Space, Pure Intuition, and Laws in the *Metaphysical Foundations*

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I. Introduction

In MAN, Kant's project is to provide the metaphysical basis for a proper science of matter. What he calls a special metaphysics of corporeal nature involves determining the powers, properties, and laws of matter. These include the fundamental forces of attraction and repulsion, along with their laws of diffusion (treated in the Dynamics Chapter), and the three laws of mechanics (treated in the Mechanics Chapter). The special metaphysics of corporeal nature given by MAN crucially involves considerations about mathematics, since any proper science, insofar as it is to be a priori, must admit of mathematical treatment (MAN 4:470-2). In an important methodological remark in the Preface, Kant indicates what resources he will utilize to execute his project:

> But in order to make possible the application of mathematics to the doctrine of body, which only through this can become natural science, principles for the *construction* of the concepts that belong to the possibility of matter in general must first be introduced. Therefore, a complete analysis of the concept of a matter in general will have to be taken as the basis, and this is a task for pure philosophy – which, for this purpose, makes use of no particular experiences, but only that which it finds in the isolated (although intrinsically empirical) concept itself, in relation to the pure intuitions in space and time and in accordance with laws that already essentially attach to the concept of nature in general, and is therefore a genuine *metaphysics of corporeal nature*. (MAN 4:472).[1]

Among other points, Kant indicates here that the pure intuition of space has an important (though not yet specified) role to play in the special metaphysics of corporeal nature. Later in the preface,

[1] There is disagreement about how to construe the role of mathematical considerations in MAN. According to one interpretation—e.g., Washburn (1975), Förster (2000), and Heis (2014)—Kant holds that the concept of matter needs to be mathematically constructed in pure intuition. For a rival interpretation, according to which mathematization has to do instead with showing how concepts of motion and matter (e.g. mass) acquire a mathematical, measurable structure, see Friedman (2013). For helpful discussion of the range of interpretive positons in the secondary literature, and for a novel interpretation of the role of mathematical construction, see McNulty (2014). My discussion has implications for this contentious issue.

Kant again mentions, without clarifying, the importance of the "form and principles of outer intuition" (MAN 4:478) for metaphysics, including special metaphysics.

I am interested in the use Kant makes of the pure intuition of space, and of properties and principles of space and space*s* (i.e. figures, like spheres and lines), in the special metaphysical project of MAN. This is a large topic, so I will focus here on an aspect of it: the role of these things in his treatment of some of the *laws* of matter treated in the Dynamics and Mechanics Chapters. In MAN and other texts, Kant speaks of space as the "ground," "condition," and "basis" of various laws, including the inverse-square and inverse-cube laws of attractive and repulsive force (Prol 4:321; MAN 4:534), and the Third Law of Mechanics (Br 11:247; KrV B293; cf. BDG 2:134). Moreover, in his proofs of all the laws just mentioned, the language of "construction" figures prominently, which suggests that Kant's proofs (somehow) rest on or involve mathematical construction in his technical sense (MAN 4:517-20, 4:546, 4:549; RL 6:232-3). [2]

Such claims give rise to a number of questions. How do properties and principles of space and spaces serve to ground this particular set of laws? Which spatial properties and principles is Kant appealing to? What, if anything, does the spatial grounding of the inverse-square and inverse-cube laws of diffusion (treated in the Dynamics Chapter) have in common with that of the Third

[2] For Kant, mathematical construction of a concept (and mathematics more generally) proceeds by means of the "presentation of the object [corresponding to a concept] in an a priori intuition" (MAN 4:469), where the intuition in question has "universal validity for all possible intuitions that belong under the same concept" (KrV A713/B741). Given this last point, Kant speaks at times of construction (=presentation a priori in intuition) of synthetic a priori propositions (KrV A718/B746), "principles" (MAN 4:471, KrV A24), and laws, like the law for the surface area of a sphere (Prol 4:320-1). Construction proves in the first instance as it were formal truths or formal laws, pertaining to mathematical objects, which as considered in pure mathematics, are not existing entities but rather possible forms for such entities (KrV A719/B748, B147, A223/B271; MAN 4:467n; KU 5:366n)—mathematical construction shows us what is formally possible (e.g. what shapes could be shapes of objects of experience [KrV A220-224/B268-B271]) and what properties are necessary consequences of what shapes and properties; but we cannot construct the following related things: existence; reality (KrV A715/B743); the matter of appearance (KrV A720/B748, KrV A723-4/B751-2); and forces (MAN 4:524-4; KrV A222-3/B269-70). Geometry supplies paradigm examples of mathematical construction. But construction also occurs in other mathematical disciplines, such as phoronomy (MAN 4:486), which combines pure spatial intuition (construction of figures) with considerations of time and motion. For helpful discussion of Kant's understanding of construction (and of interpretive debates regarding it, which I will not try to settle here), see Shabel (2006). If what I argue in this paper is correct, then Kant thinks that it is possible to mathematically construct not just formal, broadly geometric laws but also the mathematical content of some *real* laws.

Law (treated in the Mechanics Chapter)? What role—if any—does mathematical construction play in Kant's proofs of these laws? Finally, how if at all, are Kant's grounding claims consistent with his other commitments—for example, how are they consistent with his notorious denial in *Prolegomena* §38 that there are any laws that "lie in space" (Prol 4:321)?

I attempt to answers these question, and I do so by interpreting MAN's account of these matters in light of related but sometimes overlooked discussions in other Critical and pre-Critical texts. On my interpretation, Kant takes properties and principles (or laws) of space and spaces to be the grounds of the laws of diffusion and the Third Law insofar as they serve to explain the mathematical character of those laws. (Grounding is thus here an explanatory notion.) In the case of both types of laws (of diffusion, on the one hand, and of the Third Law, on the other), Kant's explanation has two stages. The first stage involves the mathematical construction of a spatial (or spatio-temporal) model that is designed to exhibit certain formal, broadly geometrical laws. In the first stage of explanation, those laws are then used to explain why, given various factors, the corresponding "real" laws of both types have the mathematical content that they do. One of the given factors is that the phenomena being modelled behave according to the model. In the second stage of explanation, Kant uses global properties of space, such as what I call below, its *efficient and equitable* character, to explain why the phenomena behave in accordance with the relevant model. I show how this account can be squared with, among other things, Kant's denial that matter and force can be mathematically constructed as well as the claim in the *Prolegomena* referred to above. My interpretation runs counter to interpretations that deny or downplay the role of mathematical construction after the Phoronomy Chapter.[3] More generally, it serves as a corrective to

[3] E.g. Friedman (2013; 32, 567-8). Plaass (1965; 112) actually denies at one point that there is *any* mathematical cognition in the *Metaphysical Foundations*. Pollok (2001, 17-18) who critiques Plaass's claim, rightly acknowledges a role (albeit limited) for mathematical construction in the Dynamics and Mechanics chapters. Brittan (1986; 64) initially grants a a role for construction in the Phoronomy and Dynamics chapters but not the Mechanics, though his analysis ultimately seems to undercut any sort of construction or mathematization of forces in the Dynamics (e.g. 90).

interpretations that emphasize the dependence of Kantian laws of nature on the categories and other *non-spatial* conditions of experience while leaving unclear just what role properties and principles of our spatial form of intuition play.[4] As we will see, appreciating that role is important for, among other things, properly understanding the differences between Kant's position and Leibniz's.

