, then we have done all that there is to be done toward fixing the sense of the word. If its syntactical function is that of a proper name, then we have fixed the sense, and with it the reference, of a proper name. If we can find a true statement of identity in which the identity sign stands between the name and a phrase of the form "the x such that Fx," then we can determine whether the name has a reference by finding out, in the ordinary way, the truth value of the corresponding sentence of the form "There is one and only one x such that Fx." There is no further philosophical question whether the name - i.e., every name of that kind - really stands for something or not. (Dummett 1956, p. 494)

This point is elegantly restated by Wright (1983):

To suppose that such a question [= does a certain term *really* refer?] does arise is exactly to suppose that it is legitimate to inquire whether such an expression genuinely denote anything in isolation from considerations from the part that it standardly plays in whole propositions. If we think that question arises, then we are asking, in effect, to have it answered by some sort of further independent investigation into the nature of the facts which makes the relevant proposition true: we are asking to show the *Bedeutung* of the expression in isolation. A major point of the Context Principle is to rule out the idea that there is any such further intelligible inquiry to be made. (Wright 1983, pp.14-15)

In Wright's view, Context Principle offers solid grounds for a thesis on the priority of truth over reference, that, following MacBride (2003), we split into three different theses, for clarity reasons:

Syntactic Decisiveness: if an expression exhibits the characteristic syntactic features of a singular term, then such an expression has the semantic function of a singular term (it's a term that "aims at referring")

Referential Minimalism: the mere fact that a referring expression figures in a true atomic sentence determines that there is an item in the world to respond to the referential probing of that expression.

Linguistic Priority: an item belongs to the ontological category of objects if it is possible that a singular term refer to it.

The first of these three theses is the less controversial one. Being a singular term is nothing but acting as singular term and 'acting as a singular terms' is something susceptible of a precise characterization. Indeed Dummett (1973) and, afterwards, Hale (2001a) have presented some effective criteria for the individuation of which sub-sentential expressions act like singular terms. These are merely syntactic criteria; one of them, to give an example, states that, in a sentence of the form Fa , 'a' is a singular term only if Fa supports its existential generalization $\exists xFx$. In other words a necessary condition that 'a' must respect in order to be a referential expression is that, from the truth of Fa , the truth of $\exists xFx$ must follow. This simple requirement excludes that an expression like 'nobody' can play the role of singular term (indeed from 'nobody is playing tennis' doesn't follow the existential generalization 'for some x, x is playing tennis'). The sole fact that an expression is the argument of a functional expression, like

a predicate, doesn't make it a singular term. The criteria presented by Dummett and Hale are able to discriminate between real referring expression and expression that simply occupy the same place of a referring expression.

The difficult job is the justification of Referential Minimalism and Linguistic Priority. As we have already seen, Wright argues in behalf of these thesis on the basis of his interpretation of Fregean Context Principle. His defence, if successful would support a kind of metaontological minimalism, particularly apt to justify a form of Platonism about abstract entities. Nevertheless a problem arises: even if we suppose that Wright's justification of the three theses above is satisfactory, the resulting view is still dubious, because of its wild ontological liberality. The first philosopher who raised this concern was Hartry Field. In his Field (1984) he argues that Context Principle can certainly support what we have called Syntactic Decisiveness, but cannot support Referential Minimalism.

For instance, I cannot see (to paraphrase part of the third paragraph of the passage quoted) how it can be 'a preconception inbuilt into the syntax of our arithmetical language' that '4' is not only a singular term but one which in fact denotes. Is it a syntactic presupposition of our historical language that 'Homer' denotes, or of our religious language that 'God' denotes? Are doubts about the existence of Homer and of God vacuous for that reason? (Field 1984, p. 646)

A justification of Referential Minimalism can come only by means of a stronger thesis.

Strong Priority: if an expression exhibit the characteristic syntactic features of a singular term, then such an expression has the semantic function of a singular term and what is true according to ordinary criteria is really true (any doubt that this is so is vacuous).

Strong Priority is simply the conjunction of Syntactic Decisiveness with the claim that the acceptability conditions that holds good within a certain domain of discourse count as plain truth conditions for every sentence belonging to it. But, again, “did the 'ordinary criteria' for truth in Ancient Greece make 'Zeus is throwing thunderbolts' true whenever there was lightning?” (Field 1984, p. 646). The enormous ontological inflation involved with Priority, as interpreted by Wright, seems to be inescapable. Indeed, some authors⁹ have pointed out that there are true sentences about fictional characters that satisfy the acceptability conditions constraining fictional discourse. For example ‘Sherlock Holmes lives in London’ is acceptable in the domain of discourse constituted by Conan Doyle’s fiction. Does Priority compel us to introduce Sherlock Holmes into our ontology? Wright (1994) has argued that fictional sentences are not properly content bearing, but this answer cannot be completely satisfactory, since a sentence like ‘Sherlock Holmes is a fictional character’ doesn’t belong to the domain of discourse of Conan Doyle’s fiction. It belongs to a domain of discourse *about* fiction and, within this domain, it is certainly acceptable.

There are two possible ways to face this predicament. One is simply to accept the wild ontological inflation imposed by whatever meaningful formulation of Priority and try to show that this doesn’t produce any bad consequence; the other is to weaken Priority with the imposition of further constraints, in order to avoid unpleasant ontological consequences. The former is the solution proposed (not without a hint of scepticism) by Matti Eklund and it will be analyzed in Chapter 3. The latter is proposed by Øystein Linnebo and Chapter 4 is dedicated to it.

⁹ See Williamson (1994b) and Divers & Miller (1995).

1.3.2 Recarving of content

In his works Frege doesn't propose a unique argument in favour of Platonism about abstract entities (and numbers in particular), but a rather variegated set of suggestions, each of them susceptible to broad developments. One of these suggestions has to do with abstraction principles and seems to be, at least at first sight, different and independent from the way of Priority. In *Grundlagen* §64 Frege famously claims that the judgement that line a is parallel to line b can be taken as an identity. This stipulation is made possible by the concept of direction: the parallelism between the two lines "amounts to" an identity of directions. We are in presence of a unique content "carved up" in two different ways. What is essential, in order to give good reasons to accept this claim, is to clarify the meaning of the generic expression 'amounts to'. Hale, in his Hale (1987) and especially (2001b), tried to address two different, but intimately related, challenges: i) on the one hand, we need to understand in what terms need to be translated the metaphorical Fregean expression 'recarving of content'; ii) on the other hand we need to assess whether Frege's theoretical proposal really achieves what it is aimed at.

The crucial question is, obviously, how 'content' should be understood. As is well known, at the time of *Grundlagen*'s composition (i.e. 1884), the distinction between *sense* and *reference* was yet to come. From a certain moment on, Frege decided to split what he previously has called, perhaps naively, 'content' in these two components. It is therefore natural to ask which of these components can be conceived as the matter of the operation of "recarving". Consider a statement like 'line a is parallel to line b ': is the statement 'the direction of a is identical with the direction of b ' a recarving of its sense or of its reference?

Consider first the reference-option. Frege notoriously takes truth values as the reference of sentences. Nevertheless truth values doesn't appear to be things that can be recarved in any meaningful sense. Moreover, in Frege's approach, there are only two truth values (true and false); they really appear to be far too few to cover the intuitively enormous variety of contents that sentences of a given language can express. If we consider the object *true* as the reference of every true sentence then, as Hale points out, 'the direction of line *a* is identical with the direction of line *b*' would bear no closer relation to 'lines *a* and *b* are parallel' than it bears to, say, 'Tuesday precedes Wednesday'. An alternative reading of the notion of content, in light of the reference-option, is to consider states of affairs as the referents of meaningful sentences. Although such a reading finds no textual support in Frege's work, it appear more attractive than the previous one, at least because states of affairs posses enough internal structure to allow sentences, whose meaning are intuitively different, to refer to different things. Moreover the idea that two sentences can constitute different conceptualization of one and the same state of affairs seems to be a seductive way to recast Frege's claim on recarving of content.

Nevertheless, in Hale's opinion, also this solution is unsatisfactory, since it is under the threat of a powerful argument, the so-called "Slingshot Argument". The first statement of such an argument can be found in Davidson (1969). It proceeds as follows: let A and B be two true sentences, then consider these two identities:

- i) the *x* such that *x* is Socrates and A = the *x* such that *x* is Socrates
- ii) the *x* such that *x* is Socrates and B = the *x* such that *x* is Socrates

Now, if we assume that the interchange of co-referential singular terms in a sentence cannot change the state of affairs it depicts, it

follows that sentences i) and ii) depict the same state of affairs. Moreover if we assume a classical notion of logical equivalence it turns out that i) is logically equivalent to A and ii) is logically equivalent to B. Indeed, since the identity between the x such that x is Socrates and the x such that x is Socrates is satisfied in every model, it follows that the set of models in which i) is satisfied is the same set in which A is satisfied and the set of models in which ii) is satisfied is the same set in which B is satisfied. But now we are compelled to conclude that A and B depict the same state of affair. Since we have put very little restriction on the choice of sentences A and B (the only request is that they are both true) it turns out that a true sentence refers to the same state of affairs every other true sentence refers to. This is an obviously unacceptable conclusion, therefore the thesis that sentences refers to states of affairs is flawed.

Hale doesn't regard this argument as a tombstone for the reference option, since some of the assumptions it is based on are contentious. In particular one can object that interchange of co-referential singular terms in a sentence *do* change the state of affairs it depicts, at least in some cases. Indeed, if we adopt a broadly Russellian conception of definite descriptions as devices of quantification, i) can be thought as saying that there is one and only one individual identical with Socrates and such that A and that individual is identical with Socrates. On the contrary ii) says that there is one and only one individual identical with Socrates and such that B and that individual is identical with Socrates. There is little temptation to see the two sentences as describing the same state of affairs. Despite the limited power of the Slingshot Argument, Hale believes that the reference option is a non-starter in virtue of a more fundamental reason. States of affairs are normally taken to be

constituted at least in part by objects; if two states of affairs are such that the objects featuring in them are not the same, then the two states of affairs cannot be identical. Unfortunately, all the interesting cases of content recarving mentioned by Frege require an identity of content between sentences whose singular terms refer to different objects.

This consideration compels Hale to endorse the sense-option, namely to identify Fregean contents with senses. The notion of sense that is in play cannot be too strong. Indeed if one assumes that senses are strongly compositional, then she faces again the problem of the identity between things composed by different parts. There are weak notions of sense that could be apt to justify a claim of identity of content between the two sides of Direction Abstraction. One of them could be the following: two sentences share the same sense if and only if they coincide in truth values at all possible worlds. The two sides of Direction Abstraction clearly coincide in truth value at all possible world. But this is too weak a notion, since two necessary truths whatsoever coincide in truth values at all possible worlds. Hale's need is to find a notion of sense of intermediate force between a strongly compositional one and a purely modal one.

He believed to find it in the notion of *compact entailment* introduced in Hale & Wright (2001) for different purposes. We say that an entailment is compact if and only if it is liable to disruption by uniform replacement of any non-logical constituent in its premises. A more precise statement:

COMPACT ENTAILMENT: A_1, A_2, \dots, A_n compactly entails B if and only if i) A_1, A_2, \dots, A_n entails¹⁰ B and ii) for any non logical

¹⁰ Here the notion of entailment appear without further specification. In cases like this, we generically mean that between two sentences A and B there's a relation such that if A is true B cannot but be true. The notion is hence taken in its full generality, not in its common model-theoretic sense.

constituent η of A_1, A_2, \dots, A_n there is some substitution η'/η which applied uniformly through A_1, A_2, \dots, A_n yields A_1', A_2', \dots, A_n' which *do not* entail B.

Hale's initial idea was to identify sameness of content with *reciprocal compact entailment*: A and B can be said to share the same sense, and therefore to be different carvings of the same content, if and only if A compactly entails B and B compactly entails A. It can be easily seen that this kind of entailment rules out all the cases in which some of the premises are irrelevant for the conclusion, therefore also all the cases of couples of necessary truths don't constitute a threat for Hale's theory.

Now let's have a closer look at the properties of compact entailment. One may wonder whether it is reflexive or not. If A is not a necessary truth then clearly A compactly entails A, since there is surely a uniform substitution of some non logical component of A such that the resulting sentence, A', doesn't entail A. What if A is necessary? In this case compact entailment clearly fails to be reflexive, since A is entailed by every sentence whatsoever. This is an undesirable feature, since it's a platitude that every sentence (whether necessary or not) has the same content of itself. An easy remedy is to refine the definition of compact entailment in this way: an entailment is compact if and only if it's a substitution-instance of an entailment that is compact according to the definition above. To avoid confusion let's rename this new definition of compact entailment 'improved compact entailment'.

IMPROVED COMPACT ENTAILMENT: A_1, A_2, \dots, A_n stands in a relation of improved compact entailment with B if and only if, for some A_1', A_2', \dots, A_n' and some B', A_1', A_2', \dots, A_n' compactly

entails B' and A_1' , A_2' , ..., A_n' and B' are uniform substitution instances of respectively A_1 , A_2 , ..., A_n and B.

Improved compact entailment is reflexive since in the expression 'A compactly entails A', in which A is a necessary truth, I can uniformly replace A with a non-necessary sentence and obtain an expression that compactly entail itself.

Unfortunately, as Michael Potter has remarked, improved compact entailment is not transitive.¹¹ This is bad news for Hale's proposal since sameness of content is supposed to be an equivalence relation, therefore the transitivity of improved compact entailment is a necessary pre-requisite. Hale made another attempt to improve on the situation, by further refinements of the notion of compact entailment, but the cost is a further complication of the notion of entailment involved. His theoretical effort shows that also the sense-option is not easy to pursue.

In such a situation one could be tempted by a radical move: giving up with both reference option and sense option and consider the possibility that sameness of content depends on the theoretical framework we are working with. In a certain theoretical framework two sentences can share the same content even if they do not stand in some complex relation of reciprocal entailment or they do not correspond, at least intuitively, to the same fact. For example, in the theoretical framework of general relativity the sentence 'object a has mass λ ' has the same content of 'object a produces a spatio-temporal camber of size μ ' (with λ and μ suitably chosen). There's no analysis of the sense of the two sentences or of the states of affairs that we intuitively associate with them that can straightforward justify their

¹¹ For a comprehensive explanation see Hale (2001b).

equivalence. Such an equivalence is assumed for general theoretical purposes: the assimilation of the two situations described allows us to explain elegantly many physical phenomena. Agustin Rayo has tried to defend the legitimacy of a view of this kind. Chapter 2 of the dissertation is dedicated to an analysis of his view.

Chapter 2

COMPOSITIONALISM AND ABSTRACT ENTITIES

In this chapter I'm going to examine Agustín Rayo's proposal. His fundamental idea is that abstraction principles should be included into a wider class of propositions called 'just is'-statements. I'll try to explain what these statements are, why we should believe that at least some of them are true and why they are so relevant for the metaontology of abstract objects. After a presentation of Rayo's notion of 'just is'-statement (sections 1-5), I'll outline my proposal (sections 6-8). My claim is essentially that the introduction, via some suitable 'just is'-statement, of abstract entities-talk into a language L is fully compatible with the application of a correspondentist notion of truth on L's statements. In order to prove this, I'll explain how a "correspondentist semantic" for a language that includes 'just is'-statement should be conceived. In the Appendix that closes the chapter, I'll show that the notion of logical consequence that results from this semantics is coherent, complete and compact.

2.1 'Just is'-statements

A very recent view about principles of abstraction and their ontological consequences is Agustín Rayo's Compositionism. Such a view allows its supporters to embrace what Rayo calls Subtle

Platonism, in opposition to Traditional Platonism. The difference between these two positions is sketched with the similitude of a creation myth:

On the first day God created light; by the sixth day, she had created a large and complex world, including black holes, planets and sea-slugs. But there was something left to be done. So on the seventh day she created mathematical objects. Only *then* did she rest. On this view, it is easy to make sense of a world with no mathematical objects: it is just like the world we are considering, except that God rested on the seventh day.

The crucial feature of this creation myth is that God needed to do something *extra* in order to bring about the existence of mathematical objects: something that wasn't already in place when she created black holes, planets and sea-slugs. According to subtle Platonists, this is a mistake. A subtle Platonist believes that for the number of planets to be eight *just is* for there to be eight planets. So when God created eight planets she *thereby* made it the case that the number of the planets was eight. (Rayo, manuscript)

While the traditional Platonist believe that a world without numbers is possible, a subtle Platonist believe that such a world is an impossible one, since for there to be no numbers just is for there to be zero numbers, but zero is a number so numbers exist after all.

Clearly the acceptance of statements of the form ' α just is β ' (where α and β are sentences) is essential to Subtle Platonism. This kind of statement can be tentatively defined as *no difference*-statements, since what they tell is substantially that there is no difference for the world to be such that α is true and to be such that β is true. Some of them are absolutely unproblematic. Consider for example:

WATER: for this glass to be full of water just is for this glass to be full of H₂O.

Here the lack of any difference in meaning between the two atomic statements that flanks the ‘just is’-operator is warranted by the facts that: 1) ‘water’ and ‘H₂O’ are synonyms; 2) ‘water’ and ‘H₂O’ are the only symbols that could have made a difference in meaning, since the remaining parts of the two sentences are identical (we obviously suppose that ‘this’ has the same reference in its two occurrences).

Other ‘just is’-statements are a little more problematic but still acceptable for the majority of people:

PHYSICALISM: for such-and-such a mental state to be instantiated just is for thus-and-such brain states and environmental conditions to obtain.

Here the lack of any difference in meaning is assured by widely accepted scientific theories and (more or less) universally accepted metaphysical assumptions.

There are finally some ‘just is’-statements that are surely controversial:

PROPERTIES: for Susan to instantiate the property of running just is for Susan to run.

Here we are in presence of a highly controversial metaphysical claim, since many philosophers believe that the left hand side member of this statement commits us to the existence of properties, while the right hand side statement doesn’t. So, how can the two sides say the same?

The relevance of the problem of the acceptability of ‘just is’-statements for the purposes of our inquiry is absolutely clear: abstraction principles could also be conceived as ‘just is’-statements. Every reason to legitimate at least some ‘just is’-statement is a reason to legitimate abstraction principles (at least some of them). Consider, for example, the ‘just is’-version of Hume Principle:

HP: for the numbers of Fs to be identical with the number of Gs just is for the Fs and the Gs to stand in one-one correspondence.

Here what is said is that what the two sides of this principle require of the world is exactly the same thing. There's no need to justify the additional ontological commitment that the left hand side of HP seems to bring with it, since there is *no real additional ontological commitment*. Indeed the right hand side of HP is already committed to the existence of numbers; the two sides doesn't differ with regard to meaning but only with regard to "appearance". Rayo thinks that this is probably the more faithful interpretation of Frege's considerations about "recarving of content" (See Frege 1884, § 64).

In Rayo's opinion this, and other metaphysically contentious statements, should be accepted. The reason why they should be accepted is rooted in a general view about meaning and reference that he calls Compositionism and that we are going to examine below. Before exploring this issue let's make the notion of 'just is'-statement a bit clearer.

2. 2 Elucidation of the notion

The best way to elucidate the notion of 'just is'-statement is to explore its relation with other semantic, metaphysical and epistemic notions. According to Rayo, a downright definition is impossible because it would require that the concepts employed to define the *definiendum* were definable independently from the *definiendum* itself. But, when it comes to the fundamental concepts that we need, in order to define 'just is'-statements, circularity is unavoidable to a certain extent. Hence the best we can do is to proceed to the elucidation of the mutual relations between the concepts at issue.

2.2.1 Inconsistency

If α just is β then we cannot consistently claim α and not- β . Here ‘inconsistency’ is not to be understood syntactically. Two sentences can be inconsistent in virtue of their sole logical forms, like in the case of “there’s water in my glass” and “there’s no water in my glass”. But they can be inconsistent also in another sense, namely as a representation of the world as being inconsistent. Consider, for example, “there’s water in my glass” and “there’s no H₂O in my glass”. Here there’s no syntactical inconsistency, but only a representation of an impossible state of affairs. Therefore if α just is β then claiming that α and not- β amounts to a description of an impossible state of affairs. The main difference between a ‘just is’-statement and what we generally call a “factual statement” (a statement like ‘Snow is white’) is that the latter rules out a consistent way for the world to be, while the former rules out only an inconsistent way for the world to be. The set of all ‘just is’-statements that we are inclined to consider true draws the limits of consistency. Since some ‘just is’-statements are clearly *a posteriori*, like in the case of ‘for this glass to be full of water just is for it to be full of H₂O’, to succeed in delineating the limits of consistency is far from being a trivial cognitive accomplishment.

2.2.2 Truth Conditions

If α just is β then the truth conditions of α are identical with the truth conditions of β . A ‘just is’-statement asserts that the two sub-statements that compose it make the same request of the world. What is required of the world, in order for the left hand side statement to be true, is exactly the same that is required for the right hand side to be true.

2.2.3 Metaphysical Possibility

A given scenario is metaphysically possible if and only if it is logically consistent with the set of the true ‘just is’-statements. When we talk about possibility, the adjective ‘metaphysical’ is intended in various ways: it can be an indicator of a particular level of strictness (metaphysical possibility is stricter than conceptual possibility) or, alternatively, it can be an indicator of a kind of possibility. In this latter sense ‘metaphysical’ is meant to indicate a possibility *de mundo*, namely a way the world can be, not a possibility *de representatione*, a way the world can be represented. ‘Just is’-statements draw the limits of *de mundo* possibilities, namely the ways that world can be, regardless of how it happens to be represented.

2.2.4 Why Closure

If α just is β then a question like “we know that α , but what reasons do we have to believe that β ?” becomes meaningless. A true ‘just is’-statement fills a explanatory gap between the two scenarios represented by its left hand side and its right hand side; what explains one of them explains the other too. We say that a sentence σ is why-closed if and only if one is unable to make sense of the question “Why is it the case that σ ?”. If someone says something like ‘I can see that things composed of water are composed of H_2O , but I wish to understand why the world is such as to satisfy this condition’, either we find a charitable interpretation of her request (capable of making the answer not trivial), or we completely reject it and just say ‘this question makes no sense, since to be composed of water is nothing but to be composed of H_2O ’. In order to highlight the connection between the notion of why-closure and that of ‘just is’-statement, we can say

that a sentence σ is why-closed if and only if σ is either a true ‘just is’-statement or a logical consequence of some ‘just is’-statement.

These are the main features of ‘just is’-statements. These features somehow define the role that ‘just is’-statements play in a domain of discourse: they add no information, they simply establish the limit of what can coherently be said. In Rayo’s view ‘just is’-statements play a role analogous to that played by *meaning postulates* in Carnap’s philosophy of language. Meaning postulates draw the line separating propositions that are true in virtue of their meaning from factual ones; ‘just is’-statements separate metaphysically necessary truths from contingent ones. In both cases the problem is to distinguish two fundamentally different class of statements: the considerable advantage of Rayo’s approach is that it doesn’t rely upon the problematic distinction between analytic and synthetic statements. Indeed, as we are going to see in the next section, the choice of the set of true ‘just is’-statements is based on considerations that are not undermined by any Quine-style objection.

2.3 Which ‘just is’-statements are true?

Until now we have made reference to *true* ‘just is’-statements or to the *set of true* ‘just is’-statements without explaining which conditions such statements must respect in order to count as true. Rayo claims that there are essentially two reasons to accept statements of this kind. If the statement in question is ‘ α just is β ’, where α and β differ only for an individual constant, then it’s true if and only if the two individual constants that make the difference have the same reference. Consider the sentence ‘for a spaceship to reach Hesperus just is for it to reach Phosphorus’; clearly the truth of this ‘just is’-statement is

grounded in the sameness of reference of the names ‘Hesperus’ and ‘Phosphorus’, which both refers to the planet Venus. Since, in some cases, sameness of reference is discovered by means of empirical investigations, ‘just is’-statements like that of our example are *a posteriori* necessary truths.¹² However, if the ‘just is’-statement in question cannot be regarded as true in virtue of coincidence of reference of two constants, things are a bit harder.

Consider the statement that we have previously called PROPERTIES, namely ‘for Susan to instantiate the property of running just is for Susan to run’. Here no empirical investigation can be of help; the task of determining its truth value must be carried out relying on entirely different considerations. The criterion that Rayo suggest to adopt is based on considerations of *framework organisation*. As we have just seen, ‘just is’-statements close a theoretical gap and, consequently, they make certain questions meaningless. This singular “power” can be extremely helpful in some cases; indeed there are metaphysical problems whose solution requires worthless theoretical effort. For example, if we believe in the existence of properties or simply we need, for our theoretical purposes, the introduction of entities like properties into our ontology, a typical objection that, fatally, we are supposed to answer is something along these lines: ‘I can see that there are good reasons to think that Susan is running, but I cannot see why we have to think that Susan is instantiating the property of running’. Naturally, there are arguments that can be employed to convince our opponent, but none of them is conclusive. This sole fact could be a good reason to think that there’s no point in trying to present a serious answer to the objection.

