Empty validity all the way up: an easy road

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Аннотация: There is a tension between the definition of empty logic as a logic with no valid arguments and no valid meta-arguments, on the one hand, and the way in which we have usually interpreted the validity of meta-arguments, on the other. Here we argue that one way to eliminate the tension is understanding the "If... then..." in a meta-argument, at least in the case of an empty logic, as a transplication (aka the de Finetti conditional) instead of an extensional or material conditional.
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Let Γ_i be a set of formulas of some formal language $\mathcal{L}$ and A_j a formula of that very language, with $1 \leq i, j \leq n$ for some natural n. An *argument* is an expression of the form $\Gamma \models_{\mathbf{L}} A$, where $\models_{\mathbf{L}}$ stands for a relation of logical consequence, and Γ is also known as a premise set and A is called 'conclusion'. A *meta-argument* is an argument between arguments that has the form "If $\Gamma_1 \models A_1, \ldots, \Gamma_n \models A_n$ then $\Delta \models A_m$".[1]

The logical validity of arguments is usually evaluated as universal truth preservation, that is, an argument $\Gamma \models A$ is *valid* if and only, in every interpretation, A is true if B is true for every $B \in \Gamma$. An argument $\Gamma \models A$ is *invalid* if and only if there is an interpretation in which B is true, for every $B \in \Gamma$, and A is not true. Such an interpretation will be considered a *countermodel* for the argument. On the other hand, a meta-argument is *valid* if and only if, if the meta-premises $\Gamma_1 \models A_1, \ldots, \Gamma_n \models A_n$ are valid, the meta-conclusion $\Delta \models B$ is valid as well.[2]

Dicher and Paoli [4], and Barrio *et al.* [1] have called *Global validity* this definition of validity for meta-arguments. This is to distinguish it from *Local validity.* The definitions of global validity and local validity work only for meta-arguments. To define local validity, it is first needed a definition of *satisfaction* of an argument. An interpretation *satisfies* an argument if and only if it is not a countermodel for it. A meta-argument is *locally valid* if and only if the meta-conclusion is satisfied in every interpretation in which the arguments of the meta-premises are satisfied.

[1] In [4], Dicher and Paoli have defined a meta-argument as a nonempty set of arguments where one of which is labeled as its conclusion. In [1], Barrio *et al.* have defined a meta-argument as an argument between a collection of arguments, and an argument.

[2] The members of the Buenos Aires logic group and many of their interlocutors use 'inference' and 'meta-inference' instead of 'argument' and 'meta-argument'. However, for reasons that cannot be fully explained here, but that resemble Zardini's [8] for using 'entailment' and 'meta-entailment', we prefer our terminology.

For some people, like Teijeiro [7], a logic is a set of valid arguments and meta-arguments. Although this is still the majority view, it is not the only one. For example, Pailos [5] says that a logic is a set of valid, anti-valid, and invalid-but-not-anti-valid arguments of all possible level. For simplicity, in this paper we will assume that a logic is indeed the set of valid arguments and meta-arguments.

Thus, a logic is *empty* if and only if its sets of valid arguments and meta-arguments are empty. Since there are no valid arguments, any meta-argument with an invalid argument as a meta-premise would be globally valid. In fact, the definitions of Global and Local validity produce problems with this characterization of empty logic. For example, consider Meta-reflexivity:

$$\text{If } A \models A \text{ then } A \models A \qquad \text{(Meta-reflexivity)}$$

In a logic without arguments, such as example **TS** [3], this meta-argument would also be both locally and globally valid. To see why this is locally valid, it is sufficient to note that in each interpretation in which $A \models A$ is satisfied, $A \models A$ is also satisfied. To see why this is globally valid, it is sufficient to note that it seems sufficient for premises to be invalid for a meta-argument to be valid. This condition is always met given our assumption. So Meta-reflexivity is valid by vacuity.

As another example, consider Monotonicity:

$$\text{If } \Gamma \models A \text{ then } \Gamma \cup \Delta \models A \qquad \text{(Monotonicity)}$$

In a logic without arguments, would be monotonic because it is always the case that $\Gamma \not\models A$; hence, it is globally valid for the reasons given above. In an empty logic, this meta-argument would also be locally valid. Suppose that $\Gamma \models A$ is valid, that is, every interpretation in which the premises in Γ are true is also an interpretation in which the conclusion is true. Suppose now that $\Gamma \cup \Delta \not\models A$. This means that there is an interpretation in which the premises in $\Gamma \cup \Delta$ are true and A is not true. But this cannot be under the assumption that $\Gamma \models A$ is valid. Therefore, $\Gamma \cup \Delta \models A$, and hence Monotonicity is locally valid.

Thus, based on the definitions of Global and Local validity, it is impossible to have a logic that lacks valid arguments and meta-arguments. In fact, to avoid the validity of any meta-argument with invalid premises, Dicher and Paoli [4], and Barrio *et al.* [1] have suggested preferring Local validity over Global validity. More recently, Teijeiro [7] has shown that there are not enough reasons to prefer Local validity over Global validity. We show the reasons for this later.

As can be seen, the problem of finding the right notion of validity for meta-arguments is still open. In the specific case of the problems raised for empty logic, we have at least two options. Either we disregard as meaningless the notion of an empty logic as a logic without valid arguments and meta-arguments, or we modify the way we understand the validity of meta-arguments. In this paper, we explore the possibility of keeping the working definition of an empty logic by (i) mantaining the definition of validity as (Global validity) but (ii) understanding the logical notions in its definition in a slightly different way. Quickly said, we will argue that as the "If... then..." in a meta-argument, at least in the case of an empty logic, should be understood as a transplication[3] instead of an extensional or material conditional.

A disclaimer is in order here. Giving a good definition of empty logic is already a problem. But we believe that if there is something like the right definition of 'empty logic', it should be along the lines of [**?**], that is, as a logic without valid, anti-valid, and invalid-but-not-invalid arguments at any level. Nonetheless, for the sake of the argument, we stick to a more conservative characterization. Part of this discussion also requires an understanding of what validity is important. As we will argue, one cannot simply take classical logic for granted at the meta-theoretical level, and much less in the very definition of validity.

The structure of the paper is as follows: first, we present some necessary preliminary definitions. Second, we present what an empty logic is and some examples. Third, we propose a new way we are understanding the validity of meta-arguments based on the evaluation conditions of transplication. Finally, we respond to some possible replies to this interpretation of validity.

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[3] See [2]; it was introduced by Reichenbach in [6], and it is nowadays more well-known as the de Finetti conditional, see [2] and the references therein.

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