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CARNAP'S CONVENTIONALISM IN GEOMETRY

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Summary

Against Thomas Mormann's argument that differential topology does not support Carnap's conventionalism in geometry we show their compatibility. However, Mormann's emphasis on the entanglement that characterizes topology and its associated metrics is not misplaced. It poses questions about limits of empirical inquiry. For Carnap, to pose a question is to give a statement with the task of deciding its truth. Mormann's point forces us to introduce more clarity to what it means to specify the task that decides between competing hypotheses and in what way such a task may be both in practice and/or in principle impossible to carry out.

1. *Introduction*

There are, possibly among others, three lines of attack against Rudolf Carnap's conventionalism in geometry. We will give a brief summary what their respective targets are and then focus on one of them to substantiate our claim that, whatever else may be said about Carnap's conventionalism in geometry, it does not run afoul of mathematical topology.

Instead, the objections reveal that there is an obscurity at the heart of Carnap's account of scientific objectivity with respect to the practical limitations an empirical inquirer faces. Empiricists sometimes reject the idea that there are areas of the world inaccessible to empirical investigation. Carnap is less clear. On the one hand, he rejects Emil Heinrich Du Bois-Reymond's *ignorabimus*, 'I shall not know.' A well-posed question is capable of an answer. On the other hand, Carnap's account explicitly suggests the existence of at least singularities of principled ignorance.

Conventionalism comes in two varieties (see chapter 1 in Ben-Menahem, 2006), both of which are strongly supported by Carnap. The first variety highlights the underdetermination of theory. There are two types

of underdetermination of theory. Reichenbach's weak version claims that a restricted body of evidence (for example, restricted with respect to time, i.e. past observations) will allow empirically equivalent but mutually incompatible theories to imply the totality of observations. Quine's strong version claims that unrestricted evidence (all observations, either past and future or all possible observations) is compatible with empirically equivalent but mutually incompatible theoretical alternatives.

The second variety is necessary truth conventionalism. Necessary truth (before Kripke, 1980, largely associated with a priori truth) cannot be refuted by experience because it does not make any assertions about the empirical world. It merely "records our determination to use words in a certain fashion" (Ayer, 1946, 84). In *The Logical Syntax of Language*, for example, Carnap seeks to show that logical and mathematical truths are grounded in linguistic convention. For an account of how conventionalism is compatible with Kripke's version of necessary truth see Sidelle, 1989.

The specific form of conventionalism in geometry that Carnap inherits from Poincaré is the platform from which, often by analogy, he launches into conventionalism in other areas. Consider, for example, Carnap's comment in *Der Raum* where the transformation of a statement from one metric into another is "aptly compared" (Carnap, 1978, 99) to the translation of a proposition from one language into another. Conventionalism in geometry serves as evidence that not only are we able to express topological facts using various metrics, but we are also able to express the meaning of a sentence using various languages. Linguistic descriptions and their underlying propositional contents are in a many-to-one relationship. Formally, there is no privilege for certain descriptions over others, and as long as they are unambiguous they are of equal rank in expressing their associated propositions (they can be differentiated by informal or pragmatic criteria such as simplicity). Conventionalism in geometry, although it is not referred to, influences the formation of the principle of tolerance in *The Logical Syntax of Language* (Carnap, 1937, 52) (see Mormann, 2007, 51).

Conventionalism is not an incidental feature of Carnap's philosophical projects, for instance in the *The Logical Structure of the World* (from now on *Aufbau*). The *Aufbau* not only pursues the reduction that subsequently was recognized to have failed both by Carnap himself as well as his critics (see Quine, 1951, 37f; Richardson, 1998, 13). A larger project behind the reduction of science to logical form on the basis of elementary experiences is "the most fundamental aim of the *Aufbau*: namely, the articulation and

defence of a radically new conception of objectivity” (Friedman, 1987, 526) (see also Richardson, 1998, 90 and *passim*).

Objectivity raises both the question of intersubjective meaning and the metaphysical nature of objects. Writing the *Aufbau*, Carnap proposes a unified answer to both of these questions. As there cannot be meaningful intersubjective agreement on phenomenal content, which is dependent on ostensive definitions, it is structural properties which provide for the only achievable objectivity in science. The metaphysical nature of objects is therefore purely conventionalist (one may speak about them from a realist, an idealist, or a phenomenalist perspective), in which science can play no arbitrate role, as there is no possible evaluative link between the different *façons de parler* in metaphysics and observation.

