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Some Pioneering Formal Reconstructions of Diodorus' *Master Argument*

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The article deals with some current pioneering formal reconstructions and interpretations of the problem well known in antiquity as *The Master Argument*. This problem is concerning with enrichment of formal logical systems with modal and temporal notions. The opening topic is devoted to reconstruction of Arthur Prior, while the other here included approach to the problem are mostly reactions, revisions or additions to this one.

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The Age of Prior - The Period of Rising of the Logic of Time

The modern history of Diodorus' M.A. has its starting point in increasing interest for topics in Ancient logic, especially that of Aristotle and Stoics. These firsts starting steps are belonging to Łukasiewicz, and his investigations he undertook during 20's. These attempts correspond with Lewis' investigations in the field of modal logic, especially in differing of the formal and material implication. This problem, intentionally or not, was the far echoes of antics debate on conditionals passing between Philo and Diodorus.

However, the first systematic approach to M.A. could be ascribed to Benson Mates. In an article on the nature of Diodorus' implication [1949], later revised and included in his study on the logic of Stoics [1961], he is following Łukasiewicz's traces, especially in the question of method that Łukasiewicz sketched earlier and applied in investigations on Aristotle's and Aristotelian logic. In his trying to give a review of Stoic achievements in logic Mates had in mind needful for modern understanding in the field. His persuasion was also grounded on believing that without modern logical tools it is impossible to understand as well as to represent the real nature of Stoic logic. The Megarian logic, and so logic of Diodorus, in its attempt covered as one stream belonging to logic of Stoics, was usual approach in many up to date commentaries. Even histories of logic appeared afterwards, including here Boheński [1961] - the next known name in investigating the history of logic - does not yet make obvious distinction between two schools, the Stoic and the Megarian. During that period, experiences of these logics are usually assorted under the frame of unique term – as the Megarian-Stoic period in logic.

The short Mates' article was followed by Prior's reaction [1955], and there he is primarily analyzing Diodorean definition of modality. This article was in fact the first modern attempt in developing such logic that works, by analogy to modal logic, with operators equipped with time relation components. However, the idea was older, and Prior himself was influenced with an article of Findley [1941][1] on the relation of time and logic. The idea was due to one short suggestion given in the footnote, where he is claiming that "*the calculus of tenses should have bend included in the modern development of logic.*"

Until the moment, analysis of Diodorus M.A., as well as development of temporal logic, are going side by side, widely helping each to other in their mutual progress. The correction of his article on Diodorus from 1955 Prior will give several years later, in his [1958]. There, he is suggesting *System D* as suitable logic for covering temporal aspects of Diodorus' M.A. In the meantime, Prior is working on further refining of his system. He is also rapt with the antics and medieval accomplishments in the field, comparing and looking them from the aspects of attainments of contemporary logic. He is also occupied with researches on McTaggart and Pierce, and the recent logical investigations of the tenses of verbs given by Reichenbach. Results of the work he maintain in [1957], and this book actuate wide interest of logicians for further researches in the field. However, many of contemporaries saw these results as problematic, querying at the same time about the question „*Is this kind of logic necessary at all?*" (Kripke in *October letter*).[2] Discussions about the last question does not obtain their final point up to date, and theoretical base for differing two orientation rised and developed into various approaches concerning the problem of treating time and tense aspects of natural language - some of authors seeing them as unique in nature while the others claim necessity to trait them as separate questions and theoretically irreducible each to other.

Short after publishing of his work, two important suggestions rose in correspondence. One was a suggestion of Kripke, that time could be represented as the '*branching*' structure (*September letter*). The another was '*Hamblin lattice*,' as the first version of a model that has to presents implicative structure for the tenses. However, the complete version of this idea coming almost ten years later, at 1965. Hamblin also suggested set of axioms and rules of inference for calculus with monadic operators, that corresponding to „*a simple interpretation in terms of two-way continuous time-scale*."

M. Dummett and E. Lemmon (Prior's listeners at his Oxford lectures during 1955-6 term), in their [1959] try to elucidate even part of the nature of Prior's above mentioned calculus, *System D* - the system which was suggested as formal representation of Diodorus' logic. They show that stronger *System D* is contained in *S4* and also contains *S4.3* (cf. also, Prior, [1962]). One year later, after the article of Dummett and Lemmon was published, O. Becker analyses Prior's interpretation of

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[1] D. Findley, *Time: A Treatment of some Puzzle*, p.160, in (ed.) Gale [1968].

[2] *Cf.* Øhrstrøm, [1993].

M.A. suited in *System D* and commented in the meanwhile in [1957], and gives the diagram [1960, s. 252] by which he wishes to show that *without an atomistic view of time, Diodorus' argument is unsound.*

Starting from these findings, Prior, in his [1962], declaring an opinion that, as it seems, *S4.3*[3] is not enough suitable to express Diodorus' modalities, since it does not expresses Diodorean assumption of *discreteness.*[4] Prior's searching for adequate system continues in the years coming after, and he gives his final account in [1967, pp.20-31.]. Since for Diodorus time has to be *discrete*, as Becker pointed out, an adequate for Prior system has to be *S4.3.1*[5] rather than *S4.3*. However, in his final version of M.A. interpretation, Prior is additionally affront with certain problematic decision that were the outcomes of his attempt in the analyse: *either to abandon own version of reconstruction as invalid, or to retain it as correct and to accuse Diodorus for invalidity of the argument alone.* His sympathies more inclined for this last option [1967, p.32-4].

Prior's version, especially that as it was given in his [1967], becomes to be standard and starting step in reconstructions that comes later. The structure and developing of the way of his construction of Diodorus' 'proof' is the following. The argument was formalised by introducing the folowing symbolism necessary for suiting it in an appropriate form that here covering rules of propositional and corresponding modal calculus as well as suitable 'time'-dependent operators.

Fp	=	'It will be the case that p'
Pp	=	'It has the case that p'
Hp	=	'It has allways been the case that p'
$\Box$p	=	'It has necessary that p'
$\Diamond$p	=	'It is possible that p'

Since „*It is necessary that* p" can be substitute here in its corresponding equvalent pair sentence to „*It is impossible that* $\neg$p" and „*It has been the case that* p" as equivalence to „*It has not always been the case that* $\neg$p", we are obtaining

[3] It is the Lewis' **S4** + $\Box(\Box(p \supset \Box q) \vee \Box(\Box q \supset \Box p))$. Shortening of this additional formula to $(\Box p \to q) \vee (\Box q \to p)$, with $\to$ used in a strict sense, is ascribed in Prior [1967] (p. 29 and p. 27) to P.T. Geach. The completeness of newly constructed system is proved (algebraically in Bull [1965]. *Vide* also Prior [1962].

[4] Bull, in his [1965] the result for *D* credited to Kripke, suggested adding of $(((p \to \Box p) \to \Box p) \to \Box p)$, where $\to$ pays in the *strict* sense, as an alternative base for the system. Historical account of these developments are given in Prior [1967], pp. 20-31.

[5] That is, **S4.3** + $\Box(\Box(p \supset \Box p) \supset p) \supset (\Diamond\Box p \supset p)$. The system obtained its name in article of Sobočińsky ('Modal system S4.4,' *NDJ* Vol. 5, 1964:305-312), where Prior's *System D* is axiomatized with shorter formula $((p \to \Box p) \to p) \supset (\Diamond\Box p \supset p)$, where $\to$ is here strict implication.

the auxillary substitution rule for inter-translating the above modal and time-dependent sentences by relationship between them that holds in the following way:

$\Box p \ \equiv_{def} \ \neg\Diamond\neg p$

$Pp \ \equiv_{def} \ \neg H\neg p$

He then restates Epictetus' quotation of Diodorus' premisses in the way that could be more elegant for formal reasoning. The first two premisses he obtained by escaping some ambiguities and problems they could rise. So that he decided to restate original text with coresponding corrections.

The **first** Diodorus' premiss - 'πᾶν παρεληλυθὸς ἀληθὲς ἀναγκαῖον εἶναι'[6] - with corresponding formal translation according above symolism, he understand as

(a) *When anything has been the case, it cannot not have been the case.*

The **second** premiss - 'τῷ δυνατῷ ἀδύνατον μὴ ἀκολουθεῖν'[7] - that contains consequence term 'ἀκολουθεῖν,' Prior takes as logical rather then temporal. He willing to emphasyse that here underlying sense of the truth of sentence can refer to momentary truth value, such one that is purported with respect to temporally unspecified propositions. So that he offer this sentence as

(b) *If anything is impossible, then anything that necessarily implies it is impossible.*

Following a sense of Becker's suggestion, that Diodorus has in mind some sentences that were in common usage and taken among contemporaries for granted,[8] Prior add to (a) and (b) two more assumptions operating with them rather as with principles than with bare translations. So that we here have instead 'There is something possible which neither is nor will be true' - 'δυνατὸν εἶναι, ὃ οὔτ' ἔστιν ἀληθὲς οὔτ' ἔσται' - (c) and (d) as result of his restating formulation of the sentences that were obtained by using substitution principle gained on the gound of interconection between condition that 'p is necessary' has its equivalence in '$\neg p$ is impossible:'

(c) *When anything is the case, it has always been the case that it will be the case.*

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[6] Reduced meaning of the premise sounds approximatelly as 'Every past truth is necessary.' Later we will see the possible ambiguities that inplies from Epictetus' formulation of the premiss and what are the outcomes in its diferent formal fittings.

[7] **LS** redaction of translation, as frequentlly is present among commentators of the place is, 'Something impossible does not follow from something possible.' They are leaving here the key term 'ἀκολουθεῖν' undecided for either possible, temporal or logical, interpretation.

[8] He supposed them as „likely to have been taken for granted both by Diodorus and by main his opponents" ([1955], p. 210).

and

(d) *When anything neither is nor will be the case, it has been the case that it will not be the case.*

As Diodorus' conclusion of M.A., which states that 'Nothing which neither is nor will be true is possible' - 'μηδὲν εἶναι δυνατόν, ὃ οὔτ' ἔστιν ἀληθὲς οὔτ' ἔσται' - he restates it as

(z) *When anything neither is nor will be the case, it is impossible.*

Giving the formalism necessary for translating of the above sentences into symbolism, he suggests as assumptions prepared for formal reasoning the following their corresponding formulations.[9]

(a)	$Pp \supset \neg\Diamond\neg Pp$	**A**
(b)	$\neg\Diamond q \supset [\Box(p \supset q) \supset \neg\Diamond p]$	**A**
(c)	$p \supset HFp$	**A**
(d)	$(\neg p \wedge \neg Fp) \supset P\neg Fp$	**A**

As a conclusion then he has the proposition

(z) $\neg p \wedge \neg Fp \supset \neg\Diamond p.$

By adding the rule of necessitation for modalities, taken over from Lewis' S4,[10]

$$\text{RL: } \vdash \alpha \rightarrow \vdash \Box\alpha$$

he proceedes with the natural deduction proof in the following way:

$\neg p \wedge \neg Fp \vdash \neg\Diamond p$

1	(1)	$\neg p \wedge \neg Fp$	**A**
1	(2)	$P\neg Fp$	from (1) and (d)
1	(3)	$\neg\Diamond\neg P\neg Fp$	from (2) and (a) $p/\neg Fp$
1	(4)	$\neg\Diamond HFp$	from (3) and the equivalence of '$\neg P\neg$' and 'H'
	(5)	$\Box(p \supset HFp)$	from (c) and the rule of necesitation : $\vdash \alpha \rightarrow \vdash \Box\alpha$
1	(6)	$\Box(p \supset HFp) \supset \neg\Diamond p$	from (4) and (b) q/ HFp
1	(7)	$\neg\Diamond p$	from (5) and (6), MPP

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[9] Prior uses Polish prefix notation. For the sake of the uniformity, we choses to give it through standard notation.

[10] Prior claims that „the rule RL asserts in Diodorean terms, that if a formula expresses something which is allways true, then the formula asserting that that thing is necessary, *i.e.* that it is and always will be true, is allways true" ([1955], p. 207).

(8) $\neg p \wedge \neg Fp \supset \neg \Diamond p$ from (1) and (7), CP

■ **Q.E.D.**

The same result in the acheiving intended conclusion (z) $\{(\neg p \wedge \neg Fp) \supset \neg \Diamond p\}$, we can obtain in a somewhat **extended procedure**, by adding, to assumptions (a) - (d) formulated above by Prior, two additional principles (1) and (2), known in antiquity,

(1) $(p \supset q) \supset ((q \supset r) \supset (p \supset r))$ *the principle of syllogism*

(2) $(p \supset (q \supset r)) \supset (q \supset (p \supset r))$ *the law of commutation*

and than we are proceeding in the following way:[11]

(3) $P\neg Fp \supset \neg \Diamond \neg P \neg Fp$ from (a): $p/\neg Fp$;

(3°) $P\neg Fp \supset \neg \Diamond HFp$ from (3), by $\neg P \neg \equiv_{def}$ H

(4) $\neg p \wedge \neg Fp \supset \neg \Diamond HFp$ from (d) and (3°) by (1): $\neg p \wedge \neg Fp/p$; $P\neg Fp/q$; $\neg \Diamond HFp/r$;

(4°) $\neg \Diamond HFp \supset [\Box(p \supset HFp) \supset \neg \Diamond p]$ from (b): q/HF;

(5) $\neg p \wedge \neg Fp \supset [\Box(p \supset HFp) \supset \neg \Diamond p]$ from (b), *i.e.* (4) and (4°) by (1): $\neg p \wedge \neg Fp/p$; $\neg \Diamond HFp/q$; $\Box(p \supset HFp) \supset \neg \Diamond p/r$

(6) $\Box(p \supset HFp)$ from (c) by RL

(6°) $\{\neg p \wedge \neg Fp \supset [\Box(p \supset HFp) \supset \neg \Diamond p]\} \supset \{\Box(p \supset HFp) \supset [(\neg p \wedge \neg Fp) \supset \neg \Diamond p]\}$ from (5) by (2): $\neg p \wedge \neg Fp/p$; $\Box(p \supset HFp)/q$; $\neg \Diamond p/r$;

(6°°) $\Box(p \supset HFp) \supset [(\neg p \wedge \neg Fp) \supset \neg \Diamond p]$ from (6) and (6°) by MPP

(z) $\neg p \wedge \neg Fp \supset \neg \Diamond p$ from (5) and (6°°) by MPP

■ **Q.E.D.**

Let we see some objections to Prior's reconstruction given across some later interpretations of MA.

Hintikka - The Discussion about the Principle of Planitude

Like Prior, Hintikka, in his [1964] and [1973] considers the consequence relation in *D2* to be *logical* rather than *temporal*, and takes truth as to refer to a 'momentary truth value' applied to the *temporally unrestricted* proposition. He criticises Prior for his interpretation of what Diodorus may count as a proposition

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[11] Numbering, given here by small letters, will be used later, in McKirahan's examination of Prior's proof.

concerning time references. He also tries to elucidate the common meaning of the terms *necessary* and *possible*, with the purpose to depict what were the common meanings of these terms for Diodorus contemporaries. Following the idea of Becker, he is searching for the answer from Aristotle as the general source of the debate on conditionals with the time references.

