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Regular Bipolar Single Valued Neutrosophic Hypergraphs

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Abstract. In this paper, we define the regular and totally regular bipolar single valued neutrosophic hypergraphs, and discuss the order and size along with properties of regular and totally regular bipolar single valued neutrosophic hypergraphs. We extend work on completeness of bipolar single valued neutrosophic hypergraphs.

Keywords: bipolar single valued neutrosophic hypergraphs, regular bipolar single valued neutrosophic hypergraphs and totally regular bipolar single valued neutrosophic hyper graphs.

1 Introduction

The notion of neutrosophic sets (NSs) was proposed by Smarandache [8] as a generalization of the fuzzy sets [14], intuitionistic fuzzy sets [12], interval valued fuzzy set [11] and interval-valued intuitionistic fuzzy sets [13] theories. The neutrosophic set is a powerful mathematical tool for dealing with incomplete, indeterminate and inconsistent information in real world. The neutrosophic sets are characterized by a truth-membership function (t), an indeterminacy-membership function (i) and a falsity membership function (f) independently, which are within the real standard or nonstandard unit interval $]^-0 , 1^+[$. In order to conveniently use NS in real life applications, Wang et al. [9] introduced the concept of the single-valued neutrosophic set (SVNS), a subclass of the neutrosophic sets. The same authors [10] introduced the concept of the interval valued neutrosophic set (IVNS), which is more precise and flexible than the single valued neutrosophic set. The IVNS is a generalization of the single valued neutrosophic set, in which the three membership functions are independent and their value belong to the unit interval [0, 1]. More works on single valued neutrosophic sets, interval valued neutrosophic sets and their applications can be found on *http://fs.gallup.unm.edu/NSS/*.

Hypergraph is a graph in which an edge can connect more than two vertices, hypergraphs can be applied to analyse architecture structures and to represent system partitions, Mordesen J.N and P.S Nasir gave the definitions for fuzzy hypergraphs. Parvathy. R and M. G. Karunambigai's paper introduced the concepts of Intuitionistic fuzzy hypergraphs and analyse its components, Nagoor Gani. A and Sajith Begum. S defined degree, order and size in intuitionistic fuzzy graphs and extend the properties. Nagoor Gani. A and Latha. R introduced irregular fuzzy graphs and discussed some of its properties.

Regular intuitionistic fuzzy hypergraphs and totally regular intuitionistic fuzzy hypergraphs are introduced by Pradeepa. I and Vimala. S in [0]. In this paper we extend regularity and totally regularity on bipolar single valued neutrosophic hypergraphs.

2 Preliminaries

In this section we discuss the basic concept on neutrosophic set and neutrosophic hyper graphs.

Definition 2.1 Let X be the space of points (objects) with generic elements in X denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by truth membership function $T_A(x)$, indeterminacy membership function $I_A(x)$ and a falsity membership function $F_A(x)$. For each point x ∈X; $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$.

Definition 2.2 Let X be a space of points (objects) with generic elements in X denoted by x. A bipolar single valued neutrosophic set A (BSVNS A) is characterized by positive truth membership function $PT_A(x)$, positive indeterminacy membership function $PI_A(x)$ and a positive falsity membership function $PF_A(x)$ and negative truth membership function $NT_A(x)$, negative indeterminacy membership function $NI_A(x)$ and a negative falsity membership function $NF_A(x)$.

For each point $x \in$ X; $PT_A(x)$, $PI_A(x)$, $PF_A(x) \in [0, 1]$ and $NT_A(x)$, $NI_A(x)$, $NF_A(x) \in [-1, 0]$.

Definition 2.3 Let A be a BSVNS on X then support of A is denoted and defined by

$Supp(A) = \{x : x \in X, PT_A(x) > 0, PI_A(x) > 0, PF_A(x) > 0, NT_A(x) < 0, NI_A(x) < 0, NF_A(x) < 0\}$.

Definition 2.4 A hyper graph is an ordered pair $H = (X, E)$, where

(1) $X = \{x_1, x_2, \ldots, x_n\}$ be a finite set of vertices.

(2) $E = \{E_1, E_2, \ldots, E_m\}$ *be* a family of subsets of

X. (3) E_j for $j= 1,2,3,...,m$ and $\bigcup_j(E_j)= X$.

The set X is called set of vertices and E is the set of edges (or hyper edges).

Definition 2.5 A bipolar single valued neutrosophic hypergraph is an ordered pair $H = (X, E)$, where

(1) $X = \{x_1, x_2, \ldots, x_m\}$ be a finite set of vertices.

(2) $E = \{E_1, E_2, \ldots, E_m\}$ be a family of BSVNSs of X.