More relevantly, conventions play an important role within science. As the construction of the space-time world, visual things, and the assignment of colour in §§125–127 show, qualities are assigned to point-instants “in such a way as to achieve the laziest world compatible with our experience” (Quine, 1951, 37) (see especially point 11 in §126). The problem what kind of simplicity guides this convention occupies Carnap already in *Der Raum* (where the rule is that “Einfachheit des Baues geht vor Einfachheit des Bauens” Carnap, 1978, 82, simplicity of the construct trumps simplicity of construction) and receives full attention in *Über die Aufgabe der Physik und die Anwendung des Grundsatzes der Einfachheit* (1923). The problem whether relativity theory is a better theory than Newtonian physics based on the conventions of simplicity or based on empirical confirmation and disconfirmation procedures is a question that may put Carnap’s view at odds with Einstein’s.

In the meantime, whereas the project of reduction in the *Aufbau* fails in Carnap’s later assessment (which does not necessarily mean that the project of the *Aufbau* fails, see Parks, 1973; Goodman, 1977; and Richardson, 1998, 73), Carnap advocates conventionalism in a similar form to his early conventionalism as late as 1966 in the *Introduction to the Philosophy of Science* (1995). This conventionalism is as firmly based on the evidence of conventionalism in geometry as the conventionalism that we see in 1922 in *Der Raum*. In a *Reply to Grünbaum* in 1963, Carnap explains that the only reason he writes so little about conventionalism in geometry between 1922 and 1966 is that he feels Hans Reichenbach has already done all the work in *The Philosophy of Space and Time* (1957) in 1928 (see Schilpp, 1963, 957).

Carnap begins his philosophical work not as a logical empiricist, but as a thinker in whom the neo-Kantian problem of the constitution of objectiv-

ity by way of the synthetic a priori and its relationship to advancing science (especially the theory of relativity) meets with Frege's predicate logic and Russell's theory of types. Carnap's position in the *Aufbau* is characterized by a large-scale attempt to replace Kant's synthetic a priori (which depends on a transcendental logic itself dependent on intuition, see Friedman, 1987, 529) by the logical structure of elements (see, for example, Carnap, 2003, 176, §106; and Carnap, 2003, 289, §179).

The essences of these elements, perceived by intuition, are no longer of consequence to objectivity, because with the new logic their formal structure can be rendered substantive without reference to phenomenal content. It is substantive in the sense that it is well-defined without reference to the elements' metaphysical or phenomenal essences but solely to the structural relations they entertain with each other, see Carnap, 2003, 24f, §13. This does not mean that science, despite its sole authority in answering wellformed questions, has to offer much insight relative to practical life and its riddles, see Carnap, 2003, 297, §183. Be that as it may, there is no need for synthetic a priori judgments as "the conventional and the empirical" (Carnap, 2003, 289, §179) exhaust the componentry of cognition.

This picture draws inspiration from the conventionalism conceived by Henri Poincaré just a few years before the advent of Albert Einstein's theory of relativity. Poincaré concludes from a result by Nikolai Lobachevsky that experiments cannot inform geometry in the sense of deciding between alternative, consistent theories (see Poincaré, 1952, 70). Lobachevsky shows that it is in principle impossible to design an experiment that leads to contradictions if interpreted in Lobachevskian geometry (or hyperbolic geometry, where given a line and a point not on the line there are more than one line going through the point that do not intersect with the original line) unless it also leads to contradictions in Euclidean geometry. Because Euclidean geometry can be shown to be consistent (Tarski, 1951), Lobachevskian geometry must be consistent as well. (Lobachevsky's proof is not difficult to grasp: hyperbolic geometry can be embedded in Euclidean geometry, and thus an inconsistency in hyperbolic geometry necessitates an inconsistency in Euclidean geometry.)