¹² At least if we endorse a classical Kripkean view on reference and necessary truths.

A different (and radical) move should be preferred: claiming that the request of the opponent cannot be satisfied, since there is no difference between the fact that Susan is running and the fact that she is instantiating the property of running. This move should not be seen as ontologically inflationary. We are not adding anything to the “furniture” of the world: the existence of the property of running amounts to the same request that we make of the world when we assert that someone is running. To summarize up: ‘just is’-statement can be introduced on the basis of a cost-benefits evaluation. If the introduction of a ‘just is’-statement σ in a theory involves a significant benefit in terms of elegance of the theory itself and a significant reduction of the effort required in order to justify what is claimed, then the truth of σ is vindicated.

2.4 The resulting picture

Now we can easily see what’s the advantage of Rayo’s approach in the case of Hume Principle. The introduction of a ‘just is’-version of HP relieves us of the burden of explaining why, if (and only if) there are pluralities of things, there are also numbers corresponding to those pluralities. Usually, Platonism about numbers (or about other abstract entities) requires justifications that very often turns out to be complicated metaphysical tour de force. If we are not equipped with a good theory of, for example, ontological dependence of numbers on pluralities of things, we might consider the possibility of endorsing Rayo’s picture of metaontology and a ‘just is’-version of HP (call it $HP\equiv$). Thus we obtain a “cheap” Platonism about numbers: their existence is nothing but the same fact that there are pluralities of things, so, when we talk about them, we are not committing ourselves

to the existence of something extra. Therefore number-talk would stand with no need of further justification.

The resulting picture is very close to the conclusions that Rudolf Carnap, in his Carnap (1950), comes to. There is no sense in which things absolutely exist; each theoretical framework requires the existence of different sets of things. Hence, questions on what there really is are to be answered saying something like “according to theoretical framework Γ there are Xs into the world”. Translated into Rayo’s words: there is no sense in which things absolutely exist; it depends on the set of ‘just is’-statements that we adopt. This set of statements (plus the set of statement that constitute a theory) establishes which things a theory is committed to. We are going to see below how exactly the ontological commitment is determined.

In this view ‘just is’-statements can play at least two other roles. First: they can be seen as “bridge principles” that translate certain statements of a certain language into statements of a different language. For example $HP \equiv$ can be seen as a bridge between a second order language with no numerical vocabulary and a second order language provided with numerical vocabulary. $HP \equiv$ translates certain statements of the former into statements of the latter and vice-versa. Second: ‘just is’-statements can help in extending a basic theory into an extended theory which employs a richer vocabulary. As is well known, the introduction of (impredicative) HP, plus some very natural definitions, into a second order theory amounts to the creation of a theory that is equivalent to full second order Peano Arithmetics. Nothing prevents $HP \equiv$ from playing the same role, since its inferential power is certainly not lower than that of HP.

2.5 Metaphysicalism and Compositionism

Some philosophers are definitely hostile towards the kind of broadly Carnapian metaontology that we have outlined in the previous section. Their perplexity is related with a problem of legitimacy of ‘just is’-statements as such. As we have said at the beginning of this chapter, some of them are unproblematic, because the sameness of meaning of their two sides is guaranteed by the sameness of reference of two singular terms and by the fact that nothing else differs. Nevertheless some other, like, for example, PROPERTIES, are not accepted by many philosophers, because it seems that their two sides make very different demands of the world. When it comes to a statement like PROPERTIES, how can the truth conditions of ‘Susan runs’ and ‘the property of running is instantiated by Susan’ be the same? While the first statement is about Susan and what Susan does, the second one seems to be about the relation between Susan and a certain property.

The simplest answer seems to be this: at least one of the two statements is deceptively formulated. More specifically: while the surface grammar of the sentences suggests a certain content, their real content is different and it is somehow disguised by the grammatical appearance. If we translate one (or both) of them into an “ontologically appropriate” language, their real content will clearly appear to be the same. Now, this kind of solution is exactly what Rayo rejects in advance. Indeed, he assumes that the logical form of a sentence can be read more or less straightforwardly from the sentence surface grammar structure. This assumption is justified by the consensus of most linguists, who think that mismatches between “surface” structure and “deep” operative semantic structure are a very limited phenomenon (see, for example, Heim and Kratzer 1998); certainly not the kind of phenomenon that would allow us to us to

claim that sentences like ‘Susan runs’ and ‘the property of running is instantiated by Susan’ have the same logical form. Therefore the perplexity of many philosophers towards statements like PROPERTIES has to be dispelled in a different way.

The first and most important question is: what’s the fundamental reason of this perplexity? The answer to this question is individuated by Rayo in a bunch of philosophical claims that he calls Metaphysicalism. Although it’s not explicitly presupposed, Metaphysicalism has a wide influence on the debate about ontological issues. It’s constituted by two thesis: one about metaphysics and one about reference.

- *Metaphysics*: there is a fundamental way of carving up reality into its constituent parts. An analogy might help: each composite natural number can be decomposed into prime numbers and, for each composite number there is one and only one factorization. Something like that holds for facts: complex facts have a structure, they are composed of simpler parts, which stands in a certain mutual relation, and there is one and only one list of these parts and their mutual relations.
- *Reference*: for an atomic sentence α to be true there must be a certain kind of correspondence between the logical form of α and the metaphysical structure of the portion of reality that α aims at describing.

Supporters of Metaphysicalism are immediately barred from accepting statements like PROPERTIES. Indeed, its two sides have a different logical form, hence they represent facts having a different structure, therefore it’s impossible for them to have the same truth conditions. What is remarkable is that Metaphysicalism rejects statements like PROPERTIES merely on the basis of syntactic considerations, namely

without taking into account their content. To summarize up, we can say that Metaphysicalism is the source of the perplexity of some philosophers towards ‘just is’-statement. To be precise, not Metaphysicalism as a whole. Indeed the principled opposition to ‘just is’-statements comes from the thesis about reference. It should be noticed that such a claim is not entailed by the metaphysical thesis; one can certainly suppose that there is one and only one correct way of carving up a fact A into its constituent parts, without claiming that a sentence that aims at being a correct description of A must have a logical structure that mirrors the structure of A. The metaphysical thesis is clearly independent of the thesis about reference and can be accepted also by those who claim that ‘just is’-statements are not acceptable for merely syntactic reasons. Thus, only the endorsement of the thesis about reference results in a principled rejection of ‘just is’-statements. But according to Rayo there is no reason to endorse such a thesis, therefore there is no room for a principled rejection of ‘just is’-statements.

Rayo claims that the Metaphysicalist thesis about reference should be rejected essentially because it’s an example of bad philosophy of language. His line of argument is essentially as follows: for the purposes of stating a fact, object-talk is optional. Indeed, we can describe one and the same state of affair using a language provided with singular terms and quantification over object-variables, as well as a language containing only predicates. For example, the content of “there is a table” can be as well expressed by the sentence “it tableize” of an hypothetic languages containing only predicates. The *only* reason why we generally prefer a language provided with singular terms and quantification over objects is that it enables us to *recursively* specify the truth conditions of a class of sentences, while a

language containing only predicates doesn't. Therefore languages provided with singular terms and quantification over objects are not preferred because of their purported metaphysical adequacy. No serious argument can show that such a language can describe reality more precisely than any other kind of language. Therefore true sentences are not true in virtue of their mirroring the metaphysical structure of the facts they talk about. Nothing excludes that the property of being true is to be identified with a sort of correspondence with reality. Rayo's point is only that an *isomorphism* between the logical form of a sentence and the metaphysical structure of the fact described is not a necessary condition.

The view about reference, that Rayo proposes, is what he calls Compositionism. Compositionism is constituted by two theses, the former about what counts as a genuine singular term, the second about reference of genuine singular terms:

1. The following three conditions are jointly sufficient for an expression t to count as a genuine singular term: a) t behaves syntactically as a singular term; b) every sentence that one wishes to use and that contains t is provided with truth conditions; c) the assignment of truth conditions is coherent with the logical relations among sentences.
2. If t is a genuine singular term and the world is such as to satisfy the truth conditions that have been associated with the sentence $\exists x(t=x)$, or with any sentence inferentially equivalent with it, then t refers to something.

Compositionism is far more "generous" a view than Metaphysicalism, when it comes to their ontological consequences. To see this, we just have to apply its principles to the controversial case of PROPERTIES. Suppose that the statement 'for Susan to instantiate

the property of running just is for Susan to run' is formulated in a language in which 'the property of running' behaves syntactically as a singular term and such that truth conditions are assigned to every sentence in which this expression occurs (so that conditions 1.a and 1.b are met). Moreover the truth conditions associated with the sentences in which the term occurs are such that if sentence α entails sentence β then what is demanded of the world for α to be true strictly includes what is demanded for β to be true (condition 1.c is met). Now, suppose that we have strong reasons to endorse PROPERTIES (obviously, reasons of framework organization, as illustrated in section 3); it turns out that the truth conditions associated with 'Susan runs' are the same that are associated with 'the property of running is instantiated by Susan'. Hence, if 'Susan runs' happens to be true, then also 'the property of running is instantiated by Susan' is true. There's no need for its logical form to mirror the structure of the same portion of reality which 'Susan runs' aims at describing. Metaphysicalism would require this kind of correspondence while, in Rayo's view, the fulfilling of such a burdening requirement is not necessary. The sentence 'the property of running is instantiated by Susan' is inferentially equivalent to 'for some x, x is instantiated by Susan and x is the property of running' that entails 'for some x, x is the property of running'. Thus also condition 2 is met. According to Compositionism, we are entitled to claim that the singular term 'the property of running' really refers to something. We can conclude that there are such things like the 'property of running', although we don't have any "robust" metaphysical justification for their existence.

Certainly, in Rayo's view, what is required for a singular term to refer to something is not so much. But the fact that the requirements for referentiality are not particularly demanding doesn't mean that the

object to which some singular terms refer are to be considered “thin” in some sense. Dummett’s considerations on this issue are widely known and there’s no need to recap them here; it’s enough to recall that he thinks that we can introduce into our language some abstract terms and the assignation of suitable truth conditions to sentences containing them is sufficient to guarantee that these terms really refer. Nevertheless such a reference is not to be interpreted in a “realist fashion” (see Dummett 1991). Abstract objects, in this sense, are a special kind of things that have been called ‘thin objects’. Although Rayo’s requirement for referentiality are light, I don’t think that, in his view, there are such things as “thin” or “lightweight” objects. Consider again PROPERTIES: the left hand side statement and the right hand side statement are treated as fully symmetric. None of them contains singular terms that refer in weaker sense than the singular terms of the other. Both are a fully accurate description of a state of affairs and the truth of ‘Susan runs’ entails not only the existence of Susan, but also the existence of the property of running since this latter thing is what a singular term of the left hand side of PROPERTIES refers to. A number of passages from Rayo (2013) seem to confirm this.¹³ I think that, in Rayo’s view, nothing prevents

¹³ Consider, for example, what he says at page 5: “One could suggest, for example, that a ‘just is’-statement should only be counted as true if the right hand side “explains the left hand side, or if it is in some sense “more fundamental”. This is not the reading that will be relevant for present purposes”. And again at page 24: “The anty-metaphysicalist is certainly committed to the view that a single feature of reality can be fully and accurately described in different ways. But this doesn’t entail that there is no fact of the matter about how the world is. On the contrary: it is strictly and literally true that the number of dinosaurs is Zero, and therefore that there are numbers. And this is so independently of which sentences are used to describe the world – or, indeed, of whether there is anyone around to describe it. The point is simply that the relevant features of the world could be also fully and accurately described in another way: by asserting ‘there are no dinosaurs’”. This latter quote is directed against a possible misunderstanding of his view (namely the misunderstanding of those who think that the truth of ‘just is’-statements involve a

abstract singular terms, that figure in a true ‘just is’-statement, from referring in a “realist fashion”.

For this reason, one could be tempted to think that he presupposes a correspondentist conception of truth and that such a conception is what grounds the Compositionalist thesis about reference. If an atomic sentence is true, then it’s an accurate description of a portion of reality and therefore every singular term that occurs in it has a reference (even if the logical form of the sentence doesn’t mirror the structure of the portion of reality it aims at describing). If an atomic sentence is such that one of its singular terms doesn’t refer, then it cannot count as a fully accurate description of a fact. I’m not completely sure that Rayo would support this latter claim, but it doesn’t matter. The point that I would like to make is not exegetic, but a rather substantial one: Rayo’s anti-metaphysicalist view is fully compatible with a correspondentist conception of truth. Such a way of conceiving truth is exactly what he needs in order to claim that the two sides of a ‘just is’-statement are descriptions of the same fact. As we already said, a supporter of anti-metaphysicalism doesn’t think that ‘ α just is β ’ is such that only one of its two sides, say β , has to be taken literally, while the other (α) is, at best, assertable on the basis of the fact that β is true. Instead she would claim that both α and β are a fully accurate description of a fact (the same). In other words she thinks their truth is to be intended as correspondence. I think that there’s a way of assigning truth conditions to sentences such that two sentences with a different logical form can have the same truth conditions and such that the truth of a sentence consist in its correspondence to a fact. As we are going to

conception of the world as a structureless blob), but is pretty clear that the point that it makes is extremely relevant also for my purposes.

see in the next section, this thesis requires a revision of the classical rules that we follow when we assign truth conditions to sentences, since these rules are a formalization of the principle according to which the truth of a sentence requires a sort of mirroring between logical form and structure of a fact. Such an alternative way of assigning truth conditions is the topic of the rest of this chapter; I will try to show that ‘just is’-statements can be true in a correspondentist sense of ‘true’ even if we drop the assumption that the ‘correspondence’ involves ‘mirroring’.

2.6 Truth conditions

What I would like to prove is that the following five claims are fully compatible:

- UNIVOCITY: the translation of a sentence of a natural language into a sentence of a formal language is (at least in the vast majority of cases) straightforward and univocal.
- CORRESPONDENCE: a sentence is true if and only if it corresponds to a fact.
- ANTI-MIRROR: for a sentence to be true there’s no need for its logical structure to mirror the structure of a fact.
- MONOTONICITY: if ϕ can be derived, in virtue of its sole logical form, from ψ , then the truth conditions associated with ψ are at least as strong a requirement as those associated with ϕ .
- ACCEPTABILITY: two sentences with a different logical form can have the same truth conditions.

In other words, we need a view of meaning such that some ‘just is’-statements are acceptable (ACCEPTABILITY), because their left hand side and their right hand side correspond to the same fact (CORRESPONDENCE). Obviously this correspondence cannot be

conceived as mirroring (ANTI-MIRROR) since these two sides really, and not only apparently, have different logical forms (UNIVOCITY). Such a view of meaning must respect the same logical requisites that model-theoretic semantics respect (MONOTONICITY).

The main problem is with the compatibility of ACCEPTABILITY and CORRESPONDENCE. Indeed the fact that two different atomic sentences can have the same truth conditions is certainly puzzling: if we follow the classical rules for the determination of truth conditions (those which are usually considered as the paradigm of a realist interpretation of sentences) we cannot but assign them different truth conditions. Take, for example, the two atomic statements that flank the ‘just is’-operator in PROPERTIES: ‘Susan runs’ and ‘the property of running is instantiated by Susan’. Their formal translations are, respectively, ‘Rs’ and ‘I(Rs)’. The classical rule that we adopt, when it comes to assign truth conditions to atomic sentences, can be stated as follows:

RULE: given a model M constituted by a domain \mathcal{D} and a function J , a sentence of the form $Rc_1c_2c_3\dots c_n$, where R is a n -ary predicate symbol and $c_1, c_2, c_3\dots c_n$ are individual constants, is true if and only if the individuals denoted by $c_1, c_2, c_3\dots c_n$ stand in the relation denoted by R , namely if and only if $J(c_1), J(c_2), J(c_3), \dots, J(c_n)$ are members of $J(R)$.

Therefore the truth conditions of our two sentences according to RULE are: 1) for ‘Rs’: ‘Rs’ is true if and only if the individual designated by ‘s’ belongs to the set of things that are R; 2) for ‘I(Rs)’: ‘I(Rs)’ is true if and only if the individuals designated by ‘R’ and ‘s’¹⁴

¹⁴ I employ to different sorts of symbols for individuals and properties: capital letters for properties and lowercase letters for individuals. As we are going to see a basic first order

stand in the relation I. Now, whatever model we adopt, there is no possibility for these two atomic statement of having the same truth conditions. The mismatch between their logical form is sufficient to exclude this eventuality. This sole example is enough to show us that the classical rules for the assignment of truth conditions are incompatible with the truth of ‘just is’-statements like PROPERTIES. No way out based on a distinction between the apparent logical form of a statement and its authentic or “deep” logical form is available. One of the *desiderata* of our theory is that the translation of a sentence into a first order formal language can be simply derived from the surface syntax of the sentence at issue (UNIVOCITY). Since many ‘just is’-statements are such that the logical forms of their left and side and right hand side are different, we face a dilemma: either renouncing most ‘just is’-statements or renouncing to the classical clauses for truth conditions (at least for atomic sentences). If we want to assume that a statement like PROPERTIES *can* be true, we need to drop out of the classical clauses for the assignment of truth conditions and replace them with something more suitable.

I think, and I would like to show, that the best way to do so is the adoption of a semantics where the truth of a sentence depends on its relation to a fact. This could seem hardly a progress, since also classical semantics assumes that a certain sentence is true if and only if there is a fact *that is shaped in such and such a way*. But this latter determination is the source of our problem: classical semantics doesn’t limit itself to assigning truth to a sentence α if and only if there is a fact A that (somehow) corresponds to α . It descend to a deeper level, “unpacking” the fact in question and establishing *how it*

language satisfying our theoretical purposes can be easily enriched with a new sort of variables, becoming able to express contents like ‘the property X is instantiated by x’.

must be shaped in order to be what corresponds to α . One of the *desiderata* that our theory should satisfy is what we have called ANTI-MIRROR, a principle that states that the logical form of a true sentence doesn't need to mirror the metaphysical structure of the fact it correspond to. The classical rule that we follow when we assign truth conditions to an atomic sentence is not consistent with the conjunction of ANTI-MIRROR, UNIVOCITY and ACCEPTABILITY. What we need is to replace RULE with something less "invasive". If we succeed in assigning a fact to a sentence without decomposing the fact itself into finer grained components and conserving the respect of the requisite that we have called MONOTONICITY, then the problem is solved.

In the semantics that I'm going to present, and that I will call *anti-metaphysicalist semantics*, the role of assigning truth conditions to sentences is played by two distinct functions: a function assigning to a sentence α a fact that *weakly satisfies* α and a function assigning to a sentence β a fact that *strongly satisfies* β . We are going to employ the symbol Φ to designate the former and the symbol Ψ to designate the latter. Both Φ and Ψ have sentences as arguments and facts as values. These two kind of semantic relations are to be conceived as fundamental and not analyzable. For the purpose of clarification, we can say that a fact A weakly satisfies a sentence α if and only if the existence of A is sufficient to make α true, and a fact B strongly satisfies β if and only if B is described by β . Two examples can be of help: 1) the fact that whales breastfeed their babies weakly satisfies the sentence 'whales are not fishes', because it is sufficient to make the sentence true; 2) the fact that 23 has no positive divisor other than 1 and 23 strongly satisfies the sentence '23 is prime', because that sentence is a full and accurate description of the fact in question.

Anyway, it's very important to bear in mind that the notions of weak satisfaction and strong satisfaction are elementary and not apt to be analyzed. The previous clarifications are not to be taken as definitions.

A sentence α is true if and only if either Φ or Ψ are defined for the argument α , otherwise is false. Intuitively, if there is no fact that makes α true or that α represents, then α doesn't correspond to any fragment of reality, therefore α is false. One could object: what about logical truths? Do your functions assigns some fact to them? If so, then you are assuming the existence of strange entities like logical facts or (even worse) you are assuming that some contingent facts make valid sentences true. If not, then you are assuming that valid sentences are false. I think that no solution can be completely satisfactory. Since, as we will see, the semantics I'm going to propose is formulated in terms of collections of facts, I assume that Φ assigns valid sentences to a sort of empty collection, a collection containing no facts. This is not to be interpreted as the introduction of a suspect entity like the null fact. I simply mean that Φ is defined for valid sentences (thus valid sentences are true), but there is no particular fact whose existence is needed for them to be true.

The introduction of two distinct functions which aims at assigning semantic values to sentences needs some explanation. Every first order theory includes or, at least, entails some negative statement. In our semantics, a sentence, also a negative one, cannot be true without corresponding to a fact. If the only possible semantic relation between sentences and facts were that of strong satisfaction, encoded by function Ψ , then it would be necessary the postulation of special facts which are fully and accurately described by negative sentences. There's no need to say that negative facts are metaphysically unappealing; the sole fact that a semantic theory requires their

existence can constitute a good reason for its rejection. To avoid this dead-end we introduce another kind of semantic relation, that of weak satisfaction, which allows us to assign semantic values to negative sentences, avoiding any undesirable ontological commitment. This different kind of semantic relation is encoded by function Φ . How can this function make true a negative sentence like, for example, $\neg\alpha$? The answer is very simple: by assigning it to a certain “positive” fact A . If $\Phi(\neg\alpha) = A$, then A is the fact whose sole existence is incompatible with what is expressed by sentence α . Nothing rules out the possibility, for a positive sentence β , of being such that $\Phi(\beta) = A$. In this case we say that the fact A makes true both $\neg\alpha$ and β by being incompatible with what is expressed by α and by being what makes true what is expressed by β . This situation can be illustrated by many examples taken from our everyday linguistic practice. Consider the sentences ‘Bacteria are not eukaryotes’ and ‘Bacteria are prokaryotes’; it’s quite intuitive that they are made true by the same fact, namely that Bacteria are single-celled organisms that lack a membrane-bound nucleus. This single fact is incompatible with their being eukaryote (hence it makes the first sentence true) and is what makes the second sentence true.

Now, what about ‘just is’-statements? We know that the two sides that compose them are in a relation of identity of truth conditions, but now the question becomes more precise: what kind of truth conditions are at issue here? Those assigned by function Φ or those assigned by Ψ ? If we think that the identity of truth conditions between the two sides of a ‘just is’-statement consist in an identity of facts that weakly satisfy those two sides, we leave open the possibility of ‘just is’-statements that are not necessarily true. Consider the previous example: ‘Bacteria are not eukaryote’ and ‘Bacteria are

prokaryote'. They are made true by the same fact, but we are not allowed to put them in a 'just is'-statement, since the result would be a statement that is only contingently true. Indeed it's only contingently true that a living being can be only prokaryote or eukaryote; a third realm, although not actually instantiated by any living being, is possible, at least logically possible. Thus, for something to be eukaryote is not just being not-prokaryote. We are allowed to formulate a 'just is'-statement only when, given two statements α and β , $\Psi(\alpha) = \Psi(\beta)$, namely when α and β are strongly satisfied by the same fact. The relation of strong satisfaction is to be conceived as strictly more demanding than that of weak satisfaction; if a fact A strongly satisfies α then A weakly satisfies α , but the converse does not hold.

The two notions that we have just introduced allow us to replace RULE with something far less demanding, something that doesn't exclude that two different atomic sentences with a different logical form can correspond to the same fact.

RULE*: an atomic sentence α is true if and only if function Φ is defined for the argument α , namely if and only if there is a fact that, even if it doesn't strongly satisfy α , at least weakly satisfies α .

What we need finally is a set of constraint on functions Φ and Ψ capable of saving MONOTONICITY. This is going to be the main purpose of the next section.

2.7 A General Anti-Metaphysicalist Semantics

Suppose we have a *uninterpreted* first order countable language with identity \mathcal{L} , enriched with the just is operator ' \equiv '. The logical operators of \mathcal{L} are \neg , \wedge , \vee , \exists , \forall , $=$ and \equiv . A deductive system is fixed by the

classical introduction and elimination rules for the for the symbols \neg , \wedge , \vee , \exists , \forall , $=$. An introduction and an elimination rule for \equiv will be presented soon. Since \mathcal{L} is countable, the set L whose members are all the sentences of \mathcal{L} is countable too. For simplicity reasons we assume that L contains only closed sentences, no open formula is allowed.¹⁵ Let the set $C=\{a, b, c, \dots\}$ contain all the constants of \mathcal{L} .¹⁶ Let $F=\{A, B, C, \dots\}$ be a set of *atomic* facts. Here the word ‘atomic’ should be understood as ‘undecomposable’. It must not be intended as ‘something that is portrayed by an atomic sentence’, since we have no reason to rule out the possibility of an atomic sentence strongly satisfied by a complex fact or weakly satisfied by a complex fact. The introduction of a set of atomic facts doesn’t necessarily amount to an ontological commitment to facts as fundamental constituent of reality; nothing prevent us from decomposing them in more fundamental entities (like, for example, individuals and properties). The introduction of a set of atomic facts is ontologically neutral; the only metaphysical assumption is that facts, whatever they are, can be decomposed until we reach a basic level, at which a further decomposition would not result in a plurality of simpler facts.

