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# Truth and the Ambiguity of Negation

## 1 Introduction

The concept of negation is intertwined with the most fundamental principles that appear to govern the notions of truth and falsehood. We need negation to express conditions for falsehood, or at least of untruth; moreover, whether the principle of bivalence holds or has genuine counterexamples depends on what truth and falsity require, and thus also on what negation is.

Semantic paradoxes like the liar, but also other cases, from presupposition failure to borderline cases of vagueness, seem to offer putative counterexamples to bivalence. But how is a putative counterexample to be described? Prima facie, a counterexample to bivalence is an item (of the relevant sort) that is neither true nor false. We need negation to express that something is a counterexample to the principle as much as we need negation to explain when something is not true, or when something is false. However, as a simple argument purports to show – we will refer to it as 'the Incompatibility Argument' –, the assumption that the very same principles for truth and falsehood are correct taken together with the assumption that there are counterexamples to bivalence leads to a contradiction.

Many believe that the principles for truth and falsehood are essential for what truth is. The idea that the schemas are essentially correct can be understood in different ways. Each of these ways requires different commitments to the role the schemas are to play. One may just hold (i) that the truth-schemas are extensionally correct, and that all their instances must be true,[1] or (ii) one can add that the schemas are not only extensionally correct, but also that they display something essential about truth, for instance that the meaning of 'true' is conveyed in instances of the schemas, or that grasping the concept of truth requires assenting to all their instances. Finally, (iii) one may maintain all of the above and furthermore add that there is nothing else to say about truth. Deflationists, in particular, will hold precisely that the meaning of 'true' is fully given (or implicitly defined) in instances of (some version of) the schemas, and that no further facts about truth are to be uncovered.

Significantly, some of the cases that seem to provide counterexamples to bivalence, in particular paradoxes like the liar, seem also to provide counterexamples to the weakest claim above, namely that all instances of the truth-schemas are correct (and a fortiori to the remaining stronger claims (ii)-(iii)). Given the centrality of negation in the principle of bivalence and in truth and falsity conditions, a tempting strategy to avoid the incompatibility argument is to hold that negation is semantically ambiguous.

This article has one aim, to reject the claim that negation is semantically ambiguous. The first section presents the putative incompatibility between gaps and the truth-schema; the second section presents the motivation for the ambiguity thesis; the third section summarizes arguments against the claim that natural language negation is semantically ambiguous; and the fourth section indicates the problems of an introduction of two distinct negation operators in natural language.

1 One may defend this view by holding that truth-schemas play a central role in semantic theories, while holding a different view about truth, say, defending a substantial account of truth as correspondence, or defending that truth is primitive and indefinable.

## 2 The incompatibility

A recognized problem for the claim that all instances of the truth-schema must be correct is that an instantiation of the truth-schema with a liar sentence offers an apparent counterexample to the claim. We can settle for simple disquotational schemas. Let '*S*' stand for a sentence; if '*S*' contains no context-dependent terms, we can just disquote it and use '*S*' itself to state what it says. Thus simplified, truth and falsity schemas can be formulated as:

$(T)$ '*S*' is true iff $S$.

$(F)$ '*S*' is false iff not $S$.

Let $\beta$ be our liar sentence:

$(\beta)$ $\beta$ is not true.

If $\beta$ instantiates $T$, we obtain:

'$\beta$ is not true' is true iff $\beta$ is not true.

Replacing '$\beta$ is not true' by its name, we can infer that

$\beta$ is true iff $\beta$ is not true.

Since a biconditional is true if both its left- and right-hand sides have the same value, this biconditional cannot be true, and $\beta$ gives hence a counterexample to the claim that all instances of disquotational schemas for truth are correct.

