[Penultimate draft: please do not cite]

# Multiple Reference and Vague Objects

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Abstract

Kilimanjaro is an example of what some philosophers would call a 'vague object': it is only *roughly* 5895 metres tall, its weight is *not* precise and its boundaries are *fuzzy* because some particles are neither determinately part of it nor determinately not part of it. It has been suggested that this vagueness arises as a result of semantic indecision: it is because we didn't make up our mind what the expression “Kilimanjaro” applies to that we can truthfully say such things as “It is indeterminate whether this particle is part of Kilimanjaro”. After reviewing some of the limitations of this approach, I will propose an alternative account, based on a new semantic relation – *multiple reference* – capable of holding in a one-many pattern between a term and several objects in the domain. I will explain how multiple reference works, what differentiates it from plural reference and how it might be used to accommodate at least some aspects of our ordinary discourse about vague objects.

Keywords: vague objects; supervaluationism; plural reference; multiple reference

## 1. Vague objects and precise objects

Many of the objects we talk and think about every day might be described as 'vague'. Consider, for instance, Kilimanjaro, the tallest mountain in the African continent. We know that Kilimanjaro is roughly 5895 metres tall. But if someone asked us to specify Kilimanjaro's exact height, down to the last millimetre, we would feel embarrassed to answer this request. Intuitively, there isn't any such thing as Kilimanjaro's *exact* height because Kilimanjaro is *not* a precise object: it has no precise weight or size and its boundaries are fuzzy, given that certain atoms are neither determinately part of it nor determinately not part of it.

Vague objects are not the only objects there are, however. In fact, wherever there is a vague object, there are also many precise objects, differing from it with respect to this or that precise property. For example, if Sparky is an atom which is neither determinately part of Kilimanjaro nor determinately not part of it, there is at least one precise mountain-like thing (call it "$K_1$") which determinately includes Sparky as one of its parts and at least another precise mountain-like thing (call it "$K_2$") which determinately fails to include Sparky as one of its parts. And the same goes for each and every particle lying on the outskirts of Kilimanjaro. Hence there are, (roughly) where Kilimanjaro is located, many mountain-like things whose weight and size are perfectly precise and whose boundaries are not fuzzy.

This familiar picture raises philosophical questions of various kinds. One question – connected with what Peter Unger (1980) called the "problem of the many" – concerns the many precise mountain-like things located (roughly) where Kilimanjaro is located: how to reconcile their existence with the commonsensical thought that there is just one mountain where Kilimanjaro is located (namely, Kilimanjaro itself)? Another question – connected with the paradox of the heap (or "sorites" paradox) – concerns Kilimanjaro: how does it manage to do the vague things it does? For example, if no precise collection of particles marks its outermost boundaries, how does Kilimanjaro manage to have boundaries at all? In this paper, I will not try to address these questions – or, at least, not directly. My interest is not so much in $K_1$, $K_2$, $K_3$... nor in Kilimanjaro itself, but in the *relationship* between the former and the latter. Most of us share the intuition that, in some important sense, Kilimanjaro is nothing 'over and above' its many precise counterparts. But we also share the intuition that Kilimanjaro is vague, while $K_1$, $K_2$, $K_3$... are not. What I will offer is a way of squaring one intuition with the other. I will remain entirely neutral on the question whether $K_1$, $K_2$, $K_3$... are mountains in their own right. And I will try to assume as little as possible about the nature of Kilimanjaro's vagueness.

## 2. Vagueness as semantic indecision

Perhaps, the simplest way of doing justice to the intuition that Kilimanjaro is nothing over and above $K_1$, $K_2$, $K_3$... would be to say that Kilimanjaro *is* one of $K_1$, $K_2$, $K_3$... We could say this if we embraced a form of *epistemicism* according to which one of $K_1$, $K_2$, $K_3$... is determinately referred to by "Kilimanjaro", though (due to our ignorance of the precise meaning of this word) we are unable to know which. The problem with this approach is that it accommodates one intuition (that Kilimanjaro is nothing over and above $K_1$, $K_2$, $K_3$...) at the expense of contradicting the other (that Kilimanjaro is *not* a precisely delimited object).[1] What if our aim is, instead, to *reconcile* the two intuitions? Then we could try something different. We might suppose that, though none of of $K_1$, $K_2$, $K_3$... is determinately referred to by "Kilimanjaro", "Kilimanjaro" is *indeterminate in reference* among them. This is the position defended by David Lewis in "Many, but Almost One":

> It is absurd to think that we have decided to apply the name "[Kilimanjaro]" to a certain precisely delimited object […]. But we needn't conclude that [this word] must rather apply

1 The observation that epistemicism contradicts some of our pre-theoretic intuitions about ordinary objects does not refute this view, of course. According to Williamson, "the sharp cut-off points for vague terms implied by the epistemic view are in a sense unimaginable, which makes the view counterintuitive without constituting an argument against it" (1997, 218).

> to [a] certain imprecisely delimited, vague [object]. Instead we should conclude that we never quite made up our minds just what [this word applies] to. […] Semantic indecision will suffice to explain the phenomenon of vagueness. We need no vague objects. (Lewis 1999, 169-170)

Along with a diagnosis – vagueness as a kind of semantic indecision – Lewis identified a therapy – a way of coping with semantic indecision. This is the method of *supervaluations*: whenever one encounters a sentence containing a term putatively referring to a vague object, one should look at the admissible interpretations of that sentence, each of which assigns to the term in question a particular precise object as a semantic value. The sentence is true if it is true under all its admissible interpretations, false if it is false under all its admissible interpretations and neither true nor false if it is true under some admissible interpretations and false under others.[2]

While this package is not without costs,[3] its advantages are well known. Even if their domain includes only precise objects, supervaluationists can vindicate large parts of our discourse about vague objects. Platitudes like "Kilimanjaro is in Africa" come out true (the sentence is true under all its admissible interpretations – *that* $K_1$ *is in Africa* , *that* $K_2$ *is in Africa*, *that* $K_3$ *is in Africa*, etc.). And so do logical truths like "Either Sparky is part of Kilimanjaro or it isn't". On the other hand, the vagueness of "Kilimanjaro" is reflected in the

2 The method of supervaluations is due to van Fraassen (1966) and its application in the context of vagueness to Fine (1975). Following Varzi (2007), I will call "supervaluationism" any view that combines the idea that a vague language admits of several precisifications and the use of the method of supervaluations for coping with the precisifications. Different versions of supervaluationism take different stands on the semantic status of the precifications. According to some versions, they are heuristic devices that have little to do with the actual meaning of the vague expression. According to Lewis's version of the view, each vague expression is indeterminate in meaning among its precisifications, so the latter can be regarded as admissible interpretations of the former. There is also a version of supervaluationism on which each precisification corresponds to a meaning that the vague expression actually has – this is what Smith (2008) calls "plurivaluationism". I will focus on Lewis's version of supervaluationism, but the concerns I will raise apply just as well to the other versions. I will say more about plurivaluationism in footnote 18.

3 Following Williamson (1994), there has been an ongoing debate about the extent to which supervaluationism requires a revision of classical logic. See Williams (2008) and Jones (2011) for discussion.

fact that claims like "Sparky is part of Kilimanjaro" come out neither true nor false (the sentence is true under at least one of its admissible interpretations – *that Sparky is part of* $K_1$ – and false under at least another – *that Sparky is part of* $K_2$). All this is – or seems to be – just as it should be.

And yet the supervaluationist strategy does not always work out as we would like it to. Consider:

(1) Kilimanjaro has no precise number of atoms

(2) Kilimanjaro has fuzzy boundaries

(3) Kilimanjaro has no precise weight.

Given that certain atoms (e.g. Sparky) are neither determinately part of Kilimanjaro nor determinately not part of it, one would expect (1) to be true. And, intuitively, (1) *should* be true. But on supervaluational semantics, (1) would seem to come out false, for the simple reason that, under every admissible interpretation, "Kilimanjaro" gets assigned an object with a precise number of atoms. The same goes for (2) and (3), which ascribe to Kilimanjaro features that no precise object can possess.