So, he understood two premises *D1* and *D2* in transcribed form of corresponding *principles*, rather than as specific premises in some particular argument. Translation of Hintikka's hypothesis sometimes seems not so much transparent, even he introduces some new light on the problems arise in an interpretation of what really is a bearer of the truth value, *i.e.* when some proposition has its truth value if it meets corresponding temporal reference. Besides, Hintikka is assorting Diodorus into defenders of '*the principle of planitude*,' ascribing to it some additional and to Diodorus unfamiliar features. The idea of 'necessity of the past (truths)' was certainly familiar for many contemporaries of Diodorus. It has the common form of its formulation in Aristotle,[12] and Hintikka tends to make an approximation of two approaches on modalities in his reformulation of Diodorus the first premiss.

According to Hintikka's final remarks, MA fails to establish its fatalistic conclusion. Retracing fashion of Prior, since he treats Diodorus' premisses (D1) and (D2) as principles rather than specific premisses, he finds that they can be read as (a) and (b), respectively,[13] where (b) is understood as principle claiming *logical* rather that *temporal* consequence, and they both will be used in the argument under the following interpretation:

(a) Any true statement concerning the past is necessary.

(b) If a possibility is assumed to be realised, no impossible conclusions follow.

Using these principles in argument, he is trying to show inconsistency of the following two propositions:

(x) It is possible that p.

(y) It is not the case that p nor will it be the case at any later moment of time.

To show inconsistency between **(x)** and **(y)**, Hintikka tries to find α) some third proposition, say r, which could be grounded on the assumption that p is the case, and β) some proposition derived from **(y)** that could asserts „It is impossible that

[12] Arist., *NE* 1139b 7-9: „It is to be noted that nothing that is past is an object of choice, *e.g.* no one chooses to have sacked Troy; for no one deliberates about the past, but about what is future and capable of being otherwise, while what is past is not capable of not having taken place; hence Agathon is right in saying: *For this alone is lacking even to God, to make undone things that have once been done*."

[13] Hintikka [1964], p. 106.

r." The result then will be to show how propositions (**x**) and (**y**) implies outcomes, derived across principle (**b**), that are inconsistent.[14]

Let t_0 and t_1 be some *future moments of time* such that t_1 is one day later than t_0.

1	(1)	At t_0 it will be true that p.	assumption that possibility p is realised
1	(2)	At t_1 it will be true that p was the case yesterday.	from (1) and definition of t_0 and t_1
3	(3)	It is not the case that p nor will it be the case at any later moment of time.	assumption that (y) is the case
3	(4)	At t_0 it will be false that p.	from (3) and the definition of t_0
3	(5)	At t_1 it will be false that p was the case yesterday.	from (4) and the definition of t_0 and t_1
3	(6)	At time t_1 it will be true that it is impossible for p to have been the case yesterday.	from (5) by virtue of principle (a)

Hintikka feels that obtaining (6) can conclude:

"This is now the imposibility Diodorus was looking for. The conclusion (2) which we obtained from the assumption that possibility (x) is realized is shown by (6) to be not only false but also impossible. Hence Diodorus was ready to conclude that the original set of premisses {(x), (y)] was inconsistent."[15]

Sutula[16] extends and continues the proof by additional steps that makes conflict between two principles more transparent:

3	(7)	It is impossible that p.	- from (1), (2), and (6) by virtue of (b)
	(8)	If it is not the case that p and it will not be the case that p at any later moment of time, than it is imposible that p.	- from (3) and (7)

The way of reasoning that yields (6) seems convincing, but looking at it more detaily, we see that it differs in some sense from the form of proposition we could expect having in mind the form of proposition (2). According to Hintikka's intention, we have to look for a formulation of the one side of inconsistency to be

[14] *Ibid.*, p. 104.

[15] *Ibid.*, p. 106.

[16] Sutula [1976], p. 331.

'It is impossible that *r*.' The proposition (6) has its form very near to intended, but it obviously does neither claim covers by

[6] *'It is impossible that* (2),'

nor its correspndeng expected form

[6'] *'It is impossible that* (At t_1 it will be true that *p* was the case yesterday),'

but just

(6) 'At time t_1 it will be true that it is impossible for *p* to have been the case yesterday.'

In sentence (2), the valuated sequent would be its part '*p* was the case (yeasterday).' So, the modal prefix is tied for, or is relative to, a moment of uttering this *partial* propositional sequent. For the purpose of illustration, let we take that valuated sequent '*p* was the case (yeasterday)' be substitued for 'α,' so that (2) we can read as

[6'] '{It *is* (*now*, or just at the moment) *impossible* that: [at t_1 it *will* be *true* that (α)]}.'

However, Hintikka's result given as (6) seems obviously different in its meaning. Following same reasoning in substitution, he serve the a mere the following:

(6) '{It *is* (*now*, or just at the moment) *true* that: [at t_1 it *will* be *impossible* that (α)]},'

what is far from intended contradiction that 'It is (*now*) impossible that (2),' and what Hintikka proposed in above quoted lines. Rather, as we saw, he is showing that 'At t_1 *will be impossible* that (2),' what does not corresponds with principle (b) [*If a possibility is assumed to be realised, no impossible conclusions follow*]. In the same sense, Sutula's suplemental step (7) of the Hintikka's reconstruction does not follow from (2) and (6) by help of the principle (b). If appropriate intervention is made, it also make no any anchoring toward obtaining desired contradiction. Even (2) and here modified [6'] are corresponding to the spirit of modified second premiss suited as the principle (b), we have to find satisfactory ground to justify implying [6'] from (6). However, if we are able to allow that (6) implies [6'], then we would have that

If (6) '{It *is* (*now*, or just at the moment) *true* that: [at t_1 it *will* be *impossible* that (α)]},' ***then*** [6'] '{It *is* (*now*, or just at the moment) *impossible* that: [at t_1 it *will* be *true* that (α)]},'

what, at the first sight, does not sound so strange, but looking at this more directly, it has as its consequence the claim of the sort 'what (is currently true that, it) will be (later) impossible, is already impossible (now).' In other words, this is the claim which states that 'afterwards imposibilities, implies those which are (actualy) present,' what evidently does not corresponds with the principle (b).

Sutula comes to the similar position using the another side of reasonong and illustrates it with an example.[17] By appropriate substitution of modal notions we have that 'what is possible *now*, will also be possible at any later date.' If it is so, let suppose t_1, t_2, and t_3 are the first, second, and third days of the month, and let take p to be a proposition '*Yesterday was the first day of the month.*' At t_2, the proposition *is* true, and so it is *possible* that p: '*Yesterday was the first day of the month* (t_1).' At t_3, the same proposition *is not true*, and it is *not possible* that p - *i.e.*, according to the principle (a): 'Any true statement concerning the past is necessary,' since at t_3 it is false that p, it is not *possible* that p. But comparing the truth and modality of the same sentence at two different, nearby moments, we have that although at t_2 it is possible that p, at t_3 it is not possible that p, what in this case lastly gives that '*what is possible at one time is not possible at all later times.*' For this reason (6) cannot imply [6'], so that Sutula's additional step (7) also fails since it does not follow from (2) and (6) by virtue of (b). At the consequence, either of two explicated ways to the intended conclusion, across principle (b) or across principle (a), are charged with serious deficiencies.

Rescher - Independence of Historical Reconstruction

Rescher's version [1966, 1968, 1971] in many aspects differs from that of Prior or Hintikka. Some of that differences are principal in nature. He allows himself a certain freedom from historical accuracy to gain logical rigor. He is concerned with giving an argument that could be more acceptable by modern logical standards, leaving aside many aspects of M.A., beside the fact that some of them are possible interpretative outcomes of available testimonies. Spirit of researches he attempts is traced on the Arabic medieval commentators of Aristotle, and are related with the work he undertook earlier in his pioneering study on the Arabic logic. These widely overtaking experiences are present and well recognisable in the papers concerning Diodorus argument. His '*medieval reading*' of the problem is consisted in specific understanding of the meaning of propositions an argument deals with, and the Diodorean sense of a proposition is here presented as *a temporally specified proposition*. The above reasoning is on the same line with his reaction against Prior's view on nature of time, and in discussion on this metaphysical assumption, imputed in his reconstruction to Diodorus, Rescher defends position *that time itself is not branching*.[18]

In the sense, he feels as required to restate Diodorus' premises as propositions with fixed time-references. That is, he takes the modalities applied to these propositions to be relative to specific time (usually, we used to name it as 'a dated

[17] *Ibid.*, p. 335.

[18] See also his [1968] pp. 211-2, and [1971] p.73.

proposition').[19] Accordingly, the first Diodorus' premiss *D1*, „Everything that is past and true is [*now*] necessary" he understood by applying the formal machinery, where he has

(1) $(t)\{[\mathrm{T}_t p \wedge (t<n)] \supset \Box_n p\}$

with sense of *temporally definite* propositions like „It rains in London on 1st January 1966," where *t* refering to *yesterday*, *n* to *today*, and *p* has the sense „It is raining *now*" (and not of temporally indefinite ones like „It rains in London today" since it would lead, as he thinks, to absurd consequences [1968, p. 204; n. 27]). The *generalised* version of the premiss then would be: „What is past and true is necessary *thereafter,*" or „If proposition *p* is true at any time *t*, then *p* is necessary at any later time *t*'." Symbolically, he states it as:

(1a) $(t)(t')\{[\mathrm{T}_t p \wedge (t<t')] \supset \Box_{t'} p\}$

In this form the premiss corresponds, on his opinion, with the theory of temporalised modalities covered by the medieval dictum „Anything, when it is, is necessary" (*Unumquodque, quando est, oportet esse*), which is here construed as „Anything, when *once* it is, is *henceforth* necessary." So, in the second step of reconstruction, he reads the term ἀκολουθεῖν and accordingly interprets Diodorus' second premise *D2* as quoted in Epictetus' source,[20] in a sense of statement concerning *temporal*, rather than logical consequence.[21] Now he understood it as the principle where the phrase '*follow from*' has to be redden as '*occur after*:' „The impossible does not *follow after* the possible." That is, what is once possible does not later become impossible, but rather is possible at all later times, or „If proposition *p* is possible at any time *t*, then *p* is possible at any later time *t*'." He suggests its following symbolical formulation:

(2) $(t)(t')\{[\Diamond_t p \wedge (t<t')] \supset \Diamond_{t'} p\}$

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[19] There are a few reasons why Rescher selected this option in his version. The sense of (1) – *i.e.* 'if *p* has been true in the past, than *p* is necessary (*now*)' – would be acceptable for statements about the past with a fixed time references, and his reasons for doing so is that without that interpretation he cannot make sense of premiss (2). However, some authors hold it as a serious limitation of his reconstruction, since it is not *in general* valid interpretation that completely corresponds to specific Diodorean 'propositions.' See, for example, McKirahan's comment of Rescher, in [1979] *p*. 250., *n*. 32.

[20] Döring, Fr. 131.

[21] This sense is due to Zeller's tradition in interpretation of the place. However, many authors are not ready to accept this approach beliving that temporal interpretation is untenable. One of the standard reasons for denial of this *temporal* reading of the second premiss consists in the reading of Diodorus' definitions of 'possible' and 'impossible.' In Rescher's case, Diodorus' proposition, being false at some time *t* and remains false thereafter, is *possible* before *t* and *impossible* after. According to this possition, this difficulty would not arise if we take 'follow' in the *logical* sense.

which has as its direct equivalent version, the following proposition

$$(t)(t')\{[\neg\Diamond_{t'} p \wedge (t<t')] \supset \neg\Diamond_{t'} p\}.$$

In other words, the last version of the premiss, sound as „What is once impossible was never possible at a prior time," and these versions of *D2* explicitly express clearly *the possibility-conservation principle* - „the once possible is always possible thereafter" - that yields 'markedly deterministic overtones [*Ibid.*, p. 205, n. 29].' Involving the duality-relationship between $\Box$ and $\neg\Diamond\neg$, we can read (2) as equivalent with $(t)(t')\{[\Box_{t'} p \wedge (t<t')] \supset \Box_t p\}$, or, in other words, as the thesis that „What is necessary at a time was always necessary theretofore."

Diodorus' *D3* - „What neither is nor will be is possible" - is transformed as statement that claims that „*Something* that neither is nor will be true is nonetheless possible" or, „Proposition p_0 is possible now but p_0 is true now and p_0 will not be true at any later time." Thus he is interpreting this as:

For some p_0: $\Diamond_n\ p_0 \wedge (t)[(n \leq t) \leftrightarrow \neg T_t\ p_0]$

For he holds as purport of M.A. to show that (1) - (3) as mutually incompatible, he treats *D3* as that one involving a form of *reductio* in argument - since if we accept (1) and (2) we must also accept denial of (3). So, it implies that we have as its denial, or exactly intended Diodorean thesis that every (present) possibility must be realised at some present or future time:

$\Diamond_n p \supset (\exists t)[(n\leq t) \wedge T_t p)$.[22]

Willing to obtain contradiction, as the fourth step of reconstruction he has the reformulating thesis (1a) that yields to the statement (4) that „If proposition *p* is not necessary at any time *t'*, then *p* is not true at any earlier time *t*." This step becomes, by substituting „$\neg p$" for „p" statement (5) that „If $\neg p$ is not necessary at any time *t'*, then $\neg p$ is not true at any earlier time *t*." By rule of equivalence between $\neg\Box_t\neg$

[22] In addition [1968] p. 205, n. 30, Rescher tries to develop somewhat transformed thesis that is ore applicable to adequate dicta, that are extracting from Boëthius source given in Döring, Fr. 138. So that this thesis can be restates and strengthening to an equivalence

$$\Diamond_n p \leftrightarrow (\exists t)[(n\leq t) \wedge T_t p]$$

and that is, by substitution, also equivalent with

$$\neg\Diamond_n p \supset (\forall t)[(n\leq t) \wedge \neg T_t p)$$

or, with the Diodorean dictum: „*The impossible is that which neither is, nor will be.*" By substitution of $\neg p$ for *p* we are obtaining another equivalence of the thesis

$$\neg\Diamond_n\neg p \supset (\forall t)[(n\leq t) \wedge \neg Tt\neg p).$$

Using duality-relationship between $\Box$ and $\neg\Diamond\neg$, and postulating alethic equivalence of *T* and $\neg T\neg$, this principle for Rescher has seen it as to become corresponding with the Diodorean dictum that *The necessity is that which is and always will be true*:

$$\Box p \supset (\forall t)[(n\leq t) \wedge T_t p).$$

α and $\Diamond_t\alpha$, he restates (5) as (6), or that „If p is possible at any time t', then $\neg p$ is not true at any earlier time t." Now, escaping immediate step from (6) to (8) he is inserting the manoeuvre concerning implying general applicability of the Law of the Excluded Middle, or which postulates thesis that, 'at any given time, any given proposition is either true, or else its contradictory is true':

$(t)[T_t p \vee T_t(\neg p)]$

which now yields to proposition (8) that „If p is possible at any time t', then p is true at any earlier time t. From (3), now he makes introductory step to obtain desired contradiction by (9) „Proposition p_0 is possible *now*" and (10) „Proposition p_0 will not be true *one day from now*." (11), or „Proposition p_0 will be possible *two days from now*" he is deriving from (2) and (9), to obtain (12) „Proposition p_0 will be true *one day from now*" from (8) and (11). Since (12) is in contradiction with (10), so that its the source premiss have to be denied, it yields him to (13) that states the statement „If a proposition p is possible now, then either p is *true now* or p will be *true at some later time*." For (13) being equivalent to Diodorus' conclusion *D3*, the argument is completed.

