Star Models and the Semantics of Infectiousness

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Abstract

The FDE family is a group of logics a many-valued semantics for each system of which is obtained from classical logic by adding to the classical truth-values *true* and *false* any subset of {*both*, *neither*, *indeterminate*}, where *indeterminate* is an *infectious* value (any formula containing a subformula with the value *indeterminate* itself has the value *indeterminate*). In this paper, we see how to extend a version of star semantics for the logics whose many-valued semantics lacks *indeterminate* to star semantics for logics whose many-valued semantics includes *indeterminate*. The equivalence of the many-valued semantics and star semantics is established by way of a soundness and completeness proof. The upshot of the novel semantics in terms of the applied semantics of these logics, and specifically infectiousness, is explored, settling on the idea that infectiousness concerns ineffability.

1 Introduction

The interesting relationships between *strong Kleene logic* (K3), the *logic of paradox* (LP), and *classical logic* are well known, as are relationships between these three logics and *first degree entailment* (FDE), the conditional-free fragment of relevant logics.[1] When ordered by strength, this quartet forms a lattice structure: the standard semantics for LP has a truth-value in between

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[1] Some classic references and contemporary discussions are Asenjo 1966; Beall 2017; Beall 2018; Beall 2019; Belnap 2019a; Belnap 2019b; Dunn 1976; Omori and Wansing 2017; Priest 1979. Note that strong Kleene logic differs from the logic of paradox only by the intermediate value not being designated (as would befit *neither true nor false*).

true and *false* interpreted as *both*, K3 an intermediate truth-value interpreted as *neither*, and FDE (in its four-valued form) has both of these.

More recently, Graham Priest has generalised this quartet to an octect he calls 'the FDE family' which contains, in addition to the four logics mentioned, *weak Kleene logic*, some other 'logics of nonsense', and a logic developed by Priest and Jay Garfield in studying Nāgārjuna's use of the catuṣkoṭi.[2] In the present paper, I extend existing star semantics (a form of semantics making use of an operator * rather than extra truth-values) to cover the whole family. This has implications for how we interpret the logics.

First, I introduce the FDE family and its many-valued semantics (§2). Then I give generalisations of existing star semantics for four of the logics in the family—first degree entailment, the logic of paradox, strong Kleene, and classical logic. The main contribution of the present paper (§3) is novel star semantics for the 'i-variants', logics in the family whose many-valued semantics involves the truth-value i. After presenting and discussing the star semantics for these logics, I show that the many-valued semantics and the star semantics are equivalent (§4). I then consider how the star semantics affects interpretations of these logics, and specifically of the idea of infectiousness (§5). I suggest that three interpretations of infectiousness (the *nonsense* interpretation, the *off-topic* interpretation, and the *emptiness* interpretation) converge with one another, with infectiousness capturing something like ineffability.

2 The FDE family and i

2.1 The FDE family

The FDE family consists of two quartets, each with a lattice structure when ordered by strength (L is properly stronger than K iff everything which is K-valid is L-valid, but not the other way round). The FDE quartet has first degree entailment (**BN**) as its weakest logic and classical logic (**Ø**) as its strongest, with strong Kleene logic (**N**) and the logic of paradox (**B**) between them. The other four logics are what I'll call the *i-variants* of each of these systems, logics obtained by adding the value i to a many-valued semantics, which are similarly arranged, with each i-variant being weaker than its i-free twin. Figure 1 shows the relationship between the logics.

Many of the logics in the FDE family are familiar to logicians. Table 1 records the details. t is the value true only, f false only, b both true and false, n neither true nor false, and i the infectious indeterminate value (some

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[2] Garfield and Priest 2009; Priest 2010; Priest 2014; Priest 2019.

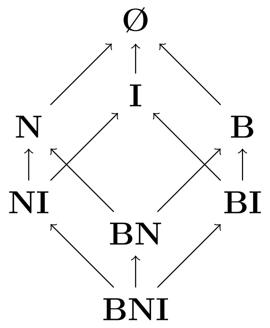

Figure 1: The FDE family ordered by strength

authors write ‘e’ for ‘empty’ for this value). The names of the logics in this paper are taken from the values added to t, f to arrive at the many-valued semantics for the logic in question.

Often called	Truth-values	Name in this paper
Classical logic	t, f	**Ø**
Logic of paradox	t, f, b	**B**
Strong Kleene	t, f, n	**N**
Weak Kleene	t, f, i	**I**
First degree entailment	t, f, b, n	**BN**
$\mathrm{S}_{\mathrm{fde}}$	t, f, b, i	**BI**
——	t, f, n, i	**NI**
FDE_{ϕ}	t, f, b, n, i	**BNI**

Table 1: Logics in the FDE family[3]

Throughout this paper, a many-valued semantics for one of these logics is marked with a ‘+’ (e.g. **BN**+ is the many-valued semantics for first degree entailment) whereas a star semantics is marked with a ‘*’ (e.g. **BN*** is the star semantics for first degree entailment). This paper is concerned only with the propositional systems.

Before we provide a many-valued semantics, let’s, for sake of explicitness, define our vocabulary (which is the same no matter the sort of semantics). The set of sentences or formulae Sent is defined inductively from the set of propositional parameters $\mathsf{Prop} = \{p, q, \ldots, p_1, \ldots\}$. Where $A, \ldots$ are metavariables standing for sentences:

- All propositional parameters are sentences.

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[3] cf. Priest 2019, p. 281

- If $\ulcorner A \urcorner$ is a sentence, then $\ulcorner \neg A \urcorner$ is a sentence.
- If $\ulcorner A \urcorner$ and $\ulcorner B \urcorner$ are sentences, then $\ulcorner (A \vee B) \urcorner$ is a sentence.
- If $\ulcorner A \urcorner$ and $\ulcorner B \urcorner$ are sentences, then $\ulcorner (A \wedge B) \urcorner$ is a sentence.

The material conditional can be defined as an abbreviation: $\ulcorner (A \supset B) \urcorner := \ulcorner (\neg A \vee B) \urcorner$. It shall not be discussed in detail. For the most part, logics in the FDE family are extended by other conditionals, e.g. strict and relevant conditionals. In all but **Ø**, it fails to satisfy at least one of $\models A \supset A$ and $A, A \supset B \models B$.

2.2 Many-valued semantics

We turn to the many-valued semantics. A **BNI+** model $m : \mathsf{Sent} \to V$ is a mapping of sentences to the truth-values V constrained by the evaluation scheme, which is shown in table 2. (V is the union of $\{t, f\}$ and some subset of $\{b, n, i\}$, as in table 1.) The scheme for any logic with less than the full set of truth-values just omits those entries in the tables containing truth-values the logic lacks. In **BN+** and stronger, the truth-values can be thought of as forming a lattice as in figure 2,[4] with conjunction as the greatest lower bound, and disjunction as the least upper bound. (Negation is a De Morgan involution with b, n, and e fixed points.)

$\neg$	
t	f
f	t
b	b
n	n
i	i

$\wedge$	t	f	b	n	i
t	t	f	b	n	i
f	f	f	f	f	i
b	b	f	b	f	i
n	n	f	f	n	i
i	i	i	i	i	i

$\vee$	t	f	b	n	i
t	t	t	t	t	i
f	t	f	b	n	i
b	t	b	b	t	i
n	t	n	t	n	i
i	i	i	i	i	i

Table 2: The connectives in **BNI+**

In systems with $b \in V$, the set of designated—roughly, *at least true*—values $D = \{t, b\}$; otherwise $D = \{t\}$.

