FUZZY LOGIC OF LUTFI ZADEH AND METHODOLOGICAL AND EPISTEMO-LOGICAL ANALYSIS OF SET THEORY*

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ABSTRACT

In order to understand the essence of Lutfi Zadeh's creativity, which introduced a number of scientific innovations into the treasury of world culture and had a great influence on contemporaries with its fundamental discoveries and amazing ideas, as well as to objectively evaluate the new impulses and directions that he brought to public consciousness, first of all, one should comprehend not only the real, but also the philosophical essence of the discoveries, to voice them in the language of philosophical thinking. The fuzzy logic of the great scientist and thinker, who has gained great prestige in modern scientific circles, illuminating theoretical thinking from a new angle and significantly elevating the horizons of human knowledge over the traditional binary logic of Aristotle, is today accepted with admiration by world scientists and is intensively studied as one of the fundamental discoveries of the twentieth century. It is no coincidence that at present this theory has become the object of extensive discussions, the subject of dialogues at international conferences, symposiums, scientific seminars periodically held in Japan, Germany, the USA, Russia, China, Turkey, France and a number of other countries. This fact itself is an important indicator of the high appreciation of Lutfi Zadeh's theories by world scientists.
***Keywords**: Lutfi Zade, fuzzy logic, methodology, theory*

The attitude to the scientific and theoretical heritage of Lutfi Zade willy-nilly is accompanied by a number of questions: what is the essence of all these scientific innovations? Why do world scientists attach so much importance to the theories of Lutfi Zadeh? What is the main difference between these theories and previous and current theories of thinking and sets? Why do their application in modern world practice and scientific innovations continue to be widely disseminated and studied? Probably, these questions, finding correct answers to which are of great scientific and ideological significance, entering the circle of interests of millions of people of science, make us think. Given this interest, we will try to shed light on them and give some explanations. First of all, we consider it appropriate to draw attention to a number of controversial issues in the theories of Lutfi Zadeh.

About the theory

In our opinion, one of these controversial issues is related to the name of the theory, expressed by the word "fuzzy". It is known that acquaintance with each theory begins with its name. In this context, the reader's first question is: "I wonder why L. Zadeh's theories are called "fuzzy" (Theory of fuzzy logic, Theory of fuzzy sets, Theory of fuzzy numbers, etc.). It is interesting that Turkish researchers present the name of the theory "Fuzzy Logic" as "Theory of Muddy Logic". As a rule, the words "fuzzy", "vague" are used to express phenomena, the meaning of which is not entirely clear or incomprehensible.

* Received: MAR 11, 2022; Accepted: MAY 13, 2022

Although the logic of L. Zade is a completely clear theory both perceived and against the background of deep reflections. Some scientists who do not understand the essence of the theory or do not want to accept its reality, taking its name as a basis, declare that since the content of the knowledge system is fuzzy, it means that it is not scientific and therefore there is no point in studying and promoting it. Since in science only exact, unambiguous, completely clear concepts are recognized. What is the truth behind this theory?

The fact is that the author of the theory, giving the name of the theory, made a certain mistake. The expression "fuzzy" in fact should not refer to the theory itself, but to thinking, sets, logic, numbers and a number of other concepts that are studied within the framework of the theory as an object of study. As for the theory itself, it is quite clear, precise and scientific. This idea was recognized by L. Zadeh himself in 2008 during his speech in Baku at ANAS. According to him, when choosing a name for his theory, he came across the word "fuzzy" in English and decided to use it, as it reflects the essence of the sets under consideration. In the future, after the theory had already been formed, specialists identified the name of the theory with the name of the sets. In the Russian lexicon, this word sounds like "fuzzy", it was translated into Azerbaijani by a close friend and colleague of L. Zadeh, corresponding member of ANAS, Professor Rafik Aliyev as “geyri-selis”. Thus, the unfortunate choice of the name of the theory was the main reason for the skeptical and cautious approach of some scientists to the theory. A number of other specialists who understand and recognize the theory, as well as those who have found its successful application in the oil industry, simply use the term "F of the set" instead of the term "fuzzy" (Altunin, 2022). Some authors call the theory as the theory of F-logic, F-sets, and this is true. The expression "fuzzy logic" is erroneous. In fact, it would be more correct to call this new theory as Continuous or Continuum Logic. Since, despite the fact that the classical logic of Aristotle is two-valued, the logic of Lukashevich is three-valued, and the logic of Post is k-valued, the logic of Lutfi Zadeh consists of infinite, continual values.

Some authors mistakenly try to call this theory multi-valued logic or give a similar name. It should be noted that such views are fundamentally erroneous. Zadeh himself repeatedly noted this idea in his speeches. Those who confuse these concepts simply do not know the difference between a continuum and countable and finite sets. They also do not take into account the fact that in second-order logic truth values consist of sets.

Misconceptions about the theory

The second controversial point related to the theories of Lutfi Zadeh "Fuzzy Logic" and "Fuzzy Sets" is that a number of specialists have formed such an erroneous opinion that supposedly the main essence of L. Zadeh's discovery is expressed in a set of numbers between zero and one . Note that the set of fractional numbers between zero and one was not a personal discovery of Lutfi Zadeh, it was discovered and used long before our era by the ancient Babylonian and Egyptian mathematicians. The concept of percentage, which makes it possible to show the degree of the constituent parts of numbers or sets, was also discovered and introduced into science long before L. Zadeh.