Considering that therefore no experiment will tell us whether space is Euclidean or non-Euclidean, a convention will have to deliver the constraint that no necessity of observation will impose on us: "one geometry cannot be more true than another; it can only be more convenient" (Poincaré, 1952, 50). Poincaré comes to the conclusion that Euclidean

geometry is the most convenient, on account of its simplicity and its sufficient agreement with the properties of natural solids.

A few years later, conventionalism is put to the test by relativity theory, which relies heavily on experiments to establish itself and its non-Euclidean view of space geometry. Ernst Cassirer, from whom Carnap inherits a deep commitment to the “logical differentiation of the contents of experience and their arrangement in an ordered system of dependencies” (Cassirer, 2004, 280), now turns Poincaré’s argument on its head and justifies why Euclidean geometry is no longer the most convenient geometry:

Pure Euclidean space stands, as is now seen, not closer to the demands of empirical and physical knowledge than the non-Euclidean manifolds but rather more removed. For precisely because it represents the logically simplest form of spatial construction it is not wholly adequate to the complexity of content and the material determinateness of the empirical. Its fundamental property of homogeneity, its axiom of the equivalence in principle of all points, now marks it as an abstract space; for, in the concrete and empirical manifold, there never is such uniformity, but rather thorough-going differentiation reigns in it. (Cassirer, 2004, 443)

Carnap largely adopts Poincaré’s conventionalism (sometimes also leaning on the more radical Hugo Dingler, although later in life Carnap calls Dingler someone who has taken conventionalism too far, see Carnap, 1995, 59) with a renewed emphasis on critical conventionalism. Critical conventionalism notes that there are parts of physics which because of their dependence on conventions cannot be verified or refuted by experience, but also insists on the ‘critical’ feature of conventions which subjects them to evaluation along simplicity considerations (for an example of this see §136 in the *Aufbau*, Carnap, 2003, 210). Edmund Runggaldier writes:

Even though there is no possibility of phenomenal verification or falsification for some of the constituent ‘content parts’ of physics, there are practical criteria for accepting or rejecting them. Carnap maintained, throughout his life, that conventions play a very great role in the introduction into physics of quantitative concepts of space, time and causality. (Runggaldier, 1984, 30)

2. *Three lines of attack*

This section does not intend to give a full account of the first two lines of attack. They are only mentioned briefly to provide context for the third one and to show that disarming its reservations has no particular implications whether or not we can make our way past the reservations of the other two.

The first attack, personified by W. V. O. Quine, maintains that the strong distinction between analytic and synthetic truths (held not only by Carnap, but also by Moritz Schlick and Reichenbach, see Howard, 1994, 47) breaks down on closer examination:

For all its a priori reasonableness, a boundary between analytic and synthetic statements simply has not been drawn. That there is such a distinction to be drawn at all is an unempirical dogma of empiricists, a metaphysical article of faith. (Quine, 1951, 34)

This conclusion rests on considerations of synonymy and artificial languages, both of which are shown by Quine to be of interest only once we have already understood the notion of analyticity, and neither of which help in gaining such an understanding. It is therefore a matter of metaphysical commitment (which is precisely what a commitment to Carnap's project must reject) to distinguish between analytic conventions, which nail down (in Carnap's words, *festsetzen*) the necessary metric (or language) to facilitate univocal structural relations that are receptive for empirical evaluation, and synthetic a posteriori scientific hypotheses. Quine identifies the *hysteron proteron* of Carnap's epistemological categorization of science as the synthetic a posteriori, contrasted with convention, and advocates in good naturalist tradition for allowing epistemology the resources of science (see Quine, 1969, 90).

For Quine, the reductionist project in the *Aufbau* is intimately related to the "cleavage between the analytic and the synthetic" (Quine, 1951, 38), and once the former fails, the latter fails as well (how this may not be the case, following Michael Friedman, see Richardson, 1998, 73). For our purposes, however, it is the intimate connection between conventionalism and the analytic/synthetic dichotomy that is relevant in Quine's critique: if the dichotomy collapses under Quine's holism, then there is no room left for Carnap's conventionalism, neither in geometry nor as it is more generally developed in *The Logical Syntax of Language*. (For this intimate connection between conventionalism and the analytic/synthetic dichotomy

see Yunez-Naude, 2003.) In the terms of Donald Davidson's 'third dogma of empiricism,' the dualism between conceptual scheme and experiential content in a theory, "of organizing system and something waiting to be organized, cannot be made intelligible and defensible" (Davidson, 1973, 11)—it is itself a dogma of empiricism.