A related, more general, difficulty for the claim that all instances of truth-schemas are correct is what Beall (2002) calls The Incompatibility Argument (cf. Beall 2002: 301): If a sentence '*S*' is gappy, i.e., if '*S*' is neither true nor false, it follows, given the schemas, that it is not the case that $S$ and that it is not the case that not $S$, i.e., not $S$ and not not $S$, which is a contradiction. Hence it seems that the supposition of gaps is incoherent, if it is assumed that all instances of the disquotational schemas are correct. In either case, the schemas appear to be incompatible with the liar and with gaps, generally.[2]

A further problem that gappy sentences (or sentences that are neither true nor false) pose to the $T$-schema is the following: if a sentence is neither true nor false, that is, if it is undetermined or gappy, then it is not true, i.e., it is false that "'*S*' is true" when '*S*' is undetermined. And if it is false that "'*S*' is true" when '*S*' is neither true nor false, then the disquotational schema, '*S*' is true iff *S*', has false instances. This a point that is made, for instance, by Dummett:

> A popular account of the meaning of 'true', also deriving from Frege, is that 'it is true that *P*' has the same sense as the sentence *P*. If, as Frege thought, there exist sentences which express propositions but are neither true nor false, then this explanation appears incorrect. Suppose that *P* contains a singular term which has a sense but no reference: then, according to Frege, *P* expresses a proposition which has no truth-value. This proposition is therefore not true, and hence the statement 'It is true that *P*' will be false. *P* will therefore not have the same sense as 'it is true that *P*', since the latter is false while the former is not. (Dummett 1959: 4–5).

How can the truth-schemas be rescued? A hypothesis is, in the first place, to discern two senses of negation. Perhaps two meanings of 'not' can be defined and introduced in the language (Beall's stance is that each negation can be introduced and learned with inference rules, Beall 2002: 303), or perhaps 'not' is anyway ambiguous in English (in the next two sections each alternative is evaluated). For clarity of exposition, we can distinguish the two alleged readings of 'not' adopting Tappenden's (1999) designations for the duo, internal and external. (Other names for the duo are weak and strong, choice and exclusion. Here 'internal' and 'external' does not point towards structural ambiguities such as the different scope readings obtained when negation interacts with other operators, which of course exist). We can, also for clarity, use $\sim$ for internal and $\neg$ for external negation.

2 A form of the same argument is notoriously advanced by Williamson 1992 and 1994 against the supposition of gaps, in general, and more particularly against gaps generated by vagueness.

The two meanings are often distinguished with the aid of truth-tables, where '*' stands for gappy or undefined:

| $A$ | $\sim A$ | $\neg A$ |
|---|---|---|
| T | F | F |
| F | T | T |
| * | * | T |

If negation is semantically ambiguous between these two readings, Beall's suggestion for evading the Incompatibility Argument becomes clear; the argument is invalid because it commits a fallacy of equivocation, conflating internal and external negation. Once the two meanings of negation are disentangled, the argument is no longer a problem for the truth-schemas. The argument is to be now understood in this way. From:

$\neg$ ('*S*' is true) and $\neg$ ('*S*' is false)

it follows that

$\neg S$ and $\neg \sim S$.

This is not a contradiction.

In the second place, we may recognize that there is a sense in which "'*S*' is true"' is not true (is false) if '*S*' is gappy; we may call this sense strong truth. But the sense of 'true' captured in the truth-schema is weak truth, according to which the equivalence thesis holds. The truth-schema can be rescued since '*S*' and "'*S*' is true" are both acceptable, rejectable or undecided in the same circumstances. The biconditional in the schema must be taken as true when both sides are true, both false, or both neither.

Strong truth can be derived from weak truth, which may be done via the two negations, internal and external. The strong sense of 'true' ($\text{true}_s$) may be defined thus:

'*S*' is $\text{true}_s$ iff $\sim \neg S$.

The appeal to strong truth, as opposed to truth, to accommodate the sense in which a truth-predication is false when the predicated sentence is gappy or undefined, depends essentially on the correctness of the account of the duo of negations, internal and external. With the duo of negations and the distinction between truth and strong truth in place, the disquotational truth-schema can be rescued, as well as the intuition that a gappy sentence is not true. The incompatibility can, it seems, be dispelled.

With respect to the liar sentence at hand, it can be held that it now provides a true instance of the disquotational schema, if it is itself a gappy sentence. If 'true' in $\beta$ is weak truth and if 'not' is $\sim$, then an instance of $T$ with $\beta$ permits inferring only that '$\beta$ is true iff $\beta$ is not true' which will itself be a gappy sentence. Beall still manifests the worry, in any case, that with 'strong negation one faces the problem of strengthened Liar-like paradox' (Beall 2002: 304). (We will return to this later).

The hypothesis of the semantic ambiguity of negation offers, then, an attractive possibility for handling paradoxical cases, for rescuing the disquotational truth-schema, and for disarming the Incompatibility Argument. As the proposed solution relies essentially on the semantic ambiguity of negation, the ambiguity thesis needs to be sufficiently well motivated, otherwise the proposal is, although convenient, ad hoc.