Definition. *A model* $m : \mathsf{Sent} \to V$ satisfies *a sentence* A *iff* $m(A) \in D$.

Definition. *A model* m satisfies *set of sentences* Γ *iff for all* $A \in \Gamma$*,* m *satisfies* A*.*

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[4] More generally, they form a bilattice (a set plus two lattice orderings) with a truth ordering (shown) and an information ordering in which b carries maximal information, n minimal information, and t, f incomparable with one another in between b and n.

[5] Belnap 2019a, p. 60.

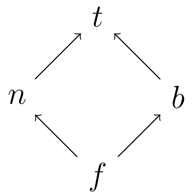

Figure 2: The *truth* (partial) order on $\{t, f, b, n\}$[5]

Validity is then informally understood as *designation preservation over all models*.[6] We can define a multiple-premiss, single-conclusion consequence relation in the ordinary way: $\Gamma \models A$ iff any model satisfying Γ satisfies A. Formally:

$$\Gamma \models_{+} A \iff \forall m(\forall B_{\in\Gamma}(m(B) \in D) \implies (m(A) \in D).$$

3 Star semantics

3.1 The basic picture: BN*

Star semantics are a form of Kripke-style semantics developed for FDE by Richard Sylvan and Val Plumwood.[7] The basic insight is that instead of adding truth-values, we add points (also called ‘situations’, ‘set-ups’, or ‘worlds’—though this term might have metaphysical undertones we want to avoid) and make negation an intensional rather than extensional operator. This results in a simpler, two-valued, evaluation scheme.

The following is a simplified form of the variant of star semantics for **BN** I’ll call **BN***.[8] (The semantics in its most general form comes later.)

A model M is a triple $\langle W,^{*}, \Vdash\rangle$ satisfying the following constraints:

- $W = \{@, \dots\}$ is a set of points (@ is a designated point);

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[6] A reviewer thought it a good idea for me to explain why I am not using the term ‘truth preservation’, which might be more familiar. *Designation preservation* concerns the designated values t, b. Since there is a truth-value t for *true only*, *truth preservation* might most naturally be thought of as preservation of the value t, but this isn’t what we want: t is merely one of the values we are interested in preserving (the *designated* ones). ‘Designation preservation’ avoids this confusion.

[7] R. Routley and V. Routley 1972.

[8] This differs in some ways from standard contemporary presentations of FDE’s star semantics. For those, see, e.g., Omori and Wansing 2017, p. 1024; Priest 2008, pp. 151–152

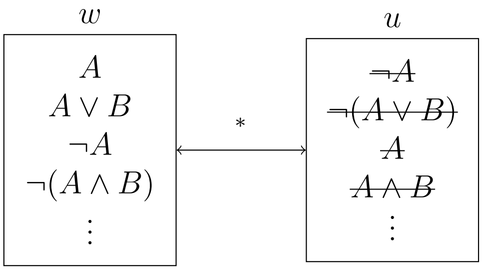

Figure 3: An example star model (~~A~~ at a point means A fails there).

- $^* : W \to W$ is a function on points satisfying
  - $w^{**} = w$ (*involution*); and
- $\Vdash \subseteq W \times \mathsf{Sent}$ such that for $w \in W$:
  - $w \Vdash (A \vee B)$ iff $w \Vdash A$ or $w \Vdash B$;
  - $w \Vdash (A \wedge B)$ iff $w \Vdash A$ and $w \Vdash B$;
  - $w \Vdash \neg A$ iff $w^* \nVdash A$.

Figure 3 gives an example.

Definition. *A point w (in some model)* satisfies *a sentence A iff $w \Vdash A$ (in that model).*

Definition. *A model $M = \langle W, N,^*, \Vdash \rangle$* satisfies *a sentence A iff $@ \Vdash A$ ($@ \in W$).*

Definition. *A model M* satisfies *set of sentences Γ iff for all $A \in \Gamma$, M satisfies A. (Same as before.)*

The definition of multiple-premiss, single-conclusion consequence is broadly the same as before—$\Gamma \models A$ iff any model satisfying Γ satisfies A—but differs in terms of satisfaction of a sentence by a model:

$$\Gamma \models_{*} A \iff \forall M (\forall B_{\in \Gamma} (@ \Vdash B) \implies (@ \Vdash A)).$$

3.2 Extensions of BN*

The semantic systems **B***, **N***, and **Ø*** are obtained from **BN*** as just presented by adding to the model structure and imposing additional constraints. A model M is in this case a quintuple $\langle W, G^+, G^-,^*, \Vdash \rangle$ constrained just as a **BN*** model, but with the further constraints that

- $G^+, G^- \subseteq W$ (G^+ and G^- are subsets of W);
- for $w \in G^+$:
  - $w \Vdash A$ or $w^* \nVdash A$ (*intensional exhaustion*);[9]
- for $w \in G^-$:
  - $w \nVdash A$ or $w^* \Vdash A$ (*intensional exclusion*).

BN* in its more general form has this quintuple model structure and satisfies these constraints, but places no further constraints on the models.

B* adds to **BN*** the constraint that

- $@ \in G^+$,

ensuring (by exhaustion) that @ satisfies at least one of A, $\neg A$ for all A, ruling out the equivalent of n.[10]

N* adds to **BN*** the constraint that

- $@ \in G^-$,

ensuring (by exclusion) that @ satisfies at most one of A, $\neg A$ for all A, ruling out the equivalent of b.

Ø* adds to **BN*** the constraint that

- $@ \in G^+ \cap G^-$,[11]

ensuring @ satisfies exactly one of A, $\neg A$ for all A.[12]

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[9] This principle comes from Beall 2009, p. 9.

[10] One will note that if w satisfies exhaustion, then w^* will satisfy exclusion, and that if w satisfies exclusion, w^* will satisfy exhaustion.

[11] One can also get **Ø*** from the simpler **BN*** semantics by imposing constraint that * satisfies $w^* = w$ (*identity*). This ensures that *every point* satisfies exactly one of A, $\neg A$ (ruling out the equivalents of b and n). Note that any identity function is an involutory function, so we needn't explicitly impose the constraint that $w^{**} = w$.

[12] 'What about $G^+ \cup G^-$?', one might ask. The constraint that $@ \in G^+ \cup G^-$ should yield *symmetric three-valued logic* (Field 2008, pp. 78–81). Importantly, $A \wedge \neg A \models B \vee \neg B$, which is valid in this logic but not **BN**, will turn out valid: any model with $@ \in G^+$ will have $@ \Vdash B \vee \neg B$, and no model with $@ \in G^-$ will have $@ \Vdash A \wedge \neg A$, so every model with $@ \in G^+ \cup G^-$ will either fail to satisfy $A \wedge \neg A$ or satisfy $B \vee \neg B$.