In §II, I consider the role that properties and principles of space play in grounding the laws of diffusion, as well as what he calls the "universal law of dynamics," all of which are treated in the Dynamics Chapter. In §III, I respond to some objections to my interpretation. In §IV, I explore the role of these things in grounding the Third Law of Mechanics. In §V, I explain how my interpretation fits with the *Prolegomena* passage. In §VI, I conclude with some thoughts on the differences between Kant and Leibniz's positions.

II. The Laws of Diffusion and the Universal Law of Dynamics

1. Preliminaries

I take to be fairly well-recognized and non-controversial some of the ways in which Kant deploys properties and principles of space and spaces in his account of matter in MAN.[5] There is, for example, his appeal to the infinite divisibility of space—something provable through geometric construction—in his proof of the infinite divisibility of matter (MAN 4:503); his appeal to the infinitude of space and to the fact that it alone cannot limit matter and its forces in the balancing argument (MAN 4:508ff); and his appeal to the properties of a line in his explanation of why there

[4] I have in mind, inter alia, the accounts of Friedman (2013); Stang (2016); and Watkins (2019, 39-40; 145; 236-7).
[5] See, e.g., Guyer (2006; 161-2) who also cites these sorts of examples.

can be at most two fundamental forces (MAN 4:498-9). These, then, are some of the accepted uses to which Kant puts the "form and principles of outer intuition" (MAN 4:478) in MAN.[6]

That Kant also makes use of properties and principles of space and spaces in his account of laws is indicated, inter alia, by the following two parallel texts. The first occurs in the General Note to the System of Principles in the B-edition of the first *Critique*: "For [space] already contains in itself a priori formal outer relations as conditions of the possibility of the real (in effect and countereffect, thus in community)" (KrV B293; cf. MSI 2:414). This remark occurs in a context in which Kant is contrasting his own view, according to which space is metaphysically *prior* to matter (and material forces and material interactions) with that of Leibniz, whom Kant takes to maintain the contrary. The parenthetical remark contains an allusion to the Third Law (the law of the equality of action and reaction), a "real" (as opposed to merely formal, geometrical law) that Kant claims has its basis in "formal" outer relations, that is, features and principles of space as the form of outer intuition.[7]

The second passage occurs in the General Remark at the end of the Dynamics Chapter when he says that "space is required for all forces of matter, and since it also contains the conditions of the laws of diffusion of these forces, it is necessarily presupposed prior to all matter" (MAN 4:534). By the "laws of diffusion," Kant means the inverse-square and inverse-cube laws of, respectively, attractive and repulsive force, which he had discussed earlier in the Dynamics Chapter. In language parallel to that used above, he claims that these laws depends on "conditions" [*Bedingungen*] contained in space. This suggests that he thinks that the grounding or conditioning relationship works in similar ways for both types of laws. To see what this grounding relationship is, and to see what similarities there are in the spatial grounding of both types of laws, we need to turn

[6] If McLear (2018)'s recent reconstruction of Kant's "affection" argument for the claim that motion is a fundamental determination of matter is correct, then this is yet another place where Kant appeals to a priori properties of space, in particular, its essentially relational character.
[7] Friedman (2013; 23) also reads the passage as alluding to the Third Law.

to the details of Kant's account of each type of law. The first sub-section below is devoted to the laws of diffusion of attractive and repulsive force; the second sub-section is devoted to the "universal law of dynamics," of which the inverse-square and inverse-cube laws are both instances.

2. The Inverse-Square and Inverse-Cube Laws

I will concentrate on Kant's treatment of the laws of diffusion in MAN but it should be noted that it is anticipated by discussions in earlier texts. In the 1756 *Physical Monadology* (1:484-5) Kant offers a geometric derivation of these laws that resembles in a number of respects his later treatment in MAN.[8] Just a few years prior to the publication of MAN, Kant discusses the inverse-square law in *Prolegomena* §38:

> If from there we go still further, namely to the fundamental doctrines of physical astronomy, there appears a physical law of reciprocal attraction, extending to all material nature, the rule of which is that these attractions decrease inversely with the square of the distance from each point of attraction, exactly as the spherical surfaces into which this force spreads itself increase, something that seems to reside as necessary in the nature of the things themselves and which therefore is customarily presented as cognizable a priori. As simple as are the sources of this law—in that they rest merely on the relation of spherical surfaces with different radii…. (Prol 4:321)[9]

What's striking here is the talk of the inverse-square law "resting on" geometric relations and geometric laws, along with apparent description of the law as knowable a priori. Considerations like these, as well as the explicit grounding and conditioning language[10] that Kant uses in MAN and elsewhere when describing the role that space plays in accounting for real laws support the idea that Kant is making (some sort of) grounding claim about the laws of diffusion.

To determine the nature and content of this groundling claim, we need to consider the details of the derivation of the inverse-cube and inverse-cube laws in the *Metaphysical Foundations*. In

[8] The *Physical Monadology*'s derivation of the inverse-square law appears to be inspired by one which can be found in Keill (1745; 5-7).
[9] Friedman (1992; Chapter 4) offers a reading of *Prolegomena* §38 according to which Kant is, appearances to the contrary, not endorsing a geometric derivation of the inverse-square law. I do not think Friedman's reading is tenable. See Messina (2018).
[10] This is not to say that Kant always uses 'ground' and its cognates interchangeably with 'condition' and its cognates. But I believe this is true in this instance.

Note 1 to Proposition 8 of the Dynamics Chapter, Kant says that, now that he has shown matter to have attractive and repulsive forces, it should be possible to derive from these forces "the possibility of a space filled to a determinate degree" (MAN 4:517). But to complete this task, and in turn to construct the "dynamic concept of matter, as that of the movable filling its space (to a determinate degree)" it is necessary to determine the ratios of attractive and repulsive force. That is to say, it requires deriving the laws of diffusion of these forces. Kant describes this derivation as a "mathematical task, which no longer belongs to metaphysics" (MAN 4:517). Since mathematics is a matter of construction, Kant is saying that deriving the laws of diffusion requires mathematical construction.[11] In Remark 1, which is divided into four points, Kant offers "an attempt at such a perhaps possible construction." In Remark 2, Kant mentions and responds to some difficulties for his construction. (I return to these difficulties in §3.1 below.)

In Remark 1, I take Kant be showing how it is possible to construct a geometric model of the diffusion of the action of attractive and repulsion forces, a model that also works for the illumination of light (which itself admits of an inverse-square law). Kant's constructed geometric models are designed to exhibit formal, geometric laws from which we can then derive the mathematical character of the corresponding laws of diffusion. The general idea is as follows: if we represent—that is, model—the action/effect (*Wirkung*) of an attractive force in such a way that it diffuses uniformly from a central point across concentric spherical surfaces so that the total quantum of action/effect is always the same[12] (with the intensity diminishing uniformly at each given distance), then the intensity of the action of the force at a point on a given spherical surface will be

[11] Friedman (2013; 28n42, 221-234) acknowledges that it is natural to read Kant as deriving the laws of diffusion from geometry and thereby constructing the concept of matter (or a partial concept of matter) in pure intuition. but he concludes this reading is untenable and offers an alternative account of what is going on. I respond to some of Friedman's objections to the natural reading below.

