

OMNIS PROPOSITIO EST AFFIRMATIVA; ERGO, NULLA PROPOSITIO EST NEGATIVA (AND THE PARADOX OF VALIDITY)

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ABSTRACT. In the first of the Insolubles in Chapter 8 of his *Sophismata*, Buridan contends that the inference *Omnis propositio est affirmativa; ergo, nulla propositio est negativa* (*PS*) is valid, even though it appeals to the self-reference in the conclusion to show that what we (following Read 2001) call the classical conception of validity (CCV) fails. This requires that we accept that there are good inferences in which a false conclusion follows from true premises. Partially following Hughes' proposal (1982), we argue that the First Sophism (*PS*) involves three different notions of validity. Two of them correspond to the ones described by Hughes (1982, 80–86), who calls them *Theory A* and *Theory B*. The third one—that will we call *Theory C*—is not mentioned by Hughes; instead, it is suggested by Buridan himself in the first three arguments in favor of the validity of *PS*. We show that: a) from what Buridan says in his *Theory C* it follows that *PS* is a *formal* and *material* consequence, and hence, a valid one. Then we show that: b) the rejection of CCV and the acceptance of *Nulla propositio est negativa* (NPN) as a (formal) consequence of *Omnis propositio est affirmativa* (OPA) leads to a paradox that bears similarities with the one put forward by Pseudo Scotus—which has been studied by Read (2001) and is related to Curry's paradox. However, there are enough differences to merit considering this paradox separately, especially in relation to the so-called *validity paradoxes*. Interestingly, our work suggests that Buridan was aware of these problems, which explains why he introduced a new criterion for validity, one that is not based on truth-preservation but on what Spade (1988) calls *firmness*, and Klima (2016) *correspondence*.

Keywords: Validity; Self-reference; Paradoxes; Buridan; Sophismata; Consequence; Formal; Material

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1. INTRODUCTION

In this paper we will discuss the first *insolubile* in Chapter 8 of Buridan's *Sophismata*:

PS: First sophism: Every proposition is affirmative; therefore, no proposition is negative.

Primum sophisma: Omnis propositio est affirmativa; ergo, nulla propositio est negativa. (2001, 952)

Clearly, this sophism involves the concepts of validity and logical consequence. However, in order to understand it properly, it will be useful to mention a few things about the context in which it appears.

The treatment of paradoxes in the Middle Ages derives from treatises that began to be produced in the thirteenth century: the *Sophismata*, *Impossibilia*, and *Insolubilia*, all of which were related to the topics in *De fallaciis*, the seventh treatise in Peter of Spain's *Summulae*, that in turn derives from Aristotle's *De Sophisticis Elenchis*, but includes some original features (see Muñoz Delgado 1964, 44). More specific works on insolubles began to appear around the early thirteenth century at the latest and continued to be produced until the end of the Middle Ages; by the fourteenth century treatises on the specific topic became a flourishing genre of logic (see Spade 2008, 246; Spade and Read 2021, §3; Yrjönsuuri 2008, 580).

Sophisms, a topic that he would approach in different ways throughout his intellectual life, occupy an important place in Buridan's work. However, “[h]is final view is described in the ninth and last treatise of the *Summulae*, with the independent title *Sophismata*, in a version from the mid-1350s” (Spade and Read 2018, sec. 3.5). In Chapter 8, Buridan declares that he is going to deal with “propositions that are self-referential [*de propositionibus habentibus reflexionem supra seipsas*] on account of the significance of their terms” (2001, 952). He then adds that the chapter “contains propositions called insolubles” (2001, 952). Even though The Liar is the most often studied among such paradoxes, it is not the only one to which medieval logicians turned their attention. “Insolubles do not only deal with the Liar paradox” (Yrjönsuuri 2011, 550) and this is the case of *PS*. The first six insolubles in *Sophismata* (which includes 30 in total) are devoted to examining the conditions of validity of certain problematic inferences. What unifies the interest of medieval logicians in such puzzling propositions (and inferences) is not the concept of truth, since their treatment of the topic was often focused on other semantic concepts, such as validity (the one we will examine in this work), or concepts of a different nature, e.g., epistemic concepts.

Nonetheless, the one characteristic that is clearly present—in various and interesting ways in each and every insoluble is (some form) of self-reference.¹ It bears recalling that in the case of *PS* this self-reference is indirect. As noted by Read (2001, 193), this happens when the propositions that make up the argument contain an expression referring to a piece of discourse of which that proposition is a part.²

It seems reasonable to assume that Buridan’s main goal in writing the treatises that comprise the *Sophismata* was providing university students with practical exercises to help them improve their argumentative skills. So, why does this topic (the very nature of the notion of validity) take such an important place within a work originally meant for the instruction of students? The answer is that this sort of meta-logical interest is in accordance with the various roles of the *Sophismata*. As Pironet and Spruyt point out, “On a basic level the *sophismata* served to illustrate a theory, but they were also used to test the limits of a theory” (2019, section 3). Scott makes a similar comment: “They were used primarily, but not exclusively, for the testing and application of logical rules” (1966, 5). Thus, the *Sophismata* proved to be a particularly fertile ground for addressing the kind of meta-theoretical questions elicited by *PS*. At the beginning of the book, Buridan explores the limits of the notion of logical consequence and the logical rules that govern it, focusing on the problems that valid inferences give rise to when one of their elements is an (indirectly) self-referential proposition, as is the case with *NPN*.

In what follows we will argue that in his analysis of this sophism Buridan presents and compares *three different notions of validity*. The first two are described by Hughes (1982, 80–86), who calls them *Theory A* and *Theory B*. We will consider a third possible theory, which we call *Theory C*, one that is not mentioned by Hughes, but instead is suggested by a reading of Buridan’s first three arguments in favor of the validity of *PS*. *Theory C* gives us logical considerations that support two different types of arguments: one to support the material validity of *PS* and others that support, using other reasons, its formal validity. Then, we will show that in his analysis of *PS* Buridan: (i) claims that *Theory A* is

¹“Insolubles were primarily certain sorts of self-referential sentences, semantic paradoxes like the ‘liar paradox’ (‘What I am now saying is false’). But few authors tried to give a rigorous definition, so that other more or less unrelated kinds of paradoxes were also treated under this heading.” (Spade 2008: 246)

²“Argument B and the others remind us that self-reference can be indirect. B’s premise contains an expression referring to a piece of discourse of which that premise is a part. Further, whether a sentence leads to paradox may depend on how the world is, on whether certain other sentences are true (as Epimenides showed).” (Read 2001, 193)

incorrect: (ii) rejects *Theory B*; and (iii) accepts *Theory C*. We will see that rejecting *Theory A* (as incorrect) while accepting *Theory C* leads to—at least two paradoxes.

