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Probabilistic Measures of Coherence and the Problem of Belief Individuation

ABSTRACT. Coherentism in epistemology has long suffered from lack of formal and quantitative explication of the notion of coherence. One might hope that probabilistic accounts of coherence such as those proposed by Lewis, Shogenji, Olsson, Fitelson, and Bovens and Hartmann will finally help solve this problem. This paper shows, however, that those accounts have a serious common problem: the problem of belief individuation. The coherence degree that each of the accounts assigns to an information set (or the verdict it gives as to whether the set is coherent *tout court*) depends on how beliefs (or propositions) that represent the set are individuated. Indeed, logically equivalent belief sets that represent the same information set can be given drastically different degrees of coherence. This feature clashes with our natural and reasonable expectation that the coherence degree of a belief set does not change unless the believer adds essentially new information to the set or drops old information from it; or, to put it simply, that the believer cannot raise or lower the degree of coherence by purely logical reasoning. None of the accounts in question can adequately deal with coherence once logical inferences get into the picture. Toward the end of the paper, another notion of coherence that takes into account not only the contents but also the origins (or sources) of the relevant beliefs is considered. It is argued that this notion of coherence is of dubious significance, and that it does not help solve the problem of belief individuation.

1. Introduction

Coherentism in epistemology has long suffered from lack of formal and quantitative explication of the notion of coherence. Epistemologists often talk about propositions “hanging together” or “cohering with one another,” but they rarely go beyond the use of such metaphoric terms to clarify what exactly it means for a set of beliefs to be coherent (or more coherent than another set of beliefs). Even Laurence BonJour, the author of an influential book-length defence of coherentism (BonJour 1985), admits, as recently as in 2002, that the notion of coherence remains obscure for lack of such an explication:

> But while the foregoing discussion may suffice to give you some initial grasp of the concept of coherence, it is very far from an adequate account, especially one that would provide the basis of *comparative* assessments of the relative degrees of coherence possessed by different and perhaps conflicting systems of beliefs. And it is comparative assessments of coherence that seem to be needed if coherence is to be the sole basis that determines which beliefs are justified or even to play a significant role in such issues. There are somewhat fuller accounts of coherence available in the recent literature, but none that come at all close to achieving this goal. Thus practical assessments of coherence must be made on a rather ill-defined intuitive basis, making the whole idea of a coherentist epistemology more of a promissory note than a fully specified alternative. (BonJour 2002: 204)

However, just about the time when BonJour was writing this passage, several interesting probabilistic measures of coherence began to emerge, such as those proposed by Shogenji (1999), Olsson (2002), Fitelson (2003), and Bovens and Hartmann (2003a, 2003b).[1] These accounts all purport to be a measure of comparison, given in purely probabilistic

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[1] As we shall explain later in Section 7, what Bovens and Hartmann offer is, strictly speaking, not a *measure* but only a *quasi-ordering*, even though they themselves call it a 'measure'. For the sake of simplicity, sometimes we shall also refer to it as a 'measure', but this should be understood loosely.

terms, between coherence degrees of different information sets – just the kind of thing BonJour thought missing from the account of coherence often given. (Indeed, the first three accounts purport to be not just *comparative*, but *absolute*, measures of coherence.) One might thus hope that they (or at least one of them) will finally help solve the problem of giving a formal and quantitative account of coherence.

This paper will show, however, that such a hope cannot be realized. These measures have a serious common problem. To put it very simply, they all take coherence of an information set (or comparative coherence between two information sets) to be determinable on the basis of probabilistic correlations among the *beliefs* (or *propositions*) that represent the set(s). But one and the same information set can be represented by different sets of beliefs; for example, one and the same information set can be represented by the *triplet* of beliefs $\{B_1, B_2, B_3\}$ and by the *pair* of beliefs $\{B_1 \,\&\, B_2, B_3\}$. And probabilistic correlations that determine the degree of coherence vary depending on how the beliefs are individuated. Indeed, it is often straightforward to construct a belief set equivalent (in the precise sense to be defined below) to any given belief set that does not have the same degree of coherence. So there can be different sets of beliefs that represent the same information set but that have different degrees of coherence. This is intuitively a quite unpalatable consequence of any of the above probabilistic measures, which we shall call the *problem of belief individuation*.