[12] As Warren (2017; 183) points out, the total quantum of action/effect is to be conceived "as the product of the intensive magnitude of the effects and the extensive magnitude of the space" (here the size of the two dimensional spherical surface. See Warren (2017; 175) for further details about what is being diffused and for the non-temporal nature of the diffusion.

inversely proportional to the square of the radius. As the areas of the concentric spherical surfaces grow in proportion to the square of the distance from the radius (this being a geometric law of the surface area of spheres), the intensity of the action of attractive force will diminish accordingly. So we get an inverse-square law of diffusion for attractive force. If we model the action/effect (*Wirkung*) of a repulsive force in such a way that it diffuses uniformly from a central point across an (infinitesimally small) spherical volume, then the intensity of the action/effect will be inversely proportional to the cube of the (infinitely small) distance. As the volume grows in proportion to the cube of the radius (this being a geometric law of the volume of spheres), the intensity of the repulsive force will diminish accordingly. So we get an inverse-cube law of diffusion for repulsive force.

In what sense do properties and laws of space and spaces ground the inverse-square and inverse-cube laws? And what is the role of construction in the derivations? As I understand it, grounding for Kant is, in this instance, a matter of explanation; this is in line with his typical usage of 'ground' and its cognates elsewhere.[13] In this particular case, Kant takes the geometric laws that are exhibited along with his models to explain why—given that matter exists and has attractive and repulsive forces, and given that these forces follow the pattern expressed in the models—these forces of matter must diffuse according to inverse-square and inverse-cube laws. That's because the necessary, formal mathematical relationships—the laws pertaining to spheres—exhibited along with the model constrain the mathematical content of the dynamical (real) laws in such a way that we can explain why they must have the mathematical character thy do. (Again, this stage of explanation takes for granted that matter has attractive and repulsive forces and that they radiate uniformly

[13] For helpful accounts of Kant's notion of a ground and of the grounding relation, see Hogan (2009), Smit (2009), Stratman (2018), Stang (2016 and 2019). As Stang shows in detail, Kant's account is part of a larger German rationalist tradition, exemplified by Wolff's definition of a ground as "that through which one can understand why something is" (Wolff (1719, §29) quoted in Stang (2019; 77).

outward according to the spatial pattern depicted in the model. We will return below to the question of whether the "fit" of forces with the model is a purely contingent matter or itself admits of an explanation.)

Explanatory grounds come in various forms for Kant. For example, grounds can be logical (where the ground and consequent are mere concepts and the grounding occurs by means of the law of identity, as in an analytic proposition) or non-logical/real; grounds can be rationes essendi (grounds of being) or rationes fiendi (grounds of becoming/actuality); and they can be causal or non-causal[14] I take it that, as explanatory grounds of the laws of diffusion, the properties and laws of space and spaces (in this case, spheres) are non-logical/real, non-causal, rationes essendi.[15] They are *non-logical* grounds because the sort of grounding at issue is not a matter of a relation between mere concepts as in an analytic truth—the connection is synthetic. They are non-causal, rationes essendi of the laws of diffusion because space, spatial figures, and their properties and laws, as object of pure mathematics, are merely formal entities. As such they lack causal efficacy and existence—as Kant says, figures have an essence but not a nature as existing things do (MAN 4:467n).[16] But as forms they, along with their properties and geometric laws, are conditions of the possibility of the interaction of material substances, as well as constraints on the kinds of "real"[17] relations and real laws material substances obey, including their mathematical content. (Recall KrV B293 and MAN 4:534.)

The interpretation I am offering of the spatial grounding of the laws of diffusion allows us to make sense of the fact that, at least at times, Kant appears to regard the laws of diffusion

[14] See Stang (2019)'s excellent discussion for these and other important distinctions, along with numerous references.
[15] In the *Metaphysics Mrongovius* Kant describes geometric objects and their properties using the language of "ratio essendi," which he there defines as "the ground of that which belongs to a thing according to its possibility." Kant gives the following example: "the three sides in the triangle are the ground of the three corners" (MM 29:809). For discussion of these passages, as well as some ways in which space serves as ratio essendi in Kant's philosophy, see Stang (2019).
[16] See the references in n.2
[17] The sense in which space, spaces, spatial properties, and spatial laws are *real* grounds differs from the sense of *realness* (linked to causal efficacy and to empirical content) of the real relations that they ground. See n.2

(especially the inverse-square law) as a priori (Prol 4:320-1; OP 21:310–1).[18] Consider the definition he Kant gives of a priori cognition in the preface: "Now to cognize something a priori means to cognize it from its mere possibility" (MAN 4:470). As rationes essendi, the formal properties and formal, geometric laws exhibited along with the constructed spatial models for the diffusion of attractive and repulsive force are grounds of the possibility of the mathematical character of the laws of diffusion. We see how the given forces could have the mathematical character they do by constructing the spherical model(s), and we also see why, assuming the forces operate in accordance with the respective spherical models and their respective geometric laws (of the surface area and volume of a sphere, respectively) they must have the character they do. Because any forces that diffuse according to the model must obey the formal laws (which themselves are constructed a priori), a priori knowledge of the formal, geometric laws yields a priori knowledge in the preface's technical sense of the mathematical character of the dynamic (real) laws. For this reason, cognition of the inverse-square and inverse-cube laws based on mathematical construction is also a priori in the sense of involving cognition of necessity (see, e.g., Prol 4:320; KrV B4)—the construction enables us to see why, given the various assumptions mentioned, their laws must be this way. Finally, it is a priori in the traditional rationalist sense of a priori knowledge as knowledge from (explanatory) grounds.[19]

My interpretation of the role of mathematical construction in Kant's derivation of the laws of diffusion is further supported by other significant passages in which Kant brings together the

[18] Some commentators have been reluctant to concede the a priori character of these laws, such as Brittan (1978, 141); Friedman (1992 and 2013); Laywine (2003, 451ff); and Stang (2016; 246n48). I think, however, that this leads to an implausible reading of both *Prolegomena* §38 (see Messina [2018]) and the notes and remarks to Proposition 8 of MAN. Warren (2017) also accepts and tries to account for the a priori character of these laws.

[19] For some important (and importantly different) takes on this tradition and Kant's appropriation of it see Smit (2009); Hogan (2009); Hebbler (2015); and Stang (2019). Hebbler's account is particularly germane, since he is concerned in part to show how Kant's views on laws, geometry, explanation, necessity, and a priori knowledge fit with this tradition. I take what I say here to largely complement his interpretation (which is largely silent on MAN).

themes of construction and laws. Consider Kant's explanation in the preface of why chemistry is not yet a science:

> So long, therefore, as there is still for chemical actions of matters on one another no concept to be discovered that can be constructed, that is, no law of the approach of withdrawal of the parts of matter can be specified according to which, perhaps in proportion to their density or the like, their motions and all their consequences thereof can be made intuitive and presented a priori in space (a demand that will only with great difficulty be fulfilled), then chemistry can be nothing more than a systematic art of experimental doctrine, but never a proper science, because its principles are merely empirical, and allow of no a priori presentation in intuition. (MAN 4:470-1)

As I understand this, chemistry does not meet the conditions for a natural science because it has been unable to mathematically construct[20]--that is present in pure intuition—a spatial model of the chemical actions of matter and thus it has been unable to exhibit the spatial principles or laws that make possible and explain the mathematical character of the chemical "laws of approach and withdrawal." Thus, what is missing from chemistry is precisely what can be achieved in the case of the laws of diffusion of attractive and repulsive force.