(3) $E_j \neq O = (0, 0, 0)$ for $j= 1,2,3,...,m$ and $\bigcup_j Supp(E_j)= X$.

The set X is called set of vertices and E is the set of BSVN-edges (or BSVN-hyper edges).

Proposition 2.6 The bipolar single valued neutrosophic hyper graph is the generalization of fuzzy hyper graphs, intuitionistic fuzzy hyper graphs, bipolar fuzzy hyper graphs and single valued neutrosophic hypergraphs.

3 Regular and totally regular BSVNHGs

Definition 3.1 The open neighbourhood of a vertex x in bipolar single valued neutrosophic hypergraphs (BSVNHGs) is the set of adjacent vertices of x, excluding that vertex and is denoted by $N(x)$.

Definition 3.2 The closed neighbourhood of a vertex x in bipolar single valued neutrosophic hypergraphs (BSVNHGs) is the set of adjacent vertices of x, including that vertex and is denoted by $N[x]$.

Example 3.3 Consider a bipolar single valued neutrosophic hypergraphs $H = (X, E)$ where, $X = \{a, b, c, d, e\}$ and $E = \{P, Q, R, S\}$, which is defined by

P = {(a, 0.1, 0.2, 0.3, -0.4, -0.6 -0.8), (b, 0.4, 0.5, 0.6, -0.4, -0.6 -0.8)}

Q = {(c, 0.1, 0.2, 0.3, -0.4, -0.4 -0.9), (d, 0.4, .5, 0.6, -0.3, -0.5 -0.6), (e, 0.7, 0.8, 0.9, -0.7, -0.9, -0.2)}

R = {(b, 0.1, 0.2, 0.3, -0.2, -0.5, -0.8), (c, 0.4, 0.5, 0.6, -0.9, -0.7 -0.4)}

S = {(a, 0.1, 0.2, 0.3, -0.7, -0.6, -0.9), (d, 0.9, 0.7, 0.6, -0.4, -0.7, -0.9)}

Then the open neighbourhood of a vertex a is the b and d, and closed neighbourhood of a vertex b is b, a and c.

Definition 3.4 Let $H = (X, E)$ be a BSVNHG, the open neighbourhood degree of a vertex x, which is denoted and defined by

$deg(x) = (deg_{PT}(x), deg_{PI}(x), deg_{PF}(x), deg_{NT}(x), deg_{NI}(x), deg_{NF}(x))$

where

$$deg_{PT}(x) = \sum_{x \in N(x)} PT_E(x)$$

$$deg_{PI}(x) = \sum_{x \in N(x)} PI_E(x)$$

$$deg_{PF}(x) = \sum_{x \in N(x)} PF_E(x)$$

$$deg_{NT}(x) = \sum_{x \in N(x)} NT_E(x)$$

$$deg_{NI}(x) = \sum_{x \in N(x)} NI_E(x)$$

$$deg_{NF}(x) = \sum_{x \in N(x)} NF_E(x)$$

Example 3.5 Consider a bipolar single valued neutrosophic hypergraphs $H = (X, E)$ where, $X = \{a, b, c, d, e\}$ and $E = \{P, Q, R, S\}$, which are defined by

P = {(a, .1, .2, .3, -0.1, -0.2, -0.3), (b, .4, .5, .6, -0.1, -0.2, -0.3)}

Q = {(c, .1, .2, .3, -0.1, -0.2, -0.3), (d, .4, .5, .6, -0.1, -0.2, -0.3), (e, .7, .8, .9, -0.1, -0.2, -0.3)}

R = {(b, .1, .2, .3, -0.1, -0.2, -0.3), (c, .4, .5, .6, -0.1, -0.2, -0.3)}

S = {(a, .1, .2, .3, -0.1, -0.2, -0.3), (d, .4, .5, .6, -0.1, -0.2, -0.3)}

Then the open neighbourhood of a vertex a contain b and d and therefore open neighbourhood degree of a vertex a is (.8, 1, 1.2, -0.2, -0.4, -0.6).