Everything we say in this work can be added to the debate on medieval problems of truth-preservation as a criterion of validity. Furthermore, our analysis suggests an interesting hypothesis, namely, that the problems posed by *PS* are not completely addressed by Buridan’s own solution, which leads him to explore a different criterion of validity, one not based on truth-preservation, but rather on the concept of signify.³

2. THE STRUCTURE OF THE INSOLUBLES AND *PS*

As time went on, medieval logicians settled on certain canonical procedures for presenting the solutions to insolubles. This was done through the highly formalized scholastic method for determining a *quaestio*, which usually consisted of three steps:

- First, one must examine the *pro* and *contra*.
- Second, one must present one’s own solution. (Sometimes this part of the discussion is preceded by some theoretical remarks or clarifications that make the terminology more precise.)
- Third, one must refute the arguments for the opposite answer. (Pironet and Spruyt 2019, 2.2)

However, Hughes notes that “this canonical way of proceeding is abandoned by Buridan, who introduces two modifications in the procedure when dealing with the First Insoluble” (Hughes 1982, 80). And he does so in two ways: first, after stating the second argument against the sophism, Buridan immediately presents a reply to that argument, and then an objection to that reply, with the argument against the sophism being resumed later [in O3] (1.2.3. in Hughes 1982; O3 in Klima, 2001; pp. 182–183 in Scott, 1966).⁴ Second—and more importantly for our

³The concept of validity linked to signify is present in different authors throughout the Middle Ages, and has been understood in different ways, depending on what each author understands by ‘signify’. Two contemporary explanations—and different from each other—to the way of understanding the relationship between signify and validity are the one we find in Spade (1988) who calls this relationship “firmness”, and the one presented by Klima (2016) who calls this relationship “correspondence”. I owe this clarification to Stephen Read.

⁴“O.3 Again, it is not a valid consequence in which the consequent, if it were added to the true antecedent, would falsify it, for such a consequent appears to be incompatible rather than compatible with this antecedent. But this is the case here: for positing that ‘Every proposition is affirmative’ is true, if we add ‘No proposition is negative’, it will be false; therefore, the consequence is not valid.” (2001, 953)

present purposes between the statement of his own views and the replies to the opposing arguments Buridan introduces five ‘conclusions’. “Although these conclusions are formally an interruption of the standard pattern, *they contain the heart of what Buridan has to say here*” (Hughes 1982, 80; emphasis added). We will examine them to make clear what Buridan’s final stance was regarding the criterion of validity that he himself presented in his analysis of *PS*.

For all of the above, and for the sake of clarity, we will present Buridan’s arguments in three stages. First, we will introduce the so-called *Theory A* and we will survey the arguments with which Buridan analyzes it. After that, we will move on to *Theory B*, which Buridan at first seems to accept, but eventually rejects. We will finally address what we call *Theory C* (the first three arguments following the enunciation of *PS*). We contend that these arguments amount to Buridan’s reasons for defending the validity of *PS*. Indeed, they are not the only reasons he offers to that effect. To show that the inference is valid, he introduces the distinction between the *possible* and the *possibly-true*, a move that, due to its subtlety and ingenuity, has attracted the attention of most logicians. However, even if not as spectacular, the arguments we focus on here are still very important—insofar as they relate to the rest of the logical theory of the Middle Ages. At some point we will need to draw from other works by Buridan in order to understand its full scope and significance.

3. *PS* AND ITS COMPONENTS

To begin with, we must not forget that we are dealing with an argument, as opposed to other sophisms which consist of only a proposition—e.g., *The Liar*.⁵ *PS* is made up of two categorical propositions. The premise is possible, i.e., we can assume it to be true, since it is not impossible that all of the *propositiones*⁶ that exist at a given moment are affirmative. Buridan claims that “Every proposition is affirmative’

⁵Although in most cases “a sophism proper is a sentence rather than an argument” (Pironet and Spruyt 2019, 2.1), in this case it is an inference (the set of a sentence-premise and a sentence-conclusion). Therefore, if we look at the distinctive features of the sophism, we notice two different topics that appear to converge on its formulation: consequence and self-reference. What makes it particularly interesting is that, in Buridan’s treatment, *self-reference is used to point out, and respond to, certain criticisms of the notion of logical consequence*.

⁶See Hughes 1982, 5: “I use the Latin word *propositiones* to make it clear that the meaning of the term is the medieval sense of the term: a proposition is a sentence-token”; and Epstein 1992, 152: “In other words: For Buridan, a proposition is a specific linguistic entity, a sentence-token that is uttered or written at a specific

would be true if God annihilated all negatives, and then the consequent would not be true, for it would not be” (2001, O.2.1, 953).

PS's conclusion (*NPN*) has several interesting logical characteristics which require a more thorough analysis than its premise. We list some of them below, each followed by a brief explanation.

- (a) *NPN is always part of an argument.* Buridan always deals with this proposition in relation to the notion of logical consequence, and this is no accident. In his works, *NPN* always appears as part of an argument. In *Tractatus de Consequentibus* it functions as a premise, whereas in *Sophismata* it is the conclusion of the inference.⁷
- (b) *NPN is self-referential in an indirect sense.* That is to say, *NPN* is self-referential because it is an expression referring to a piece of discourse of which that proposition is a part. When such an expression appears in an argument, the argument itself is self-referential.
- (c) *NPN is syntactic in character.* The self-reference in *NPN* does not depend on the use of *true*, *validity* or any other semantic predicate.
- (d) *NPN is always false.* Roughly, Buridan argues that *NPN* cannot be true, since in order to be true it must exist, and if it exists, then there is at least one negative proposition—namely, *NPN* itself.⁸
- (e) *NPN does not behave in a classical way regarding immediate inferences.* If *NPN* is false, its contradictory (“Some propositions are negative”) should be true, and this is what happens. But

time.” In the rest of the text, whenever we talk about “propositions” we do so in this sense.

⁷(1) “No proposition is negative; therefore, no ass is running” (2015, 67); (2) “Every proposition is affirmative; therefore, no proposition is negative” (2001, 952). On (1), see Klima 2004; Dutilh Novaes 2005a; Read 2012; Johnston 2015. On (2), see Scott 1966; Prior 1969; Hughes 1982; Yrjönsuuri 2008; Uckelman 2012. For papers that deal with self-reference and validity but do not focus explicitly on *Sophismata*, see Read 2001 and Archambault 2017.