This problem becomes more palpable if we take into account the fact that one can move from the belief state represented by one belief set to another belief state represented by another, equivalent belief set by making purely logical inferences (or by conducting purely formal transformations of the propositions). Consequently, according to the above probabilistic measures, one can raise or lower the coherence degree of one's belief set by purely logical reasoning, without adding anything new to the set or dropping anything old from it. This is a violation of the Stability Principle – a constraint, to be formulated shortly, that any epistemologically significant notion of coherence must satisfy. All of the above measures are mainly intended to apply to beliefs obtained non-inferentially, such as those obtained from witness testimonies and direct observations. Since these measures all violate the Stability Principle, they cannot be expanded to deal with coherence of *all* beliefs, and, in particular, beliefs obtained by inferences.

In what follows, the above argument is given in full details. Specifically, in Section 2, the Stability Principle is presented, the notion of *equivalent belief set* is formulated, and the so-called Equi-Coherence Principle is derived from the Stability Principle. Shogenji's and Fitelson's measures, which may be considered to be remote descendants of the well-known non-quantitative account given by C. I. Lewis (1946),[2] not only determine the coherence degree of a belief set, but also give a verdict as to whether the set is coherent *tout court* (depending on whether the degree is higher than a certain number). In Sections 3 to 5, these three accounts are shown to violate the Equi-Coherence Principle (and, consequently, the Stability Principle as well). Then, in Sections 6 and 7, the probabilistic measures proposed by Olsson and by Bovens and Hartmann are examined, and they are also shown to violate the Equi-Coherence Principle. Finally, in Section 8, an argument is given against Bovens' and Hartmann's and Shogenji's view that not only the probabilities of the contents

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[2] Fitelson himself considers his measure to be based on Kemeny's and Oppenheim's (1952) quantitative account.

but also the origins or sources of the relevant beliefs ought to be taken into account for determining the degree of coherence. It is argued that the difference in origin does not account for the large fluctuation in the degree of coherence as a result of logical reasoning.

2. Stability and Equi-Coherence

Let us call the following principle the Stability Principle, or, for short, (Stability):

> (Stability) No belief set changes its degree of coherence unless the believer adds any essentially new information to the set or drops any essentially old information from it.

In this paper we shall deal with the *ideal* believer who does not make logical mistakes and can see clearly the implicit consequences of her explicit beliefs. So adding a logical consequence to a set of beliefs does not count as adding essentially new information. We consider (Stability) as one of the essential principles of coherence, the principles that any epistemologically significant measure of coherence must obey. It implies that one cannot raise or lower the coherence degree of one's belief set by making purely logical inferences, formally transforming the beliefs in the set without obtaining any essentially new information from the outside world or giving up any old information. If (Stability) does not hold, one could gain or lose coherence *cheaply*, by simply manipulating the beliefs one already has. We do not need to deny that such a notion of coherence *might* be of *some* use for certain theoretical purposes.[3] But on such a notion, there would be nothing particularly attractive in obtaining coherence; coherence would have no role to play in one's rational choices. A notion of coherence unable to meet (Stability) can hardly be considered an epistemic virtue.

Again, Lewis' account of coherence and Shogenji's and Fitelson's measures give a verdict, for any belief set, as to whether it is coherent *tout court* (in addition, in the last two cases, to assigning a degree of coherence). If such a qualitative notion of coherence makes sense, the following qualitative version of the Stability Principle also seems reasonable:

(Stability*) No coherent belief set can be turned into a non-coherent[4] set, and no non-coherent belief set can be turned into a coherent set, unless the believer adds any essentially new information to the set or drops any essentially old information from it.