Another thing to note about the chemistry passage is how closely Kant links all of the following together: (1) construction of formal geometrical concepts and formal principles/laws; (2) construction of principles/laws of chemical action; and (3) construction of a corresponding concept of (a type of) matter. In fact, we have also seen Kant link together (1); (2') construction of (the mathematical content of) the laws of diffusion; and (3') construction of "the dynamical concept of matter, as that of the movable filling its space to a determinate degree." This occurred in Note 1 of Proposition 8—immediately before his geometric derivation of the inverse-square and inverse-cube laws.

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[20] That mathematical rather than metaphysical construction is at issue in this passage is emphasized by Pollok (2001, 88–89n139) and McNulty (2014). For a gloss on how mathematical construction works for Kant, see n. 2.

Especially interesting for our purposes—which include seeking connections between Kant's account of the laws of diffusion and the Third Law of Mechanics—is that in an intriguing passage in, of all places, the Doctrine of Right in the *Metaphysics of Morals*, Kant links together (1), (2") construction of the Third Law, and (3") construction of an unspecified "dynamical concept":

> The law of a reciprocal coercion necessarily in accord with the freedom of everyone under the principle of universal freedom is, as it were, the *construction* of that concept, that is, the presentation of it in pure intuition a priori, by analogy with presenting the possibility of bodies moving freely under the *law of the equality of action and reaction*. In pure mathematics we cannot derive the properties of its objects immediately from concepts but we can discover them only by constructing concepts. Similarly, it is not so much the *concept* of right as rather a fully reciprocal and equal coercion brought under a universal law and consistent with it, that makes the presentation of that concept possible. Moreover, just as a purely formal concept of pure mathematics (e.g. of geometry) underlies this dynamical concept, reason has taken care to furnish the understanding as far as possible with a priori intuitions for constructing the concept of right. (RL 6:232-3)

Kant says here that the concept of right can be constructed by constructing the law of reciprocal coercion, just as an (unspecified) dynamic concept of matter can be constructed by presenting in pure intuition (that is, constructing) the law of the equality of action and reaction (that is, the Third Law). He suggests that we construct the latter law, thereby exhibiting its possibility, by constructing an underlying "purely formal concept" of geometry—along with, I take it a, formal principle/law in virtue of which the mathematical content of the Third Law (its equal and opposite character) is both possible and necessary. Kant doesn't fill in the details here; will have to look to Kant's various formulations of the proof of the Third Law to determine, e.g., what formal concept and formal, broadly geometric law is being constructed. Given, however, that the Mechanics Chapter is devoted to the concept of matter as the "movable, insofar as it, as such a thing, has moving force" (MAN 4:526), it is plausible that this is the dynamical concept he is referring to, the one whose construction is enabled by the construction of the Third Law.[21] The upshot here is that in all three of the passages

[21] Note that Kant distinguishes between a mechanical and a dynamical law of the equality of action and reaction (MAN 4:548). See n. 46 below.

discussed Kant indicates that by mathematically constructing a formal concept of space and formal spatial/principles we are able to construct in a fashion a *real* law (or at least its mathematical content) and exhibit its real possibility. This, in turn, enables a sort of construction of a concept of matter. (I return to this point in §3.1.)

C. Universal Law of Dynamics

In our treatment of Kant's geometric construction cum derivation of the laws of diffusion and repulsion, we saw that in each case Kant constructs a model—a figure—whose formal properties and laws are used to demonstrate the possibility of the mathematical character of the corresponding law of diffusion and moreover to explain why they are as they are—given a number of things, including that there are forces, and that the forces follow the pattern of the model. Now consider the following question: does the fact that the forces obey the pattern (as opposed to, say, behaving like shrapnel from an exploding grenade, to use the example in Laywine 2003; 452) admit of even a partial explanation, from Kant's standpoint, or is it a purely contingent fact? I will approach this question by considering Kant's account of the grounding of what he calls the "universal law of dynamics"[22]

> The action of the moving force, exerted by a point on every other point external to it, stands in inverse ratio to the space in which the same quantum of moving force would need to have diffused, in order to act immediately on this point at the determinate distance. (MAN 4:522)

According to this more general law, every action of a fundamental force that acts immediately on others[23] is inversely proportional to the distance from the center of force ($F \approx 1/r^x$). What, if anything, is the ground of *this* more general dynamical law, of which the inverse-cube and inverse-square laws are species?

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[22] For other treatments of this law see Pollok (2001; 334) and Warren (2017)

[23] Warren (2017; 184-185) emphasizes the restriction of the law to fundamental, immediate forces; on Warren's view, which I find plausible, the immediacy conditions is what is important for ruling out additional causal factors that would, as it were, disturb the diffusion of a force.

Warren (2017) says that Kant doesn't "give any sense here of what kind of grounds might be put forward to support it" (174) though he then goes on to argue that it is grounded in Kant's broader account of intensive magnitudes. I think, though, that Kant takes it to be grounded in features of space (indeed, Warren himself also ends up appealing to features of space, including its inertness, in his account of the law (182)).[24] As I read Kant, he holds that global features of space, features that attach to the whole of space (as opposed to features of particular constructed spaces/figures) ground the fact that, given that there are fundamental forces that act immediately at various distances, so that their action is limited only by space, these forces must obey the universal law of dynamics. One relevant feature of space is its three-dimensionality. If space were two- or four-dimensional, this law would presumably not hold. Another relevant feature is what I will call the "efficient and equitable"' character of space. While space, as Kant conceives it, is causally inert, it nevertheless enables and favors certain kinds of efficient and equitable distributions of force and motion. In the case of the fundamental forces of action and repulsion, which, as fundamental, immediately acting forces, are limited in their action only by space, the three-dimensional, efficient and equitable character of space grounds their behaving in accordance with the spherical models. For, in these models, force is distributed equally and efficiently in all directions in a shape that maximizes area (as opposed to, say, behaving according to the grenade model). In this way, the efficient and equitable character of space, together with its inert and 3-D character, explain why, given that matter possesses essentially fundamental, immediately acting forces of attraction and repulsion, these forces operate in accordance with the universal law of dynamics (that is, why $F \approx 1/r^x$).

[24] One key difference between Warren's treatment of these issues (from which I have learned a great deal) and my own is that he does not appeal to anything like what I am calling the efficient and equitable character of space. As a result, it remains unclear (at least to me) whether and how his Kant could explain why fundamental, immediate forces must diffuse evenly from a central point. Moreover, it is only by incorporating this element that we can fully appreciate the connection (which Warren doesn't explore) between Kant's treatment of the laws of diffusion and the Third Law.

Admittedly, there is little direct textual evidence in MAN that Kant thinks that the universal law of dynamics—and, in turn, the fact that his fundamental forces operate in accordance with the spherical models—is to be explained by citing these features of space (though perhaps MAN 4:519 and 4:534 come closest).