Pluralities of facts can be taken as a whole. For this purpose we introduce the operation \sqcup of union among facts. We are going to write $A \sqcup B$ to indicate the fact constituted by A and B taken as a whole; in some occasions we are going to write $\sqcup(A, B, C, \dots)$ if we are considering the union of many facts. Analogously we can also define an operation of overlap (in symbols, \sqcap). Two unions of atomic

¹⁵ Rayo (2013) p. 67 declares that there can be ‘just is’-statements with free variables, so the constraint that we impose is needed only to simplify our formalization.

¹⁶ If someone wishes to make this language capable of expressing second order sentences, she can add a further set of constants, turning \mathcal{L} into a two-sorted language. For simplicity reasons I restrict myself to the presentation of a first order language.

facts overlap if and only if there is at least one atomic fact that is a proper part of both unions. Unions of atomic facts are complex facts. We introduce now a sort of analogous of the power set: if F , as we have said previously, is the set of all facts, $\wp(F)$ is the set of all the possible unions among the members of F . Let Φ be a partial function $L \rightarrow \wp(F)$ assigning to each sentence that is included into its domain a *minimal* fact that weakly satisfy it. ‘Minimal’ because it’s the minimum union of atomic facts that weakly satisfy α . Suppose that α is weakly satisfied by the fact $A \sqcup B$; then it is of course made true by $\sqcup(A, B, C)$ and also by $\sqcup(A, B, C, D)$. Moreover we can simply say that, for each sentence α such that Φ is defined for α , $\Phi(\alpha) = F$. To avoid this trivialization we establish that, for every α , such that Φ is defined for α , $\Phi(\alpha)$ is the *overlap* of all the members of $\wp(F)$ that weakly satisfy α . The assumption that we make is obviously that this overlap exists and is sufficient to weakly satisfy α ; in other words we assume that a sentence α cannot be made true by two (complex) facts $\sqcup(A, B, C, \dots)$ and $\sqcup(A', B', C', \dots)$ such that their overlap doesn’t exist or is unable to weakly satisfy α .

The function Φ doesn’t need to be injective, since it can be the case that two different sentences are weakly satisfied by the same fact. Moreover it doesn’t need to be surjective, since there’s no reason to assume that the domain of Φ is a set of sentences that correspond to every fact of the world (we don’t rule out the possibility of facts that doesn’t satisfy any sentence of \mathcal{L}).

Function Φ must respect these constraints in order to fulfil
MONOTONICITY:

1. If a sentence α is $a=b$ and Φ is defined for the argument α then, for every sentence $\sigma(a) \in L$ such that Φ is defined for $\sigma(a)$, Φ is defined for $\sigma(b)$ and *vice versa*.
2. If a sentence α is identical with $\neg\beta$ and Φ is defined for the argument α , then Φ is not defined for the argument β and *vice versa*.
3. If α is identical with $\beta\wedge\gamma$ and Φ is defined for the argument α , then Φ is defined for both the arguments β and γ and $\Phi(\alpha) = \sqcup(\Phi(\beta), \Phi(\gamma))$.
4. If Φ is defined for the arguments α and β then it's defined also for the argument $\alpha\wedge\beta$ and $\Phi(\alpha\wedge\beta) = \sqcup(\Phi(\alpha), \Phi(\beta))$.
5. If α is identical with $\beta\vee\gamma$ and Φ is defined for the argument α , then if Φ is not defined for γ then Φ is defined for β .
6. If Φ is defined for α then it is defined also for $\alpha\vee\beta$.
7. If α is identical with $\exists x\beta$ and Φ is defined for the argument α , then: 1) if β contains no variable, then Φ is defined for β and $\Phi(\alpha) = \Phi(\beta)$; 2) if $\beta \leftrightarrow \sigma(x)$, then, for some $c \in C$, Φ is defined for the argument $[\sigma(x)](x/c)$.
8. If, for some $c \in C$, Φ is defined for $[\sigma(x)](x/c)$, then: 1) Φ is defined for $\exists x[\sigma(x)](x/c)$; 2) Φ is defined for $\exists x\sigma(x)$.
9. If α is identical with $\forall x\beta$ and Φ is defined for the argument α , then: 1) if β contains no variable, then Φ is defined for the argument β and $\Phi(\alpha) = \Phi(\beta)$; 2) if $\beta \leftrightarrow \sigma(x)$, then, for every $c \in C$, Φ is defined for the argument $[\sigma(x)](x/c)$ and $\Phi(\alpha)$ is identical with the union of the facts that weakly satisfy these sentences.
10. If, for every $c \in C$, Φ is defined for $[\sigma(x)](x/c)$ then: 1) Φ is defined for $\forall x[\sigma(x)](x/c)$ for every $c \in C$ and $\Phi(\forall x[\sigma(x)](x/c)) =$

$\Phi([\sigma(x)](x/c))$; 2) Φ is defined for $\forall x\sigma(x)$ and $\Phi(\forall x\sigma(x))$ is the union of the facts that weakly satisfy all the sentences $[\sigma(x)](x/c)$

11. If α is identical with $\beta \equiv \gamma$ and Φ is defined for the argument α , then if Φ is defined for the argument β so is for γ and *vice versa*, and $\Phi(\beta) = \Phi(\gamma)$.

Clauses 5, 6 about disjunction and clauses 7, 8 about existential quantification deserve some comments. I take a disjunction like $\alpha \vee \beta$ to be made true by a fact that can be identified neither with what makes true α , nor with what makes true β (and not even with the union of these two verifiers). Indeed if it were the case that $\Phi(\alpha \vee \beta) = \sqcup(\Phi(\alpha), \Phi(\beta))$ then the truth of α would not entail the truth of $\alpha \vee \beta$, since $\alpha \vee \beta$ asks Φ to be defined also for β , not only for α . Clauses 5 and 6 constraint function Φ in such a way that if Φ is defined for α , then it's defined also for $\alpha \vee \beta$ and if Φ is defined for $\alpha \vee \beta$, but not for α , then it's surely defined for β . This says nothing on the nature of facts making true α , β and $\alpha \vee \beta$. Analogous considerations apply to clauses 7 and 8. This move avoids problems with the notion of logical consequence that we are going to introduce later.

Let Ψ be a partial function $L \rightarrow \wp(F)$ that associates each sentence that belongs to its domain to the fact that strongly satisfies it. Also in this case the function at issue is not injective nor surjective.

Here we have the requirements that the function Ψ must satisfy in order to fulfil MONOTONICITY:

12. If sentence α is identical with $a=b$ and Ψ is defined for the argument α then, for every sentence $\sigma(a) \in L$ such that Ψ is defined for $\sigma(a)$, Ψ is defined for $\sigma(b)$ and *vice versa*.

13. If α is identical with $\neg\beta$ and Ψ is defined for the argument α , then: 1) Ψ is not defined for the argument β ; 2) if α is not logically

equivalent with a positive sentence (= a sentence that doesn't start with a negation operator), then there is a $\gamma \in L$ such that γ is positive and $\Psi(\alpha) = \Psi(\gamma)$.

14. If α is identical with $\beta \wedge \gamma$ and Ψ is defined for the argument α , then Ψ is defined for both the arguments β and γ and $\Psi(\alpha) = \sqcup(\Psi(\beta), \Psi(\gamma))$.

15. If Ψ is defined for both the arguments α and β , then it's defined also for the argument $\alpha \wedge \beta$.

16. If $\alpha \leftrightarrow \forall x \beta$ and Ψ is defined for the argument α , then: 1) if β contains no variable, then Ψ is defined for the argument β and $\Psi(\alpha) = \Psi(\beta)$; 2) if $\beta \leftrightarrow \sigma(x)$, then, for every $c \in C$, Ψ is defined for the argument $[\sigma(x)](x/c)$ and $\Psi(\alpha)$ is identical with the union of facts that strongly satisfy these sentences.

17. If, for every $c \in C$, Ψ is defined for $[\sigma(x)](x/c)$ then: 1) Ψ is defined for $\forall x [\sigma(x)](x/c)$ for every $c \in C$ and $\Psi(\forall x [\sigma(x)](x/c)) = \Psi([\sigma(x)](x/c))$; 2) Ψ is defined for $\forall x \sigma(x)$ and $\Psi(\forall x \sigma(x))$ is the union of facts that strongly satisfy all the sentences $[\sigma(x)](x/c)$.

18. If $\alpha \leftrightarrow \beta \equiv \gamma$ and Ψ is defined for the argument α , then if Ψ is defined for the argument β so is for γ and *vice versa*, and $\Psi(\beta) = \Psi(\gamma)$.

19. If α and β are such that $\Psi(\alpha) = \Psi(\beta)$, then Ψ is defined for $\alpha \equiv \beta$.

Moreover there is a further constraint that Φ and Ψ have to respect:

20. If Ψ is defined for the argument α then also Φ is defined for α and $\Psi(\alpha) = \Phi(\alpha)$.

If a couple (Φ, Ψ) satisfies all these 20 constraints then it constitute an *acceptable* interpretation of \mathcal{L} . If an acceptable couple (Φ, Ψ) is such that, for every sentence $\alpha \in L$, Φ is either defined for α or for $\neg \alpha$ we say that (Φ, Ψ) is a *complete* interpretation of \mathcal{L} .

Finally we can define the notion of logical consequence:

LOGICAL CONSEQUENCE: Let \mathcal{J} be the set of all couple of functions that constitute a complete interpretation of a language \mathcal{L} . We say that α is a logical consequence of a set of sentences Γ (in symbols $\Gamma \models \alpha$) if and only if there is no $(\Phi, \Psi) \in \mathcal{J}$ such that Φ is defined for every sentence $\beta \in \Gamma$ and not defined for α .

In the Appendix below I'll prove the theorems of consistency, completeness and compactness for this notion of logical consequence.

2.8 Some conclusive remarks

The anti-metaphysicalist semantics that we have just introduced has the remarkable virtue of satisfying all the five desiderata that we set at the beginning of section 6. It allows us to regard as true all and only the sentence that correspond to a fact (**CORRESPONDENCE**), since the functions Φ and Ψ represent the two different ways in which a sentence can correspond to a fact. It's possible, for two statements with a different logical form, to correspond to the same fact (**ACCEPTABILITY**). The requisite of **MONOTONICITY** is respected by every couple (Φ, Ψ) that satisfies the 20 constraints indicated in section 2.7. Finally, truth conditions are assigned independently of any kind of isomorphism between logical form of sentences and structure of corresponding facts (**NO-MIRROR**); thus the reading of the logical form of a sentence from its surface semantics is allowed and totally harmless (**UNIVOCITY**).

Three final remarks deserve to be made:

Remark 1: Anti-metaphysicalist semantics adopt substitutional quantification. This cannot be avoided, since we don't employ any domain of object over which one can quantify.

Remark 2: The two semantic notions that we employ are left without a definition. I assume that they are primitive concepts. In most cases, when we grasp what a sentence α says, we are able to identify the fact it correspond to and to establish whether it weakly satisfies α true or strongly satisfies α . The criteria that we adopt in doing such a cognitive task are complex and maybe not easily definable. For our purposes this is not a problem, since here we deal only with semantic notions. Indeed our question is ‘how can α mean something?’ and not ‘how do we know what α means?’

Remark 3: The classical assignment of truth conditions to atomic sentences somehow contributes to explain how we manage to understand (and employ appropriately) sentences that we have never heard before. The well known Generality Constraint claims that “if a subject can be credited with the thought that a is F , then he must have the conceptual resources for entertaining the thought that a is G , for every property of being G of which he has a conception” (Evans 1982, p. 104). The classical (model theoretic) assignment of truth conditions fits perfectly with the Generality Constraints, since it clearly shows how the semantic value of an atomic sentence depends on the semantic values of its components. Thus it contributes to explain how a subject that can understand ‘ a is F ’ is able to understand also ‘ a is G ’: the meaning of the first sentence depends on its components and once someone gets the meaning of these components is also able to understand a statements where one of these components is replaced by another one. A supporter of anti-metaphysicalist semantics must be inclined to accept that his semantic apparatus cannot do that. Since, in this view, the smaller meaningful part of the discourse is individuated in a complete sentence, there’s no way in which the meaning attributed to a sentence like ‘ a is F ’ can be of help in explaining why

people who understand 'a is F' are also able to understand 'a is G'. A supporter of anti-metaphysicalism can say that the task of semantics is not to ease the explanation of the mental processes of language understanding. Semantics is exclusively about meaning, not about "what we get when we understand a sentence". We could even say that there are two different notion of meaning of a sentence α : 1) how the world must be in order for α to be true and 2) what we get when we understand α . The supporter of anti-metaphysicalism deals only with the first interpretation of the notion of meaning and is not concerned with the second. This might be considered a remarkable cost, depending on how we are inclined to define meaning.

Appendix

1. Consistency

We adopt the usual inference rules for the logical connectives \neg , \wedge , \vee , \exists , \forall , $=$. The proof of their consistency is completely straightforward and largely overlapping with the classical proof of consistency. The only rule that doesn't appear in classical consistency proofs and that we need to justify explicitly is that associated with the operator \equiv . Such a rule establishes that $\alpha, \alpha \equiv \beta \vdash \beta$.

Proof: whatever function Ψ , that is member of an acceptable pair $(\Phi, \Psi) \in \mathcal{J}$, and that is defined for α and for $\alpha \equiv \beta$ cannot but be defined for β , because whatever acceptable pair (Φ, Ψ) has to respect constraint 18, which says that if Ψ is defined for α and for $\alpha \equiv \beta$, then Ψ is defined for β .

2. Completeness

We want to prove that, for every theory $\Gamma \subseteq L$, and every $\alpha \in L$, if $\Gamma \models \alpha$ then $\Gamma \vdash \alpha$.

Theorem 1

These two statements are equivalent:

- a) if $\Gamma \models \alpha$ then $\Gamma \vdash \alpha$.
- b) if Γ is consistent then there is a pair $(\Phi, \Psi) \in \mathcal{J}$ that satisfies¹⁷ Γ .

Proof

- Proof of a) \rightarrow b): suppose (for *reductio*) that if $\Gamma \models \alpha$ then $\Gamma \vdash \alpha$, that Γ is consistent and that there is no pair $(\Phi, \Psi) \in \mathcal{J}$ that satisfies Γ .

¹⁷ Here the notion of satisfaction is not that of weak satisfaction, nor strong satisfaction that hold only for single sentences, but a different notion: a theory Γ is satisfied by a couple $(\Phi, \Psi) \in \mathcal{J}$ if and only if (Φ, Ψ) is such that Φ is defined for every sentence belonging to Γ .

If there is no pair $(\Phi, \Psi) \in \mathcal{J}$ capable of satisfying Γ , then there must be a sentence β such that $\Gamma \models \beta$ and $\Gamma \models \neg\beta$. Therefore, since we are assuming that if $\Gamma \models \alpha$ then $\Gamma \vdash \alpha$, we have $\Gamma \vdash \beta$ and $\Gamma \vdash \neg\beta$. But this contradicts our previous assumption that Γ is consistent. Hence, if we assume that if $\Gamma \models \alpha$ then $\Gamma \vdash \alpha$ and that Γ is consistent, it turns out that there is a pair $(\Phi, \Psi) \in \mathcal{J}$ that satisfies Γ . From this follows that a) \rightarrow b).

- Proof of b) \rightarrow a): suppose (for *reductio*) that if Γ is consistent then there is a pair $(\Phi, \Psi) \in \mathcal{J}$ that satisfies Γ , that $\Gamma \models \alpha$ and that $\Gamma \not\vdash \alpha$. If $\Gamma \not\vdash \alpha$ then $\Gamma \cup \{\neg\alpha\}$ is consistent. Therefore, since we assume that if Γ is consistent then there is a pair $(\Phi, \Psi) \in \mathcal{J}$ that satisfies Γ , there must be a pair $(\Phi, \Psi) \in \mathcal{J}$ that satisfies $\Gamma \cup \{\neg\alpha\}$. But this contradicts our previous assumption that $\Gamma \models \alpha$. Thus, if we assume that if Γ is consistent then there is a pair $(\Phi, \Psi) \in \mathcal{J}$ that satisfies Γ and that $\Gamma \models \alpha$, it turns out that $\Gamma \vdash \alpha$. From this follows that b) \rightarrow a).

Thus we can prove that, for every theory $\Gamma \subseteq L$, and every $\alpha \in L$, if $\Gamma \models \alpha$ then $\Gamma \vdash \alpha$, simply by proving that if Γ is consistent, then there is a pair $(\Phi, \Psi) \in \mathcal{J}$ that satisfies Γ .

Definition 1

A theory Γ is *complete* relative to L if and only if, for every $\alpha \in L$, either $\alpha \in \Gamma$ or $\neg\alpha \in \Gamma$.

Theorem 2

Every consistent theory $\Gamma \in L$ can be extended to a theory Γ' consistent and complete.

Proof: consider a consistent theory Γ ; we can extend it by adding a sentence $\alpha \in L$ which doesn't belong to Γ and such that $\neg\alpha$ doesn't belong to Γ . The result of the implementation of this procedure is a set of sentences Γ that is consistent (since, in each step

of our procedure the raising of a contradiction is blocked) and complete (since the process doesn't stop until every sentence that doesn't make Γ inconsistent is included).

Definition 2

We define Γ as *saturated* relative to L if and only if Γ satisfies these conditions:

1. $\neg\neg\alpha \in \Gamma$ entails $\alpha \in \Gamma$,
2. $\alpha \wedge \beta \in \Gamma$ entails $\alpha \in \Gamma$ and $\beta \in \Gamma$,
3. $\neg(\alpha \wedge \beta) \in \Gamma$ entails $\neg\alpha \in \Gamma$ or $\neg\beta \in \Gamma$,
4. $\alpha \vee \beta \in \Gamma$ entails $\alpha \in \Gamma$ or $\beta \in \Gamma$,
5. $\neg(\alpha \vee \beta) \in \Gamma$ entails $\neg\alpha \in \Gamma$ and $\neg\beta \in \Gamma$,
6. $\forall x\alpha \in \Gamma$ entails $\alpha(x/c) \in \Gamma$ for every $c \in C$,
7. $\neg\forall x\alpha \in \Gamma$ entails $\neg\alpha(x/c) \in \Gamma$ for some $c \in C$,
8. $\exists x\alpha \in \Gamma$ entails $\alpha(x/c) \in \Gamma$ for some $c \in C$,
9. $\neg\exists x\alpha \in \Gamma$ entails $\neg\alpha(x/c) \in \Gamma$ for every $c \in C$,
10. for every sentence α , Γ doesn't contain both α and $\neg\alpha$,
11. for every formula $\alpha(a, b, c, \dots) \in \Gamma$, if $a=a'$, $b=b'$, $c=c'$, ..., then $\alpha(a', b', c', \dots) \in \Gamma$,
12. $\alpha \equiv \beta \in \Gamma$ and $\alpha \in \Gamma$ entail $\beta \in \Gamma$.

Theorem 3

If Γ is consistent and complete then Γ is saturated.

Proof:

1. If Γ is complete, then either α or $\neg\alpha$ belongs Γ . But $\neg\neg\alpha \in \Gamma$ and, since Γ is coherent α belongs to Γ . Indeed if $\neg\alpha$ belonged to Γ (instead of α) then Γ would include both $\neg\alpha$ and $\neg\neg\alpha$ losing, thus, its coherence.

2. If $\alpha \wedge \beta \in \Gamma$ and Γ is complete, then one between α and $\neg\alpha$ and one between β and $\neg\beta$ belong to Γ . But neither $\neg\alpha$ nor $\neg\beta$ belongs to Γ , since Γ is coherent. Therefore Γ includes α and β .
3. If $\neg(\alpha \wedge \beta) \in \Gamma$ and Γ is complete, then one between α and $\neg\alpha$ and one between β and $\neg\beta$ belong to Γ . But if both α and β belonged to Γ , then Γ would not be coherent. Therefore at least one between $\neg\alpha$ and $\neg\beta$ belongs to Γ .
4. If $\alpha \vee \beta \in \Gamma$ and Γ is complete, then one between α and $\neg\alpha$ and one between β and $\neg\beta$ belong to Γ . But if both $\neg\alpha$ and $\neg\beta$ belonged to Γ , then Γ would not be coherent. Therefore at least one between α and β belongs to Γ .
5. If $\neg(\alpha \vee \beta) \in \Gamma$ and Γ is complete, then one between α and $\neg\alpha$ and one between β and $\neg\beta$ belong to Γ . But if at least one between α and β belonged to Γ , then Γ would not be coherent. Therefore both α and β belong Γ .
6. If $\forall x \alpha \in \Gamma$ and Γ is complete then, for every $c \in C$, either $\alpha(x/c)$ or $\neg\alpha(x/c)$ belongs to Γ . But if there were a constant $k \in C$ such that $\neg\alpha(x/k)$ belongs to Γ , then Γ would not be coherent. Therefore $\alpha(x/c) \in \Gamma$ for every $c \in C$.
7. If $\neg\forall x \alpha \in \Gamma$ and Γ is complete then, for every $c \in C$, either $\alpha(x/c)$ or $\neg\alpha(x/c)$ belongs to Γ . But if, for every $c \in C$, $\alpha(x/c)$ belonged to Γ , then Γ would be incoherent. Therefore, at least for one constant $k \in C$, $\neg\alpha(x/k)$ belongs to Γ .
8. If $\exists x \alpha \in \Gamma$ and Γ is complete then, for every $c \in C$, either $\alpha(x/c)$ or $\neg\alpha(x/c)$ belongs to Γ . But if there were no constant $k \in C$ such that $\alpha(x/k)$ belonged to Γ , then Γ would not be coherent. Therefore $\alpha(x/c) \in \Gamma$ for some $c \in C$.

9. If $\neg\exists x\alpha \in \Gamma$ and Γ is complete then, for every $c \in C$, either $\alpha(x/c)$ or $\neg\alpha(x/c)$ belongs to Γ . But if, for some $c \in C$, $\alpha(x/c)$ belonged to Γ , then Γ would be incoherent. Therefore, for all constant $k \in C$, $\neg\alpha(x/k)$ belongs to Γ .
10. If $\alpha \in \Gamma$ and Γ is coherent and complete, then Γ cannot include $\neg\alpha$.
11. If $\alpha(a, b, c, \dots) \in \Gamma$ and Γ is complete, then, if $a=a'$, $b=b'$, $c=c'$ either $\alpha(a', b', c', \dots) \in \Gamma$ or $\neg\alpha(a', b', c', \dots) \in \Gamma$. But if $\neg\alpha(a', b', c', \dots)$ belonged to Γ , then Γ would be inconsistent. Therefore $\alpha(a', b', c', \dots) \in \Gamma$.
12. If $\alpha \equiv \beta \in \Gamma$ and $\alpha \in \Gamma$ and Γ is complete, then either β or $\neg\beta$ belongs to Γ . But if $\neg\beta$ belonged to Γ then Γ would be inconsistent. Therefore $\beta \in \Gamma$.

Theorem 4

If a theory Γ is saturated then there is a couple $(\Phi, \Psi) \in \mathcal{J}$ that satisfies Γ .

Proof: suppose Γ is saturated, but there is no couple $(\Phi, \Psi) \in \mathcal{J}$ that satisfies Γ . Then there must be at least one sentence γ such that no member of \mathcal{J} can assign a fact to γ . But if no member of \mathcal{J} can do so, then the assignation of a fact to γ would violate at least one of the 20 constraints that functions Φ and Ψ must respect. But the violation of one of these constraints would necessarily lead a complete theory (like Γ) to inconsistency. But if Γ were inconsistent, then Γ would not be saturated. This would contradict our hypothesis. Therefore if Γ is saturated then there is a couple $(\Phi, \Psi) \in \mathcal{J}$ that satisfies Γ .

Theorem 5

If there is a couple $(\Phi, \Psi) \in \mathcal{J}$ that satisfies a theory Γ then such a couple satisfies every Λ such that $\Lambda \subseteq \Gamma$.

Proof: if there were a set of sentences $\Lambda \subseteq \Gamma$ such that a couple $(\Phi, \Psi) \in \mathcal{J}$ that satisfies Γ doesn't satisfy Λ , then there would be at least a sentence γ belonging to Λ such that γ doesn't fall into the domain of one between Φ or Ψ . But, if $\Lambda \subseteq \Gamma$, then $\gamma \in \Gamma$. Since (Φ, Ψ) satisfies Γ , then it cannot but satisfy γ . Therefore if (Φ, Ψ) satisfies Γ then (Φ, Ψ) satisfies every Λ such that $\Lambda \subseteq \Gamma$.

Theorem 6 (Completeness)

If Γ is consistent then there is a couple $(\Phi, \Psi) \in \mathcal{J}$ that satisfies Γ .