## 3 The motivation for the duo of negations

The appeal to the ambiguity of negation is relatively popular. Wolfgang Künne (2005), for instance, tries to rescue the coherency of the truth- and falsity-schemas and the existence of gaps precisely by drawing the distinction between internal and external negation (*ibid*: 41)[3], although the way he attempts to motivate the distinction is not the most felicitous, since he illustrates the apparent true readings of external negation with what he takes to be 'non-significant sentences', as in 'adverbs don't hibernate' (whether or not category mistakes

3 As do Pelletier & Stainton (2003).

are non-significant is independently disputable, but not our concern here). Künne's distinction between internal and external negation is meant to capture the internal reading where negation attaches to a verb phrase, or perhaps to a predicate directly. Falsity is to be defined as truth of the internal negation. External negation is in turn expressible as 'it is not true that...'/ 'it is not the case that...', where the sentence embedded under negation may lack sense, as in the example considered by Künne, 'adverbs hibernate'.

However, there are clear cases of external readings of negation in Künne's sense (precisely when negation takes wide scope over the whole sentence, for instance because the negated sentence contains other operators with narrower scope) where we want to say, against Künne, that the truth of the negation is tantamount to the falsity of the unnegated sentence, for instance in 'it is not the case/true that everything is made of water'. Moreover, there should be clear examples of the distinction between internal and external negation that do not depend on some felicitous uses of negated non-significant sentences. After all, the point of having a lexically distinct negation operator must be, in the context of this debate, that sometimes the negation of a significant sentence (that could have been truth-valued) not only yields another significant sentence, but also one with a truth-value. It seems implausible to introduce an operator that produces sense from non-sense. Furthermore, felicitous uses of negated non-significant sentences, if they exist, can probably be subsumed under the distinct general phenomenon of metalinguistic negation (Horn 2001), a pragmatic process where some aspect of a previous utterance is corrected, as in 'he didn't flaunt Grice's maxims, he flouted them'. And it should be clear that metalinguistic negation is of no avail to rescue the joint consistency of gaps and the truth-schemas, where gaps are assumed to be significant and possibly evaluable sentences.[4]

This does not mean there is no motivation for the ambiguity thesis. Historically there is, and Horn (2001, for instance ch. 2) covers it in

4 Arguing that this is so is beyond the scope of this article, since here I am only concerned with the semantic ambiguity of negation, and not with pragmatic processes by which some aspect of an utterance, independent of the literally asserted content (if any), is objected to.

detail. Three types of case have motivated the ambiguity of negation during the last century. Initially, the ambiguity between external and internal readings was indeed seen as structural, as noted by Russell, in cases like:

(1) a. The king of France is not bald – the king of France does not exist.

b. The king of France is not bald – he has long dark hair.

In (a), the negation of 'the king of France is bald' is meant to be external, or to take wide scope, whereas in (b) it is meant to be internal, or to take narrow scope, but this is explained by the claim that in 'the king of France is not bald' there are two operators interacting, the negation operator and the definite article, which is treated as a quantificational operator.[5]

Now, it seems that the same distinction can be drawn in sentences containing proper names instead of descriptions, as in 'Santa Claus is not coming for Christmas':

(2) a. Santa Claus is not coming for Christmas – Santa Claus does not exist.

b. Santa Claus is not coming for Christmas – you were a naughty boy this year.

Russell would have held that proper names function like abbreviated descriptions, and hence allow for the same scope ambiguities, with a wide and a narrow scope reading. However, there are good (semantic and modal) arguments not to treat proper names like descriptions, but rather as genuine singular terms (Kripke 1980). Given this, it is arguable that proper names are insensitive to scope distinctions in (at least) extensional contexts, in particular, in the context of negated

5 Clearly, 'internal' and 'external' here do not mean the same as what the truth table above means to capture; it merely describes whether negation takes narrow or wide scope. On the Russellian analysis, both disambiguations are truth-valued, in (a) the negated sentence is true and in (b) the negated sentence is false.

sentences (cf. Neale 1990).[6] So, the scope ambiguity of negation seems to be unjustified in simple sentences with proper names.