In order to avoid these results, we could suspend the supervaluationist rule when evaluating sentences like (1) – (3) – an approach that Lewis applies to other problematic cases (Lewis 1999, 173-4). But what's the alternative to the method of supervaluations? And how are we to decide which sentences should be supervaluated and which not? Lewis doesn't say.

An alternative option is to interpret predicates like "has a precise number of atoms", "has fuzzy boundaries" and "has a precise weight" in terms of determinacy, i.e. lack of semantic indecision. Let 'Determinately p' ($\Delta$p) be true if and only if 'p' is true under all admissible interpretations. Then (1)-(3) could be regimented as follows:

(1*) $\sim\exists n\ \Delta\ Tnk$

There is no number n such that n is determinately the number of atoms of Kilimanjaro.

(2*) $\exists r\ Okr\ \&\ \sim\exists r\ \Delta\ Okr$

Kilimanjaro occupies a spatiotemporal region but there is no spatiotemporal region it determinately occupies.

(3*) $\sim\exists v\ \Delta\ Gvk$

There is no value v such that v is determinately the weight value of Kilimanjaro.

Since the candidate referents of Kilimanjaro $K_1$, $K_2$, $K_3$... differ from one another (even if only slightly) in the number of their atoms, their boundaries and their weight, (1*)-(3*) come out supervaluationarily true, as desired.[4]

It is not obvious that, in making such claims as (1)-(3), ordinary speakers are implicitly employing a notion of semantic determinacy. But even if this is assumed to be so, the problem is not completely solved. Under the proposed regimentation, it remains true that each of $K_1$, $K_2$, $K_3$... has a precise number of atoms, precise boundaries and a precise weight. Moreover, even if “Kilimanjaro” gets assigned different objects under different admissible interpretations, the object it gets assigned is always one or another of $K_1$, $K_2$, $K_3$... Hence it is true (and determinately so) that Kilimanjaro is identical to *something* with a precise number

4 Keefe (2000, 186-88) offers a suggestion along these lines, though she focuses more on vague predicates than on vague singular terms. In principle, one could also interpret statements like (1)-(3) metalinguistically, with a truth-predicate in the place of the 'Determinately' operator. But the metalinguistic regimentation is less plausible: on the face of it, when we say “Kilimanjaro has no precise number of atoms”, we are *using* the term “Kilimanjaro”, not *mentioning* it.

of atoms, precise boundaries and a precise weight:

(1**) $\exists x\ \exists n\ x = k\ \&\ \Delta\ Tnx$

Kilimanjaro is something which has a precise number of atoms.

(2**) $\exists x\ \exists r\ x = k\ \&\ \Delta\ Oxr$

Kilimanjaro is something which has precise boundaries.

(3**) $\exists x\ \exists v\ x = k\ \&\ \Delta\ Gvx$

Kilimanjaro is something which has a precise weight.

This is puzzling. Surely, to the extent that I believe Kilimanjaro to have no precise number of atoms I also believe it to be something which has no precise number of atoms. But if we adopt the proposed interpretation of “has a precise number of atoms”, I am right in believing Kilimanjaro to have no precise number of atoms (because (1) is equivalent to (1*), which is true) and wrong in believing it to be something which has no precise number of atoms (because (1**) is also true). So my original intuitions about Kilimanjaro are, at best, only partly accommodated. Meanwhile, I have to accept a distinction between 'F-ing' and 'being something which F-ies' for which my intuitions make hardly any room.[5]

Faced with these complaints, supervaluationists might offer a claim which closely resemble (1**), but, unlike it, comes out false on the proposed regimentation, namely:

(1***) $\exists x\ \exists n\ \Delta\ x = k\ \&\ \Delta\ Tnx$

Something which has a precise number of atoms is determinately identical to

5 Note that (1**) - (3**) are typical cases of existential statements that come out supervaluationarily true even if none of their substitution instances does. For instance, (1**) comes out supervaluationarily true even if there is no object *o* for which “Kilimanjaro is identical to *o* and *o* has a precise number of atoms” is supervaluationarily true. For discussion of this 'semantic anomaly', see Keefe (2000, 181-3).

Kilimanjaro.

The falsity of (1***) only helps up to a point, though. If (1**) is true, my belief that Kilimanjaro is *not* something with a precise number of atoms is flat-out wrong. One can suggest that I hold that belief because I conflate (1**) and (1***). But this is effectively giving up on trying to vindicate a certain intuition, while offering an error theory for why we have it – which is not exactly what we were after.[6]

Claims about what is or is not determinately the case are the source of other intuitive difficulties. Define 'It is indeterminate whether p' ($\nabla$p) as 'Neither determinately p nor determinately not p' and consider claim that it is indeterminate whether Sparky is part of Kilimanjaro, which is supervaluationarily true:

(4) $\nabla$ P*sk*

It is indeterminate whether Sparky is part of Kilimanjaro

Insofar as one finds (4) plausible, one is naturally tempted to say that there exists something (Kilimanjaro, of course) such that it indeterminate whether Sparky is part of it:

(5) $\exists$x $\nabla$ P*s*x

There is something such that it is indeterminate whether Sparky is part of it

6 The supervaluationist's acceptance of (1**)-(3**) closely parallels her denial of the soritical premise "For no *n*, *n* grains make a heap but *n* - 1 grains do not make a heap", which Keefe describes as "one of the least appealing aspects of [supervaluationism]" (2000, 183). According to Keefe, we endorse the premise because we confuse the (wrong) thought that no number of grains makes the difference between a heap and a non-heap with the (right) thought that no number of grains *determinately* makes the difference between a heap and a non-heap (2000, 185).

But supervaluationists cannot accept the truth of (5). By hypothesis, their domain of quantification contains only precise object. And how could a precise object have Sparky as a borderline part? Sparky is itself precise and, plausibly, if x and y are precise, there can be no indeterminacy in whether or not x is part of y. So *nothing* in the domain is such that it is indeterminate whether Sparky is part of it, which makes (5) false.

Similarly, consider the inference from (6) to (7):

(6) $\Delta\, k = k$

It is determinately the case that Kilimanjaro is self-identical

(7) $\exists x\, \Delta\, k = x$

There is something which is determinately identical to Kilimanjaro

On supervaluational semantics, the sentence “Kilimanjaro is self-identical” is true on every admissible interpretation, so (6) is true. But “Kilimanjaro” refers to different objects under different admissible interpretations. Assuming the objects in question to be determinately self-identical and determinately distinct from one another, none of them can be determinately identical to Kilimanjaro. (7) must therefore therefore false.[7]

The failure of these inferences does not show that the supervaluationist view is logically incoherent. But it does make one worry about its implications for the ontological status of Kilimanjaro. Supervaluationists can vindicate the truth of “Kilimanjaro exists” (after all, it is true of all of “Kilimanjaro”'s candidate referents that they exist). But if – contrary to

7 Even in a supervaluationist setting, (5) and (7) follow from (respectively) (4) and (6) on a substitutional account of quantification. But note that we want the inferences to be valid in natural languages like English and it is unclear whether natural language quantification is interpretable as substitutional. For discussion of the failure of the existential quantifier to commute with the 'Determinately' operator, see McGee and McLaughlin (1994, 212).