We consider two belief sets, **E** and **E***, *equivalent* if and only if the believer who holds all beliefs $B \in \mathbf{E}$ can obtain every belief $B^* \in \mathbf{E}^*$, and *vice versa*, by simply deducing the propositional content of B^* from the propositional contents of Bs, and, conversely, by deducing the propositional content of B from the propositional contents of B^*s. Thus, if **E** and **E*** are equivalent, the believer can move from the information state represented by **E** to

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[3] Similarly, we do not need to deny that a certain limited notion of coherence that does not apply to the beliefs obtained by inferences *might* be of *some* use for certain theoretical purposes.

[4] On Shogenji's measure, a belief set is neither coherent nor *in*coherent if its coherence degree is 1, and it is incoherent if its coherence degree is lower than 1. We consider any set with coherence degree not higher than 1 on Shogenji's measure to be *non*-coherent (or *not* coherent).

the information state represented by **E*** by first deriving each $B^* \in \mathbf{E}^*$ from **E**, and then dropping every $B \in \mathbf{E}$. Obviously, no essentially new information is added in the process. No old information is lost either, for the **E*** thus constructed can easily be turned back into **E** in the converse process (that is, without adding any new information). Equivalent belief sets, thus, can be considered to represent the same information set. This observation reveals that (Stability) and (Stability*) respectively entail the following quantitative and qualitative versions of the Equi-Coherence Principle:

(Equi-Coherence) If **E** and **E*** are equivalent sets of beliefs, **E** and **E*** have the same degree of coherence.

(Equi-Coherence*) If **E** and **E*** are equivalent sets of beliefs, **E** is coherent if and only if **E*** is coherent.

We shall prove in Section 3 that Lewis' account of coherence is incompatible with (Equi-Coherence*), and, in Sections 4 and 5, that Shogenji's and Fitelson's coherence measures are incompatible with (Equi-Coherence). All these accounts are thus untenable. Then, in Sections 6 and 7, we shall prove that (Equi-Coherence) makes both Olsson's and Bovens' and Hartmann's coherence measures trivial and inconsistent. Also these measures will thus prove untenable.

3. Lewis

C. I. Lewis defined a coherent (or "congruent")[5] set of statements as follows:

> A set of statements, or a set of supposed facts asserted, will be said to be [coherent] if and only if they are so related that the antecedent probability of any one of them will be increased if the remainder of the set can be assumed as given premises. (Lewis 1946: 338)

This claim can be turned into the following precise definition of coherence:

C_L: If $\mathbf{E}$ is a set of beliefs $B_1, ..., B_n$, $\mathbf{E}$ is coherent if and only if, for any $B_i \in \mathbf{E}$, $\Pr(B_i) < \Pr(B_i | \mathbf{E}!\{B_i\})$, where $\mathbf{E}!\{B_i\}$ is the conjunction of all members of $\mathbf{E}$ except B_i.[6]

Lewis' notion of coherence is not quantitative: it does not give degrees of coherence, nor does it tell which of any two belief sets is more coherent than the other. It only determines whether a belief set is coherent *tout court*. Still, from our viewpoint it is worth

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[5] Lewis used the term 'congruence' rather than 'coherence' to distinguish himself from the British idealist advocates of the coherence theory of truth.

[6] In this paper, we assume that the notion of coherence is applicable only to belief sets that have at least two members – that is, it is *not* applicable to *singletons* (or single beliefs). This is not an assumption we would like to commit ourselves to; rather, we are just playing safe here. When Akiba (2000) showed the untenability of Shogenji's measure by using the notion of *self-coherence* (i.e., coherence of singletons), some, such as Shogenji (2001) and Fitelson (2003), called the notion into question. If the notion of self-coherence is admissible – that is, if the notion of coherence is applicable also to singletons – we indeed can simplify our proofs a great deal, taking a cue from Akiba.

proving that Lewis' notion is incompatible with (Equi-Coherence*), because the proof is very similar to the proofs against Shogenji's and Fitelson's measures, to be given in the next two sections. Shogenji's and Fitelson's measures are shown to have inherited the same crucial deficiency from Lewis' notion.