Consequently, the discovery of L. Zadeh was neither a fraction nor a percentage. The main objects of his discovery are sets with indefinite boundaries. He discovered such sets, gave meaning to them, and created a new mathematical theory for their application. It should be noted that the fundamental scientific innovation and discoveries made by Lutfi Zadeh, revealing a new scientific picture of the world, have a philosophical content. These new scientific ideas, which arose along with two such key theories as "Fuzzy Sets" and "Fuzzy Logic", were included as paradigms in a number of fundamental discoveries of modern science. Later, until the end of his life, Lutfi Zadeh developed these theories and, on their basis, created a number of even more grandiose theories.

Set theory by Georg Cantor

As you know, set theory was created in the 19th century by the famous German mathematician Georg Cantor. This theory laid the foundation for a new trend in mathematics. This scientist, with a brilliant mind, showed the possibility of depicting all mathematical operations as operations performed on sets. He proved that all mathematical objects created before this theory can be recreated and depicted as structures (Vopenka, 1983).. Thus, if we express the thought of a number of well-known mathematicians, then we can say that G. Kantor's set theory is such a grandiose world in which all mathematics can fit (Vopenka, 1983).

Although in the following century the theory of G. Cantor was significantly developed by his followers, however, the sets described by him were classical sets with clear boundaries. According to G. Kantor, each element of a set can be either inside it or outside it, but not in an intermediate position. In his theory, G. Kantor gave the following definition of the concept of a set: a set arises from the combination of various elements into a single whole. Here, many elements are understood as elements connected under a single name. A multitude can be seen as an act of thought. Here, under one name, a number of elements are defined and, conversely, a number of elements are combined under one name on the basis of some law (Frenkel, 1969; Hausdorf, 2014).

Note that the concept of a set is a fundamental concept, constantly used by human thinking in the course of its activity, and an important tool for this activity. Lutfi Zade, touching upon the concept of a set, intervened in the foundation of thinking and the entire mathematical science, having managed to bring novelty to them. This, in turn, led to the creation of new mathematics and logic.

fuzzy sets

L. Zadeh's article "Fuzzy sets" [2], published by him in 1965, was the first article that presented his new theory to the scientific community. In this article, the concept of "fuzzy-sets" was first introduced into scientific use, and in this context the foundations of a grandiose theory were laid. The first step towards studying the theory of L. Zadeh is connected with the study of such a question as "What is a fuzzy-set?". Fuzzi or fuzzy set is such a set, each element of which is included in it with a certain degree. In other words, there is a degree of membership that characterizes each of its elements.

The set theory of L. Zade differs significantly from previous theories. Here the transition of quantity into quality is more clearly manifested. With an increase in the degree of membership (quantity) of each element of the set, its right to represent the set (quality) also increases. According to a common principle in the content of the concept of a set, the elements of quantity and quality form a unity and it is impossible to separate them from each other. Each element occupies a position between two such opposite poles as presence and absence in the set. The concept of a set, like all phenomena in the world, is not unipolar, and is a unity of dialectical opposites.

Thus, we are witnessing that in the history of theoretical thought, the idea of the unity of quantitative and qualitative sides (characteristics) in the content of the concept of "set" was first voiced in the theory of "Fuzzy Sets" by L. Zadeh. And this means that when describing a fuzzy set, it must be indicated how many times and how each element entered the set.

It is known that the study of the content of any theory begins with an analysis of its concepts that make up its scientific apparatus. Without violating this tradition accepted in science, in order to acquaint the respected reader with the content of the theory of "fuzzy sets", first of all, we turn to its conceptual apparatus and try to study its philosophical essence. The first concept to which we would like to turn for this purpose is the concept of "universe", which occupies a central place in the theory.

universe

Lutfi Zade, revealing in the history of science a new meaning of the concept of set, directed the concept of Cantor's sets in a new direction. To this end, he used new concepts. One such concept is the concept of the universe. It is generally accepted that human thinking, in order to solve a problem, first of all, highlights all the possible elements that can participate in this problem. Then, dividing this set of common elements into separate subsets, performs various operations on them and gets the result. Thus, the concept of "universe" is the first concept used by L. Zadeh. By "universe" he meant a set of various elements used to solve problems posed in a certain context. L. Zade, using this concept as a starting point, built his theory on it (Zadeh, 1976).

Let's use some examples to clarify our idea. For example, if we are looking for the best student in the group, then in this case the concept of "universe" will include all the students of the group, but if we are looking for the best student in the faculty, then this concept will include all students of the faculty, but if the search is conducted among students university, then, accordingly, the "universe" will include all students of the university.

As is known, due to the fact that the main object of study of L. Zadeh's theory is the model of human thinking, the concept of "universe" reflects precisely the processes occurring in thinking. Therefore, a person, thinking about the solution of a particular problem, first of all, determines a universal set containing all the elements related to this problem, and begins to generate all thoughts within it. For this reason, L. Zadeh, in his theory of fuzzy sets, having determined the entire set of elements of the universe, developed it precisely on the basis of this set.

From the above judgments it becomes clear that all the considered sets are inside the universe. And now let's consider an arbitrary subset A inside the universe. In this case, all elements of the universe with respect to the subset A can be divided into three groups. The first group will include elements that are not included in subset A, the second group will include elements included in this subset, and the third group will include elements that are partially included in subset A.