(5) – there is literally *nothing* of which Sparky is a borderline part, in what sense is Kilimanjaro *something*? Conversely, if Kilimanjaro exist, how can there fail to be an object which is determinately identical to it?[8]

Related worries arise in connection with identity statements like:

(8) $k = k_1$

Kilimanjaro is identical to $K_1$

If it is indeterminate whether Sparky is part of Kilimanjaro, Kilimanjaro is such that it is indeterminate whether Sparky is part of it. But $K_1$ is *not* such that it is indeterminate whether Sparky is part of it. So Kilimanjaro has a property that $K_1$ doesn't have, which should be reason enough for thinking that Kilimajaro and $K_1$ are *not* identical and that (8) is false.[9] Not so for supervaluationists. On their account, (8) is neither true nor false (because it is false on all admissible interpretations, except the one on which “Kilimanjaro” gets assigned $K_1$ as semantic value) and the simple argument I've just given for the falsity of (8) is fallacious (because it involves a fallacious use of Leibniz's law).[10] Here as elsewhere, the supervaluationist's viewpoint is theoretically well-motivated. But doubts arise all the same. Kilimanjaro is vague and $K_1$ isn't. So how can the proposition that they are the same thing be indeterminate, rather than outright false? Conversely, if it is indeterminate whether Kilimanjaro is identical to a precise object like $K_1$, shouldn't it also be indeterminate whether

8 It has been noticed that these difficulties have repercussions elsewhere, for example when it comes to provide a semantics for vague terms occurring in speech reports (Schiffer 2000) and belief reports (McGee and McLaughlin 2000).

9 The idea of using Leibniz's law to show that a vague objects is distinct from any of its precise counterparts was first discussed by Gareth Evans (1978).

10 The fallacy consists in thinking that since “It is indeterminate whether Sparky is part of Kilimanjaro” is true, there must be an x such that it is indeterminate whether Sparky is part of it. On the supervaluationist account, there is no such x (this is why the inference from (4) to (5) fails). So no such x can be shown to be distinct from $K_1$, $K_2$, $K_3$... using Leibniz's Law. See Varzi (2001) for discussion.

Kilimanjaro is vague (rather than precise) and $K_1$ precise (rather than vague)? And yet it isn't.

None of these difficulties is fatal to the supervaluationist treatment of “Kilimanjaro” (let alone to supervaluationism as such). What each of them shows, however, is that the idea of indeterminate reference can only get us so far when it comes to accommodating our ordinary talk of vague objects. If our aim is to *explain*, rather than *eliminate*, the phenomenon of vagueness, I don't think that we can be completely satisfied with interpreting talk of vague objects as indeterminate talk of precise objects. The question is: what else can we do?

## 3. Vagueness and multiple reference

Lewis says that we need no vague objects. But insofar as we ordinarily say and think that Kilimanjaro is something with fuzzy boundaries and no precise weight or number of atoms (something that, because of its vagueness, seems quite distinct from each of $K_1$, $K_2$, $K_3$...) there is a sense in which we *do* need vague objects – alongside precise objects, they *are* part of the world we live in, as ordinarily conceived.[11] What is true is that, when providing a semantics for our ordinary discourse about the world, we shouldn't need to make 'special provision' for vague objects – once precise objects have been included in our ontology, vague objects should, as it were, result automatically. A vague object is not a nothing, but, intuitively, it is nothing *over and above* its many precise counterparts.

This is where the crux of the problem lies. When we say 'nothing over and above', we mean that facts involving the vague object (including, of course, its existence) *require no*

11 Saying that vague objects are part of our ordinary conception of the world is *not* saying that vague identity is part of our ordinary conception of the world. Pace Evans (1978), the existence of vague objects does not immediately require matters of identity to be vague. For discussion of this point, see Williamson 1994, 255-6.

*more of the world* than facts involving the object's precise counterparts.[12] On the supervaluationist account, it is quite clear how this can be so: if talk of Kilimanjaro is understood as indeterminate talk of of $K_1$, $K_2$, $K_3$..., there is an obvious sense in which Kilimanjaro is nothing over and above $K_1$, $K_2$, $K_3$... But what happens if we set the idea of indeterminate reference aside? Remember that we want to say that some things are true of Kilimanjaro without being true of each of $K_1$, $K_2$, $K_3$... (for example, it is true of Kilimanjaro that it has fuzzy boundaries, but the same is not true of each $K_1$, $K_2$, $K_3$...). Conversely, we want to say that some things are true of each of $K_1$, $K_2$, $K_3$... without being true of Kilimanjaro (for example, it is true of each of $K_1$, $K_2$, $K_3$... that they have a precise weight, but the same is not true of Kilimanjaro). How can we say such things without introducing a dubious dualism of vague objects and precise objects in our domain?

Put this way, the problem might seem unsolvable. But then an analogy suggests itself. Some things are true of the Beatles without being true of each of Paul, John, George and Ringo (for example, the Beatles may be famous in Austria without Paul, John, George and Ringo being famous in Austria). And some things are true of each of Paul, John, George and Ringo without being true of the Beatles (for example, each of Paul, John, George and Ringo has done a solo album, but the Beatles have never done a solo album). And yet the Beatles are nothing 'over and above' Paul, John, George and Ringo. We don't need to conceive of the Beatles as a social entity (e.g. the band called “the Beatles”) or an abstract object (e.g. the set including all and only the Beatles) or a mereological composite (e.g. the fusion of the Beatles). Instead, we can say that the Beatles *just are* Paul, John, George and Ringo: the term

12 The idea of a fact or truth requiring 'no more of the world' than another fact or truth can be found in Thomasson (2007) and deRosset (forthcoming). One natural way of glossing this idea is in terms of grounding: roughly, a fact requires nothing more of the world than another fact if full grounds for the latter are also full grounds for the former (Thomasson (2007, 16) offers a suggestion along these lines, but she speaks of 'sufficient truth-makers' instead of 'full grounds'). Sider (2015) discusses alternative ways of precisifying the slogan 'nothing over and above', though he focuses mainly on its applications in mereology.

"the Beatles" *plurally refers* to Paul, John, George and Ringo.[13] And the reason why certain predicates that apply to the Beatles do not apply to the referents of "the Beatles" (and vice versa) is simply that these predicates are – as it is usually put – *non-distributive*.

Of course, the point of the analogy is not to suggest that we should take "Kilimanjaro" to be a plural term – this much should be obvious, given that, unlike the Beatles, Kilimanjaro is one, not many. The hypothesis I want to explore is, rather, the following:

- There is, alongside plural reference, another way in which certain expressions of our language – what I will call *multiple* terms – can refer to many items in the domain, instead of just one.
- What should be said of the term "Kilimanjaro" is that it multiply refers to $K_1$, $K_2$, $K_3$...
- Predicates ascribing vague features to Kilimanjaro are *non-distributive*, in much the same way as predicates ascribing collective features to the Beatles are non-distributive.
- Predicates ascribing precise features to $K_1$, $K_2$, $K_3$... are also non-distributive, in much the same way as predicates ascribing individual features to Paul, John, George and Ringo are non-distributive.

The task for the next two sections is to get clear on what multiple reference is and how it differs from plural reference. In § 6, I will return to the problem of the relationship between Kilimanjaro and $K_1$, $K_2$, $K_3$... and explain how multiple reference might help us to solve it.

## 4. Multiple reference versus plural reference

13 I take this to be the case with respect to at least one way of using the term "the Beatles" in English, but clearly nothing crucial hinges on the correctness of this particular analysis. Other plausible (though no less controversial) examples of plural terms include the locution "Russell and Whitehead" as it occurs in "Russell and Whitehead wrote *Principia Mathematica*" or the pronoun "they" as it occurs in "They carried the piano upstairs".

Let me start by introducing a distinction between two kinds of terms:

- A term is *singular* if and only if it denotes one and only one thing in each context in which it is used.
- A term is *non-singular* if and only if it is not singular.

In principle, a term could be non-singular by being empty, i.e. by referring to no object at all. However, my focus hereafter will be exclusively on non-singular non-empty terms. These are terms that denote more than one thing in at least some of the contexts in which they are used. My working hypothesis is that there are two kinds of non-singular non-empty terms: plural terms like “the Beatles” and multiple terms like “Kilimanjaro”. What's the difference between them? To answer this question, two notions need to be introduced.