The second line of attack criticizes Carnap's conventionalism in geometry on an altogether different level, its relationship to Einstein's theory of relativity. It has been articulated by both Thomas Ryckman (2005) and Friedman (1999), although we will focus on Ryckman's version. For a defence of Carnap against the second line of attack see Ben-Menahem, 2006, 80–136. Ryckman skillfully locates Einstein's position between Hermann Weyl's and Reichenbach's. At the time (there will be an ironic reversal of Weyl's and Einstein's position later on), Weyl pursues a 'broadened relativity theory' seeking to explain rods and clocks as derived from field equations and not "stipulated as independent primitive 'facts' licensed in the physical definition of metrical notions" (Ryckman, 2005, 79). Reichenbach, on the other hand, defends Schlick's new empiricism of coordination between mathematical representations and concrete physical objects, thus basing geometry on stipulations regarding rigid measuring rods and uniform clocks.

Einstein is in this conflict squarely on Reichenbach's side, relying on the work of the much younger physicist Wolfgang Pauli, who identifies empirical contradictions in Weyl's work. Weyl's theory predicts the perihelion precession of Mercury and the bending of light rays in the solar gravitational field as well as Einstein's theory of relativity, but it also turns out to predict, according to Pauli's calculation, a widely varying spectral signature of hydrogen atoms at far distances. Unfortunately for Weyl's theory, astronomical observation confirms the homogeneity of this signature even at far distances. Einstein had followed his intuition for 'practical geometry,' which in his view had not been possible without the assumption of rigid measuring rods and uniform clocks, and Pauli had, for the time being, proven him right.

On another point, however, Einstein disagrees with "Reichenbach's method of analysis that proposes to cleave a physical theory into its empirical and its non-empirical parts (to be designated, after the 'linguistic turn' prefigured in Schlick, its synthetic and its analytic statements)" (Ryckman, 2005, 95). For Einstein, it is the observation of uniformity that brings about his empirical belief in rigid measuring rods and uniform clocks, while for Reichenbach in his opposition to Weyl they are postulates vul-

nerable at best to evaluation on non-empirical grounds. This is also the breeding ground for a sharp disagreement between Einstein's position and Carnap's conventionalism in geometry.

In the general theory of relativity, physics and geometry are entangled in a way that geometric conventionalism had not previously envisaged: the metric of space-time is no longer accounted as a globally rigid structure, fixed for all time, but as dynamically dependent in a given region, according to the Einstein field equations, upon surrounding matter and energy distributions. (Ryckman, 2005, 78)

In a 1921 lecture, entitled *Geometry and Experience*, Einstein refers to Riemann's 'audacious idea' "that the geometric behavior of bodies might be conditioned by physical realities or forces" (in Ryckman, 2005, 91). In Einstein's later words,

If we imagine the gravitational field, i.e. the functions g_{ik} , to be removed, there does not remain a space of the type (1) [Minkowski spacetime], but absolutely *nothing*, and also no 'topological space' (Einstein, 1952, 155).

This is clearly not what Carnap has in mind, explicitly in *Der Raum*, where topological space is a type of space to which projective and metrical space stand in a relationship of species and subspecies (Carnap, 1978, 17), which is characterized by the mathematical relationships of curves and surfaces lying in or upon one another (Carnap, 1978, 45), and which comes in three 'meanings of space,' formal, intuitive, and physical (Carnap, 1978, 5). Topological space is here not "entangled" (Ryckman, 2005, 78) with metrical or physical space, nor is physical space constitutive of it. On the contrary, philosophers, mathematicians, and physicists are admonished to keep them properly differentiated (Carnap, 1978, 95).

The third line of attack picks up where the second one leaves off in our discussion, with the question of how enmeshed topologies are with the metrics that can be defined on them. As we have seen, Carnap suggests already in the *Aufbau* that Kant's division of judgments into synthetic a priori and other variants of synthetic/analytic and a priori/a posteriori judgments can be completely replaced by the conventional and the empirical (see Carnap, 2003, 289, §179). This foreshadows how Carnap's conventionalism eventually culminates in the principle of tolerance (for the principle of tolerance see Carnap, 1937, 52), which considers even logic to be conventional.