Proof: if Γ is coherent, then it can be extended until it becomes complete (Theorem 2); if such extension is complete, then it's saturated (Theorem 3); if it's saturated then there is a couple $(\Phi, \Psi) \in \mathcal{J}$ that satisfies it (Theorem 4) and such a couple satisfies also the original theory Γ (Theorem 5).

3. Compactness

A couple $(\Phi, \Psi) \in \mathcal{J}$ satisfies Γ if and only if it satisfies every finite subset of Γ 's sentences.

The theorem's enunciate is a biconditional, so we are going to give a proof for both the conditional and the converse.

If a couple $(\Phi, \Psi) \in \mathcal{J}$ satisfies Γ then it satisfies every finite subset of Γ 's sentences.

Proof: same of Theorem 5 of Completeness.

If a couple $(\Phi, \Psi) \in \mathcal{J}$ satisfies every finite subset of Γ 's sentences then it satisfies Γ .

Proof: suppose that a couple $(\Phi, \Psi) \in \mathcal{J}$ satisfies every finite subset of Γ 's sentences, but doesn't satisfy Γ . Then there must be at least one sentence γ belonging to Γ such that γ doesn't fall into the domain of one of the functions Φ, Ψ . But, since $\gamma \in \Gamma$ there must be at

least one finite subset $\Lambda \subseteq \Gamma$ such that $\gamma \in \Lambda$. Therefore (Φ, Ψ) doesn't satisfy Λ . But this contradicts our assumption that every finite subset of Γ is satisfied by (Φ, Ψ) . Therefore if a couple $(\Phi, \Psi) \in \mathcal{J}$ satisfies every finite subset of Γ 's sentences, then it satisfies Γ .

Chapter 3

MAXIMALISM AND VAGUE EXISTENCE

In this chapter I'm going to present an important view on the Priority Thesis and its metaontological consequences. This view is known as Maximalism. In section 1 I will illustrate the ontological inflation caused by the endorsement of Priority Thesis. Section 2 is devoted to Maximalism and its main problem: the existence of incompatible objects. A possible solution, consisting in the adoption of a notion of vague existence, is outlined. In section 3 I will present the two main difficulties that Maximalism, enriched with a theory of vague existence, has to overcome. Section 4 is devoted to a detailed presentation of a framework for vague quantification. In section 5 I will show how this framework overcomes the difficulties that threaten the notion of vague existence. In section 6 I will draw some provisional conclusions.

3.1 Priority and Metaontology

A widely held opinion about the Priority Thesis is that it commits us to a rather promiscuous ontology. Indeed, if truth is constitutively prior to reference, whenever an atomic sentence satisfies the criteria of truth that *ordinarily* rule the domain of discourse which it belongs to, its singular terms really refer. For example, if a numerical statement

like ‘ $7+5=12$ ’ is true according the criteria of arithmetic, then its singular terms really refer. We are compelled to maintain that terms like ‘5’, ‘7’ and ‘12’ stand for really existent things. Mathematical Platonism is a natural consequence of the endorsement of Priority Thesis. Those who hold a theory like this generally think that nominalists deal with the problem of ontology of mathematics in a completely wrong way. Nominalists generally think that reference is prior to truth; the latter depends on the former. For example, Field (1980) claims that arithmetical statements are simply false, because the singular terms featuring into them are empty. The reason why arithmetic (and mathematics in general) is useful has nothing to do with truth or falsity, but with a different and more complex characteristic that all the demonstrated mathematical statements share: conservativity. Wright (1992) argues against nominalism claiming, among many other things, that within mathematical discourse the ordinary criterion of truth is in fact conservativity. For a mathematical statement, to be true is nothing but to be conservative. Hence mathematical statements are true, after all. The fact that they are true and the endorsement of the Fregean context principle (“never ask for the meaning of a word in isolation; only in the context of a sentence does a word have meaning”; see Frege (1884)) compel us to regard the singular terms featuring in true atomic mathematical sentences as really referring ones. In general, a supporter of Priority Thesis thinks that the right order of explanation is from truth to reference, while the nominalist proceed erroneously in the reverse order.

A natural question is: “why is mathematics so special? Why don’t we employ the same approach with other domains of discourse?” The final landing place of a this train of thoughts is that, for every domain of discourse, if a sentence satisfies the relevant norm

of acceptability, then its singular terms really refer. This position leads to a form of *ontological maximalism*. It's not surprising that Divers & Miller (1995) and Williamson (1994b) claim that the adoption of Priority leads to the ontological commitment to fictional characters. Even fictional discourse has a relevant norm of correctness, a norm in virtue of which, for example, it's true that Sherlock Holmes lived in London and false that he lived in Paris. A supporter of Priority seems to have no robust reason to refuse the commitment to Sherlock Holmes and similar entities. Most philosophers believe that this is a really unpleasant consequence. Hale & Wright (2009) and Wright (1994) deny that fictional contexts are properly content bearing; there cannot be true sentences in fictional discourses. They argue that the presence (even if implicit) of quotational, modal or fictional operators somehow disturbs the referential mechanism, preventing singular terms from working properly. Here we do not want to discuss this issue. We simply concede, for the sake of the argument, that Wright and Hale are right in thinking that fictional discourse cannot be properly true, even if it apparently displays its own norm of correctness. The ontology a supporter of Priority Thesis is committed with is still a luxurious one, since it includes, at least, every kind of abstract object that we can coherently define. The sensation is that this can be a source of not negligible philosophical problems.

3.2 Maximalism and Incompatible Objects

Eklund (2006) has the merit of showing that this is not just a sensation. He defines metaontological maximalism in a quite intuitive way:

MAXIMALISM: for a given sortal F, Fs exist just in case (a) the hypothesis that Fs exist is consistent and (b) Fs do not fail to exist simply as a matter of contingent empirical fact.

Condition (b) shows the endorsement of Hale and Wright's argument against the commitment to fictional entities. A supporter of Priority may be committed to intuitively strange objects, but she is not committed to believing in objects that we have empirical reasons not to believe in. Condition (a) is an alternative way of expressing the idea that every sentence that respect the norm of correctness that rule the domain of discourse it belongs to is such that its singular terms really refer. Indeed, a discourse about abstract entities has consistency as its unique norm of correctness. Every theory about a kind of abstract objects, no matter how weird they could appear, needs only to be consistent in order to be acceptable. Therefore, according to Maximalism, every coherently definable sortal whatsoever is instantiated by some individual. Maximalism can be considered a consequence, at metaontological level, of Priority.

Eklund shows that such a metaontological theory is flawed, since there is an entire class of convincing counterexamples to the thesis that everything is coherently definable must exist. This class is constituted by incompatible objects.

INCOMPATIBLE OBJECTS: sortals F and G are incompatible if and only if (a) if there are Fs then there are no Gs and (b) if there are Gs there are no Fs.

A couple of incompatible sortals is such that the fact that one of them is instantiated is sufficient to exclude that the other has instantiations. Now, suppose that both the mutually incompatible sortals are coherently defined. It's easy to see that this would turn out to be a

serious problem for Maximalism. According to this metaontological view, if two sortals are coherently defined they must be both instantiated. But incompatible sortals cannot be both instantiated. Therefore either there are no convincing examples of coherently defined incompatible objects, or Maximalism is unacceptable. If Maximalism is unacceptable, then Priority has to be rejected, since the former is a consequence of the latter. Unfortunately for the supporter of Maximalism there seems to be plenty of convincing examples of mutually incompatible (and individually consistently definable) sortals.

A well known example can be found in Boolos (1990). Consider the following statement:

HUME PRINCIPLE: $N(F) = N(G) \leftrightarrow \text{EqFG}$

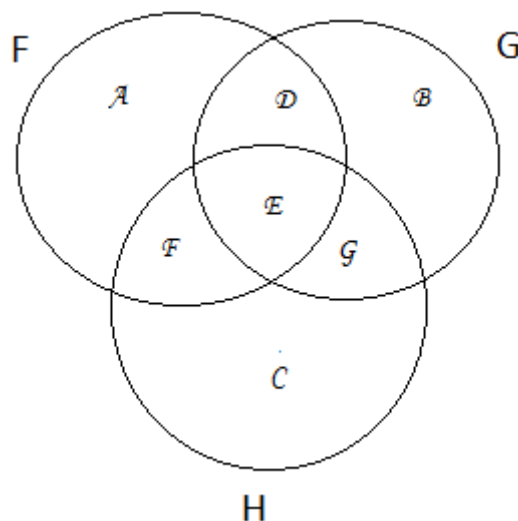
It says that the number of Fs is identical with the number of Gs if and only if the Fs and the Gs are in one-one correspondence. This principle has no finite model. If we interpret it in a model whose domain includes only a finite number of objects it becomes inconsistent. To see this, consider a domain containing three objects: a, b, c. The extensions of the concepts over which our second order quantifiers quantify over are the following ones: \emptyset , {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}. The relation of equinumerosity groups them into four classes: 1) concepts with empty extension; 2) concepts one-instantiated; 3) concepts two-instantiated; 4) concepts three-instantiated. To each class Hume's Principle makes it correspond a number. Therefore the domain should include at least four objects, but it has only a, b and c. A generalization of this argument on a domain constituted by n objects (where n is finite) is completely straightforward. We will obtain a grouping of concepts into n+1

classes: the class of concepts zero-instantiated, that of concepts one-instantiated, ..., that of concepts n-instantiated. Therefore there is no finite domain in which Hume's Principle holds.

Now consider another abstraction principle:

PARITY PRINCIPLE: $P(F) = P(G) \leftrightarrow \text{Diff}FG$.

It says that the parity of the concept F is identical with the parity of the concept G if and only if the Fs and the Gs differ evenly. The relation of evenly differing is defined as follows: two concepts F and G differ evenly if and only if the number of objects falling under F, but not under G, or falling under G, but not under F, is finite and even. The relation of evenly differing is an equivalence relation since it is: i) Reflexive: F and F differ evenly because they differ of 0 objects and 0 is a finite and even number; ii) Symmetric: if F and G differ of a even number of objects then obviously G and F differ of an even number of objects too; iii) Transitive: if F and G differ evenly and G and H differ evenly, then F and H differ evenly too. Since it's not immediately evident that the relation of evenly differing is transitive look at the picture below.



The picture represents the three concepts with their mutual intersections. The letters into the various areas of the picture represent the number of objects included in that area. If the concepts F and G differ evenly then $A+F+B+G$ is an even number. Moreover if the concepts G and H differ evenly then $B+D+F+C$ is an even number. The sum of two even numbers is an even number too, so $A+F+B+G+B+D+F+C$ is an even number. If we subtract from it the factor $2(F+B)$, we are subtracting an even number from an even number, obtaining an even number again. We obtain $A+G+D+C$ which is the number of things falling under F but not under H or falling under H but not under F. Therefore F and H differ evenly. The relation of evenly differing is transitive.

Now, suppose we interpret this principle with a model whose domain is an infinite set, for example, the set of natural numbers. In every case in which two concepts differ of an infinite numbers of elements they don't have the same parity, since the numbers of elements that they don't share is infinite, hence neither even nor odd. Consider a subset Σ of the set of natural numbers constituted by the powers of each prime number:

2 2^2 2^3 2^4 2^5 2^6 2^7 2^8 ...

3 3^2 3^3 3^4 3^5 3^6 3^7 ...

5 5^2 5^3 5^4 5^5 5^6 ...

7 7^2 7^3 7^4 7^5 ...

11 11^2 11^3 ...

13 13^2 ...

...

Consider the concept of *being into the series of p*, where p is a prime number. A member of Σ is into the series of p if and only if it's a power of p. In the representation above, each line is the extension of a

concept of this kind: the first line represent the extension of the concept of being into the series of 2, the fourth line represent the extension of the concept of being into the series of 7, and so on. It's easy to see that the parity of each of these concepts is different from the parity of every other, since two different series cannot but differ by infinite elements, hence they cannot differ evenly. So there are at least as many parities as prime numbers. But there's more. Nothing prevents us from setting concepts like 'being into the series of 7 or into the series of 53 or into the series of 217'. Such a concept has a different parity from the concept of 'being into the series of 19 or the series of 29 or the series of 73. The same hold for every analogous disjunctive concept whatsoever. Since the cardinality of prime numbers is \aleph_0 , there are 2^{\aleph_0} disjunctive concepts like those of the example.¹⁸ This claim is an obvious consequence of Cantor Theorem on the cardinality of power sets. Each disjunctive concept of the envisaged kind is associated by the Principle of Parity with a different parity, since there are no couples of such concepts that differ evenly. So there are 2^{\aleph_0} parities. But the members of Σ are \aleph_0 , hence there are more parities than members of Σ and this is impossible. We can conclude that the Principle of Parity has no model of infinite cardinality. It can hold only into the context of a finite domain.¹⁹

Therefore Hume's Principle and Principle of Parity are incompatible. There cannot be a domain (or, if you prefer, a possible world) where both the concept of number and the concept of parity are instantiated. The problem for the supporter of Maximalism is that both Hume's Principle and Parity Principle are consistent. Indeed they both

¹⁸ At least if the Continuum Hypothesis is true.

¹⁹ This sketch of proof of the unsatisfiability of Parity Principle within an infinite domain differs from that of Boolos (1990), which is more compact but far less intuitive.

have a model. Therefore there should exist both parities and numbers but, as we have just shown, these sortals are incompatible.

This example shows that Maximalism has problem with incompatible objects. A way of saving this metaontological view from collapse is to show that, properly speaking, there are no incompatible objects. Consider again the example of Hume Principle vs Parity Principle. One could say that thinking of the universe as constituted by a finite number of things is incompatible with the exigencies of Maximalism. Since Parity Principle requires a finite number of objects in the universe, it cannot be considered true, although it's consistent. Thus there are numbers, but there are no parities. This response is far from being convincing. Maximalism clearly establishes that, for an object to exist, very little is required. The only thing required is consistency, not compatibility with the exigencies of a certain metaontological view (Maximalism itself in this case). A more refined response could be that Hume Principle, unlike Parity Principle, satisfies some good requisite. A good requisite could be Conservativity.

CONSERVATIVITY: A statement α is conservative, with respect to a theory T , if and only if $T \cup \{\alpha\}$ doesn't prove any theorem, expressible in T , that T alone was not able to prove.²⁰

The introduction of Conservativity as a standard that good definitions of abstract objects must respect is not an *ad hoc* move aimed at saving Maximalism from collapse. Conservativity can be independently motivated; for example Field (1980) shows that all the classical mathematical definitions are conservative. As demonstrated by Wright

²⁰ A precise definition of Conservativity is a bit different from this one, since it requires the mention of the language in which T and $T \cup \{\alpha\}$ are formulated. Consider the definition above as an intuitive approximation, sufficient for our purposes.

(2001) Hume Principle is conservative, while Parity Principle is not. This could appear a very good point in favour of the claim that there are numbers but not parities, but unfortunately it's not. Shapiro and Weir (1999) provide an example of a couple of mutually incompatible abstraction principles, each of which is consistent and conservative.²¹ It seems that appeal to Conservativity cannot be a solution.

A drastic strategy could be employed: since the problem of incompatible objects seems to involve essentially objects defined by abstraction principles, we could declare abstraction principles illegitimate. A Neo-Logicist philosopher would not be happy about that, but this is not a problem in the context of our discourse. We are not dealing with foundation of Arithmetic, but only with metaontological problems. Unfortunately also this drastic move is insufficient. There are various incompatible objects that can be defined with no appeal to abstraction principles. Eklund (2006) proposes some examples. Consider the concept of *anti-number*, an entity whose existence both supervenes on anything one likes and rules out the existence of numbers. Numbers and anti-numbers are clearly incompatible objects and their definition doesn't require the employment of abstraction principles. Another example: let *xhearts* to be almost like hearts, except that they exist only if *xlivers* do not and *xlivers* to be almost like livers, except that they exist only if *xhearts* do not. This latter example could be dubious because the *xhearts* and *xlivers* are defined circularly. Nevertheless Eklund notices that there are examples of successful circular definitions (see Yablo (1993)) and, moreover, that, since Maximalism is a radically promiscuous ontology, it offers no basis for a rejection of circular definitions.

²¹ I omit the details because it's a very complex example and it would require a long explanation.

Definitions like those of xhearts and xlivens suffer from an absence of *ground*,²² but they are not properly inconsistent. Hence, from a Maximalist point of view, they are perfectly acceptable.

It should be clear that there's no way to reject incompatible objects, once Maximalism is endorsed. The only possible escape seems to be the theorization of a special kind of *indeterminacy*. One could claim that some objects are borderline cases of existent things. Incompatible objects could be an example of things whose existence suffers from a sort of vagueness; it's indeterminate whether, for example, xhearts exist and xlivens do not or xlivens exist and xhearts do not.

Here is a suggestion: what we have here is a special kind of *indeterminacy*. Suppose we have a purported case of incompatible objects to which the maximalist has no other satisfactory answer. For concreteness suppose the xhearts/xlivens case to be such. Then the maximalist can say that it is simply indeterminate whether it is xhearts or xlivens that exist. She can even insist that she has independent reason to say this. Can xhearts consistently exist, given the empirical facts? That is, do xhearts satisfy the minimal conditions that some purported objects must satisfy to exist, given maximalism? It can be said: they do only if xlivens do not exist. *Mutatis mutandis* for xlivens. There is nothing that determines whether it is xhearts or xlivens that exist. So it is indeterminate which exist. But indeterminacy the maximalist can live with. [Eklund (2006), p. 113]

Eklund thinks that this solution doesn't look promising, since it encounters at list two obstacles: on the one hand, it seems to be very hard to find a precise and coherent notion of existential indeterminacy; on the other hand, even if such a notion is available, there is a further example of incompatible objects that poses a serious difficulty for a

²² I use the term 'ground' in analogy with Kripke (1975)'s use. Here we are in a situation of absence of ground, because the existence of xhearts relies on the inexistence of xlivens, that, in turn, relies on the existence of xhearts.

version of Maximalism equipped with existential indeterminacy. In the next section we are going to explain what these two difficulties consist in.

3.3 Two challenges for vague existence

3.3.1 Sider's Objection

The notion of vague existence is still controversial, not because it's difficult to give intuitively convincing examples of borderline cases of existent things, but because the coherence of the notion itself is disputed. The most known and discussed objection against vague existence is due to Sider (2003). It's a very refined argument and it deserves to be thoroughly examined. To my knowledge, the best semi-formal reconstruction of Sider's argument is provided by Torza (forthcoming), so my exposition will follow closely his presentation.

The first part of the argument says that quantifier vagueness would be completely different from the already known cases of vagueness. It would be a completely *sui generis* case of vagueness; it would require an entirely different model from the usual one. The usual model is characterized by *precisifications*. Those who have thought hard on vagueness converge on the claim that its formalization requires precisifications, no matter whether they are supervaluationist or epistemicist or nihilist. There are alternative accounts of vagueness that doesn't involve claims about precisifications (e.g. degree-theoretic construal with fuzzy logic), but they can be refuted on the basis of independent considerations.²³

When I meet a vague predicate and I'm in the necessity of saying something like 'Sam is bald' I can always retreat to a relatively precise background language that allows me to precisify my assertion.

²³ See Williamson (1994a).

The background language is more precise than the ordinary one at least in the relevant aspect. But Sider claims this mechanism breaks down in the case of quantifiers. Suppose it's definitely the case that there is exactly one F and exactly one G but it's indeterminate whether the following is true:

(E) $\exists x(x \text{ is composed of F and G})$

Now suppose that the vagueness at issue is to be ascribed to the existential quantifier, not to the predicates F and G, nor to 'is composed'. Therefore there are at least two precisifications of the existential quantifier to the effect that (E) is true if the quantifier at issue is \exists_1 and false if the quantifier is \exists_2 . So *there is* an object that belongs to the domain of the first quantifier but not to the domain of the second one. Our use of 'there is' cannot but be absolutely unrestricted.

ABSOLUTENESS: if on some precisification there are Fs, then $\exists xFx$ is true.

As we are going to see in the presentation below, this premise plays a crucial role in Sider's argument against vague existence.

Let \mathcal{L} be an ordinary first order language and let \mathcal{L}_Δ be the same language enriched with a determinacy operator ' Δ ' and an indeterminacy operator ' I '. In the course of the argument we are going to present, we will employ also a metalanguage to talk about some sentences of \mathcal{L}_Δ . The argument is a *reductio ad absurdum* and it begins with the premise (E) to the effect that it's indeterminate that there is an object composed by two other objects.

1) $I\exists z(x \neq z \wedge y \neq z)$

Now we deduce an elementary consequence in our metalanguage:

2) ' $\exists z(x \neq z \wedge y \neq z)$ ' is true on some precisification of \mathcal{L}_Δ and false in some other.

To say this is nothing but to say that:

3) on some precisification of \mathcal{L}_Δ there's an object other than x and y and in other precisification there's no further object.

With a conjunction elimination performed on the previous passage we deduce:

4) on some precisification of \mathcal{L}_Δ there's an object other than x and y .

Now, in virtue of the principle of absoluteness of existential quantifier, we deduce:

5) if, on some precisification of \mathcal{L}_Δ there's an object other than x and y then ' $\exists z(x \neq z \wedge y \neq z)$ ' is true.

Then, by Modus Ponens on 4) and 5) we get:

6) ' $\exists z(x \neq z \wedge y \neq z)$ ' is true.

A widely accepted principle in vagueness frameworks is the rule of Δ -introduction. It says that when, in the object language, a sentence is true, such a sentence is determinately true. Therefore, from 6) we get:

7) ' $\Delta \exists z(x \neq z \wedge y \neq z)$ ' is true.

By a universally accepted principle of disquotation we deduce:

8) $\Delta \exists z(x \neq z \wedge y \neq z)$

From 1) and 8) we finally get:

9) $I \exists z(x \neq z \wedge y \neq z) \wedge \Delta \exists z(x \neq z \wedge y \neq z)$

Since one and the same sentence cannot be indeterminate and determinate 9) is a contradiction. Therefore 1) must be false. In

Sider's opinion, we cannot but conclude that the notion of vague existence is irremediably flawed.

3.3.2 A Contradiction

There's another problem with existential indeterminacy taken as a solution for the problem of incompatible objects. Suppose that a coherent notion of vague existence is available. In Eklund's opinion, there is still room for an objection against the coherence of the solution of enriching our language with vague quantification.²⁴

Let dheartes be almost like hearts except for the fact that they exist only if ddivers *determinately* do not exist and let ddivers be almost like livers except for the fact that they exist only if dheartes *determinately* do not exist. The definitions of dheartes and ddivers allow us to say that if it's not determinate that ddivers do not exist, then dheartes do not exist. The same holds for ddivers.

$$1) \neg\Delta\neg\exists xL_d x \rightarrow \neg\exists xH_d x$$

$$2) \neg\Delta\neg\exists xH_d x \rightarrow \neg\exists xL_d x$$

The solution that a supporter of Maximalism should adopt in case of incompatible objects is to claim that the existence of each of them is indeterminate.

$$3) \neg\Delta\exists xL_d x \wedge \neg\Delta\neg\exists xL_d x$$

$$4) \neg\Delta\exists xH_d x \wedge \neg\Delta\neg\exists xH_d x$$

Applying Modus Ponens on 1) and on the second conjunct of 3) we get:

$$5) \neg\exists xH_d x$$

²⁴ The argument that follows is a personal reworked version of a suggestion that can be found at page 114 of Eklund (2006). I believe that my version of the argument is an accurate development of Eklund's idea.

Applying Modus Ponens on 2) and the second conjunct of 4) we get:

$$6) \neg\exists xL_d x$$

By the rule of Δ -introduction, from 5) and 6) we get:

$$7) \Delta\neg\exists xH_d x$$

$$8) \Delta\neg\exists xL_d x$$

By \wedge -introduction applied on 8) and on the second conjunct of 3) we finally get:

$$9) \neg\Delta\neg\exists xL_d x \Delta \wedge \Delta\neg\exists xL_d x$$

By \wedge -introduction applied on 7) and on the second conjunct of 4) we finally get:

$$10) \neg\Delta\neg\exists xH_d x \wedge \Delta\neg\exists xH_d x$$

Since 9) and 10) are contradictions we have just proved that the assumptions 1) and 2) are incompatible with the assumptions 3) and 4). In Eklund's opinion, this should be a knock-out argument against Maximalism. Even if it is equipped with devices for vague quantification, Maximalism is still inconsistent. In the next section we will elaborate a framework for vague quantification that should be able to dispose of the contradiction pointed out by Eklund and to answer the challenge constituted by Sider's objection.