3.3 Generalisation to the i-variants

We are now in a position to present the semantics for **BNI***. It is similar to **BN***, though with different constraints. A model M is a sextuple $\langle W, N, G^+, G^-, ^*, \Vdash\rangle$ satisfying the following constraints:

- $W = \{@, \ldots\}$;
- $N, G^+, G^- \subseteq W$;
- $^* : W \to W$ is a function on points satisfying
  - $w^{**} = w$ (*involution*);
  - $w \in N \implies w^* \in N$ (*closure*: N is closed under *);
- $\Vdash \subseteq W \times \mathsf{Sent}$ such that
  - for $w \in G^+$:
    - for $w \in N$:
      - $w \Vdash A$ or $w^* \nVdash A$;
  - for $w \in G^-$:
    - $w \nVdash A$ or $w^* \Vdash A$;
  - for $w \in N$:
    - $w \Vdash (A \vee B)$ iff $w \Vdash A$ or $w \Vdash B$,
    - $w \Vdash (A \wedge B)$ iff $w \Vdash A$ and $w \Vdash B$,
    - $w \Vdash \neg A$ iff $w^* \nVdash A$; and
  - for $w \notin N$:
    - $w \Vdash (A \vee B)$ iff $w \Vdash A$ or $w \Vdash B$ and w *acknowledges* A and B,
    - $w \Vdash (A \wedge B)$ iff $w \Vdash A$ and $w \Vdash B$ and w *acknowledges* A and B,
    - $w \Vdash \neg A$ iff $w^* \nVdash A$, and
    - $w \Vdash A$ iff $w^* \nVdash A$.

Note that G^+'s exhaustion condition holds for points in N (that is, $G^+ \cap N$).

Definition. *A point w* acknowledges *a sentence A iff w stands in $\Vdash$ to A or $\neg A$.*

The definition of validity is unaffected, and business is as usual for *normal points* (those in N). Let's unpack those conditions on the points outside of N (*abnormal points*). The constraint that $w \Vdash A$ iff $w^* \nVdash A$ ensures that whenever A holds at w (w abnormal), A fails at w^*, and whenever A holds at w^*, A fails at w. By the truth-conditions of negation, this means that whenever A holds at an abnormal point, so does $\neg A$ and whenever $\neg A$ holds, so does A.

The acknowledgement condition requires that a point w stands in $\Vdash$ to A, $\neg A$. By the constraint on negation, the condition simplifies to:

Definition. *A point w* acknowledges *a sentence A iff w stands in $\Vdash$ to A.*

The above conditions for conjunction and disjunction are therefore equivalent to:

- $w \Vdash (A \vee B)$ iff $w \Vdash A$ and $w \Vdash B$,
- $w \Vdash (A \wedge B)$ iff $w \Vdash A$ and $w \Vdash B$.

The result is that for a formula B containing any of the connectives as a major connective, an abnormal point w satisfies B just in case it satisfies all of the subformulae which are arguments of the main connective, and each of those are satisfied just in case the same condition holds with respect to their main connective (or, if the subformula in question is a propositional parameter p, w satisfies p). Conversely, should any propositional parameter p occurring in a formula B be unsatisfied at an abnormal point w, the smallest subformula of B containing p as an argument of a connective will be unsatisfied by w (if there is one—if not, B fails at w trivially), and the smallest subformula of B containing this subformula as an argument of a connective shall likewise be unsatisfied by w (if there is one, else the buck stops here), $\ldots$, and so B shall itself be unsatisfied by w.

All this is to say that nonsatisfaction by an abnormal point is *infectious*—the failure of any subformula of a formula at an abnormal point ensures the failure of that formula there.

Note well that this more general model structure holds for **BN***, **B***, and **N*** too in their most general form, but is moot, since there are no abnormal points ($N = W$ so $W - N = \emptyset$)—hence we can use the simpler semantics without any worries. From these more general models we obtain those of **BN*** and its extensions by imposing the constraint that $@ \in N$.

To obtain **BI***, **NI***, and **I*** from **BNI***, impose the same constraints that when imposed on **BN*** yield **B***, **N***, an **Ø***, respectively:

logic	constraint
BI*	$@ \in G^+$
NI*	$@ \in G^-$
I*	$@ \in G^+ \cap G^-$

4 Equivalence of many-valued and star semantics

In this section, we see that the star semantics and the many-valued semantics are equivalent for the logics **BNI**, **BI**, **NI**, and **I**. (I don't discuss the other logics in detail for sake of brevity.)

4.1 Natural deduction for the FDE family

In recent work, Graham Priest provides natural deduction systems for the FDE family.[13] Let us mark a natural deduction system of this kind with a subscript 'G' (e.g. $\mathbf{BN}_G$ is the natural deduction system for **BN**). Priest has proved these systems sound and complete relative to the many-valued semantics for all the logics in the FDE family. Soundness and completeness results relative to the star semantics then establish the equivalence of the star and many-valued semantics for the logic in question.

In the natural deduction systems, a *basic deduction* in the system is of the form A; *complex deductions* are formed by applying rules to basic deductions and other complex deductions. Then $\Gamma \mid\!\!\!- B$ iff B is at the end of a deduction whose undischarged assumptions (if there are any) are all in Γ. For example, $\{A\} \mid\!\!\!- A$ since A is a deduction whose undischarged assumptions are only in $\{A\}$.

The rules are given in table 3, in which a double line means that a rule goes both ways, $\phi(A)$ may be any sentence containing all of the propositional parameters that occur within A (Priest's notation for this is '$A^\dagger$'), and $[A]^n$ is an assumption discharged by the rule labelled with 'n'. Systems for stronger i-variants add rules from table 4:

logic	extra rules
$\mathbf{BI}_G$	wxm
$\mathbf{NI}_G$	efq
$\mathbf{I}_G$	wxm, efq

In what follows, I shall write the inverse of the star function thus: ${}^*w = u$ iff $u^* = w$—sc. $({}^*w)^* = {}^*(w^*) = w$.

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[13] Priest 2019.

$$\textsf{dn:}\ \begin{array}{c} A \\ \hline\hline \neg\neg A \end{array} \qquad \textsf{dem:}\ \begin{array}{c} \neg(A \wedge B) \\ \hline\hline \neg A \vee \neg B \end{array} \qquad \begin{array}{c} \neg(A \vee B) \\ \hline\hline \neg A \wedge \neg B \end{array}$$

$$\textsf{adj:}\ \frac{A \qquad B}{A \wedge B} \qquad \textsf{s:}\ \frac{A \wedge B}{A} \qquad \frac{A \wedge B}{B}$$

$$\textsf{wad:}\ \frac{A \qquad \phi(B)}{A \vee B} \qquad \frac{\phi(A) \qquad B}{A \vee B}$$

$$\textsf{sc:}\ \frac{A \vee B \qquad \begin{array}{c} [A]^1 \\ \vdots \\ C \end{array} \qquad \begin{array}{c} [B]^2 \\ \vdots \\ C \end{array}}{C}\,{}^{1,2}$$

Table 3: Priest's rules for $\mathbf{BNI}_G$

$$\textsf{efq:}\ \frac{A \wedge \neg A}{B} \qquad \textsf{wxm:}\ \frac{\phi(A)}{A \vee \neg A}$$

Table 4: The extra rules for stronger i-variants

4.2 Soundness

Theorem. $\Gamma \vdash_{\boldsymbol{BNI}_G} A$ *only if* $\Gamma \vDash_{\boldsymbol{BNI}^*} A$. *Soundness for* ***BNI****.*

Proof. By recursion: the base case shows that the basic deduction ($A \vdash A$) is valid, and the step cases show that satisfaction carries over each rule. Base case: $A \vdash A$ only if $A \vDash A$. Take arbitrary model M; if M satisfies A then $@ \Vdash A$, which is the conclusion.