Other authors, like Strawson (1950), take a Fregean line, and regard the existence clause as a presupposition of a (use of a) sentence with a singular term, rather than part of the content of the sentence, or of its truth-conditions. If the existence presupposition fails, then the utterance of the sentence containing the singular term is neither true nor false. The presupposition is, for Strawson, a condition for either a sentence or its negation being successfully used in making an assertion or statement. Yet, it is unclear whether Strawson accepted presupposition cancelling readings of negations as in the (a) sentences above.

Presuppositionalists after Strawson, however, allowed for ambiguous readings of the negation of sentences like 'the King of France is bald' or 'Santa Claus is coming for Christmas'. The idea is that there is a marked reading, captured in the (a) sentences, which is true even if the presupposition fails, and an unmarked natural reading, captured in the (b) sentences, which is truth-valueless when the presupposition fails. The first marked and unnatural reading is taken to be the presupposition-cancelling one.

The existence of marked, presupposition-cancelling readings, as well as the unmarked or natural uses of negation, is claimed to occur in further cases not involving reference failure, as in:

(3) a. John didn't stop *smoking* – he never smoked in his life.

b. John didn't stop smoking – he smokes more than ever.

6 Nevertheless, some authors still believe that there are scope differences in negated sentences with proper names, for instance Sainsbury (2005: 71). Undoubtedly, there are semantic models where there are scope differences in the negation of a simple sentence with a singular term. The question, however, is what should lead us to accept that such models capture how natural language negation interacts with proper names. After all, the modal argument against descriptivism depends on the fact that proper names are insensitive to scope differences even in modal contexts. A very strong case would have to be made in favour of the claim that in one extensional context, as that of negation, proper names are not scope insensitive.

(4) a. I don’t regret going to *the* dinner – it was cancelled in the last minute.

b. I don’t regret going to the dinner – I am happy I went.

Internal negation is argued to be the more natural one, and to reveal a general feature of presuppositions, namely that they are preserved under embeddings (not only under negation, but also in conditionals, disjunctions, etc.). If one accepts that presupposition failure entails that a sentence is neither true nor false, and at the same time accepts that there are marked true readings, one has arrived at the motivation to distinguish between internal and external negation, as captured in the truth table above.

Finally, category mistakes have also motivated the ambiguity thesis. To use Künne’s example, ‘adverbs don’t hibernate’ can be argued to have a reading that is neither true nor false – where negation is internal – and a true external reading, as in ‘neither adverbs hibernate nor they don’t’.

Different formal multivalued systems were developed in the past century (for instance Kleene 1938, 1952; Smiley 1962; Herzberger 1970) that are compatible with a distinction between the two negation operators, one presupposition preserving and a presupposition cancelling (internal/weak and external/strong negation.) [7]

The two negations can be applied to further problematic cases. Take vagueness first, with an example from Tappenden 1999.[8] Suppose you have to sort out colour samples. In one bin you are to put samples that look red to you. In another bin you are to put samples that do not look red to you. Moreover, if there are samples that you cannot distinguish between the ones that look red and the ones that

7 For a summary, cf. Horn (2001: 122-132).

8 The motivation for the ambiguity thesis is also discussed by Tappenden (1999: 214). Tappenden offers a comprehensive discussion of the topic. Although I agree with much of his criticism against the thesis of the semantic ambiguity of negation, I have reasons to disagree with his account of how negation semantically functions and his support of the pragmatic ambiguity of negation. The discussion of pragmatic side of the issue is, as mentioned before, beyond the scope of this article.

do not look red, you are to put them on a separate shelf. Cases like the sorting of samples that look red from the rest gives rise to two ways in which we can deny that a sample looks red. We place a sample in the bin with non-red things, asserting ‘this samples does not look red’. We can also say ‘this sample does not look red’ by placing it neither with the things that look red nor with the things that don’t look red, because it looks indistinguishable from either. In one case, ‘this does not look red’ seems as incorrect to assert as ‘this looks red’. In the other case, ‘this does not look red’ seems correct to assert whenever ‘this looks red’ does not seem to be correct to assert.

The semantic ambiguity of ‘not’ may also be applied to the liar paradox. Consider again the simplest form of the Strengthened Liar mentioned earlier:

($\beta$) $\beta$ is not true.