The first is the notion of a *distributive* predicate. Taking inspiration from Oliver and Smiley (2013, 112), we can say that:

- a predicate '*F*' is *distributive* if and only if it is analytic that '*F'* is true of *b* iff each of the things among *b* is *F*
- A predicate is *non-distributive* if and only if it is not distributive.[14]

Analytic claims are those that are true in virtue of the meaning of their constituents. Thus, what this characterization of distributivity says is that, if a predicate '*F*' is distributive, it is true in virtue of the meaning of '*F*' that '*F*' is true of *b* if and only if each of the things among *b* is *F.* One could put this by saying that, when a predicate is distributive, the very meaning of that predicate *mandates* the distribution, both 'downwards' (from *b* to the single things among *b*) and 'upwards' (from the single things among *b* to *b*). For example, the predicate “be on the stage” is distributive, because its meaning mandates the distribution in both directions: it is analytic that “...are on the stage” is true of the Beatles if and only if each of the individuals

14 See also Linnebo (2014) for a definition along similar lines.

among the Beatles is on the stage. By contrast, when a predicate is non-distributive, the distribution is *not* mandated. This does not mean that the distribution is prohibited or ruled out. It only means that the meaning of the predicate does not settle the matter one way or the other. For example, consider the sentence "The Beatles wrote a song". If the Beatles collectively wrote a song, the sentence is true. But this tells us nothing about whether the predicate "write a song" is *also* true of each the Beatles individually: one could equally well continue the sentence "The Beatles wrote a song" with "...whereas Paul did not" or with "...and so did Paul". In other words, the predicate "write a song" – like other non-distributive predicates – *allows,* but does not mandate the distribution.[15]

The second notion we need is the the notion of *exact denotation.* For present purposes, this notion can be taken as primitive. Intuitively, the exact denotation of a term '*b*' is the best answer one can give to the question "What does '*b*' denote?". To illustrate, suppose someone were to ask you what the term "the Beatles" denotes. Arguably, the best answer you could give is that "the Beatles" denotes Paul, John, George and Ringo. If you were to answer only "Paul", your answer wouldn't be quite as good. The term "the Beatles", then, exactly denotes Paul, John, George and Ringo, without exactly denoting Paul. This shows that the predicate "... exactly denotes..." is non-distributive (more precisely, it is non-distributive at its second place). Other respectable notions of denotation that do not have this feature. For example, we can define a notion of *partial denotation,* such that:

- '*b*' partly denotes x, y, z,... iff x, y, z... are among the things that '*b*' exactly denotes

It follows from this definition that "The Beatles" partly denotes Paul, John, George and Ringo if and only if each of Paul, John, George and Ringo is partly denoted by "The Beatles". This

15 It is a well-known fact that a sentence like "The Beatles wrote a song" is ambiguous between a reading on which the Beatles wrote a song together and a reading on which each of the Beatles wrote a song on his own (Lasersohn 1995 offers a helpful discussion of this topic as well as compelling reasons for locating the source of the ambiguity in the predicate). But that is not the point here. The point is, rather, that the first reading of the sentence (i.e. the non-distributive one) neither requires nor rules that each the Beatles wrote a song on his own. It is in this sense that the non-distributive reading allows, but does not mandate the distribution.

is just to say that, unlike the predicate “...exactly denotes...”, the predicate “...partly denotes...” is distributive (more precisely, it is distributive at its second place).

The non-distributive nature of exact denotation is the key to the distinction between plural and multiple reference, or so I want to suggest. We've seen earlier that, when a predicate is non-distributive, it can, but need not distribute. This means that there is room for distinguishing at least two ways in which many things can be exactly denoted by a term: they can be exactly denoted *collectively* or they can *each* be exactly denoted (the predicate “...exactly denotes...” allows but does not mandate the distribution). So a distinction can be drawn between two kinds of non-singular non-empty terms:

- '*b*' is a *plural* term if and only if many things are *collectively* exactly denoted by '*b*'
- '*b*' is a *multiple* term if and only if many things are *each* exactly denoted by '*b*'

To appreciate the difference, another analogy may be of some help. Metaphysicians often use the notion of an object *exact location*: x's exact location is, roughly, the best answer to the question “Where is x located?”. When x is a spatially extended object (like my body, for instance), many spatial points are collectively occupied by x and the best answer to the question “Where is x located” will mention all of them. In principle, though, it seems conceivable that the question “Where is x located?” could have many equally best answers, each of each may well mention a single spatial point – these cases define what some metaphysicians call *multiple location*.[16] Well, multiple reference is to good-old plural reference what multiple location is to good-old extended location. When a term '*b*' *plurally refers*, the question “What does '*b*' denote?” has a single best answer, mentioning several things. When a term '*b*' *multiply refers*, the question “What does '*b*' denote?” has several equally best answers, each mentioning a single thing

Someone might take issue with this way of setting things up. In particular, the claim

16 For a discussion of multiple location, see Hudson (2005).

that a term is plural if and only if many things are *collectively* exactly denoted by it might be thought to be in tension with Oliver and Smiley's recent argument that plural denotation is neither determinately distributive nor determinately collective (2008; 2013, 93-104). The argument goes like this. Oliver and Smiley start by assuming that plural denotation is whatever notion combines with truth and satisfaction to deliver correct truth conditions for plural predications (2013, 96). Then they go on to show that several notions of denotation combine with truth and satisfaction to deliver correct truth conditions for plural predications, and that some of them are distributive and some of them are not.[17] From this, they conclude that “the extension of plain “denotes” is indeterminate, and there is no fact of the matter whether “Anne, Charlotte, and Emily” just denotes the three of them together, or also denotes any things among them” (Oliver and Smiley 2013, 103).

Now, I am not sure I accept the major premise of Oliver and Smiley's reasoning: combining with truth and satisfaction to deliver correct truth conditions for plural predications is *one* of the things that we should expect a good notion of plural denotation to do, but it might not be the only one. Fitting with our patterns of use is another. And so is being more intrinsically 'natural' than other candidate notions (at least if something in the ballpark Lewis's (1983) reference magnetism is correct). Ultimately, however, all this does not matter too much. For even if Oliver and Smiley's indeterminacy thesis is correct, that does not affect my point about the distinction between plural terms and multiple terms. Let the extension of plain “...denotes...” be as indeterminate as it can be between exact denotation, partial denotation and other one-many denotation relations. My point concerns exclusively exact denotation, not

17 One non-distributive notion of denotation that Oliver and Smiley explicitly define (2013, 99) is the notion of *maximal denotation*:

- '*b*' maximally denotes x, y, z,... if and only if all the things that '*b*' denotes are among x, y, z....

I note in passing that exact denotation and maximal denotation are *not* to be equated with one another. If a term '*b*' maximally denotes x, there is nothing else apart from x that '*b*' denotes. But if a term exactly denotes x, that does not mean that there are no other things that the term denotes – in fact, in the case of multiple terms what happens is precisely that the exact denotation of the term does not coincide with its maximal denotation.

the plain "...denotes...". For me the exact denotation of a term '*b*' is the best (though, perhaps, not the only) answer one can give to the question "What does '*b*' denote?". And since it is plain that in the case of some terms (terms like "the Beatles") no answer mentioning a single item is as good as an answer mentioning several items, it is also plain that the notion of exact denotation, as understood here, is non-distributive. But – to reiterate the same point – non-distributive notions *allow* the distribution, even if they do not mandate it. So the possibility arises of some terms doing with respect to *each* of many items what terms like "the Beatles" only do with respect to many items *taken together*. The hypothesis I am putting forward is simply that there actually are terms doing that – these are what I call "multiple terms". Clearly nothing in this hypothesis conflicts with Oliver and Smiley's claim that the extension of plain "...denotes..." is indeterminate.

