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Multi-attribute Decision Making based on Rough Neutrosophic Variational Coefficient Similarity Measure

Kalyan Mondal[1] Surapati Pramanik[2] and Florentin Smarandache[3]

[1]Department of Mathematics, Jadavpur University, West Bengal, India. Email: kalyanmathematic@gmail.com

[2]Department of Mathematics, Nandalal Ghosh B.T. College, Panpur, PO-Narayanpur, and District: North 24 Parganas, Pin Code: 743126, West Bengal, India. Email: sura_pati@yahoo.co.in,

[3]University of New Mexico. Mathematics & Science Department, 705 Gurley Ave., Gallup, NM 87301, USA. Email: fsmarandache@gmail.com

Abstract: The purpose of this study is to propose new similarity measures namely rough variational coefficient similarity measure under the rough neutrosophic environment. The weighted rough variational coefficient similarity measure has been also defined. The weighted rough variational coefficient similarity measures between the rough ideal alternative and each alternative are calculated to find the best alternative. The ranking order of all the alternatives can be determined by using the numerical values of similarity measures. Finally, an illustrative example has been provided to show the effectiveness and validity of the proposed approach. Comparisons of decision results of existing rough similarity measures have been provided.

Keywords: Neutrosophic set, Rough neutrosophic set; Rough variation coefficient similarity measure; Decision making.

1 Introduction

In 1965, L. A. Zadeh grounded the concept of degree of membership and defined fuzzy set [1] to represent/manipulate data with non-statistical uncertainty. In 1986, K. T. Atanassov [2] introduced the degree of non-membership as independent component and proposed intuitionistic fuzzy set (IFS). F. Smarandache introduced the degree of indeterminacy as independent component and defined the neutrosophic set [3, 4, 5]. For purpose of solving practical problems, Wang et al. [6] restricted the concept of neutrosophic set to single valued neutrosophic set (SVNS), since single value is an instance of set value. SVNS is a subclass of the neutrosophic set. SVNS consists of the three independent components namely, truth-membership, indeterminacy-membership and falsity-membership functions.

The concept of rough set theory proposed by Z. Pawlak [7] is an extension of the crisp set theory for the study of intelligent systems characterized by inexact, uncertain or insufficient information. The hybridization of rough set theory and neutrosophic set theory produces the rough neutrosophic set theory [8, 9], which was proposed by Broumi, Dhar and Smarandache [8, 9]. Rough neutrosophic set theory is also a powerful mathematical tool to deal with incompleteness.

Literature review reflects that similarity measures play an important role in the analysis and research of clustering analysis, decision making, medical diagnosis, pattern recognition, etc. Various similarity measures [10, 11, 12, 13, 14, 15, 16, 17, 18] of SVNSs and hybrid SVNSs are available in the literature. The concept of similarity measures in rough neutrosophic environment [19, 20, 21] has been recently proposed.

Pramanik and Mondal [19] proposed cotangent similarity measure of rough neutrosophic sets. In the same study [19],Pramanik and Mondal established its basic properties and provided its application to medical diagnosis. Pramanik and Mondal [20] also proposed cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis. The same authors [21] also studied Jaccard similarity measure and Dice similarity measures in rough neutrosophic environment and provided their applications to multi attribute decision making. Mondal and Pramanik [22] presented tri-complex rough neutrosophic similarity measure and its application in multi-attribute decision making. Together with F. Smarandache and S. Pramnik, K. Mondal [23] presented hypercomplex rough neutrosophic similarity measure and its application in multi-attribute decision making. Mondal, Pramanik, and Smarandache [24] presented several trigonometric Hamming similarity measures of rough neutrosophic sets and their applications in multi attribute decision making problems.

Different methods for multiattribute decision making (MADM) and multicriteria decision making (MCDM) problems are available in the literature in different environment such as crisp environment [25, 26, 27, 28, 29], fuzzy environment [30, 31], intuitionistic fuzzy environment [32, 33, 34, 35, 36, 37, 38, 39, 40], neutrosophic environment [41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62], interval neutrosophic environment [63, 65, 66, 67, 68], neutrosophic soft expert environment [69], neutrosophic bipolar environment [70, 71], neutrosophic soft environment [72, 73, 74, 75, 76], neutrosophic hesitant fuzzy environment [77, 78, 79], rough neutrolsophic environment [80, 81], etc. In neutrosophic environment Biswas, Pramanik and Giri [82] studied hybrid vector similarity measure and its application in multi-attribute decision making. Getting motivation from the work of Biswas, Pramanik and Giri [82], for hybrid vector similarity measure in neutrosophic envionment, we extend the concept in rough neutrosophic environment.

In this paper, a new similarity measurement is proposed, namely rough variational coefficient similarity measure under rough neutrosophic environment. A numerical example is also provided.

Rest of the paper is structured as follows. Section 2 presents neutrosophic and rough neutrosophic preliminaries. Section 3 discusses various similarity measures and varional coefficient similarity measure in crisp environment. Section 4 presents various similarity measures and variational similarity measure for single valued neutrosophic sets. Section 5 presents variational coefficient similarity measure and weighted variational coefficient similarity measure for rough neutrosophic sets and establishes their basic properties. Section 6 is devoted to present multi attribute decision making based on rough neutrosophic variational coefficient similarity measure. Section 7 demonstrates the application of rough variational coefficient similarity measures to investment problem Finally, section 8 concludes the paper with stating the future scope of research.

2 Neutrosophic preliminaries

Definition 2.1 [3, 4, 5] Neutrosophic set

Let X be a space of points (objects) with generic element in X denoted by x. Then a neutrosophic set A in X is denoted by $A=\{x(T_A(x), I_A(x), F_A(x)): x \in X\}$ where, $T_A(x)$ is the truth membership function, $I_A(x)$ is the indeterminacy membership function and $F_A(x)$ is the falsity membership function. The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]^-0, 1^+[$. There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ i.e. $^-0 \le T_A(x)+I_A(x)+F_A(x) \le 3^+$.

Definition 2.2 [6] (**Single-valued neutrosophic set**).

Let X be a universal space of points (objects), with a generic element $x \in X$. A single-valued neutrosophic set (SVNS) $N \subset X$ is denoted by

$N=\int_x \langle T_N(x), I_N(x), F_N(x)\rangle / x, \forall x \in X$, when X is continuous;

$N=\sum_{i=1}^{m}\langle T_N(x), I_N(x), F_N(x)\rangle / x, \forall x \in X$, when X is discrete.

SVNS is characterized by a true membership function $T_N(x)$, a falsity membership function $F_N(x)$ and an indeterminacy function $I_N(x)$ ith $T_N(x)$, $F_N(x)$, $I_N(x)$ $\in$ [0, 1] for all $x \in X$. For each $x \in X$, of a SVNS N $0 \le T_N(x)+I_N(x)+F_N(x) \le 3$.

2.1 Some operational rules and properties of SVNSs

Let $N_A=\langle T_A, I_A, F_A\rangle$ and $N_B=\langle T_B, I_B, F_B\rangle$ be two SVNSs in *X*. Then the following operations are defined as follows:

I. Complement: $N_{A^c}=\langle F_A, 1\text{-}I_A, T_A\rangle \ \forall x \in X$.

II. Addition: $N_A \oplus N_B=\langle T_A+T_B-T_AT_B, I_AI_B, F_AF_B\rangle$

III. Multiplication:

$N_A \otimes N_B=\langle T_AT_B, I_A+I_B\text{-}I_AI_B, F_A+F_B\text{-}F_AF_B\rangle$

IV. Scalar Multiplication:

$\lambda N_A=\langle 1-(1-T_A)^\lambda, I_A^\lambda, F_A^\lambda\rangle$ for $\lambda>0$.

V. $\langle N_A\rangle^\lambda=\langle (T_A)^\lambda, 1-(1-I_A)^\lambda, 1-(1-F_A)^\lambda\rangle$ for $\lambda>0$.

Definition 2.3 [6]

Complement of a SVNS N is denoted by N^c and is defined by

$T_{N^c}(x)=F_N(x)$; $I_{N^c}(x)=1-I_N(x)$; $F_{N^c}(x)=T_N(x)$

Definition 2.4 [6]

A SVNS N_A is contained in the other SVNS N_B, denoted as $N_A \subseteq N_B$, if and only if

$T_{N_A}(x) \le T_{N_B}(x)$; $I_{N_A}(x) \ge I_{N_B}(x)$; $F_{N_A}(x) \ge F_{N_B}(x) \ \forall x \in X$

Definition 2.5 [6]

Two SVNSs N_A and N_B are equal, i.e. $N_A= N_B$, if and only if $N_A \supseteq N_B$ and $N_A \subseteq N_B$

Definition 2.6 [6]

Union of two SVNSs N_A and N_B is a SVNS N_C, written as $N_C=N_A \cup N_B$. Its truth membership, indeterminacy-membership and falsity membership functions are related to those of N_A and N_B by

$T_{N_C}(x)=\max(T_{N_A}(x), T_{N_B}(x))$; $I_{N_C}(x)=\min(I_{N_A}(x), I_{N_B}(x))$; $F_{N_C}(x)=\min(F_{N_A}(x), F_{N_B}(x))$ for all *x* in *X*.