Recapitulating Rescher's reconstruction in its formalised version, MA goes on as follows:

1	(1) $(t)(t')\{[T_t p \wedge (t<t')] \supset \Box_{t'} p\}$	A
2	(2) $(t)(t')\{[\Diamond_t p \wedge (t<t')] \supset \Diamond_{t'} p\}$	A
3	(3) $\Diamond_n p_0 \wedge (t)[(n \le t) \supset \neg T_t p_0]$ - for some p_0	A
1	(4) $(t)(t')\{[\neg\Box_{t'} p \wedge (t<t')] \supset \neg T_t p\}$	from (1)
1	(5) $(t)(t')\{[\neg\Box_{t'}\neg p \wedge (t<t')] \supset \neg T_t \neg p\}$	from (4), $p/\neg p$
1	(6) $(t)(t')\{[\Diamond_{t'} p \wedge (t<t')] \supset \neg T_t \neg p\}$	from (5) replacing $\neg\Box_t\neg\alpha$ by its equivalent, $\Diamond_t\alpha$
	(7) $(t)(T_t p \vee T_t \neg p)$	the law of excluded middle is here taken as a theorem
1	(8) $(t)(t')\{[\Diamond_{t'} p \wedge (t<t')] \supset T_t p\}$	from (6) and (7)
3	(9) $\Diamond_n p_0$	from (3)
3	(10) $\neg T_{n+1}\, p_0$	from (3)
2,3	(11) $\Diamond_{n+2}\, p_0$	from (2) i (9)
1,2,3	(12) $T_{n+1}\, p_0$	from (8) and (11)
1,2	(13) $\Diamond_n p_0 \supset (\exists t)[(n \le t) \wedge T_t p_0]$ for all p_0	by RAA

■ **Q.E.D.**

Rescher gives his own interpretation across *unrestricted* version of the law of excluded middle as it was shown in form of his step (7). It is contrary to usual interpretation of Aristotle's conception stated above in discussion of the 'see-battle example,' where Aristotle is presented as an advocate of restricted version of the principle, and an opponent of its unrestricted version, for his intention to save free will and to escape deterministic consequences appointed by the Lazy Argument.

In his reconstruction, willing to evade deterministic conclusion of M.A., Rescher wishes to show that rejection of the *possibility-conservation* principle (2) was not the path Diodorus himself chose. His intention was to signify that the way Diodorus selected, was to resolve the incompatibility of given premisses by abondoning (3), and retaining (1) and (2), which yields his option toward a *logical determinism*. In that sense, the key Rescer's step is consisted in the claim that we could not in Diodorus' case both „*p*-at-*t*" and „not *p*-at-*t*." Hence, Rescer advocates a position, that here, „the truth status of proposition with respect to the *time-of-reference t* must be decisive consideration" [1971, p. 197]. This idea was formed in contrast with understanding of temporally relativised propositions given at Kneales, where it is maintained that „Diodorus' definitions of the modal notions are based on the assumption that truth-values change" [1962, p. 121]. Even a truth-value of *temporally indefinite* statement „It rains in Athens today" will indeed change if it is made on January 1st, 1966, its bounding to the strict or explicit time reference, as in the case of its *temporally definite propositional* counterpart like it is here „It rains in Athens on January 1st, 1966," will not change over time. So, Rescher asserts that „what can change with time *t* is not the truth-value of the absolute proposition, *p*-at-*t'*, but that of its temporally-modalized necessary, or possible or actual counterparts, $\Box_t$(*p*-at-*t'*), or $\Diamond_t$(*p*-at-*t'*) or T_t(*p*-at-*t'*) [*cf. ibid.* p.167. n. 9]. Diodorus' proposition is understood in the reconstruction as of this last type.

Rescher's *theory*, based on the idea of a single course of time, on his view, has several advantages. Beside its *simplicity*, or transparency at the first sight, the main problem remarking favour of the theory is in its differing *epistemic* and *ontological* status of truth in the relativised proposition with ingredients of time references. This item is an outcome of Rescher's criticism of the Prior-type approach, that if contingency of the future is inherent in the ontological structure of the time itself, then a future contingency cannot even be specified, and so, a future contingent proposition must be set apart into truth-status limbo.

Rescher believes, contrary to Prior, that contingency of the future is inherent in the causal structure of the course of events, not in the nature of time as such. For this reason, the future contingency is plausible to characterise as true or false. This opinion is far reaction of debate on Aristotle's *de Interpretatione Ch. 9*, in the Antics and the Medieval commentators on how to read 19a36-38, where the wellknown phrase '*not yet determinately true*' had been usually understood in the *epistemic* context. Even his conception is fresh and stimulative in many aspects for further study of the problem, it seems very far from acceptable, not just from a standpoint of suggested historical context that is involved in discussion, but also if we are looking for satisfactory formal interpretation of M.A.

Sutula - Objections to Rescher's Reconstruction

Rivality between Diodorus and Aristotle, as it was seen in Rescher's reconstruction, run around the key premiss (7), or unrestricted version of the law of excluded middle. To escape deterministic consequences that implies from Diodorus' argument we have to reject this version of the principle and to accept *restricted* one, commonly ascribed to Aristotle - 'even every proposition about the past or present must be such that either it is true or its denial is true, a proposition about future need not be such.'

Sutula, in his [1976, p. 338.] tried to proceed Rescher's idea on how to escape deterministic conclusion of M.A. To block an inference that leads to Diodorus' conclusion in (13), we have to adopt its restricted, Aristotle-type form of the law

(XM1) $(t)(\mathrm{T}_t p \vee \mathrm{T}_t \neg p)$, for all t such that $n \geq t$,

and, since it is equivalent with

(XM2) $(t)[(t<n) \supset \mathrm{T}_t p \vee \mathrm{T}_t \neg p)]$,

substitution of (XM2) instead (7) would be block the inference to step (12), hence it would be also blocked the inference to the conclusion (13).For these reasons we would naw have the steps that follow retained steps (1)-(6). Let we change assumption (7) by (7') that is grounded on restricted version of the law, and its above form (XM2). The inference we have now is the following:

	(7')	$(t)[(\mathrm{t}<\mathrm{n}) \supset \mathrm{T}_t \mathrm{p} \vee \mathrm{T}_t \neg p)]$	the (XM2) is here taken as a theorem
1	(8')	$(t)(t')\{[\Diamond_{t'} p \wedge (t<t') \wedge (t<n)] \supset \mathrm{T}_t\, p\}$	from (6) and (7').

The next tree steps are the same as before:

3	(9)	$\Diamond_{\mathrm{n}}\, p_0$	from (3)
3	(10)	$\neg \mathrm{T}_{\mathrm{n}+1}\, p_0$	from (3)
2,3	(11)	$\Diamond_{\mathrm{n}+2}\, p_0$	from (2) i (9).

But now we have no (12), for $\mathrm{T}_{n+1} p_0$ does not follow from from (8') and (11), since the condition ($\mathrm{t} < \mathrm{n}$), aded in (8'), is not satisfied when $t = n+1$. Even Rescher was not recalling *this* Sutula's procedure, it seems as the way we are escaping Diodorus' fatalistic conclusion, the same one used in 'common' interpretation of Aristotle's solution to the future contingence problem illustrated by his the 'sea-battle' example.

Sutula tries to develop Rescher's idea more explicitely, to show some inconsistencies in tacit assumptions of Rescher's intended reconstruction. He is willing to base his claim that by restricting the law of excluded middle, made out in the fashion of Rescher's attempt, it is not possible to successfully escape the fatalistic conclusion given in (13). We said above that Rescher's formulation of 'temporally specified propositions,' with which his argument is concerned, has to be of the form '*X*-at- t_0' where, given *X* as some case happend at t_0, t_0 is the time of reference for some proposition. Accordingly, we can derive a wider version of his unrestricted form of the law of excluded middle as

$(t)\,[\mathrm{T}_t\,(X\text{-at-}\,t_0) \lor \mathrm{T}_t\,(\text{NOT: } X\text{-at-}\,t_0)]$.

As „the most convenient exit from the sphere of necesitation" [1966, p. 444; 1968, p. 209; 1971, p. 197], he has in mind „the denial of applicability of the law of the Excluded Middle in the context of a temporally relativized conception of truth." For these reasons he is placing some additional conditions so that maintains $\mathrm{T}_t\,(X\text{-at-}t_0)$ whenever $t \geq t_0$, but whenever $t < t_0$ he rejects both this and $\mathrm{T}_t\,(\text{NOT: } X\text{-at-}\,t_0)$. By this way, the cversion of *restricted* version of the law of excluded middle becomes:

(XM3) $(t)\,[\mathrm{T}_t\,(X\text{-at-}t_0) \lor \mathrm{T}_t\,(\text{NOT: } X\text{-at-}t_0)]$ for all t such that $t \geq t_0$,

what is equivalent to

(XM4) $(t)\,\{\neg\,(t<t_0) \supset [\mathrm{T}_t\,(X\text{-at-}t_0) \lor \mathrm{T}_t\,(\text{NOT: } X\text{-at-}t_0)]\}$

Now, we have in reconstruction somewhat different organisation of steps, and it seems that, as Sutula supposed, this way of approach is „*not* sufficient to block the chain of inferences leading to the conclusion in Rescher's original argument" [1976, p. 340]. If we replace step (7) by newly formed step (XM4) as a theorem, leaving the first six steps to preceed as before, then step (8) is necessary to reformulate in respect to the new version of the law. So that now we have

1 (8") $(t)\,(t')\{[\Diamond_{t'}\,(X\text{-at-}t_0) \land (t<t') \land \neg(t<t_0)] \supset \mathrm{T}_t\,(X\text{-at-}t_0)\}$ from (6) and (XM4).

The next tree steps looks like those formerlly given except '$n+1$' is replaced by '$n+\Delta$' and '$n+2$' is replaced by '$n+2\,\Delta$'. The condition here required by (3), is that $n+\Delta > n$, while the condition required by (2) is that $n+2\,\Delta > n > \Delta$, so that Δ can be any arbitrarily large positive number. With these changes, we have the following reworked steps:

3 (9') $\Diamond_n\,(X\text{-at-}t_0)$ from (3)

3 (10') $\neg\mathrm{T}_{n+\Delta}\,(X\text{-at-}t_0)$ from (3)

2,3 (11') $\Diamond_{n\,+\,2\Delta}\,(X\text{-at-}t_0)$ from (2) i (9').

The step that follows here is not blocked as it was previously, 'for since Δ can be arbitrarily large it can be chosen so as to satisfy the added condition in (8") that $\neg(t<t_0)$, where $t = n + \Delta$. So that we have as (12'):

1,2,3 (12) $\mathrm{T}_{n+\Delta}(X\text{-at-}t_0)$ from (8") and (11').

Here we have intended contradiction - (12') and (10') - so that the conclusion (13) follows as before, beside introduction of, in this way reformulated, the restricted law of excluded middle.

We saw above how it is possible to avoid deterministic conclusion by adding (XM2). However, Sutula serves additional reconstruction based on this thesis,[23] to show that some variation of the argument, 'with only a few adjustments' could be constructed in a way that still lead to (13). Here steps (1) through (11) would be unchanged and alike to those in the argument formed by help of (XM2) above. Instead of infering (12) that lays on (11) and (XM2) we could continue it as follows.

1,2,3	(11a)	$T_{n-1} p_0$	from (11) and (8')
1,2,3	(11b)	$T_{n+1} p_0$	from (11a) and (1)
1,2,3	(12)	$T_{n+1} p_0$	from (11b) and the principle $\Box_t \alpha \supset T_t \alpha$
1,2	(13)	$\Diamond_n p_0 \supset (\exists t)[(n \leq t) \wedge T_t p_0]$ for all p_0	by RAA, from (3), (10), and (12).

Since the step (8') contains the added condition $t < n$, and the condition is satisfied when $t = n - 1$, the step (11a) is not blocked, and directly follows from (11) and (8'). (11b) than follows from application of premiss (1) to (11a), and now (12) follows on the basis of the principle that '*whatever is necessarilly true is true*'. For we have contradiction of (12) and (10), the conclusion would be stated as before.

The result of idea served here by Sutula, is that whatever form of the law of excluded middle would be, restricted or unrestricted, one can not avoid the fatalistic conclusion if reconstruction is grounded on this principle.

However, Sutula gives in addtion a version of the reconstruction to show further that Rescher's assumption, based on the temporal translation of *D2* ("*The impossible does not follow from the possible*") into (2) ("*If proposition* p *is possible at any time* t, *then* p *is possible at any later time* t'"), is 'most likely false'.[24] The reason for this is for (2) implies that 'whatever is possible now will also be possible at any later date.' He thinks that there is an easier way of escaping fatalistic conclusion (13), without rejecting the law of excluding middle, but with rejecting Rescher's formulation of his second premiss, based on 'temporally specified' reading of the phrase 'follow from.'

Here, we will recount the example Sutula served as illustrative for outcomes of Rescher's reading of Diodorus' premiss (2) and reasons for its abondoning:[25]

"Suppose that today is the first day of the month and it is possible today that I will eat meat on the second day of the month and also possible today that I will not eat meat on the second day of the month. (Let us say that I have meat available and

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[23] *Ibid.* p. 340.

[24] *Ibid.* p. 341.

[25] *Ibid.* p. 342.

the outcome tomorrow depwnds on whether or not I decide to become a vegeterian tonight.) Now premiss (2) implies that on the third day of the month is still possible that I eat meat on the second day of the month and also still possible that I do not eat meat on the second day of the month. On the second day of the month either I will have eaten meat or I will not (recall that we are attempting here to avoid the fatalistic conclusion while not rejecting the law of the excluded middle). Now according to Rescher's first premiss (If p is true at t then p is necessary at any later time t') either it is necessary on the third day that I ate meat on the second day or it is necessary on the third day that I did not eat meat on the second day. In eitther case, it is not still possible on the third day that I ate meat on the second day and also still possible on the third day that I did not eat meat on the second day."