3.4 A framework for vague existence

3.4.1 A mixed approach

The discussion on predicate-vagueness has been historically dominated by two different approaches: a semantic one and an epistemic one. Other approaches are possible, but they have attracted less attention. The difference between these two fundamental

approaches depends on which thing is identified as the source of the phenomenon of vagueness: according to semantic approach, the source is language itself, because it includes many predicates whose extension is not perfectly defined; according to epistemic approach the source of vagueness is our ignorance of some facts of the world. For the first approach, for example, there are borderline cases of baldness because the threshold between the extension and the antiextension of the predicate <is bald> is fuzzy; it's not a neat line, but rather a wide grey area. For the second approach, there's no grey area between what is a clear case of baldness and what falls certainly into the antiextension of <is bald>. No part of our language has a vague meaning, there's always a sharp boundary between the extension and the antiextension of a predicate. The origin of all the known vagueness phenomena is our ignorance about where such a boundary lies. With respect to our example, there's a precise number n of hairs, such that whoever has more than n hairs is not bald and whoever has n hairs or less is bald. Since now, these two approaches has always proceeded separately, as two different and incompatible ways of addressing the problem of vagueness.

Nevertheless a fruitful union of these two approaches, although not yet carefully explored, is not unconceivable. The idea that, in some cases, predicate-vagueness comes from an intrinsic indeterminateness of the meaning of the predicate at issue and in some other cases it comes from our ignorance of the precise meaning of such a predicate is not a non-starter. One could easily find some convincing examples. The often cited predicate <is bald> seems to constitute a good example of predicate whose extension is vague because of the intrinsic vagueness of its meaning. A clear evidence of the indeterminateness of its meaning is the fact that any answer

whatsoever to the question ‘what’s the exact number of hair below which a human being can be considered bald?’ cannot but sound totally arbitrary. There are also examples of predicates whose vagueness is clearly due to our ignorance of their precise meaning. Consider the predicate, taken from cosmological domain of discourse, <indefinitely expands>. The extension of this predicate is indeterminate; indeed it’s not clear whether our universe will expand forever or, once it will have reached a critical volume, it will progressively narrow. The vagueness of this predicate doesn’t come from its meaning. Indeed it has a perfectly determined meaning: a (possible) universe indefinitely expands if and only if its energy density is below λ . The value of λ is perfectly determinate and it constitute a precise threshold. We simply ignore the exact amount of λ . Here the meaning of the predicate at issue is precise and so is its extension; vagueness comes from our ignorance.

A detailed development of such a view is outside the scope of my dissertation; my only point is that an account along these lines has some intuitive underpinning and it probably deserves further investigation. When it comes to vague quantification, I believe that the best approach is, again, a mixed one. Indeed, also in the case of existential vagueness we can distinguish between two distinct kinds of existential indeterminateness. These two kinds don’t constitute two alternative and mutually exclusive ways of conceiving existential vagueness, but they can be integrated in a unique theory. The best way to outline the distinction is by means of two examples.

First, consider the much discussed notion of *mereological fusion*. If A and B are two objects, their mereological fusion is a further object C such that A is part of C, B is part of C and if I subtract from C both A and B then C completely vanishes (no other

parts of it remain). Now, consider this example taken from Putnam (1987). In a certain portion of universe there are the following absolutely simple objects: a, b, c. If we ask ourselves how many objects there are in that portion of universe, the answer could significantly vary, depending on the answer we give to the Special Composition Question:²⁵ under what circumstances two (or more) things constitute a unique thing, namely a mereological fusion? A supporter of mereological nihilism would say that, in that portion of universe, there are three things since her answer to Special Composition Question is “never”. On the contrary, a mereological universalist would say that there are seven things, namely, a, b, c, ab, ac, bc, abc, since her answer to the Special Composition Question is “always”. Other, more complex answers are possible. Indeed one would argue that many things constitute one thing if and only if they are tightly connected, i.e. they constitute an uninterrupted stretch of solid stuff. Someone else would argue that they constitute a unique thing if and only if they constitute a functional organism. The important thing to notice is that the question ‘are there mereological fusions?’ admits a wide range of answers, not limited to the extreme two (‘yes’ or ‘not’). An answer according to which some mereological fusion exist and some other not is perfectly meaningful (even though not necessarily true).

Now, consider the familiar case of natural numbers as defined by Hume Principle. One could meaningfully ask whether there really are entities like these; in other words one could be unsure of the effectiveness of a stipulation like Hume Principle: if we have a domain composed by infinitely many individuals, are there necessarily natural numbers among them? But suppose that someone is pretty

²⁵ See Van Inwagen (1990)

persuaded of the effectiveness of Hume Principle; then her answer to the question ‘are there numbers?’ would be undoubtedly ‘yes’. More complex answers, maintaining, for example, that some natural number exist and some other do not exist are simply meaningless. Once I endorse Hume Principle I accept the existence of all natural numbers; a question like ‘which natural numbers really exist?’ would be rejected as unintelligible.

These two examples show that there are two different ways of being existentially indeterminate: some sortals are such that it’s indeterminate which instantiation exist and which don’t exist and some other are such that if they are instantiated, then *all* their conceivable instantiations really exist (and if not, none of them really exist). The model of vague quantification that I would like to present here is mixed, because it allows for these two different kind of vagueness. As we are going to see, this is the feature allowing the model to be safe from Eklund’s indeterminacy paradox.

3.4.2 Supervaluationism reconceived

The model of vague quantification that I would like to present is inspired by the classical account of Fine [1975]. My aim is to modify the classical supervaluationist machinery in order to make it suitable to interpret a language whose *sole* vague constituent are quantifiers. Nothing rules out the possibility of a completely generalized model for vague languages, capable of accounting for both predicate vagueness and quantifier vagueness. I think the endeavour is feasible, even though it would result in a notational nightmare. Thus I prefer to present a model capable of interpreting a language that is perfectly precise, except for its quantifiers. Once the model is settled, a generalization would be tedious, but completely straightforward.

Fine's account of predicate-vagueness is well known; we only need to recall its essential features. A specification space for a language \mathcal{L} is a set of specification points that constitutes a partial order under an accessibility relation. Each specification point is a partial model for \mathcal{L} ; for every predicate it determines which objects (belonging to a unique domain) are included in its extension and which of them are part of its anti-extension; the union of extension and anti-extension doesn't always coincide with the entire domain. Indeed, at least for some predicate, there are objects that do not belong neither to its extension nor to its anti-extension. Each specification point precisifies a point from which it is accessible; it is able to assign to the extension or to the anti-extension of a predicate some objects that were in the grey area at a precisification point from which it is accessible. The base point is not accessible from any other specification point and constitute the root of the tree of specification points. It's the less determined model of \mathcal{L} . On the opposite side, the complete specification points are the peaks of the tree; they are classical models in which, for every predicate and every object, it is always determined whether the object belongs to the extension or to the anti-extension of the predicate in question.

The fundamental insight of my model of vagueness is to treat specification points as models, each one equipped with a domain that doesn't include all the objects that language \mathcal{L} is able to refer to or quantify over. An atomic sentence can be such that it cannot be interpreted at a certain specification point, since at that point there's no reference for the individual constant featuring in it. In this case the sentence cannot be true; it can be indeterminate or false (we are going to see how to determine the truth value in these cases soon). Similarly, an existentially quantified sentence can be such that, at a certain

specification point, there's no object satisfying the predicate into the scope of the quantifier. In this case the truth value of the sentence can be only false or indeterminate. If a specification point p is accessible from a specification point s then the domain of p strictly includes the domain of s . The process or progressive specification, i.e. the process of progressive extension of the domain, goes on until it reaches the complete specification points. These points are models of \mathcal{L} such that *no sentence of \mathcal{L} has an indeterminate truth value*. It's important to stress that, in the framework I would like to introduce, complete specification points are not classical models, i.e. models in which, for example, every constant of \mathcal{L} has necessarily a reference. It's simply a model at which every sentence has a determined truth value, even the atomic sentences in which there are empty names. Indeed, in my framework, there are rules that allows the determination of truth value for this kind of sentences. Complete specification points can still be defined *classical* in a different sense: they are ruled by classical bivalent logic. They are models whose domain could be potentially further extended. These domains are not all-inclusive, they are simply able to assign to each sentence of \mathcal{L} a determinate truth value. In the technical exposition, that will follow shortly, these features will appear clearer.

The most heretical characteristic of my account is the plurality of base points that it contemplates. While in classical supervaluationism there is one and only one base point liable to various precisifications, in my framework there are various base points, each of them liable to various precisifications. This feature allows us to distinguish the two kind of existential indeterminacy that we have previously outlined: existential vagueness of sortals whose instantiation happens in a "all or nothing" fashion and existential

vagueness of sortals which admit a partial instantiation. The first kind of indeterminacy is expressed by the plurality of base points and the second kind by the plurality of specification points that each base point bears. Indeed the first kind of indeterminacy is related with the problem of incompatible objects. Consider again the incompatibility between natural numbers (as defined by Hume Principle) and parities (as defined by Parity Principle). A specification point cannot contain parities and be such that some of the specification points accessible from it contain numbers (and vice versa). Moreover there cannot be a specification point without numbers and without parities and such that numbers and parities appear in separate branches of the tree generated by its accessible specification points. Each specification point is such that either it contains numbers or it contains parities in its domain, because each domain is either infinite or finite. If it is finite it must contain parities, if it is infinite it contains numbers.²⁶ If a base point contains numbers, its various specification points enrich its domain in various different ways, but they never add parities. The general idea is that different base points constitute alternative (and in some cases incompatible) pictures of the world. No precisification can remove the incompatibilities between them. On the contrary, from a certain base point with no mereological fusions we can access specification points with some mereological fusions.

Therefore the two kinds of existential vagueness are mirrored by a plurality of base points, each of which is precisified by a plurality of specification points.

²⁶ Recall: we suppose that Maximalism is true. The framework we are going to outline is designed to save the coherence of Maximalism.

3.4.3 The framework

Let's start with the technical details. A specification space S is a structure composed by:

- i. a set of specification points P ;
- ii. a function D assigning to each member of S a domain of objects;
- iii. an accessibility relation R over the members of S . Such a relation is reflexive, transitive and anti-symmetric. Given a specification point p , the set of all points accessible from p is called *shade* of p .
- iv. A set of base points B , such that B is a subset of P . Each member b of B is such that, for no p belonging to P , Rpb and, for every p belonging to P , there is one and only one b , belonging to B , such that Rbp .
- v. An interpretation function I assigning to each predicate of \mathcal{L} a set of objects and to each singular term of \mathcal{L} an object. But from which domain are taken these objects? An appropriate definition of an interpretation I requires the definition of a set that we call *total domain of I* and we represent by the symbol \mathbb{D}_I . The total domain \mathbb{D}_I is the union of all the domains that function D associates to the members of P . An interpretation I maps every individual constant of \mathcal{L} with a member of \mathbb{D}_I and every n -ary predicate of \mathcal{L} to a member of \mathbb{D}_I^n .

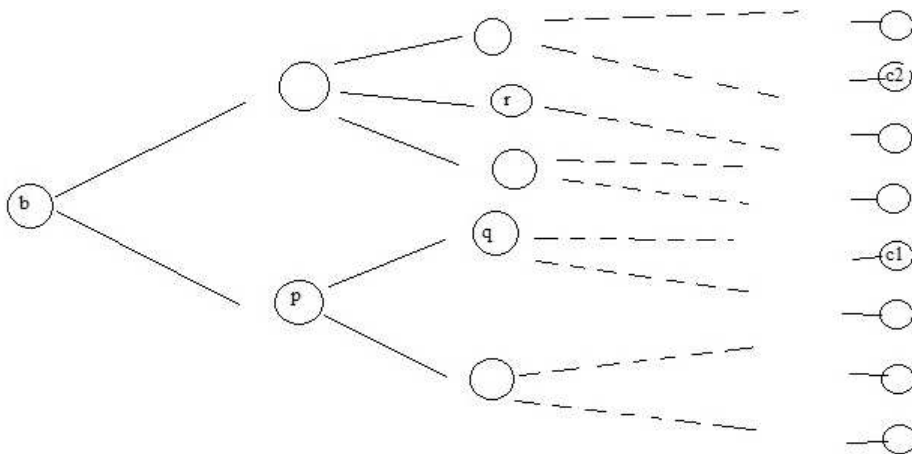
A specification space $S = \langle P, D, R, I, B \rangle$ is *acceptable* if and only if function D and relation R are such that, for every couple of specification points p and q , Rpq if and only if $D(p) \subset D(q)$. This condition assures that no specification point q can be a precisification of a specification point p unless q expands the domain of p . From this moment on, when we speak of specification spaces, we always mean acceptable specification spaces.

From this moment on, we will focus exclusively on how a certain portion of our system works, for simplicity reasons. We will consider one and only one base point b and the specification points accessible from it, ignoring the fact that there are various base points. At the end of this explanation we will see how this momentarily neglected fact enriches our framework. Thus, let's pretend we are examining a specification space provided with a unique base point.

A precise definition of the notion of specification point requires the notion of *range restriction*. As is well known, every binary function is liable to restriction; given a function $f: A \rightarrow B$, a restriction $f \upharpoonright_{A^*}$ is a function identical to f , except for the fact that its domain is not A but a set A^* such that $A^* \subseteq A$. In other words the graph of $f \upharpoonright_{A^*}$, in symbols $G(f \upharpoonright_{A^*})$, is $\{(x,y) \in G(f) \mid x \in A^*\}$. This kind of restriction (that is the most common) is sometimes called left-restriction or domain restriction. A range restriction, or right-restriction, of a binary function $f: A \rightarrow B$, in symbols $f \upharpoonright_{B^*}$ is a function identical to f except for the fact that its range is not B but a set B^* such that $B^* \subseteq B$. Therefore $G(f \upharpoonright_{B^*}) = \{(x,y) \in G(f) \mid y \in B^*\}$. A *specification point* p is a partial model of \mathcal{L} constituted by a domain $D(p)$ and a range restriction of the interpretation function I to the domain $D(p)$, in symbols I_p . In a rather simplistic fashion: I_p is the interpretation of the portion of \mathcal{L} that speaks of the objects included in $D(p)$. In a more sophisticated way: i) I_p assigns to a constant c of \mathcal{L} a referent r if and only if $I(c) = r$ and $r \in D(p)$. If r is not a member of $D(p)$ then I_p is not defined for the argument c ; ii) I_p assigns to a n -ary predicate F of \mathcal{L} the intersection between $I(F)$ and $D(p)$. Such an intersection can be empty: in this case I_p is indefinite for the argument F . It can be identical with $I(F)$: in this case I_p is completely defined for the argument F . In all the other cases I_p is partially defined for the

argument F . In the rest of our discourse we will call I a *global interpretation* of \mathcal{L} and its restrictions (like I_p, I_q, I_s, \dots) *local interpretations* of \mathcal{L} . The notion of local interpretation allow us the introduce another helpful distinction, that between *global anti-extension* and *local anti-extension* of a predicate. If $I(F)$ is the set of n-ples of objects of \mathcal{D} satisfying F , $^A I(F)$ is the set of all the n-ples of objects of \mathcal{D} that don't satisfy F . Similarly, if $I_p(F)$ is the set of n-ples of objects of $D(p)$ satisfying F , $^A I_p(F)$ is the set of all the n-ples of objects of $D(p)$ that don't satisfy F . It's easy to see that every local extension of a predicate is a subset of its global extension and the same holds for anti-extensions.

The determination of truth value of sentences is designed to be such that, ascending from the base point toward the complete specification points, every sentence of \mathcal{L} will receive a determinate truth value (true or false) sooner or later. Once this truth value is assigned, it cannot change ascending toward the complete specification points. Indeed each specification point *precisifies* the specification points from which it's accessible, but it can never be inconsistent with them (we will call this property "Stability"). A precisification is not a *correction*. I will soon give the precise truth condition for each kind of sentence, but before doing that, I think that an example could be of help. Look at the picture below:



Suppose that sentence ‘Fa’ is true at specification point q. This means that i) the function of local interpretation for specification point q, namely I_q , is defined for the argument ‘a’; ii) the function of local interpretation for q is defined (at least partially defined) for the argument ‘F’; iii) $I_q(a) \in I_q(F)$. Now let’s step back at specification point p, from which q is accessible. Suppose that I_p is not defined for the argument ‘a’, i.e. there’s no member of $D(p)$ that local interpretation I_p assigns to ‘a’. In this case the sentence ‘Fa’ cannot be true at p. Moreover it cannot be false, because there is a specification of p, namely q, at which ‘Fa’ is true. If ‘Fa’ were false at p, then q would correct p and not simply precisify it. Hence ‘Fa’ is indeterminate at p.

Now suppose that I_q is not defined for the argument ‘a’ and the same holds for every specification point accessible from p. In other words, for every specification point z, such that z is into the shade of p, $D(z)$ doesn’t include $I(a)$. Now, into the shade of p there are also many complete specification points (c1 is one of them). At these points sentence ‘Fa’ cannot be indeterminate, since complete specification points are ruled by classical bivalent logic (this requisite will be called “Classicality” below). Since it cannot be true, nor

indeterminate, the sentence is false at all the complete specification points into the shade of p . In a case like this, we establish that ‘ Fa ’ was already false at point p . If a sentence is not true at any of the points that precisify p , then it’s false at p . Sentence ‘ Fa ’ can be true at some specification point outside the shade of p , but this is irrelevant for the determination of its truth value at p . Look at the picture again. On a different branch of our tree of specification points there’s a point s such that I_s is defined for the argument ‘ a ’ and for the argument ‘ F ’ too. But s is not accessible from p and p is not accessible from s ; they are part of two branches whose last common node, namely b_1 , is far behind p and s . If s is not a precisification of p , whatever happens at s is irrelevant for the determination of the truth value at p of whatever sentence.

Now we are ready to examine the truth conditions of our framework. The following conditions establish in which cases a sentence α of \mathcal{L} is *true simpliciter* at a specification point p :

1. If α is an identity like $a = b$: i) α is true simpliciter at p if and only if $I_p(a) = I_p(b)$; ii) false if and only if $I_p(a) \neq I_p(b)$ or at least one between a and b belongs exclusively to the domains of specification points s such that $\neg Rps$; iii) indeterminate otherwise.
2. If α is an atomic sentence constituted by a n -ary predicate and n individual constants, like $Fa_1a_2a_3\dots a_n$: i) α is true simpliciter at p if and only if $I_p(\langle a_1, a_2, a_3, \dots, a_n \rangle) \in I_p(F)$; ii) false if and only if $I_p(\langle a_1, a_2, a_3, \dots, a_n \rangle) \in {}^A I_p(F)$, or if some member of $I(\langle a_1, a_2, a_3, \dots, a_n \rangle)$ belongs exclusively to domains of specification points s such that $\neg Rps$; iii) indeterminate otherwise.
3. If α is a negative sentence like $\neg\beta$: i) α is true simpliciter at p if and only if β is false at p ; ii) false if and only if β is true simpliciter at p ; iii) indeterminate otherwise.

4. If α is a conjunction like $\beta \wedge \gamma$: i) α is true simpliciter at p if and only if both β and γ are true simpliciter at p ; ii) false if and only if at least one between β and γ is false at p ; iii) indeterminate if and only if both β and γ are indeterminate at p .
5. if α is a disjunction like $\beta \vee \gamma$: i) α is true simpliciter at p if and only if at least one between $\beta \vee \gamma$ is true simpliciter at p ; ii) false if both β and γ are false at p ; iii) indeterminate otherwise.
6. if α is an existentially quantified sentence like $\exists x\beta$: i) α is true simpliciter at p if and only if there's at least one member of $D(p)$ such that β is true simpliciter at p ; ii) false if and only if there's no point s such that Rps and such that at least one member of $D(s)$ makes β true simpliciter at s ; iii) indeterminate otherwise.
7. If α is a universally quantified sentence like $\forall x\beta$: i) α is true simpliciter at p if and only if, for every s such that Rps , s is such that every member of $D(s)$ makes β true simpliciter at s ; ii) false if at least one member of $D(p)$ is such that β is false at p ; iii) indeterminate otherwise.

A specification point p_c is complete if and only if it assigns a definite truth value (true or false) to each sentence of \mathcal{L} . Unlike classical supervaluationism, in my semantics, complete specification points are not classical models of \mathcal{L} , since no complete specification point contains the referents of every individual constant of \mathcal{L} and none is such that every predicate of \mathcal{L} is completely interpreted by the local restriction of I . Nevertheless there is a perfectly legitimate sense in which these specification points are classical. As we have just said, they are such that no sentence of \mathcal{L} is left without a truth value, while all the specification points that are not complete are characterized by

truth value gaps. Complete specification points are classical because they are ruled by classical bivalent logic.

A sentence is *supertrue* at p if and only if it's true at all complete specification points that are accessible from p . If truth is supertruth then all the classically valid formulas are true at all specification points. Our model respect the same constraints that Fine [1975]'s model respects:

- *Fidelity*: complete specification points are classical. As we have already said they are ruled by classical bivalent logic.
- *Completeness*: for every p belonging to P there's an s belonging to P such that Rps and such that s is a complete specification point. The progressive expansion of a domain ends at some point. There are maximal domains. This maximality is not to be intended as universal inclusivity; indeed no complete specification point has a domain identical with \mathcal{D}_I . Maximality is relative to the capacity of assigning a definite truth value (true or false) to each sentence of \mathcal{L} .
- *Stability*: for every p belonging to P and for every sentence α of \mathcal{L} , if α is true at p then, for every s belonging to P , such that Rps , α is true at s . The conditions 1) - 7) are such that if a sentence is true at p , it is also true at every point into the shade of p ; if it's false at p , it's also false at every point into the shade of p ; if it's indeterminate then at some point accessible from p it is true, at some other it is false.

We can see now in what sense a plurality of base points enriches our model. In Fine (1975) a sentence that is true at the base point is determinately true. In his framework, since there is one and only one base point, the determinacy of sentence is an absolute quality. In my framework a sentence α is *determinately true relatively to base point b* (in symbols $\Delta_b\alpha$) if and only if it's true at the base point b , i.e. if and

only if it's true at every specification point into the shade of b. This relativization allows us to introduce two further operators: 'Π' and 'Σ'. They mean, respectively, 'for every base point' and 'for some base point'. Πα means that, whatever base point we choose, α is true, while Σα means that for some base point α is true. More complex combinations are possible and meaningful: ΠΔα means that whatever base point we choose α is determinately true, namely true at every base point; Π¬Δα means that whatever base point we choose α is not determinate, i.e. there's no base point at which α is true; ΣΔα means that for at least one base point α is determinately true; Σ¬Δα means that for at least one base point α is not determinately true. It's easy to see that ¬ΠΔα is equivalent to Σ¬Δα and ¬ΣΔα is equivalent to Π¬Δα.

A sentence α is a *supervaluational consequence* of a set of sentences Γ (in symbols, $\Gamma \models \alpha$) if and only if every specification point at which all the sentences that compose Γ are true is a point at which α is true. A sentence α is *supervaluational valid* if and only if it's true at every specification point.

In the next section we are going to see how the presented apparatus can be of help in disposing of the contradiction pointed out by Eklund and in answering the challenge posed by Sider's objection.

3.5 Answering the challenges

3.5.1 Dhearts-dlivers problem

Now we can try to rephrase our paradox of dhearts and dlivers using our new conceptual apparatus. Let's recall the argument leading to contradiction:

$$1) \neg\Delta\neg\exists xL_d x \rightarrow \neg\exists xH_d x \quad \text{Incompatibility dhearts-dlivers}$$

- | | |
|--|----------------------------------|
| 2) $\neg\Delta\neg\exists xH_d x \rightarrow \neg\exists xL_d x$ | Incompatibility ddrivers-dhearts |
| 3) $\neg\Delta\exists xL_d x \wedge \neg\Delta\neg\exists xL_d x$ | Vague existence of ddrivers |
| 4) $\neg\Delta\exists xH_d x \wedge \neg\Delta\neg\exists xH_d x$ | Vague existence of dhearts |
| 5) $\neg\exists xH_d x$ | Modus Ponens on 1,3 |
| 6) $\neg\exists xL_d x$ | Modus Ponens on 2,4 |
| 7) $\Delta\neg\exists xH_d x$ | Δ -introduction on 5 |
| 8) $\Delta\neg\exists xL_d x$ | Δ -introduction on 6 |
| 9) $\neg\Delta\neg\exists xL_d x \wedge \Delta\neg\exists xL_d x$ | \wedge -introduction on 3,8 |
| 10) $\neg\Delta\neg\exists xH_d x \wedge \Delta\neg\exists xH_d x$ | \wedge -introduction on 4,7 |

Passages 9) and 10) are clearly contradictory. Now, our aim is to rephrase this argument using our conceptual apparatus and to show that, when the premises are suitably restated, the contradiction is disposed of.