⁸“If propositions are atemporal, they exist timelessly, that is, there is no time at which they do not exist. So, whenever a timeless proposition is expressed by a temporally occurring sentence-token, then the proposition expressed by that sentence-token exists. Therefore, whenever I form a token of the sentence ‘No proposition is negative’ the proposition that no proposition is negative expressed by this sentence-token exists. But its existence entails that some proposition is negative, so the proposition cannot be true.” (Klima 2004, 98)

the inference from the truth of *NPN* to the falseness of its contradictory (“Some propositions are negative”) is not valid, since *NPN* can never be true. Neither can we consider the falsity of “Some propositions are negative”, because *that* proposition is always true (its own utterance makes it true that there is at least one affirmative proposition).

- (f) *NPN* (and other propositions of the same type) describe characteristics that apply to themselves, such as “being negative” or “being affirmative”. That is, they can describe facts about their own syntax without confusing the object language and the meta-language (see Uckelman 2012, 495).
- (g) *NPN* is a token, and different tokens of the same type can have different truth-values. Notably, this particular semantic feature allows every proposition to count as evidence for itself. In this way, we can allow that the token, considered as evidence, fulfills the task of informing the semantic content of the proposition. This gives us the representation that models the “scenario” where the inference takes place.⁹
- (h) *NPN* is possible but not true. *NPN* can be true about a state of affairs, but it cannot be true in a state of affairs—since, by (d), its mere existence renders it false. (see Prior 1966, sec. 2, 485–487)

4. *PS* AND THE CLASSICAL CONCEPTION OF VALIDITY (THEORY A)

What Read calls the Classical Conception of Validity (*CCV*) and Hughes calls *Theory A* “states that an argument is valid if and only if it

⁹“Each term is associated with a particular group of shapes, which it may be said to connote, though this means no more than that the presence on a sheet of marks of certain shapes will determine [...] whether or not sentences containing certain terms are to be counted as ‘true on their sheets’.” (Prior 1969, 483)

Definition 3.5 : The connotation of a term is defined as follows:

The term *propositio* connotes all \mathcal{L} -sentences.

The term *negativa* connotes all \mathcal{L} -sentences whose sign of quantity is *nulla* or whose copula is *non est*.

The term *affirmativa* connotes all \mathcal{L} -sentences which are not connoted by the term *negativa*. (Uckelman 2012, 491)

We can see, then, how a sentence can connote the set to which it belongs. “Some propositions are affirmative” is evidence of the fact that there exist affirmative propositions.

is impossible for its premises to be true while its conclusion is false”.¹⁰ Such is the definition of logical consequence that follows from adopting truth-preservation as the criterion of validity. In other words, a necessary condition for validity is that the premises cannot be true while the conclusion is false. And this was indeed the dominant view in the fourteenth century.¹¹

In *Tractatus de Consequentibus*, Buridan gives his definition of consequence and then introduces two additional definitions meant to improve upon it:

Hence, many say that of two propositions one is antecedent to the other if it is impossible for the one to be true without the other being true, and one is consequent to the other if it is impossible for the one not to be true when the other is true, so that every proposition is antecedent to every other proposition for which it is impossible for it to be true without the other being true. (2015, 67)

We know that the point of *PS* was to test the adequacy of *CCV*. According to Hughes, Buridan argues that *Theory A* cannot be correct, and then proceeds to use “the established validity of the sophism as a reason for rejecting *Theory A*” (1982, 82). Buridan puts it this way:

O.2. Again, a consequence is not valid if the antecedent can be true without the truth of the consequent. But this is the case here, for since the antecedent can be true and the consequent cannot be true, it is clear that the antecedent can be true without the truth of the consequent. (2015, 67)

But is *PS* a valid argument? To establish its validity we need to start from the fact that both premise and conclusion have the same meaning. In order to see that, as noted by Sara Uckelman (2012, 489), it suffices to replace the quantifiers according to well known equivalences of first-order logic, so that, analytically, $\neg A(x)$: ‘*x* is not affirmative’ entails

¹⁰“An inference is valid if and only if it is impossible for the premise(s) to be true without the conclusion also being true. Let us call this Theory A.” (Hughes 1982, 80)

¹¹“Validity itself was truth-preservation. So too in the English tradition: in Ockham, for example, we find many divisions of consequence, but what is common to them is that in a valid consequence the premises cannot be true without the conclusion.” (Read 2020, 283)

$N(x)$: ‘ x is negative’.¹² If *OPA* and *NPN* have the same meaning, then the inference from one to the other is a material consequence. Indeed, from the perspective of *CCV*—in its ‘representational’ or ‘modal’ variety (see Read 1994; Etchemendy 1990) there cannot be a scenario in which all propositions are affirmative but it is not the case that no proposition is negative.¹³ We know that this kind of consequences, in which validity depends on the meaning of the terms and on meeting *CCV*, were considered valid by medieval logicians. Buridan, in particular, takes them to be materially valid consequences.¹⁴ And if a consequence is materially valid, it is possible to turn it into an enthymeme (of a valid form) by adding a necessary proposition, which makes it formally valid (as is the case with *PS*, as we will see in more detail later). For now, it is enough to say that, in addition to seeming intuitive, *PS* appears to be a formally and materially valid argument, notwithstanding the fact that it has a true premise and a false conclusion. That is the most interesting point, since the argument is invalid according to *CCV* (*Theory A*); but, as we’ve seen above, its validity is one of Buridan’s reasons to reject *CCV*. The First and Second conclusions express this most clearly:

¹²This is founded on the Aristotelian conception presented in *Categories*, which entails assuming that things have opposites, that is, “that those things are contraries which, within the same class, are separated by the greatest possible distance” (*Categories*, ch. 6). Although opposites only apply to objects, for medieval logicians, propositions are exactly that: physical objects.

¹³“John Etchemendy (1990) contrasts ‘interpretational’ with ‘representational semantics’. In representational semantics we describe a situation, perhaps different from how things actually are, in which the propositions take various values. In interpretational semantics, we interpret certain expressions differently from their actual interpretation to much the same effect. [...] A more perspicuous terminology might be to speak of substitutional semantics on the one hand, where we substitute different expressions within a substitution-class for certain expressions, to see if truth results; and of modal semantics on the other, evaluating the statements in different possible situations. [...] Nonetheless, provided it is properly understood, the term ‘representational’ does significantly demarcate a semantic approach in which the interpretation of the language is fixed while the situations represented vary, different from one where one varies the interpretation within a fixed world.” (Read 1994, 248–249)

¹⁴“A material consequence, however, is one where not every proposition similar in form would be a good consequence, or, as it is commonly put, which does not hold in all terms retaining the same form; e.g., ‘A human is running, so an animal is running,’ because it is not valid with these terms: ‘A horse walks, so wood walks’.” (John Buridan 2015, 68)

First conclusion

The first conclusion is that some consequence is valid whose antecedent can be true without the truth of the consequent and without the consequent. For this is a valid consequence: ‘A man runs; therefore, an animal runs’, and yet the first [proposition, i.e., the antecedent of this consequence] would be true, even if the second [proposition, i.e., the consequent of the same consequence] did not exist but were annihilated.