Sutula claims that this outcome contradicts with what is implied by Rescher's second premiss, and that for this reason it should be rejected. Reading p as to be proposition „I eat meat on the second day of the month," that stressing temporally specified reading of the proposition in question, he suggests the next formalised steps of the argument:[26]

1	(1)	$(t)(t')\{[\mathrm{T}_t p \wedge (t<t')] \supset \Box_{t'} p\}$	**A** of Rescher's first premiss
a	(a)	$\Diamond_1 p$	**A**
b	(b)	$\Diamond_1 \neg p$	**A**
	(c)	$\mathrm{T}_2 p \vee \mathrm{T}_2 \neg p$	law of excludded middle
1	(d)	$\Box_3 p \vee \Box_3 \neg p$	from (1) and (c)
1	(e)	$\neg\Diamond_3 \neg p \vee \neg\Diamond_3 p$	from (d) replacing $\Box_t \alpha$ by its equivalent $\neg\Diamond_t \neg \alpha$
1,a,b	(f)	$(\Diamond_1 p \wedge \neg\Diamond_3 p) \vee (\Diamond_1 \neg p \wedge \neg\Diamond_3 \neg p)$	from (a), (b), and (e)
1,a,b	(g)	$(\exists t)(\exists t')\{[\Diamond_t q \wedge (t<t')] \wedge \neg\Diamond_t p\}$ for some q	from (f)
1,a,b	(h)	$\neg(t)(t')\{[\Diamond_t q \wedge (t<t')] \supset \Diamond_{t'} q\}$ for all q	from (g)

■ **Q.E.D.**

Sutula's refreshen reconstruction of Rescher's attempt pursues to approve that even we are leaving the law of excluded middle the argument can be considered unsound, and the fatalistic conclusion that is in question could be avoided without rejecting any version of this law and „without saacrificing anithing else of importance."[27] In addition, Rescher's argument implies that 'any proposition which is possible is also true and necessary, hence it would follow that any proposition which is possible is such that its denial is impossible.' The circle implied by

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[26] *Ibid.*

[27] *Ibid.* p. 343.

introducing Rescher's reading of the second premiss 'can only be broken if premiss (2) is supported on some other grounds, or if premiss (2) is intuitively more plausible than the assumption that there is some proposition (e.g. *p* above) which is such that it is possible and its denial is possible.'

Sutula's suggestions are again opening a question, among some others, concerning the 'legal' number of premises and what we could allow and accept as some tacit assumptions that were present in the original version of the argument. His critique of Rescher leaving these problems aside and gives us no more instructions on how to form such acceptable remade of the argument with steps in the reconstruction that would be appropriate not just in logical sense but also satisfactory from historical standpoint. Related question - consisting the correspondence between intention of the argument and its appropriate formal reconstruction, the nature of Diodorus' statements concerning time references, the nature of conditional employed in the argument, *et c.* -stays open further, and almost constantly repeats through some other reconstructions that appears simultaneously or in later approaches.

Michael and the Reduction in Number of Premises

Rescher's reconstruction was a starting point in another trying for solution of M.A. F. Michael, in his [1976] choose slightly different way than Rescher. The difference consists in the fact that the predicates for modalities, such as possibility and necessity, in Rescher's reconstruction have *temporal indices*, just as predicates for truth do. In Michael's reconstruction, on the other hand, modal predicate does not have such indices. His reconstruction is flowing without *D2*, since he wishes to show that it would be quite superfluous to assert it as a premise. Also, Michael showing his historical reasons for omitting *D2*, trying to analyse it in a light of its acceptability from immediately Stoical followers in interpreting the argument.

On this place we have to note that above reconstructions turning us back to the problem of *methodological criteria* for accepting some historical reconstruction as *satisfying*. As we saw, Becker and Prior were the first who choose additional premises as necessary to Diodorus argumentation. They recognise the step due to an assumption on the underlying *principles* allowed and acceptable from contemporaries. Becker showed that these two premises were known in antiquity and were in the common pool of information available to Diodorus. Rescher's continues it with this attempt willing to find acceptable 'metaphysical reasons' that could cover argument to be complete, partly leaving aside its historical accuracy. Handling with it in a different manner he operates by help of different additional principles. Michael reduces even Rescher's set of premises for obtaining an intended result in Rescher's *program* of reconstruction. All these reasons were of technical importance and far from the sense that is historically grounded.

Beside that fact, they were of help in throwing light to places where are rising the problems with its *overlogical* aspects and also, with the question in what direction we have to search for an answers of open questions in philosophical background of M.A. These steps could serve us in completing our representation of the possible domain of M.A., and also where to put some hypothetical restrictions in free making of our assumptions we are using in reconstruction of this argument.

Michael intention is to give a reconstruction according to which premises of the argument do constitute an inconsistent set. Premises are:

P_1. What is past and true is necessary

P_2. The impossible does not follow (*from?, after?*) the possible.

P_3. What neither is nor will be is possible.

The third premiss is in Michael's interpretation slightly reformulated, for he claims that P_3 solely said nothing about the past. The reasons for transforming of P_3 into new form is explaind on the base of following reasons, that what is not now true it has not in the past been true. From this, recalling P_1, we have that P_3 is claiming existence of a proposition which is possible even being not true now, and will not be true. So, it can be infered that

P_3*. There is a proposition which is possible but at no time true.

His formal representation of reconstruction, called *Reconstruction D*, is relative to that of Rescher, unless introducing the time reference indices for modal predicates, since he is showing that result obtained in such a way makes the argument to develop 'pretty much' as in his version.[28] The argument is based on the following notation. Symbolically, n will be that value of t standing for the present, d will be variable ranging over increments of time greater than zero, while $n - d$ will, for each value of d, represent a point of time that is prior to n, or a point of time that is in the past.

The reconstruction sketched by words then would seem as follows. According to P_3*, there is a proposition which is possible, but at no time true, and such proposition would be p_0. For p_0 is at no time true, its negation would be true at all times, so that being true at all times, it is true in the past and, applying to it P_1, we have that p_0 is necessary. If p_0 is necessary, then its negation is impossible. For by P_3* p_0 is possible, we have contradiction between P_1 and P_3*. Formaly, the steps are following. We have as assumptions

D_1. $T_{n-d}(p) \rightarrow \Box(p)$, for all values of p and d;

D_3*. $\Diamond(p_0) \wedge \neg(\exists t)T_t(p_0)$, for some p_0.

Here, D_1 represents P_1 with a meaning „*A proposition true in the past is necessary*," while D_3* is formal presentation of P_3* which here has a meaning

[28] Michael [1976], p. 232.

„*There is a proposition which is possible but at no time true.*" So formulated, D_3*, having in mind that p_0 is bivalent proposition, asserts the existence of a proposition that happens to be false. Hence, D_3* says that there are propositions that are *contingently false*, since not every false proposition is impossible.

The context of bivalent propositions, in the proposed notation the would be formulated by:

T_1	$T_t(\neg p) \leftrightarrow \neg T_t(p)$	for all values of t;
T_2	$\Box(p) \leftrightarrow \neg\Diamond(\neg p)$.	

The *Reconstruction D* than proceeds in the following way

D_1	$T_{n\text{-}d}(p) \to \Box(p)$,		for all values of p and d;
D_3*	$\Diamond(p_0) \wedge \neg(\exists t)T_t(p_0)$,		for some p_0.
D_3*	1.	$\Diamond(p_0)$	by the rule of elimination for $\wedge$;
D_3*	2.	$\neg(\exists t)T_t(p_0)$	by the rule of elimination for $\wedge$;
D_3*	3.	$(\forall t)\neg T_t(p_0)$	from 2;
D_3*	4.	$\neg T_{n\text{-}d}(p_0)$	from 3;
D_3*	5.	$T_{n\text{-}d}(\neg p_0)$	from 4, by T_1;
D_1, D_3*	6.	$\Box(\neg p_0)$	from 5;
D_1, D_3*	7.	$\neg\Diamond(p_0)$	from 6, by T_2.

Michael also criticises Rescher's approach of reconstruction across recalling of the unrestricted law of excluded middle which is above considered as a principle presupposed in the argument, but that would be unacceptable for its convincingness. Michael, on the other side, states that the argument can be obtained and be convincing without help of the principle, since it occurs inessentially for reconstruction. He is trying to serve an alternative, modified Rescher's approach (that is, one opposite to his), without recalling of the principle in the argument.

Showing that Rescher's R_1a, obtained by unrestricted version of the law, can be alternativelly be obtained by using T_1, and that for this reasons R_1a holds explicitelly only for bivalent propositions, what is not obvious in Rescher's attempt, he believes that there is no reason 'to postulate that R_1a holds of *all* [V.M.] propositions, and so to suppose that all propositions are bivalent,'[29] as it is case with Rescher's attempt. By making an explicit assumption that p_0 is *bivalent*, he modified Rescher's reconstruction, refered to as *Reconstruction R*, by postulating R_1a *only* for bivalent propositions, what makes it unnecessary also to assume the unrestricted version of the law.

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[29] *Ibid.*, p. 233.

One of the reasons for this motivation is consisted in assumed a fact that there is no historical reasons (supported by some tracks of evidence that would be show maintenance to this suggestion), for introducing this principle as acceptable and reasonable on this ground. For if it would be different, inviting of this obvious principle will be in some way historically supported by some evidence in available sources.

Several additional problems arise with Michael's way out in finding an appropriate reconstruction of MA, and also with his attack to that of Rescher. The most obvious, is connected with a problem of *number in premises* that would be enough for the argument. The additional problem, of no less importance, concerning a way of interpreting the first premiss, here taken over from Rescher, and suited in Michael form as D_1.

As the first, Michael believes that if we are reading Diodorus' second premiss in a temporal sense, than we can derive it as a logical outcome from the first premiss, and so, we do not need it in further reconstruction.[30] To show inconsistency of the premises of the M.A., it is well enough to use just formal representation of P_1 and P_3 in its derivation. For this reason, just P_1 and P_3 are *substantive* premises, whereas P_2 'is just a principle of logic' and 'just because it is a principle of logic, it is quite superfluous to assert it as a premiss.'[31] Even we could read P_2 in a logical sense, Michael defences a claim that there is no historical ground to do that, and that it is more appropriate to ascribe this attempt to Chrysippus, than to Diodorus himself.

In demonstration of P_2 as a logical consequence of R_1, he is following Rescher's *Reconstruction R*, where from R_1 we have derived

$R_1a \quad (t)(t')\{[\Diamond_{t'}p \wedge (t<t')] \supset T_t p\}$,

what is Rescher's above shoved step (8), that lays on applying the law of excluded middle to the first premiss. Then, Michael is adding a thesis of modal logic, $\Box(p) \supset \Diamond(p)$ - for all p, suited in Rescher's time-indexical notation as

$T_3 \quad \Box_t(p) \supset \Diamond_t(p) \quad$ for all p.

Now, he wants to demonstrate

$R_2 \quad (t)(t')\{[\Diamond_t p \wedge (t<t')] \supset \Diamond_{t'}p\}$

as an translation of P_2, given in its *temporal* form, as the statement implying from P_1. He gives this just for the case of derivation of $\Diamond_{t'}p$ from $\Diamond_t p$. Writing $t+d$ for t', given $\Diamond_t p$, he will derive $\Diamond_{t+d}\, p$ by a following procedure:

1. $\Diamond_t p$

[30] *Ibid.*, p. 235.

[31] *Ibid.*, p. 234.

2. $T_{n-d}p$ from 1, by R_1a
3. $\Box_{n+d}p$ from 2, by R_1
4. $\Diamond_{n+d}p$ from 3, by T_3.

For this reason, to reject temporal interpretation of P_2 would be to deny P_1, and, in effect, to reject Rescher's *Reconstruction R.*

The idea of inconsistency among Diodorus' premises, is laying on a simple supposition that not all premises can be true at the same time. This is basically failure on which is based starting Michael's assumption of the reconstruction, that there is mutual inconsistecy of the premises. Confussion arised in mistaken reading of Epictetus' source, where it is transparentlly stated that inconsistency is intended to mark the third proposition to be false, what he did by relying on the plausibility of the first two premises. We saw above that his result is obtained from problematic omission of the basic principle Diodorus' argument is intended – to produce a sound argument from two palusible premisses to the negation of the third. Even if his reasonong could be reflected as logically satisfactory, this failure left as at the same begining of the problem, that is, the problem of correspondence of some reconstruction with Diodorus' background intention. For these reasons Michael's reconstruction seems to be implausible and his result may be evaluated as questionable.

The Reconstruction of Denyer

Following Alexander[32] as a source for reconstruction, Denyer [1981] is accepting as the purpose of the argument to establish the biconditional „$\Diamond p \equiv (p \vee Fp)$," where one half, on his opinion, is more dubious than another, *e.g.,* $\Diamond p \rightarrow (p \vee Fp)$. Reading it through the information accessible from Epictetus, where „*Nothing is possible that neither is true nor will be,*"[33] as immediate conclusion of the MA, we have formally transformed last half of the biconditional, which is now $\neg[\Diamond p \wedge \neg(p \vee Fp)]$. For this form of conclusion is what is gained by refuting the third premiss, then, the third premiss is to be formalised in the way similar to its Prior's reading, as $\neg\neg[\Diamond p \wedge \neg(p \vee Fp)]$ (or simply, $\Diamond p \wedge \neg(p \vee Fp)$, that

[32] Döring, Fr. 135: „For **Diodorus** set down as possible only what either is or, in any event, will be (τὸ γὰρ ἢ ὂν ἢ ἐσόμενον πάντως δυνατὸν μόνον ἐκεῖνος ἐτίθετο). According to him, for me to be in Corinth was possible if I was in Corinth or if I was, in any event, going to be; if not, it was not even possible. And for the child to become literate was possible if he was, in any event, going to be. It is to establish this that **Diodorus'** Master Argument is posed."