For example, consider the set "Azerbaijanis of the world". For this set, the universe will be the totality of all people living in the world (approximately 7 billion). There are people in the world who have no family ties with Azerbaijanis. They are representatives of other peoples (nations). For example, they include the British, French, Russians and representatives of other nations. Therefore, we can categorically state that representatives of these peoples are not included in the set of “Azerbaijanis”. The second set includes Azerbaijanis who have no family ties with other peoples. Finally, the third group includes Azerbaijanis, in whose clan other nations participated to one degree or another. For example, it may be that some Azerbaijani has a father, mother, grandfather, grandmother or even great-grandmother and great-grandfather were representatives of another nation. We must include such people in the third group. This means that each element of the third group has a degree of inclusion

in the set "Azerbaijanis". This power is a real number between zero and one. For example, if someone has a Russian mother, an Azerbaijani father, this degree is equal to one second, if both parents are Azerbaijanis, but the grandmother is Russian, then the degree is one fourth, etc.
Thus, it can be argued that the set "Azerbaijanis" is a fuzzy, fuzzy set. The degree of membership of the elements of most of this set is equal to one. This part is called the core of the fuzzy set. The degree of belonging of the elements of the other part of the set to the fuzzy-set "Azerbaijanis" will be equal to some real number between zero and one. These elements are called the boundary elements of the set. They are represented by people who, to a certain extent, are related to other nations.
Our example clearly demonstrates that fuzzy sets are really existing sets. Some of the elements of this set are included in this set completely, and some - to a certain extent.

Membership function

The most important, central and key concept of the theory of fuzzy sets is the membership function (Zadeh, 1976). All theories of L. Zadeh are based on this function. In fact, L. Zadeh's theories can be called a miracle of one function. This function, having a deep meaning, can be considered the key of human thinking and the foundation of knowledge. Any person's knowledge of the surrounding world can be modeled and described by means of membership functions. The decisions a person makes every moment depend directly on the membership functions. These functions are updated every moment. As people learn about the world around them, acquire knowledge, these functions get more accurate values.
Persons with deep knowledge of a particular science or practical experience are called experts. In their thinking, the functions of belonging, in comparison with other people, are more accurate and adequate. The membership function shows the degree of occurrence of each element of the universe in the considered set. It should be noted that the main strength and meaning of L. Zadeh's theories is provided precisely by this function. The discovery, giving meaning and application of this function by the famous scientist L. Zadeh contributed to a serious leap in world science.

The membership function can be linear or non-linear, discontinuous or continuous. Often, for the sake of simplicity, this function is assumed to be linear and continuous. In the theory of Fuzzy numbers (fuzzy numbers), functions that are continuous from above are also used. In practical calculations, triangular, trapezoidal, bell-shaped, Gaussian, S-shaped and other membership functions are most often used, taken as a template.

Singleton

One of the concepts that enrich the mathematical apparatus of fuzzy sets is also the term singleton. To get acquainted with this term, let us recall once again that the logic of L. Zadeh is not classical or traditional logic. It is a post-non-classical logic with a synergetic essence. Singletons are indivisible elements of a set, and the membership function is a means that combines these elements into a set. Singletons are the smallest indivisible parts of the universe. The universe is a collection of singletons. The participation of each singleton in the set is provided by the membership function. For example, in the "friends" set, every person we know is a singleton. Together they all form the universe. In this case, the membership function is a function that reflects our inner, subjective thought.
Thus, as we can see, fuzzy sets are sets that are actually formed from synergy, that is, the unity of singletons. In these sets, each singleton participates only to a certain extent. Fuzzy sets, on the other hand, consist of connections of segments, represented by sets of singletons with an identical membership function. Such sets are sets that change from presence to absence not abruptly, but gradually. For example, consider the structure of a place where an earthquake occurred. Here, the degree of occurrence of elements subjected to an earthquake at the epicenter of the earthquake is equal to one, a little further from the epicenter it is 0.9, even further - 0.8, 0.5, 0.2, etc. This degree gradually decreases in the direction from the epicenter to the place that was not subjected to an earthquake. Some number α between zero and one, forms one segment of the set "earthquake". In other words, the degree of relation to the earthquake of all places located on a given segment is equal to α. A fuzzy set is a set formed from a combination of segments α. In general, we can say that all sets formed from explosions and waves are fuzzy. An example is the Earth's atmosphere. As you know, the atmosphere into space passes gradually, it has fuzzy boundaries.

Transition point

Some scientists argue that in Aristotle's logic there is no transition between hot and cold, wet and dry, truth and lies, etc. That is, Aristotle did not think about what is day and night and that something exists between them. In fact, the logic of Aristotle can be regarded as approximate. For example, in the interval [0,1], numbers close to zero can be taken as 0, and numbers close to one can be taken as 1. This means that there is nothing between 0 and 1. It's just that some of them are called zero, and some are called one. If we start pointwise moving from 0 to 1, then when will we, having finished with points close to zero, reach points close to one? This transition occurs