Second conclusion

The second conclusion is that *the antecedent of a valid consequence can be true while its consequent cannot be true.* And this is clear in the present case: for ‘Every proposition is affirmative’ can be true and ‘No proposition is negative’ cannot be true, and yet the second follows from the first. And the same would hold in many other cases, for example, ‘Every syllable is several letters; therefore, no syllable is a single letter’. (2001, 954; emphasis added)

The moral of all this—as it pertains to the present work—is that Buridan comes to reject *CCV* because he considers *PS* to be valid; and that means there is at least one valid argument with true premises and a non-true conclusion (see Read 2020; Klima 2004; Klima 2016; Dutilh Novaes 2005a). Here Buridan is challenging the truth-preservation criterion.¹⁵

5. *PS* AND *Theory B*

Theory B expresses the definition of consequence that in *Tractatus de Consequentibus* comes immediately after—and is offered as a refinement or elaboration on—*Theory A*:

So some say the given definition should be supplemented like this: the one proposition is antecedent to the other proposition if it is impossible that it be true the other not being true when they are formed together. (2015, 67)¹⁶

¹⁵More schematically, the argument is as follows: Either every argument with true premises and a false conclusion is invalid, or *CCV* is inadequate. But there is a valid argument *PS* with a true premise and a false conclusion. Therefore, *CCV* is inadequate as a criterion for validity.

¹⁶This is why Buridan considers the previous definition to be “defective”, although he uses another example (see 6.3 below).

The main difference between *Theory A* and *Theory B* is that the latter adds the clause “when the premise(s) and conclusion are formulated at the same time (*ipsis simul formatis*)”. This idea depends on the medieval conception of propositions as tokens.¹⁷ Simultaneous valuation requires that both propositions exist at the same time.¹⁸ This is how Buridan states it in *Sophismata*:

But it is replied that a consequence is not said to be valid because the antecedent cannot be true without the truth of the consequent, *but because it cannot be true without the truth of the consequent when they are formed together*—but this is the case here. (2001, 95; emphasis added)

The problem is that *PS* falls short of this added requirement. Indeed, “it is essential to the notion of validity of an argument that it guarantees to take one from truth to truth” (Read 2001, 183). But in this case whenever the antecedent (*OPA*) is true, the consequent (*NPN*) is false; whereas if *NPN* is assumed to be true (and, accordingly, is assumed to

¹⁷“It is important to realize, however, that all such expressions, including mental expressions, are for Buridan concrete contingent particulars. They exist only if they are uttered or inscribed (spoken or written down) or thought. In particular, things might be as some proposition signifies (e.g., Buridan might be running) but unless someone says or thinks that Buridan is running, the proposition that Buridan is running would not be true, since it would not exist.” (Read 2015, 7)

¹⁸Languages in which the word ‘now’ occurs (implicitly or explicitly) are of two types: those in which the presence or absence of the word ‘now’ works vacuously, i.e., those in which ‘now’ makes no difference to establishing the truth value of the sentence; and those in which the presence or absence of the word ‘now’ does not work vacuously, i.e., in which it is not indifferent to establishing the truth value of proposition. The language of medieval logic is a language that is not vacuous with respect to the particle ‘now’, which is implicitly present in all sentences where the copula ‘is’ occurs in the present tense. (Dahlquist 2021, 380)

exist), it immediately entails the falsity of *OPA*.¹⁹ Buridan also offers another example, which obviously applies to *PS* as well.

Contra: if a consequence were said to be valid for this reason, then it would follow that this consequence is valid: ‘No proposition is affirmative; therefore, a stick stands in the corner’, for it is impossible, if these are formed together, for the antecedent to be true; and if it cannot be true, then it follows that it cannot be true without the consequent. (2001, 953)

The premise of this argument states that there are no affirmative propositions, but its conclusion is an affirmative proposition (‘a stick stands in the corner’), so if the premise were true, the conclusion could not be true, at least not simultaneously. In the Fifth Conclusion Buridan elaborates on the insufficiency of *Theory B*:

Fifth conclusion

The fifth conclusion is that for the validity of a consequence *it does not suffice for it to be impossible for the antecedent to be true without the consequent if they are formed together*, as has been correctly argued above about the stick in the corner. (2001, 955; emphasis added)

However, in the second part of the conclusion Buridan adds a new element:

And this is also obvious from another example, for this is not valid: ‘No proposition is negative; therefore, no proposition is affirmative’. *And this is clear because the opposite of the consequent does not entail the opposite of the antecedent*. Yet, the first cannot be true without the truth of the second, for it cannot be true.

¹⁹Existence of the propositions will not affect inferential validity in the ordinary way, though it will affect truth. As Buridan notes in the Fourth Conclusion:

[...] in a valid consequence it is impossible for the antecedent to be true without the truth of the consequent if they are formed together. This notwithstanding, however, it has to be conceded that something true can entail something false. For let us posit that the following proposition is true: ‘Every proposition is affirmative’; then it can entail a false one, namely, ‘No proposition is negative’. But when this is concluded, the former is no longer true, but false. (2001, 953)

Therefore, something more is required, namely, that things cannot be as the antecedent signifies without being as the consequent signifies. (2001, 955; emphasis added)

In *Tractatus de Consequentibus* Buridan gives an example that is similar to the one in *Sophismata*:

But I say that this definition is even now not good, because “No proposition is negative, so no ass is running” is not a good consequence, [p. 22] but according to the second definition given one must concede that it is good. *I prove the main claim because the opposite of the antecedent does not follow from the opposite of the consequent*, that is, this does not follow: some ass is running, so some proposition is negative. (2015, 67; emphasis added)

In both cases, Buridan’s objections have two things in common: (1) the premise and the conclusion cannot be true at the same time, and (2) they do not comply with the rule of Contraposition. Read explains it as follows:

One might, therefore, suggest that consequence is valid if the premise cannot be true without the conclusion’s being true, when both are formed together. That will not work, says Buridan: ‘No proposition is negative, so no ass is running’ is not valid (for its contrapositive is not valid), but the premise cannot be true, and so cannot be true without the conclusion’s being true. (2012, 906)

We draw attention to this point because, as we will see in the next section, *PS* cannot have a true premise and a true conclusion at the same time, but neither does it comply with the rule of Contraposition. This is one of its most interesting characteristics, as well as one of Buridan’s main reasons to conclude that the argument is valid.