## 5. Semantics for multiple reference

So here's my proposal. "Kilimanjaro" is not a case of semantic indecision. It is a case of semantic abundance: rather than being indeterminate in reference among $K_1$, $K_2$, $K_3$..., the term "Kilimanjaro" *multiply refers* to $K_1$, $K_2$, $K_3$..., meaning that *each one* of $K_1$, $K_2$, $K_3$... is exactly denoted by "Kilimanjaro" (equivalently: "$K_1$", "$K_2$", "$K_3$"... are all equally best possible answers to the question "What does the term "Kilimanjaro" denote?"). In the relevant sense, then, Kilimanjaro is nothing over and above the precise things in it (I use the locution "x is *in* something" as the analogue of the locution "x is *among* some things", when multiple rather than plural reference is involved – more about this in a moment).[18] At the same time,

18 As I suggested in § 3, for Kilimanjaro to be nothing over and above $K_1$, $K_2$, $K_3$... is for facts involving Kilimanjaro to *require no more of the world* than facts involving $K_1$, $K_2$, $K_3$..., where this notion can naturally be glossed in terms of grounding (see footnote 11). I take it to be relatively uncontroversial that, whenever '*a*'

we should not be surprised if some predicates can be true of Kilimanjaro without being true of the precise things in Kilimanjaro, while other predicates can be truth of each of the precise things in Kilimanjaro without being true of Kilimanjaro itself – this is just an instance of the familiar fact that not all predicates are distributive.[19, 20]

To flesh out my proposal, let me explain how one might provide a semantics for a language that contains multiple terms alongside singular and plural ones. There are various ways of doing this, but the approach I will pursue here is inspired by three ideas. First, that even if plural terms do not refer to sets, they can conveniently modelled as doing so. For example, even if "the Beatles" does not strictly speaking refer to the set {Paul, John, George, Ringo}, pretending that it does is heuristically useful, because it allows us to understand locutions like "Paul is among the Beatles" in set-theoretic terms (compare: even if "Possibly p" does not strictly speaking involve quantification over possible worlds, pretending that it does is heuristically useful, because it allows us to understand this and other modal locutions in simple quantificational terms). Second, that singular, plural and multiple terms should all be treated on a par – so if we model the semantics of plural terms using sets, we should do the same with singular and multiple terms. In particular, if we pretend the reference of "the Beatles" to be the set {Paul, John, George, Ringo}, we should pretend the reference of "Paul" to be the singleton {Paul}.[21] Third, that whatever kind of entities we pretend singular terms to

refers to $a_1$, $a_2$, $a_3$… (either multiply or plurally), full grounds for facts involving $a_1$, $a_2$, $a_3$… will also be full grounds for facts involving *a*.

19 Notice that, in order to accommodate the possibility of multiply referring terms, the definition of distributivity I offered in the last section has to be amended by replacing "among" with "in or among".

20 The recognition that distributivity may fail in the presence of multiple reference marks a major difference between the approach I am advocating in this paper and the view Smith (2008) calls "plurivaluationism". Plurivaluationists treat expressions like "Kilimanjaro" as having multiple semantic values, but since they deal with semantic multiplicity in the same way in which other supervaluationists deal with semantic indecision (i.e. using the method of supervaluations) their account shares the same limitations as any other form of supervaluationism. In addition, the application of the method of supervalutions to sentences involving multiple terms raises difficulties of its own: given the *prima facie* analogy between plural terms and multiple ones, it is not entirely clear what justifies the application of the method to sentences involving the latter, but not to sentences involving the former.

21 Scha (1981) makes exactly this move.

refer to, we should pretend multiple terms to refer to several entities of that kind. In particular, if we pretend “$K_1$” to refer to a singleton (i.e $\{K_1\}$) we should pretend “Kilimanjaro” to refer to several singletons (i.e.$\{K_1\}$, $\{K_2\}$, $\{K_3\}$,...).

To put these ideas into practice, we can consider a language L that contains:

- singular terms (*a*, *b*, *c*,...), plural terms (*aa*, *bb*, *cc*,...) and multiple terms ($\underline{a}$, $\underline{b}$, $\underline{c}$,...)
- singular variables (*x*, *y*, *z*,...), plural variables (*xx*, *yy*, *zz*,...) and multiple variables ($\underline{x}$, $\underline{y}$, $\underline{z}$,...)
- predicates (*F*, *G*,...) that can combine with any term and any variable – in particular, a predicate of inclusion (<, to be read as “is or are among”) and a predicate of inherence (◄, to be read as “is or are in”);
- an existential quantifier (∃) than can bind singular, plural or multiple variables
- the familiar truth-functional connectives (~, &, v, , →)

Given what I said in the last paragraph and assuming a domain D of precise objects, one natural way of interpreting L is to use a valuation function *V* that assigns to each singular term a singleton of some element of D, to each plural term a subset of D with more than one element and to each multiple term several singletons of elements of D. Slightly more formally:

For any singular term $\alpha$, $V(\alpha)$ is a singleton of some element of D

For any plural term $\alpha\alpha$, $V(\alpha\alpha)$ is a subset of D with two or more elements

For any multiple term $\underline{\alpha}$, $V(\underline{\alpha})$ are two or more singletons of elements of D

Notice that *V* assigns to *each* multiple term *several* singletons of elements of D (for any multiple term $\underline{\alpha}$, '$V(\underline{\alpha})$' is to be read as 'the valuation*s* of $\underline{\alpha}$'). Thus *V* is what Oliver and Smiley call a “multi-valued function”, i.e. a function that can yield multiple values or outputs for the same argument or input (Oliver and Smiley 2013, 140).[22] In Oliver and Smiley's semantics, it is plural terms that get assigned multiple items as their valuations. But I find it more natural to reserve this possibility for multiple terms and let *V* assign each plural term a single set having several items as its members. Dealing with plural terms in this way is in line with the heuristic nature of our model, besides being a fairly widespread practice among linguists.[23]

In principle, singular, plural and multiple variables could be treated in exactly the same way, i.e. by letting *V* assign to each singular variable one singleton, to each plural variable a single set of two or more elements and to each multiple variable two or more singletons.[24] However, a more convenient choice is to allow multiples variables to be assigned by *V* even just one singleton as a limit case:

> For any singular variable $\xi$, $V(\xi)$ is a singleton of some element of D
>
> For any plural variable $\xi\xi$, $V(\xi\xi)$ is a subset of D with more than one element
>
> For any multiple variable $\underline{\xi}$, $V(\underline{\xi})$ are one or more singletons of elements of D.

In this way, the multiple variables of L will function roughly in the same way as English words like “it” or “that”, which can be used indifferently as devices of singular reference (e.g. to denote $K_1$) or as devices of multiple reference (e.g. to denote Kilimanjaro).

Given that we are taking the semantic values of the terms and variables of our language to be sets, it is natural to take the semantic value of predicates to be properties of

22 Oliver and Smiley offer many examples of multi-valued functions, from *the square root of* (both 2 and -2 are square roots of 4) to *the husbands of* (seven distinct individuals are the husbands of Elizabeth Taylor). As an alternative to using a multi-valued valuation function, one could decide to assign each multiple term a single set having several singletons of elements of D as members. However, I find it somewhat inelegant to assign multiple terms semantic values of a different kind than singular and plural terms. Better to treat singular, plural and multiple terms uniformly – among other things, this allows us to see singularity as a limit case of both plurality and multiplicity.

23 See, among others, Landman (1989a; 1989b) and Schwarzschild (1996).

24 For simplicity of exposition, I follow Oliver and Smiley (2013) in using just one function that assigns values to both terms and variables, instead of distinguishing a valuation function for terms and a variable assignment function for variables.

sets:

For any n-place predicate $\Phi$, $V(\Phi)$ is an n-place relation on the subsets of D

Among the properties we need, there will be properties that can hold of a set of only one element (e.g. the property of containing tall elements), properties that can only hold of a set with more than one element (e.g. the property of containing elements that wrote *Principia Mathematica*), but also, crucially, properties that can only hold of more than one set (e.g. the property of being sets that differ from one another in the number of their elements). In particular, the predicate of inclusion ('<') will be assigned by *V* the relation that holds between two sets if and only if the first is a proper subset of the second. This is the relation that holds between the singleton {Paul} and the set {Paul, John, George, Ringo}, i.e. a property of the second kind. By contrast, the predicate of inherence ('◀') will be assigned by *V* the relation that holds between one set and several sets when the former is among the latter. This is a relation that holds between the singleton {Paul} and the sets {Paul}, {John}, {George}, {Ringo}, i.e. a property of the third kind.