Definition 2.7 [6] Intersection of two SVNSs N_A and N_B is a SVNS N_D, written as $N_D=N_A \cap N_B$, whose truth membership, indeterminacy-membership and falsity membership functions are related to those of N_A and N_B by $T_{N_C}(x)=\min(T_{N_A}(x), T_{N_B}(x))$; $I_{N_C}(x)=\max(I_{N_A}(x), I_{N_B}(x))$; $F_{N_C}(x)=\max(F_{N_A}(x), F_{N_B}(x))$ for all *x* in *X*.

Definition 2.8 Rough Neutrosophic Sets [8, 9]

Let Z be a non-null set and R be an equivalence relation on Z. Let P be neutrosophic set in Z with the membership function T_P indeterminacy function I_P and non-membership function F_P. The lower and the upper approximations of P in the approximation (Z, R) denoted by $\underline{N}(P)$ and $\overline{N}(P)$ are respectively defined as follows:

$\underline{N}(P) = \left\langle <x, T_{\underline{N}(P)}(x), I_{\underline{N}(P)}(x), F_{\underline{N}(P)}(x) > / z \in [x]_R, x \in Z \right\rangle,$

$\overline{N}(P) = \left\langle <x, T_{\overline{N}(P)}(x), I_{\overline{N}(P)}(x), F_{\overline{N}(P)}(x) > / z \in [x]_R, x \in Z \right\rangle,$

Here, $T_{\underline{N}(P)}(x) = \wedge_z \in [x]_R\, T_P(z)$, $I_{\underline{N}(P)}(x) = \wedge_z \in [x]_R\, I_P(z)$, $F_{\underline{N}(P)}(x) = \wedge_z \in [x]_R\, F_P(z)$, $T_{\overline{N}(P)}(x) = \vee_z \in [x]_R\, T_P(z)$, $I_{\overline{N}(P)}(x) = \vee_z \in [x]_R\, I_P(z)$, $F_{\overline{N}(P)}(x) = \vee_z \in [x]_R\, F_P(z)$

So, $0 \le \sup T_{\underline{N}(P)}(x) + \sup I_{\underline{N}(P)}(x) + \sup F_{\underline{N}(P)}(x) \le 3$

$0 \le \sup T_{\overline{N}(P)}(x) + \sup I_{\overline{N}(P)}(x) + \sup F_{\overline{N}(P)}(x) \le 3$

Here $\vee$ and $\wedge$ denote "max" and "min'' operators respectively. $T_P(z)$, $I_P(z)$ and $F_P(z)$ denote respectively the membership, indeterminacy and non-membership function of z with respect to P. It is easy to see that $\underline{N}(P)$ and $\overline{N}(P)$ are two neutrosophic sets in Z.

Thus NS mappings $\underline{N}$, $\overline{N}$: $N(Z) \rightarrow N(Z)$ are, respectively, referred to as the lower and the upper rough NS approximation operators, and the pair $(\underline{N}(P), \overline{N}(P))$ is called the rough neutrosophic set [8, 9] in (Z, R).

From the above definition, it is seen that $\underline{N}(P)$ and $\overline{N}(P)$ have constant membership on the equivalence classes of R. if $\underline{N}(P) = \overline{N}(P)$ i.e. $T_{\underline{N}(P)}(x) = T_{\overline{N}(P)}(x)$, $I_{\underline{N}(P)}(x) = I_{\overline{N}(P)}(x)$ and $F_{\underline{N}(P)}(x) = F_{\overline{N}(P)}(x), \forall x \in Z.$

P is said to be a definable neutrosophic set in the approximation (Z, R). It can be easily proved that zero neutrosophic set (0_N = (0, 1, 1)) and unit neutrosophic sets (1_N = (1, 0, 0)) are definable neutrosophic sets.

Definition 2.9 [8, 9]

If $N(P) = (\underline{N}(P), \overline{N}(P))$ is a rough neutrosophic set in (Z, R) , the rough complement [8, 9] of $N(P)$ is the rough neutrosophic set denoted by $\sim N(P) = (\underline{N}(P)^c, \overline{N}(P)^c)$ where $\underline{N}(P)^c$, $\overline{N}(P)^c$ are the complements of neutrosophic sets of $\underline{N}(P)$, $\overline{N}(P)$ respectively.

$\underline{N}(P)^c = \left\langle <x, F_{\underline{N}(P)}(x), 1 - I_{\underline{N}(P)}(x), T_{\underline{N}(P)}(x) > / x \in Z \right\rangle$ and

$\overline{N}(P)^c = \left\langle <x, F_{\overline{N}(P)}(x), 1 - I_{\overline{N}(P)}(x), T_{\overline{N}(P)}(x) > / x \in Z \right\rangle$

Definition 2.10 [8, 9]

If $N(P_1)$ and $N(P_2)$ are the two rough neutrosophic sets of the neutrosophic set P respectively in Z, then the following definitions [8, 9] hold:

$N(P_1) = N(P_2) \Leftrightarrow \underline{N}(P_1) = \underline{N}(P_2) \wedge \overline{N}(P_1) = \overline{N}(P_2)$

$N(P_1) \subseteq N(P_2) \Leftrightarrow \underline{N}(P_1) \subseteq \underline{N}(P_2) \wedge \overline{N}(P_1) \subseteq \overline{N}(P_2)$

$N(P_1) \cup N(P_2) = < \underline{N}(P_1) \cup \underline{N}(P_2), \overline{N}(P_1) \cup \overline{N}(P_2) >$

$N(P_1) \cap N(P_2) = < \underline{N}(P_1) \cap \underline{N}(P_2), \overline{N}(P_1) \cap \overline{N}(P_2) >$

$N(P_1) + N(P_2) = < \underline{N}(P_1) + \underline{N}(P_2), \overline{N}(P_1) + \overline{N}(P_2) >$

$N(P_1).N(P_2) = < \underline{N}(P_1).\underline{N}(P_2), \overline{N}(P_1).\overline{N}(P_2) >$

If N, M, L are the rough neutrosophic sets in (Z, R), then the following proposition are stated from definitions [8, 9].

Proposition 1 [8, 9]

1. $\sim(\sim N) = N$
2. $N \cup M = M \cup N, N \cap M = M \cap N$
3. $(L \cup M) \cup N = L \cup (M \cup N)$, $(L \cap M) \cap N = L \cap (M \cap N)$
4. $(L \cup M) \cap N = (L \cup M) \cap (L \cup N)$, $(L \cap M) \cup N = (L \cap M) \cup (L \cap N)$

Proposition 2 [8, 9]

De Morgan's Laws are satisfied for rough neutrosophic sets .

1. $\sim (N(P_1) \cup N(P_2)) = (\sim (N(P_1)) \cap (\sim N(P_2))$
2. $\sim (N(P_1) \cap N(P_2)) = (\sim (N(P_1) \cup (\sim N(P_2))$

Proposition 3 [8, 9]

If P_1 and P_2 are two neutrosophic sets in U such that $P_1 \subseteq P_2$ *then* $N(P_1) \subseteq N(P_2)$

1. $N(P_1 \cap P_2) \subseteq N(P_1) \cap N(P_2)$
2. $N(P_1 \cup P_2) \supseteq N(P_1) \cup N(P_2)$

Proposition 4 [8, 9]

1. $\underline{N}(P) = \sim \overline{N}(\sim P)$
2. $\overline{N}(P) = \sim \underline{N}(\sim P)$
3. $\underline{N}(P) \subseteq \overline{N}(P)$

3 Similarity measures and variational coefficient similarity measure in crisp environment

The vector similarity measure is one of the important tools for the degree of similarity between objects. However, the Jaccard, Dice, and cosine similarity measures are often used for this purpose. Jaccard [83] , Dice [84] , and cosine [85] similarity measures between two vectors are stated below.

Let $X = (x_1, x_2, \ldots, x_n)$ and $Y = (y_1, y_2, \ldots, y_n)$ be two n-dimensional vectors with positive co-ordinates.