6. THEORY C: BURIDAN’S (FORGOTTEN) FIRST THREE ARGUMENTS FOR THE VALIDITY OF *PS*

At the very beginning of his analysis of *PS*, Buridan gives three arguments in favor of its validity. Our goal is to show that these arguments—which, together, we call *Theory C*—allow for *PS* to be considered valid for reasons of logical theory, albeit not intuitively. Since Buridan rejects *Theory A* but accepts *Theory C*—which leads to paradoxes—he is left

with no other way out than to offer a new criterion of validity. We will present and examine his arguments one by one.

P.1: This [that is, *PS*] is proved first by the locus from contraries.

For just as ‘Every man is ill; therefore, no man is healthy’ is valid because it is impossible for the same [person] to be both ill and healthy, so is the above, because it is impossible for the same proposition to be both affirmative and negative.

P.2: Again, an enthymeme is valid if by the addition of a necessary proposition it can be completed into a formally valid syllogism—for it is by such additions that we usually prove our enthymemes. For example, we say that this is a valid consequence: ‘A donkey flies; therefore, a donkey has wings’, for this is necessary: ‘Everything flying has wings’, and if we add this as the major, then we get a valid syllogism in the third mode of the first figure. So also in connection with the sophism, this is true: ‘No affirmative is a negative’, and if this is made the major in this enthymeme, then we shall get a valid syllogism, in the second mode of the first figure.

P.3: Again, the opposite of the consequent entails the opposite of the antecedent; therefore, the consequence is valid. For this rule is common to every valid consequence.

P.3.1: But the antecedent [suggested in P.3] is obvious: for ‘Some proposition is negative; therefore, not every proposition is affirmative’ is obviously valid. (2001, 952)

Thus, the validity of *PS* is established by the following arguments: by the locus from contraries (P.1); by the possibility of generating an enthymeme (P.2); and by the rule of Contraposition (P.3). As Hughes puts it, “The arguments in favor of the sophism (which Buridan clearly accepts) are straightforward, but some comments on matters of detail seem to be called for” (1982, 80). By going into such comments we seek to make it clear that accepting *Theory C* is incompatible with rejecting *Theory A* or, at least, that the price of doing so is running into paradoxes.

6.1. **P.1: The locus from contraries.** In discussing Buridan’s first argument, Uckelman points out that

The first one turns on an equivalence representable in modern notation as

$$\forall x(Px \rightarrow Ax) \equiv \neg \exists x(Px \wedge \neg Ax)$$

This is just the interdefinability of the quantifiers. (2012, 489)

This is fine, but it should be noted that *interdefinability is not enough*; justification is not complete without accepting that there are contrary predicates, i.e., pairs of predicates such that if one is a predicate of one thing, the other is excluded from that possibility; only then can we accept that Non-Affirmative is equivalent to Negative. This addition makes only the first part of the argument purely logical. In other words, the two sentences cannot be equated in an entirely extensional way; to establish that it is *bona inferentia* we need to consider the meaning of the contrary predicates ‘affirmative’ and ‘negative’; e.g., we must take into account certain intensional characteristics of the components of the expression (understanding that the intension of an expression explicates what we understand when we understand it). If we grant that, then not only can either proposition be inferred from the other one, we can go one step further: we can claim that *OPA* and *NPN* have the same meaning.²⁰ These reasons make it clear that the argument is intuitively valid. More importantly, they show that we are dealing with a material consequence (as we argued in section 4), since that is exactly what a material consequence is.

A material consequence, however, is one where not every proposition similar in form would be a good consequence, or, as it is commonly put, which does not hold in all terms retaining the same form; e.g., “A human is running, so an animal is running,” because it is not valid with these terms: “A horse walks, so wood walks.” (John Buridan 2015, 68)

PS is materially valid, that is, its validity depends not on its form alone, but on the content of (some of) its terms. *PS* is valid on account of the meaning of ‘affirmative’ and ‘negative’.²¹ If “not all valid arguments are valid in virtue of form, but are materially valid,” then knowing that *PS* is materially valid is extremely important. But what about its *formal*

²⁰Inference *PS* is basically an instance of the syllogistic law of so-called obversion stating that a universal affirmative proposition is equivalent to a universal negative proposition with negated predicate. This makes the propositions syllogistically equivalent, and in this sense they “mean the same thing”. I am aware that some people would insist that logically equivalent propositions may still have (slightly) different meanings (although I personally don’t think that is the case here). I owe this point to Dr. Wolfgang Lenzen.

²¹We are paraphrasing Read 1994, 250.

validity? That is dealt with in Buridan's next argument—although he does not make it explicit in *Summulae de Dialectica*.

6.2. P.2: Syllogism and formal consequence. Buridan's second argument is very interesting, since it connects the material validity of *PS* with its formal validity. The latter has its own characteristics for the Paris school:

During the fourteenth century, two doctrinally quite clearly separable traditions developed: Oxford and Paris. *The main doctrinal difference in question is that whereas the Parisian tradition tied the notion of formal validity to truth-preservation under all substitutions of non-logical terms*, the English tradition (in line with the earlier Parisian tradition from before the fourteenth century) required a containment principle, often described in psychological terms (requiring that the understanding of the antecedent should contain the understanding of the consequent). (Klima 2016, 318)

In the second argument Buridan provides the method for transforming *PS* into a syllogism. But what does it mean that we can syllogize? It means that there is a formal consequence, since “the syllogism was brought under a general theory of consequence. Buridan, in particular, treats the syllogism as a special case of formal consequence” (Read 2012, 908). The procedure is straightforward: all we need to do is add the necessary sentence *Nulla affirmativa est negativa* to get a syllogism in *Celarent*.