If we think that $\beta$ must be neither true nor false, then $\beta$ is not true. Since ‘$\beta$ is not true’ is $\beta$ itself, it follows that $\beta$ is after all true, and we land back in the paradox. We may adapt Tappenden’s point here:

> Clearly ($\beta$) has something wrong with it – it cannot be correctly asserted… [and it is wrong to assert ($\beta$)] because the facts are not as ($\beta$) says they are. But how is this to be conveyed? The suggestion that ($\beta$) is neither true nor false attempts to convey what is wrong with ($\beta$), but can we say this without falling back into the liar cycle? (Tappenden 1999: 264)

The inclination to deny the liar, accompanied by the unwillingness to land back in paradox, can lead one to make use of the two readings for ‘not’. So, consider the result of applying the duo of negations to $\beta$; $\beta$ says it is not true. But if there are, in general, two ways to read negated sentences, one in which ‘$\beta$ is not true’ can be as incorrectly asserted as ‘$\beta$ is true’ and another in which ‘$\beta$ is not true’ is correctly asserted whenever ‘$\beta$ is true’ is incorrectly asserted, then $\beta$ could be said to be not true without thereby asserting $\beta$ itself.

## 4 Against the semantic ambiguity of negation

Although the idea that negation is semantically ambiguous seems reasonably motivated, and would be useful if true, it faces considerable obstacles. This section summarizes some reasons why natural language negation is not lexically ambiguous.

Consider (3a) and (3b), 'John didn't stop smoking – he never started' and 'John didn't stop smoking – he smokes more than ever'. Although cases of this sort have been taken to motivate the ambiguity of negation, several studies have indicated that natural language is not semantically ambiguous – in particular, that negation is not lexically ambiguous.

In the first place, the claim of the semantic ambiguity of negation should pass a Kripkean test (Kripke 1977).[9] The test is not a proof, but it should be taken as giving strong evidence for or against any semantic ambiguity claim. Here is the test: if 'not' were lexically ambiguous in English, then there should be a natural language in which the two meanings are expressed by distinct expressions. For instance, if 'not' allowed for the semantic ambiguity marked in the truth-tables for internal and external negation, we would expect this difference to come out in translations of, say, 'this is not red' into other languages. But this is not the case, even in cases where it is more plausible to claim the existence of an ambiguity, for instance in the difference that comes out between marked and unmarked readings of negated sentences carrying presuppositions.

As noted by Horn (2001: 366), Tappenden (1999: 270), and argued by Gazdar (1979: 65–66), among others,[10] no natural language has two negation operators corresponding to the external and internal readings given earlier that offer a disambiguation of 'John didn't stop smoking'. In fact, translations of such sentences like this into

9 For more on ambiguity tests and the difference between semantic ambiguity and multiple understandings of particular sentences for other reasons (vagueness, unspecifity, indeterminacy, etc.) based on identity-of-sense tests, cf. Zwicky & Sadock (1975). Negation also fails these tests, although I will not cover the details here.

10 For instance, Alwood (1972), Atlas (1977) or Kempson (1977).

other languages are as likely to allow a presupposition cancelling as well a presupposition preserving reading. If the explanation for both readings rested in the existence of a lexical ambiguity, then it would be expected that it should be revealed if negation passed the Kripkean test. In particular, it would be expected that a translation of ‘John didn’t stop smoking’ into Portuguese – ‘O João não deixou de fumar’ – would disambiguate between the two readings. But this is precisely what does not happen. The sentence carries the same presupposition, and raises the same issue, as the original sentence in English does. If the Kripkean test had been passed, a phenomenon analogous to that verified in (5) below (which translates into Portuguese either as (b) or as (c)) would occur:

(5) a. John sat at the board on the bank.
b. O João sentou-se na tábua na margem do rio.
c. O João ocupou uma posição na direcção do banco.

So, negation fails the Kripkean test for semantic ambiguity.[11]

In the second place, as Horn (2001: 365) remarks, although internal and external readings are meant to be captured in the difference between ‘it’s not true/the case that the John stopped smoking’ and ‘John didn’t stop smoking’, the fact is that either form allows a presupposition cancelling and a presupposition preserving reading. So, it is not clear that the phrases ‘it is not true that’/ ‘it is not the case that’ capture the desired external reading of negation.