That said, I think it is supported by other texts. One of these is a 1791 letter that we will consider in conjunction with the Third Law of Mechanics. Another is an intriguing passage in the 1763 *Only Possible Argument*, which concludes with the statement

> the variety of spatial relations is so infinite and yet it yields a cognition which is so certain and an intuition which is so clear that…these same spatial relations can also enable us to recognize, from the simplest and most universal principles, the rules of perfection present in naturally necessary causal laws, *in so far as they depend upon relations*. (BDG 2:134; my emphasis)

Kant's idea that laws of nature depend on spatial relations is one that carries over into the Critical period. Consider again B293 in this regard. In fact, already in the *Only Possible Argument* we find Kant giving the Third Law as an example of a "naturally necessary causal law" that depends on spatial relations (BDG 2:134).[25] To show how laws of nature like this one rest on spatial relations, Kant turns to geometry (the science of space and spatial relations), in particular to a discussion of efficient shapes. Kant notes that circles encompass more space relative to their perimeter than other figures because of the equal distance of the points to the center, while the polygons that have the largest areas relative to their perimeters are ones that are more equal (e.g. regular polygons). Kant concludes from this that the way to achieve the most bang for one's buck (when it comes to the ratio of area to perimeter) is to maximize "equality":

> If one were to ask about the use which could be made of the great unity which prevails among the many different relations of space and which are investigated by geometry, I suspect that the universal concepts of the unity of mathematical objects might also reveal the grounds of the unity and perfection of nature. For example, of all figures, the circle is the one in which the circumference encloses the greatest possible area which can be enclosed by a line of that length. The reason, namely, is that the distance between the center and the

[25] Note that this doesn't preclude the pre-Critical Kant from also holding that this law is essential to matter. I think the same is true for the Critical Kant.

> circumference is strictly constant throughout the figure. If a figure is to be bounded by straight lines, then the greatest possible equality in respect of the distance between the sides and the center of the figure can only occur if the following conditions are satisfied: not only must the distances between the angles and the center of the figure be exactly equal to each other, but the perpendicular lines extended from the center to the sides must also be exactly equal to each other. If these conditions are satisfied, a regular polygon is the product…. The following observation immediately suggests itself: the reciprocal relationship between the greatest and smallest depends upon equality [*Gleichheit*]. And since nature offers many others cases of such a necessary equality, it follows that the rules derived from the aforementioned geometric cases relating to the universal grounds of such a reciprocal relation between the greatest and smallest in space, may be applied to the necessary observance of the law of parsimony in nature. In the laws of impact, a certain equality is always necessary…. (BDG 2:134)

Kant's view as expressed in this passage can be fruitfully compared with that of Leibniz, who similarly finds in various geometric and physical laws (such as those of optics) confirmation "of a principle of determination in nature which must be sought by maxima and minima; namely, that a maximum effect should be achieved with a minimum outlay, so to speak" (L 487/AG 150).[26] [27] In addition to the optical laws of reflection and refraction, Leibniz cites, inter alia, the fact that "liquids placed in a different medium compose themselves in the most spacious figure, a sphere" (L 488/AG 151). The point I wish to emphasize for the moment is that it is not farfetched to ascribe to both the pre-Critical and Critical Kant the view that space has an efficient and equitable character that grounds the diffusion of fundamental force in spherical patterns and thus the universal law of dynamics. Due to the efficient and equitable character of space, forces and motions exhibit maximally efficient and equitable distributions, or at least they do so when space is the only relevant factor for deciding their distribution. In §4, we will this same feature of space playing a key role in Kant's Critical account of the Third Law.[28]

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[26] For discussion of Leibniz's commitment to the "Most Determined Path Principle" (especially in the context of his optics) and its connection to his views on God, see McDonough (2009).

[27] I cite works of Leibniz as follows:
AG=*Philosophical Essays*. L=*Philosophical Papers and Letters*. T=*Theodicy*, cited by section number (e.g. §3).

[28] While the pre-Critical Kant and Leibniz appeal to a broadly teleological principle of efficiency or economy in their explanations of some laws, I think that the pre-Critical Kant differs from Leibniz in taking the principle to be an essential property of space itself (built into it as it were) and to possess thereby a strong form of (geometric) necessity, a

I will synthesize the points I have made so far. I have been trying to show that and how, for Kant, spatial properties and geometric laws ground the laws of diffusion, as well as the universal law of dynamics. In the case of the universal law of dynamics, he appeals to global features of space, including its efficient and equitable character, to explain why these forces diffuse in an even, spherical pattern, rather than in a scattershot sort of way.[29] In light of this analysis, we can discern two distinct stages in Kant's explanation of the laws of diffusion (though admittedly Kant himself does not sharply separate them). In the initial stage, we construct spatial, geometric models (which exhibit formal, geometric laws) for the diffusion of attractive force and repulsive force. At this stage of explanation, we take for granted not only that there are attractive and repulsive forces that are fundamental and essential to matter and act immediately,[30] but we also take as given that they behave in accordance with the model. Given these things, the formal laws constructed along with the models serve to explain why the laws of diffusion are inverse-square and inverse-cube laws. At the second stage, we appeal to global properties of space (in particular, its 3-Dimensionality and its efficient and equitable character) to explain why forces that are fundamental and act immediately in

necessity that laws of nature that depend upon space and its laws thereby inherit. For the pre-Critical Kant this aspect of space has its ultimate ground in the divine essence (not in the divine will). This distinguishes his position from that of Leibniz who links the principle to God's will, and who relatedly, sees laws of nature as contingent in a way that geometric laws are not. I think the comparison with Leibniz helps to at least partly defuse the worry that Kant's appeal to the efficient and equitable character of space is a virtus dormitiva. Both Kant and Leibniz see a need to posit a general principle to account for some similarities between various laws. While Kant differs from Leibniz in linking the principle to the essence of space (rather than to God's preference for the best), he might well have thought that if Leibniz is not invoking a virtus dormitiva than neither is he, and moreover that geometric and metaphysical considerations favored his explanation over Leibniz's.

[29] That said, it is not clear that spatial considerations can explain why attractive force must be modelled in terms of spherical *surfaces*. Warren (2017; 190-1) suggests that experience plays a key role in showing that attractive force obeys that particular pattern; by contrast, the need for repulsive force to be modelled in terms of a volume would follow simply from its character as a space filling force.

[30] I take it that Kant thinks that these features of the forces rule out their being influenced by an outside causal factor which would leas them to depart from, as it were, the default pattern that space privileges (see n. 23). This leaves room for the fact that others kinds of forces can be anisotropic. Thanks to Bennett McNulty for pushing me on this.

all directions conform to the spherical models. In this way, we see then that their conformity with the spherical models is not contingent.[31]

III. Some objections

There are some pressing objections to my interpretation that it will be useful to consider before turning to the Third Law (and the ways in which Kant's account of it resembles that of the laws of diffusion).[32]

1. First Objection: Kant's Ambivalence

One concern has to do with the apparent "hesitation" and "ambivalence" (to use Friedman's terms (2013; 222-3) with which Kant offers these derivations. Consider that Kant offers the derivations as something of an aside, describing them as a:

> purely mathematical task, which no longer belongs to metaphysics – nor is metaphysics responsible if the attempt to construct the concept of matter in this way should perhaps not succeed. For it is responsible only for the correctness of the elements of the construction granted to our rational cognition, not for the insufficiency and limits of our reason in carrying it out. (MAN 4:517-8)

Consider, too, Kant's modest talk of a "a *perhaps possible construction*" (MAN 4:518) in Remark 1, and the way he appears to distance himself especially from the derivation of the inverse-cube law of repulsion in Remark 2: "I declare, furthermore, that I do not want the present exposition of the law

[31] Notice that the constructed models/laws (and appeal to spatial features) only serve to explain and derive the laws when they are, as it were, discursively thought in conjunction with the various assumptions at the different stages. So while Kant can be said to prove (and explain) these real laws by means of construction, and while he speaks of constructing the (mathematical content of) such laws, one also needs to discursively reason through the results of the construction together with assumptions and concepts that cannot themselves be constructed (or given in pure intuition), in order to grasp the proof/explanation of the mathematical content of the real law, and to see how it is in effect constructed when we construct the formal law corresponding to it. Thus, Kant's proofs can be said to combine construction with discursive reasoning in a distinctive way. Thanks to Houston Smit for prompting clarification on this point.