Definition 3.1 [83]

Jeccard index of two vectors (measuring the "similarity" of these vectors) can be defined as follows:

$$J(X, Y) = \frac{X.Y}{\|X\|^2 + \|Y\|^2 - X.Y} = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{n} y_i^2 - \sum_{i=1}^{n} x_i y_i} \quad (1)$$

where $\|X\|^2 = \sum_{i=1}^{n} x_i^2$ and $\|Y\|^2 = \sum_{i=1}^{n} y_i^2$ are the Euclidean norm of X and Y, $X.Y = \sum_{i=1}^{n} x_i y_i$ is the inner product of the vector X and Y.

Proposition 5 [83]

Jaccard index satisfies the following properties:

1. $0 \le J(X, Y) \le 1$

2. J(X, Y) = J(Y, X)
3. J(X, Y) = 1, for $X = Y$ i.e, $x_i = y_i$ (i = 1, 2, …, n) for every $x_i \in X$ and $y_i \in Y$

Definition 3.2 [84]

The Dice similarity measure can be defined as follows:

$$E(X, Y) = \frac{2\,X.Y}{\|X\|^2+\|Y\|^2} = \frac{2\sum_{i=1}^{n} x_i\, y_i}{\sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{n} y_i^2} \quad (2)$$

Proposition 6 [84]

The Dice similarity measure satisfies the following properties:
1. 0 ≤ E(X, Y) ≤ 1
2. E(X, Y) = E(Y, X)
3. J(X, Y) = 1, for $X = Y$ i.e, $x_i = y_i$ (i = 1, 2, …, n) for every $x_i \in X$ and $y_i \in Y$.

Definition 3.3 [85]

The cosine similarity measure between two vectors X and Y is the inner product of these two vectors divided by the product of their lengths and can be defined as follows:

$$C(X, Y) = \frac{X.Y}{\|X\|.\|Y\|} = \frac{\sum_{i=1}^{n} x_i\, y_i}{\sqrt{\sum_{i=1}^{n} x_i^2}\sqrt{\sum_{i=1}^{n} y_i^2}} \quad (3)$$

Proposition 7 [85]

The cosine similarity measure satisfies the following properties
1. 0 ≤ C(X, Y) ≤ 1
2. C(X, Y) = C(Y, X)
3. C(X, Y) = 1, for $X = Y$ i.e, $x_i = y_i$ (i = 1, 2, …, n) for every $x_i \in X$ and $y_i \in Y$.

These three formulas are similar in the sense that they take values in the interval [0, 1]. Jaccard and Dice similarity measures are undefined when $x_i = 0$, and $y_i = 0$ for i = 1, 2, …, n and cosine similarity measure is undefined when $x_i = 0$ or $y_i = 0$ for i = 1, 2, …, n.

Definition 3.4 [86]

Variational co-efficient similarity measure can be defined as follows:

$$V(X, Y) = \lambda\frac{2\,X.Y}{\|X\|^2+\|Y\|^2} + (1-\lambda)\frac{X.Y}{\|X\|.\|Y\|}$$

$$= \lambda\frac{2\sum_{i=1}^{n} x_i\, y_i}{\sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{n} y_i^2} + (1-\lambda)\frac{\sum_{i=1}^{n} x_i\, y_i}{\sqrt{\sum_{i=1}^{n} x_i^2}\sqrt{\sum_{i=1}^{n} y_i^2}} \quad (4)$$

Proposition 8 [86]

Variational co-efficient similarity measure satisfies the following properties:
1. 0 ≤ V(X, Y) ≤ 1
2. V(X, Y) = V(Y, X)
3. V(X, Y) = 1, for $X = Y$ i.e, $x_i = y_i$ (i = 1, 2, …, n) for every $x_i \in X$ and $y_i \in Y$.

4. Various similarity measures for single valued neutrosophic sets.

Assume $N_A = \langle T_A, I_A, F_A \rangle$ and $N_B = \langle T_B, I_B, F_B \rangle$ be two SVNSs in a universe of discourse $X = (x_1, x_2,…, x_n)$. $T_A, I_A, F_A \in [0,1]$ for any $x_i \in X$ in N_A or $T_B, I_B, F_B \in [0,1]$ for any $x_i \in X$ in N_B can be considered as a vector representation with three elements. Let $w_i \in [0,1]$ be the weight of each element x_i for i = 1, 2, …, n such that $\sum_{i=1}^{n} w_i = 1$, then Jaccard, Dice and cosine similarity measures can be presented as follows:

Definition 4.1[10] Jaccard similarity measure between $N_A = \langle T_A, I_A, F_A \rangle$ and $N_B = \langle T_B, I_B, F_B \rangle$ can be defined as follows:

$Jac(N_A, N_B)$ =

$$\frac{1}{n}\sum_{i=1}^{n}\frac{(T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i))}{\left\langle \begin{array}{l}\left[(T_A(x_i))^2+(I_A(x_i))^2+(F_A(x_i))^2\right]\\ +\left[(T_B(x_i))^2+(I_B(x_i))^2+(F_B(x_i))^2\right]\\ -\{T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)\\ +F_A(x_i)F_B(x_i)\}\end{array}\right\rangle} \quad (5)$$

Proposition 9 [10]

Jaccard similarity measure satiefies the following properites:
1. $0 \le Jac(N_A, N_B) \le 1$;
2. $Jac(N_A, N_B) = Jac(N_B, N_A)$;
3. $Jac(N_A, N_B) = 1$; if $N_A = N_B$ i.e., $T_A(x_i) = T_B(x_i)$, $I_A(x_i) = I_B(x_i)$, and $F_A(x_i) = F_B(x_i)$, for every x_i (i = 1, 2, …, n) in X.

Definition 4.1.1 [10] Weighted Jaccard similarity measure between $N_A = \langle T_A, I_A, F_A \rangle$ and $N_B = \langle T_B, I_B, F_B \rangle$ can be defined as follows:

$Jac_w(N_A, N_B)$=

$$\sum_{i=1}^{n} w_i\frac{(T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i))}{\left\langle \begin{array}{l}\left[(T_A(x_i))^2+(I_A(x_i))^2+(F_A(x_i))^2\right]\\ +\left[(T_B(x_i))^2+(I_B(x_i))^2+(F_B(x_i))^2\right]\\ -\{T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)\\ +F_A(x_i)F_B(x_i)\}\end{array}\right\rangle} \quad (6)$$

Proposition 10 [10]

Weighted Jaccard similarity measure satisfies the following properties:
1. $0 \le Jac_w(N_A, N_B) \le 1$;
2. $Jac_w(N_A, N_B) = Jac_w(N_B, N_A)$;
3. $Jac_w(N_A, N_B) = 1$; if $N_A = N_B$ i.e., $T_A(x_i) = T_B(x_i)$, $I_A(x_i) = I_B(x_i)$, and $F_A(x_i) = F_B(x_i)$, for every x_i (i = 1, 2, …, n) in X.

Definition 4.2 [11]

Dice similarity measure between $N_A=\langle T_A, I_A, F_A\rangle$ and $N_B=\langle T_B, I_B, F_B\rangle$ is defined as:

$Dic(N_A, N_B)$ =

$$\frac{1}{n}\sum_{i=1}^{n}\frac{2\begin{bmatrix}T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)\\+F_A(x_i)F_B(x_i)\end{bmatrix}}{\left\langle\begin{matrix}\left[(T_A(x_i))^2+(I_A(x_i))^2+(F_A(x_i))^2\right]\\+\left[(T_B(x_i))^2+(I_B(x_i))^2+(F_B(x_i))^2\right]\end{matrix}\right\rangle} \quad (7)$$

Proposition 11 [11]

Dice similarity measure satisfies the following properties:

1. $0\le Dic(N_A, N_B)\le 1$;
2. $Dic(N_A, N_B) = Dic(N_B, N_A)$;
3. $Dic(N_A, N_B)=1$; if $N_A=N_B$ i.e., $T_A(x_i)=T_B(x_i)$, $I_A(x_i)=I_B(x_i)$, and $F_A(x_i)=F_B(x_i)$, for every x_i (i = 1, 2, …, n) in X.