The general idea for the step cases is that we start with a deduction

$$\frac{\begin{array}{c} \Gamma_1 \\ \vdots \\ \mathit{input}_1 \end{array} \quad \begin{array}{c} \Gamma_2 \\ \vdots \\ \mathit{input}_2 \end{array} \quad \cdots}{\mathit{rule\ output}}$$

and for each rule show that any model M satisfies $\Gamma = \Gamma_1 \cup \Gamma_2 \cup \cdots$ (and the *rule inputs* thereby) only if M satisfies the *rule output.* I will (except in the case of sc) leave the deduction from Γ to the rule inputs implicit for sake of brevity and simplicity, and show that M satisfies the inputs only if it satisfies the outputs. (Another way to think of this is to restrict M to models satisfying Γ.)

- dn$\uparrow$: $\neg\neg A \vDash A$. Suppose arbitrary M satisfies $\neg\neg A$. Then $@ \Vdash \neg\neg A$. By the truth-conditions of $\neg$, $@ \Vdash \neg\neg A$ iff $@^* \nVdash \neg A$ iff $@^{**} \Vdash A$. By involution, $@^{**} = @$, so $@ \Vdash A$.

- $\mathsf{dn}\!\downarrow$: $A \models \neg\neg A$. Suppose arbitrary M satisfies A. Then $@ \Vdash A$. By the truth-conditions of $\neg$, $@ \Vdash A$ iff $^{*}@ \nVdash \neg A$ iff $^{**}@ \Vdash \neg\neg A$. By involution, $^{**}@ = (^{**}@)^{**} = @$, so $@ \Vdash \neg\neg A$.

- $\mathsf{dem}_L \downarrow$: $\neg(A \wedge B) \models \neg A \vee \neg B$. Take arbitrary M such that $@ \Vdash \neg(A \wedge B)$. If $@ \notin N$, $@ \Vdash \neg(A \wedge B)$ iff $@ \Vdash A \wedge B$ iff $@ \Vdash A$ and $@ \Vdash B$ iff $@ \Vdash \neg A$ and $@ \Vdash \neg B$ iff $@ \Vdash \neg A \vee \neg B$. In case $@ \in N$, we have $@^{*} \nVdash A \wedge B$; so, by the truth-conditions of $\wedge$, one of A, B must fail at $@^{*}$: $@ \nVdash A$ or $@ \nVdash B$. By the truth-conditions of $\neg$, in former case $@ \Vdash \neg A$, so by the truth-conditions of $\vee$, $@ \Vdash \neg A \vee \neg B$; in latter case, $@ \Vdash \neg B$ and thus $@ \Vdash \neg A \vee \neg B$.

- $\mathsf{dem}_L \uparrow$: $\neg A \vee \neg B \models \neg(A \wedge B)$. Take arbitrary M such that $@ \Vdash \neg A \vee \neg B$. The abnormal case is trivial. If $@ \in N$, either $@ \Vdash \neg A$ or $@ \Vdash \neg B$. From these follow, by the truth-conditions of $\neg$, $@^{*} \nVdash A$ and $@^{*} \nVdash B$, respectively. In either case, the truth-conditions for $A \wedge B$ are not met at $@^{*}$, so $@^{*} \nVdash A \wedge B$. But $@ \Vdash \neg(A \wedge B)$ iff $@^{*} \nVdash A \wedge B$.

- $\mathsf{dem}_R \downarrow$: $\neg(A \vee B) \models \neg A \wedge \neg B$. Take arbitrary M such that $@ \Vdash \neg(A \vee B)$. The abnormal case is trivial. If $@ \in N$, we have $@^{*} \nVdash A \vee B$, so, by the truth-conditions of $\vee$, $@^{*} \nVdash A$ and $@^{*} \nVdash B$. By the truth-conditions of $\neg$, $@ \Vdash \neg A$ and $@ \Vdash \neg B$, and consequently $@ \Vdash \neg A \wedge \neg B$.

- $\mathsf{dem}_R \uparrow$: $\neg A \wedge \neg B \models \neg(A \vee B)$. Take arbitrary M such that $@ \Vdash \neg A \wedge \neg B$. The abnormal case is trivial. If $@ \in N$, truth-conditions of $\wedge$ yield that $@ \Vdash \neg A$ and $@ \Vdash \neg B$, so $@^{*} \nVdash A$ and $@^{*} \nVdash B$. Therefore the truth-conditions of $A \vee B$ cannot be met at $@^{*}$, so $@^{*} \nVdash A \vee B$, and thus, by the truth-conditions of $\neg$, $@ \Vdash \neg(A \vee B)$.

- adj: $\{A, B\} \models A \wedge B$. Take arbitrary M satisfying $\{A, B\}$; M then satisfies A and satisfies B, so $@ \Vdash A$ and $@ \Vdash B$. By the truth-conditions of $\wedge$, $@ \Vdash A \wedge B$.

- s: $A \wedge B \models A$. Take arbitrary M such that $@ \Vdash A \wedge B$. By the truth-conditions of $\wedge$, $@ \Vdash A$. The B case is analogous.

- wad: $\{A, \phi(B)\} \models A \vee B$. Take arbitrary M such that $@ \Vdash A$ and $@ \Vdash \phi(B)$. If $@ \in N$, then, by the truth-conditions of $\vee$, $@ \Vdash A \vee B$. If $@ \notin N$, then, by definition of $\phi()$, $@$ stands in $\Vdash$ to all propositional parameters occurring in B (call this set of sentences Φ_0). Let Φ_{n+1} be defined inductively as the union of Φ_n with the set of sentences formed by negating, conjoining, or disjoining any of the sentences in Φ_n, and let Φ be the union of Φ_n for all n. By the truth-conditions of operators at abnormal points (given the acknowledgement condition and the constraint that $w \Vdash A$ iff $w^* \nVdash A$), if @ stands in $\Vdash$ to all the sentences in Φ_n, @ stands in $\Vdash$ to all the sentences in Φ_{n+1}. Since @ stands in $\Vdash$ to everything in Φ_0 and $B \in \Phi$, $@ \Vdash B$, and thus, by the truth-conditions of $\vee$, $@ \Vdash A \vee B$. The $\{B, \phi(A)\}$ case is analogous.