The semantic clauses for predication and quantification will be the usual ones (and so will be the ones governing the truth-functional connectives, which I omit for the sake of brevity):

For any n-place predicate $\Phi$ and any singular, plural or multiple term or variable $\tau$, $V(\Phi\tau) = 1$ iff $V(\Phi)$ holds of $V(\tau)$

For any singular, plural or multiple variable $\zeta$, $V(\exists\zeta F\zeta) = 1$ iff there is some $\zeta$-variant $V'$ of $V$ such that $V'(F\zeta) = 1$

Notice that, given what I said in the last paragraph concerning properties that can only hold of more than one set, 'hold of' is effectively a *non*-distributive predicate of the meta-language (more precisely, it is non-distributive at its second place).[25]

To illustrate how this semantics work, let '$k_1$' and 'p' be singular terms that are assigned by *V* , respectively, the singleton $\{K_1\}$ and the singleton {Paul}. Let 'bb' be a plural term that is assigned by *V* the set {Paul, George, John, Ringo}and '$\underline{k}$' a multiple term that is assigned by *V* the singletons $\{K_1\}$, $\{K_2\}$, $\{K_3\}$,... Finally, let 'T' be a predicate that is assigned by *V* the property of being set(s) of elements of D that are tall. Here are some sentences of L that come out true on the semantics, along with their English translations:

| | |
|---|---|
| $T\underline{k}$ | Kilimanjaro is tall |
| Tbb | The Beatles are tall |
| $\exists xx\ Txx$ | There are some tall things |
| $\exists x\ Tx$ | There is something precise which is tall |
| $\exists \underline{x}\ T\underline{x}$ | There is something which is tall |
| $k_1 \blacktriangleleft \underline{k}$ | $K_1$ is in Kilimanjaro |
| $p < bb$ | Paul is among the Beatles |

I translate '$\exists \underline{x}$' with “There is something” because I take English quantification to be multiple quantification (I take this to be shown by the fact that one can infer “There is something F” from “*a* is F” independently of whether “*a*” is a singular or a multiple term, just as one can

25 The same can be said of the predicates 'true of' and 'hold of' in the semantics Oliver and Smiley offer for plural logic (see Oliver and Smiley 2013, 96, 146 and 217). Notice that, since 'hold of' is non-distributive, it cannot be defined in terms of distributive notions such as membership – instead, it is best seen as a primitive. We could avoid having a primitive non-distributive predicate in the meta-language (and instead state the truth-condition for predication in terms of membership) if we decided to assign each multiple term a *single* set having several singletons of elements of D as members. But there are good reasons not to do that: see footnote 20.

infer '∃$\underline{x}$ F$\underline{x}$' both from 'F$\underline{a}$' and from 'Fa'). I do not think that any quantifier expression in English ranges exclusively over precise objects. That is why I think that an accurate translation of L's '∃x' might be something along the lines of “There is something precise”.

## 6. Kilimanjaro

Let us now go back to Kilimanjaro. One nice feature of the semantics I described in the last section is that it allows us to see very clearly why some things can be true of Kilimanjaro without being true of the precise things in Kilimanjaro. For example, take the predicate 'F' and suppose its English translation is “...has fuzzy boundaries”. It seems plausible to think that something has fuzzy boundaries if and only if the things in it have roughly but not exactly the same boundaries. If so, *V* will assign to 'F' the property of being singletons of elements D that have roughly but not exactly the same boundaries, and we will have:

| | |
|---|---|
| F$\underline{k}$ | Kilimanjaro has fuzzy boundaries |
| ~F$k_1$ | $K_1$ does not have fuzzy boundaries |
| ~F$k_2$ | $K_2$ does not have fuzzy boundaries |
| ~F$k_3$ | $K_3$ does not have fuzzy boundaries |
| … | ... |

Conversely, there will be things true of each of the things in Kilimanjaro that are not true of Kilimanjaro itself. For example, take the predicate 'W' and suppose its English translation is

“...has a precise weight”. It seems plausible to suppose that something has a precise weight if and only if the things in it all have the same weight. If so, *V* will assign to 'W' the property of being set(s) that (i) contain elements of D that have a weight, but (ii) do not differ from one another in the weight of their elements, and we will have:

| | |
|---|---|
| $Wk_1$ | $K_1$ has a precise weight |
| $Wk_2$ | $K_2$ has a precise weight |
| $Wk_3$ | $K_3$ has a precise weight |
| … | ... |
| $\sim W\underline{k}$ | Kilimanjaro does not have a precise weight |

There is a similarity with the supervaluationist proposal I discussed in § 2, according to which “Kilimanjaro has a precise weight” must be regimented as '$\exists v\ \Delta\ Gvk'$ (i.e. “There is a value v such that v is determinately the weight value of Kilimanjaro”). On that interpretation, the sentence expresses agreement in weight value among “Kilimanjaro”'s candidate referents (some value v is *determinately* Kilimanjaro's weight value only if *all* of “Kilimanjaro”'s candidate referents have v as weight value). On the present proposal, the sentence expresses agreement in weight value among “Kilimanjaro”'s multiple referents (some value v is Kilimanjaro's *precise* weight value only if *all* the things in Kilimanjaro have v as weight value). One noteworthy difference is that, while that supervaluationist interpretation is essentially involved with a notion of semantic determinacy, the present interpretation is somewhat more naïve, for it treats “...has a precise weight” as expressing a non-semantic property of the same kind as most other ordinary-language predicates.

So far, I have not said anything about whether L and other languages containing multiple terms will respect the principle of bivalence, according to which every meaningful sentence must be either true or false. My view is that, all by itself, the existence of multiple reference does not give us any special reason to deny bivalence (or, at least, it gives us no more reason to deny bivalence than the existence of plural reference does). In particular, it seems to me that someone who thinks that language contains no “semantic indecision” (in the supervaluationist's sense of the term) might accept the existence of “semantic multiplicity” (in my sense of the term) while insisting that every meaningful sentence has a perfectly determinate truth-value. A defender of this view will have to deny that “Sparky is part of Kilimanjaro” (or, for that matter, any other meaningful sentence involving “Kilimanjaro”) is indeterminate in truth-value. But she might accept that “Kilimanjaro has fuzzy boundaries” is true and “Kilimanjaro has a precise weight” false if (as I have been suggesting) the truth of the first sentence requires only that the precise things in Kilimanjaro have roughly but not exactly the same boundaries, whereas the truth of the second sentence requires that the precise things in Kilimanjaro all have the same weight. While I am not especially drawn to this position, I find it interesting that multiple reference might be used to do (at least, partial) justice to the thought that Kilimanjaro is vague rather than precise, even in a setting where language is assumed to be fully bivalent and meaning-facts fully determinate.

Personally, I think there is more attractiveness in the view that language contains *both* semantic multiplicity *and* semantic indecision, with the latter resulting in at least some cases of truth-value indeterminacy. Since I began this paper by discussing the supervaluationist treatment of semantic indecision and truth-value indeterminacy, I want to conclude by showing how supervaluationists could improve their view – in exactly those respects in which I argued it to be lacking in § 2 – by incorporating the idea of multiple reference into their account. The main difference with the standard supervaluationist account is that we will supervaluate over (bivalent) interpretations that allow for singular, plural and multiple terms. Terms purportedly referring to vague objects will *not* be treated as indeterminate in reference. Instead, they will be treated as semantically determinate multiple terms – for instance, “Kilimanjaro” will be assigned the singletons $\{K_1\}$, $\{K_2\}$, $\{K_3\}$,... under every admissible interpretation. The rest of the supervaluationist account remains unchanged: when we encounter an expression whose meaning is indeterminate (for example, a semantically indeterminate predicate), we use the method of supervaluations to determine the truth value of the sentence in which the expression occurs. As before, '$\Delta p$' will express truth under all interpretations, while '$\nabla p$' will be defined as '$\sim \Delta p \ \& \sim\Delta \sim p$'.