Nulla affirmativa est negativa
Omnis propositio est affirmativa
 Nulla propositio est negativa

We can add something else (derived from Prior's analysis) to further strengthen the argument: *Nulla affirmativa est negativa* is always true

(or true on all sheets).²² As we saw before, *NPN* is always false.²³ This connects with the previous argument (P.1), since another way of proving the formal validity—albeit not mentioned in *Summulae de Dialectica*—is related to Buridan’s method for transforming material consequences into formal consequences: “Simple material consequences reduce to formal consequences by adding a necessarily true premise” (Read 2012, 905). We have already shown that *PS* is a material consequence, so we can follow the rule given by Buridan himself in *Tractatus de Consequentibus* to get a formal consequence, i.e., to reason enthymematically:

It seems to me that no material consequence is evident in inference except by its reduction to a formal one. Now it is reduced to a formal one by the addition of some necessary proposition or propositions whose addition to the given antecedent produces a formal consequence. E.g., if I say “A human is running, so an animal is running,” I will establish the consequence by adding that every human is an animal; for if every human is an animal and a human is running, it follows in a formal consequence that an animal is running. For everyone arguing enthymematically endeavors to prove his consequences in this way if they are not formal. (2015, 68)

Following Dutilh Novaes, we can claim that if *PS* is a formal consequence, then it can be considered a logical consequence as well.

Is formal consequence equivalent to logical consequence?—
So far I have been considering the two notions as roughly equivalent; in particular, I claim that Buridan’s notion of formal consequence is a very good model of the intuitive notion of logical consequence. (Dutilh Novaes 2005b, 122)

²²“We may say similarly that a sentence is *necessary* on the sheet on which it occurs if it is true of every sheet, and that it is *necessarily-true* on any sheet on which it occurs if it is true on every sheet on which it occurs. Whatever is in this sense necessary on a sheet is necessarily-true on that sheet, but not always vice versa. For example, 4. *Quaedam propositio est affirmativa* is true on any sheet on which it occurs (its presence there being its own verification), and so is “necessarily-true” on any sheets on which it occurs, but it would fail to be “necessary” in any set of sheets which included one on which nothing was written but, say, *Nulla affirmativa est negativa*; for *Quaedam propositio est affirmativa* would not be true of that sheet, and so not true of all sheets, and so not necessary.” (Prior 1969, 487)

²³In this context, the necessity for *NPN* (*Nulla affirmativa est negativa*) comes from its condition of being the negation of a tautology (*Quaedam propositio est affirmativa*), that is, *NPN* is a contradiction.

Thus, P.2 guarantees that between *NPN* and *OPA* there is, at least, a formal consequence. Furthermore, if Buridan is correct when he opines (in the quote above) “It seems to me that no material consequence is evident in inference except by its reduction to a formal one,” then the undeniable formal validity of *PS* is an argument in favor of its material validity.

6.3. P.3: The rule of Contraposition. Buridan’s third and final argument for the validity of *PS* is based on the fact that some inferences comply with a certain logical rule. Fourteenth-century logicians used rules to formulate a theory of inference,²⁴ but they also used rules to investigate the nature of logical consequence and inference.

But 14th-century logicians were probably the first to attempt a systematization of the propositional rules of inference that we now take as fundamental, such as contraposition, *ex falso*, the behavior of conjunctions, disjunctions, etc. Medieval logicians sought not only to establish the validity of such basic rules; they also made inquiries on the very nature of logical consequence and inference. In this sense, their investigations overlap not only with modern ‘proof theory’, but also with modern philosophy of logic (as exemplified by modern discussions on the nature of logical consequence such as [Etchemendy, 1990]). (Dutilh Novaes 2008, 468)

For these reasons, we must think of the rule of Contraposition, when presented by medieval logicians, in terms of what (the young) Gentzen himself understood as a structural rule, that is, a rule linked to logical consequence.

By structural reasoning I mean reasoning based on structural rules which affect the way formulas are arranged in consequence relations expressed by sequents, without reference to their internal articulation by means of logical connectives and quantifiers. (Moriconi 2015, 662)

²⁴“The philosophical achievement realized in these various writings was no less than a formulation of a theory of inference: the rules for consequences given by these mediaeval authors spell out a natural deduction system in the sense of Jaśkowski and Gentzen.” (King 2001, 117)

As we saw, the third argument in favor of the validity of *PS* is that it complies with the rule of Contraposition.²⁵ Buridan considers that “this rule is common to every valid consequence” (see above, section 6, P.3), and this is by no means the only place in his work where he makes such a claim. In *Tractatus de Consequentibus, Conclusion of Book I, Consequences in General and Consequences between Assertoric Propositions*, we read:

Third Conclusion: In every good consequence, the contradictory of the antecedent must follow from the contradictory of the consequent, and every proposition formed as a consequence is a good consequence if the contradictory of the antecedent of the said consequence follows from the contradictory of the consequent of the said consequence. (John Buridan 2015, 76)

Moreover, when discussing “No proposition is negative, so no ass is running” (2015, 67), Buridan says it is invalid, because it does not comply with the rule of Contraposition (see Dutilh Novaes 2005a; Klima 2004; Read 2012).

“No proposition is negative, so no ass is running” is not a good consequence, [p. 22] but according to the second definition given one must concede that it is good. I prove the main claim because the opposite of the antecedent does not follow from the opposite of the consequent, that is, this does not follow: some ass is running, so some proposition is negative. (John Buridan 2015, 67)

²⁵“The contradictory of the antecedent must follow from the contradictory of the consequent, and every proposition formed as a consequence is a good consequence if the contradictory of the antecedent of the said consequence follows from the contradictory of the consequent of the said consequence.” It is a rule expressed by Buridan and countless medieval logicians, and called interchangeably contraposition or contrapositive. To avoid confusion—especially with contemporary formulations—here we will call it the rule of Contraposition and it responds to the definition that we have just presented.

As we saw above, another of the examples submitted for scrutiny and marked as invalid is “No proposition is negative; therefore, a stick stands in the corner”, which also does not comply with the rule. Although the previous quotes should make it sufficiently clear, we would like to call attention to the extreme generality that this rule possesses.²⁶ In other words, *no bad inference complies with the rule of Contraposition*, which must be fulfilled by “every good consequence” (i.e., the rule of Contraposition is a necessary and sufficient condition for validity). This is confirmed, to some extent, by the fact that Buridan argues in favor of the validity of *PS* by claiming that its contrapositive (‘Some proposition is negative; therefore, not every proposition is affirmative’) is a valid inference (P.3.1.).

Summing up, Buridan’s three arguments for the validity of the insoluble are as follows: by P.1 we can affirm that in *PS* there is a material consequence between premise and conclusion; by P.2 we can claim that there is both a formal and a material consequence (and, if Dutilh Novaes is right, a logical consequence); by P.3 we can claim that *PS* meets the most general rule for an inference to be valid—the rule that expresses the necessary and sufficient conditions for validity.

7. TWO PARADOXES

In the last section we showed that from what Buridan says in *Theory C* it follows that *PS* is a material and formal consequence (which makes it a valid consequence). In what follows we will show that rejecting *Theory A* and accepting *Theory C*—accepting *NPN* as a valid consequence of *OPA*—leads to a paradox.