- sc: Priest has a proof that works just as well for us.[14] To summarise, suppose we have $\Gamma_1 \vdash A \vee B$, $\Gamma_2 \cup \{A\} \vdash C$, and $\Gamma_3 \cup \{B\} \vdash C$. Assume for recursion that $\Gamma_1 \cup \Gamma_2 \cup \Gamma_3 \subseteq \Delta$ and that $\Gamma_1 \models A \vee B$, $\Gamma_2 \cup \{A\} \models C$, and $\Gamma_3 \cup \{B\} \models C$. $\Delta \models A \vee B$ (since $\models$ is monotonic), so, by the truth-conditions of $\vee$, $\Delta \models A$ or $\Delta \models B$, from each of which follows $\Delta \models C$ (since $\Delta \cup \{A\} \models C$ and $\Delta \cup \{B\} \models C$). ☐

Theorem. $\Gamma \vdash_{\boldsymbol{BI}_G} A$ *only if* $\Gamma \models_{\boldsymbol{BI}^*} A$. *Soundness for **BI**.*

Proof. We extend $\mathbf{BNI}_G$ with wxm:

- wxm: $\phi(A) \models A \vee \neg A$. Suppose arbitrary M satisfies $\phi(A)$, so $@ \Vdash \phi(A)$. If $@ \notin N$, by definition of $\phi()$, @ stands in $\Vdash$ to all propositional parameters occurring in A (this set Φ_0). Construct Φ as in wad. Then since @ stands in $\Vdash$ to all the sentences in Φ_0, and $A \in \Phi$ (and thus $\neg A \in \Phi$), we have $@ \Vdash A$ and $@ \Vdash \neg A$ and, by the truth-conditions of $\vee$, $@ \Vdash A \vee \neg A$.

  In the case of $@ \in N$, we start with the fact that, since $@ \in G^+$, either $@ \Vdash A$ or $@^* \nVdash A$. If $@ \Vdash A$, $@ \Vdash A \vee \neg A$ follows by the truth-conditions for $\vee$. If $@^* \nVdash A$, $@ \Vdash \neg A$ and so $@ \Vdash A \vee \neg A$. ☐

Theorem. $\Gamma \vdash_{\boldsymbol{NI}_G} A$ *only if* $\Gamma \models_{\boldsymbol{NI}^*} A$. *Soundness for **NI**.*

Proof. We extend $\mathbf{BNI}_G$ with efq:

- efq: $A \wedge \neg A \models B$. Suppose not: then there's some model M such that $@ \Vdash A \wedge \neg A$, but $@ \nVdash B$. By the truth-conditions of $\wedge$, $@ \Vdash A$ and $@ \Vdash \neg A$. But $@ \Vdash \neg A$ only if $@^* \nVdash A$. Since $\mathbf{NI}^*$ models require $w \nVdash A$ or $w^* \Vdash A$ (since $@ \in G^-$), M isn't a model. ☐

Theorem. $\Gamma \vdash_{\boldsymbol{I}_G} A$ *only if* $\Gamma \models_{\boldsymbol{I}^*} A$. *Soundness for **I**.*

Proof. We extend $\mathbf{BNI}_G$ with efq and wxm. ☐

For soundness results for **BN**, **B**, **N**, and **Ø**, we'd need to check the soundness of the extra rules relative to the relevant models.[15]

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[14] Ibid., pp. 282–283.

[15] See ibid., pp. 286–289.

4.3 Completeness

Lemma. *Henkin construction:* $\Gamma \nvdash A$ *only if there's some* $\Pi \supseteq \Gamma$ *such that* $\Pi \nvdash A$*;* $\Pi \vdash B$ *only if* $B \in \Pi$ *(closure); and* $\Pi \vdash B \vee C$ *only if* $\Pi \vdash B$ *or* $\Pi \vdash C$ *(primeness).*

Proof. Here I'll summarise Priest's proof.[16] The formulae B_i are enumerated and Π (a Henkin theory) is constructed from $\Gamma = \Pi_0$ by taking the union of each Π_i defined as $\Pi_{n+1} = \Pi_n \cup \{B_n\}$ iff $\Pi_n \cup \{B_n\} \nvdash A$, and $\Pi_{n+1} = \Pi_n$ otherwise. $\Pi \nvdash A$ follows from the fact that Π is compact. For closure, suppose for reductio that $\Pi \vdash B_n$ but $B_n \notin \Pi$; but then, by construction, $\Pi_n \cup \{B_n\} \vdash A$, so $\Pi \vdash A$. For primeness, suppose $\Pi \vdash B_n \vee B_m$ but $B_n \notin \Pi$ and $B_m \notin \Pi$. Then $\Pi_n \cup \{B_n\} \vdash A$ and $\Pi_m \cup \{B_m\} \vdash A$, so $\Pi \vdash A$. ☐

Lemma. *Antitheory construction: Let* Π *be a Henkin theory. There's some* Σ *extending* $\{\neg B \mid B \notin \Pi\}$ *such that* $\Sigma \nvdash C$ *for all* C *in* $\{C \mid \neg C \in \Pi\}$*;* $\Sigma \vdash D$ *only if* $D \in \Sigma$ *(closure); and* $\Sigma \vdash D \vee E$ *only if* $\Sigma \vdash D$ *or* $\Sigma \vdash E$ *(primeness). Call* Σ *the* antitheory twin *of* Π.

Proof. Enumerate the formulae B_i and construct Σ from $\{\neg C \mid C \notin \Pi\} = \Sigma_0$ as in the Henkin construction but the with inductive definition of Σ_n changed to $\Sigma_{n+1} = \Sigma_n \cup \{B_n\}$ iff for all $D \in \{D \mid \neg D \in \Pi\}$, $\Sigma_n \cup \{B_n\} \nvdash D$, and $\Sigma_{n+1} = \Sigma_n$ otherwise. Closure, primeness, and that $\Sigma \nvdash D$ for all $D \in \{D \mid \neg D \in \Pi\}$ are analogous to the Henkin case. ☐

Lemma. *Let* Π *be a Henkin theory and* Σ *its antitheory. Then* Π *is the antitheory of* Σ.

Proof. Let Φ be the antitheory of Σ, so we need to show that $\Phi = \Pi$: $A \in \Pi$ iff $A \in \Phi$. For the left-to-right conditional, suppose $A \in \Pi$: then, by closure via dn, $\neg\neg A \in \Pi$. By the construction of Σ, $\neg A \notin \Sigma$ and hence, by the construction of Φ, $\neg\neg A \in \Phi$, from which we get $A \in \Phi$ by closure via dn. The right-to-left conditional is similar. ☐

Theorem. $\Gamma \models_{\boldsymbol{BNI^*}} A$ *only if* $\Gamma \vdash_{\boldsymbol{BNI}_G} A$. *Completeness for* ***BNI***.

Proof. By contraposition: $\Gamma \nvdash A$ only if $\Gamma \nvDash A$. Let Π be a Henkin theory extending Γ, and let Σ be the antitheory twin of Π. We then construct a model $M = \langle W, N, G^+, G^-, ^*, \Vdash \rangle$ with $W = \{@, @^*\}$ constrained in the following way:[17]

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[16] Ibid., p. 283.

[17] For discussion on the size of models, I'm grateful to an anonymous reviewer.

- $@ \Vdash B$ iff $B \in \Pi$;
- $@^* \Vdash B$ iff $B \in \Sigma$;
- $@ \in N$ iff there's some C such that $\phi(C) \in \Pi$ but $C \notin \Pi$;
- $@^* \in N$ iff there's some C such that $\phi(C) \in \Pi$ but $C \notin \Sigma$;

We now need to check that M really is a model. If so, M is a model satisfying Γ but not A, so $\Gamma \not\models A$ (as wanted). Specifically we need to show that that $\Vdash$ satisfies the truth-conditions of the connectives. We do so by recursion on the truth-conditions of $\wedge$ and $\neg$, with $A \vee B$ equivalent to $\neg(\neg A \wedge \neg B)$.