One advantage of the resulting account should be apparent already: if “Kilimanjaro” multiply refers to $K_1$, $K_2$, $K_3$... instead of being indeterminate in reference among them, sentences like “Kilimanjaro has fuzzy boundaries”, “Kilimanjaro has no precise weight” and “Kilimanjaro has no precise number of atoms” will be *determinately* true (i.e. true on every admissible interpretation), on the simple grounds I explained above. No adjustment or suspension of the supervaluationist rule will be needed to do justice to these commonsensical claims. Moreover, no distinction will have to be posited between 'F-ing' and 'being something which F-ies', because – parallel to the truth of “Kilimanjaro has fuzzy boundaries”, “Kilimanjaro has no precise weight” and “Kilimanjaro has no precise number of atoms” – we will also have:

(1') $\exists \underline{x}\ \underline{x} = k \ \& \sim A\underline{x}$

Kilimanjaro is something which has no precise number of atoms

(2') $\exists \underline{x}\ \underline{x} = k \ \& \ F\underline{x}$

Kilimanjaro is something which has fuzzy boundaries

(3') $\exists \underline{x}\ \underline{x} = k\ \&\ {\sim}W\underline{x}$

Kilimanjaro is something which has no precise weight.

One might worry that, if “Kilimanjaro” is not indeterminate in reference, “Sparky is part of Kilimanjaro” will not be indeterminate in truth-value and we will not have what seemed like a natural expression of Kilimanjaro's vagueness, namely:

(4') $\nabla$ P$s\underline{k}$

It is indeterminate whether Sparky is part of Kilimanjaro

But the worry is misguided. First of all, if “Kilimanjaro” multiply refers, we can perfectly well express the idea that Sparky is a borderline part of Kilimanjaro in terms of a simple predicate '...is a borderline part of...' true of two objects if and only if the first is part of some but not all the things in the second:

(4'') B$s\underline{k}$

Sparky is a borderline part of Kilimanjaro

Second, the truth of (4') can also be vindicated. For, even if there is no semantic indeterminacy in “Kilimanjaro”, the indeterminacy of “Sparky is part of Kilimanjaro” can still arise from the interaction of the term “Kilimanjaro” with the predicate “...is part of...”. In particular, one may plausibly suppose that – while we made up our minds as to when the predicate “...is part of...” should true of Sparky and objects like $K_1$ – we never quite made up our minds as to when exactly it should be true of Sparky and objects like Kilimanjaro: does

Sparky have to be part of *each* of the precise things in Kilimanjaro? Does it have to be part of *most* of the precise things in Kilimanjaro? Or is it enough if it is part of at least *some* of them? Given this semantic indecision concerning the application-conditions of "...is part of..." to multiple terms, "Sparky is part of Kilimanjaro" will be neither true nor false and the truth of (4') will not be jeopardized – in fact, (4') will be true whenever (4") is, which is exactly what one would expect.[26]

A second important advantage of treating "Kilimanjaro" as a multiple term has to do with the inference from "It is indeterminate whether Sparky is part of Kilimanjaro" to "There is something such that it is indeterminate whether Sparky is part of it". Recall that (given plausible assumptions about parthood and identity) the traditional supervaluationist view was bound to treat that inference as invalid. But, intuitively, the inference *is* valid, and its intuitive validity can be vindicated if "Kilimanjaro" is understood as a multiple term and "There is something" as a multiple quantifier. For then (4') will entail (5') even if it does not entail (5):

(5) $\exists x\ (\nabla\ Psx)$

There is something precise such that it is indeterminate whether Sparky is part of it

(5') $\exists \underline{x}\ (\nabla\ Ps\underline{x})$

There is something such that it is indeterminate whether Sparky is part of it.

The same goes with the seemingly impeccable inference from "It is determinately the case

26 Notice that there is nothing special about "...is part of..." here. Presumably the application-conditions of "...is the weight value of..." and "...is the boundary of..." to multiple-terms are also indeterminate. As a result, whenever "Kilimanjaro does not have a precise weight" and "Kilimanjaro has fuzzy boundaries" come out true, "*n* is the weight value of Kilimanjaro" and "*b* is the boundary of Kilimanjaro" come out indeterminate for any *n* and *b* that correspond (respectively) to the weight value and boundary of some but not all of the things in Kilimanjaro.

that Kilimanjaro is self-identical" to "There is something which is determinately identical to Kilimanjaro": it is prohibited by the traditional supervaluationist view and validated by the present one. More precisely, what we can infer from (6') on the present view is not (7), but (7'):

(6') $\Delta\, \underline{k} = \underline{k}$

It is determinately the case that Kilimanjaro is self-identical

(7) $\exists x\, \Delta\, \underline{k} = x$

There is something precise which is determinately identical to Kilimanjaro

(7') $\exists \underline{x}\, \Delta\, \underline{k} = \underline{x}$

There is something which is determinately identical with Kilimanjaro.

At this point, it might be objected that, while there may be good reasons to treat "Kilimanjaro" as a multiple term, there are equally good reasons to treat it as indeterminate in reference. After all, just as it would seem unprincipled to pick one amongst $K_1$, $K_2$, $K_3$... as the one and only referent of "Kilimanjaro", it would seem equally unprincipled to treat the particular singletons $\{K_1\}$, $\{K_2\}$, $\{K_3\}$... as its semantic values: why not instead pick a slightly more inclusive set of singletons, or a slightly less inclusive one? Isn't it absurd (as Lewis would put it) to think that we decided to apply the term "Kilimanjaro" to a *determinate* multiplicity of precise objects? In response to this worry, one might decide to treat "Kilimanjaro" as indeterminate in reference among several multiplicities. But if "Kilimanjaro" is a semantically indeterminate multiple term, the inferences from (4') to (5') and from (6') to (7') will again be invalidated. On the other hand, if one insists that – contrary to first impressions – there is a determinate multiplicity that “Kilimanjaro” refers to, one runs the risk of undermining the original motivation for treating “Kilimanjaro” as a multiple term: why not make the same move from the start, and claim that one amongst $K_1$, $K_2$, $K_3$... is the unique referent of “Kilimanjaro”?

As far as I can see, there are two possible ways of responding to this objection. According to the first response, it is far more absurd to insist that *nothing* has Sparky as a borderline part while conceding that Kilimanjaro does (or that *nothing* is determinately identical to Kilimanjaro while conceding that Kilimanjaro is) than to accept a view on which “Kilimanjaro” refers to a determinate multiplicity of objects. The inferences from “It is indeterminate whether Sparky is part of Kilimanjaro” to “There is something such that it is indeterminate whether Sparky is part of it” and from “It is determinately the case that Kilimanjaro is self-identical” to “There is something which is determinately identical to Kilimanjaro” *are* good inferences and this should be enough to convince us that “Kilimanjaro” is *not* semantically indeterminate. What's more, denying that “Kilimanjaro” is semantically indeterminate does *not* undermine our initial motivation for treating it as a multiple term. For our original motivation for treating “Kilimanjaro” as a multiple term had nothing to do with the idea that it would be “unprincipled” or “absurd” to pick one amongst $K_1$, $K_2$, $K_3$... as its unique referent. Our original motivation had to do with the need to reconcile two prima facie conflicting, but equally compelling intuitions: that Kilimanjaro is nothing over and above $K_1$, $K_2$, $K_3$... and that $K_1$, $K_2$, $K_3$... are precise, whereas Kilimanjaro is not. The present view does a pretty good job at achieving the reconciliation – if the price to be paid for this is the revision of our prior views about the amount of semantic indeterminacy there is in natural language, it might well be a price worth paying.