[32] The objections below are based on those that lead Friedman (1992) and (2013) to offer an alternative interpretation of the laws of diffusion.

of an original repulsion to be viewed as necessarily belonging to the goals of my metaphysical treatment of matter" (MAN 4:522).

While the considerations behind Kant's cautious language are complex, I do not think they are inconsistent with Kant's believing in his constructions, taking them to be crucial to understanding the mathematical content of the laws, and seeing mathematical construction as essential to the project of MAN. With regard to the last two points, Kant indeed emphasizes in numerous texts that metaphysics is not itself mathematical construction (MAN 4:469, 4:473; Krv A713/B741).[33] But this claim does not preclude mathematical construction being not only relevant to but indeed essential to the aims of special metaphysics. Kant says of special metaphysics that it is concerned with the application of mathematics to body, that it contains "principles of the construction"[34] (MAN 4:472-3) of concept*s* (note the plural) belonging to matter and its possibility, and that it is "responsible… for the correctness of the elements of construction granted to our rational cognition" (MAN 4:518). As I understand this, special metaphysics provides principles and elements of construction by, as the methodological remark (MAN 4:472) quoted in my introduction suggests, analyzing the concept of matter into concepts of matter (each associated with a heading in the table of the categories) and then showing how those partial concepts admit of a sort of mathematical construction i*n principle*. The partial concepts into which matter is analyzed are, for example, "the movable in space" (where motion is treated as "pure quantum" (MAN 4:477)), "the movable insofar as it fills space" (MAN 4:496) and "the movable insofar as it, as such a thing, has a moving force" (MAN 4:536). In the case of movable in space, Kant shows in the Phoronomy Chapter how this concept can be in principle constructed by showing how composite motion admits of construction in pure intuition. In the case of the movable insofar as it fills space, I think Kant is

[33] For some historical background on Kant's distinction, see Shabel (2006).
[34] I am in agreement with McNulty (2014; 396) and Heis (2014; 354) that the construction here is mathematical construction. Cf. Stang (2016; 248).

trying to show how it is in principle possible to construct the concept, insofar as it is in principle possible to construct a model and formal laws that explain how the mathematical content of the dynamical laws is both possible and necessary. Kant is plausibly read as doing something very similar for the Third Law and the corresponding concept of matter treated in the Mechanics Chapter (as the passage from the *Metaphysics of Morals* quoted above [RL 6:232-3] suggests and which we will see borne out in §4).

The point of the "in principle" qualification is that special metaphysics qua metaphysics does not concern itself overly with messy technical details of actual constructions—the success of Kant's metaphysical-dynamical analysis of matter does not stand or fall with such details. What matters is that he has done enough to show construct*ibility* along the lines of the account above.[35] Kant is particularly keen to emphasize this in the lead up to the actual constructions that he cannot "forebear" giving because of some technical issues having to do with his derivation of the inverse-cube law.[36] A key "difficulty" (MAN 4:521) that Kant refers to in Remark 2 to Proposition 8 is that matter is in fact a continuum—there are no discrete centers of force, as in his pre-Critical *Physical Monadology*. This generates a problem, because the sort of model we construct for repulsive force is naturally taken to involve point-centers that diffuse across a finite volume (and are separated from others by a finite distance).[37] Nevertheless, Kant suggests that the gap between the (somewhat) idealized model we construct and the physical reality can be bridged by taking the distances and volumes in the model to be infinitely small. Since infinitely small distance/volume is "not different from contact," according to Kant, the idealized aspects of our model do not in fact present an insuperable difficulty (MAN 4:522).

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[35] That the philosopher must establish construct*ibility* is a point also made by Brittan (1986; 86) and Foerster (2000; 60).
[36] For other discussions of this point, see Förster (2000, 63-65); Friedman (2013, 228ff.); and Warren (2017, 173n9). As Warren notes, Kant does not seem to have the sort of concerns about the inverse-square law he has about the inverse-cube law.
[37] For similar descriptions of the problem (though differing in their construal of its severity for establishing mathematical constructibility), see Friedman (2013; 228-229); Pollok (2001; 329-333); and Förster (2000; 63-65).

Now, evidently Kant thinks that readers will be skeptical about this point, and he doesn't want their skepticism to extend to the larger project of the MFNS. But it is far from obvious that Kant takes this technical, mathematical worry, or the empirical worry he raises immediately after about Mariotte's law,[38] to seriously call into question that these forces obey these laws, and that they are in principle constructible and explicable from properties and laws of space and spaces (something Kant had claimed as early as the 1756 *Physical Monadology*).

2. Second Objection: The Non-Constructibility of Force and Matter

Another worry that one might have about the account I have attributed to Kant is that it may appear to conflict with his denial that force, including his fundamental forces, can be constructed in pure intuition (KrV A223/B270; MAN 4:525). This in turn is connected to his denial that matter as it is conceived through his own metaphysical-dynamical approach admits of mathematical construction (MAN 4:525).[39]

Rather than try to fully explicate Kant's reasons for denying that we can construct the concept of matter and fundamental forces, I merely wish to note the following. First, our inability to construct the forces doesn't obviously entail our inability to construct the laws that they stand in, or at least the mathematical content of the laws. Second, Kant's denials about the constructibility of matter are consistent with his holding that (1) the partial concepts[40] into which the metaphysician analyzes matter themselves admit of a sort of construction, (2) the sort of construction that these partial concepts admit of is very different from the kind of construction that mathematical-mechanical investigators of nature can undertake on behalf of their preferred concept of matter. The

38 See Friedman (2013) for discussion.

39 Friedman (2001, 2013) emphasizes such passages. I take Kant's reasons for denying that force can be constructed to be connected with the non-constructibility of existence, causality, and reality (see n.2 above).

40 Friedman (2013; 30-1) also emphasizes the distinction between the concept <matter> and these partial concepts (*Theilbegriffe*), though he denies that, apart from the Phoronomy, they admit of construction in pure intuition. It is arguably a weakness of Friedman's reading that it introduces this asymmetry between the Phoronomy Chapter and later chapters.

latter do not have forces to worry about, and can, working with their concepts of solidity and empty space, more directly and straightforwardly construct in pure intuition properties of matter, like density (MAN 4:525). By contrast, for Kant, the only sort of construction that partial concepts of matter admit of occurs in the manner I have described above.

3. Objection 3: The need for experience

A final worry one might have about the account I have attributed to Kant is that it appears to conflict with his insistence that we need experience to find out these laws: "For, aside from this, no law of either attractive or repulsive force may be risked on a priori conjectures. Rather, everything, even universal attraction as the cause of weight, must be inferred, together with its laws, from data of experience…." (MAN 4:534; cf. MAN 4:525).