These results indicate that natural language negation is not semantically ambiguous. This could be a defect of natural languages – there should be, it may be claimed, two distinct uses for negation corresponding to the specified internal and external negations, and a theorist is free to introduce two such distinct operators. If a distinction between the internal and the external reading of negation

11 Gazdar (1979) uses examples like these to make the same point. Gazdar draws further considerations against the claim of the ambiguity of negation (Gazdar 1979: 65-66). Cf. also Tappenden (1999: 270), with a similar claim against the semantic ambiguity of negation.

makes the truth-schema compatible with truth-value gaps, and is moreover useful to reject particular uses of sentences, for instance some paradoxical or vague ones, then this in itself is sufficient to introduce (perhaps, as Beall suggests, via the respective inference rules) the two distinct negation operators. The next section argues that this move is unjustified: it simply reproduces new versions of the problematic cases.

## 5 Resilient Problems

The duo of negations could be useful to handle the Incompatibility Argument if there were any evidence that such a semantic ambiguity exists. But there is, instead, evidence to the contrary. Yet, it may be argued two meanings of negation can be defined and introduced in our language. This requires that there be some independent reasons in support of the introduction – that the duo of negations can cope with semantic paradoxes, for instance. The remaining cases – semantic paradoxes like the liar, or borderline vague utterances – are part of the problem the duo of negations is meant to cope with. Unless the ambiguity thesis is successful in coping with these cases, there is no justification to introduce the two negation operators.

Once we have internal and external negation, and weak and strong truth, a liar sentence like:

($\beta$) $\beta$ is not true.

can be read in four ways:

($\beta_1$) $\sim$ ($\beta_1$ is true)
($\beta_2$) $\neg$ ($\beta_2$ is true)
($\beta_3$) $\neg$ ($\beta_3$ is true$_s$)
($\beta_4$) $\sim$ ($\beta_4$ is true$_s$)

We may suppose that if the negation in $\beta$ is internal – and we have $\beta_1$ – then we can reject $\beta_1$ by saying: $\neg \sim$ ($\beta_1$ is true). Yet, $\beta_2$ is a stronger liar than $\beta_1$:

($\beta_2$) $\neg$ ($\beta_2$ is true)

'$\neg$ ($\beta_2$ is true)' is true if $\beta_2$ is either gappy or false, and false when $\beta_2$ is true. But since '$\neg$ ($\beta_2$ is true)' is $\beta_2$ itself, we have landed back in paradox. So, the ambiguity of negation does nothing to prevent the resilient strengthened liar. Likewise paradoxical are $\beta_3$ and $\beta_4$, because 'true$_s$' is defined with weak and strong negation: '$S$' is true$_s$ iff $\sim \neg S$. The result is that strong truth, unlike weak truth, is a well-defined predicate: each sentence is either true$_s$ or not true$_s$. It is easily verifiable that $\beta_3$ is a further form of the liar paradox: a sentence that says of itself that it is not strongly true. The same happens with $\beta_4$. The ambiguity of negation is here of no help – there is no further negation operator that would allow for the correct non-paradoxical denial of $\beta_2$, $\beta_3$ or $\beta_4$. Since the alleged recognition of the distinction between two meanings of 'not' is at best used to discard one of the readings of the strengthened liar, $\beta_1$, and is useless against the other readings, the introduction of a duo of negations to cope with the liar is here unjustified. Beall's fears of a new strengthened liar were well grounded.

Similar worries arise for the success of the duo of negations in rescuing the truth-schemas. A liar sentence seems to be a counterexample to $T$. The semantic ambiguity of negation would have been useful only if the negation operator in a liar sentence were internal negation, and if the relevant notion of truth were weak truth. In that case, the instance of schema $T$ with the liar is true, even though both sides of the biconditional "'$\beta$ is not true' is true iff $\beta$ is not true" are gappy. But since there are several other strengthened liar sentences in the vicinity, the question arises as to whether these offer false (or, at least, untrue) instances of the truth-schema.