at a certain point that exists between zero and one. This point is called the transition point. In general, it can be taken equal to 0.5. However, this point is subjective. For each person, the transition point may be different. And now let's find the answer to the following question: “Until what hours do we call the time of day day, and from what hours we call it night? If we ask this question to different people, we will get different answers. Someone will say 6, someone 10, and someone 12 hours. So this time is subjective. I mean, it depends on the person. But this point exists. The transition point is the point where day turns into night, dry into wet, truth into lies, good into bad, and so on. Now let's look at the immediate environment of the transition point. Suppose someone says that for him six hours is the transition point between day and night. Then it means that without one millionth of a second, six is considered day, and one millionth of a second of the seventh is considered night. The time between these moments is so short that the human senses cannot detect it. Consequently, the day at once, in an instant, turns into night. And in other examples, at the transition point, the truth turns into a lie, good into bad, dry into wet, etc. That is, a certain being at this point from one qualitative state passes into another not gradually, but suddenly, with a jump. In Aristotle's logic, intermediate states are rounded up and transferred to either this or the other side. This logic is based on the law of the excluded middle. That is, any sentence is either "true" or "false." The third is ruled out. That is, a certain being at this point from one qualitative state passes into another not gradually, but suddenly, with a jump. In Aristotle's logic, intermediate states are rounded up and transferred to either this or the other side. This logic is based on the law of the excluded middle. That is, any sentence is either "true" or "false." The third is ruled out. That is, a certain being at this point from one qualitative state passes into another not gradually, but suddenly, with a jump. In Aristotle's logic, intermediate states are rounded up and transferred to either this or the other side. This logic is based on the law of the excluded middle. That is, any sentence is either "true" or "false." The third is ruled out.

The main essence of Lutfi Zade's logic is that, according to this logic, processes from one qualitative state to another do not pass immediately, but gradually. That is, the transition point is not really a point, but an interval. In other words, the day gradually turns into night, dry into wet, good into bad, etc. to denote this transition, a person uses in colloquial speech such words as "a little", "almost", "about", "close", etc. In the theory of Lutfi Zadeh, they are called terms that can be adequately depicted in the numerical axis as fuzzy sets.

Comparison of fuzzy logic and Aristotle's logic

The logical basis of the theory of fuzzy sets of L. Zadeh is fuzzy, fuzzy-logic, which runs like a red line through his work and differs from the binary logic of Aristotle. Therefore, we present fuzzy logic as one of the important elements of a fuzzy set.

As is known, in the history of the theoretical thought of mankind, the classical binary logic of Aristotle, which gave rise to the development of logical representations, was linear, formal in its content and recognized only the qualitative state of logical concepts, without affecting their quantitative degrees. This feature of Aristotle's logic, which sees the world in two colors, was clearly manifested in the idea of "non-contradiction", which is one of its highest principles. For example, according to binary logic, a body cannot be both large and small at the same time, it must be either large or small. This means that, based on binary logic, we must perceive the body not in a state of unity of opposites, but only in one state: either in a large or in a small one. From our example, it is clear that that in the classical logic of Aristotle, the quantitative characteristics of such qualitative states as “large” or “small”, in other words, the degree of amplification, did not find their expression. Let's give another example, "Ali is my friend" (thesis) - "Ali is not my friend" (antithesis). The concept of “friend” used in these expressions can have different degrees: “Ali is my closest friend”, “Ali is one of my friends”, “Ali is my distant friend” and other judgments express different degrees of the concept of “friend” . Although the binary logic of Aristotle studies a linear, simple, primitive world, the lower levels of human thinking, which are the subject of knowledge of logical science, gives a description of the general state of the concept of “friend” used here, neglecting its various degrees. Thus, the logic of Aristotle does not take into account the spectral nature of the concept and is silent about the diversity of their degrees. For example, according to the logic of Aristotle, "red" is only red, "black" is only black. As can be seen, due to the fact that here is a linear approach to concepts, their classification by degrees is not required. In other words, with the help of this logic, it is impossible to adequately model these concepts by translating them into mathematical language. L. Zade introduced a new concept of a linguistic and fuzzy variable into science (Zadeh, 1976).

However, in the theory of fuzzy logic, L. Zadeh pays attention not only to the semantic shades of identical concepts, but also to their various quantitative characteristics, which allows them to be grouped according to degrees of difference. 134 L.Zade introduced new concepts of linguistic and fuzzy variables into science. The study of the concept, considered in the field of knowledge created by it, from the perspective of nonlinearity, allows us to identify its various degrees and depict them mathematically on the segment [0,1]. This means that Aristotle's logic studies the linear world, while L. Zadeh's logic studies the non-linear world. However, despite a number of sharp differences between them, both logical theories are true, thus the world is a unit and it is not linear. But it can be studied by resorting to a linear approach. The regularities of the linear world are studied in classical rationality, while the regularities of the non-linear world are studied in non-classical rationality. In