Definition 3.6 Let $H = (X, E)$ be a BSVNHG, the closed neighbourhood degree of a vertex x is denoted and defined by

$deg[x] = (deg_{PT}[x], deg_{PI}[x], deg_{PF}[x], deg_{NT}[x], deg_{NI}[x], deg_{NF}[x])$

which are defined by

$$deg_{PT}[x] = deg_{PT}(x) + PT_E(x)$$

$$deg_{PI}[x] = deg_{PI}(x) + PI_E(x)$$

$$deg_{PF}[x] = deg_{PF}(x) + PF_E(x)$$

$$deg_{NT}[x] = deg_{NT}(x) + NT_E(x)$$

$$deg_{NI}[x] = deg_{NI}(x) + NI_E(x)$$

$$deg_{NF}[x] = deg_{NF}(x) + NF_E(x)$$

Example 3.7 Consider a bipolar single valued neutrosophic hypergraphs *H = (X, E)* where, *X = {a, b, c, d, e}* and *E = {P, Q, R, S}*, which is defined by

P = {(a, 0.1, 0.2, 0.3, -0.1, -0.2, -0.3), (b, 0.4, 0.5, 0.6, -0.1, -0.2, -0.3)}

Q = {(c, 0.1, 0.2, 0.3, -0.1, -0.2, -0.3), (d, 0.4, 0.5, 0.6, -0.1, -0.2, -0.3), (e, 0.7, 0.8, 0.9, -0.1, -0.2, -0.3)}

R = {(b, 0.1, 0.2, 0.3, -0.1, -0.2, -0.3), (c, 0.4, 0.5, 0.6, -0.1, -0.2, -0.3)}

S = {(a, 0.1, 0.2, 0.3, -0.1, -0.2, -0.3), (d, 0.4, 0.5, 0.6, -0.1, -0.2, -0.3)}

The closed neighbourhood of a vertex *a* contain *a*, *b* and *d*, hence the closed neighbourhood degree of a vertex *a* is (0.9, .1.2, 1.5, -0.3, -0.6, -0.9).

Definition 3.8 Let *H = (X, E)* be a BSVNHG, then *H* is said to be an *n*-regular BSVNHG if all the vertices have the same open neighbourhood degree $n = (n_1, n_2, n_3, n_4, n_5, n_6)$

Definition 3.9 Let *H = (X, E)* be a BSVNHG, then *H* is said to be *m*-totally regular BSVNHG if all the vertices have the same closed neighbourhood degree $m = (m_1, m_2, m_3, m_4, m_5, m_6)$.

Proposition 3.10 A regular BSVNHG is the generalization of regular fuzzy hypergraphs, regular intuitionistic fuzzy hypergraphs, regular bipolar fuzzy hypergraphs and regular single valued neutrosophic hypergraphs.

Proposition 3.11 A totally regular BSVNHG is the generali-zation of totally regular fuzzy hypergraphs, totally regular intuitionistic fuzzy hypergraphs, totally regular bipolar fuzzy hypergraphs and totally regular single valued neu-trosophic hypergraphs.

Example 3.12 Consider a bipolar single valued neutrosophic hypergraphs *H = (X, E)* where, *X = {a, b, c, d}* and *E = {P, Q, R, S}* which is defined by

P = {(a, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3), (b, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3)}

Q = {(b, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3), (c, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3)}

R = {(c, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3), (d, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3)}

S = {(d, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3), (a, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3)}

Here the open neighbourhood degree of every vertex is *(1.6, 0.4, 0.6, -0.2, -0.4, -0.6)* hence *H* is regular BSVNHG and closed neighbourhood degree of every vertex is *(2.4, 0.6, 0.9, -0.3, -0.6, -0.9)*, Hence *H* is both regular and totally regular BSVNHG.

Theorem 3.13 Let *H = (X, E)* be a BSVNHG which is both regular and totally regular BSVNHG then *E* is constant.

Proof: Suppose *H* is an *n*-regular and *m*-totally regular BSVNHG. Then $deg(x) = n = (n_1, n_2, n_3, n_4, n_5, n_6)$ and $deg[x] = m = (m_1, m_2, m_3, m_4, m_5, m_6)$ $\forall x \in E_i$. Consider *deg[x] = m.* Hence by definition, $deg(x) + E_i(x) = m$ this implies $E_i(x) = m - n$ for all $x \in E_i$. Hence *E* is constant.

Remark 3.14 The converse of above theorem need not to be true in general.

Example 3.15 Consider a bipolar single valued neutrosophic hypergraphs *H = (X, E)* where, *X = {a, b, c, d}* and *E = {P, Q, R, S}*, which is defined by

P = {(a, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3), (b, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3)}

Q = {(b, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3), (d, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3)}

R = {(c, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3), (d, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3)}

S = {(d, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3), (a, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3)}

Here *E* is constant but *deg(a) = (1.6, 0.4, 0.6, -0.2, -0.4, -0.6)* and *deg(d) = (2.4, 0.6, 0.9, -0.3, -0.6, -0.9)* i.e deg(a) and deg(d) are not equals hence *H* is not regular BSVNHG. Next *deg[a] = (2.4, 0.6, 0.9, -0.3, -0.6, -0.9)* and *deg[d]= (3.2, 0.8, 1.2, -.4, -0.8, -1.2)*, hence deg[a] and deg[d] are not equals hence *H* is not totally regular BSVNHG, Thus that *H* is neither regular and nor totally regular BSVNHG.