Specifically, Lewis' notion is flawed because the conjunction of C_L and (Equi-Coherence*) entails these statements:

(1) Every belief set **E** such that $0 < \Pr(\mathbf{E}) < 1$[7] is coherent.

(2) No belief set **E** such that $0 < \Pr(\mathbf{E}) < 1$ is coherent.

Obviously, either of (1) and (2) makes the notion of coherence trivial. Furthermore, they are inconsistent with each other. Thus, Lewis' notion of coherence is untenable.

Proof of (1). Consider any belief set $\mathbf{E} = \{B_1, ..., B_n\}$ such that $0 < \Pr(\mathbf{E}) < 1$. For any such **E**, it is always possible to construct the equivalent set $\mathbf{E}^* = \{B_1 \,\&\, ... \,\&\, B_n, B_i\}$ such that $B_i \in \mathbf{E}$ and $\Pr(B_i) < 1$. Since $0 < \Pr(B_1 \,\&\, ... \,\&\, B_n) < 1$, $\Pr(B_i) < 1$, and $B_1 \,\&\, ... \,\&\, B_n$ entails B_i, it follows that $\Pr(B_i \mid B_1 \,\&\, ... \,\&\, B_n) = 1 > \Pr(B_i)$ and $\Pr(B_1 \,\&\, ... \,\&\, B_n \mid B_i) > \Pr(B_1 \,\&\, ... \,\&\, B_n)$. So $\mathbf{E}^*$ is coherent on C_L. Since **E** is equivalent to $\mathbf{E}^*$, given (Equi-Coherence*), **E** is coherent on C_L, too. QED.

Proof of (2). Consider any belief set $\mathbf{E} = \{B_1, ..., B_n\}$ such that $0 < \Pr(\mathbf{E}) < 1$. For any such **E**, it is always possible to construct the equivalent set $\mathbf{E}^* = \{B_1 \,\&\, ... \,\&\, B_n, B \vee {\sim}B\}$.

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[7] Henceforth '**E**' in Pr(**E**) stands for the conjunction of all members of **E**. That $0 < \Pr(\mathbf{E}) < 1$ entails that the conjunction in question is contingent (i.e., neither logically true nor inconsistent).

Pr(B_1 & ... & B_nïB Ú ~B) = Pr(B_1 & ... & B_n). So **E*** is not coherent on C_L. Since **E** is equivalent to **E***, given (Equi-Coherence*), **E** is not coherent on C_L, either. QED. [8]

4. Shogenji

Again, Lewis' notion of coherence is not quantitative. It is wanting in this regard, as many of us think that coherence is a matter of degree. That is, many of us believe that coherence of belief sets should be evaluated on the basis of a *measure* rather than a *qualitative* definition like C_L. Shogenji (1999) and Fitelson (2003) have proposed measures of coherence that take up Lewis' basic insight. We are now in a position to consider those measures.

Shogenji emphasizes that the intuitive idea of coherence entails that coherent beliefs "hang together" (Shogenji 1999: 338). Since coherence comes in degrees, this plausibly means that "the *more* coherent beliefs are, the *more* likely they are true together" (338). Accordingly, Shogenji proposes the following measure of coherence for a belief set {B_1, ..., B_n}:

$$C_S(B_1,\ldots,B_n) = \frac{\Pr(B_1 \,\&\ldots\&\, B_n)}{\Pr(B_1)\times\ldots\times\Pr(B_n)}.$$

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[8] One might find this example objectionable because it involves a tautology. One can however obtain an alternative proof of (2) by replacing the set **E*** containing a tautology with the set **E**** = {(B_1 & … & B_n) Ú ~B_i, B_i} such that B_i Î **E** and Pr(B_i) < 1. We are thus not resorting to the peculiarity of belief sets that contain tautologies; we are using the present example just for the sake of simplicity. Indeed, this is true for any proof we give in this paper that uses a tautology: we can obtain alternative proofs of (2) by using the set **E**** also for Shogenji's and Fitelson's coherence measures, as we shall demonstrate in following notes.