Let's start from the premises that we have dubbed "incompatibility dhearts-dlivers". In our conceptual framework they can be properly restated in the following way:

$$1^*) \prod((\neg\Delta\neg\exists xL_d x \rightarrow \neg\exists xH_d x)$$

$$2^*) \prod((\neg\Delta\neg\exists xH_d x \rightarrow \neg\exists xL_d x)$$

The idea is that, whatever base point happens to be the right (partial) model for our language, there cannot be dhearts (dlivers) unless there are determinately no ddrivers (dhearts). Therefore, for every base b point there is no specification point p, accessible from b, such that $\exists xH_d x$ is true simpliciter at p and $\exists xL_d x$ is true or indeterminate at p. The paraphrase of the next two assumptions, namely "vague existence of ddrivers (dhearts)", is a bit more problematic. We cannot adopt the

same device that we used before, since the plain introduction of the operator ‘ Π ’ before the sentence doesn’t dispel the contradiction. To see this, look at the following lines:

$$3+) \Pi(\neg\Delta\exists xL_d x \wedge \neg\Delta\neg\exists xL_d x)$$

$$4+) \Pi(\neg\Delta\exists xH_d x \wedge \neg\Delta\neg\exists xH_d x)$$

These premises simply say that there’s no base point b such that $\exists xL_d x$ is true at b or $\neg\exists xL_d x$ is true at b . Whatever base point happen to be the right one, these two sentence are always indeterminate. From these premises, plus 1*) and 2*), it follows that

$$5+) \Pi(\neg\exists xH_d x)$$

$$6+) \Pi(\neg\exists xL_d x)$$

Then the path that leads to contradiction is open and even wider than before, since we have added the specification that, whatever base point happens to be the right one, dhearts (dlivers) must determinately exist and, at the same time, they must be determinately inexistent. If we want to avoid this contradiction we need to rephrase premises 3) and 4) in a less straightforward way. The solution I would like to propose is quite simple: since we cannot maintain that, for every base point b , the existence of dhearts (dlivers) is indeterminate, we have to claim that, for some base points dhearts determinately exist and dlivers are determinately inexistent and for some other base points dlivers determinately exist and dhearts are determinately inexistent. In other words, the existential indeterminacy in which dhearts and dlivers are involved is of the same kind of the indeterminacy involving numbers and parities. This solution is not at odd with our intuitions; on the contrary, it’s natural to think that, if Maximalism is the correct metaontological view, couples of sortals like dhearts-dlivers must be

such that one of them is determinately instantiated (and, therefore, the other determinately do not exist) but we don't know which one. If dhearts exist only if ddivers determinately do not exist and *vice versa*, then only two cases are possible: either there are dhearts and determinately no ddivers or there are ddivers and determinately no dhearts. Our conceptual apparatus allows us to express this in a rather synthetic way:

$$3^*) \sum \Delta \exists x L_d x \wedge \sum \Delta \neg \exists x L_d x$$

$$4^*) \sum \Delta \exists x H_d x \wedge \sum \Delta \neg \exists x H_d x$$

From the two premises above and from 1*) and 2*) follows that:

$$5^*) \sum \Delta \neg \exists x H_d x$$

$$6^*) \sum \Delta \neg \exists x L_d x$$

From the two sentences above and from 3*) and 4*) we can conclude:

$$7^*) \sum \Delta \neg \exists x L_d x \wedge \sum \neg \Delta \neg \exists x L_d x$$

$$8^*) \sum \Delta \neg \exists x H_d x \wedge \sum \neg \Delta \neg \exists x H_d x$$

As we can clearly see, here there's no contradiction. We simply came to the conclusion that, at certain base points it's true that there are ddivers (dhearts) and at other base points it's true that there are no ddivers (dhearts). The application of our conceptual framework has shown that the contradiction pointed out by Eklund (2006) is essentially due to an imprecise way of expressing the ontological relation between determinately incompatible objects. As soon as we adopt a framework able to express some more subtle distinctions the contradiction disappears.

3.5.2 Response to Sider's Objection

In Sider's argument a crucial role is played by the assumption of the absoluteness of quantifiers: if on some precisification of a language \mathcal{L} there are Fs, then $\exists xFx$ is true. The most important feature of the framework I've just presented is the presence of a plurality of base points. We could ask ourselves what would be the result of the application of absoluteness principle to a framework with this peculiar feature. If the objects that exist at a whichever precisification exist in general, then there is a sort of super-domain identical with the union of the domains of all the specification points of the specification space. Certainly, in our framework there is a set \mathbb{D}_1 containing all the members of the domains of all specification points. Nevertheless such a set doesn't constitute a domain over which we quantify. Indeed, how is it possible to quantify over a domain containing both dheartes and dlivers, both numbers and parities, both numbers and anti-numbers? The sole existence of incompatible objects is a valid reason to doubt that this is possible. Indeed, the domain over which we quantify should be, for example, finite and infinite at the same time. I think there is an axiom that is widely accepted, although tacitly: a domain is not only a set. It's part of a model and a model must be consistent. If a set is such that no model can interpret a language that quantify over all the members of that set, then such a set is not a domain. Since Sider's objection relies crucially on the absoluteness of quantifiers, the fact that such a principle cannot hold in the framework I've presented is a sufficient reason to think that his powerful objection cannot threaten the coherence of the presented notion of vague existence.

3.6 Conclusions

In this chapter I have presented a metaontological view known as Maximalism. It's worth noticing that I've avoided to deeply analyze the connection between Priority Thesis and Maximalism. I've simply assumed that Maximalism is a natural consequence, on a metaontological level, of Priority Thesis. In the next chapter I'll show that this is far from being obvious and that there are other ways to understand the relation between Priority Thesis and a certain indubitable abundance of existent entities. Maximalism is not the only option, but, I think, is an acceptable one. Its main problem is with incompatible object. I've shown how to solve it by means of a framework allowing vague quantification. Such a framework is able to save Maximalism from the threat posed by incompatible objects and moreover it's able to resist to a well known objection against the idea of vague existence.

Chapter 4

THE POSSIBILITY OF THIN OBJECTS

4.1 Introduction

The question that Linnebo (forthcoming) tries to answer is a fundamental one for the ontology of abstract entities: are there objects whose existence doesn't impose very demanding requirements on reality? Such objects are called *thin*, in opposition to thick objects, whose existence requires, for example, that some physical conditions obtain. Objects can be thin in at least two different senses: i) *absolutely* thin, i.e. their existence puts no significant requirements on reality. An example could be that of pure sets, since pure sets are supposed to exist even if nothing else exists; ii) *relatively* thin, i.e. given a set of objects whose existence is undisputed, they do not require very much *else* to reality. For example a mereological sum is thin relative to its constituent parts. The question of the existence of thin objects is tightly related with a certain metaontological theory that has already been mentioned in the course of this thesis: metaontological minimalism.

Metaontological Minimalism is, fundamentally, the thesis that there are thin objects (no matter if they are relatively or absolutely thin). It's a metaontological view that sets the bar of the existence very low; a minimalist claims that our concept of object allows for

thin objects. Minimalism tends to be connected with *Ontological Maximalism*, a thesis according to which everything we quantify over in every kind of discourse really exist. Indeed if very little is required for something to be a really existent object, then the commitment with a generous ontology is unavoidable.

The idea of thin object is appealing for an obvious reason: it allows for a face-value reading of mathematical statements and therefore it lends support to Mathematical Platonism. Nevertheless the very idea of thin object has, since now, proven quite slippery, for at least two reasons: i) their introduction seems to justify extravagant ontologies (roughly: if we allow for the existence of thin objects why should we put constraints on the existence of whatever weird entity an ontologist can imagine?); ii) the epistemic access to thin objects seems to be problematic (here, obviously, the old problem pointed out by Benacerraf (1973) is in play).

Linnebo claims that his approach is able to solve both these problems. His fundamental idea is to make clear what's the link between a triad of concepts that Frege believed to be tightly related, those of *object*, *reference* and *identity criteria*. This triangle can show us how thin objects can exist and how they can be understandable. In Frege's view, reference is a sufficient condition for objecthood.²⁷ Moreover a sufficient condition for a singular term t to refer is that there are identity conditions available, namely, a formula saying in which cases, for some x , $x = t$. If we put together the two theses we should get what we seek: an easy way to introduce abstract objects in ontology, namely, a good defence of Metaontological Minimalism. This, apparently easy, Fregean road to thin objects requires a theorist

²⁷ But not a necessary condition, since there are objects which we cannot refer to with singular terms, like, for example, complex numbers. See Linnebo (forthcoming) pp. 19-20.

to pass through three different steps. The first consist in a justification of the legitimacy of abstraction principles as ways to introduce light additional commitment into our ontology. In particular, which conditions must respect the relation occurring between the two sides of an abstraction principle in order to assure its acceptability? Section 2 is dedicated to the attempt to answer this question. The second step is a clarification of the notion of criterion of identity. How can a criterion of identity for t assure us that t really refer to something? Section 3 addresses this problem. The third step must be a clear explanation of the mechanism by which, in a given language, singular terms really referring to thin objects are introduced. Section 4 is dedicated to this task. Finally section 5 explains the notion of thin objects that results from this complex theory.

4.2 The possibility of recarving of content

In § 62 of *Grundlagen der Arithmetik* Frege famously suggests that the two sides of an abstraction principle are to be considered as different carvings of one and the same content. As already said in the first chapter, the notion of content recarving is quite elusive, since it seems to be very difficult to respect all the constraints which such a notion is subject to. For example, if propositional content is cognitively understood, then it seems that no philosophically interesting abstraction principle is an example of content recarving. Indeed, in the case of direction abstraction, we cannot seriously claim that the two sides have the same cognitive content, at least because they talk of different objects. Linnebo believes that it's not easy to find a satisfactory notion of content recarving, but the endeavour is not desperate.

In his view the notion must be analyzed in terms of mutual sufficiency. In other words, ϕ and ψ are different carvings of one and the same content if and only if ϕ is sufficient for ψ (in symbols $\phi \Rightarrow \psi$) and ψ is sufficient for ϕ . Intuitively, the idea of sufficiency is that, if ϕ suffices for ψ , then when we assert ϕ we are also asserting ψ . With a theological metaphor: if God makes it the case that ϕ He is also making the case that ψ , without this requiring an extra creative/determinative effort. At this stage we don't need a more refined notion, an intuitive grasp is enough.

The notion of mutual sufficiency is clearly different from that of content recarving, because to say that ϕ and ψ are mutually sufficient is not to say that they share the same content (however intended). Nevertheless there's still a sort of equivalence that preserves some interesting philosophical properties. Which ones? To answer this question we need to consider what mutual sufficiency is supposed to do into the theoretical framework we are working with. First of all, this notion is supposed to be of help in solving Benacerraf (1973) problem about the epistemic access to mathematical objects. Indeed, if we take numbers, functions, sets, and other things populating the mathematical universe to be self-subsistent non-material objects, we will have hard time to explain how we get acquainted with them. Since our cognitive apparatus is known to be able to deal exclusively with material beings, knowledge of mathematical objects looks impossible. The notion of mutual sufficiency is a promising way of answering the challenge only if its primitive (sufficiency) is able to transmit an epistemic status from the antecedent to consequent.

Epistemic Constraint

$$\phi \Rightarrow \psi$$

$$K\phi$$

$$\diamond K\psi$$

The operator K represent an epistemic status whatsoever (being known, being plausible, and the like). This inference rule states that if ϕ suffices for ψ and ϕ has the epistemic property K then it's *possible* for ψ to have the property K too. Here the possibility at stake is real possibility, not a highly idealized one; an *ordinary* knower who knows/believes/etc ϕ must be able to know/believe/etc also ψ . Moreover this possibility ($\diamond K\psi$) must be grounded in the fact that $K\phi$ happens to be true. The possibility of knowing/believing/etc ψ obtains *in virtue* of ϕ 's being known/believed/etc.

A second task that sufficiency is supposed to carry out is to justify certain properties of mathematical universe, especially those regarding its ontological luxury. As already remarked, a principle like Direction Abstraction is supposed to justify the introduction of directions in a universe containing straight lines. In particular a//b it's supposed to count as an explanation of the identity $D(a) = D(b)$. Therefore we need a further constraint on the notion of sufficiency. Here it's presented in Linnebo's version.

Explanatory Constraint

$$\phi \Rightarrow \psi$$
$$E\phi$$

$$E\psi$$

If ϕ is sufficient for ψ and ϕ admits an explanation, then ψ admits an explanation too. Linnebo idea is that what explains ϕ , explains (or, at

least, give rise to an explanation of) ψ too. Therefore, if we want a better formalization of this constraint we need a two-places explanation-predicate Σ , such that $\Sigma(A,B)$ means that B is explained by A. Here is the version of Explanatory Constraint that I favour.

Explanatory Constraint*

$\phi \Rightarrow \psi$

$\Sigma(\gamma, \phi)$

$\Sigma(\gamma, \psi)$

If we consider only Epistemic and Explanatory Constraints, we might be struck by the fact that there's an obvious candidate for the role of sufficiency: the notion of logical consequence. Surely this notion satisfies both constraints; indeed if $A \models B$ then if A enjoys a certain epistemic status then, presumably, we are entitled to suppose that it's possible for B to enjoy the same status; moreover if γ explains A and $A \models B$, then γ explains B too. Nevertheless the identification of sufficiency with logical consequence would completely trivialize this notion and, what counts more, it would make it unfit for our purposes. Indeed if we aim at justifying some philosophically significant abstraction principles (like Direction Abstraction for example) logical consequence is of no help. The identity of D(a) with D(b) is clearly not a logical consequence of a//b. Therefore we need to impose a further constraint on the notion of sufficiency.

Non-Triviality Constraint

There must be systematic range of couples of sentences ϕ and ψ such that

i) $\phi \Rightarrow \psi$

ii) ϕ and ψ are to be taken at face value

iii) the ontological commitment of ψ exceed that of φ

The quest for a notion able to respect these three constraints doesn't immediately appear hard. For example, a possible candidate is *analytic entailment*.²⁸ It could appear a good option, but when it comes to abstraction principles like, for example, direction abstraction, we find ourselves into troubles. Suppose that $a//b$ analytically entails $D(a) = D(b)$. From $D(a) = D(b)$ we derive the existential claim $\exists x(x = D(a))$. If we suppose that logical consequence is subsumed under sufficiency (we are going to do so below), we get that $a//b$ analytically entails $\exists x(x = D(a))$. We can write $A[a//b \rightarrow \exists x(x = D(a))]$ and, since there's no free occurrence of x into $a//b$ we can write $A\exists x[a//b \rightarrow x = D(a)]$. So we are in presence of an analytical existence claim. No need to say that this cannot but be a very problematic consequence of this approach.

Therefore we need something less demanding. This less demanding notion can be thought to be *strict implication*. In other words the idea is that $\varphi \Rightarrow \psi$ is nothing but $\Box(\varphi \rightarrow \psi)$. It's easy to see that if φ and ψ are two necessary truths, say $a=a$ and $p \rightarrow p$, we can legitimately claim that the former suffices for the latter. Here the explanatory constraint is clearly not respected. Moreover the strict implication between a certain necessary truth and the existence of a necessary God would not conserve the epistemic status nor it would be explanatory. We need a notion less demanding than analytic entailment and more demanding than strict implication. Is there such a notion? Before trying to answer let's examine some other requirements that the such a notion must respect.

²⁸ Which is, by the way, Frege's favourite candidate.

There must be also some logical constraints on the notion of sufficiency. Indeed if φ and ψ are mutually sufficient, they must entertain the same logical relationships with all the other sentences. For the exposition of logical constraints we'll employ the symbol Γ for a set of sentences, so, when we write $\Gamma \Rightarrow \varphi$ we mean that the sentences of Γ are collectively sufficient for φ or, if the language we adopt allows for infinite conjunction, that the infinite conjunction of all the γ belonging to Γ .

First of all it's convenient to subsume logical consequence under sufficiency.

Subsumption

$\Gamma \models \varphi$

—————

$\Gamma \Rightarrow \varphi$

In other words, if φ is a logical consequence of a set of sentences Γ , then Γ suffices for φ .²⁹ The notion of logical consequence at issue must be first order or, if intended for a higher order language, it must be defined according to Henkin's semantics. This restriction is

²⁹ I believe that there could be some tension between this rule and the inference rule that we called Epistemic Constraint. Linnebo is well aware of the risk connected with so strong a requirement (see pp. 28-29 of Linnebo forthcoming) but it seems that he didn't notice that Classical Logical Consequence allows us to assert $\varphi \models \psi \vee \neg\psi$. For Subsumption we get $\varphi \Rightarrow \psi \vee \neg\psi$. Now suppose that φ enjoys the epistemic status of being known (let's represent it by means of the operator K); from Epistemic Constraint we get $\diamond K(\psi \vee \neg\psi)$. In principle there's nothing wrong with this, but, as Linnebo clearly states, Epistemic Constraint establishes that, if $A \Rightarrow B$ then there's a *transmission* of epistemic status from A to B. Therefore, if A is known it's possible that B is known and this is true *in virtue of A being known*. But, in the present case, the possibility, for $\psi \vee \neg\psi$, of being known doesn't depend on the fact that φ is known. Indeed $\psi \vee \neg\psi$ is a logical truth; we are allowed to suppose that it's known a priori and not in virtue of knowledge of any other statement whatsoever. I don't believe that this is a serious problem for Linnebo's view. A possible solution is to weaken Epistemic Constraint replacing Classical Logical Consequence with a suitable version of Relevant Logical Consequence. This move seems to be rather innocuous; indeed we can weaken the constraint in question avoiding any other substantial modification of Linnebo's approach.

motivated by a well known concern: standard second order logical consequence is badly incomplete. Indeed not every logical consequence of a sentence φ can be deduced from φ .³⁰ But if standard second order logical consequence is subsumed under sufficiency and Epistemic Constraint holds, then there are cases in which a consequence of φ that cannot be deduced from φ is (possibly) known in virtue of φ . This would be a serious problem for the coherence of this theoretical proposal. The adoption of Henkin's semantics for second order logic disposes of this threat since Henkin's style logical consequence is demonstrably complete.

Another indispensable logical property of sufficiency is transitivity.

Cut

$\Gamma_i \Rightarrow \varphi_i$ for each $i \in I$

$\{\varphi_{i \in I}\} \Rightarrow \psi$

$\bigcup_{i \in I} \Gamma_i \Rightarrow \psi$

If a set of sentences Γ_i suffices for φ_i (for every i belonging to I) and the set of all φ_i such that i belongs to I suffices for ψ , then the union of all the sets Γ_i (such that i belongs to I) suffices for ψ .

Moreover, sufficiency must interact properly with the material conditional.

³⁰ To be more precise: there's no sound deductive system for second order logical which is also complete under standard semantics and effective. This is a consequence of Gödel Incompleteness Theorem.

Conditionalization

$$\begin{array}{ccc} \Gamma, \varphi \Rightarrow \psi & & \Gamma \Rightarrow \varphi \rightarrow \psi \\ \hline & & \\ \Gamma \Rightarrow \varphi \rightarrow \psi & & \Gamma, \varphi \Rightarrow \psi \end{array}$$

This is clearly an analogous of Deduction Theorem applied to sufficiency; if Γ and φ suffice for ψ then Γ suffices for $\varphi \rightarrow \psi$, and vice versa.

Finally we cannot allow sufficiency to ensure everything.

Non-trivialization

$$\not\Rightarrow \perp$$

No set of sentences suffices for a contradiction.

These four logical constraints are sufficient to ensure that if φ and ψ are mutually sufficient, then they entertain the same logical relationships with all the other sentences. Indeed all the classical introduction and elimination rules for the logical connectives can be deduced from these four constraints.

These four logical constraints are not so difficult to satisfy, they leave room for a multiplicity of solutions.³¹ The problem is only to reconcile the strictures posed on the one hand by epistemic and explanatory constraint and, on the other hand, by the non-triviality constraint. The difficulty is not easy to overcome but there are two reasons not to despair: i) the notion of content we are looking for must not necessarily be an already known one; 2) two sentences can have the same content with respect to the features that are relevant for us, i.e. it's not a problem if two sentences are not cognitively equivalent, it's enough if they are explanatorily and epistemically equivalent. In

³¹ In Linnebo (forthcoming) p. 38 it is shown that every consistent theory T is such that if we identify sufficiency with \models_T , sufficiency respects the four logical constraints.

section 5 it will be shown what's the notion of sufficiency must be identified to.

4.3 A metasemantic conception of criteria of identity

Criteria of identity are the key to get the desired theory of thin objects. Quine (1948) suggested that a necessary condition for an act of reference to take place is the availability of identity conditions for the things we aim at referring to. The importance of such identity conditions is not negligible. For example, in order to refer univocally to the river Cayster we need to distinguish it from what is not the river Cayster and what is merely a part (spatial or even spatio-temporal) of it. Moreover identity criteria allow us to answer the question 'what's so special about abstraction principles?'. The answer is simple: they are special because they provide identity conditions for abstract objects. They are nothing but identity criteria for directions, numbers and everything is liable to be defined this way. A nice feature of an account of reference based on identity conditions is its full generality. Unlike accounts based on causal relationships between an object x and a term standing for x , whose validity cannot cover cases of terms whose reference is not a typical middle-size dry good, this account would be fully general.

Before examining Linnebo's reflections on the relation between criteria of identity and reference, it's worth remarking that what is needed is only a *sufficient* condition for reference. Nobody claims that, in every case, reference must take place as described below. The aim of the presented model is only to show that what is *minimally* required for a singular term to have reference is so easy to achieve that also an abstract entity term can successfully refer.

Linnebo presents a toy model of how identity criteria and reference interact. His point is essentially that reference to objects can be implemented by a very rudimental system if such a system is able to apply criteria of identity. Imagine a robot that is provided with two of our senses: sight and touch. It is capable of some very fundamental cognitive processes, not highly complex ones such as consciousness. Suppose this robot is capable to see objects, i.e. to detect light reflected by their surface, and to touch them, i.e. to entertain a physical contact with a spatiotemporal part of them. The robot needs also the ability to understand when two nucleuses of information come from the same body and when they come from two different bodies; otherwise the robot would simply be a thing capturing light from other things and bumping into them. What it needs is to know that different parts constitute an object if and only if they are spatiotemporally connected, i.e. if and only if there's an uninterrupted stretch of solid stuff connecting its various parts. If two parts are so connected we can say that they belongs to the same body. The relation in play is obviously symmetric and transitive (there's no need to suppose that it's also reflexive). The relation holding in a domain of parts is therefore *partial equivalence relation*. This relation is such that not every object belonging to the domain is included into an equivalence class. We can write

$$B(u) = B(v) \leftrightarrow u \sim v$$

In words: the body which u belongs to is identical with the body which v belongs to if and only if u and v are connected by an uninterrupted stretch of solid stuff. Notice that this statement has the form of an abstraction principle. If the robot's software is provided with such a principle it should be able to refer to a body. Indeed it's not easy to imagine what else is needed for machine to do so. If a

cognitive system can do what this robot can do then it is able to refer to objects. The envisaged situation displays an interesting reductionist aspect that in the next section will receive more attention: the robot should be able to refer to a body without any computation involving the notion of body.

A possible objection to this account of reference is that it is at odd with Kripke's account of reference of proper names. He famously claimed, in Kripke (1972), that the reference of a proper name is direct, not mediated by any sort of description. It could seem that an account of reference for names based on criteria of identity calls for the existence of something mediating between a name and its reference. Despite the appearances it's only a superficial concern. The account of reference at issue is explicitly metasemantic; in other words, it's not an account of the meaning of proper names, but an account of what are the minimal conditions that a linguistic expression must respect in order to be able to refer. The problem is not what reference consists in, but how reference is possible. Kripke's famous claims address only the semantic problem and are perfectly compatible with any metasemantics whatsoever. Nothing prevents us from maintaining that proper names directly refer to objects and that the reason why they can do so is that they satisfy certain identity criteria.

Now, the problem for Linnebo is to justify his metasemantic conception of criteria of identity. Since now, we have treated his conception as the only one, but, as usual, reality is more complex. There are two different conceptions of criteria of identity other than the metasemantic one that we have adopted: a metaphysical one and an epistemic one. According to the metaphysical conception of criteria of identity, such criteria provides information about the *nature* of the

things they are about. In this interpretation the principle known as Direction Abstraction gives answer, at least partially, to the question ‘what are directions?’. According to the epistemic conception, criteria of identity are what allows us to *decide* whether items are the same or not.

Linnebo believes that both conceptions are correct and that criteria of identity have both a metaphysical and an epistemic side, but, in his opinion, their fundamental trait is their metasemantic role. Let’s see how he shows this. Both epistemic and metaphysical conceptions suffer from some serious limitations. Epistemic conception treats criteria of identity simply as means that we need to recognise two presentations as belonging to one and the same object or as belonging to distinct objects. In order to do so they simply direct our attention to some distinctive marks, whose detection should lead us immediately to the right conclusion. These marks cannot tell us so much about the identified thing itself. Certainly the typical marks of a thing depends on its features but, in many cases they are not so informative about the thing itself. Consider, for example, the case of a certain disease and its symptoms. Strong chest pain is a typical mark of heart attack and it certainly allows a physician to detect it and to distinguish it from other diseases, but it doesn’t say so much about what the heart attack itself is or about what’s going on into the heart.