Recall that the weakest thesis on the role the truth-schema plays that is considered in the beginning of this article is that all instances of the schema are correct – i.e, true. Inserting any sentence in the place of $S$ in the schema must yield a true equivalence. But a sentence like:

($\beta$) $\beta$ is not true.

can now be read in four different ways. So, consider all the alternatives $\beta_1$ to $\beta_4$. Unsurprisingly, the only of these sentences that provides a true instance of the $T$-schema is $\beta_1$, that is formulated with weak truth and weak (internal) negation:

(T$_1$) '$\sim$ ($\beta_1$ is true)' is true iff $\sim$ ($\beta_1$ is true).

which entails:

$\beta_1$ is true iff $\sim$ ($\beta_1$ is true),

which again will not be false if $\beta_1$ is gappy, as the proposed solution intends. All other instances of $T$, namely:

(T$_2$) '$\neg$ ($\beta_2$ is true)' is true iff $\neg$ ($\beta_2$ is true)
(entailing: $\beta_2$ is true iff $\neg$ ($\beta_2$ is true)),

(T$_3$) '$\neg$ ($\beta_3$ is true$_s$)' is true iff $\neg$ ($\beta_3$ is true$_s$)
(entailing: $\beta_3$ is true iff $\neg$ ($\beta_3$ is true$_s$)),

(T$_4$) '$\sim$ ($\beta_4$ is true$_s$)' is true iff $\sim$ ($\beta_4$ is true$_s$)
(entailing: $\beta_4$ is true iff $\sim$ ($\beta_4$ is true$_s$)),

are false, as can be easily verified.

So, the duo of negations cannot save the truth-schema from the liar paradox. Even if there were an ambiguity of 'not' in English, it would not save the truth-schemas, since the existence of several negations just results in the existence of several forms of the liar. Moreover, if the introduction of a distinct meaning of 'not' generates a different version of the same paradox, then handling the liar and rescuing the truth-schema from paradox does not justify the introduction of two semantically distinct negation operators.

Now, can the duo of negations help handling vague cases? Suppose that we have a similar task of sorting out colour samples as was described earlier. We have to sort out those that look red to us, and all the remaining samples. The question is: does the existence of a negation operator, that allows us to truly assert that a given sample is either red or not, help us in any way in deciding whether a given sample (about which we are undecided) should go with the red things or not? The question posed is ‘Does this look red?’ You are to answer ‘yes’ or ‘no’, bearing in mind that when you say ‘no’ you are separating that sample from all the remaining red samples. The expected outcome of this task is, it seems, that there will be a good number of coloured samples about which you are less than certain of what the answer should be. And, if that is right, the usefulness of a further operator that allows a sharp division between red things and all the rest should be reconsidered. One is not in a better position to discriminate red samples, say, from all others by having a use for negation that in principle permits truly making the discrimination.

The use that is left for two negation operators is their application to disarm the Incompatibility Argument. But we cannot introduce the duo of negations to show that the argument commits a fallacy of equivocation. The burden of proof – establishing that there is indeed some ambiguity of negation, or that distinguishing two readings is well justified – rests with the ambiguist. It is ad hoc to claim that there is such an ambiguity in the absence of independently grounded reasons. So, the argument cannot commit a fallacy that rests on an ambiguity that does not exist, as all evidence indicates, and that is ill justified, as we have just argued. As Burge (1979) puts it, concerning the liar:

> Now one might appeal to the restrictions on the truth-schema, which all gap theorists appeal to, to treat the ‘ordinary’ paradoxes (and pathologies like ‘this is true’), and a hierarchy of negations (and of material conditionals!) to deal with the strengthened versions. But such an approach, though technically feasible, promises little philosophical illumination. The semantical paradoxes are remarkable in their similarity. The Strengthened Liar does not appear to have sources fundamentally different from those of the ordinary Liar. What is wrong with the proposed account is that it gives no insight into the general phenomenon of semantical pathology and offers instead a hodgepodge of makeshift and merely technical remedies. A theory of semantical paradox should focus on semantical notions. (Burge 1979: 177)

It seems that the thesis of the ambiguity of negation – in the absence of independent justification – gives no insight into the general phenomena at stake and offers a merely technical remedy.

The question of whether or not there are counterexamples to bivalence, and whether the truth-schema can be rescued, must depend on other issues, for instance, on whether paradoxical sentences can instantiate the schemas in the first place (are they sentences that say that something is the case? This is disputed by Laurence Goldstein 2000, 2001), or on whether there is some pragmatic form to reject given troublesome sentences that is not tantamount to negating them (this is defended by Parsons 1984, Richard 2008, Smiley 1996, and Tappenden 1999, for instance).[12] The issue remains open, furthermore, as to whether the very assumption that every instance of the truth-schema is correct should be dropped.[*]

## References

Allwood, J. (1972). Negation and the strength of presuppositions. In Dahl, Ö. (Ed.). *Logic, pragmatics and grammar*, (pp. 11–57). University of Göteborg Department of Linguistics.