[33] Döring, Fr. 131: „δυνατὸν εἶναι, ὃ οὔτ' ἔστιν ἀληθὲς οὔτ' ἔσται."

is, in Epictetus' words: '*There is something possible which neither is nor will be true*').[34]

Denyer intention is to show how Diodorus established the other half of his biconditional definition of possibility. Before he realised this task, he serves his 'the simplest reconstruction'[35] he can devise of the argument. Let we give his formal elaboration of the argument with comment of particular steps. The first side of 'Diodorus' biconditional' in his reconstruction has the following formal structure. The first premiss, which says that „*every past truth is necessary*," he is assuming in its common modern, as we saw in Prior's reconstruction. He believes that it „does not lead to any bizarre conclusion,"[36] for if we understood it as $Pp \rightarrow \Box Pp$, it leads just to known thing of tense logic such as $Pp \rightarrow GPp$. Hence, he used this form as

1 (1) $Pp \rightarrow \neg\Diamond\neg Pp$ **A**

while the second proposition now starts to be (2),

2 (2) $[H\Box(p \rightarrow q) \wedge \Box(p \rightarrow q)] \rightarrow (\Diamond p \rightarrow \Diamond q)$ **A**

which states that „*the impossible does not follow from the possible*." Now, he is deriving step (3) from (2),

2 (3) $[H\Box(p \rightarrow \neg q) \wedge \Box(p \rightarrow \neg q)] \rightarrow (\Diamond p \rightarrow \Diamond\neg q)$

$2, \neg q / q$

by substituting $\neg q$ instead of q throughout, to obtain it in such formulation which says that „*if negation of a proposition follows from possibility, then the negation of that proposition is also possible*." From (3), by contraposition and by application of the rule of double negation, he derives

2 (4) $[H\Box(q \rightarrow \neg p) \wedge \Box(q \rightarrow \neg p)] \rightarrow (\neg\Diamond\neg q \rightarrow \neg\Diamond p)$

$3, p \rightarrow q / \neg q \rightarrow \neg p$, DN

which says that „*if negation of a proposition follows from a necessity, then that proposition is impossible*." Now he tends to find a proposition from which $\neg p$ follows and which is, if true, then necessary. Such proposition is here assumed as

5 (5) $PG\neg p$ **A**

By rules of tense logic (he says 'of any sensible logic for tenses'[37]), he derives

5 (6) $\neg p$ $5, p \rightarrow HFp$

Steps (5) and (6) are laid on assumption that „*if it was once true that* p *would always be false thereafter, it follows that* p *is false now*."[38] The step (7) that

[34] *Ibid.*: „ὅτι ἔστι τέ τι δυνατόν, ὃ οὔτ' ἔστιν ἀληθὲς οὔτ' ἔσται."

[35] Denyer [1981], p. 42.

[36] *Ibid.* p. 37.

[37] *Ibid.* p. 42.

follows, is obtained from antecedent of (2), as a version of the principle of conditional proof 'adjusted to accord with Diodorus' account of conditionals,'[39] and also with appropriate substitution that incorporates (5) and (6) in it. His meaning is that (6) follows from (5)

2,5 (7) $H\Box(PG\neg p \rightarrow \neg p) \wedge \Diamond(PG\neg p \rightarrow \neg p)$ S.[5+6], Diod. $\rightarrow$

and its purpose is to obtain the consequent from (4) by rule of detachment and with appropriate substitution, what will become

2,5 (8) $\neg\Diamond\neg PG\neg p \rightarrow \neg\Diamond p$ 4,7, MPP, S. $q / PG\neg p$.

Now, he introduces as an assumption

9 (9) $\neg(p \vee Fp)$ **A**

to infer

9 (10) $PG\neg p$ 9, $p \wedge Gp \rightarrow PGp$

Transition from (9) to (10) is not possible without an additional assumption grounded on theorem-hypothesis of time as *discrete* and *without branching*.[40] From

……………………………………………

[38] We will show here explicitelly a transition from step (5) to step (6), which is obtained by the following way:

(5)	$\vdash$	$PG\neg p$	A
	$\vdash$	$p \rightarrow HFp$	A, *td*
	$\vdash$	$p \rightarrow H\neg\neg Fp$	DN
	$\vdash$	$p \rightarrow \neg P\neg Fp$	S.H/$\neg P\neg$
	$\vdash$	$p \rightarrow \neg PG\neg p$	S.$\neg F/G\neg$
	$\vdash$	$PG\neg p \rightarrow \neg p$	$p\rightarrow q / \neg q \rightarrow \neg p$
(6)	$\vdash$	$\neg p$	D.

[39] *Ibid.*, p. 43.

[40] The transition is obtained by the following procedure:

(9)	$\vdash$	$\neg(p \vee Fp)$	A
	$\vdash$	$p \wedge Gp \rightarrow PGp$	A, td
	$\vdash$	$\neg p \wedge G\neg p \rightarrow PG\neg p$	$p / \neg p$
	$\vdash$	$\neg p \wedge \neg Fp \rightarrow PG\neg p$	$G\neg / \neg F$
	$\vdash$	$\neg(p \vee Fp) \rightarrow PG\neg p$	$\wedge / \vee$
(10)	$\vdash$	$PG\neg p$	MPP

This last step is not possible without additional assumption of the *discreteness* of time. For, if we are asserting the last step $PG\neg p$, we are asserting that $G\neg p$ *was* true at some moment in the past. If we are assuming the assumption of *continuity*, between that moment and the present one, there would be a third moment. But since $G\neg p$ *was* true at some

(...)

(1) and (10), by rule of detachment and suitable substitution in (1), we can now infer

1,9 (11) $\neg\Diamond\neg PG\neg p$ 1,10 S.$Pp/PG\neg p$, MPP

what we will use to detach the consequent of (8), what now gives

1,2,9 (12) $\neg\Diamond p$ 8,11 MPP

that rests on undischarged assumptions (1), (2) and (9), that could now be discharged by the principle of conditional proof, what gives

1,2 (13) $\neg(p \vee Fp) \rightarrow \neg\Diamond p$ 12 CP

which rests just on assumptions (1) and (2). The result is equivalent to $\Diamond p \rightarrow (p \vee Fp)$, and this is the half of the definition of possibility Denyer was trying to establish. The result is also equivalent to negation of the third premiss, *e.g.*, $\neg[\Diamond p \wedge \neg(p \vee Fp)]$ – „NOT: *What neither is nor will be is possible*."

Denyer's 'innovation' involved in reconstruction is especially related with his reading of the second premiss. The second premiss of Denyer is constructed in opposition to that of Rescher and his 'temporal' approach to interpretation of the term 'follow.' Denyer bases his standpoint on Döring, Frgs. 141, 142, where – conditional is true *iff* neither *was* possible nor *is* possible for it to begin with a truth and end with a falsehood.

So, the claim that *q follows* (in Diodorean sense) from *p*, can be formalised as

$\neg P\Box(p \wedge q) \wedge \neg\Box(p \wedge \neg q)$,

what can be expressed in the more near and readable form as

$H\Box(p \rightarrow \neg q) \wedge \Box(p \rightarrow \neg q)$.

The second premiss also can be presented as

$[H\Box(p \rightarrow q) \wedge \Box(p \rightarrow q)] \rightarrow (\Diamond p \rightarrow \Diamond q)$,

... ...

moment before this third moment, than $\neg p$ was true at the intervening one. Now, if we have given only that $\neg(p \vee Fp)$, then it is not possible to assert that $\neg p$ *was* true in the past. It means that, on the assumption of *denseness* or *continuity*, $\neg(p \vee Fp)$ does not entails the last step in above inferring, *e.g.* $PG\neg p$. This reasoning obligate us to conclusion that there is no *third moment* between the last moment of the past and the present one at which we would be in position to claim that $\neg p$ was true.

Above transition from step (9) to step (10) also pressuposes not only the *discreteness* of time, but also its *linearity*. This two assumptions have to be taken together if we wish to assess its validity. If we are supposing first, but not second, *.i.e.* discreteness but not linearity, we could not obtain the step (10). Suppose that although time is discrete but that it has branching structure towards the future. Then at a node we could have $\neg(p \vee Fp)$ true without having truth of $PG\neg p$ at the last step of that node. At that node $\neg p$ could be *true* just as at all others which branches from it, but we also could have $\neg p$ as *false* at some nodes branshing out from the one that is immediately before that node.

what is here just a weaker its form derived from the commonplace thesis of modal logic

$\Diamond(p \rightarrow q) \rightarrow (\Diamond p \rightarrow \Diamond q)$.[41]

Denyer took this way in interpreting the second premiss since his opinion is based on Sextus' source[42] what he is reading as one that makes no strict difference between question on 'when conditional statements are true' and 'when the consequent of conditional follows from its antecedent.' The idea for this was grounded on the assumption that such formulation of Diodorus' thesis would be more favorable than simple claim with stright fatalistic association that q follows from p *iff* $\Box(p \rightarrow q)$.

The claim is made in opposition to Rescher's approach which sense has as its outcome '*permanent possibility*' – if proposition was once possible it remains possible forever after. It means that if '*Caesar is dictator*' was once true, then it was once possible. But if this proposition was once possible, it does mean that it is *now* possible. The claim that proposition which was once true is and will be true thereafter does not corresponding with Diodorus' account on possibility given in some other fragments,[43] where he renders possible as 'that which either is or will be true.' For in Rescher's form it would lead us to some bizarre consequences that 'anything that once was true either is or will be true again' – that is, $\mathrm{P}p \rightarrow (p \vee \mathrm{F}p)$. To be once true would be mean once possible; if anything was true, it was once possible; if it was once possible it either is possible now or will be possible; if it is not now possible than it will be possible thereafter, so 'If Caesar was once dictator, it is ether dictator now or will be dictator once more. For these reasons Denyer choses to connect Diodorus' definition of necessity with his account on conditionals and to amplify simple claim that 'q follows from p *iff* $\Box(p \rightarrow q)$' with the expanded clause (2) that includes $\mathrm{H}\Box(p \rightarrow q)$ „which is liable to seem redundant to those who have not yet been brought to accept his definition of necessity," for reduced expression would have „the grossly implausible consequence that validity changes with time."[44]

The rest half of the equivalence given above than would be as following:

1	(1)	$p \vee \mathrm{F}p$	**A**
2	(2)	p	**A**
2	(3)	$\Diamond p$	2, $\mathrm{A}td{:}p \rightarrow \Diamond p$, MPP

[41] It is obtained from definition of the strict implication, *Def.* $⥽ : a ⥽ b \equiv \Box(a \rightarrow b)$, and from the modal logic claim $(p ⥽ q) \rightarrow (\Diamond p \rightarrow \Diamond q)$

[42] Döring, Fr. 142.

[43] Döring, Frgs. 132, 134, 135, 138, 139.

[44] Denyer, *ibid.*, p. 40-1.

4	(4)	$\mathrm{F}p$	**A**
4	(5)	$\Diamond p$	4
1	(6)	$\Diamond p$	1(2,3),(4,5) $\vee$E
	(7)	$(p \vee \mathrm{F}p) \rightarrow \Diamond p$	1,6 CP

This was the way Denyer consolidates the other half and intended validity of equivalence $\Diamond p \equiv p \vee \mathrm{F}p$. Now he is able to compare the results with testimonies regarding relationship of this definition to other modal notions, that can be find in Boëthius.[45] The additional, corresponding list is as following:

$$\neg\Diamond p \equiv \neg p \wedge \neg \mathrm{F}p,$$
$$\Box p \equiv p \wedge \mathrm{G}p,$$
$$\neg\Box p \equiv \neg p \vee \mathrm{F}\neg p.$$

From these definitions Denyer wishes to emphasise two things. As the *first*, the reconstruction given above corresponds with Boëthius quotation that differentiates contrast between Diodorus and Philo approach to the question of valid conditionals. As the *second*, the reconstruction sayng us something more about assumed Diodorus' position concerning determinism and the future contingence problem. Denyer is here making obvious distinction between modal notions by showing that, in Diodorus system, p, $\Box p$ and $\Diamond p$ are not equivalent.

Suggested formulations makes apparent difference from the alleged 'common' Megarian conception reported in Aristotle,[46] concerning his attack to the conception holding that 'what is actual is possible.' Further, it also eliminates ascribing to Diodorus M.A. an intention of identifying the possibility with necessity, what is usual to those who attaching to Diodorus position more inclining toward defense the fatalistic conception. This is the base for Denyer to claim that Diodorus advocates position according to which 'the past is necessary' while 'the future is contingent.' He supports it by the fact that Diodorus hypothetical system has the thesis $\mathrm{P}p \rightarrow \Box\mathrm{P}p$ but it lacks corresponding symmetry with future oriented

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[45] Döring, Fr. 138 [= Boëthius, *Comm. in Aristotelis de interpr.*, sec. ed. 234, 10-235, 9 Meiser (1880); Diodorus Fr. 28 Giannantoni (part); **FDS** 988]: „Diodorus defines ***the possible*** as 'what is or will be', ***the impossible*** as 'what, being false, will not be true', ***the necessary*** as 'what, being true, will not be false', and ***non-necessary*** as 'what either is now (*false*), or will be, false'."

[46] In *Met.* Θ 1046 b 29-32 = Döring Fr. 130A: „E"σὶ δέ τινες οἵ φασι, οἷον Μεγαρικοί, ὅταν ἐνεργῇ μόνον δύνασθαι, ὅταν δὲ μὴ ἐνεργῇ οὐ δύνασθαι..." The same lines are present in Alexander's comment on the place [*in met.* 570, 25-30 Hayd. = Döring Fr. 130B], where Megarians are those who are equalising the actual and possible: „... οἱ Μεγαρικοὶ δύναμιν καὶ ἑνέργειαν ταὐτὸν ποιοῦσιν..."

thesis $Fp \rightarrow \Box Fp$ which, regarding definition of necessity, would immediately entail the thesis $Fp \rightarrow \Box GFp$.

For this position he serves several anchorages derived from the historical sources. Even having in mind that above claim has its opposition in Cicero's quotation of Diodorus in *De fato*,[47] where Diodorus standpoint is contrasted to that of Chrysippus, he believes that Cicero and Hieronymus are sole and mistaken places that serves us less persuasive and insignificant testimony about Diodorus' modal definitions. Denyer found more reeasonable to take into account some other sources that are agree in ascribing to Diodorus such definitions which does not pushing him toward 'fatalistic' side in the conditional debate and which are far from accepting the thesis that $Fp \rightarrow \Box Fp$. Hence, he gives advantage to three other ancient sources – Plutarch,[48] Alexander[49] and Boëthius[50] – where Diodorus account on the problem is given in contrast to the Stoics, Philo and Aristotle.

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[47] Döring, Fr. 132A; Cicero, *De fato*, 7, 13; **LS**38E; **FDS** 437: „For Diodorus says that only that is possible which either is true or will be true; that ***whatever will be is necessary***; and that whatever will not be is impossible. You [Chrysippus] say that even things which will not be are possible..." *Cf.* also, *ibid.* 9, 17 and compare it with Döring Frs. 133 (Cic. *ad fam.* 9, 4 < **FDS** 990) and 132B (Hieron., *adv. Pelag.* i 702, PL 23 p. 502 C-D; **FDS** 991). Döring Fr. 132B is almost a bare transcription of Dörong Fr. 133, where Hieronymus uncritically rescribes Cicero's words.

[48] Döring, Fr. 134 Plutarch, *De Stoic. repugn.*, 1055D-E; < **FDS** 1008; **SVF** ii, 202: „Is it not clear that [Chrysippus'] doctrine of the possible contradicts his doctrine of fate? For if the possible is not what either is or will be true, as **Diodorus** states, but everything is possible that admits of happening even if it will not happen, then many of those things will be possible which according to insuperable and unviolable and victorious Fate will not happen. Thus either fate's power will dwindle, or, if fate is that what Chrysippus believes it to be, that which admits of happening will often become impossible. For all that is true will necessarily be, being compelled by supreme necessity, but all that is false will be impossible, the strongest cause preventing it from becoming true."