The problem that Linnebo sees with the metaphysical conception of criteria of identity is that, if so conceived, criteria of identity cannot be distinguished from other metaphysical truths. Indeed a criterion of identity is presumably a metaphysically necessary sentence that has a certain typical logical form. Look at this example: we customarily claim that the identity of sets is given by the principle of Extensionality (namely, A and B are the same set if and

only if for every x , x is a member of A if and only if it's a member of B). Nevertheless there's another metaphysical truth about sets that, for its logical form, is apt to be a criterion of identity and it's the following one:

$$\text{SET}(a) \wedge \text{SET}(b) \leftrightarrow [a = b \leftrightarrow \forall x(a \in x \leftrightarrow b \in x)]$$

In words: a and b are sets if and only if they are the same set if and only if they belongs to the same sets. This sentence is necessarily true, it displays a logical form compatible with that of an identity criteria and, finally, it certainly says something about the identity of sets. Nevertheless we still prefer the principle of extensionality as a criterion of identity. A supporter of metaphysical conception of criteria of identity cannot but find difficult to explain why we accord our preference to Extensionality. Linnebo can finally concludes:

I believe that my metasemantic conception of criteria of identity provides a clearer and more satisfactory account of how criteria of identity are distinguished from more humdrum metaphysical truths. Criteria of identity explain how certain fundamental forms of reference are constituted. And through that, they also come to govern the referents in questions. (Linnebo forthcoming p. 126)

4.4. Abstraction and reference

4.4.1 A community of speakers

As already said, criteria of identity have an important role to play in Frege's account of reference. Linnebo believes that this is a fundamental feature that a good account must preserve, since it favours the elaboration of a fully general explanation of reference, capable of justifying reference to both material and abstract objects. In order to illustrate his Fregean conception of reference, Linnebo proposes a sort of thought experiment in which he asks to imagine a

community of speakers who, at same point, starts talking about some abstract entities, somewhat extending its language. He then tries to show that we should interpret their use of abstract terms as really referring to abstract entities and not as an indirect or metaphorical way to refer to concrete entities.

Let's start with the thought experiment. A community of speakers communicates by means of a language \mathcal{L}_0 , that we will call *base language*, capable of referring to and quantifying over various concrete objects. For simplicity reasons Linnebo assumes that \mathcal{L}_0 is a first order language with identity and that its interpretation I is shared by each member of the community. Therefore it's a priori excluded the possibility of linguistic disagreement. The domain of I is D_0 and it consist of concrete objects whose existence is not matter of any dispute. The base language is able to refer to (and quantify over) letter-tokens, namely concrete *inscriptions* (on physical supports) of alphabetic letters. Now suppose that, at a certain point, this community of speakers becomes able to talk about letter-types, namely those abstract entities we commonly call *letters*. They can do this employing an *extended language* \mathcal{L}_1 which is identical with \mathcal{L}_0 , except for the fact that it allows for reference to and quantification over a new sort of objects, namely letters. The extended language \mathcal{L}_1 is a two sorted language; it's constituted by a base sort covering all the constants, variables and predicates of \mathcal{L}_0 and by an extended sort, reserved for letters talk. The latter is constituted by: 1) a set of constants referring to letters; 2) a set of variables ranging over letters; 3) a set of predicates suitable for letter talk; 4) a functional operator \S assigning a letter to each inscription; 5) two identity signs, $=$ and $=_1$, that can be flanked respectively by singular terms referring to

inscriptions and by singular terms referring to letters.³² It should be noticed that this example can be easily generalized and hence cover other cases of abstract entities talk. Nevertheless, the fact itself that the presented model requires a many sorted language poses a restriction on the generality of Linnebo's model: it can be applied only to cases of predicative abstraction. This restriction may affect the mathematical power of a theory of abstraction, but, since we are dealing exclusively with the ontological aspects of the problem, we are free from this concern.

The members of this hypothetical community use the extended language to talk about letters; they act *as if* they are referring to and quantifying over letters. They can do so in virtue of abstraction on inscription. Indeed we suppose that they master some criteria for when two inscriptions count as different inscriptions of the same letter. It's worth noticing that this ability doesn't require any cognitive grasp of the notion of letter (indeed there are electronic devices with this capacity); the speakers of this community are able to assign (or to refuse to assign) two different inscription to the same letter even if they are not able to explain what letters are. We use the symbol ' \sim ' to design the relation, holding on D_0 , between inscriptions that count as inscriptions of the same letter. This relation is clearly *symmetric* and *transitive*, but *not reflexive* since not every member of D_0 is an inscription of a letter. In other words \sim is a partial equivalence relation that divides a subset of D_0 into equivalence classes.

The principle of abstraction on inscription is the following:

Letter Abstraction: $\S(x_1) =_1 \S(x_2) \leftrightarrow x_1 \sim x_2$

³² Identities between terms belonging to different sorts are deemed as ill-formed. This is aimed at avoiding the so called Cesar Problem of mixed identities.

The free variables x_1 and x_2 belong to the base sort, while the singular terms $\xi(x_1)$ and $\xi(x_2)$ and, obviously, the identity sign $=_1$ belong to the extended sort. This principle enables speakers to formulate assertible statements about letters. Before examining the assertibility conditions, it's worth noticing that, in considering a principle like Letter Abstraction as a norm of assertibility, and not as a norm of truth, Linnebo is claiming that the hypothetical community simply regards sentences respecting it as correct, as valid moves in their everyday linguistic practice. Here no substantial semantic claim is in play: it simply makes no sense to say that $\xi(x_1) =_1 \xi(x_2)$ expresses the same content of $x_1 \sim x_2$ or that the former can be reduced to the latter. The principle at issue is simply a linguistic practice accepted into the community and the same holds for every other assertible sentence formulated in \mathcal{L}_1 . Here is a semiformal presentation³³ of the assertibility conditions for sentences of \mathcal{L}_1 :

1. Every sentence of \mathcal{L}_1 that is also a sentence of \mathcal{L}_0 is assertible if and only if it's true under the interpretation I.
2. $\xi(x_1) =_1 \xi(x_2)$ is assertible if and only if $x_1 \sim x_2$.
3. Each n -place predicate P is associated with an assertibility condition ϕ_P such that $P(\xi(x_1), \xi(x_2), \dots, \xi(x_n))$ is assertible if and only if $\phi_P(x_1, x_2, \dots, x_n)$
4. The clauses for truth functional connectives are the obvious compositional ones.
5. $\forall x\theta$ is assertible if and only if, for every value that x can assume, θ is assertible.

³³ For a more precise presentation see Linnebo (forthcoming) pp. 154-155. For the purpose of the present exposition the semiformal characterization is sufficient.

The assertibility condition ϕ_p mentioned at condition 3 must be subject to two constraints. First we need to assume that \sim is a congruence relatively to assertibility condition ϕ_p :

$$(x_1 \sim y_1 \wedge x_2 \sim y_2 \wedge \dots \wedge x_n \sim y_n) \rightarrow \phi_p(x_1, x_2, \dots, x_n) \leftrightarrow \phi_p(y_1, y_2, \dots, y_n)$$

In other words ϕ_p must be unable to distinguish items that are equivalent relatively to relation \sim . If two inscriptions x_1 and y_2 count as inscriptions of the same letter, then the assertibility conditions of two sentences, that differ exclusively because in the former x_1 features in a certain position and in the latter there's y_2 in that same position, are the same. Second we have to assure that each object to which ϕ_p applies is an inscription (recall that the domain of the extended language doesn't include only inscription but also whatever object is included into domain D_0).

$$\phi_p(x_1, x_2, \dots, x_n) \rightarrow (x_1 \sim x_1 \wedge x_2 \sim x_2 \wedge \dots \wedge x_n \sim x_n)$$

In other words if ϕ_p holds for x_1, x_2, \dots, x_n then each x_i (with i ranging from 1 to n) must be an inscription. This characterization of \mathcal{L}_1 should suffice to show that the speakers belonging to our envisaged community speak *as if* they were referring to and quantifying over letters. In the next section we will explain why Linnebo claims that they are *really* referring to and quantifying over letters.

4.4.2 Reductionism vs Non-Reductionism

The question is what's the best interpretation of the extended language \mathcal{L}_1 . As already said, the interpretation of \mathcal{L}_0 poses no problem, since every member of the envisaged community agrees on I. But when it comes to \mathcal{L}_1 at least two different stances are possible: on the one hand someone could argue that, since the assertibility conditions for the

sentences of \mathcal{L}_1 don't mention letters, the most appropriate (because the closest to the linguistic data) interpretation is a *reductionist* one, according to which a model for \mathcal{L}_1 doesn't require a domain more extended than D_0 ; on the other hand someone else could argue that the behaviour of the speakers suggests that they are really referring to and quantifying over letters and therefore an acceptable model for \mathcal{L}_1 cannot but require a domain wider than D_0 . This latter position, that we'll call *non-reductionist* takes at face value the assertible sentences of \mathcal{L}_1 and, since there are functional operators that, when saturated with an inscription-term, generate a singular term that behaves like a letter-term, then \mathcal{L}_1 really allows for reference to letters. On the contrary the reductionist reads off her interpretation from the assertibility conditions and instead of saying that a term like $\S(x_1)$ is associated with x_1 , she claims that it refers to x_1 . From her point of view, assertibility conditions become truth conditions.

The supporter of non-reductionism, as already said, needs to add letters to the set of existing things. This addition is obtained introducing a separate domain D_1 for letters. Such a domain can be considered the range of function \S .

Letters Domain: $b \in D_1 \leftrightarrow \exists x(x \in D_0 \wedge x \sim x \wedge \S(x) = b)$

In words: an item b belongs to D_1 if and only if there is an x belonging to D_0 and being an inscription such that function \S associates b to x . According to the non-reductionist view, identities between items belonging to D_1 are true if and only if they respect the condition imposed by the already presented principle Letter Abstraction. The truth conditions for an atomic sentence, in which a predicate P is applied to letter-terms, requires that assertibility conditions for P are defined as follows:

$$\mathbf{A-Equivalence:} \quad \forall x_1, x_2, \dots, x_n \{ \phi_P^* [\xi(x_1), \xi(x_2), \dots, \xi(x_n)] \leftrightarrow \phi_P(x_1, x_2, \dots, x_n) \}$$

This constraint simply says that the assertibility condition (that we represent with the symbol ϕ_P^*) for a predicate P applying to a certain sequence of letters is equivalent to the assertibility condition for the same predicate applying to the corresponding sequence of inscriptions. A-Equivalence enable us to say that an atomic sentence $P(b_1, b_2, \dots, b_n)$, with $b_1, b_2, \dots, b_n \in D_1$, is true, under the non-reductionist view, if and only if $\phi_P^*(b_1, b_2, \dots, b_n)$. The truth conditions for sentences in which logical connectives or quantifiers feature are construed in the obvious way (just recall that the quantifiers of sentences of \mathcal{L}_1 range over D_1 , not D_0).

These two interpretations, reductionist and anti-reductionist, differ for their truth conditions and obviously for the ontological commitment that they assign; nevertheless they are equivalent under an important respect: as it's clear from the two principles above (Letters Domain and A-Equivalence) the “request of the world” that the two interpretations make are perfectly equivalent. For example: for an identity of letters to be true what is required is just the fact that makes true an identity of inscriptions. There's a precise sense according to which also the non-reductionist interpretation is reductionist: the truth conditions it assigns to \mathcal{L}_1 sentences never make an irreducible reference to entities belonging to D_1 . One may ask what's the fundamental source of the disagreement between a reductionist and a non-reductionist. Also to this question there's a precise answer: the disagreement lies in the different conception that the two disputer have of semantics and what semantics involves. The reductionist believes that his foe's semantics requires a problematic and purposeless introduction of abstract semantics values; the non-

reductionist believes that his foe's semantics includes some metaphysical principles alien to genuine semantics, principles according to which, for example, identity of letters is nothing but identity of inscriptions. In a sense, the disagreement lies in what they believe is the boundary of semantics.

Before looking at Linnebo's arguments in favour of non-reductionist interpretation, it's worth observing that such an interpretation is really available and not simply an imaginary possibility. The truth conditions given by a non-reductionist interpretation doesn't require to the world something that could be required only after a serious empirical inquiry. Consider the case of a term like 'phlogiston': its introduction as a constant of \mathcal{L}_1 would require scientific knowledge at least of its chemical structure and its role in a combustions process. The assertibility conditions of an atomic sentence containing such a constant cannot avoid to mention what the term purport to refer to. Unlike the introduction of letter terms, the introduction of the term 'phlogiston' and its use in atomic sentences makes some substantial request to reality. The non-reductionist interpretation, as is characterized in the present chapter, is really available.

Let's see what are the arguments that Linnebo proposes to show the superiority of non-reductionist interpretation. A first argument is related with a problem of compositionality and it is about the treatment of some generalized quantifiers. Consider a language \mathcal{L}_M identical with \mathcal{L}_1 except for the fact that it's enriched with the quantifier 'most'; its standard semantic treatment the following: if we say that most x are P we mean that, given a collection of relevant x (our domain), more than half of them is P . Now suppose that five inscriptions are drown on a blackboard: A, B, A, A, C. Taking this

collection of inscriptions as our domain, one can truly say, in \mathcal{L}_M , that most inscriptions are vowels. There's no reason to doubt that a reductionist and a non-reductionist would agree on the truth value of this sentence. But now consider the sentence 'most letters are vowels'. Such a sentence is intuitively false in \mathcal{L}_M , since only one letter is a vowel, while the other two are consonant. A non-reductionist has no difficulty in recognising the falsity of such a sentence and therefore in upholding our intuitions. Indeed the sentence at issue is:

$$(1) \exists_M x(Lx \rightarrow Vx)$$

The symbol ' \exists_M ' stands for the quantifier 'most'. The semantic analysis of a non-reductionist would take the quantifier to range over a subset of domain D_1 (contextual restriction of D_1) containing three letters, only one of which is a vowel. Hence, according to a non-reductionist interpretation, the sentence is false (and our intuition are vindicated). What about a reductionist interpretation? According to reductionism, the quantifier doesn't range over a subset of D_1 , but over a subset of D_0 , a set containing precisely the inscriptions on the blackboard. The reductionist semantic analysis can be represented semi-formally as follows:

$$(2) \text{For most } x \text{ of the blackboard}(x \text{ is an inscription} \rightarrow x \text{ is a vowel inscription})$$

The analysis cannot but give as output the truth of sentence (2). Indeed most inscriptions on the blackboard are vowel-inscription. Therefore a reductionism semantics has an important limit: it's not able to deal with the generalized quantifier 'most' in the desired way.

It should be noticed (and Linnebo notices it) that the reductionist can rebut that her semantics can be easily enriched with the resources needed for the treatment of these cases in accordance

with our intuition and without being unfaithful to its reductionist vocation. For example, we can introduce, into the metalanguage that we use to express truth conditions for the sentences of \mathcal{L}_M , a function π associating each singular term (constant or free variable) t of the extended sort (hence a term standing for a letter) with one and only one member x of D_0 such that $x \sim t$, namely with an inscription. Function π respects the constraint imposed by Letter Abstraction; it never associates two distinct letter-terms with the same inscription, but each letter term with a different inscription. Such inscription is one of the many inscriptions that Letter Abstraction associates with a certain letter and it acts like a proxy of the equivalence class it belongs to. A reductionist can claim that letters talk is nothing but proxy-inscriptions talk. Her analysis of (1) becomes:

(3) For most x of the blackboard(x is a proxy-inscription \rightarrow x is a vowel inscription)

Since the proxy inscriptions are three and only one of them is a vowel inscription (3) is false. The accordance with our intuitions is regained.

Linnebo remarks that this strategy works in the present case, but cannot be extended to all the conceivable cases. Indeed, in some cases the entities of the extended domain largely outstrip the entities of the base domain. Consider, for example, Predicative Basic Law V. Unlike its impredicative cousin, it's perfectly consistent, since the domain of extensions is separate from the domain of basic entities (no extension can be included into an equivalence classes to which abstraction assigns an extension). In virtue of Cantor Theorem we know that the cardinality of the extended domain is bigger than the cardinality of the base domain. Therefore the reductionist gambit cannot work here: there cannot be enough proxies among the objects of the base domain.

Nevertheless there's a last option: taking the pluralities generated by Predicative BLV themselves as proxies. Linnebo believes that also this strategy cannot but lead to a dead end. In his words:

Since pluralities are not objects, however, this will result in type clashes. For instance, when 'most' is applied to talk about ordinary objects, then the truth conditions involve cardinality comparisons among objects, but when 'most' is applied to talk about sets, then we need to make cardinality comparisons among pluralities. (Linnebo forthcoming p.145)

I believe that the problem that he sees cannot be avoided. One could well imagine a strategy allowing the reductionist to both treat extensions as objects and conserving fidelity to the essential ambition of a reductionist semantics, but this wouldn't work for different reasons. Let's examine what's the extreme move that a reductionist could try. Extension b can be identified with the mereological fusion of all the objects x such that $\xi(x) = b$. The number of possible mereological fusions of objects belonging to the base domain equals the number of objects belonging to the extended domain, so we avoid cardinality problems. Moreover the acceptance of mereological fusions is presumably not a scandal for a reductionist. Under the standard account of mereology,³⁴ the mereological fusion $\langle bc \rangle$ of two (unproblematic) entities b and c doesn't count as an additional ontological commitment, since $\langle bc \rangle$ is nothing more than b and c taken as a whole. Even if someone might contend that the thesis of the ontological innocence of mereology is dubious, since it relies on a not universally accepted characterization of the notion of object,³⁵ there seems to be room for a reductionist to accept mereological fusion without twisting her view. Finally the problem of type clashes is

³⁴ See, for example, Lewis (1991), and more recently, Hawley (2014).

³⁵ See, for example, Carrara (manuscript).

avoided since mereological fusions count as objects. But there's a serious problem: to which domain these mereological fusions belong to? If we take them as belonging to the extended sort, then a reductionist cannot make use of them when she formulates the truth conditions for sentences talking about them. Recall that reductionist truth conditions don't mention entities belonging to the extended domain (that's the point of reductionism). If, alternatively, she takes them as belonging to the base domain, she faces a cardinality problem again. Indeed Predicative BLV is such that the quantifiers featuring on its right hand side ranges over all the entities of the base domain. Therefore it generates a number of extensions that largely outstrips the number of mereological fusions, which constitutes a subset of the base domain. I claim that the reductionist faces a dilemma: either renouncing to an essential trait of a reductionist semantics or falling into inconsistency. I believe we should conclude that Linnebo is right in believing that non-reductionism fares better with respect to compositionality.

A second argument Linnebo appeals to for the vindication of the superiority of non-reductionist approach to the interpretation of \mathcal{L}_1 , has to do with the cognitive constraints that interpretations must satisfy. In general we believe that there is a certain cognitive transparency of meaning, that can be loosely expressed like this:

Cognitive Constraint: the truth conditions that an interpretation assign to a sentence must be the ones adequately grasped by a speaker who understand the sentence.

As Linnebo puts it, a plausible semantic analysis of 'snow is white' must mirror the cognitive content that a competent speaker grasps when reading it, namely that a thing called 'snow' enjoys a property called 'being white'. This constraint seems to be more

straightforwardly met by a non-reductionist interpretation. Indeed a speaker who understands an atomic sentence of \mathcal{L}_1 talking about letters tends to grasp a content according to which a certain property is attributed to a certain letter. It's very unlikely that what she grasps is that a corresponding property is attributed to a certain inscription that is related to the mentioned letter by a relation that is not completely transparent to her mind. Indeed the relation \sim that allows us to count different inscriptions as inscriptions of the same letter is very difficult to analyze. The mental mechanisms involved acts largely at a sub-personal level. We all are able to see that two different inscription of a letter correspond to the same letter, but, if asked to explain why, we will find ourselves in a predicament. The mental processes involved are surely important, but they leave no significant trace on our consciousness. Non-reductionist interpretation seems to fare better also with regard to cognitive aspects.

I find this argument to be less convincing than the previous one, since, as Rayo (2013) has pointed out there is, at least in some cases, a significant gap between what a competent speaker grasp and what a sentence mean. Indeed some sentences can have the same meaning without them being associated with the same cognitive content. I believe that the critical considerations of Rayo about that philosophical prejudice that he dubs 'Metaphysicalism' undermine Cognitive Constraint or, at least, show that it's far too demanding. In a number of cases (recall, for example, the couple of sentences 'Susan runs' and 'the property of running is instantiated by Susan') we can legitimately assign the same truth conditions to two sentences that the cognitive apparatus of a competent speaker might take as very different. Rayo's arguments successfully explain why there's no need to assume that the face-value reading of a sentence must be mirrored

by its semantic analysis. Therefore I believe that Linnebo's cognitive argument in behalf of the superiority of non-reductionist interpretation is not conclusive. Nevertheless we can say that, in general, the availability of a non-reductionist interpretation and its advantage over a reductionist one is well established in his theory.

4.5 Thin Objects

The consequences of the considerations exposed in the previous section are ontologically relevant. It is shown that an extension of a base language by means of abstraction principles is legitimate and a non-reductionist interpretation of the extended language is legitimate and preferable to a reductionist one. Moreover there are true existential sentences in the extended language \mathcal{L}_1 that, according to both the reductionist and the non-reductionist interpretation, are true and such that, in virtue of the latter, we can consider as really committing to letters as abstract objects. Linnebo remarks that

This way of introducing objects into one's rational discourse will no doubt strike some philosophers as too easy. Surely, such philosophers will think, the view that there are abstract objects is a substantive thesis whose truth requires the cooperation of reality and not just the adoption of some language. I disagree. I believe that letters and other abstract objects that can be introduced in an analogous way are thin objects. [...] the idea is that thin objects do not require much of reality for their existence. Their existence requires only the obtaining of some condition which does not mention the objects in question and which is thus comparatively unproblematic. For instance, the existence of a letter requires nothing more than that there be an appropriate inscription. (Linnebo forthcoming, p.151)

Linnebo's theory has clearly a reductionist aspect. Each sentence of \mathcal{L}_1 has assertibility conditions that doesn't mention letters. This is needed in order to guarantee the legitimacy of the introduction of letter talk:

nothing radically new has been introduced. Nevertheless reductionism is confined to assertibility conditions. Truth conditions assigned by the non-reductionist interpretation are classical, hence, sentences which mention letters are interpreted as really referring to letters.

We are now in a position to clarify the notion of sufficiency presented in section 2 of the present chapter.

Sufficiency: $\varphi \Rightarrow \psi$ if and only if there is a legitimate extension of our linguistic resources of the sort described in section 4 of the present chapter such that φ provides the assertibility conditions for ψ .

Linnebo shows that this definition satisfies all the constraints which the notion of sufficiency is subject to. Epistemic and explanatory constraints are satisfied or, at least, we are in good position to claim that they are satisfied. Consider the example given at section 4 of the present chapter: the assertibility conditions associated with sentences of language \mathcal{L}_1 guarantee that a sentence talking about letters can be legitimately asserted if and only if there is a corresponding sentence about inscriptions that is classically true. Therefore there are good reasons to claim that if φ is explained by α and $\varphi \Rightarrow \psi$ then ψ is explained by α too. The same holds for the transmission of epistemic status. For example, the knowability of φ and $\varphi \Rightarrow \psi$ are enough to claim that ψ is (possibly) knowable, since its assertibility conditions are equivalent to those of φ . The non-triviality constraint is obviously respected in virtue of the essential characteristics of a non-reductionist interpretation. Indeed atomic sentences of \mathcal{L}_1 talking about letters present an additional ontological commitment relatively to the sentences of \mathcal{L}_0 they are equivalent to. As it has been already shown in section 2 of the present chapter the logical constraints are easily

satisfied by various notions of sufficiency and, among them, also by Sufficiency.

4.6 Conclusions

At this point it should be clear the Linnebo's theory is able to pass through the three explanatory steps that we outlined in section 1 of the present chapter. Though some passages of his argument are tricky, the resulting picture is rather simple: the concepts of reference, object and identity criteria are tightly related. An object is whatever we are able to refer to in virtue of a criterion of identity. Moreover, if a given language is enriched with the means needed for reference to abstract objects (essentially abstraction principles) and if non-reductionist truth conditions for the sentences of the enriched language are available, then reference to abstract objects really succeeds. I have pointed out that not all the arguments that Linnebo presents in behalf of non-reductionist semantics are fully convincing, nevertheless at least the availability of such a semantic is fully established.

In the next chapter we are going to compare this theory with the other two (Eklund and Rayo's) and to show its superiority over at least one of them.

Chapter 5

CONCLUSIONS

NeoFregeanism is a complex and original conception of the relation between language and reality. The analysis of the three theories of Rayo, Eklund and Linnebo has given us an idea of how different are the ways in which such a conception can be articulated. These different formulations face some difficulties; I've tried to show how they can be overcome. I hope my discourse has given some good reason to consider these theories as something worth of serious reflection.