terms of content, Aristotle's logic is a metaphysical theory, which of the two contradictory states of the body (the body is black - the body is not black), recognizes one (for example, the body is black) and denies the second state (the body is not black). Consequently, the logic of Aristotle is a binary logic that perceives a bipolar world, and thereby leads it away from dialectical contradictions. In contrast to the logic of L. Zadeh, the law of the excluded middle is fulfilled here. Conscious human thinking can be studied in many ways, using simple and complex methods. In our opinion, the main difference between the logics of L. Zadeh and Aristotle lies in the choice of the method of approach to the processes of thinking. The logic of Aristotle studies the world in a simple way, and the logic of L. Zade - in a more complex way. which of the two contradictory states of the body (the body is black - the body is not black), recognizes one (for example, the body is black) and denies the second state (the body is not black). Consequently, the logic of Aristotle is a binary logic that perceives a bipolar world, and thereby leads it away from dialectical contradictions. In contrast to the logic of L. Zadeh, the law of the excluded middle is fulfilled here. Conscious human thinking can be studied in many ways, using simple and complex methods. In our opinion, the main difference between the logics of L. Zadeh and Aristotle lies in the choice of the method of approach to the processes of thinking (Belohlavek, 2018). The logic of Aristotle studies the world in a simple way, and the logic of L. Zade - in a more complex way. which of the two contradictory states of the body (the body is black - the body is not black), recognizes one (for example, the body is black) and denies the second state (the body is not black). Consequently, the logic of Aristotle is a binary logic that perceives a bipolar world, and thereby leads it away from dialectical contradictions. In contrast to the logic of L. Zadeh, the law of the excluded middle is fulfilled here. Conscious human thinking can be studied in many ways, using simple and complex methods. In our opinion, the main difference between the logics of L. Zadeh and Aristotle lies in the choice of the method of approach to the processes of thinking. The logic of Aristotle studies the world in a simple way, and the logic of L. Zade - in a more complex way. Aristotle's logic is a binary logic that perceives a bipolar world, and thereby leads it away from dialectical contradictions. In contrast to the logic of L. Zadeh, the law of the excluded middle is fulfilled here. Conscious human thinking can be studied in many ways, using simple and complex methods. In our opinion, the main difference between the logics of L. Zadeh and Aristotle lies in the choice of the method of approach to the processes of thinking. The logic of Aristotle studies the world in a simple way, and the logic of L. Zade - in a more complex way. Aristotle's logic is a binary logic that perceives a bipolar world, and thereby leads it away from dialectical contradictions. In contrast to the logic of L. Zadeh, the law of the excluded middle is fulfilled here. Conscious human thinking can be studied in many ways, using simple and complex methods. In our opinion, the main difference between the logics of L. Zadeh and Aristotle lies in the choice of the method of approach to the processes of thinking. The logic of Aristotle studies the world in a simple way, and the logic of L. Zade - in a more complex way. Zadeh and Aristotle lies in the choice of the method of approach to the processes of thinking. The logic of Aristotle studies the world in a simple way, and the logic of L. Zade - in a more complex way. Zadeh and Aristotle lies in the choice of the method of approach to the processes of thinking (Schang, 1974). The logic of Aristotle studies the world in a simple way, and the logic of L. Zade - in a more complex way.

Probability and Possibility

Many specialists confuse L. Zadeh's logic with the theory of probability and do not see the difference between them. For example, according to the conclusion of the American professor Myron Tribus (1993), in reality, fuzziness is a masked probability. However, Professor L. Zadeh in his articles repeatedly proved the fallacy of this idea. In the late 70s of the last century, having given a new interpretation of his theory, he created the theory of possibilities. Although this theory is similar to the theory of probability, however, it differs from it both in meaning and in mathematical apparatus (Mizumoto, 1981).

A person who has set himself the goal of studying the patterns of the world, reflecting on the processes and phenomena of reality, thinks both in the form of possibilities and in the form of probabilities. If the theory of probability has a 300-year history, then the theory of possibilities has existed for only 50 years.

Probability theory is concerned with events and estimates the probability of their occurrence. Possibility theory, on the other hand, explores the possibilities of distributing events in accordance with the ideas of a person. For example, consider the following event. Imagine that a gray-haired teacher with an average level of knowledge should come to the seminar. In this event, the field of science that studies the arrival of a teacher and calculates the probability of his arrival is the theory of probability, and the scientific direction that studies how much the teacher will be gray-haired and how average the level of knowledge he will have is the theory of possibilities.

In general, we can say that these areas of knowledge cover two branches of uncertainty. To build more adequate and realistic models, it is necessary to use these theories simultaneously and in parallel. For example, if you ask any specialist in probability theory about the probability of getting 6 points when throwing a dice, he will say without thinking that the probability will be equal to 1/6. But, in fact, this is an erroneous idea. Because for this probability to be 1/6, the die must be perfectly symmetrical. And this is not so. In reality, the probability of getting a 6 is around 1/6. To be more precise, it is a fuzzy number 1/6.

Mathematics and Synergetics

The theory of fuzzy logic has not bypassed the development of mathematical science. Thus, a fuzzy analogue of numbers and geometric figures appeared, which are the main objects of classical mathematics. This increased the power of mathematics and created excellent conditions for its study of more complex systems.