Theorem 3.16 Let *H = (X, E)* be a BSVNHG then *E* is constant on *X* if and only if following are equivalent,

(1) *H* is regular BSVNHG.

(2) *H* is totally regular BSVNHG.

Proof: Suppose *H = (X, E)* be a BSVNHG and *E* is constant in *H*, that is $E_i(x) = c = (c_1, c_2, c_3, c_4, c_5, c_6)$ $\forall x \in E_i$.

Suppose H is n-regular BSVNHG, then $deg(x) = n = (n_1, n_2, n_3, n_4, n_5, n_6)$ $\forall x \in E_i$, consider $deg[x] = deg(x) + E_i(x) = n + c$ $\forall x \in E_i$, hence H is totally regular BSVNHG.

Next suppose that H is m-totally regular BSVNHG, then $deg[x] = m = (m_1, m_2, m_3, m_4, m_5, m_6)$ for all $x \in E_i$, that is $deg(x) + E_i(x) = m$ $\forall x \in E_i$, this implies that $deg(x) = m - c$ $\forall x \in E_i$. Thus H is regular BSVNHG, thus *(1)* and *(2)* are equivalent.

Conversely: Assume that *(1)* and *(2)* are equivalent. That is H is regular BSVNHG if and only if H is totally regular BSVNHG. Suppose contrary E is not constant, that is E_i(x) and E_i(y) not equals for some x and y in X. Let $H = (X, E)$ be n-regular BSVNHG, then $deg(x) = n = (n_1, n_2, n_3, n_4, n_5, n_6)$ for all x $\in E_i$. Consider

$$deg[x] = deg(x) + E_i(x) = n + E_i(x)$$

$$deg[y] = deg(y) + E_i(y) = n + E_i(y)$$

Since E_i(x) and E_i(y) are not equals for some x and y in X. Hence $deg[x]$ and $deg[y]$ are not equals, thus H is not totally regular BSVNHG, which contradict to our assumption.

Next let H be totally regular BSVNHG, then $deg[x] = deg[y]$, that is $deg(x) + E_i(x) = deg(y) + E_i(y)$ and $deg(x) - deg(y) = E_i(y) - E_i(x)$, since RHS of last equation is nonzero, hence LHS of above equation is also nonzero, thus $deg(x)$ and $deg(y)$ are not equals, so H is not regular BSVNHG, which is again contradict to our assumption, thus our supposition was wrong, hence E must be constant, this completes the proof.

Definition 3.17 Let $H = (X, E)$ be a regular BSVNHG, then the order of BSVNHG H is denoted and defined by

$O(H) = (p, q, r, s, t, u)$, where $p = \sum_{x \in X} PT_{E_i}(x)$, $q = \sum_{x \in X} PI_{E_i}(x)$, $r = \sum_{x \in X} PF_{E_i}(x)$, $s = \sum_{x \in X} NT_{E_i}(x)$, $t = \sum_{x \in X} NI_{E_i}(x)$, $u = \sum_{x \in X} NF_{E_i}(x)$. For every $x \in X$ and size of regular BSVNHG is denoted and defined by $S(H) = \sum_{i=1}^{n}(S_{E_i})$, where $S(E_i) = (a, b, c, d, e, f)$ which is defined by

$$a = \sum_{x \in E_i} PT_{E_i}(x)$$

$$b = \sum_{x \in E_i} PI_{E_i}(x)$$

$$c = \sum_{x \in E_i} PF_{E_i}(x)$$

$$d = \sum_{x \in E_i} NT_{E_i}(x)$$

$$e = \sum_{x \in E_i} NI_{E_i}(x)$$

$$f = \sum_{x \in E_i} NF_{E_i}(x)$$

Example 3.18 Consider a bipolar single valued neutrosophic hypergraphs $H = (X, E)$ where, $X = \{a, b, c, d\}$ and

$E = \{P, Q, R, S\}$, which is defined by

P = {(a, .8, .2, .3, -.1, -.2, -.3), (b, .8, .2, .3, -.1, -.2, -.3)}

Q = {(b, .8, .2, .3, -.1, -.2, -.3), (c, .8, .2, .3, -.1, -.2, -.3)}

R = {(c, .8, .2, .3, -.1, -.2, -.3), (d, .8, .2, .3, -.1, -.2, -.3)}

S = {(d, .8, .2, .3, -.1, -.2, -.3), (a, .8, .2, .3, -.1, -.2, -.3)}

Here order and size of H are given (3.2, .8, 1.2, -.4, -.8, -1.2) and (6.4, 1.6, 2.4, -.8, -1.6, -2.4) respectively.