Carnap often justifies his conventionalism with respect to language and logic by analogy to conventionalism in geometry. Expressing a proposition

in a natural language is analogous to expressing a topological fact in a conventional metric. Translating natural language sentences from German to French, for example, compares to translating a statement belonging to one metrical spatial form into another (see Carnap, 1978, 99). In Mormann's words, metrical conventionalism is the paradigm for conventionalism in general. Mormann's contention is that, whatever may be true about conventionalism in general, the mathematical discipline of differential topology does not support conventionalism in geometry:

for purely mathematical reasons geometry fails to be a stronghold for conventionalism. One can show that Poincaré's result concerning the metrical structure of Euclidean spaces is not representative for manifolds in general: differential topology and related mathematical disciplines of 20th century mathematics have shown that the relation between the topological and geometrical structure of manifolds is extremely intricate. It is quite misleading to describe this relation in terms of a hierarchical conventionalism à la Carnap, according to which there is a bedrock of topological facts ('topologischer Tatbestand') dealing with the topological structure of space-time, and then there are different 'Euclidean' and 'non-Euclidean languages' in which these facts are expressed. (Mormann, 2007, 51)

It is now up to us to parse what Mormann means by the intricacies of the relationship between topological and geometrical structures of manifolds. Carnap's guiding example is the compatibility of a hyperbolic (Lobachevskian) and parabolic (Euclidean) metric with the same underlying topology, provided by Poincaré (see Poincaré, 1952, 74). There is a sense, however, in which Poincaré's example provides only the argument for an existence claim, i.e. that it is possible for different metrics to arrange themselves with a topology so that within the topology no experiment can decide between those particular metrics. Carnap wants a more universal claim indicating that in general experiments cannot decide between suitably chosen metrics for any or at least most given topologies.

For this purpose, Carnap seeks to convince us by providing two more scenarios pointing in the same direction as Poincaré's example. Let us agree on the convention that the Earth's surface has zero curvature everywhere. Mormann's topological interpretation of this claim is (where S^2 is the surface of a two-dimensional sphere)

$$S^2 \text{ can be endowed with a metric } l_1 \text{ with curvature } K = 0 \quad (\text{C1})$$

This topology, according to Carnap, does not contradict any geodetic measurements or physical observations. He is not satisfied with this scenario, however, because the metric l_1 must give preference to a particular point in S^2 . This requirement does not sit well with our need for simplicity. Instead of postulating curvature $K = 0$ everywhere, we have the choice of postulating $K = k$ everywhere, where $k > 0$ is the curvature corresponding to the curvature of S^2 given l_0 , the Euclidean distance measure we are used to. Now we no longer need a privileged point to define a distance measure $K = 0$ for this topology (also extending it from S^2 to \mathbb{R}^3), which has a positive curvature $k > 0$ everywhere:

$$l_2(A, B) = l_0(A, B)(1 - \sin b)$$

We need a postulate on how to measure h , which Carnap provides with the following rule:

$$\int_0^b \frac{1}{1 - \sin x} dx = a$$

where a is the length of a measuring rod measuring b transferred to S^2 . Again, Carnap claims that accepting this topology and metric will not put us at odds with any empirical observations or measurements. Mormann translates his claim in terms of differential topology into

$$\begin{aligned} \mathbb{R}^3 \text{ can be endowed with a metric } l_2 \\ \text{with constant positive curvature } K = k \end{aligned} \tag{C2}$$

Mormann now provides a proof that, under suitable conditions, both (C1) and (C2) are false.

For polyhedra, the Euler-Poincaré characteristic $\chi(T)$ is known as the number of vertices minus the number of edges plus the number of areas. The theorem of Gauss-Bonnet states that for a compact two-dimensional Riemannian manifold M without a boundary (such as S^2), the total Gaussian curvature is (A being the area element of M)

$$\iint_M K dA = 2\pi\chi(M)$$

The Euler-Poincaré characteristic for an orientable compact surface homeomorphic to a sphere with some handles attached is $2 - 2g$, g being the number of handles. Consequently, $\chi(S^2) = 2$, and (C1) is false.