The base case is trivial, since $\Vdash$ imposes no constraints on the assignment of propositional parameters. The connectives need to satisfy:

- $w \Vdash A \wedge B$ iff $w \Vdash A$ and $w \Vdash B$
- $w \Vdash \neg A$ iff $w^* \nVdash A$

In the following I use @ and Π (matters are similar for $@^*$ and Σ).

- $\wedge$: $@ \Vdash A \wedge B$ iff $@ \Vdash A$ and $@ \Vdash B$—viz. $A \wedge B \in \Pi$ iff $A \in \Pi$ and $B \in \Pi$. Left-to-right is by closure via s. Right-to-left is by closure via adj.
- $\neg$: $@ \Vdash \neg A$ iff $@^* \nVdash A$—viz. $\neg A \in \Pi$ iff $A \notin \Sigma$. Left-to-right is from the construction of Σ by closure: since $\Sigma \nvdash A$ for $\{A \mid \neg A \in \Pi\}$, each such $A \notin \Sigma$. Right-to-left is by the fact that Π and Σ are antitheory twins.

Since $A \vee B$ is equivalent to $\neg(\neg A \wedge \neg B)$, we can define $\vee$ in terms of $\wedge$ and $\neg$, meaning there's no need for a separate case for $\vee$. The deductions are simple.[18] Since the rules are sound, they show semantic equivalence too. $\square$

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[18]

$$\dfrac{\dfrac{\neg(\neg A \wedge \neg B)}{\neg\neg A \vee \neg\neg B} \qquad \dfrac{[\neg\neg A]^1}{A} \qquad \dfrac{[\neg\neg B]^2}{B}}{A \vee B}\,{\scriptstyle 1,2}$$

$$\dfrac{\dfrac{A \vee B \qquad \dfrac{\dfrac{[A]^1}{\neg\neg A}}{\neg\neg A \vee \neg\neg B} \qquad \dfrac{\dfrac{[B]^2}{\neg\neg B}}{\neg\neg A \vee \neg\neg B}}{\neg\neg A \vee \neg\neg B}\,{\scriptstyle 1,2}}{\neg(\neg A \wedge \neg B)}$$

Theorem. $\Gamma \models_{\boldsymbol{BI^*}} A$ *only if* $\Gamma \vdash_{\boldsymbol{BI}_G} A$. *Completeness for* ***BI***.

Proof. For `wxm`, we add the constraint to M that $@ \in G^+$. The truth conditions are unaffected. ☐

Theorem. $\Gamma \models_{\boldsymbol{NI^*}} A$ *only if* $\Gamma \vdash_{\boldsymbol{NI}_G} A$. *Completeness for* ***NI***.

Proof. For `efq`, we add the constraint to M that $@ \in G^-$. The truth conditions are unaffected. ☐

Theorem. $\Gamma \models_{\boldsymbol{I^*}} A$ *only if* $\Gamma \vdash_{\boldsymbol{I}_G} A$. *Completeness for* ***I***.

Proof. For both `wxm` and `efq`, we add both constraints, so $@ \in G^+ \cap G^-$. ☐

5 From pure to applied semantics

5.1 What does it all mean?

Up until now, we have been broadly concerned with the 'pure' semantics of the FDE family and of infectiousness. Let us now turn to the 'applied' semantics. A pertinent motivation for considering this is given by reflecting on what the semantics mean—*really mean*. Johan van Benthem puts this concern bluntly for star semantics when he says: '[p]ending further explanation of the nature of [the * operator], one cannot even begin to say if [star semantics] is more than just a formal trick'.[19]

What is at stake in the choice of semantics? Briefly, one important difference between the many-valued and star semantics concerns what is the most natural interpretation of negation. On the star semantics it is natural to think of negation as an intensional *exclusion operator*: the fact that $\neg A$ holds at w is grounded in the fact that A fails at w^*, w^* being the point ('world', with all its metaphysical import, is perhaps appropriate here) recording what is *compatible* with w.[20] The many-valued semantics would seem to retain the classical interpretation of negation as the operator that makes the (at least) true (at least) false and the (at least) false (at least) true, but admits as logical possibilities those cases where sentences are both true and false and those where sentences are neither true nor false (and the strange cases, too).

Detailed discussion of each of these interpretations is outside the scope of this paper, but I would like to take the opportunity to briefly gesture in the direction of negation as an exclusion operator, which I hope will frame

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[19] Van Benthem 1979, p. 341.

[20] Meyer and Martin 1986, pp. 306–310.

discussion in the rest of this section. We first note that the two interpretations are equivalent in terms of when something's true/false, at least when we think of falsity as truth of negation (which is widely-held).[21] The consideration in favour of negation as exclusion is then the fact that this interpretation *explains why* falsity is truth of negation, whereas the classical interpretation says nothing beyond that falsity is truth of negation, and is thus open to the charge of adhocery. A is false at w when $\neg A$ is true at w because $w \Vdash \neg A$ means $w^* \nVdash A$—that is, A fails to be compatible with the way things are at w, so it's false.

So, the star semantics has something going for it. But how much has it going for it? Let us consider some interpretations of the i value, and try to make sense of them in terms of the semantics presented in this paper.

Three of the candidates for an interpretation of logics containing i are:

- the *nonsense* interpretation,
- the *off-topic* interpretation, and
- the *emptiness* interpretation.

5.2 Nonsense

Perhaps the most prominent interpretation of i is due to Bochvar.[22] On the *nonsense* interpretation, a sentence assigned i is meaningless or senseless. This senselessness, it's thought, is inherited by any sentence in which it occurs. Let's take the liar sentence ('this sentence is false') as our candidate for the bearer of our infectious value. Infectiousness means the following are meaningless:

- 'It's not the case that this setence is false'
- 'Hillary climbed Everest and this sentence is false'
- 'Hillary climbed Everest or this sentence is false'

How is this thought extended to the star semantics? A natural thought is as follows. To fail at one of the abnormal points is to be senseless—it's just like being i. When we are in a context in which the meaningfulness of our expressions is guaranteed, then, we can constrain our models to those in which @ is normal. When our expressions may lack meaning/sense, @ may be abnormal.

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[21] Ibid., 308, do not hold this, thinking of falsity as failure at a point.

[22] Bochvar and Bergmann 1981.

But what grounds have we to say that this nonsense value is infectious? In the case of paradoxes like the liar, this is far from obvious. Bochvar takes these paradoxes as paradigm examples of meaningless sentences (the logic of his concern being **I**),[23] but in **N** or weaker or **B** or weaker *indeterminate* doesn't seem the most obvious assignment. The liar sentence looks like it should be both true and false, and its twin ('this sentence is true') looks like it should be neither true nor false, and the thought that those things which walk and quack like ducks are probably ducks is a compelling one.[24]

Even if the liar and related puzzles are not genuinely antinomous in the sense *indeterminate* requires, there may well be other candidates, but it's not altogether obvious what these would be. This would seem to leave i-variant infectious logics insufficiently motivated with respect to their i-free uninfectious twins, offering little in the way of explanatory resources we didn't already have—that pigs *could*, for all we know, fly is not a strong reason to revise our folk theory of porcine aviation. (But a flying pig is.)