What we need to ask in each of these cases is: What is the relationship between *Theory A*—which Buridan rejects based on the validity of

²⁶We know that establishing a system of rules was the natural way of expressing the notion of consequence among the medievals. Furthermore, there is a hierarchy among the rules: they go from the most general to the most specific. As Boh says:

How did men come to possession of principles of logic, such as rules of consequences? Buridan says that others had treated the consequences in a posteriori manner but that he was investigating the “cause” of the validity of laws of inference. (see Moody 1953, 8.) In fact, his *Tractatus de Consequentibus* is a quasi-deductive system of rules with a clear distinction between principal rules and those derived from them. His system of principles of deduction, however, is restricted to assertoric (propositional) and alethic modal areas. (2001, 171; emphasis added)

PS—and the reasons given in P.1–P.3 (i.e., *Theory C*) that he takes as demonstrating the validity of *PS*?

Let's begin with P.3 and the rule of Contraposition. This already generates a paradox. According to Dutilh Novaes, both *CCV* and the rule of Contraposition define the notion of logical consequence, since the latter is simply “another formulation of the same idea.”

Now, *as for the general definition of consequence, most authors of the 14th century accept at least as a necessary condition for a (valid) consequence that the antecedent cannot be true while the consequent is false; many accept this as a sufficient condition as well. This is, of course, the very familiar modal definition of consequence, present in Aristotle and also widely accepted in current (philosophy of) logic (at least as a necessary condition). Another formulation of the same idea is that a consequence is valid if from the contradictory of the consequent the contradictory of the antecedent follows [Ockham, 1974, 728], or similarly that the contradictory of the consequent is incompatible with (the truth of) the antecedent [Burley, 2000, 149]. In fact both Ockham and Burley give several equivalent formulations of this core idea, but the key point is obvious: for our authors, the most fundamental characteristic of a consequence is that the truth of the antecedent is incompatible with the falsity of the consequent. (Dutilh Novaes 2008, 472; emphasis added)*

Dutilh Novaes is right, and these two equivalent formulations can be found in the Middle Ages as well as among our contemporaries. Let us see two examples of the first one:

The first rule of inferences is this: In every good simple inference, the antecedent cannot be true without the consequent. (Burley 2000, 3)

... consider any class *K* of sentences and a sentence *X* which follows from this class. From an intuitive standpoint, it can never happen that both the class *K* consists of only true sentences and the sentence *X* is false. (Tarski 1983, 414–15)

We can add two examples of the second formulation of the same idea, according to the rule of Contraposition.

A solid inference is that in which the opposite of the consequent is repugnant to the antecedent. (Paul of Venice 1986, 167)

An argument is valid only if it would be inconsistent (or self contradictory) to assert the premises while denying the conclusion; or, in other words, only if the truth of the premises is inconsistent with the falsity of the conclusion. (Strawson 1952, 2)

From all this it follows that *Theory A* and *Theory C* are equivalent. But if we mark *Theory A* as incorrect and accept *Theory C* as good, which is what Buridan does, then the following paradox arises:

Paradox 1. If *Theory A* and *Theory C* are equivalent (in virtue of their being different representations of the same notion of logical consequence), then both should entail that the same inferences are valid. In other words, the sets of valid inferences according to each theory should have the same extension. However, *PS* is classified as a valid inference by *Theory C* but is not valid according to *Theory A*, since it has a true premise and a false conclusion. Therefore, there is a sentence that both is and is not a consequence of what is basically the same theory. Hence, the contradiction and, by *reductio*, the paradox.

We will now address P.1 and P.2 and their relationship to *Theory A* in order to show that this also leads to a paradox. Issues of space prevent us from going into further details about the thesis that need to be assumed for this part of the argument, but interested readers can find a clear defense in Read 1994. Basically, valid consequences are of two kinds, formal and material. The former hold for arguments that are valid by virtue of their form (which, in turn, is established by the logical components of the language), whereas material consequences are characterized thus: (Thesis 1) an argument is materially valid if and only if “its validity depends not on any form it exhibits, but on the content of certain expressions in it” (Read 1994, 250). So, given Thesis 1, arguments like *Iain is a bachelor; so Iain is unmarried* are valid.

Although among our contemporaries there are those who refuse to consider material validity as a topic to be addressed by logic, this was not the case in the Middle Ages, when inferences like *A human is running, so an animal is running* were common examples of logical consequence. As we know, Buridan admits material consequences, which he distinguishes from formal consequences (see *Tractatus de Consequentibus*, Chap. 4). Our second thesis affirms that material consequences are logically valid arguments. (Thesis 2) “Not all valid arguments are valid in

virtue of form, but are materially valid” (Read 1994, 251). And this is so because materially valid arguments meet the necessary and sufficient conditions for validity, that is, they meet *CCV*. In a sense, formal validity presupposes material validity.²⁷

Finally, Thesis 3 states that “An argument is valid if and only if there is no possible situation where the premises are true and the conclusion is false” (Read 1994, 256).

Let us now turn to P.1 and P.2. Buridan holds that *PS* is a materially valid argument because it can be made into a syllogism through the addition of a necessary premise. Yet, Buridan knows (as does Read) that this procedure does not *transform* an invalid argument into a valid one.²⁸ “The extra premise adds clarification, but it does not serve to turn an invalid argument in a valid one” (Read 1994, 262). Buridan understood this in the same way; the addition of a premise serves no other purpose than making it evident that there is the valid material consequence:

It seems to me that no material consequence is evident in inference except by its reduction to a formal one. Now it is reduced to a formal one by the addition of some necessary proposition or propositions whose addition to the given antecedent produces a formal consequence, e. g., if I say “A human is running, so an animal is running,” I will establish the consequence by adding that every human is an animal; for if every human is an animal and a human is running, it follows in a formal consequence that an animal is running. (John Buridan 2015, 68)

Summing up, the most important conclusion that we can draw from P.2 is that *PS* is materially valid. On the other hand, P.1, which establishes the validity of *PS* by the *locus* from contraries, also indicates in a less technical (or more intuitive) way the material validity of *PS*. This is so because every *locus* is the expression of a predicative relationship between terms of two or more propositions, with respect to some subject

²⁷“I adopt here the view that there are such things as ‘material consequences’, which are in fact the primitive kind of consequences, and that formal consequences are a particular subset of the set of valid consequences.” (Dutilh Novaes 2005a 281)

²⁸“The extra premise is strictly redundant. For if the original argument were invalid, the added premise would not be logically true. Given that it is logically true, it follows that the unexpanded argument was already valid. Hence it was (logically) unnecessary to add the extra premise.” (Read 1994, 259)

matter.²⁹ ‘Contraries’ here refers to a relationship between terms (not between propositions) and this is, in essence, the material consequence (i.e., the relationship between ‘bachelor’ and ‘unmarried’ in the example above). In this way, P.1 also indicates that *PS* is a (valid) material consequence.