Now let M be a complete connected Riemannian manifold with curvature $K \geq a > 0$ (call this last condition (*)). Bonnet's theorem states that then M must be compact. Because \mathbb{R}^3 fulfills all these conditions except (*) and is not compact, (C2) is false. (Both of these proofs see Mormann, 2004, 820f.)

What Mormann initially mentions only in footnotes (footnote 9 and footnote 12) and eventually discusses in a section toward the end of his article is that his idealized mathematical conditions do not necessarily match the pragmatic constraints Carnap assumes to be true for the physicists doing the work of finding empirical disconfirmation of physical theories with respect to applicable conventions.

Mormann clearly disagrees with Carnap on the admissibility of limitation in empiricist inquiry. This disagreement explains their mathematical disagreement. (C1) and (C2) are not false, Carnap just never makes clear that he admits limitations and the Riemannian manifolds may not be complete (a space X is complete if every Cauchy sequence in it converges). Mormann complains that completeness is "indispensable from an empiricist point of view" (Mormann, 2004, 817), that incompleteness "lacks empirical significance" (Mormann, 2004, 820), that "it would be a desperate move to attempt to rescue Carnap's thesis by allowing him to fall back on incomplete metrics" (Mormann, 2004, 821), and, most relevantly, that

for an empiricist it is meaningless to be engaged in investigating the global structure of the world under the presupposition that large areas of that world are principally inaccessible to empirical investigation. (Mormann, 2004, 823)

In reply to Mormann, first off we need to note that completeness is not the issue for (C1). Let a plane F go through a point on the radius between the centre of the Earth and the North Pole (say 6000km away from the centre of the Earth) and be parallel to the equatorial plane. Then define T^2 , a spherical cap with a height greater than the radius of the underlying sphere, as the intersection of \mathbb{R}^3 south of F (including F) and S^2 . Think of it as a punch bowl or a spherical decapitated eggshell (see figure 1). T^2 fulfills the conditions of the Gauss-Bonnet theorem, and there is no longer a problem with Carnap's claim that T^2 can be endowed with a metric whose curvature is 0, as Gauss-Bonnet's theorem for a space with a boundary runs like this (see Chavel, 2006, 260):

$$\int_{\partial M} k_g ds + \iint_M K dA = 2\pi - \sum_{j=1}^m \alpha(p_j)$$

where ∂M is the boundary of M , k_g is the geodesic curvature of ∂M , and the $\alpha(p_j)$ are the exterior angles of the corners p_1, \dots, p_m of ∂M .

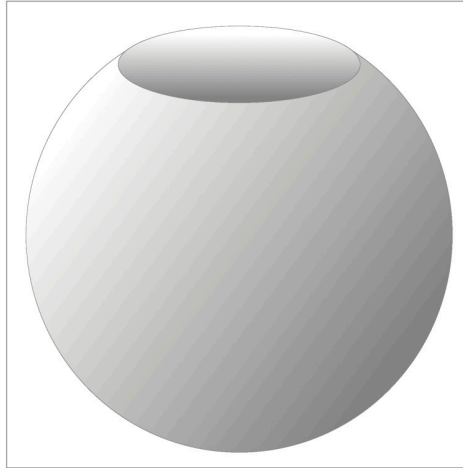


Figure 1:

A compact two-dimensional topological space T^2 that is complete, has a boundary, and can be endowed with a metric whose curvature is 0. We do not need incompleteness to save (C1) from Mormann's attack. Because of the boundary, however, the limitation of inquiry (for ant physicists, for example) is greater than in the case of a singularity.

Despite its intimidating looks this formula makes good sense. Our boundary (the intersection of F and S^2) has no corners, so we can ignore the sum of exterior angles. The concavity of the boundary, however, makes up for the convexity of the sphere so that it is possible to endow T^2 with a metric with constant curvature $K = 0$. You may ask why we did not keep T^2 open and exclude the boundary, which would also provide us with the possibility of a metric with constant curvature $K = 0$. Such a space would be homeomorphic to \mathbb{R}^2 , very close to what Carnap had in mind, but it lacks the completeness we were hoping for. In any case, T^2 as defined is complete and fulfills Carnap's criteria.