Atlas, J. D. (1977). Negation, ambiguity and presupposition. *Linguistics and Philosophy, 1*, 321–36.

Beall, J. C. (2002). Deflationism and gaps: untying 'not's in the debate. *Analysis, 62,* 299–304.

Burge, T. (1979). Semantical paradox. *Journal of Philosophy, 76*, 169–198.

Dummett, M. (1959). Truth. In Dummet, M. *Truth and other enigmas,* (pp. 1–28). London: Duckworth.

12 The question of whether negation is pragmatically ambiguous, perhaps signalling a distinct speech act of denial, as these authors hold, or some distinct pragmatic process, is the subject of another article.

∗ I am grateful for the support of the research projects C-project: Truth, Semantic Confusion and Embodiment, Code: HUM 2006-08236/FISO (C-CONSOLIDER) and PERSP – Philosophy of Perspectival Thoughts and Facts, CSD2009-00056 from the Spanish Ministry of Education and Science.

Frege, G. (1892). On sense and reference. In Richard, M. (Ed.) (2003). *Meaning,* (pp. 36–56). Oxford: Blackwell.

Gazdar, G. (1979). *Pragmatics: Implicature, presupposition and logical form.* New York: Academic Press.

Goldstein, L. (2000). A unified solution to some paradoxes. *Proceedings of the Aristotelian Society, 100*, 53–74.

——— (2001). Truth-bearers and the liar: A reply to Alan Weir. *Analysis, 61*, 115–126.

Herzberger, H. (1970). Truth and modality in semantically closed languages. In Martin, R. L. (Ed.). *The paradox of the liar,* (pp. 25–46). Reseda, CA: Ridgeview.

Horn, L. (2001). *A natural history of negation,* (reissue ed.) Stanford: CSLI Publications.

Kempson, R. (1977). *Semantic theory*. Cambridge: Cambridge University Press.

Kleene, S. C. (1938). On a denotation for ordinal numbers. *Journal of Symbolic Logic, 3*, 150–55.

——— (1952). *Introduction to metamathematics.* New York: Van Nostrand.

Kripke, S. (1977). Speaker's reference and semantic reference. In French, P., Vehling Jr., T. & Wettstein, H. (Eds.). *Contemporary perspectives in the philosophy of language,* (pp. 6–27). University of Minnesota Press.

——— (1980). *Naming and necessity.* Oxford: Blackwell.

Künne, W. (2005). *Conceptions of truth.* Oxford; New York: Oxford University Press.

Neale, S. (1990). *Descriptions*. Cambridge, MA: MIT Press.

Parsons, T. (1984). Assertion, denial and the liar paradox. *Journal of Philosophical Logic 13*(2), 137–152.

Pelletier, F. & Stainton, R. (2003). On 'The denial of bivalence is absurd'. *Australasian Journal of Philosophy, 81*, 369–382.

Richard, M. (2008). *When truth gives out*. Oxford; New York: Oxford University Press.

Russell, B. (1918-19). Lectures on the philosophy of logical atomism. In Marsh, R. C. (Ed.) (1956). *Bertrand Russell: Logic and Knowledge Essays 1902–1950*. George Allen and Unwin.

Sainsbury, M. (2005). *Reference without referents.* Oxford; New York: Oxford University Press.

Smiley, T. (1960). Sense without denotation. *Analysis, 20*, 125–135.

——— (1996). Rejection. *Analysis, 56*, 1–9.

Strawson, P. F. (1950). On Referring. *Mind, 59*, 320–344.

Tappenden, J. (1999). Negation, denial and language change in philosophical logic. In Gabbay, D. & Wansing, H. (eds.). *What is negation?* (pp. 261–298). Kluwer Academic Press.

Williamson, T. (1992). Vagueness and ignorance. *Proceedings of the Aristotelian Society, Supplementory Vol. 66,* 145–162.

——— (1994). *Vagueness.* London; New York: Routledge.

Zwicky, A & Saddock, J. M. (1975). Ambiguity tests and how to fail them. In Kimball, J. P. (Ed.). *Syntax and semantics 1*, (pp. 1–36). New York: Seminar Press.