Definition 4.2.1 [11]

Weighted Dice similarity measure between $N_A=\langle T_A, I_A, F_A\rangle$ and $N_B=\langle T_B, I_B, F_B\rangle$ can be defined as follows:

$Dic_w(N_A, N_B)$ =

$$\sum_{i=1}^{n} w_i\frac{2\begin{bmatrix}T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)\\+F_A(x_i)F_B(x_i)\end{bmatrix}}{\left\langle\begin{matrix}\left[(T_A(x_i))^2+(I_A(x_i))^2+(F_A(x_i))^2\right]\\+\left[(T_B(x_i))^2+(I_B(x_i))^2+(F_B(x_i))^2\right]\end{matrix}\right\rangle} \quad (8)$$

Proposition 12 [11]

Weighted Dice similarity measure

1. $0\le Dic_w(N_A, N_B)\le 1$;
2. $Dic_w(N_A, N_B)= Dic_w(N_B, N_A)$;
3. $Dic_w(N_A, N_B)=1$; if $N_A = N_B$ i.e., $T_A(x_i)=T_B(x_i)$, $I_A(x_i)=I_B(x_i)$, and $F_A(x_i)=F_B(x_i)$, for every x_i (i = 1, 2, …, n) in X.

Definition 4.3 [12]

Cosine similarity measure between $N_A=\langle T_A, I_A, F_A\rangle$ and $N_B=\langle T_B, I_B, F_B\rangle$ can be defined as follows:

$$Cos(N_A, N_B) = \frac{1}{n}\sum_{i=1}^{n}\frac{(T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i))}{\sqrt{(T_A(x_i))^2+(I_A(x_i))^2+(F_A(x_i))^2}\sqrt{(T_B(x_i))^2+(I_B(x_i))^2+(F_B(x_i))^2}} \quad (9)$$

Proposition 13 [12]

Cosine similarity measure satisfies the following properties:

1. $0\le Cos_w(N_A, N_B)\le 1$;
2. $Cos_w(N_A, N_B)= Cos_w(N_B, N_A)$
3. $Cos_w(N_A, N_B)=1$; if $N_A = N_B$ i.e., $T_A(x_i)=T_B(x_i)$, $I_A(x_i)=I_B(x_i)$, and $F_A(x_i)=F_B(x_i)$, for every x_i (i = 1, 2, …, n) in X.

Definition 4.3.1 [12]

Weighted cosine similarity measure between $N_A=\langle T_A, I_A, F_A\rangle$ and $N_B=\langle T_B, I_B, F_B\rangle$ can be defined as follows:

$Cos_w(N_A, N_B)$=

$$\sum_{i=1}^{n} w_i\frac{(T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i))}{\sqrt{(T_A(x_i))^2+(I_A(x_i))^2+(F_A(x_i))^2}\sqrt{(T_B(x_i))^2+(I_B(x_i))^2+(F_B(x_i))^2}} \quad (10)$$

Proposition 14 [12]

Weighted cosine similarity measure satisfies the following properties:

1. $0\le Cos_w(N_A, N_B)\le 1$;
2. $Cos_w(N_A, N_B)= Cos_w(N_B, N_A)$
3. $Cos_w(N_A, N_B)=1$; if $N_A = N_B$ i.e., $T_A(x_i)=T_B(x_i)$, $I_A(x_i)=I_B(x_i)$, and $F_A(x_i)=F_B(x_i)$, for every x_i (i = 1, 2, …, n) in X.

Jaccard and Dice similarity measures between two neutrosophic sets $N_A=\langle T_A, I_A, F_A\rangle$ and $N_B=\langle T_B, I_B, F_B\rangle$ are undefined when $T_A(x_i)=I_A(x_i)=F_A(x_i)=0$ and $T_B(x_i)=I_B(x_i)=F_B(x_i)=0$ for all i = 1, 2, …, n. Similarly the cosine formula for two neutrosophic sets $N_A=\langle T_A, I_A, F_A\rangle$ and $N_B=\langle T_B, I_B, F_B\rangle$ is undefined when $T_A(x_i)=I_A(x_i)=F_A(x_i)=0$ or $T_B(x_i)=I_B(x_i)=F_B(x_i)=0$ for all i = 1, 2, …, n.

5 Variational similarity measures for rough neutrosophic sets

The notion of rough neutrosophic set (RNS) is used as vector representations in 3D-vector space. Assume that $X = (x_1, x_2,…, x_n)$ and $Y = (y_1, y_2, …, y_n)$ be two n-dimensional vectors with positive co-ordinates. Jaccard, Dice, cosine and cotangent similarity measures between two vectors are stated as follows.

Definition 5.1 [21] Jaccard similarity measure under rough neutrosophic environment

Assume that $A=\left\langle(\underline{T}_A(x_i),\underline{I}_A(x_i),\underline{F}_A(x_i)),(\overline{T}_A(x_i),\overline{I}_A(x_i),\overline{F}_A(x_i))\right\rangle$ and $B=\left\langle(\underline{T}_B(x_i),\underline{I}_B(x_i),\underline{F}_B(x_i)),(\overline{T}_B(x_i),\overline{I}_B(x_i),\overline{F}_B(x_i))\right\rangle$ in $X = (x_1, x_2,…, x_n)$ be any two rough neutrosophic sets. Jacard similarity measure [21] between rough neutrosophic sets A and B can be defined as follows:

$Jac_{RNS}(A, B)$ =

$$\frac{1}{n}\sum_{i=1}^{n}\frac{(\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)+\delta F_A(x_i)\delta F_B(x_i))}{\left\langle\begin{array}{l}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\+\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\\-[\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)\\+\delta F_A(x_i)\delta F_B(x_i)]\end{array}\right\rangle} \quad (11)$$

Proposition 15 [21]

Jaccard similarity measure [21] between A and B satisfies the following properties:

1. $0\le Jac_{RNS}(A,B)\le 1$;
2. $Jac_{RNS}(A,B)=Jac_{RNS}(B,A)$;
3. $Jac_{RNS}(A,B)=1$; iff $A = B$
4. If C is a RNS in Y and $A\subset B\subset C$ then, $Jac_{RNS}(A, C)\le Jac_{RNS}(A, B)$, and $Jac_{RNS}(A, C) \le Jac_{RNS}(B, C)$

Definition 5.1.1 [21]

If we consider the weights of each element x_i, weighted rough Jaccard similarity measure [21] between rough neutrosophic sets A and B can be defined as follows:

$Jac_{WRNS}(A, B)$ =

$$\sum_{i=1}^{n}w_i\frac{(\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)+\delta F_A(x_i)\delta F_B(x_i))}{\left\langle\begin{array}{l}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\+\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\\-[\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)\\+\delta F_A(x_i)\delta F_B(x_i)]\end{array}\right\rangle} \quad (12)$$

$w_i\in[0,1]$, i = 1, 2,…, n and $\sum_{i=1}^{n}w_i=1$. If we take $w_i=\frac{1}{n}$, i = 1, 2,…, n, then $Jac_{WRNS}(A, B) = Jac_{RNS}(A,B)$

Proposition 16 [21]

The weighted rough Jaccard similarity [21] measure between two rough neutrosophic sets A and B also satisfies the following properties:

1. $0\le Jac_{WRNS}(A,B)\le 1$;
2. $Jac_{WRNS}(A,B)=Jac_{WRNS}(B,A)$;
3. $Jac_{WRNS}(A,B)=1$; iff $A = B$
4. If C is a WRNS in Y and $A\subset B\subset C$ then, $Jac_{WRNS}(A, C) \le Jac_{WRNS}(A, B)$, and $Jac_{WRNS}(A, C) \le Jac_{WRNS}(B, C)$

Definition 5.2 [21] **Dice similarity measure under rough neutrosophic environment**

In this section, Dice similarity measure and the weighted Dice similarity measure for rough neutrosophic sets have been stated due to Pramanik and Mondal [21].

Suppose that

$A=\left\langle\left(\underline{T}_A(x_i),\underline{I}_A(x_i),\underline{F}_A(x_i)\right),\left(\overline{T}_A(x_i),\overline{I}_A(x_i),\overline{F}_A(x_i)\right)\right\rangle$ and

$B=\left\langle\left(\underline{T}_B(x_i),\underline{I}_B(x_i),\underline{F}_B(x_i)\right),\left(\overline{T}_B(x_i),\overline{I}_B(x_i),\overline{F}_B(x_i)\right)\right\rangle$ be any two rough neutrosophic sets in $X = (x_1, x_2,…, x_n)$. Dice similarity measure between rough neutrosophic sets A and B can be defined as follows:

$DIC_{RNS}(A, B)$=

$$\frac{1}{n}\sum_{i=1}^{n}\frac{2\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]}{\left\langle\begin{array}{l}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\+\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\end{array}\right\rangle} \quad (13)$$

Proposition 17 [21]

Dice similarity measure [21] satisfies the following properties.

1. $0\le DIC_{RNS}(A,B)\le 1$;
2. $DIC_{RNS}(A,B)=DIC_{RNS}(B,A)$;
3. $DIC_{RNS}(A,B)=1$; iff $A = B$
4. If C is a RNS in Y and $A\subset B\subset C$ then, $DIC_{RNS}(A, C) \le DIC_{RNS}(A, B)$, and $DIC_{RNS}(A, C) \le DIC_{RNS}(B, C)$,

For proofs of the above mentioned four properties, see [21].