Paradox 2. In *Summulae de Dialectica* Buridan uses a vocabulary typical of those who adhere to the representationalist conception of logical consequence.³⁰ For him, as for Read, material validity is basic and, in a sense, presupposes formal validity. *PS* complies with *CCV* by virtue of being a materially valid consequence. But if it complies with *CCV*, there can be no situation where its premises are true and its conclusion is false. However, in a situation where there are no negative propositions, *PS* has a true premise and a non-true conclusion. Therefore, it does not comply with *CCV*. So, *PS* complies with *CCV* and does not comply with *CCV*; by *reductio*, we have a paradox.

8. CONCLUSION: FIRMNESS OR PARADOX

We believe that Buridan—like other logicians of the Middle Ages—was aware that the problems generated by self-reference for the notion of validity could not be solved within the framework of a notion of validity as truth-preservation (*CCV*). According to Klima, it is a notion that Buridan intends to abandon for a more general one, which can account for problems such as those raised by *PS*.

Take the proposition ‘No proposition is negative; therefore, no donkey is running’. The antecedent of this consequence is a negative proposition, whence it cannot be true. But then, it is not possible for the antecedent to be true and the consequent not to be true; therefore, it would seem that the consequence is valid. However, it is certainly a possible situation in which there are no negative propositions (as was actually the case, for example, before the first human being formed the first negative proposition in the first human language), in which, however, some donkey is running, which would be precisely

²⁹“Propositions involving different kinds of predicative relation and different kinds of subject matter will necessitate different kinds of advice; the basic unit of advice is referred to as a topic, or, in the Latin, a *locus*.” (Wilks 2008, 122)

³⁰“And this is also clear, because ‘Every proposition is affirmative’ would be true if God annihilated all negatives, and then the consequent would not be true, for it would not be.” (John Buridan 2001, 953, emphasis added)

the scenario that would have to be excluded by the consequence in question, if it were valid. So, *Buridan reformulates the requirement for the validity of a consequence in terms of the correspondence-conditions* of the propositions it involves. (Klima 2016, 320–321; emphasis added)
 31

However, this does not imply that he completely rejects the idea that a consequence is valid if and only if it is impossible that the antecedent be true without the truth of the consequent. Buridan, as the rest of the fourteenth century logicians, considers this notion of validity to be adequate. As Ashworth notes:

Third, when we consider validity, it turns out that we need to distinguish between truth, which can only belong to an existent proposition, and the possibility that things are as signified by a proposition. Otherwise, we will be trapped into rejecting such valid consequences as “Every proposition is affirmative, therefore no proposition is negative,” for if the antecedent is true, the consequent cannot be true, yet it is impossible for things to be as signified by the antecedent without being as signified by the consequent (SD *Sophismata*, c. 8, First sophism, 952–956, *SDSOPH*, 140–143). Sometimes, however, Buridan does present the definitions “‘*A* therefore *B*’ is valid if and only if it is impossible for *B* to be false when *A* is true” and “‘*A* therefore *B*’ is valid if and only if it is impossible for things to be as *A* signifies without things being as *B* signifies” simply as alternatives (*QANPR* II, q. 5). Klima has argued that Buridan needs only the satisfaction of correspondence conditions, and so has a logic without truth, writing “Buridan’s logic *does not have and does not need* a definition of truth” (2009, 225). This may be so, but Buridan was clearly quite happy to

³¹“Therefore, some give a different definition, saying that one proposition is antecedent to another, which is such that it is impossible for things to be altogether as it signifies unless they are altogether as the other signifies when they are proposed together” (John Buridan 2015, 67). In this same sense, he expresses it in the fifth and final conclusion derived from *PS*’s analysis in *Sophismata*; see sec. 5.

speak of truth in accordance with conventional language.
(Ashworth 2017, 247–248)³²

We would like to suggest that the PS problem, and Buridan’s proposed solution (well stated in the two quotes above), is ultimately a problem about the very nature of logic. Can there be more than one valid notion of consequence? Can there be cases that escape the intuitive notion of logical consequence? Can the laws of logic have counterexamples (i.e. a restricted scope)? In other words, giving an answer to the first insoluble, can be the same as giving an answer to the previous questions, and this is nothing more than to raise in the framework of the current controversy between monists, pluralists and nihilists in the framework of the philosophy of logic.³³ This will be one of our future investigations.

Finally, let us add that the case of *PS* can be considered (and this increases its logical value) as an analogous case other than that which Read points out in Roger Swyneshed, in which truth, validity, and signify are linked:

In his treatise on insolubles, that is, the logical paradoxes, written in Oxford in the early 1330s, Roger Swyneshed, one of the Oxford Calculators, put forward three surprising, even paradoxical, claims:

- There is a false proposition which principally signifies as things are;
- There is a formally valid inference with true premises and false conclusion;
- There is a pair of contradictory propositions both of which are false.

The third claim, (3), if correct, is a counter-instance to the Rule of Contradictory Pairs (RCP), that of any pair of mutually contradictory propositions, one is true and the other false. (Read 2020, 276)

³²Dutilh Novaes (2005a, 296) expresses the same idea: “Buridan himself says that, in practice, the final definition of consequence is needed in only a few cases; for most cases, the familiar definition in terms of truth-values is perfectly sufficient.”

³³Cotnoir (2018, 302):

Logical Monism: There is exactly one logical consequence relation that correctly represents natural language inference.

Logical Pluralism: There is more than one logical consequence relation that correctly represents natural language inference.

Logical Nihilism: There is no logical consequence relation that correctly represents natural language inference.

We can rewrite those claims in terms of *PS*, assuming that in this sophism the premise and the conclusion have the same meaning (as we showed in section 6.1):

- There is a false proposition which principally signifies as things are, (*No proposition is negative* or *NPN*);
- There is a formally valid inference with true premises and false conclusion, *Every proposition is affirmative; therefore, no proposition is negative* (*OPA*; therefore *NPN*);
- There is a pair of equivalent propositions (whose *significata* are equivalent) that have different truth-values.

Thus, we have here three statements that are as surprising as the ones proposed by the Oxford Calculators, but which derive from the work of our Master of Arts in Paris.

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