The second response is more conciliatory. It might be conceded that treating "Kilimanjaro" as a multiple term (i.e. as a term referring to a particular multiplicity of objects) involves a certain amount of *idealization*, or deliberate simplification of things. A less idealized account would treat "Kilimanjaro" as a multiple term of second degree (i.e. as a term multiply referring to several multiplicities of objects), assign it several sets of singletons as semantic values and modify the rest of the semantics accordingly. An even less idealized account would treat "Kilimanjaro" as a multiple term of third degree (i.e. as a term multiply referring to several multiplicities of multiplicities of objects), assign it several sets of sets of singletons as semantic values and modify the rest of the semantics accordingly. And so on and so forth. The idea is that, even if we cannot attain a fully realistic and unidealized semantics for "Kilimanjaro", we can indefinitely *approximate* it: we cannot make "Kilimanjaro" vague through and through, but we can make it vague to whatever degree we want, and we can do this while preserving the truth of claims like "Kilimanjaro has fuzzy boundaries" and the validity of the inferences  from "It is indeterminate whether Sparky is part of Kilimanjaro" to "There is something such that it is indeterminate whether Sparky is part of it" and from "It is determinately the case that Kilimanjaro is self-identical" to "There is something which is determinately identical to Kilimanjaro".

I have explained how supervaluationists who accept the existence of multiple reference can account for the fact that things that are true of Kilimanjaro can fail to be true of $K_1$, $K_2$, $K_3$... (and viceversa). I also have shown how they can vindicate certain intuitively valid inferences that the traditional supervaluationist view is bound to treat as invalid. Let me conclude my discussion by considering again the identity statement:

(8') $\underline{k} = k_1$

Kilimanjaro is identical to $K_1$

Recall that, if “Kilimanjaro” is indeterminate in reference among $K_1$, $K_2$, $K_3$..., all we can say about “Kilimanjaro is identical to $K_1$” is that it is neither true nor false – a result that does not sit very comfortably with the intuition that Kilimanjaro is vague and $K_1$ is not. What if “Kilimanjaro” multiply refers to $K_1$, $K_2$, $K_3$...? On the semantics I outlined above, the semantic values of singular, multiple and plural terms and variables are sets of elements of D. Thus, two terms or variables will co-refer if some set(s) is (or are) the semantic value(s) of both:

For any singular, plural or multiple variables or terms $\tau$ and $\sigma$, $V(\tau = \sigma) = 1$ iff $V(\tau)$ is or are the same as $V(\sigma)$

If this is correct, what we should say about (8) is that it is flat-out false: Kilimanjaro is distinct from $K_1$, and determinately so. Since the same applies to each and every precise object in Kilimanjaro, we reach what seems to me to be a surprisingly natural conclusion: Kilimanjaro is nothing 'over and above' $K_1$, $K_2$, $K_3$... without being numerically identical to any of them, just as the Beatles are nothing 'over and above' their four members without being numerically identical to any of Paul, John, George and Ringo.

7. Conclusions

Some philosophers believe that vague objects should be dispensed with. On the face of it, though, ordinary thinking seems to have no issue with them: we ordinarily believe Kilimanjaro to be something with fuzzy boundaries, no precise weight or number of atoms, so, in a sense, we ordinarily believe it to be a vague object (though, perhaps, we wouldn't ordinarily call it so). Treating “Kilimanjaro” as a multiple term allows us to do justice to this aspect of ordinary thinking while maintaining that, in an important sense, Kilimanjaro is nothing over and above $K_1$, $K_2$, $K_3$... This strikes me as an attractive 'third way' between eliminating vague objects altogether and treating them as wholly independent of their precise counterparts.

Of course, one may still worry about the ontological import of the multiple-reference account I have outlined: given their acceptance of claims like “Kilimanjaro is something with fuzzy boundaries” and their outright denial of identity statements like “Kilimanjaro is identical to $K_1$”, isn't there a clear sense in which multiple-reference theorists committed to *more* entities than standard supervaluationists? I cannot hope to address this delicate issue here – what entities a philosophical theory is 'committed to' is a meta-philosophical question whose discussion would take us too far afield. I limit myself to observing that, just as a strong case has been made for the 'ontological innocence' of plural quantification,[27] perhaps an equally strong case can be made for the 'ontological innocence' of multiple quantification. It may well be true, as Quine famously argued, that a theory is committed to all the objects which are quantified over by its first-order variables. The crucial point remains that a theory of multiple quantification commits us to more first-order variables, not to more objects. If this is right, multiple-reference might offer us the best of both worlds – a way of thinking about vague objects in a vagueness-free ontological framework. It might not be the answer to all our questions about vague objects. But it might be the beginning of an answer.

27 See, for instance, Boolos (1984).

*References*

Boolos, G. (1984). “To be is to be the value of a variable (or to be some values of some variables)”. *Journal of Philosophy* 81, 8: 430-439.

DeRosset, L. (forthcoming). “Analyticity and ontology”. *Oxford Studies in Metaphysics*, vol. 9.

Evans, G. (1978). “Can There Be Vague Objects?”. *Analysis* 38: 208.

Fine, K. (1975). “Vagueness, truth and logic”. *Synthese* 54: 235–59.

Hudson, H. (2005). *The Metaphysics of Hyperspace*. Oxford: Oxford University Press.

Keefe, Rosanna. (2000). *Theories of Vagueness*. Cambridge: Cambridge University Press.

Jones, N. K. (2011). “Williams on supervaluationism and logical revisionism”. *The Journal of Philosophy* 108 (11): 633-641.

Landman, F. (1989a). “Groups, I”. *Linguistics and Philosophy* 12 (5): 559 - 605.

Landman, F. (1989b). “Groups, II”. *Linguistics and Philosophy* 12 (6):723 - 744.

Lasersohn, P. (1995). *Plurality, conjunction and events*. Dordrecht: Kluwer.

Lewis, D. (1983). “New Work for a Theory of Universals”. *Australasian Journal of Philosophy* 61, 4: 343-377.

Lewis, D. (1999). “Many, but Almost One” in Lewis, D., *Papers in Metaphysics and Epistemology,* Cambridge: Cambridge University Press (pp.164-82).

Linnebo, Ø. (2014). “Plural Quantification” in Zalta, E. N. (Ed.), *The Stanford Encyclopedia of Philosophy*, URL = <http://plato.stanford.edu/archives/fall2014/entries/plural-quant/>.

McGee, V. and B. McLaughlin. (1994). “Distinctions without a difference”. *The Southern Journal of Philosophy* 33: 203-51.

McGee, V. and B. McLaughlin. (2000). “The lessons of the many”. *Philosophical Topics* 28, 1: 129-151.

Oliver, A. and T. Smiley. (2008). “Is Plural Denotation Collective?”. *Analysis* 68 (1): 22-34.

Oliver, A. and T. Smiley. (2013). *Plural Logic*. Oxford: Oxford University Press.

Parsons, J. (2007). “Theories of location.” *Oxford Studies in Metaphysics* 3: 201–232.

Scha, R. (1981). “Distributive, collective and cumulative quantification” in J. A. G. Groenendijk, T. M. V. Janssen & M. B. J. Stokhof (Eds.), *Formal Methods in the Study of*

*Language*, Mathematical Center, Amsterdam.

Schiffer, S. (2000). “Replies”. *Philosophical Issues* 10: 320 – 43.

Schwarzschild, R. (1996). *Pluralities*. Dordrecht: Kluwer.

Sider, T. (2015). “Nothing over and above”. *Grazer Philosophische Studien* 91: 191-216.

Smith, N. J. J. (2008). *Vagueness and degrees of truth*. Oxford: Oxford University Press.

Thomasson, A.L. (2007). *Ordinary objects*. Oxford: Oxford University Press.

Unger, P. (1980). “The Problem of the Many”. *Midwest Studies in Philosophy* 5: 411-67.

Van Fraassen, B. (1966). “Singular Terms, Truth-Value Gaps, and Free Logic”. *Journal of Philosophy*, 63: 481–495.

Varzi, A. (2001). “Vagueness, logic and ontology”. *The Dialogue. Yearbooks for Philosophical Hermeneutics* 1: 135–154.

Varzi, A. (2007). “Supervaluationism and its logics”. *Mind* 116 (2007), 633-676.

Williams, J. R. G. (2008). “Supervaluationism and logical revisionism”. *The Journal of Philosophy* 105 (4): 192-212.

Williamson, T. (1994). *Vagueness*. London: Routledge.

Williamson, T. (1997). “Imagination, stipulation and vagueness”. *Philosophical Issues* 8, Truth: 215-228.