We need experience to acquire concepts of matter and force (and indeed to be able to represent their presence). Moreover, Kant thinks one could not hope to initially discover and justify the inverse-square law from the armchair without any empirical data, just by considering geometric relationships in pure intuition. Kant allows that one needs to draw on the sorts of observations that Newton did (Kepler's Laws, etc.). However, this doesn't preclude him from thinking that there is a two stage procedure involving construction of the sort we have described that *explains why and how* the inverse-square law holds.[41] The properties and laws of space and spaces described above are not the ratio cognoscendi of the inverse-square law.[42] However, if I am right, they are the ratio essendi of this law (as well as the inverse-cube law). As I explained above, this is consistent with our having a priori cognition of the laws, where one sense of a priori cognition is cognition from (explanatory) grounds.

[41] Warren (2017; 188–89) makes a similar claim, as does Hebbler (2015; 53-64).
[42] There may be an asymmetry in this regard between the inverse-square and inverse-cube law. See n. 29 above.

IV. The Third Law of Mechanics

It is now time to consider the Third Law of Mechanics. We've now encountered three passages in which Kant points to a grounding role for space (KrV B293; BDG 2:134; RL 6:232-3). But Kant makes his view particularly explicit in a 1791 letter to Christoph Friedrich Hellwag, a German doctor and physicist. Kant's letter includes a response to a question from Hellwag about the basis of the laws of inertia and the equality of action and reaction (Kant's Second and Third Laws, respectively). Hellwag proposes that these laws have their source in some "unnamed real cause of all free motion and of all mechanical resistance against moving forces" (Br 11:242)— a kind of inertial force that is in space but not identical with it (Br 11:243). In his response, Kant re-formulates Hellwag's question as follows: "What is the ground of the law that matter, in all its changes, is dependent on outer causes and also the law that requires the equality of action and reaction in these changes occasioned by outer causes?" (Br 11: 246) Against Hellwag's proposal, Kant argues that "the general and sufficient ground of these laws lies in the character of space" (Br 11:247).

How does this grounding work? Before we turn to the account in Kant's letter, it will be helpful to first review the official proof in MAN. Kant's Third Law states that "In all communication of motion, action and reaction are always equal to one another" (MAN 4:544). In his proof, Kant "borrows" from the Third Analogy the idea that communication of motion involves a causal community (a community of moving forces), so that when A communicates motion to B, both A and B must act (MAN 4:544, 4:558). Kant then claims that we must "represent" communication of motion in such a way that when there is a communication of motion between A and B, considered in absolute space "the change of relation (and thus the motion) between the two is completely mutual" (MAN 4:545). That is, we should "construct the action in the community of the bodies" (MAN 4:545-6) in a case of impact in such a way that (a) leading up to the impact both are moving towards each other with equal and opposite momenta, (b) in the impact, the momenta transferred or communicated are equal and opposite.[43] We do this by viewing the motions of the impacting bodies "in absolute space," that is relative to their common center of mass—relative to that frame, (a) and (b) are satisfied.[44]

Kant's "construction" of such a community of motion, which I take to be a mathematical construction in the technical sense, involves a spatial diagram—a model—with A and B, whose respective masses are depicted by circles of differing sizes, connected by a line, and with a point C on the line depicting absolute space (their center of mass) with respect to which they move. I take it the constructed model is supposed to be one where it is intuitively obvious that conditions (a) and (b) would be met.[45] In that respect, the constructed model exhibits a formal law of the equality of momenta and transfer of momenta in cases of communication of motion.[46]

What does any of this have to do with the equality of *action* and *reaction* (the Third Law)? Kant says in Remark 1 that the Third Law is a "necessary condition" of this construction. I take him to mean by this that if we apportion motion in cases of communication of motion in accordance with the model (that is, we assume actual cases of communication of motion work in accordance with the model), and if we assume further that (1) both A and B are causally active in the communication of motion, with each acting on the other rather than itself (Kant here draws on the Third Analogy, and perhaps also the Second Analogy and Second Law[47]), and (2) the causal action of

[43] Kant also identifies a further condition—namely, that the motion of A and B in absolute space is cancelled through their impact (the impact would thus be a perfectly inelastic one). This appears to me to be a consequence of the other two conditions.
[44] For further details, see Stan (2013); Friedman (2013); and Watkins (2019)
[45] Friedman (2013; 352n115) explains the first condition as follows: "The distances from the center of mass are always inversely as the masses, by definition, so the velocities relative to this center – i.e. the changes of distance from the center in a given time – must also be inversely as the masses."
[46] Following Watkins (2019), I take this step of the argument to establish what Kant (in Note 2) calls the "mechanical law" of equality and reaction; the next establishes the "dynamical law" of action and reaction (MAN 4:548).
[47] Watkins' (2019; 86) reconstruction invokes the Second Analogy and Second Law rather than the Third Analogy.

each is measured by the change of momentum in the other it effects, then the equality of action and reaction in the communication of motion follows.

One thing that is not particularly clear in this proof is why we are entitled to assume, and what would account for the fact that, actual motions of interacting bodies conform to the constructed model? After all, in the Phoronomy Chapter, Kant had said that assignments of motion are arbitrary when we are treating matter as a point and abstracting from force (MAN 4:487-8). Why are we now required, once we are talking about communication of motion, to apportion the motion according to the equality of momenta law exhibited in the model, and what would explain this fact? The letter to Hellwag sheds light on this issue:

> As for the second law [Note: Kant actually means here the third], it is based on the relationship of active forces in space in general, a relationship that must necessarily be one of reciprocal opposition and must always be equal (actio est aequalis reactioni), for space makes possible only reciprocal relationships such as these, precluding any unilateral relationships. Consequently it makes possible change in those spatial relationships, that is, motion and the action of bodies in producing motion in other bodies, requiring nothing but reciprocal and equal motions. I cannot conceive of a line drawn from body A to every point of body B without drawing equally as many lines in the opposite direction, so that I conceive the change of relationship in which body B is moved by the thrust of body A as a reciprocal and equal change. Here, too, there is no need for a special positive cause of reaction in the moved body, just as there was no such need in the case of the law of inertia, which I mentioned above. The general and sufficient ground of these laws lies in the character of space, viz., that spatial relationships are reciprocal and equal (which is not true of the relations between successive positions in time). (Br 11:246-7)

Kant claims that space makes possible only reciprocal relationships, a point which he illustrates (as he did in the official proof in MAN) with a line model. As he notes, when there are two points A and B on a line, A's changing distance to B always requires B's changing distance to A. This constructed model allows us to see how it is *possible* to distribute motions in the way Kant's proof of the Third Law requires—and in that sense (given the role that this way of apportioning motion plays in Kant's proof) also how us to see how it is at least *possible* for the Third Law to have the equal and opposite character it does. (Recall in this regard the Doctrine of Right passage, where Kant speaks of "presenting the *possibility* of bodies moving freely under the law of the equality of action and reaction" (RL 6:232-3; my emphasis.)