Definition 5.2.1

If we consider the weights of each element x_i, a weighted rough Dice similarity measure between rough neutrosophic sets A and B can be defined as follows:

$DIC_{WRNS}(A, B)$ =

$$\sum_{i=1}^{n}w_i\frac{2\left[\begin{array}{l}\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)\\+\delta F_A(x_i)\delta F_B(x_i)\end{array}\right]}{\left\langle\begin{array}{l}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\+\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\end{array}\right\rangle} \quad (14)$$

$w_i\in[0,1]$, i = 1, 2,…, n and $\sum_{i=1}^{n}w_i=1$. If we take $w_i=\frac{1}{n}$, i = 1, 2,…, n, then $DIC_{WRNS}(A, B) = DIC_{RNS}(A,B)$

Proposition 18 [21]

The weighted rough Dice similarity [21] measure between two rough neutrosophic sets A and B also satisfies the following properties:

1. $0\le DIC_{WRNS}(A,B)\le 1$;
2. $DIC_{WRNS}(A,B)=DIC_{WRNS}(B,A)$;
3. $DIC_{WRNS}(A,B)=1$; iff $A = B$
4. If C is a RNS in Y and $A\subset B\subset C$ then, $DIC_{WRNS}(A, C) \le DIC_{WRNS}(A, B)$, and $DIC_{WRNS}(A, C) \le DIC_{WRNS}(B, C)$.

For proofs of the above mentioned four properties, see [21].

Definition 5.3 [20]

Cosine similarity measure can be defined as the inner product of two vectors divided by the product of their lengths. It is the cosine of the angle between the vector representations of two rough neutrosophic sets. The cosine similarity measure is a fundamental measure used in information technology. Pramanik and Mondal [20] defined cosine similarity measure between rough neutrosophic sets in 3-D vector space.

Assume that $A=\langle(\underline{T}_A(x_i),\underline{I}_A(x_i),\underline{F}_A(x_i)),(\overline{T}_A(x_i),\overline{I}_A(x_i),\overline{F}_A(x_i))\rangle$ and $B=\langle(\underline{T}_B(x_i),\underline{I}_B(x_i),\underline{F}_B(x_i)),(\overline{T}_B(x_i),\overline{I}_B(x_i),\overline{F}_B(x_i))\rangle$ in $X = (x_1, x_2,\ldots, x_n)$ be any rough neutrosophic sets. Pramanik and Mondal [20] defined cosine similarity measure between rough neutrosophic sets A and B as follows:

C_{RNS}(A, B) =

$$\frac{1}{n}\sum_{i=1}^{n}\frac{\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)+\delta F_A(x_i)\delta F_B(x_i)}{\sqrt{\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]}} \quad (15)$$

Here, $\delta T_A(x_i)=\frac{\underline{T}_A(x_i)+\overline{T}_A(x_i)}{2}$, $\delta T_B(x_i)=\frac{\underline{T}_B(x_i)+\overline{T}_B(x_i)}{2}$, $\delta I_A(x_i)=\frac{\underline{I}_A(x_i)+\overline{I}_A(x_i)}{2}$, $\delta I_B(x_i)=\frac{\underline{I}_B(x_i)+\overline{I}_B(x_i)}{2}$, $\delta F_A(x_i)=\frac{\underline{F}_A(x_i)+\overline{F}_A(x_i)}{2}$, $\delta F_B(x_i)=\frac{\underline{F}_B(x_i)+\overline{F}_B(x_i)}{2}$

Proposition 19 [20]

Let A and B be rough neutrosophic sets. Cosine similarity measure [20] between A and B satisfies the following properties.

1. $0\le C_{RNS}(A,B)\le 1$;
2. $C_{RNS}(A,B)=C_{RNS}(B,A)$;
3. $C_{RNS}(A,B)=1$; iff $A = B$
4. If C is a RNS in Y and $A\subset B\subset C$ then, $C_{RNS}(A, C) \le C_{RNS}(A, B)$, and $C_{RNS}(A, C) \le C_{RNS}(B, C)$.

Definition 5.3.1 [20]

If we consider the weights of each element x_i, a weighted rough cosine similarity measure between rough neutrosophic sets A and B can be defined as follows:

$$C_{WRNS}(A,B)=\sum_{i=1}^{n}w_i\frac{\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)+\delta F_A(x_i)\delta F_B(x_i)}{\sqrt{\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]}} \quad (16)$$

$w_i\in[0,1]$, i = 1, 2,…, n and $\sum_{i=1}^{n}w_i=1$. If we take $w_i=\frac{1}{n}$, i = 1, 2,…, n, then $C_{WRNS}(A, B) = C_{RNS}(A, B)$

Proposition 20 [20]

The weighted rough cosine similarity measure [20] between two rough neutrosophic sets A and B also satisfies the following properties:

1. $0\le C_{WRNS}(A,B)\le 1$;
2. $C_{WRNS}(A,B)=C_{WRNS}(B,A)$;
3. $C_{WRNS}(A,B)=1$; iff $A = B$
4. If C is a WRNS in Y and $A\subset B\subset C$ then, $C_{WRNS}(A, C) \le C_{WRNS}(A, B)$, and $C_{WRNS}(A, C) \le C_{WRNS}(B, C)$.

For proofs of the above mentioned four properties, see [20].

Definition 5.4 [19] **Cotangent similarity measures of rough neutrosophic sets**

Assume that $A=\langle(\underline{T}_A(x_i),\underline{I}_A(x_i),\underline{F}_A(x_i)),(\overline{T}_A(x_i),\overline{I}_A(x_i),\overline{F}_A(x_i))\rangle$ and $B=\langle(\underline{T}_B(x_i),\underline{I}_B(x_i),\underline{F}_B(x_i)),(\overline{T}_B(x_i),\overline{I}_B(x_i),\overline{F}_B(x_i))\rangle$ in $X = (x_1, x_2,\ldots, x_n)$ be any two rough neutrosophic sets. Pramanik and Mondal [19] defined cotangent similarity measure between rough neutrosophic sets A and B as follows:

$COT_{RNS}(A, B)$ =

$$\frac{1}{n}\sum_{i=1}^{n}\left\langle\cot\left(\frac{\pi}{12}\left(\begin{array}{l}3+|\delta T_A(x_i)-\delta T_B(x_i)|\\+|\delta I_A(x_i)-\delta I_B(x_i)|\\+|\delta I_A(x_i)-\delta I_B(x_i)|\end{array}\right)\right)\right\rangle \quad (17)$$

Here, $\delta T_A(x_i)=\frac{\underline{T}_A(x_i)+\overline{T}_A(x_i)}{2}$, $\delta T_B(x_i)=\frac{\underline{T}_B(x_i)+\overline{T}_B(x_i)}{2}$, $\delta I_A(x_i)=\frac{\underline{I}_A(x_i)+\overline{I}_A(x_i)}{2}$, $\delta I_B(x_i)=\frac{\underline{I}_B(x_i)+\overline{I}_B(x_i)}{2}$, $\delta F_A(x_i)=\frac{\underline{F}_A(x_i)+\overline{F}_A(x_i)}{2}$, $\delta F_B(x_i)=\frac{\underline{F}_B(x_i)+\overline{F}_B(x_i)}{2}$

Proposition 21 [19]

Cotangent similarity measure satisfies the following properties:

1. $0\le COT_{RNS}(A,B)\le 1$;
2. $COT_{RNS}(A,B)=COT_{RNS}(B,A)$;
3. $COT_{RNS}(A,B)=1$; iff $A = B$
4. If C is a RNS in Y and $A\subset B\subset C$ then, $COT_{RNS}(A, C) \le COT_{RNS}(A, B)$, and $COT_{RNS}(A, C) \le COT_{RNS}(B, C)$.