[49] Döring, Fr. 135 = Alex. Aphr., *In an. pr. i* 183,34-184,10 [Wallies] = Diodorus Fr. 27 Giannantoni (part); **LS** 38B: „He [Aristotle, *An. pr.* i 34*a* 12] may possible be talking also about the issue 'What things are possible?,' and about the so-called 'Diodorean' answer (ὃ Διοδώρειον λέγεται), 'What either is or will be' (ὃ ἢ ἔστιν ἢ ἔσται). For **Diodorus** set down as possible only what either is or, in any event, will be (τὸ γὰρ ἢ ὂν ἢ ἐσόμενον *πάντως δυνατὸν μόνον ἐκεῖνος ἐτίθετο*). According to him, for me to be in Corinth was possible if I was in Corinth or if I was, in any event, going to be; if not, it was not even possible. And for the child to become literate was possible if he was, in any event, going to be. It is to establish this that **Diodorus'** Master Argument is posed. And likewise about **Philo's** answer. This was: 'That which is predicated in accordance with the bare fitness of the subject, even if it is prevented from coming about by some necessary external factor.' On this basis he said that it was possible for chaff in atomic dissolution to be burnt, and

(...)

The way of Denyer's assortment of the sources is based on the analogy principle, as well as on his logical intuition, rather than on the rendering some particular source on the certain convincing scale of authenticity. Correspondence in three authors, he chooses as more appropriate for his purposes, could be equally hard to assert as suitable for Diodorus' position. It is known fact that Boëthius, opposite to Cicero, wrote from memory rather than with many books available,[51] and that Cicero, with his approach to sources, sometimes serve us more direct informations about persons and opinions he deals with.[52] Denyer's reasons are formal and he holds that Cicero's attributes an absurd and not appropriate view to Diodorus. For Cicero is taking as Diodorus' position $\mathrm{F}p \rightarrow \Box\mathrm{F}p$ to be equivalent to $\neg\mathrm{F}p \rightarrow \Box\neg\mathrm{F}p$, even the last indeed can be incorporated as a thesis in Diodorus' 'system,' while the corresponding thesis about the past, $\neg\mathrm{P}p \rightarrow \Box\neg\mathrm{P}p$, does not belong to it. The mistake is indisputable for Diodorus starting assumption consists in making demarcation line between nature of the past and future oriented 'propositions.'

However it is, according to Denyer, in the question of time dependent propositions and in the problem concerning the passing of time, excepting Cicero, other three sources are unique in ascribing to Diodorus the asymmetry of time conception, and thereby such standpoint on the determinism that encloses in the system the concept of contingence. As it seems, the nature of contingence in Diodorus' case corresponds to an approach that covers conception of contingence as unactualised possibilities, or such possibilities that are 'not yet' realised. It looks as straight implication of the way he is basing his interpretation of the second Diodorus' premiss. So that the system he is exposing represents the 'synthesis of determinism with indeterminism,'[53] for it contains both deterministic thesis that $\Diamond$

... ...

likewise chaff at the botom of the sea, while it was there, even though the circumstances necessarily prevented it."

[50] Döring *Frgs.* 138 and 139.

[51] For the circumstances under which Boëthius wrote, *cf.* Sharples [1991], p. 41. Also, *cf. fns.* 2 and 3 at the same place, where there are references to wider informations concerning the problem of Boëthius originality.

[52] Some authors saw Boëthius' source (Döring, Frg. 138) as misleading, for his misunderstanding of Diodorus' definition of modality. For example, McKirahan, following modern tradition of 'the normal interpretation' (the one accepted by Kneale [1962] and Mates [1961]), claims that this interpretation as one that provides compatible definitions of the modal 'squere' while Boëthius went wrong in his interpretation. The explanation McKirahan assumes is consisting in possible fact that Boëthius „may well misunderstod the force of the particles, taking them as causal and not simply attributive," since „he was faced with a Greek account which gave Diodorus' definition of 'necessary' as ὅ, ἀληθὲς ὄν, οὐκ ἔσται ψευδές and that of 'impossible' as ὅ, ψευδές ὄν, οὐκ ἔσται ἀληθὲς" (McKirahan [1979], p. 225 and p. 248. *fn.* 8).

[53] Denyer, *ibid.*, p. 52.

$\mathrm{F}p \equiv \mathrm{F}p$, as well as indeterministic one, for it lacks its necessity correspondence that is covered with the thesis $\Box\mathrm{F}p \equiv \mathrm{F}p$ – the system, according to Denyer, „holds that the only propositions about future which are possible are those which are true, but it does not hold that only propositions about the future which are true are those which are necessary."[54]

At the same place, as an illustration for this sort of contingency, Denyer serves the following example. Consider the proposition '*I shall die at noon on 27 September 2020*' which can be formalised as $\mathrm{F}(p \wedge q)$, where p is an abbreviation for '*I am now dying*' while q covers fixed date proposition '*It is now noon on 27 September 2020.*' If above complex proposition is indeed true, then, according to Denyer, there are at least three ways that are corresponding to the ***indeterministic*** side of Diodorus' position, which includes in the system and expresses the contingent nature of my death: $\Diamond(p \wedge q) \wedge \Diamond\neg(p \wedge q)$, $\Diamond\mathrm{F}(p \wedge q) \wedge \Diamond\neg\mathrm{F}(p \wedge q)$, $\Diamond\mathrm{F}(p \wedge q) \wedge \Diamond\mathrm{F}\neg(p \wedge q)$. From the other side, or the ***deterministic*** side of system, given proposition can be expressed by the following five ways that does not coincide with *my death* as some contingent matter:

$\neg\Diamond(p \wedge \neg q)$, $\neg\Diamond(\neg p \wedge q)$, $\neg\Diamond\mathrm{F}(p \wedge \neg q)$, $\neg\Diamond(\neg p \wedge q)$, $\Box\mathrm{FP}(p \wedge q)$.

In many points Denyer's reconstruction seems more convincing than that of Hintikka, Rescher or Michael. He tries to incorporate in his reconstruction Diodorus' intended purpose of The Master Argument. Besides, he includes in it wider scope of his logical 'system' as well as relative metaphysical background of his assumed 'theory.' The reconstruction well covers Diodorus conception concerning the nature of conditionals together with relevant aspects of his understanding of time as discrete and linear, even in this last item it is hard to give appropriate answer to the question – what could be Diodorus' representation of the contingency in such ordered and 'dual' system of truth that permits unactualised events to be sorted as contingent matters.

From the other side, we are partly loosing the direct answer to the problem concerning the nature of Aristotelian and Diodorean controversy on modalities if, at the end, according to this reconstruction, such controversy is present further at all. The idea of reformulation of the second premiss, even seems as the right way of escaping insufficiencies of former reconstructions, leave open many shady places that lead us back to toward the revision of the legitimate principles we may suppose that Diodorus operated with, or that were at his disposition in forming the argument.

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[54] Ibid.

McKirahan - Critical Revision of Prior's Conception

In his reconstruction [1979], McKirahan wishes to show that Diodorus failure was not in the fact of defending the conception of temporal atomism, as Prior used to hold, but rather that insufficiencies of MA are contended in the presumptions covered by other premises. Mainly following the reconstruction of Prior, and willing „to set out Prior's reconstruction somewhat more fully than he did," and „to show ambiguities in two his premises."[55]

McKirahan deals with number of the time situations for which Diodorus' premises are obtaining, or failing to obtain, their value. According to these steps in analysis, he is coming to the concluding point, that there is a flaw in the MA, as it was reconstructed by Prior, and that Prior's approach pays just partly, for *restricted* domain of Diodorus' intended time situations. Analysing these time situations, as well as the logical interconnections among those with a relevance for the argument, he also throws more light to possible Diodorus' meaning of 'proposition' that he is using in own constructing the way of reasoning in the argumentation.

Another fact of value is in McKirahan's trying to find some helpful remarks that could be obtained across comparing MA with The Moving Argument of Diodorus. He saw that these two arguments have many items in common, what could serve us to better understand them as different aspects of a unique theoretical conception.

As the last achievement of interest that McKirahan did in his analysis concerning the question of MA as an argument for determinism. Here he is willing to show how far is the conception of Prior successful reconstruction and does it really implies deterministic outcomes. In that sense, giving his concluding remarks, McKirahan agrees with M. Blanché [1965], repeating his words that this Diodorus argument *does not imply any 'metaphysical' necessity*. For these reasons, the form of it we could find in Prior, fails if it was given with intention to support the deterministic conception.

McKirahan starting position is consisting in the claim that Diodorus was concerned with specifical sense of 'proposition,' or bearers of truth-values, that differs from currently use of the term. The main difference consists in ability of propositions to take on the *different truth-values at different times*. Accordingly, Diodorus' definition covers not just elements of time, but also appropriate relation to truth of modalities.

In his interpretation McKirahan tends to emphasise and to show relevance of the time-ingredients for truth of propositions. His main idea is in a modeling having in mind Mates' understanding of Diodorean-type of 'proposition' as considered of

[55] McKirahan [1979], p. 225.

the corresponding *functions* formed by adding 'at *t*' to each proposition.[56] The main problem with such interpreted Diodorus' propositions is, according to McKirahan, that 'It is now day' is not shorthand for a function which yields different propositions at different times, but that „it is a single thing which may have different truth-values at different times, depending on the time at which it is expressed."[57]

According to the above, McKirahan gives a symbolic expressions of, in a such way understood, Diodorus' idea of 'truth-at-a-time:'

$T_t(p)$ - 'at time *t*, "proposition" *p* is true;'

$\neg T_t(p)$ - 'at time *t*, "proposition" *p* is false;'

$T_t T_{t'}(p)$ - 'at time *t*, it is true that "proposition" *p* is/was/will be **true** at time *t*;'

Since the truth-values of Diodorus' 'propositions' can change over the time, so can their modalities. Hence, he is introducing notation that covers attaching to modalities the notion of their being true 'at a time.'

$T_t\Box_{t'}(p)$ - 'at time *t*, it is true that "proposition" *p* is/was/will be **necessary** at time *t'*;'

Above proposition is based on modalities that are symbolically formed by the following modal expressions and their first-order equivalencies, equipped with attached indexical reference of their 'being at a time:'

$\Box_t(p) \equiv (t')[t<t' \wedge T_{t'}(p)]$ - 'at time t "proposition" *p* is **necessary**;'

$\Diamond_t(p) \equiv (\exists t')[t<t' \wedge T_{t'}(p)]$ - 'at time t "proposition" *p* is **possible**;'

$\neg\Box_t(p) \equiv (\exists t')[t<t' \wedge \neg t_{t'}(p)]$ - 'at time t "proposition" *p* is **non-necessary**;'

$\neg\Diamond_t(p) \equiv (t')[t<t' \wedge \neg T_{t'}(p)]$ - 'at time t "proposition" *p* is **impossible**;'

Prior's symbols are translated so to fit the basic McKirahan intention to make transparent the different first-order outcomes of Priorean assumptions. Here, '*n*' will represent the time of expressing the 'proposition,' and conventionally has to be understood as analogous to 'now.'

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[56] Mates [1961], p. 36. In addition, Mates gives several examples. For example, such proposition of Diodorean-type would be 'Snow is white at *t*,' where '(*t*)(Snow is white at *t*)' would represent the proposition „'Snow is white' is always [ἀεί] true."

[57] McKirahan *ibid.*, *p*. 226.

Priorean formulation		The first-order equivalencies in McKirahan's interpretation
F*p*	'It *will* be the case that *p*'	$(\exists t)[n<t \wedge T_t(p)]$
P*p*	'It *has been* the case that *p*'	$(\exists t)[t<n \wedge T_t(p)]$
H*p*	'It *has always been* the case that *p*'	$(t)[t<n \supset T_t(p)]$

In making explicit the implicit time-references of Prior's reconstruction, McKirahan obtains several possible interpretations of assumptions Prior supposed as necessary for inferring its conclusion, where some of them has to be eliminated as untenable, while the others could be understood in their more or less general sense.

Beside the first-order translations of Prior's assumptions as we know it from the chapter on his reconstruction, McKirahan adds some further inferences that are valid in Diodorus 'system' of tense logic, but which make no use of his definition of the modalities. Some of them are used in developing his own analyse of Prior's reconstruction:

(e) $T_{t'}(p) \supset (t)[T_t T_{t'}(p)]$

(f) $T_{t''}T_{t'}(p) \supset T_{t'}(p)$

(g) $T_t\{(\exists t')[n<t' \wedge T_{t'}(p)]\} \equiv (\exists t')[n<t' \wedge T_t T_{t'}(p)]$

(h) $T_t\{(\exists t')[t<t' \wedge T_{t'}(p)]\} \equiv (\exists t')[t\langle t' \wedge T_t T_{t'}(p)]$

(i) $T_n\{(t)[t<n \supset T_t(p)]\} \equiv (t)[t<n \supset Tt(p)]$

A) Priorean first assumption

(a) $Pp \supset \neg\Diamond\neg Pp$ — *When anything has been the case, it cannot not have been the case.*

according to the opening its explicit reading then can be presented as:

(a') $(\exists t)[t<n \wedge T_t(p)] \supset \neg\Diamond_n\neg(\exists t)[t<n \wedge T_t(p)]$.

This verson[58] of Priorean (a) takes as its sense the following: 'If *p* has ever previous *to now* been true, then it cannot be the case that *p* has never previous *to now* been

[58] This is not the only one possible way of translating Priorean (a). There is a version due to Kneale where she interprets it as statement where truth and necessity *coincide*, so that she takes (a) to be $[(t < n) \wedge \neg T_t(p)] \supset \neg\Diamond_t(p)$, which is equivalent to $[(t < n) \wedge \neg T_t(p)] \supset \Box_t(p)$, what McKirahan [*ibid*., p. 251] holds as not a plausible way to interpret it, since,

(...)

true.' If we wish to make this statement generalised for *all* times, and not just '*now*,' then we have as its appropriate form 'If *p* has ever previous *to any time* been true, then it cannot be the case that *p* has never previous *to any time* been true' and its corresponding symbolical expression:

(a'') $(t)(\exists t)[t'<t \wedge T_{t'}(p)] \supset \neg\Diamond_t\neg(\exists t')[t'<t \wedge T_{t'}(p)]$.

B) The same way of approach is applicable to Prior's second premiss

(b) $\neg\Diamond q \supset [\Box(p \supset q) \supset \neg\Diamond p]$ *If anything is impossible, then anything that necessarily implies it is impossible.*

The premiss has also its more and less general version depending on the way of reading its time-reference. In the same manner as above, the less general version than would be

(b') $\neg\Diamond_n(q) \supset \{\Box_n[T_n(p) \supset T_n(q)] \supset \neg\Diamond_n(p)\}$

or, 'If anything *is now* impossible, then anything that *now* necessarily implies it *is now* impossible.' In its more general version, Prior's (b) would be sounds as 'If anything *is ever* impossible, then anything that then necessarily implies it is then impossible,' or in its corresponding symbolical translation,

(b'') $(t)[\neg\Diamond_t(q) \supset \{\Box_t[T_t(p) \supset T_t(q)] \supset \neg\Diamond_t(p)\}]$.