Intuitively, C_S measures the degree to which the beliefs $B_1, ..., B_n$ are more likely to be true together than they would be if they were related neutrally, namely, if the truth of one belief had no consequence on the truth of any other. The set $\{B_1, ..., B_n\}$ is coherent/incoherent *tout court* if and only if the ratio is higher/lower than 1; the set is neither coherent nor incoherent *tout court* if the ratio is equal to 1.

C_S has been variously criticized by Akiba (2000), Olsson (2001, 2002), Fitelson (2003), and Bovens and Hartmann (2003b, Ch. 2). Here we shall not stop to consider those criticisms; instead, we shall just show that, in a way analogous to Lewis' C_L, Shogenji's C_S, in conjunction with (Equi-Coherence), entails both (1) and (2), and is thus untenable. (We shall touch upon some of the criticisms in the next section. Our proofs are closest to Akiba's (2000) refutation. We shall also briefly discuss Shogenji's (2001) reply to Akiba near the end of the paper.)

Proof of (1). Consider any belief set $\mathbf{E} = \{B_1, ..., B_n\}$ such that $0 < \Pr(\mathbf{E}) < 1$. For any such $\mathbf{E}$, it is always possible to construct the equivalent set $\mathbf{E}^* = \{B_1 \,\&\, ... \,\&\, B_n, B_i\}$ such that $B_i \in \mathbf{E}$ and $\Pr(B_i) < 1$. Then the numerator of the ratio of $C_S(\mathbf{E}^*)$ will be $\Pr(B_1 \,\&\, ... \,\&\, B_n \,\&\, B_i) = \Pr(B_1 \,\&\, ... \,\&\, B_n)$, and its denominator will be $\Pr(B_1 \,\&\, ... \,\&\, B_n) \times \Pr(B_i)$. Thus, $C_S(\mathbf{E}^*) = 1/\Pr(B_i) > 1$, as $\Pr(B_i) < 1$. Since $\mathbf{E}$ is equivalent to $\mathbf{E}^*$, given (Equi-Coherence), $C_S(\mathbf{E}) > 1$, too. So $\mathbf{E}$ is coherent on C_S. QED.

Proof of (2). Consider any belief set $\mathbf{E} = \{B_1, ..., B_n\}$ such that $0 < \Pr(\mathbf{E}) < 1$. For any such $\mathbf{E}$, it is always possible to construct the equivalent set $\mathbf{E}^* = \{B_1 \,\&\, ... \,\&\, B_n, B$ Ú $\sim B\}$. Then the numerator of the ratio of $C_S(\mathbf{E}^*)$ will be $\Pr((B_1 \,\&\, ... \,\&\, B_n) \,\&\, (B$ Ú $\sim B)) = \Pr(B_1$

& ... & B_n), and its denominator will be Pr(B_1 & ... & B_n) × Pr(B Ú ~B) = Pr(B_1 & ... & B_n), as Pr(B Ú ~B) = 1. Thus, C_S(**E***) = 1. Since **E** is equivalent to **E***, given (Equi-Coherence), C_S(**E**) = 1, too. So **E** is not coherent on C_S. QED.[9]

5. Fitelson

We now move on to consider Fitelson's measure of coherence. Again, let **E** be a set of n beliefs B_1, ..., B_n. According to Fitelson, an adequate measure C_F of the coherence of **E** should be "a quantitative, probabilistic generalization of the (deductive) logical coherence of **E**" (Fitelson 2003: 194). This means, according to Fitelson, that C_F should satisfy the following intuitive general desiderata:

$$(3)\ C_F(\mathbf{E}) \text{ is } \begin{cases} \text{Maximal (positive, constant)} & \text{if the } B_i \text{ are logically equivalent (and } \mathbf{E} \text{ is satisfiable)} \\ > 0 & \text{if } \mathbf{E} \text{ is positively dependent} \\ 0 & \text{if } \mathbf{E} \text{ is independent} \\ < 0 & \text{if } \mathbf{E} \text{ is negatively dependent} \\ \text{Minimal (positive, constant)} & \text{if all subsets of } \mathbf{E} \text{ are unsatisfiable.} \end{cases}$$

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9 As was alluded to in the previous note, we can give an alternative proof of (2) that involves no tautology, using **E**** = {(B_1 & … & B_n) Ú ~B_i, B_i} such that B_i Î **E** and Pr(B_i) < 1. Since Pr((B_1 & … & B_n) Ú ~B_i|B_i) < Pr((B_1 & … & B_n) Ú ~B_i),

Thus **E**** is not coherent on C_S.