What, interestingly, distinguishes T^2 from manifolds usually considered is the inclusion of a boundary. Around a boundary point, a scientist would no longer be able to draw a circle open to empirical investigation.

These points are odd in the sense that one would be able to go left, for example, but unable to go right. The cosmology is reminiscent of the ancient idea (see the map of Hecataeus of Miletus) that the world has a defined perimeter beyond which it plunges into chaos. Carnap obviously never says that this is the world as it presents itself to its examiner. We are only pointing it out to show that it is not incompleteness as such (for T^2 is perfectly complete) that is the problem, but more broadly the limitations that an empirical investigator may face. These limitations could be of various natures, of which incompleteness is only one.

Thus, when Mormann says that with incomplete metrics, while “(C2) could be saved, (C1) remains false” (Mormann, 2004, 821), it remains false because we do not even need to go as far as retreating to incomplete metrics. We can keep (C1) by introducing a boundary, or, as Carnap would say, a limitation. Carnap takes precisely this line of defence against Grünbaum, who has reservations similar to Mormann’s in (1963) (although Mormann dismisses Grünbaum’s argument, Mormann, 2004, 819). The limitation Carnap introduces, however, is just one point: the projection point of the stereographic projection (accordingly, Carnap’s limitation does not address Mormann’s concerns which presume completeness, a property not available to Carnap’s account of a limitation). This limitation

has no consequences for any possible observational results, since every observation involves a spatial region with a positive extension, however small, but never a single point. (Schilpp, 1963, 957)

This approach raises doubts. Reichenbach, for one, disagrees with it when he notes that singularities, while admissible in topology, should not be admitted in physics (see Reichenbach, 1957, 80). Carnap rejects this worry (see Schilpp, 1963, 958). Singularities, however, have radical implications for the topological features of a space (for example, rendering it incomplete) and, according to Carnap, are inaccessible to observation. This conjunction makes them hard to swallow.

Mormann’s criticism is more to the point: should the empiricist accept principled limitations to her inquiry? We have shown that we do not have to give up on completeness to save (C1), but in this case we can no longer retreat to the observational indifference of singularities. We need a boundary (foremost in the topological sense, but figuratively the topological boundary indeed introduces a boundary to our inquiry). Carnap articulates the question of limitations to scientific inquiry using

Du Bois-Reymond's famous *ignorabimus* speech about the 'Grenzen des Naturerkennens' (1872).

In §183 of the *Aufbau*, Carnap pronounces that "for us there is no *ignorabimus*" (Carnap, 2003, 297) because *ignorabimus* would mean that "there are questions to which it is in principle impossible to find answers." 'In principle impossible' in the *Aufbau* means that the map exhaustively representing the structural properties of scientific objects makes the answers to the questions indistinguishable (see Carnap, 2003, 27, §15). It is dubious whether pragmatic limitations such as those imposed on ant physicists have the required impact on this map. Carnap asserts that in connection with structural correlation properties, when we encounter competing hypotheses, we can at least "indicate which empirical data would be required to decide in favor of one hypothesis or another" (Carnap, 2003, 37). It remains unclear, however, whether this set of data needs to be associated with an executable task on part of the inquirer or not.

3. *Conclusion*

Mormann means to show that we do not have to go as far as Quine, Ryckman, or Friedman, to reveal the weaknesses of Carnap's conventionalism in geometry. A look at the mathematical foundations of Carnap's claim identifies serious shortcomings. Consequently, conventionalism in geometry is weak evidence for conventionalism in general, but conventionalism in general is highly significant in Carnap's lifelong philosophical quest for scientific objectivity.

Our claim, contra Mormann, is that it is not so much mathematical inconsistency that is at the heart of this problem, but rather a lack of clarity to what extent the limitations of scientific observation enter into which questions it is in principle possible to answer. Our impression, unfortunately not based on elucidation by Carnap himself, is that he includes practical limitations in his account of the limits of science. To pose a question, Carnap says in §180 of the *Aufbau*, "is to give a statement together with the task of deciding whether this statement or its negation is true" (Carnap, 2003, 290). If the task is 'in principle' impossible to carry out, which it very well may be (unless 'in principle' means just the opposite of 'in practice'), then it remains open whether the question is properly posed.

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