With Prior's (c) and (d) there are connected several questions, what had been shown in previous chapters, concerning the substantive number of premisses that are familiar and necessary for adequate and primeval looking of the argument. Even, for example, Rescher shares an opinion that items offered by Sextus are well enough to cover presentation of the original form of the argument, reconstructions vary in namber of used assumptions. Version here discussing relies on accepting Becker's observation about necessity of introducing the additional elements in infering the intended purpose of the argument, what Prior includes in later versions of his proposal. Such additional assumptions, as premisses discussed earlier, also admits rival approaches.

C) Prior is attributing to Diodorus as familiar an extra assumption that serves several versons of its first-order reading, and involves ambiguity, according to McKirahan, where it is „one that proves fatal to the validity of the Master Argument as reconstructed by Prior."[59] This is

... ...

according to him, the *necessity*, the *true* and the *possible* coincide only to the following extent: $T_{t'}(p) \supset (t)[\Box_t T_{t'}(p)]$, $T_{t'}(p) \supset (t)[T_t T_{t'}(p)]$ and $T_{t'}(p) \supset (t)[\Diamond_t T_{t'}(p)]$.

[59] McKirahan, *ibid.*, p. 231.

(c) $p \supset \mathrm{HF}p$ *When anything is the case, it has always been the case that it will be the case.*

In the *less* **general** its first-order version, by adding appropriate indexicals rendering over the different domains of time, we have two interpretations. That is, (c') 'If anything is *now* true, then before *now* it was always going to be true' and (c") 'If anything is at *any* time true, then before *that time* it was always going to be true.' Hence, we have also different interpretations corresponding to answer that can be given to the question 'Future *to when* was it always going to be true?' Hence, from the standpoint of less general version, we here have two answers to the question – 'future to *any time before now*' and '*future to now*.' That is,

$\mathrm{T}_n(p) \supset (t)[t<n \supset \mathrm{T}_t\{(\exists t')[t<t' \wedge \mathrm{T}_{t'}(p)]\}]$,

by the first version of substituting part contained in the consequent concerning Prior's F*p*, by (h) that is generalised version, we are obtaining

(c'$_1$) $\mathrm{T}_n(p) \supset (t)[t<n \supset (\exists t')[t<t' \wedge \mathrm{T}_t\mathrm{T}_{t'}(p)]\}$,

and in respect to the '*future to now*,'

$\mathrm{T}_n(p) \supset (t)[t<n \supset \mathrm{T}_t\{(\exists t')[n<t' \wedge \mathrm{T}_{t'}(p)]\}$,

by substituting the consequent part in expression by (g), we have

(c'$_2$) $\mathrm{T}_n(p) \supset (t)[t<n \supset (\exists t')[n<t' \wedge \mathrm{T}_t\mathrm{T}_{t'}(p)]\}$.

The ***more*** **general** versions, in accordance with the above questions, then would be

(c"$_1$) $(t)[\mathrm{T}_t(p) \supset (t')[t'<t \supset \{(\exists t'')[t'<t'' \wedge \mathrm{T}_{t'}\mathrm{T}_{t''}(p)]\}]$,

and

(c"$_2$) $(t)[\mathrm{T}_t(p) \supset (t')[t'<t \supset \{(\exists t'')[t<t'' \wedge \mathrm{T}_{t'}\mathrm{T}_{t''}(p)]\}]$.

(c'$_1$) saying that 'If *p* is *now* true, it *has always been* true that *p would be* true,' and is true even for those 'propositions' and is true even for those propositions which have specified the time of occurrence they are referring to, as well as if they are spoken precisely at that moment. The same thing pays for (c"$_1$), while (c'$_2$) and (c"$_2$), saying that 'If *p* is *now* true, it *has always been* true that *p will be* true at some future time, that is future *to now*,' are not valid for some cases of 'Diodorean' propositions that can change their truth values, since they have the last moment of their truth.

D) The same ambiguity that rising in the previous case with (c), can be found with Prior's assumption (d) formulated as

(d) $(\neg p \wedge \neg\mathrm{F}p) \supset \mathrm{P}\neg\mathrm{F}p$ *When anything neither is nor will be the case, it has been the case that it will not be the case.*

The *less* **general** version of interpretation concerning this proposition is 'If anything neither *is* nor *is going to be* true, then at some time before now it *was* true to say that it *was* never *going to be* true.' Respecting the question '*After what* *time* was it never going to be true?' if we understand it in the sense '*after that time before now,*' we have as the first

$\neg T_n(p) \wedge \neg(\exists t)[n<t \wedge T_t(p)] \supset (\exists t)[t<n \wedge T_t\{\neg(\exists t')[t<t' \wedge T_{t'}(p)]\}$

what, by applying the substitution based on (h), gives ***less*** **general** version

(d'$_1$) $\neg T_n(p) \wedge \neg(\exists t)[n<t \wedge T_t(p)] \supset$
$(\exists t)\{t<n \wedge \neg(\exists t')[t<t' \wedge T_t T_{t'}(p)]\}$,

while, as the second, corresponding to the sense based on the phrase '*after now,*' will give

$\neg T_n(p) \wedge \neg(\exists t)[n<t \wedge T_t(p)] \supset (\exists t)[t<n \wedge T_t\{\neg(\exists t')[n<t' \wedge T_{t'}(p)]\}$

or, by applying the substitution based on (g)

(d'$_2$) $\neg T_n(p) \wedge \neg(\exists t)[n<t \wedge T_t(p)] \supset$
$(\exists t)\{t<n \wedge \neg(\exists t')[n<t' \wedge T_t T_{t'}(p)]\}$.

The ***more*** **general** version of it is 'If anything is *at any time* not true and future to that time will not be true, than at some time *prior* to that time it *was* true to say that it *was* never *going to be* true,' has its corresponding forms

(d''$_1$) $(t)(\neg T_n(p) \wedge \neg(\exists t')[t<t' \wedge T_{t'}(p)] \supset$
$(\exists t')\{t'<t \wedge \neg(\exists t'')[t'<t'' \wedge T_{t'} T_{t''}(p)]\}$,

and, in respect to the second,

(d''$_2$) $(t)(\neg T_n(p) \wedge \neg(\exists t')[t<t' \wedge T_{t'}(p)] \supset$
$(\exists t')\{t'<t \wedge \neg(\exists t'')[t<t'' \wedge T_{t'} T_{t''}(p)]\}$.

Quite like as above, in the cases (c'$_1$) and (c''$_1$), propositions that have their form corresponding to (d'$_2$) and (d''$_2$), are true. (d'$_2$) says that 'If *p* is not *now* true and never *will* be true, then at some time *prior to now* it *was* true to say that *p* would never be true *later than now*.' The nature of its value is similar to related cases noted for (c'$_1$), where the reference is fixed with some strictly quoted moment, and which truth value changes after that moment. The same situation is with (d''$_2$), while (d'$_1$) – which says that 'If *p* is not *now* true and never *will be* true, then *at some later time prior to now* it *was* true to say that *p would never be* true *later than that time*' – is not generally true for there can be *no earliest time* from which a 'proposition' is false.

(**Z**) The conclusion given at Prior,

(z) $\neg p \wedge \neg Fp \supset \neg \Diamond p$ — *When anything neither is nor will be the case, it is impossible,*

like previously reconsidered assumptions, has also two its possible reading, so that the ***less*** **general** version would be 'What is not *now* and is not *going to be* true is *now* impossible,' or (z') $\neg T_n(p) \wedge \neg(\exists t)[n<t \wedge T_t(p)] \supset \neg\Diamond_n(p)$,

while ***more*** **general** version would state that 'What is *at some time* not true *will* never *future to that time* be true, it is *at that time* impossible.' Or, symbolically,

(z'') $(t)\{\neg T_t(p) \wedge \neg(\exists t')[t<t' \wedge T_{t'}(p)] \supset \neg\Diamond_t(p)\}$.

THE PROOF OF:[60]

PRIOR - EXTENDED VERSION -	**MCKIRAHAN** - THE LESS GENERAL VERSION -
(a) $Pp \supset \neg\Diamond\neg Pp$	(a') $(\exists t)[t<n \wedge T_t(p)] \supset \neg\Diamond_n\neg(\exists t)[t<n \wedge T_t(p)]$
(b) $\neg\Diamond q \supset [\Box(p \supset q) \supset \neg\Diamond p]$	(b') $\neg\Diamond_n(q) \supset \{\Box_n[T_n(p) \supset T_n(q)] \supset \neg\Diamond_n(p)\}$
(c) $p \supset HFp$	(c) $T_n(p) \supset (t)[t<n \supset T_t\{(\exists t')[t<t' \wedge T_{t'}(p)]\}]$
(d) $(\neg p \wedge \neg Fp) \supset P\neg Fp$	(d'$_2$) $\neg T_n(p) \wedge \neg(\exists t)[n<t \wedge T_t(p)] \supset (\exists t)\{t<n \wedge \neg(\exists t')[n<t' \wedge T_tT_{t'}(p)]\}$
RL: $\vdash \alpha \rightarrow \vdash \Box\alpha$	**RL'**: $(t)[T_t(p)] \supset \Box_n(p)$

(1) $(p \supset q) \supset [(q \supset r) \supset (p \supset r)]$, the principle of syllogism

(2) $[p \supset (q \supset r)] \supset [q \supset (p \supset r)]$, the law of commutation

(3) $P\neg Fp \supset \neg\Diamond\neg P\neg Fp$	(3') $(\exists t)\{t<n \wedge \neg(\exists t')[n<t' \wedge T_tT_{t'}(p)]\} \supset \neg\Diamond_n(t)\{t<n \supset (\exists t')[n<t' \wedge T_tT_{t'}(p)]\}$
from (a): $p/\neg Fp$;	from (a') by sub. $\neg\exists/\forall\neg$, and from equival. $\neg(\exists t)[t<n \wedge T_t(p)] \equiv (t)[t<n \supset T_t(\neg p)]$, and then by sub. $p/\neg(\exists t')[n<t' \wedge T_tT_{t'}(p)]$
(3°) $P\neg Fp \supset \neg\Diamond HFp$ from (3), by $\neg P\neg \equiv_{def} H$	
(4) $\neg p \wedge \neg Fp \supset \neg\Diamond HFp$	(4') $\neg T_n(p) \wedge \neg(\exists t)[n<t \wedge T_t(p)] \supset \neg\Diamond_n(t)\{t<n \supset (\exists t')[n<t' \wedge T_tT_{t'}(p)]\}$

[60] The steps suffixed by sign '°' are those additionally explcitely developed.

from (d) and (3°) by (1): $\neg p \wedge \neg Fp/p$; $P\neg Fp/q$; $\neg\Diamond HFp/r$;

Step (4') we have from (d'_2) and (3') by (1), where: $p/\neg T_n(p) \wedge \neg(\exists t)[n<t \wedge T_t(p)]$; $q/(\exists t)\{t<n \wedge \neg(\exists t')[n<t' \wedge T_tT_{t'}(p)]\}$; $r/\neg\Diamond_n(t)\{t<n \supset (\exists t')[n<t' \wedge T_tT_{t'}(p)]\}$

(4°) $\neg\Diamond HFp \supset [\Box(p \supset HFp) \supset \neg\Diamond p]$

(4'°) $\neg\Diamond_n(t)\{t<n \supset (\exists t')[n<t' \wedge T_tT_{t'}(p)]\} \supset \{\Box_n[T_n(p) \supset (t)\{t<n \supset (\exists t')[n<t' \wedge T_tT_{t'}(p)]\}] \supset \neg\Diamond_n(p)\}$

from (b): q/HF;

Step (4'°) we have from (b') and (i): first we do sub. in (b') $q/(t)\{t<n \supset (\exists t')[n<t' \wedge T_tT_{t'}(p)]\}$, and then $\neg\Diamond_n(t)\{t<n \wedge (\exists t')[n<t' \wedge T_tT_{t'}(p)]\} \supset \{\Box_n[T_n(p) \supset T_n(t)\{t<n \supset (\exists t')[n<t' \wedge T_tT_{t'}(p)]\}] \supset \neg\Diamond_n(p)\}$, and then we are applying equivalence (i) $T_n\{(t)[t<n \supset T_t(p)]\} \equiv (t)[t<n \supset T_t(p)]$, *i.e.* here it is: $T_n(t)\{t<n \supset (\exists t')[n<t' \wedge T_tT_{t'}(p)]\}] \equiv (t)\{t<n \supset (\exists t')[n<t' \wedge T_tT_{t'}(p)]\}]$

(5) $\neg p \wedge \neg Fp \supset [\Box(p \supset HFp) \supset \neg\Diamond p]$

(5') $\neg T_n(p) \wedge \neg(\exists t)[n<t \wedge T_t(p)] \supset \{\Box_n[T_n(p) \supset (t)\{t<n \supset (\exists t')[n<t' \wedge T_tT_{t'}(p)]\}] \supset \neg\Diamond_n(p)\}$

from (b), *i.e.* (4) and (4°) by (1): $\neg p \wedge \neg Fp/p$; $\neg\Diamond HFp/q$; $\Box(p \supset HFp) \supset \neg\Diamond p/r$

Step (5') we have from (4') and (4'°) [obtained from (b')] by following sub.'s in (1): $p/\neg T_n(p) \wedge \neg(\exists t)[n<t \wedge T_t(p)]$; $q/\neg\Diamond_n(t)\{t<n \supset (\exists t')[n<t' \wedge T_tT_{t'}(p)]\}$; $r/\{\Box_n[T_n(p) \wedge (t)\{t<n \wedge (\exists t')[n<t' \wedge T_tT_{t'}(p)]\}] \wedge \neg\Diamond_n(p)\}$

(6) $\Box(p \supset HFp)$

($6'_1$) $\Box_n[T_n(p) \supset (t)\{t<n \supset (\exists t')[t<t' \wedge T_tT_{t'}(p)]\}]$

from (c) by RL

Step ($6'_1$) we have by applying RL' to (c'_1)

($6'_2$) $\Box_n[T_n(p) \supset (t)\{t<n \supset (\exists t')[n<t' \wedge T_tT_{t'}(p)]\}]$.