Thus, Lutfi Zadeh, in connection with the study of the laws of development of complex systems of nature and society, including self-organizing, nonlinear, open, dissipative systems, through intensive and extensive research since the 60s of the last century, showed that to understand the laws of such systems, neither binary logic of Aristotle, nor in general classical mathematics based on this system. In this regard, L. Zade, who tried to create a connection between his theory of fuzzy logic and synergetics, put forward the idea that a new mathematics should be formed to study self-organizing nonlinear systems. As the dynamics of the development of modern scientific knowledge shows, this should be the theory of fuzzy sets. However, the achievement of scientific knowledge of this level does not mean that that binary logic and linear mathematics, including the famous Cantor's set theory, should be removed from the thinking of human culture and included in a number of archaic knowledge that no longer has a place in scientific knowledge. The essence of the issue lies in the fact that the problems of nature and society surrounding a person can be both linear and non-linear, both clear and fuzzy. For many years, linearity and clarity, as attributes of classical logic and classical mathematics, have been sufficiently studied. These theoretical systems can only explore the regularities of simple and not very complex self-organizing systems. Since the 70s of the last century, mankind has switched to the study of complex, open, nonlinear, dissipative (scattering matter, energy and information) systems and won new amazing achievements in the field of studying new patterns of the world (Cao, 2018). Complex, self-organizing systems have a particular logic and specific mathematics, and, in addition, these theoretical systems are non-linear. While simple and relatively complex systems can be explained using the principles of fuzzy logic, such a description is impossible for complex, self-organizing systems. A similar description for complex systems can be built on fuzzy logic and fuzzy mathematics. Fuzzy logic L. Zadeh, having paved the path from the linear world to the non-linear one, significantly expanded the modeling of the logical possibilities of thinking, created ample opportunities for a person to enter into a new dialogue with thinking. self-organizing systems have a particular logic and specific mathematics, and, in addition, these theoretical systems are non-linear. While simple and relatively complex systems can be explained using the principles of fuzzy logic, such a description is impossible for complex, self-organizing systems. A similar description for complex systems can be built on fuzzy logic and fuzzy mathematics. Fuzzy logic L. Zadeh, having paved the path from the linear world to the non-linear one, significantly expanded the modeling of the logical possibilities of thinking, created ample opportunities for a person to enter into a new dialogue with thinking. self-organizing systems have a particular logic and specific mathematics, and, in addition, these theoretical systems are non-linear. While simple and relatively complex systems can be explained using the principles of fuzzy logic, such a description is impossible for complex, self-organizing systems. A similar description for complex systems can be built on fuzzy logic and fuzzy mathematics. Fuzzy logic L. Zadeh, having paved the path from the linear world to the non-linear one, significantly expanded the modeling of the logical possibilities of thinking, created ample opportunities for a person to enter into a new dialogue with thinking. for complex, self-organizing systems, such a description is impossible. A similar description for complex systems can be built on fuzzy logic and fuzzy mathematics. Fuzzy logic L. Zadeh, having paved the path from the linear world to the non-linear one, significantly expanded the modeling of the logical possibilities of thinking, created ample opportunities for a person to enter into a new dialogue with thinking. for complex, self-organizing systems, such a description is impossible. A similar description for complex systems can be built on fuzzy logic and fuzzy mathematics. Fuzzy logic L. Zadeh, having paved the path from the linear world to the non-linear one, significantly expanded the modeling of the logical possibilities of thinking, created ample opportunities for a person to enter into a new dialogue with thinking.

The theory of fuzzy sets by L. Zadeh is a fundamental theory that gave a new impetus to the development of mathematical science. It is no coincidence that already in most areas of mathematics the ideas of L. Zadeh's logic have found wide application. In this regard, I would like to note that at the end of the last century, the well-known Russian mathematician, academician V.T. Arnold proposed using "soft" models to describe social processes. Although, long before this proposal, the whole world had already recognized the possibility of the most perfect and rational method for creating soft models using the theory of fuzzy sets proposed by L. Zade.
The main idea that glorified L. Zadeh and further creatively developed by world scientists is to accept fuzzy sets as a whole as an object of study, build various theories on their basis and describe the deep meaning of such sets and at the same time their great applied value.

Fuzzy sets of the second order

One of the amazing ideas that reinforce the theories of L. Zadeh is that the degrees of membership of elements of fuzzy sets themselves can consist of fuzzy sets. Such sets in science are called fuzzy sets of the second order. Naturally, the concepts of fuzzy sets of the third, fourth and, in general, n-th order also entered science. However, the sets that make it possible to better model the uncertainties in the human brain are precisely the fuzzy sets of the second order. Sets of higher order are relatively abstract.

For example, if someone says that "Gasan is my friend", this means that Gasan is in the "friends" fuzzy set. However, the very degree of Gasan's entry into this set is also a fuzzy set. Human thinking perceives such sets and their degrees of membership through intuition and sense organs and performs operations on them. This process can be modeled and studied using such concepts as a linguistic variable and a term introduced into science in the theories of L. Zade (1976). As a rule, we express such sets in speech by means of the words “many”, “few”, “medium”, “large”, etc., as well as particles that change their degrees of amplification as non-linear operators (very, almost, somewhat, approximately, excessively, etc.) For example, we say: “Gasan is my closest friend”, “Gasan and I are a little friendly”, “Gasan and I are almost friends”, etc.

Thanks to the theory of L. Zadeh, such subjective thoughts can be objectified, in other words, expressed in symbols and accurately marked on the numerical axis. This, in turn, provides ample opportunities to manage them, put meaning into them and thus create artificial intelligence. On the other hand, it should be especially noted that these theories are able to study the mechanisms of perception, understanding and cognition of people with high intelligence and knowledge, and thus pave the way for human thinking.

The main strength of L. Zadeh's logic lies in the fact that here the truth values of sentences in the scale [0,1] can be represented not by a point, but by certain intervals. Using the membership function, each point of such intervals is assigned a specific degree, which in the colloquial language is modeled by such expressions as “a little true”, “very true”, “almost true”, etc.