The more underlying worry here is that if we can't sensibly talk about something (in the way required for infectiousness to apply), why are there sentences in our language about it? Why can we conjoin, disjoin, and negate it? We leave this thought for now, and shall return to it later.

5.3 Incongruity

The *off-topic* interpretation is a more recent suggestion of Jc Beall's, formulated in response to weaknesses in the *nonsense* interpretation.[25] On this account, the truth-values go:

- t: true (and not false) and on-topic;
- f: false (and not true) and on-topic;
- b: both true and false, and on-topic;
- n: neither true nor false, and on-topic;
- i: off-topic.

Suppose we have it that Marmite is tasty. Does it follow that Marmite is tasty or Wellington is in New Zealand? In the i-variants, no. But why not? On the *off-topic* interpretation, while *truth* is preserved over the inference,

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[23] Ibid., pp. 105–107.

[24] Beall 2018, pp. 48–49.

[25] Beall 2016.

topic is not. The topic of 'Marmite is tasty' is *Marmite* or *tasty food* or similar, whereas the topic of 'Marmite is tasty or Wellington is in New Zealand' is some compound or product of the topics of the disjuncts.

The application to the star semantics is analogous to the *nonsense* interpretation case. The normal points are ones where everything is on-topic, and the abnormal points ones where things may be off-topic. When @ is constrained to the normal points, we are guaranteed to be on-topic; when it isn't, we aren't.

On such an interpretation, it is natural to think of points (normal points, at any rate) as *theories* concerning some topic, and classes of points as collections of theories concerning that topic. We might add the qualification that a theory may not be exhaustive with respect to its topic, so let's concern ourselves only with exhaustive theories.[26]

It is then natural to think of a model as a an *exhaustive theoretical position* with respect to some topic: @ is the correct theory of the topic according to that position, other normal points are rival theories concerning the topic, regarded by that position as incorrect, and abnormal points are theories which are off-topic by the lights of that position.

Let us consider the topic that encompasses everything. There will be a class of exhaustive theoretical positions (models) concerned with such a topic, and among such a class of exhaustive theoretical positions, there will be a correct one—the *theory of everything.* @ will be the way that everything is and other normal points will be ways that everything isn't. But what are we to say about abnormal points?

When faced with such a question, two options seem salient. The first is to say that such a model will have no abnormal points, since it concerns everything, and everything means *everything.* Such an answer would yield that the correct logic, insofar as logic is concerned with 'absolute generality' or 'universal closure', is **BN** (FDE) or one of its extensions—at any rate, not an infectious logic. Indeed, Beall has made an argument along such lines for **BN**.[27] (Note that this wouldn't make infectious logics useless, since we are nearly always concerned with less than everything, so things might still be off-topic with respect to what we're interested in.)

The second is to countenance the idea that there are certain matters which are off-topic with respect to everything. This is difficult to get one's

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[26] What to say about inexhaustive theories on this sort of account is not totally clear. We could have it such that claims w doesn't decide A are such that $w \nVdash A$ and $w^* \Vdash A$ (neither true nor false), but then it is unclear how to draw the distinction between matters a theory makes no decision on, and matters a theory holds are underdetermined or otherwise neither true nor false.

[27] Beall 2018; Beall 2019.

head around, but it's to be expected that the theory of everything seems strange in certain respects. But even stranger is the idea that @ could be abnormal (as the *i*-variant models allow). To these thoughts we shall return.

5.4 Emptiness

It has been suggested that the *catuṣkoṭi* ('four corners' or tetralemma) of classical Buddhist philosophy corresponds to **BN**.[28] The catuṣkoṭi's exclusive corners (and their **BN** counterparts) are:

corner	**BN+**	**BN***
being A	$m(A) = t$	$@ \Vdash A$, $@^* \Vdash A$;
not being A	$m(A) = f$	$@ \nVdash A$, $@^* \nVdash A$;
both being and not being A	$m(A) = b$	$@ \Vdash A$, $@^* \nVdash A$;
neither being nor not being A	$m(A) = n$	$@ \nVdash A$, $@^* \Vdash A$

But Nāgārjuna, founder of the Madhyamaka school, sometimes rejects all the corners—the *fourfold negation*—:

> Having passed into nirvana, the Victorious Conqueror
> Is neither said to be existent
> Nor said to be nonexistent.
> Neither both nor neither are said.[29]

Garfield and Priest analyse this as demanding another truth-value—our *i*, yielding **BNI**+—to formally capture the Madhyamaka concept of *śūnyatā* (emptiness, the absence of *svabhāva* or essence).[30] Madhyamaka metaphysics holds that *ultimate reality* (linked to *ultimate truth*) exhibits emptiness in this sense—everything is grounded (in a certain sense) in other things. *Conventional reality* (linked to *conventional truth*) isn't empty, since we speak and think of things as having essence. The picture is nihilistic with respect to the ultimate truth of our views while allowing them some sort of (conventional) truth, forming a *middle way* (three guesses what 'Madhyamaka' translates to). So the trick, roughly, on the many-valued semantics is to assign *i* when we're speaking of ultimate reality, which defies theorisation, and assign the normal four truth values when we're talking about conventional matters.[31]

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[28] Priest 2010.

[29] Nāgārjuna and Garfield 1995, §21.17.

[30] Garfield and Priest 2009; Priest 2010, §4.

[31] Ibid., §5, thinks the picture is ultimately more complicated, and we end up with a five-valued plurivalent logic.

The story here for the star semantics is similar too. Normal points are linked to effable conventional reality, and abnormal points to ineffable ultimate reality. A sentence evaluated at a normal point concerns conventional reality, and, at an abnormal point, ultimate reality. Since ultimate reality is ineffable, abnormal points won't satisfy sentences said of it. Hence infectiousness is motivated. So, when we allow ourselves to speak of ultimate reality, @ may be abnormal.

5.5 Disjunction and its simulacra

Hitoshi Omori and Damian Szmuc have argued that one interesting feature of infectious logics (when they are given a plurivalent semantics) is this: while their conjunction operator does capture genuine conjunction, their disjunction operator *does not* capture genuine disjunction.[32] To see the force of this claim, consider the fact that $A \wedge B$ is at least true iff A is at least true and B is at least true, but it's not necessarily the case that $A \vee B$ is at least true iff A is at least true or B is at least true, since one of A, B could be at least true, while the other is infectiously untrue—in which case $A \vee B$ wouldn't be at least true.

That the disjunction operator doesn't capture genuine disjunction—doesn't respect the truth-conditions of disjunction—seems even more stark on the star semantics, since the truth-conditions for $A \vee B$ at abnormal points are

- $w \Vdash A \vee B$ iff $w \Vdash A$ and $w \Vdash B$,

equivalent to to those of $A \wedge B$. To put it baldly, disjunctions at abnormal points are effectively conjunctions:

- $w \Vdash A \vee B$ iff $w \Vdash A \wedge B$.[33]

5.6 Putting these thoughts together

We saw that infectiousness (and that to which it applies), on the pure semantics given in this paper, can be interpreted as capturing some sort of meaninglessness or senselessness (on the nonsense interpretation), falling outside the scope of the most general topic (on the off-topic interpretation), and taking a view on ineffable ultimate reality (on the emptiness interpretation). But we were left with two headscratchers: what are the candidates for senseless

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[32] Omori and Szmuc 2017, pp. 279–281; for discussion of plurivalence, see Priest 2014.