Definition 5.4.1

If we consider the weights of each element x_i, a weighted rough cotangent similarity measure [19] between rough neutrosophic sets A and B can be defined as follows:

$COT_{WRNS}(A, B)$ =

$$\sum_{i=1}^{n}w_i\left\langle\cot\left(\frac{\pi}{12}\left(\begin{array}{l}3+|\delta T_A(x_i)-\delta T_B(x_i)|\\+|\delta I_A(x_i)-\delta I_B(x_i)|\\+|\delta I_A(x_i)-\delta I_B(x_i)|\end{array}\right)\right)\right\rangle \quad (18)$$

$w_i\in[0,1]$, i = 1, 2,…, n and $\sum_{i=1}^{n}w_i=1$. If we take $w_i=\frac{1}{n}$, i = 1, 2,…, n, then $COT_{WRNS}(A, B) = COT_{RNS}(A, B)$

Proposition 22 [19]

The weighted rough cosine similarity measure between two rough neutrosophic sets A and B also satisfies the following properties:

1. $0\le COT_{WRNS}(A,B)\le 1$;
2. $COT_{WRNS}(A,B)=COT_{WRNS}(B,A)$;
3. $COT_{WRNS}(A,B)=1$; iff $A = B$
4. If C is a WRNS in Y and $A\subset B\subset C$ then, $COT_{WRNS}(A, C) \le COT_{WRNS}(A, B)$, and $COT_{WRNS}(A, C) \le COT_{WRNS}(B, C)$

Definition 5.5 (Variational co-efficient similarity measure between rough neutrosophic sets)

Let $A=\left\langle\left(\underline{T}_A(x_i),\underline{I}_A(x_i),\underline{F}_A(x_i)\right),\left(\overline{T}_A(x_i),\overline{I}_A(x_i),\overline{F}_A(x_i)\right)\right\rangle$ and $B=\left\langle\left(\underline{T}_B(x_i),\underline{I}_B(x_i),\underline{F}_B(x_i)\right),\left(\overline{T}_B(x_i),\overline{I}_B(x_i),\overline{F}_B(x_i)\right)\right\rangle$ be two rough neutrosophic sets. Variational co-efficient similarity measure between rough neutrosophic sets can be presented as follows:

$Var_{RNS}(A, B) =$

$$\frac{1}{n}\left[\begin{array}{l}\lambda\sum_{i=1}^{n}\dfrac{2\left\{\begin{array}{l}\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)\\+\delta F_A(x_i)\delta F_B(x_i)\end{array}\right\}}{\left\{\begin{array}{l}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\+\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\end{array}\right\}}\\+(1-\lambda)\sum_{i=1}^{n}\dfrac{\begin{array}{l}\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)\\+\delta F_A(x_i)\delta F_B(x_i)\end{array}}{\sqrt{\begin{array}{l}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\end{array}}}\end{array}\right] \quad (19)$$

Here, $\delta T_A(x_i)=\frac{\underline{T}_A(x_i)+\overline{T}_A(x_i)}{2}$, $\delta T_B(x_i)=\frac{\underline{T}_B(x_i)+\overline{T}_B(x_i)}{2}$,

$\delta I_A(x_i)=\frac{\underline{I}_A(x_i)+\overline{I}_A(x_i)}{2}$, $\delta I_B(x_i)=\frac{\underline{I}_B(x_i)+\overline{I}_B(x_i)}{2}$,

$\delta F_A(x_i)=\frac{\underline{F}_A(x_i)+\overline{F}_A(x_i)}{2}$, $\delta F_B(x_i)=\frac{\underline{F}_B(x_i)+\overline{F}_B(x_i)}{2}$

Proposition 23

The variational co-efficient similarity measure $Var_{RNS}(A, B)$ between two rough neutrosophic sets A and B, satisfies the following properties:

1. $0\le Var_{RNS}(A,B)\le 1$;
2. $Var_{RNS}(A,B)=Var_{RNS}(B,A)$;
3. $Var_{RNS}(A,B)=1$; if $A = B$ i.e.,

$\delta T_A(x_i)=\delta T_B(x_i)$, $\delta I_A(x_i)=\delta I_B(x_i)$, and $\delta F_A(x_i)=\delta F_B(x_i)$, for every x_i (i = 1, 2, …, n) in X.

Proof.

(1.) It is obvious that $Var_{RNS}(A,B)\ge 0$. Thus it is required to prove that $Var_{RNS}(A,B)\le 1$.

From rough neutrosophic dice similarity measure it can be witten that

$$0\le\frac{1}{n}\sum_{i=1}^{n}\frac{2\left[\begin{array}{l}\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)\\+\delta F_A(x_i)\delta F_B(x_i)\end{array}\right]}{\left\langle\begin{array}{l}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\+\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\end{array}\right\rangle}\le 1 \quad (20)$$

and from rough neutrosophic cosine similarity measure it can be written that

$$0\le\frac{1}{n}\sum_{i=1}^{n}\frac{\begin{array}{l}\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)\\+\delta F_A(x_i)\delta F_B(x_i)\end{array}}{\sqrt{\begin{array}{l}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\end{array}}}\le 1 \quad (21)$$

Combining Eq.(20) and Eq.(21) , we obtain

$Var_{RNS}(A, B) =$

$$\frac{1}{n}\left[\begin{array}{l}\lambda\sum_{i=1}^{n}\dfrac{2\left\{\begin{array}{l}\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)\\+\delta F_A(x_i)\delta F_B(x_i)\end{array}\right\}}{\left\{\begin{array}{l}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\+\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\end{array}\right\}}\\+(1-\lambda)\sum_{i=1}^{n}\dfrac{\begin{array}{l}\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)\\+\delta F_A(x_i)\delta F_B(x_i)\end{array}}{\sqrt{\begin{array}{l}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\.\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\end{array}}}\end{array}\right] \quad (22)$$

$\le\lambda+(1-\lambda)=1$

Thus, $0\le Var_{RNS}(A,B)\le 1$;

(2.) $Var_{RNS}(A, B) =$

$$\frac{1}{n}\left[\begin{array}{l}\lambda\sum_{i=1}^{n}\dfrac{2\left\{\begin{array}{l}\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)\\+\delta F_A(x_i)\delta F_B(x_i)\end{array}\right\}}{\left\{\begin{array}{l}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\+\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\end{array}\right\}}\\+(1-\lambda)\sum_{i=1}^{n}\dfrac{\begin{array}{l}\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)\\+\delta F_A(x_i)\delta F_B(x_i)\end{array}}{\sqrt{\begin{array}{l}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\end{array}}}\end{array}\right]$$

$$=\frac{1}{n}\left[\begin{array}{l}\lambda\sum_{i=1}^{n}\dfrac{2\left\{\begin{array}{l}\delta T_B(x_i)\delta T_A(x_i)+\delta I_B(x_i)\delta I_A(x_i)\\+\delta F_B(x_i)\delta F_A(x_i)\end{array}\right\}}{\left\{\begin{array}{l}\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\\+\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\end{array}\right\}}\\+(1-\lambda)\sum_{i=1}^{n}\dfrac{\begin{array}{l}\delta T_B(x_i)\delta T_A(x_i)+\delta I_B(x_i)\delta I_A(x_i)\\+\delta F_B(x_i)\delta F_A(x_i)\end{array}}{\sqrt{\begin{array}{l}\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\\\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\end{array}}}\end{array}\right]$$

$=Var_{RNS}(B,A)$

(3.) If $A = B$ i.e., $\delta T_A(x_i)=\delta T_B(x_i)$, $\delta I_A(x_i)=\delta I_B(x_i)$, and $\delta F_A(x_i)=\delta F_B(x_i)$, for every x_i (i = 1, 2, …, n) in X,

$Var_{RNS}(A, A) =$

$$\frac{1}{n}\left[\begin{array}{l}\lambda\sum_{i=1}^{n}\frac{2\left\{\begin{array}{l}\delta T_A(x_i)\delta T_A(x_i)+\delta I_A(x_i)\delta I_A(x_i)\\+\delta F_A(x_i)\delta F_A(x_i)\end{array}\right\}}{\left\{\begin{array}{l}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\+\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\end{array}\right\}}\\+(1-\lambda)\sum_{i=1}^{n}\frac{\delta T_A(x_i)\delta T_A(x_i)+\delta I_A(x_i)\delta I_A(x_i)+\delta F_A(x_i)\delta F_A(x_i)}{\sqrt{\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]}}\end{array}\right]$$

$$=\frac{1}{n}\left[n\lambda+n(1-\lambda)\right]=1$$

These results show the completion of the proofs of the three properties.