Proposition 3.19 The size of an n-regular BSVNHG $H = (H, E)$ is $nk/2$, where $|X| = k$.

Proposition 3.20 If $H = (X, E)$ be m-totally regular BSVNHG then $2S(H) + O(H) = mk$, where $|X| = k$.

Corollary 3.21 Let $H = (X, E)$ be a n-regular and m-totally regular BSVNHG then $O(H) = k(m - n)$, where $|X| = k$.

Proposition 3.22 The dual of n-regular and m-totally regular BSVNHG $H = (X, E)$ is again an n-regular and m-totally regular BSVNHG.

Definition 3.23 A bipolar single valued neutrosophic hypergraph (BSVNHG) is said to be complete BSVNHG if for every x in X, $N(x) = \{x: x \text{ in } X-\{x\}\}$, that is $N(x)$ contains all remaining vertices of X except x.

Example 3.24 Consider a bipolar single valued neutrosophic hypergraphs $H = (X, E)$, where $X = \{a, b, c, d\}$ and $E = \{P, Q, R\}$, which is defined by

P = {(a, 0.4, 0.6, 0.3, -0.5, -0.2, -0.3), (c, 0.8, 0.2, 0.3, -0.1, -0.8, -0.3)}

Q = {(a, 0.8, 0.8, 0.3, -0.1, -0.6, -0.3), (b, 0.8, 0.2, 0.1, -0.1, -0.2, -0.3), (d, 0.8, 0.2, 0.1, -0.1, -0.9, -0.3)}

R = {(c, 0.4, 0.9, 0.9, -0.1, -0.2, -0.3), (d, 0.7, 0.2, 0.1, -0.5, -0.9, -0.3), (b, *0.4, 0.2, 0.1, -0.8, -0.4, -0.2)}*. Here *N(a) = {b, c, d}* , *N(b) = {a, c, d}*, *N(c) = {a, b, d}*, *N(d) = {a, b, c}* hence *H* is complete BSVNHG.

Remark 3.25 In a complete BSVNHG *H = (X, E),* the cardi-nality of *N(x)* is same for every vertex.

Theorem 3.26 Every complete BSVNHG *H = (X, E)* is both regular and totally regular if *E* is constant in *H*.

Proof: Let *H = (X, E)* be complete BSVNHG, suppose *E* is constant in *H*, so that $E_i(x) = c = (c_1, c_2, c_3, c_4, c_5, c_6)$ $\forall x \in E_i$, since BSVNHG is complete, then by definition for every vertex *x* in *X*, *N(x) = {x: x in X-{x}}*, the open neighbourhood degree of every vertex is same. That is *deg(x)* = *n* = $(n_1, n_2, n_3, n_4, n_5, n_6)$ $\forall x \in E_i$. Hence complete BSVNHG is regular BSVNHG. Also, *deg[x] = deg(x) +* $E_i(x)$ = *n + c* $\forall x \in E_i$. Hence *H* is totally regular BSVNHG.

Remark 3.27 Every complete BSVNHG is totally regular even if *E* is not constant.

Definition 3.28 A BSVNHG is said to be *k*-uniform if all the hyper edges have same cardinality.

Example 3.29 Consider a bipolar single valued neutrosophic hypergraphs *H = (X, E),* where *X = {a, b, c, d}* and *E = {P, Q, R},* which is defined by

P = {(a, 0.8, 0.4, 0.2,-0.4, -0.6, -0.2), (b, 0.7, 0.5, 0.3, -0.7, -0.1, -0.2)}

Q = {(b, 0.9, 0.4, 0.8, -0.3, -0.2, -0.9), (c, 0.8, 0.4, 0.2, -0.4, -0.3, -0.7)}

R = {(c, 0.8, 0.6, 0.4, -0.3, -0.7, -0.2), (d, 0.8, 0.9, 0.5, -0.4, -0.8, -0.9)}

4 Conclusion

Theoretical concepts of graphs and hypergraphs are utilized by computer science applications. Single valued neutrosophic hypergraphs are more flexible than fuzzy hypergraphs and intuitionistic fuzzy hypergraphs. The concepts of single valued neutrosophic hypergraphs can be applied in various areas of engineering and computer science. In this paper, we defined the regular and totally regular bipolar single valued neutrosophic hyper graphs. We plan to extend our research work to irregular and totally irregular on bipolar single valued neutrosophic hyper graphs.

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