[33] One might wonder whether something funny is going on with negation, too. Omori and Szmuc point out that it satisfies the $\neg A$ is at least true/false iff A is at least false/true condition, and it satisfies $w \Vdash \neg A$ iff $w^* \nVdash A$ too. So all seems fine here.

sentences, if not the traditional paradoxes, and how are we to understand abnormal points in our theory of everything?

These thoughts, which seemed puzzling in isolation, seem to fit together now. This is to say that the three interpretations considered, on the star semantics, seem to converge: What is senseless? Answer: that which is off-topic with respect to the absolutely general topic according to the correct absolutely general exhaustive theoretical position. But what could *that* be? Answer: ineffable ultimate reality. (According to the real world, on such a picture, @ would be abnormal.)

What this would seem to suggest is that there is something in this picture, and infectious logics are modelling something interesting. Three broken clocks do not often agree on the time.

However, interesting as that which is infectious seems, so too does there just seem to be something wrong with it—if something is infectious, you probably don't want it. This intuition would seem vindicated by the first option we considered about what to say about abnormal points with respect to the theory of everyting, and by Omori and Szmuc's worries about disjunction. These thoughts fit together too: disjunction in infectious logic is not the right account of disjunction because infectious logic is not the right account of logic: logic is interested in relations between sentences in true theories about some topic or other (or perhaps about the absolutely general topic in particular, depending on how we think of topic-neutrality).[34]

Here we find ourselves back with the distinction between many-valued and star semantics, for an interesting difference here emerges in terms of what it might be appropriate to call their *quarantine strategies.* How do we keep this abnormal infectious stuff from tearing down the logical edifice (and everything else with it)? In less dramatic language, how do we *quarantine* such a pathosis?

The star semantics, in a sense, handles quarantine all by itself. As one will recall, points in a star model for an infectious logic are split into the normal points, where disjunction is disjunction and all is well with the world, and the abnormal points, where things get quite strange. Mathematicians like to describe this sort of distinction as that between the *well-behaved* and the *pathological*, the latter of which seeming particularly appropriate terminology to describe infectious logics.

In the star semantics a sharp line is drawn between well-behaved points and pathological points—the abnormality is confined to $W - N$. In the many-valued semantics, however, models lack such a structural difference, and sentences assigned i are treated like everything else—they pathologise

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[34] For an account of logic along these lines, see Beall 2018; Beall 2019.

the whole model, so to speak. Quarantining then must be done at the level of models. (Maybe this difference counts as some sort of reason in favour of a star semantical treatment of infectiousness, and, by extension, negation as an exclusion operator, or perhaps it's just an interesting observation.)

6 Conclusion

In this paper, I've extended star semantics to the infectious logics in the FDE family, mirroring the existing many-valued semantics. Discussion of the interpretation of infectiousness started with the idea that both what infectiousness is and what is infectious seem of significant philosophical interest but also very difficult to pin down, and came to rest on the idea that three prominent interpretations of infectious logics (when adapted to match the star semantics) seem to converge on taking infectiousness to concern something like ineffability. There's gold in them thar hills, but there aren't really any hills.[35]

References

Asenjo, F. G. (1966). 'A Calculus for Antinomies'. *Notre Dame Journal of Formal Logic*, *16*(1), 103–105.

Beall, J. (2009). *Spandrels of Truth*. Oxford, Oxford University Press.

Beall, J. (2016). 'Off-Topic: A New Interpretation of Weak-Kleene Logic'. *The Australasian Journal of Logic*, *13*(6).

Beall, J. (2017). 'There is no Logical Negation: True, False, Both, and Neither'. *The Australasian Journal of Logic*, *14*(1).

Beall, J. (2018). 'The Simple Argument for Subclassical Logic'. *Philosophical Issues*, *28*(1), 30–54.

Beall, J. (2019). 'FDE as the One True Logic', In *New Essays on Belnap-Dunn Logic*, edited by H. Omori and H. Wansing. Cham, Springer International Publishing.

Belnap, N. D. (2019a). 'A Useful Four-Valued Logic'. In *New Essays on Belnap-Dunn Logic*, edited by H. Omori and H. Wansing (pp. 55–76). Cham, Springer International Publishing.

Belnap, N. D. (2019b). 'How a Computer Should Think'. In *New Essays on Belnap–Dunn Logic*, edited by H. Omori and H. Wansing (pp. 35–53). Cham, Springer International Publishing.

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[35] My thanks are due to three anonymous *UPJA* reviewers, whose comments have proved invaluable.

Bochvar, D. A., & Bergmann, M. (1981). 'On a Three-valued Logical Calculus and Its Application to the Analysis of the Paradoxes of the Classical Extended Functional Calculus'. *History and Philosophy of Logic*, *2*(1-2), 87–112.

Dunn, J. M. (1976). 'Intuitive Semantics for First-Degree Entailments and "Coupled Trees"'. *Philosophical Studies*, *29*(3), 149–168.

Field, H. (2008). *Saving Truth from Paradox.* Oxford, Oxford University Press.

Garfield, J. L., & Priest, G. (2009). 'Mountains are Just Mountains', In *Pointing at the Moon: Buddhism, Logic, Analytic Philosophy*, edited by Jay L. Garfield, T. J. F. Tillemans, and M. d'Amato. Oxford, Oxford University Press.

Meyer, R. K., & Martin, E. P. (1986). 'Logic on the Australian Plan'. *Journal of Philosophical Logic*, *15*(3), 305–332.

Nāgārjuna, & Garfield, J. L. (1995). *The Fundamental Wisdom of the Middle Way: Nāgārjuna's Mūlamadhyamakakārikā.* Oxford, Oxford University Press.

Omori, H., & Szmuc, D. (2017). 'Conjunction and Disjunction in Infectious Logics', In *Logic, Rationality, and Interaction*, edited by A. Batlag, J. Seligman, and T. Yamada. Berlin/Heidelberg, Springer Berlin Heidelberg.

Omori, H., & Wansing, H. (2017). '40 years of FDE: An Introductory Overview'. *Studia Logica*, *105*(6), 1021–1049.

Priest, G. (1979). 'The Logic of Paradox'. *Journal of Philosophical Logic*, *8*(1), 219–241.

Priest, G. (2008). *An Introduction to Non-classical Logic: From If to Is* (2nd ed.). Cambridge, Cambridge University Press.

Priest, G. (2010). 'The Logic of the Catuskoti'. *Comparative Philosophy*, *1*(2), 51–51.

Priest, G. (2014). 'Plurivalent Logics'. *The Australasian Journal of Logic*, *11*(1).

Priest, G. (2019). 'Natural Deduction Systems for Logics in the FDE Family', In *New Essays on Belnap–Dunn Logic*, edited by H. Omori and H. Wansing. Cham, Springer International Publishing.

Routley, R., & Routley, V. (1972). 'The Semantics of First Degree Entailment'. *Noûs*, *6*(4), 335–359.

van Benthem, J. F. A. K. (1979). 'What is Dialectical Logic?' *Erkenntnis*, *14*(3), 333–347.