Definition 5.6 (Weighted variational co-efficient similarity measure between rough neutrosophic sets)

Let $A=\left\langle\left(\underline{T}_A(x_i),\underline{I}_A(x_i),\underline{F}_A(x_i)\right),\left(\overline{T}_A(x_i),\overline{I}_A(x_i),\overline{F}_A(x_i)\right)\right\rangle$ and $B=\left\langle\left(\underline{T}_B(x_i),\underline{I}_B(x_i),\underline{F}_B(x_i)\right),\left(\overline{T}_B(x_i),\overline{I}_B(x_i),\overline{F}_B(x_i)\right)\right\rangle$ be any two rough neutrosophic sets. Rough variational co-efficient similarity measure between rough neutrosophic setsA and B in 3-D vector space can be presented as follows:

$Var_{WRNS}(A, B)$ =

$$\left[\begin{array}{l}\lambda\sum_{i=1}^{n}w_i\frac{2\left\{\begin{array}{l}\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)\\+\delta F_A(x_i)\delta F_B(x_i)\end{array}\right\}}{\left\{\begin{array}{l}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\+\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\end{array}\right\}}\\+(1-\lambda)\sum_{i=1}^{n}w_i\frac{\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)+\delta F_A(x_i)\delta F_B(x_i)}{\sqrt{\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right].\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]}}\end{array}\right] \quad (23)$$

If $w=\left[\frac{1}{n},\frac{1}{n},...,\frac{1}{n}\right]^T$, then Eq.(23) is reduced to Eq.(19).

Proposition 24

The weighted variational co-efficient similarity measure also satisfies the following properties:

1. $0\le Var_{WRNS}(A,B)\le 1$;
2. $Var_{WRNS}(A,B)=Var_{WRNS}(B,A)$;
3. $Var_{WRNS}(A, B) = 1$; if $A = B$ i.e., $\delta T_A(x_i)=\delta T_B(x_i)$, $\delta I_A(x_i)=\delta I_B(x_i)$, and $\delta F_A(x_i)=\delta F_B(x_i)$, for every x_i (i = 1, 2, …, n) in X.

Proof:

(1.) It is obvious that $Var_{WRNS}(A,B)\ge 0$. Thus it is required to prove that $Var_{WRNS}(A,B)\le 1$.

From rough neutrosophic weighted dice similarity measure, it can be written that

$$0\le\frac{1}{n}\sum_{i=1}^{n}w_i\frac{2\left[\begin{array}{l}\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)\\+\delta F_A(x_i)\delta F_B(x_i)\end{array}\right]}{\left\langle\begin{array}{l}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\+\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\end{array}\right\rangle}\le 1 \quad (24)$$

and from rough neutrosophic weighted cosine similarity measure it can be written that

$$0\le\frac{1}{n}\sum_{i=1}^{n}w_i\frac{\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)+\delta F_A(x_i)\delta F_B(x_i)}{\sqrt{\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]}}\le 1 \quad (25)$$

Combining Eq.(24) and Eq.(25), we obtain

Var_{WRNS}(A, B) =

$$\left[\begin{array}{l}\lambda\sum_{i=1}^{n}w_i\frac{2\left\{\begin{array}{l}\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)\\+\delta F_A(x_i)\delta F_B(x_i)\end{array}\right\}}{\left\{\begin{array}{l}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\+\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\end{array}\right\}}\\+(1-\lambda)\sum_{i=1}^{n}w_i\frac{\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)+\delta F_A(x_i)\delta F_B(x_i)}{\sqrt{\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]}}\end{array}\right] \quad (26)$$

$\le\lambda+(1-\lambda)=1$

Thus, $0\le Var_{WRNS}(A,B)\le 1$;

(2.) $Var_{WRNS}(A, B)$ =

$$\left[\begin{array}{l}\lambda\sum_{i=1}^{n}w_i\frac{2\left\{\begin{array}{l}\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)\\+\delta F_A(x_i)\delta F_B(x_i)\end{array}\right\}}{\left\{\begin{array}{l}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\+\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\end{array}\right\}}\\+(1-\lambda)\sum_{i=1}^{n}w_i\frac{\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)+\delta F_A(x_i)\delta F_B(x_i)}{\sqrt{\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]}}\end{array}\right]$$

$$=\left[\begin{array}{l}\lambda\sum_{i=1}^{n}w_i\frac{2\left\{\begin{array}{l}\delta T_B(x_i)\delta T_A(x_i)+\delta I_B(x_i)\delta I_A(x_i)\\+\delta F_B(x_i)\delta F_A(x_i)\end{array}\right\}}{\left\{\begin{array}{l}\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\\+\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\end{array}\right\}}\\+(1-\lambda)\sum_{i=1}^{n}w_i\frac{\delta T_B(x_i)\delta T_A(x_i)+\delta I_B(x_i)\delta I_A(x_i)+\delta F_B(x_i)\delta F_A(x_i)}{\sqrt{\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]}}\end{array}\right]$$

$=Var_{WRNS}(B, A)$

(3.) If $A = B$ i.e.,

$\delta T_A(x_i)=\delta T_B(x_i)$, $\delta I_A(x_i)=\delta I_B(x_i)$, and $\delta F_A(x_i)=\delta F_B(x_i)$, for every x_i (i = 1, 2, …, n) in X,

$$Var_{WRNS}(A, A)=\left[\begin{array}{l}\lambda\sum_{i=1}^{n} w_i\dfrac{2\left\{\begin{array}{l}\delta T_A(x_i)\delta T_A(x_i)+\delta I_A(x_i)\delta I_A(x_i)\\+\delta F_A(x_i)\delta F_A(x_i)\end{array}\right\}}{\left\{\begin{array}{l}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\+\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\end{array}\right\}}\\+(1-\lambda)\sum_{i=1}^{n} w_i\dfrac{\begin{array}{l}\delta T_A(x_i)\delta T_A(x_i)+\delta I_A(x_i)\delta I_A(x_i)\\+\delta F_A(x_i)\delta F_A(x_i)\end{array}}{\sqrt{\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]}\sqrt{(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2}}\end{array}\right]$$

$$=\left[\lambda\sum_{i=1}^{n} w_i+(1-\lambda)\sum_{i=1}^{n} w_i\right]=1$$

These results show the completion of the proofs of the three proiperties.

6. Multi attribute decision making based on rough neutrosophic variational coefficient similarity measure

In this section, a rough variational co-efficient similarity measure is employed to multi-attribute decision making in rough neutrosophic environment. Assume that $A=\{A_1, A_2,..., A_m\}$ be the set of alternatives and $C=\{C_1, C_2,..., C_n\}$ be the set of attributes in a multi-attribute decision making problem. Assune that w_j be the weight of the attribute C_j provided by the decision maker such that each $w_i\in[0,1]$ and $\sum_{j=1}^{n} w_j=1$ However, in real situation decision maker may often face difficulty to evaluate alternatives over the attributes due to vague or incomplete information about alternatives in a decision making situation. Rough neutrosophic set can be used in MADM to deal with incomplete information of the alternatives. In this paper, the assessment values of all the alternatives with respect to attributes are considered as the rough neutrosophic values (see Table 1).

Table1: Rough neutrosophic decision matrix

$D_{RNS}=\langle \underline{d}_{ij}, \overline{d}_{ij}\rangle_{m\times n}=$

	C_1	C_2	.	C_n
A_1	$\langle \underline{d}_{11}, \overline{d}_{11}\rangle$	$\langle \underline{d}_{12}, \overline{d}_{12}\rangle$	...	$\langle \underline{d}_{1n}, \overline{d}_{1n}\rangle$
A_2	$\langle \underline{d}_{21}, \overline{d}_{21}\rangle$	$\langle \underline{d}_{22}, \overline{d}_{22}\rangle$	...	$\langle \underline{d}_{2n}, \overline{d}_{2n}\rangle$
.	...	...	...	...
.	...	...	...	...
A_m	$\langle \underline{d}_{m1}, \overline{d}_{m1}\rangle$	$\langle \underline{d}_{m2}, \overline{d}_{m2}\rangle$	...	$\langle \underline{d}_{mn}, \overline{d}_{mn}\rangle$

(27)

Here $\langle \underline{d}_{ij}, \overline{d}_{ij}\rangle$ is the rough neutrosophic number for the i-th alternative and the j-th attribute.

Definition 6.1: Transforming operator for SVNSs [80]

The rough neutrosophic decision matrix (27) can be transformed to single valued neutrosophic decision matrix whose ij-th element α_{ij} can be presented as follows:

$$\alpha_{ij}=\left\langle\frac{\underline{d}_{ij}+\overline{d}_{ij}}{2}\right\rangle_{m\times n}, \text{ for } i=1,2,3,...,m;$$

j = 1, 2, 3, ..., n. . (28)

Step1. Determine the neutrosophic relative positive ideal solution

In multi-criteria decision-making environment, the concept of ideal point has been used to help identify the best alternative in the decision set.

Definition 6.2 [51].