Comparison with other theories

In 1967, the American scientist R. E. Moore created a theory called "Interval Analysis". Here, intervals were taken as numbers and all mathematical operations were performed on them. However, due to the fact that in this mathematical system the intra-interval points do not differ from each other, the membership function for all of them took the same unit value. In this aspect, R.E.Moore's mathematical system can be considered as a special case of L.Zadeh's mathematics. On the other hand, studies have shown that the mathematical systems of Aristotle, Lukashevich, Post and a number of other systems are special cases of L. Zadeh's logic. Thus, we can say that with his theory L. Zadeh generalized and expanded the mathematical systems that existed before him, and created a new and strong system with even greater capabilities.

What kind of logic does a person think?

According to some experts, most people think in the logic of Aristotle. In our opinion, this opinion is not true. So, we will not be able to find even one person who would think only with the logic of Aristotle. This property can only be attributed to cybernetic machines, robots. Although today in a number of advanced countries of the world robots have been made that think with the logic of L. Zadeh. On the other hand, if all people thought only by Aristotelian logic, then they would not be able to distinguish from each other their close people, friends, whose degree of belonging is equal to one. In this case, the human decision-making mechanism would be significantly weakened and would not reflect reality. In fact, human logic is not the logic of either Aristotle, or Lukashevich, or Post, or Zadeh. Simply put, it's safe to say

It should be noted that there are a number of other thoughts about the logic of L. Zadeh. For example, according to UC Berkeley professor William Kahan (Kosko, 1993), “Fuzzy logic is fallacious, flawed, and harmful. We actually need more logical thinking. The danger of fuzzy logic is that it will promote fuzzy thinking, which will lead to a number of problems. Fuzzy logic is cocaine for science.

On the other hand, some authors erroneously note that with the advent of L. Zadeh's theories, the hopes placed on the formation of absolute, precise and clear mathematics, on the observance of a strict and clear logical sequence in mathematical judgments, collapsed.

We must note with regret that such erroneous approaches and views are associated with an incorrect understanding of the meaning of L. Zadeh's theories. In fact, the main point of these theories is to more accurately model and study the uncertainties that exist in real life. In other words, the main goal of L. Zadeh's ideas is the transformation of fuzzy sets into clear ones. Where the strength of classical mathematics runs out and processes can only be studied by rounding off, fuzzy mathematics allows us to build more accurate models. Thus, in fact, the theories of L. Zadeh do not at all contradict the basic principle of mathematical science, namely, accuracy, but, on the contrary, by giving accuracy a new meaning, it introduces accuracy into obscure, vague processes and, multiplying the forces of mathematics, further expands its possibilities. Contrary to the opinion of the above-mentioned authors, judgments here did not become fuzzy, but, on the contrary, fuzzy thoughts and processes became clear. That is, in other words, one of the goals of this theory is to transform

fuzzy, incomprehensible, indefinite for human thinking sets, processes and phenomena into clear ones, bringing clarity to them.

According to the famous physicist Albert Einstein, “as far as the laws of mathematics approach the real world, they are so uncertain and, conversely, as far as they are certain, they do not reflect the real world” (Kosko, 1993). As you can see, L. Zade's mathematics serves to better reflect the abstract artificial world built on symbols of the real world around us. The main strength of this mathematics lies in its adequacy. As American scholar Bart Kosko put it (Kosko, 1993), “The mathematical world does not correspond to the real world he describes. These worlds are different from each other. One is artificial, the other is the real world, one is ordered, the other is disordered.” L. Zadeh's theories provide us with the conditions for more adequate modeling and research of the disordered real world, for its ordering and better management for the benefit of society. On the other hand, the application of L. Zadeh's theories allows us to transform operations occurring in the form of subjective thinking into mathematical language. And this, in turn, contributes to the objectification of human thoughts, the practical application of computer-built models, the study of cause-and-effect relationships in the human thought process.

Z numbers

By the end of his life, L. Zade introduced a new perfect concept into science. This concept is Z numbers, which are a new model of human thoughts. As you know, a person, expressing some thought, tries to intuitively determine both the degree of its truth and confidence in it. The degree of certainty of a thought is the probability of its being true. For example, if someone confirms that Ramiz is a good boy, then there are two uncertainties. The first is how good Ramiz is, the second is how confident the person who expressed this thought is. In fact, the probability itself, given on the interval [0,1], is a fuzzy number. Thus, every human thought has a dimension of uncertainty in two directions. One is the degree of truth, the other is the degree of certainty. Each of these degrees can be represented by fuzzy numbers.

Basic philosophical essence

The theory of fuzzy sets of Lutfi Zadeh has a multifaceted philosophy. To reveal the deeper meanings of this theory, first of all, it is necessary to trace the connection between the forms of movement known to science and mathematics. It is known that the famous German philosopher F. Engels gave a classification of 5 forms of movement (Engels, 1968), where he placed them according to the degree of complexity in the following sequence: mechanical, physical, chemical, biological, social. Mechanical movement is the simplest, social movement is the most complex form. Classical mathematics used by man is effective only for the lower forms of movement (mechanical, physical, chemical), for more complex, higher forms of movement, including biological, mental, mental, i.e. for the individual, and for society as a whole, it loses its power. The system analysis of the theory of fuzzy sets by L. Zadeh shows that in the study of higher forms of movement this theory is more rational and capable of giving adequate results.