E is positively or negatively dependent if and only if each of its members is positively or negatively supported by all remaining members and their conjunctions. **E** is independent if and only if each of its members is neither positively nor negatively supported by all remaining members and their conjunctions. **E** is coherent if $C_F(\mathbf{E}) > 0$, and not coherent otherwise.

To characterize precisely the support that each member of **E** can receive from the other members, Fitelson defines the two-place function $F(X, Y)$. $F(X, Y)$ gives the degree to which one belief Y supports another belief X relative to a finitely additive, regular Kolmogorov (1956) probability function Pr. Such a function assigns probability 1 only to necessary truths and probability 0 only to necessary falsehoods.

$$F(X,Y) = \begin{cases} \dfrac{\Pr(Y|X) - \Pr(Y|{\sim}X)}{\Pr(Y|X) + \Pr(Y|{\sim}X)} & \text{if } X \text{ is contingent and } Y \text{ is not a necessary falsehood} \\ 1 & \text{if } X \text{ and } Y \text{ are necessary truths} \\ 0 & \text{if } X \text{ is a necessary truth and } Y \text{ is contingent} \\ -1 & \text{if } Y \text{ is a necessary falsehood.} \end{cases}$$

By appealing to F, Fitelson defines the notions of probabilistic dependence and independence of a belief set **E**. Let $\mathbf{P}_i$ be the power set (excluding the null set) of the set **E**! $\{B_i\}$. And for each $x \in \mathbf{P}_i$, let X be the conjunction of all members of x. Then:

$$\mathbf{E} \text{ is } \begin{cases} \text{positively dependent} & \text{iff for all } B_i \in \mathbf{E} \text{ and for all } x \in \mathbf{P}_i,\ F(B_i, X) > 0 \\ \text{independent} & \text{iff for all } B_i \in \mathbf{E} \text{ and for all } x \in \mathbf{P}_i,\ F(B_i, X) = 0 \\ \text{negatively dependent} & \text{iff for all } B_i \in \mathbf{E} \text{ and for all } x \in \mathbf{P}_i,\ F(B_i, X) < 0. \end{cases}$$

To define C_F, Fitelson introduces the set **S**, where $\mathbf{S} = \boxed{?}\{\{F(B_i, X)ïx \in \mathbf{P}_i\}ïB_i \in \mathbf{E}\}$. For instance, if $\mathbf{E} = \{B_1, B_2\}$, then $\mathbf{S} = \{F(B_1, B_2), F(B_2, B_1)\}$. If $\mathbf{E} = \{B_1, B_2, B_3\}$, then $\mathbf{S} = \{F(B_1, B_2), F(B_1, B_3), F(B_1, B_2 \& B_3), F(B_2, B_1), F(B_2, B_3), F(B_2, B_1 \& B_3), F(B_3, B_1), F(B_3, B_2), F(B_3, B_1 \& B_2)\}$. Finally, the measure C_F is defined as follows:

$$C_F(\mathbf{E}) = \text{mean}(\mathbf{S}).$$

That is, C_F is the straight average of **S**.[10] It is easy to see that C_F satisfies (3).

Fitelson's measure of coherence C_F is apparently superior to Shogenji's C_S (even setting aside the fatal flaw of C_S discussed in the last section). For instance, while C_S makes the coherence degree of a belief set **E** depend on the probabilistic correlations existing among each $B_i \in \mathbf{E}$ and $\mathbf{E}!\{B_i\}$, C_F makes it depend, more exhaustively, on the probabilistic correlations among each $B_i \in \mathbf{E}$ and *any* subset of $\mathbf{E}!\{B_i\}$ (except the null set). Furthermore, as Akiba (2000) points out (and as we have seen in the proof of (1)), when B_1 entails B_2, C_S makes the coherence degree of $\{B_1, B_2\}$ depend, implausibly, only on B_2's prior probability. In contrast, C_F seems to make such a coherence degree depend on more plausible probabilistic correlations between B_1 and B_2 (whenever B_1 and B_2 are contingent).