Let H be the collection of two types of attributes, namely, benefit type attribute (P) and cost type attribute (L) in the MADM problems. The relative positive ideal neutrosophic solution (RPINS) $Q_S^+=[\delta_{q_S}^{+},\delta_{q_S}^{+},...,\delta_{q_S}^{+}]$ is the solution of the decision matrix $D_S=\langle \delta T_{ij},\delta I_{ij},\delta F_{ij}\rangle_{m\times n}$ where, every component of Q_S^+ has the following form:

for benefit type attribute, every component of Q_S^+ has the following form:

$$q_S^+=\langle \delta T_j^+,\delta I_j^+,\delta F_j^+\rangle =\left\langle \max_i\{\delta T_{ij}\},\min_i\{\delta I_{ij}\},\min_i\{\delta F_{ij}\}\right\rangle \text{ for } j\in P \quad (29)$$

and for cost type attribute, every component of Q_S^+ has the following form

$$q_S^+=\langle \delta T_j^+,\delta I_j^+,\delta F_j^+\rangle =\left\langle \min_i\{\delta T_{ij}\},\max_i\{\delta I_{ij}\},\max_i\{\delta F_{ij}\}\right\rangle \text{ for } j\in L \quad (30)$$

Step 2. Determine the weighted variational co-efficient similarity measure between ideal alternative and each alternative.

The variational co-efficient similarity measure between ideal alternative Q_S^+ and each alternative A_i for i = 1, 2, …, m can be determined by the following equation as follows:

$$Var_{WRNS}(Q_S^+, D_S)=\left[\begin{array}{l}\lambda\sum_{i=1}^{n} w_i\dfrac{2\left\{\delta T_j^+.\delta T_{ij}+\delta I_j^+.\delta I_{ij}+\delta F_j^+.\delta F_{ij}\right\}}{\left\{\begin{array}{l}\left[(\delta T_j^+)^2+(\delta I_j^+)^2+(\delta F_j^+)^2\right]+\\\left[(\delta T_{ij})^2+(\delta I_{ij})^2+(\delta F_{ij})^2\right]\end{array}\right\}}\\+(1-\lambda)\sum_{i=1}^{n} w_i\dfrac{\left\{\delta T_j^+.\delta T_{ij}+\delta I_j^+.\delta I_{ij}+\delta F_j^+.\delta F_{ij}\right\}}{\sqrt{\left[(\delta T_j^+)^2+(\delta I_j^+)^2+(\delta F_j^+)^2\right]}\sqrt{(\delta T_{ij})^2+(\delta I_{ij})^2+(\delta F_{ij})^2}}\end{array}\right] \quad (31)$$

Step3. Rank the alternatives.

According to the values obtained from Eq.(31), the ranking order of all the alternatives can be easily determined. Highest value indicates the best alternative.

Step 4. End.

7 Numerical example

In this section, rough neutrosophic MADM regarding investment problem is considered to demonstrate the applicability and the effectiveness of the proposed approach. However, investment problem is not easy to solve. It not only requires oodles of patience and discipline, but also a great deal of research and a sound understanding of the market, mathematical tools, among others. Suppose an investment company wants to invest a sum of money in the best option. Assume that there are four possible alternatives to invest the money: (1) A_1 is a computer company; (2) A_2 is a garment company; (3) A_3 is a telecommunication company; and (4) A_4 is a food company. The investment company must take a decision based on the following three criteria: (1) C_1 is the growth factor; (2) C_2 is the environmental impact; and (3) C_3 is the risk factor. The four possible alternatives are to be evaluated under the attribute by the rough neutrosophic assessments provided by the decision maker. These assessment values are given in the rough neutrosophic decision matrix (see the table 2).

Table2. Rough neutrosophic decision matrix

$$D=\left\langle \underline{N}_{ij}(P),\overline{N}_{ij}(P)\right\rangle_{4\times 3}=$$

$$\begin{array}{c|ccc} & C_1 & C_2 & C_3 \\ \hline A_1 & \left\langle\begin{array}{c}(0.1,0.2,0.2),\\(0.3,0.2,0.2)\end{array}\right\rangle & \left\langle\begin{array}{c}(0.6,0.4,0.3),\\(0.8,0.2,0.3)\end{array}\right\rangle & \left\langle\begin{array}{c}(0.3,0.2,0.3),\\(0.5,0.2,0.1)\end{array}\right\rangle \\ A_2 & \left\langle\begin{array}{c}(0.2,0.4,0.3),\\(0.4,0.2,0.3)\end{array}\right\rangle & \left\langle\begin{array}{c}(0.6,0.3,0.3),\\(0.8,0.1,0.1)\end{array}\right\rangle & \left\langle\begin{array}{c}(0.1,0.4,0.3),\\(0.3,0.2,0.3)\end{array}\right\rangle \\ A_3 & \left\langle\begin{array}{c}(0.3,0.2,0.3),\\(0.5,0.2,0.1)\end{array}\right\rangle & \left\langle\begin{array}{c}(0.5,0.2,0.3),\\(0.7,0.2,0.1)\end{array}\right\rangle & \left\langle\begin{array}{c}(0.0,0.2,0.4),\\(0.2,0.2,0.2)\end{array}\right\rangle \\ A_4 & \left\langle\begin{array}{c}(0.0,0.4,0.4),\\(0.2,0.2,0.2)\end{array}\right\rangle & \left\langle\begin{array}{c}(0.5,0.4,0.4),\\(0.7,0.2,0.2)\end{array}\right\rangle & \left\langle\begin{array}{c}(0.2,0.3,0.3),\\(0.4,0.1,0.1)\end{array}\right\rangle \end{array} \quad (32)$$

The known weight information is given as follows: $W = [w_1, w_2, w_3]^T = [0.3, 0.3, 0.4]$ and $\sum_{i=1}^{3} w_i = 1$.

Step1. Determine the types of criteria.

First two types i.e. C_1 and C_2 of the given criteria are benefit type criteria and the last one criterion i.e. C_3 is the cost type criteria.

Step2. Determine the relative neutrosophic positive ideal solution

Using Eq. (29), Eq.(30), the relative positive ideal neutrosophic solution for the given matrix defined in Eq.(32) can be obtained as:

$Q_S^+=[(0.4,0.2,0.2),(0.7,0.2,0.2),(0.1,0.3,0.3)]$

Step3. Determine the weighted variational similarity measure

The weighted variational co-efficient similarity measure is determined by using Eq.(28), Eq.(31) and Eq.(32). The results obtained for different values of have been shown in the Table-3.

Table-3. Results of rough variational similarity measure for different values of , $0 \le \lambda \le 1$

Similarity measure method	Values of s	Measure values	Ranking order
$Var_{WRNS}(Q_S^+, D_S)$	0.10	0.8769; 0.9741; 0.9917; 0.8107	$A_3 > A_2 > A_1 > A_4$
	0.25	0.8740, 0.9739 0.9905 0.8078	$A_3 > A_2 > A_1 > A_4$
	0.50	0.8692; 0.9735; 0.9887; 0.8028	$A_3 > A_2 > A_1 > A_4$
	0.75	0.8643; 0.9730; 0.9868; 0.7979	$A_3 > A_2 > A_1 > A_4$
	0.90	0.8614; 0.9728; 0.9857; 0.7949	$A_3 > A_2 > A_1 > A_4$

Step 4. Rank the alternatives.

According to the different values of , the results obtained in Table-3 reflects that A_3 is the best alternative.

8. Comparisons of different rough similarity measure with rough variation similarity measure

In this section, four existing rough similarity measures - namely: rough cosine similarity measure, rough dice similarity measure, rough cotangent similarity measure and rough Jaccard similarity measure - have been compared with proposed rough variational co-efficient similarity measure for different values of λ. The comparison results are listed in the Table 3 and Table 4.

Table-4. Results of existing rough neutrosophic similarity measure methods.

Rough similarity measure methods	Values of s	Measure values	Ranking order
$JAC_{WRNS}(Q_S^+, D_S)$ [21]	...	0.7870, 0.9471; 0.9739; 0.6832	$A_3 > A_2 > A_1 > A_4$
$DIC_{WRNS}(Q_S^+, D_S)$ [21]	...	0.8595; 0.9726; 0.9873; 0.7929	$A_3 > A_2 > A_1 > A_4$
$C_{WRNS}(Q_S^+, D_S)$ [20]	...	0.8788; 0.9738; 0.9920; 0.9132	$A_3 > A_2 > A_4 > A_1$
$COT_{WRNS}(Q_S^+, D_S)$ [19]	...	0.8472; 0.9358; 0.9643; 0.8103	$A_3 > A_2 > A_1 > A_4$

Conclusion

In this paper, we have proposed rough variational coefficient similarity measures. We also proved some of their basic properties. We have presented an application of rough neutrosophic variational coefficient similarity measure for a decision making problem on investment. The concept presented in the paper can be applied to deal with other multi attribute decision making problems in rough neutrosophic environment.

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