What I would like to do now is to reassess these theories in light of the four theses, characterizing NeoFregeanism, that I have enunciated in the introduction and that I restate here for the reader's ease:

1) PRIORITY: the following two facts:

a) the singular term 'a' in the atomic sentence 'Fa' refers to an existing object

b) the existence of objects satisfying the condition ϕ that features in the sentence ' $\exists x\phi(x)$ '

are *grounded* respectively in the following two facts:

a') 'Fa' is a true sentence

b') ' $\exists x\phi(x)$ ' is a true sentence

2) PLATONISM : there are self-subsistent abstract objects

3) ABSTRACTION EFFECTIVENESS: some abstraction principles are effective stipulations, i.e. they are such that their two sides share the same content.

4) ABSTRACTION ESSENTIALITY: an argument in behalf of Platonism about a certain kind of abstract entities requires the employ of effective abstraction principles.

As we have said in the introduction, Platonism is the conclusion of an argument that can employ some (or all) of the three other theses. The question that I believe it's worth answering is how each of the three examined theories performs with respect of these theses. Can each justify all of them? If not, which are justified and which are not? Why? The answers to these questions should significantly clarify our view of NeoFregeanism.

5.1 Rayo's Compositionism

Let's start our analysis from Rayo's theory. The main part of Rayo's theoretical effort is devoted to the justification of the legitimacy of 'just is'-statements. He tries to show that there are perfectly acceptable 'just is'-statements, and that those who believe that there is something wrong with them are deceived by a poorly motivated philosophical preconception. Abstraction principles are a subset of 'just is'-statements, so their acceptability is supported by the very same arguments. A 'just is'-statement is acceptable if and only if its two sides are depictions of the same state of affairs, that is, if and only if they constitute two different reconceptualization of one and the same content. If one is able to argue in favour of the acceptability of 'just is'-statements then she can argue in favour of Abstraction

Effectiveness too. As I have shown in Chapter 2 Rayo's defence of 'just is'-statements, as ways of describing one and the same state of affairs in two different ways, requires the appeal to a truthmaker semantics. In a language in which there are acceptable 'just is'-statements, sentences cannot receive their truth conditions from the classical tarskian clauses. Indeed these clauses are too fine grained; they are not able to assign the same truth conditions to different atomic sentences. But this is what some acceptable 'just is'-statements require. Therefore the defence of Abstraction Effectiveness comes at a cost: a coarse grained semantics for the statements of the envisaged language. This fact, as we observed in Chapter 2 may appear at odds with Evans' Generality Requirement. The only solution that I can see to this concern is to distinguish two different aspect of meaning: the state of affair that makes a sentence true and the cognitive content that a competent speaker associates to it. 'Just is'-statements are true when their two sides depict the same state of affair, no matter how different is the cognitive content that a competent speaker would attribute them. If we are ready to accept this distinction, then we can consider Abstraction Effectiveness as adequately justified by Rayo's Compositionism.

What about Priority? Compositionism is certainly a generous view when it comes to assigning a reference to expressions behaving as singular terms. If a 'just is'-statement is true (and it can be so also in virtue of simple framework organization reasons) and one of its two sides is true for the ordinary criteria of truth, then also the other side is true, since the truth of one of the two sides of a 'just is'-statement counts as an ordinary truth condition for the other. And, Compositionism says, if an atomic statement is true according to the ordinary truth conditions, then its singular terms really refer. This

strong thesis, substantially equivalent with Priority, is justified, as we have seen in Chapter 2 in a *negative* way. Rayo's argument is that there's no metaphysical strong reason to favour a certain conceptualization and to employ exclusively that one to depict states of affair. The fact that two lines stand in certain reciprocal relation can be described equally well by means of parallelism between them and by means of identity of their direction. Believing that only one of these two presentations carves reality at the joints is a philosophical prejudice (what Rayo calls Metaphysicalism). If we accept his argument we can say that also Priority is defended by his approach.

When it comes to Abstraction Essentiality things change. As we have already remarked, abstraction principles constitute a subset of 'just is'-statements. Nevertheless nothing, in Rayo's theory, gives us reasons to think that they are essential for the justification of Platonism. Consider for example this 'just is'-statement:

DINOSAURS: for the number of dinosaurs to be zero just is for there to be no dinosaurs.

This sentence can hardly be considered an abstraction principle. Its logical form is $N(D) = 0 \Leftrightarrow \neg \exists x Dx$. Clearly it's not the form of an abstraction principle. Now the truth of Dinosaurs plus the truth of 'there are no dinosaurs' is sufficient for the truth of 'the number of dinosaurs is zero'. If we accept the Compositionalist doctrine, then the singular term 'zero' has a reference. Hence there are numbers (at least one). Nothing prevent us from introducing also other numbers in our ontology in the same way. Consider the following 'just is'-statement:

POPES: for the number of Popes to be two just is for there to be exactly two Popes.

The logical form of this statement is $N(P) = 2 \Leftrightarrow \exists x,y[Px \wedge Py \wedge \forall z(Pz \rightarrow z=x \vee z=y)]$ and this too is obviously not an abstraction principle. It should be clear from these examples that abstraction principles play no essential role, in Rayo's framework, for the justification of Platonism. The thesis of acceptability of some 'just is'-statement and Compositionism provide enough basis for it, there's no need to attribute abstraction principles a privileged role.

5.2 Eklund's Maximalism

As we have explained in Chapter 3 of this dissertation, Maximalism is the view according to which a sortal concept F is instantiated by some individual if and only if F is consistent and it doesn't fail to be instantiated simply as a matter of empirical facts. This view seems to offer robust support to Priority. Consider for example the notion of temporal part. It is a consistent notion; its definition is not contradictory and doesn't entail any contradiction. If we assume Maximalism, then there are temporal parts. From this, assuming the classical disquotational schema, we can deduce that 'there are temporal parts' is true. Now, what Priority ask for is that, if 'there are temporal parts' is true, according to ordinary criteria, then there really are things such that they exemplify the concept of temporal part. But this has been already guaranteed, thanks to Maximalism, by the fact that the notion of temporal part is consistent. Therefore, at least in the case of temporal parts, Maximalism offer a robust justification for Priority. Examples might certainly be multiplied and, among them, there might be many examples of abstract objects whose existence is justified in virtue of the coherence of their notion.

Nevertheless I don't believe that we are entitled to conclude that, in general, Maximalism entails Priority. I believe that there are

entities whose existence could be justified by Priority, but not by Maximalism. Consider the following example. Mathematical Analysis is an immense field of mathematics and it includes a number of theorems about analytic functions, integrals, limits, differential equations and the like. All of these theorems are true according to the ordinary standards of acceptability that rule mathematical discourse. Among the theorems of analysis, there are statements like this:

$$\lim_{n \rightarrow +\infty} \sum_{i=1}^n \frac{1}{2^i} = 1$$

It states that the infinite sum $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$ equals 1. More precisely: the limit, for n which tends to infinity of the summation of the inverses of all the powers of 2 is 1. Since it is a true statement in which ‘lim...’ figure as a singular term, then, if we assume Priority, ‘lim...’ really refers to an item of reality. Hence there are limits. One could ask whether Maximalism is able to justify the same conclusion. The answer can hardly be fully affirmative. There’s no proof of the coherence of mathematical analysis. Consequently one cannot be sure of the compatibility of the existence of limits with all the other theorems of mathematical analysis. The notion of limit, individually taken, is consistent, but there’s no guarantee that it doesn’t clash with other notions of the same theory. Maximalism, as defined by Eklund (2006) requires, for the existence of Fs, only the coherence of the notion of ‘being an F’ individually taken, but, as shown in Chapter 3, there are couple of concepts which are coherent, if taken individually, but inconsistent if taken together. The emendation that we have proposed and defended requires the introduction of vague existential quantification. Such an emendation allows us to solve the problem of individually consistent, but collectively incompatible sortal concepts,

by claiming that both the incompatible sortals only vaguely exist. The case of sortal concepts involved in mathematical analysis might be certainly dealt with the help of vague existential quantification. We can say that, according to Maximalism, limits vaguely exist, i.e. it's not perfectly determined whether they exist or not. But this claim (that is the strongest that Maximalism can support) is definitely weaker than that supported by Priority. Indeed, the truth of $\lim_{n \rightarrow +\infty} \sum_{i=1}^n \frac{1}{2^i} = 1$ plus Priority entails the plain existence of limits, not their vague existence. I think that we can conclude that Maximalism offers only a partial support to Priority.

It should be perfectly clear that, in Eklund's theoretical framework, abstraction principles play no role at all. Maximalism doesn't defend Abstraction Effectiveness, since it doesn't articulate any notion of sameness of content. Moreover no justification is offered for Abstraction Essentiality. We could even say that Maximalism is a perfect disproof of Abstraction Essentiality, since it shows that the introduction of abstract entities in our ontology can be justified without any reference whatsoever to abstraction principles or to statements expressing the relation of sameness of content between two statements (like 'just is'-statements do).

Maximalism can be considered the simplest and the most extreme option that a NeoFregean can choose. It supports Platonism, at least if we are able to provide an acceptable solution for the problem of incompatible objects (and I believe we are), but at the cost of a significant departure from the classical NeoFregean view.

5.3 Linnebo's Metaontological Minimalism

In Chapter 4 we have analyzed an hypothetical situation in which a community of speakers, whose language allows reference to and quantification over inscriptions, extends its language, stipulating that two inscriptions count as inscriptions of the same letter if and only if a certain condition is met. It is argued that this stipulation allows a speaker of this community to really refer to (and quantify over) letters. The truth conditions associated with sentences in which letter-terms feature are such that, for an atomic sentence to be true, the letter-term must really refer to an item of reality. But for an atomic sentence to be true what is requested to reality is nothing more than a completely unproblematic condition involving only objects whose existence is undisputed. Hence Linnebo's theory offers a good defence of Priority. It should be noticed that his defence doesn't entails a wild ontological proliferation (as in Wright's version of Priority), since the constraints imposed on the process of introduction of referring abstract term in a language are rather strict. In particular there must be identity conditions for the newly introduced entities and the truth conditions for the sentences of the extended language must be reducible to truth conditions valid for sentences of the base language. These two restrictions can be satisfied with the help of abstraction principles. Indeed abstraction principles are able to provide both identity conditions for the newly introduced entities and the root for the formulation of truth condition of the appropriate reductive fashion. Abstraction principles play an essential role in Linnebo's justification of Platonism, therefore we can conclude that also Abstraction Essentiality is vindicated.

On the contrary I believe it's dubious that Metaontological Minimalism can adequately defend Abstraction Effectiveness. This

might appear surprising, since Linnebo's effort to the end of defining an appropriate notion of sufficiency is noteworthy and fruitful. Let's reconsider his final definition of sufficiency:

SUFFICIENCY: $\varphi \Rightarrow \psi$ (= φ suffices for ψ) if and only if there is a legitimate extension of our linguistic resources of the sort described in the example of the community of speakers such that φ provides the assertibility conditions for ψ .

Abstraction Effectiveness requires the sameness of content of the two sides of an abstraction principle. Using Linnebo's conceptualization, we could say that the relation between the two sides of an abstraction principle must be of *mutual* sufficiency, not only of sufficiency of one side for the other. Now, his definition of sufficiency seems to be apt to justify only the sufficiency of one side of an abstraction principle (the "concrete" one) for the other (the "abstract" one). It's hard to imagine how a language containing abstract terms to design letters, but no term for inscriptions, could ever be extended in the way described by Linnebo in order to enable us to speak of inscriptions. If sameness of content is explained in terms of mutual sufficiency, then abstraction principles like those presented by Linnebo or like Direction Abstraction cannot be such that their two sides share the same content. We cannot but conclude that Metaontological Minimalism doesn't support Abstraction Effectiveness.

5.4. Priority and Abstraction

The following table summarizes the "performances" of the three examined theories relative to the four characteristic theses of NeoFregeanism.

	Priority	Abs. Eff.	Abs. Ess.	Platonism
Compositionalism	Yes	yes	No	Yes
Maximalism	Partially	no	No	Yes
Metaont. Minimalism	Yes	no	Yes	Yes

As we can see from this schematic recap, all the three theories are able to defend Platonism. Indeed, as I hope to have shown in the course of this dissertation, no objection is fatal for these theories and they all give sufficient justification for a Platonism about abstract entities.

Eklund Maximalism has the worst performance when it comes to justification of the characteristic theses of NeoFregeanism. Abstraction is neither justified as a method of introduction of abstract concepts, nor it plays an essential role towards the justification of Platonism. Moreover, as we have shown before, Priority doesn't seem to be adequately defended.

The performances of Rayo's Compositionalism and Linnebo's Metaontological Maximalism are far better, since they both adequately justify Priority. The remarkable difference between them is about Abstraction Effectiveness and Abstraction Essentiality. While the former is well defended by Compositionalism, but not by Metaontological Minimalism, the latter is well defended by Metaontological Minimalism, but not by Compositionalism. A reasonable question is whether this is only a contingent fact or there is some substantial theoretical reason why it's extremely difficult (if not impossible) to justify both.

It's not hard to see that there are deep theoretical reasons under this difficult, perhaps impossible, reconciliation. Consider first Abstraction Essentiality. A theory in which abstraction principles play

an essential role in the justification of Platonism is certainly a theory that employs one of the peculiar properties of abstraction principles to this end. As Metaontological Maximalism clearly shows, this helpful property is their asymmetry of ontological commitments. The right hand side of an abstraction principle is committed to the existence of (relatively) concrete and unproblematic objects, while its left hand side is committed to abstract entities. The justification of Platonism via abstraction requires that there are abstract entities *in virtue of* there being concrete ones. The right hand side has, loosely speaking, a foundational role. The truth of the right end side of an abstraction principle requires the obtaining of a certain state of affairs; the abstraction principle as a whole assures that nothing more than the obtaining of that very state of affairs is sufficient for the truth of the left hand side. Hence the additional ontological commitment borne by the left hand side comes for free.

If we adopt an argumentative strategy along these lines, then it clearly becomes hard to show that the two sides of an acceptable abstraction principle have the same content, unless we adopt a very weak notion of content. The strategy above requires a different explanatory power for the two sides: one of them grounds the other, but not vice-versa. If we adopt such a strategy (and we *must* adopt it if we want to defend Abstraction Essentiality) then it becomes hard to justify Abstraction Effectiveness.

Obviously this general observation is not meant to prove that a conciliation of the two theses is impossible, but only to show that here we face a substantial theoretical problem and therefore the fact that, so far, no broadly NeoFregean theory is able to maintain both of them should not surprise us. NeoFregeanism is not *per se* a coherent and structured theory, but rather a set of interrelated theses, still in need of

a systematization and nevertheless capable of offering a radical and fascinating view of the relation between language and reality.

Bibliography

- Audi, Robert (2012), 'A clarification and defense of the notion of grounding', in Correia & Schnieder (2012), 101-121.
- Benacerraf, Paul (1965), 'What numbers could not be', *The Philosophical Review*, 74 (1), 47-73.
- (1973) 'Mathematical Truth', *Journal of Philosophy*, 70 (19), 661-679.
- Boolos, George (1986), 'Saving Frege From Contradiction', *Proceedings of the Aristotelian Society*, 87, 137–151.
- (1987), 'The Consistency of Frege's *Foundations of Arithmetic*', in J. Thomson (ed.), *On Being and Saying*, Cambridge, MA: The MIT Press, 3–20.
- (1990), 'The Standard of Equality of Numbers', in G. Boolos (ed.), *Meaning and Method: Essays in Honor of Hilary Putnam*, Cambridge: Cambridge University Press, 261–77.
- (1998) *Logic, Logic, and Logic*, Harvard University Press, Cambridge (MA).
- Burgess, John (2005), *Fixing Frege*, Princeton University Press, Princeton.

- Carnap, Rudolf (1950), 'Empiricism, Semantics and Ontology', *Revue Internationale de Philosophie* 4, 20-40. Reprinted in the Supplement to *Meaning and Necessity: A Study in Semantics and Modal Logic*, enlarged edition (University of Chicago Press, 1956).
- Carrara, Massimiliano (Manuscript), 'On the Alleged Innocence of Mereology'.
- Coffa, J.A. (1991), *The Semantic Tradition from Kant to Carnap*, L. Wessels (ed.), Cambridge University Press, Cambridge.
- Demopoulos, William (ed.), (1995), *Frege's Philosophy of Mathematics*, Harvard University Press, Cambridge (MA).
- Chalmers, David, Manley, David & Wasserman, Ryan (Eds.) (2009), *Metametaphysics: New Essays on the Foundation of Ontology*, Clarendon Press, Oxford.
- Correia, Fabrice & Schnieder, Benjamin (Eds.) (2012), *Metaphysical Grounding: Understanding the Structure of Reality*, Cambridge University Press, Cambridge.
- Davidson, Donald (1969), 'True to the Facts', *Journal of Philosophy*, 66, 748–764.
- Divers, John & Miller, Alexander (1995), 'Minimalism and the Unbearable Lightness of Being', *Philosophical Papers* 24, 127–39.
- Dorr, Cian (2005), 'What we disagree about when we disagree about ontology?', in Mark Eli Kalderon (ed.), *Fictionalism in Metaphysics*, Oxford University Press, Oxford, 234-286.
- Dummett, Michael (1956), 'Nominalism', *The Philosophical Review*, 65 (4), 491-505.

- (1973), *Frege: Philosophy of Language*, Duckworth, London.
 - (1991), *Frege: Philosophy of Mathematics*, Harvard University Press, Cambridge (MA).
- Eklund, Matti (2006), 'Neo-Fregean Ontology', *Philosophical Perspectives*, 20 (1), 95–121.
- (2008), 'The Picture of Reality as an Amorphous Lump', in Theodore Sider, John Hawthorne & Dean W. Zimmerman (eds.), *Contemporary Debates in Metaphysics*. Blackwell Pub. 382-396.
 - (2014), 'Rayo's Metametaphysics', *Inquiry*, 57 (4), 483-497.
- Fine, Kit (1975), 'Vagueness, Truth and Logic', *Synthese*, 30 (3-4), 265-300.
- (1998) 'The limits of abstraction', in *Philosophy of Mathematics Today*, Mathias Schirn, ed., Clarendon Press, Oxford, 503–629.
 - (2002), *The Limits of Abstraction*, Clarendon Press, Oxford.
 - (2012), 'Guide to Ground', in Correia & Schneider (2012), 37-80.
- Frege, Gottlob (1884), *Die Grundlagen der Arithmetik: eine logisch-mathematische Untersuchung über den Begriff der Zahl*, Breslau: W. Koebner. Translated as *The Foundations of Arithmetic: A logico-mathematical enquiry into the concept of number*, by J.L. Austin, Oxford: Blackwell, second revised edition, 1974.
- (1893-1903), *Grundgesetze der Arithmetik*, Jena: Verlag Hermann Pohle, Band I/II. Partial translation of Band I, *The Basic Laws of Arithmetic*, by M. Furth, Berkeley: U. of California Press, 1964.

- Field, Hartry (1980), *Science without Numbers: the Defence of Nominalism*, Princeton University Press, Princeton.
- (1984), 'Critical Notice of Crispin Wright's 'Frege's Conception of Numbers as Objects'', *Canadian Journal of Philosophy*, 14 (4), 637-662.
- Hale, Bob (1987), *Abstract Objects*, Basil Blackwell, Oxford.
- (2001a), 'Singular Terms (1)', in Hale & Wright (2001), 31-47.
 - (2001b), 'Grundlagen § 64', in Hale & Wright (2001), 91-116.
- Hale, Bob & Wright, Crispin (2001), *The Reason's Proper Study: Essays towards a NeoFregean Philosophy of Mathematics*, Clarendon Press, Oxford.
- (2009), 'The Metaontology of Abstraction', in Chalmers, Manley & Wasserman (2009), 178-212.
- Hawley, Katherine (2014), 'Ontological Innocence', in A. J. Cotnoir & Donald L. M. Baxter (eds.), *Composition as Identity*, Oxford University Press, Oxford, 70-89.
- Heck, Richard (1992), 'On the Consistency of Second-Order Contextual Definitions', *Noûs*, 26 (4), 491-494.
- (1993), 'The Development of Arithmetic in Frege's Grundgesetze der Arithmetik', *Journal of Symbolic Logic*, 58 (2), 579-601.
 - (ed.) (1997), *Language, Thought, and Logic: Essays in Honour of Michael Dummett.*, Oxford University Press, Oxford.
 - (2011), *Frege's Theorem*, Oxford University Press, Oxford.
- Heim, Irene & Kratzer, Angelika (1998), *Semantics in Generative Grammar*, Blackwell, Oxford.

- Kripke, Saul (1975), 'Outline of a Theory of Truth', *The Journal of Philosophy*, 72 (19), 690-716.
- (1980), *Naming and Necessity*, Harvard University Press, Cambridge (MA).
- Lewis, David (1991), *Part of Classes*, Blackwell, Oxford.
- Linnebo, Øystein (2004a), 'Predicative Fragments of Frege Arithmetic', *Bullettin of Symbolic Logic*, 10 (2), 153-174.
- (2004b), 'Frege's Proof of Referentiality', *Notre Dame Journal of Formal Logic*, 45 (2), 73-98.
 - (2009a), 'Introduction', *Synthese*, 170 (3), 321-329.
 - (2009b), 'Bad Company Tamed', 170 (3), 371-391.
 - (2010), 'Some Criteria for Acceptable Abstraction', *Notre Dame Journal of Formal Logic*, 52 (3), 331-338.
 - (2012a), 'Reference by Abstraction', *Proceedings of the Aristotelian Society*, 112 (1pt 1), 45-71.
 - (2012b), 'Metaontological Minimalism', *Philosophy Compass*, 7 (2), 139-151.
 - (2013), 'Platonism in the Philosophy of Mathematics', *The Stanford Encyclopedia of Philosophy* (Winter 2013 Edition), Edward N. Zalta (ed.), URL = <http://plato.stanford.edu/archives/win2013/entries/platonism-mathematics/>.
 - (2014), "'Just is'-Statements as generalized identities', *Inquiry*, 57 (4), 466-482.
 - (forthcoming), *Thin Objects: an Abstractionist Account*.

- Linnebo, Øystein & Uzquiano, Gabriel (2009), 'Which Abstraction Principles are Acceptable? Some Limitative Results', *British Journal for the Philosophy of Science*, 60 (2), 239-252.
- Lynch, Michael (2009), *Truth as One and Many*, Oxford University Press, Oxford.
- MacBride, Fraser (2003), 'Speaking with Shadows: a Study of NeoLogicism', *British Journal of Philosophy of Science*, 54, 103-163.
- Putnam, Hilary (1987), *Representation and Reality*, MIT Press, Cambridge (MA).
- Quine, William V.O. (1948), 'On what there is', *Review of Metaphysics* (1948). Reprinted in 1953 *From a Logical Point of View*, Harvard University Press, Cambridge (MA).
- (1951), 'Two Dogmas of Empiricism', *The Philosophical Review*, 20-43.
- Rayo, Agustin (2013), *The Construction of Logical Space*, Oxford University Press, Oxford.
- (manuscript), 'NeoFregeanism Reconsidered'.
- Shapiro, Stewart (2003), 'Prolegomenon to Any Future Neo-Logicist Set Theory: Abstraction and Indefinite Extensibility', *British Journal for the Philosophy of Science*, 54 (1), 59-91.
- Shapiro, Stewart and Weir, Alan (1999), "New V, ZF, and Abstraction," *Philosophia Mathematica*, 7, 293–321.
- Sider, Theodore (2003), 'Against Vague Existence' *Philosophical Studies*, 114 (1-2), 135 – 146.

- Torza, Alessandro (forthcoming), 'Vague Existence', in Dean Zimmerman & Karen Bennett (eds.), *Oxford Studies in Metaphysics*, Oxford University Press, Oxford.
- Van Inwagen, Peter (1990), *Material Beings*, Cornell University Press, Ithaca (NY).
- Weir, Alan (2003), 'Neo-Fregeanism: An Embarrassment of Riches', *Notre Dame Journal of Formal Logic*, 44 (1), 13–48.
- Williamson, Timothy (1994), *Vagueness*, Routledge, London & New York.
- (1994b), 'Crispin Wright, Truth and Objectivity', *International Journal of Philosophical Studies*, 2 (1), 130.
- Wright, Crispin (1983), *Frege's Conception of Numbers as Objects*, Aberdeen University Press, Aberdeen.
- (1992), *Truth and Objectivity*, Harvard University Press, Cambridge (MA).
 - (1994), 'Realism, Pure and Simple: A Reply to Timothy Williamson', *International Journal of Philosophical Studies*, 2 (2), 327.
 - (2001), 'On the Harmless Impredicativity of Hume Principle', in Hale & Wright (2001), 229-255.
- Yablo, Stephen (1993), 'Definitions: Consistent and Inconsistent', *Philosophical Studies*, 72, 147–75.
- Zalta, Edward & Linsky, Bernard (2006), 'What is NeoLogicism?', *Bullettin of Symbolic Logic*, 12 (1), 60-99.