In particular, L. Zade notes that most of the methods (Zadeh, 1973), applied to the analysis of "humanistic" systems (where a person participates), are obtained as a result of improvement in one form or another of the methods used for mechanical systems. Therefore, their effectiveness was not high. For such complex systems, a new look is needed, a new kind of approach on which the theory of fuzzy sets would be based. The author, proceeding from this position, step by step created his new theory and laid the foundation for an amazing trend in science.

Principle of incompatibility

One of the valuable discoveries of L. Zadeh's approach was the principle of incompatibility. The essence of this principle lies in the fact that as the level of complexity of the system increases, it becomes more difficult to judge how it will behave and what decisions it will make (Zadeh, 1973). For systems, the degree of complexity of which has passed a certain limit, the accuracy and the value of the practical effect become properties that refute each other. For example, when establishing air communication between two cities, it makes no sense to indicate the distance between them with millimeter and centimeter accuracy. In general, accuracy with respect to the system is relative. For every complex system, accuracy loses its meaning after a certain limit.

Thinking Model

Summing up our research related to the theory of fuzzy sets by L. Zadeh, I would like to touch upon the issue of the model of human thinking. The question we want to answer is that a number of researchers who draw a parallel between the logic of Aristotle and the logic of Zade consider binary logic to be weak, harmful, and fuzzy logic to be strong, rational. Aristotelian logic, being the first logical model in the history of logical science, is not able to accurately describe human thinking. And the theory of L. Zade differs from other logical models (Lukashevich's logic, Post's logic, etc.) by a more adequate reflection of human thinking and thought processes. In this regard, according to some authors, fuzzy logic completely destroyed Aristotelian logic. In our opinion, this idea is fundamentally wrong. In fact, the logic of L.

Taken as a whole, Aristotle's logic can be considered as a special case of L. Zadeh's logic. Both logics reflect the logical truth, the only difference is that while the Aristotelian logic for the processes of logical thinking is a slightly crude and simple model, L. Zadeh's logic is a richer, more accurate, adequate and convincing model. In this model, by eliminating the sharp difference between randomness and orderliness of thought, an even greater approximation of chaotic thoughts to ordered ones is provided.

A person has a system of thinking given to him at birth on a genetic basis and developed as a result of gradual improvement. This is precisely the system of thinking that thinks, carries out activities, makes decisions, steadily fights to achieve the intended goal. All human creative activity, gaining success, solving problems are based on this system. Thinking is a thinking spirit, a moral process, a relatively adequate reflection of reality in the human brain. Logic, on the other hand, is a fundamental science that studies thinking, reveals its patterns, and selects a method of approach to it. People in different periods of historical development approached thinking with different methods and, depending on the degree of rationality of the method, studied its objective laws at different levels.

The first step in this area was made by the ancient Greek philosopher Aristotle, who outlined the first contours of the science of thinking. Throughout history, scientists have studied this theory and improved it. And, finally, Lutfi Zade, approaching the study of thinking from a new prism, created a more perfect theory about it.

This means that there is a succession between the binary logic of Aristotle and the fuzzy logic of Zadeh, which preserves the intellectual merits of both theories. The purpose of each of the logics is to create an exemplary model of human logic to explain the principles of its work. Undoubtedly, the best model of thinking that exists today is the fuzzy logic model of Lutfi Zadeh. Despite this, the role of Aristotelian logic in the intellectual development of mankind can neither belittle nor deny the role of Aristotelian logic. The logic of Aristotle had a great influence on the historical development of science. Until now, most of the scientific literature is based on this logic. This suggests that at the present time both logics serve the intellectual development of mankind: there are such questions, the solution of which requires Aristotelian logic, and there are those that

Approximation of thought in words

Each person, based on the experience acquired by him throughout his life, has a conceptual system with a finite number of clear and fuzzy concepts. He tries with the help of these concepts to know the world, to study the causal relationships of events. However, as you know, the number and strength of events occurring in the world is at least equal to the continuum. Thus, a system with a finite number of concepts explains events whose strength is equal to the continuum. It follows from this that such an explanation is absolutely unsuitable for a conceptual system containing complete, indivisible concepts, taken without degrees. It becomes obvious that a person in the process of mental activity uses each basic concept to a certain extent to analyze various events. And this means that the logic of L. Zadeh is already being launched here and the cognitive process is taking place. On the other hand, human thought is approximated by words and concepts. The quality of approximation depends on the knowledge of a person, and on his vocabulary, and on the richness of the language. For example, usually writers and poets are better than others at expressing their thoughts in words. Thanks to their experience in this area, the approximation process is more successful for them. The scope and meanings of words and concepts form fuzzy sets. Thus, in the thought process, a person, in order to express his thought, actually approximates it with the help of fuzzy sets. Thanks to their experience in this area, the approximation process is more successful for them. The scope and meanings of words and concepts form fuzzy sets. Thus, in the thought process, a person, in order to express his thought, actually approximates it with the help of fuzzy sets. Thanks to their experience in this area, the approximation process is more successful for them. The scope and meanings of words and concepts form fuzzy sets. Thus, in the thought process, a person, in order to express his thought, actually approximates it with the help of fuzzy sets.

Summing up, we can say that the main philosophical essence of L. Zadeh's theories lies in their more accurate reflection of human thinking and greater adequacy to the real world. Lutfi Zadeh, by virtue of his amazing mindset and abilities, better understood the principles of the work of human thinking and the objective laws operating in it, and therefore he was able to adequately reflect their mathematical models with the appropriate symbols.

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