(6°) $\{\neg p \wedge \neg Fp \supset [\Box(p \supset HFp) \supset \neg\Diamond p]\} \supset \{\Box(p \supset HFp) \supset [(\neg p \wedge \neg Fp) \supset \neg\Diamond p]\}$

from (5) by (2): $\neg p \wedge \neg Fp/p$; $\Box(p \supset HFp)/q$; $\neg\Diamond p/r$;

(6°°) $\Box(p \supset HFp) \supset [(\neg p \wedge \neg Fp) \supset \neg\Diamond p]$

from (6) and (6°) by MPP

(z) $\neg p \wedge \neg Fp \supset \neg\Diamond p$

from (5) and (6°°) by MPP

Step (6_2') we have by applying RL' to (c'_2)

(6'°) $\neg T_n(p) \wedge \neg(\exists t)[n<t \wedge T_t(p)] \supset \{\Box_n[T_n(p) \supset (t)\{t<n \supset (\exists t')[n<t' \wedge T_tT_{t'}(p)]\}] \supset \neg\Diamond_n(p)\} \supset \Box_n[T_n(p) \wedge (t)\{t<n \wedge (\exists t')[n<t' \wedge T_tT_{t'}(p)]\}] \supset \neg T_n(p) \wedge \neg(\exists t)[n<t \wedge T_t(p)] \supset \neg\Diamond_n(p)$

from (5') by (2), by applying (6_2') for *q*;

(6'°°) $\Box_n[T_n(p) \wedge (t)\{t<n \wedge (\exists t')[n<t' \wedge T_tT_{t'}(p)]\}] \supset \neg T_n(p) \wedge \neg(\exists t)[n<t \wedge T_t(p)] \supset \neg\Diamond_n(p)$

MPP from (5') i (6'°)

(z') $\neg T_n(p) \wedge \neg(\exists t)[n<t \wedge T_t(p)] \supset \neg\Diamond_n(p)$

The conclusion (z) $\neg p \wedge \neg Fp \supset \neg\Diamond p$, according to Prior, is obtained from (2) by sub. $\neg p \wedge \neg Fp/p$; $\Box(p \supset HFp)/q$; $\neg\Diamond p/r$ to yield $(x \supset (y \supset z))$, where $x =_{def}$ formula (5), $y =_{def}$ formula (6), $z =_{def}$ formula (z);

In McKirahan's interpretation of Prior we have two, here framed, versions of (6), obtained by applying RL' to (c'_1) and (c'_2) accordingly. However, following the same reasoning, as was given in the comment on the item that pay for extended versons of (c) and (d), we can see that ($6'_1$) is in general true, while ($6'_2$) is not.

We saw that (5') is equivalent to $p \supset (q \supset r)$ in (2) so that the result of appropriate substitution has to be equivalent to $p \supset r$, what represents intended form of (z). Since *q* in (5') is equivalent to ($6'_2$), we are obtaining (z') across form of *q* which is false, so that the truth of (z') is not well established. In the case where *q* is replaced by generally true sequence ($6'_1$), we have, as result of appropriate replacing in (2), a valid implication ($p \supset r$) as consequent, *i.e.* $(6'_1) \supset (z')$. However, the antecedent is no more equivalent to (5'), based on acceptable consequence of (d'_2), but on unacceptable one given in (d'_1). So that the kernel that generates the problem of acceptability of reconstruction MA is not possible to escape in any of both alternative ways in Prior's, extended and explicated, reconstruction.

As we said earlier, the another contribution of McKirahan is consisting in the fact that he saw the chain that connects MA with other aspects of Diodorus' possible declarations concerning time. Here it is an argument against existence of motion, presented by Sextus,[61] imputing him ideas familiar to the tradition of Parmenides and Melissus. Even Sextus notes there that Diodorus 'is in agreement' with such predecessors in sharing an opinion 'that no moving thing will exist,' he adds in addition Diodorus 'slight' distinction on the mater, that „according to him something *has moved* [κεκινῆσθαι μέν τι κινεῖσθαι δὲ μηδὲ ἕν]." The quoted place is in accordance with another, where Sextus repeats Diodorus' claim that „although nothing *is* moving it none the less *is moved* (or, has moved)."[62] Sextus' text continues with a few examples that illustrates Diodorus' position.

The kernel of this conception is also in agreement with testimonies we have about comments concerning introducing of some necessary speech prescriptions for escaping ambiguities. Besides, this makes open way in the approaching to interpretations that assorts him under the cover of *temporal atomism*, usually ascribed as familiar to the Epicureans,[63] even some questions, concerning this direction of reading the available testimonies, still stay widely open in McKirahan interpretation.

An interesting thing that McKirahan recognised under the scope of his (extended) interpretation of Prior's reconstruction, is that Diodorus left elbow room for conflict between (c'_1) and (c'_2) and further outcomes, suited under $(6'_1)$ and $(6'_2)$. In example with the ball touching the roof, Diodorus probably wishes to express his position of what he understood as legitimate meaning of 'proposition.' This also seems that fills the gap rises in Prior's reconstruction. Elimination of (some) the present-tense uses of verbs in favour of the past-tense, gives us more light on his standpoint. If we substitute the present-tense 'proposition,' ('*The ball is touching the roof*,' that indexically corresponds with the time of utterance n), with its adequate past-tense form ('*The ball has touched the roof*'), then we have in effect concordance between two conflict sides. Then we have sequences associated in (c'_1). That is, $T_n(p)$ and $(t)[t<n \supset (\exists t')[t<t' \wedge T_t T_{t'}(p)]\}$. But, as

[61] Döring, Fr. 122 < SE *M* x 47-48; „For what moves must complete a certain interval, but every interval is incapable of being completed because it admits of division *ad infinitum*, so that no moving thing will exist. |48| And with these men Diodorus Cronos also is in agreement, unless it should be said that according to him something *has moved* [κεκινῆσθαι μέν τι κινεῖσθαι δὲ μηδὲ ἕν] but not a single thing is moving — as we shall explain later in the course of argument, when we come to examine his view more closely."

[62] Döring, Fr. 123 (= SE *M* x 85-101) >: „Another weighty 'remainder' [argument] of the nonexistence of motion is provided [adduced] by Diodorus Cronus, trough which he shows that although nothing *is* moving it none the less *is moved* (or, has moved)."

[63] Simplicius, *Comm. in Phys.* (Diels) 934.25.

the point of Diodorus' trick, we can see that we also have as valid sequence associated in (c'_2) the line $(t)[t<n \supset (\exists t')[n<t' \wedge T_t T_{t'}(p)]\}$. So, in this context, both (c'_1) and (c'_2) hold: *once the ball has touched the roof, it will always be true that it has touched the roof.* This is the way McKirahan hopes that „the crucial ambiguity in interpretation of (c) becomes irrelevant."[64]

McKirahan, as we did note, has certain suspicions about testimonies that comes to us from Cicero as the source. The base of his plentiful reconstruction is primarily Sextus. His logical outbuilding is reposed on outcomes that Prior's reconstruction serves. Even he does not show us the complete domain of aspects that Diodorus' possible held concerning the nature of 'propositions' used in M.A. argumentation, undoubtfully that McKirahan elevate the level of investigations about M.A.

His researches on M.A. conduce him to the concluding point of this debate, that regards the nature of Diodorus determinism. He shows more sympathy to the option of scoping Diodorus logical 'system' as *indeterministic,'* even with some its obvious restrictions. Possible the most interesting thing that can be read out from McKirahan's reconstruction is consisting in his recognising the main, or basic method Diodorus used in constructing the argument(s). We clearly can see that he is willing to provoke rival conceptions and to put their representants in situation to misinterpret their own thesis, that he used to incorporate in argumentation, as his own – since his intention was just to show how far his opponents advocates conceptions that are not well-founded and to present what are the outcomes of their strategies.

In such light McKirahan understood Cicero's testimonies, where the theories of modal concepts are accompanied by the conception of determinism. Cicero's evidence is without mentioning the peculiarity connecting with the nature of above discussed kind of 'propositions,' or temporal aspect of sentences referring to such events as were Diodorus' balls that are touching the roof. So, according to McKirahan, he fails to recognise real Diodorus' conception, while identifies future truth with future necessity, or even future truth with necessity at all.[65] Making no

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[64] McKirahan [1979] p. 244.

[65] This hypothesis is based on Döring Fr. 132A (Cicero, *De fato*, 7.13 and 9.17), where this interpretation has its sense familiar to defenders of determinism: „For **Diodorus** says that only that is possible which either is true or will be true; **that whatever will be is necessary; and that whatever will not be is impossible...**" or in Rackham's translation, „whatever will be, he [Diodorus] says, must necessarily happen, and whatever will not be, according to him cannot possibly happen." In addition, the claim seems more stronger a few lines later, where Cicero says that „(9,17) this possition is connected with the argument that **nothing happens which was not necessary**, and that whatever is possible either is now or will be, and that it is no more possible for things that will be alter than it is for things that have happened."

distinction between 'to be necessarily true' and 'to be necessary,' deterministic thesis ascribing in Cicero to Diodorus is not valid in the system McKirahan tried to depict as Diodorean. If we are omitting the specifications derived from his arguments about 'moving,' as Cicero did, his position naturally looks as deterministic one. McKirahan does neither setting further questions about the nature of Diodorus' 'propositions,' nor about his theory of meaning and reference. Having in mind primary critique of Prior reconstruction, he is not dealing with other aspect of Diodorus' physical theory, that can be find principally in Sextus, which taste of exposition is that to one McKirahan more inclines. However, he is the rare one among other critics of the problem who has well-based intuition in fixing either the common features in different testimonies we can obtain from incomplete and dispersed pieces of Diodorus' 'theory,' or his wider philosophical conception that implies such problems as it did M.A. during his long life in antiquity.

Bibliography

Becker, O. [1960]; Zur Rekonstruktion des 'kyrieuon logos' des Diodoros Kronos, *Erkenntnis und Verantwortung: Festschrift fur Theodor Litt*, eds. J.Derbolav & F.Nicolin, Dusseldorf, s.250-263.

Boheński, I.M. [1961]; *A History of Formal Logic*, (trans. by Ivo Thomas), Notre Dame.

Bull, R.A. [1965]; An Algebraic Study of Diodorian Modal Systems, *The Journal of Symbolic Logic*, (30):58-64.

Bull, R.A. [1968]; An Algebraic Study of Tense Logics with Linear Time, *The Journal of Symbolic Logic*, (33):27-38.

Denyer, N. [1981]; Time and Modality in Diodorus Cronus, *Theoria*, (47/p1):31-53.

Döring, K. [1972]; *Die Megariker - Kommentierte Sammlung der Testimonien*, Amsterdam.

Döring, K. [1989]; Gab es eine Dialektische Schule?, *Phronesis*, (34/3):293-310.

Dummett, M.A.E. & Lemmon, E.J. [1959]; Modal Logics Between S4 and S5, *Zeitschrift fur Mathematische Logik und Grundlagen der Mathematik*, (4):250-264.

Findley, D. [1941]; Time: A Treatment of some Puzzle, (first time published in *Australian Jurnal for Philosophy*, No. 19:216-35), reprinted in *ed.* Gale [1968].

Gale, R.I. [1968]; *The Philosophy of Time*, London: Macmillan.

Hintikka, J. [1964]; Aristotle and the 'Master Argument' of Diodorus, *American Philosophical Quarterly*, (1):101-144.

Hintikka, J. [1973]: *Time and Necessity*, Oxford.

Kneale, W.& M. [1962/1984]: *The Development of Logic*, Oxford: Clarendon Press.

Long, A.A. & Sedley, D.N. [1987]: *The Hellenistic Philosophers: Translations of the Principal Sources with Philosophical Commentary*, Vol. i, Cambridge, New York: Cambridge University Press.

Long, A.A. & Sedley, D.N. [1987]: *The Hellenistic Philosophers: Greek and Latin Texts with Notes and Bybliography*, Vol. ii, Cambridge, New York: Cambridge University Press.

Łukasiewicz, J. [1930]: Philosophical Remarks on Many-Valued Systems of Propositional Logic, (first time appeared as Philosophische bemerkungen zu mehrwetigen Systemen des Aussagenkalküls, in *Comptes rendus des séances de la Société des Sciences et des Letters de Varsovie*, 23, cl.iii, pp.51-77), reprinted in: (ed.) Borkowski, pp.153-178.

Łukasiewicz, J. [1934]: On the History of the Logic of Propositions (first time appeared as Z historii zdań *Przegląd filozoficzny*, (37):417-437) reprinted in (ed.) Borkowski, pp.197-217.

Łukasiewicz, J. [1941]: Logic And the Problem of the Foundations of Mathematics, (first time appeared as Die Logik und das Grundlagenproblem in *Les Entretiens de Zürich sur les fondements et la méthode des sciences mathematiques* 6-9, 12 (1938) Zürich, 1941, pp. 82-100), reprinted in (ed.) Borkowski, pp.278-294.

Łukasiewicz, J. [1951[1]/1957[2]]: *Aristotle's Syllogistic from the Standpoint of Modern Formal Logic* (2nd ed. enlarged), Oxford: Oxford University Press.

Łukasiewicz, J. [1961]: On Determinism (first time published as *O determinizmie*, in (ed.) Słupecki) reprinted in (ed.) Borkowski, pp.110-128.

Massey, G. [1969]: Tense Logic! Why Bother?, *Noûs* (3):17-32.

Mates, B. [1949]: Diodorean Implication, *Phylosophical Review*, 58:234-42.

Mates, B. [1961]: *Stoic Logic*, Berkeley and Los Angeles.

McKirahan, R. [1979]: Diodorus and Prior and the Master Argument, *Synthese*, (42/2):223-253.

Michael, F.S. [1976]: What is Master Argument of Diodorus Cronus, *American Philosophical Quarterly*, (13/30):229-235.

Øhrstrøm, P. [1980]: A New Reconstruction of the Master Argument of Diodorus Cronus, *International Logical Review*, (11):60-65.

Øhrstrøm, P. & Per Hasle [1993]: A. N. Prior's Rediscovery of Tense Logic, *Erkenntnis*, (39):23-50.

Prior, A.N. [1955]: Diodorian Modalities, *Philosophical Quarterly*, (5):203-213.

Prior, A.N. [1957]: *Time and Modality*, Oxford: Clarendon Press.

Prior, A.N. [1958]: Diodorus and Modal Logic: A Correction, *Philosophical Quarterly*, (8):226-230.

Prior, A.N. [1962]: Tense-Logic and the Continuity of Time, *Studia Logica*, (12):133-148.

Prior, A. N. [1967]: *Past, Present and Future*, Oxford: Clarendon Press.

Prior, A. N. [1968]: *Papers on Time and Tense*, Oxford: Clarendon Press.

Purtill, R. L. [1973]: The Master Argument, *Apeiron*, (7):31-36.

Rescher, N. [1966]: A Version of the Master Argument of Diodorus, *Journal of Philosophy*, (63):438-445.

Rescher, N. [1968]: Truth and Necessity in Temporal Perspective, in (ed.) Gale, pp.183-220.

Rescher, N. & Urquhart, A. [1971]: *Temporal Logic*, New York: Springer.

Sedley, D. [1977]: Diodorus Cronus and Helenistic Philosophy, *Proceedings of the Cambridge Philological Society*, (203):74-120.

Sutula, M. [1976]: Diodorus and the Master Argument, *Southern Journal of Philosophy*, (14/3):323-343.

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