But again: why must actual cases of communication of motion occur in accordance with the model—couldn't they instead involving just one body moving, or bodies moving in something other than an equal and opposite way? (Granted, this would mean for Kant that the Third Law would not be true, but why should we think it is true?) This, I take it, is where Kant's appeal to the "character of space, viz., that spatial relationships are reciprocal and equal" is relevant. I take Kant to be alluding to what I referred to as the efficient and equitable character of space. The efficient and equitable character grounds—in the sense of explaining—why, given that there are bodies that causally communicate motion to each other, they do so in accordance with the constructed spatial (or spatio-temporal) model, in which momenta are equal and opposite.

This reading allows us to understand the significance of Kant's claim in the official proof in MAN that "there is no more reason to ascribe more of the motion to one than to the other." Kant is not appealing to the principle of sufficient reason, as Marius Stan and various other commentators have suggested (2013, 502)[48] (MAN 5:545)—or more carefully, he is not appealing to this principle *rather than* considerations about space.[49] The reason the quoted remark is relevant is that, due to the efficient and equitable character of space, where there are no special, additional causes[50] for deviation, the default distributions of motion and force are maximally efficient and equitable.

[48] See e.g, Watkins (2019, 86.n46) and Adickes (1924; 316-317). The latter mentions Kant's appeal to space in the letter to Hellwag (1924; 323, n2) but dismisses it quickly as a "mere assertion without any proof".
[49] This more careful parenthetical formulation leaves it open that the structure of space might be itself a result or a manifestation of the principle of sufficient reason. (Thanks to Colin McLear for pushing me here.) Schopenhauer, who thinks that spatial relations are a specific form of grounds (rationes essendi) and that space as the form of intuition is itself a specific form of the PSR, is an example of such a view. See Schopenhauer [1999, §36-37] and Schopenhauer [2014, §§14-15]).
[50] In the case of the laws of diffusion, the absence of these is secured by the fundamentality and immediacy of the force. It is less clear what in the present case secures their absence.

If this correct, then we can distinguish two stages of explanation in Kant's construction-cum-derivation of the Third Law, just as we did with the laws of diffusion. In the first stage we construct a spatial model (with a temporal element[51])—in this case a line with geometric points taken to be changing their distances to each other and to the point standing in for their center of mass. In doing so, we simultaneously construct a formal, broadly geometric law, in this case a law of the equality of momenta.[52] By means of this model and the law expressed through it, we can explain why, given that bodies communicating motion to each other conform to the model, and given the other assumptions laid out above, it is not only possible but also necessary that "in all communication of motion, action and reaction are always equal to one another" (MAN 4:544). In the second stage, we appeal to properties of space, namely its efficient and equitable character, to explain why, given that there are bodies that communicate motion by means of force, the motion must conform to the model.

V. Another objection: laws don't lie in space

Let me now consider an important passage that may seem inconsistent with the sort of view I have ascribed to Kant. It occurs in *Prolegomena* §38, where Kant says that "space is something so uniform and so indeterminate with respect to all specific properties that certainly no one will look for a stock of natural laws within it" (Prol 4:321). How can this be reconciled with the fact that Kant explicitly ascribes a grounding role to space?

[51] Notice that, while there appears to be such a temporal element, Kant consistently highlights the spatial, geometric aspects of the model.
[52] I take it that the line model corresponds to the formal, geometric concept that Kant spoke of in the Doctrine of Right passage. Note that in that passage goes on to discuss lines.

In this difficult passage, I take Kant to be rejecting a kind of view—to which he thinks some transcendental realist are committed—that takes space *all on its own*, apart from the other conditions that make experience possible, to be *sufficient* for the existence of laws, including geometric laws, transcendental laws of nature, laws of diffusion, and laws of mechanics. On my interpretation, Kant denies the sufficiency claims.[53] The space that we are given in pure intuition does not alone suffice for any laws. In the case of geometric laws (like the law of the surface area of spheres), Kant thinks they depend not just on pure space but also on constructive acts of the geometer (which themselves involve a synthesis in accordance with the mathematical categories). In the case of the transcendental laws (like the Third Analogy), Kant thinks they depend not just on pure space but also on the dynamical categorial synthesis of empirical intuitions in space and time in virtue of which experience of a unitary, objective space and time is possible. Consider in this regard Kant's statement later in the *Prolegomena*: "space and time (*in combination with the pure concepts of the understanding*) prescribe their law a priori to all possible experience" (Prol 4:375; my emphasis).

As for the laws of diffusion and the Third Law, they depend not just on pure space but also on constructed spaces and their formal geometric laws—which means they also depend on the constructive acts and categorial synthesis they presuppose. They depend further on non-spatial conditions of the experience of matter: these include intellectual conditions (e.g. the categories of reality, and also the relational categories and Third Analogy in the case of the Third Law) as well as empirically given ones, like the presence of forces. All of this, I think, is compatible with holding that properties and laws of space and spaces explain in the two stages (each stage of which involves certain "givens" associated with various intellectual and sensible conditions of matter/the experience of matter) the mathematical character of the laws of diffusion and the Third Law.

[53] See Messina (2018) for details.

VI. Conclusion

We began our discussion by noting two parallel passages that suggested some fundamental similarities between the grounding role of space in both the laws of diffusion and the Third Law. We are now in a position to describe the similarities. The grounding of the laws of diffusion as well as of the Third Law, incorporate construction in a very similar two stage explanation. The first stage involves the construction of a spatial model (a figure) along with a formal law. The second stage appeals to the efficient and equitable character of space to explain why the phenomena in question must accord with the model and its formal law. In both cases, the result of the explanation is a priori knowledge in the three senses noted above. In both cases, we come to see how the mathematical content of a real law (in one case the laws of diffusion, in the other the Third Law) admits of a kind of mathematical construction in virtue of the constructibility of the a formal spatial (or spatio-temporal) law that corresponds to it. Moreover, as we seen, the constructibility of the mathematical character of the real law is supposed to vindicate the mathematical constructibility of various "dynamical" concepts into which the concept of matter is analyzed.

I will conclude with some—all-too-brief—implications of this interpretation for Kant's relationship to Leibniz. As we noted above, space, as Kant conceives it, is able to play a role in grounding laws because it is taken to be, pace Leibniz, prior to matter and its forces (KrV B292-3, A264-5/B320-1, A273-4/B329-30). A further difference is that Leibniz maintains a sharp distinction between the kind of (logical) necessity that he thinks is possessed by the truths of geometry and that of laws of nature, which he takes to be contingent and to rest on metaphysical laws like the PSR and the principle of perfection (AG 53-54, 124, 321-322; T §345)[54]. By contrast, as I read Kant, the

[54] Thanks to Katherine Dunlop for calling my attention to these points and passages.

contrast between geometric necessity (which Kant in contrast to Leibniz takes to be non-logical) and at least some of the a priori laws treated in MAN is not as sharp a one, since geometric considerations (along with other a priori features of space) serve to explain why the latter must have the content they do. Finally, while Kant relies in his proofs of the Laws of Mechanics on principles involving the categories (which he takes to be "metaphysical") [MAN 4:474]), his principles are not of the abstract metaphysical sort that Leibniz relies on. Kant does not (or not to the exclusion of the intrinsic and special structural characteristics of space) invoke the PSR, or a Principle of Perfection, at least not if what I say about the Third Law above is correct.[55] [56]

[55] For discussion of other similarities and differences between Kant and Leibniz (and the Leibnizian-Wolffians) see Stan (2013); Friedman (2013); and Watkins (2019).

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