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[10] Actually, C_F seems formally defective. It makes little sense to calculate the average of the *set* of values **S**. For, if **S** contains identical values, they will count as eliminable reiterations of one and the same value. C_F should rather be defined as the average of a *sequence* of *F*-values, $\mathbf{S} = \langle F(B_i, X)\rangle_{x\in\mathbf{P}i,\, Bi\in\mathbf{E}}$. This formal amendment would leave our objections to C_F unaffected.

However, Fitelson's measure of coherence suffers from essentially the same problem as Shogenji's,[11] for the conjunction of C_F and (Equi-Coherence) entails both (1) and (2). That is, all sets of beliefs are coherent and non-coherent on C_F. This is an absurd consequence.

Proof of (1). Consider any belief set $\mathbf{E} = \{B_1, ..., B_n\}$ such that $0 < \Pr(\mathbf{E}) < 1$. For any such $\mathbf{E}$, it is always possible to construct the equivalent set $\mathbf{E}^* = \{B_1 \,\&\, ... \,\&\, B_n, B_i\}$ such that $B_i \in \mathbf{E}$ and $\Pr(B_i) < 1$. If the set $\mathbf{S}$ defined as above is built out of $\mathbf{E}^*$, $\mathbf{S} = \{F(B_i, B_1 \,\&\, ... \,\&\, B_n), F(B_1 \,\&\, ... \,\&\, B_n, B_i)\}$.

(i) Let us first calculate the value of $F(B_i, B_1 \,\&\, ... \,\&\, B_n)$. Since B_i is contingent and $B_1 \,\&\, ... \,\&\, B_n$ is not a necessary falsehood, it follows from F's definition that:

$$F(B_i, B_1 \,\&\, ... \,\&\, B_n) = \frac{\Pr(B_1 \,\&\, ... \,\&\, B_n | B_i) - \Pr(B_1 \,\&\, ... \,\&\, B_n | {\sim}B_i)}{\Pr(B_1 \,\&\, ... \,\&\, B_n | B_i) + \Pr(B_1 \,\&\, ... \,\&\, B_n | {\sim}B_i)}.$$

Since $\Pr(B_1 \,\&\, ... \,\&\, B_n) > 0$, and $B_1 \,\&\, ... \,\&\, B_n$ entails B_i, it follows that $\Pr(B_1 \,\&\, ... \,\&\, B_n | B_i) > 0$ and $\Pr(B_1 \,\&\, ... \,\&\, B_n | {\sim}B_i) = 0$. Consequently, the denominator of this ratio is always greater than 0. The numerator is also greater than 0, as it is always the case that $\Pr(B_1 \,\&\, ... \,\&\, B_n | B_i) > \Pr(B_1 \,\&\, ... \,\&\, B_n | {\sim}B_i)$. Thus, the whole ratio is greater than 0; that is, $F(B_i, B_1 \,\&\, ... \,\&\, B_n) > 0$.

[11] Bovens and Hartmann (2003b, Ch. 2) and Siebel (2004) also offer apparent counterexamples to Fitelson's measure.

(ii) Let us now calculate the value of $F(B_1 \,\&\, ... \,\&\, B_n, B_i)$. Since $B_1 \,\&\, ... \,\&\, B_n$ is contingent and B_i is not a necessary falsehood, it follows from *F's* definition that:

$$F(B_1 \,\&\, ... \,\&\, B_n, B_i) = \frac{\Pr(B_i | B_1 \,\&\, ... \,\&\, B_n) - \Pr(B_i | {\sim}(B_1 \,\&\, ... \,\&\, B_n))}{\Pr(B_i | B_1 \,\&\, ... \,\&\, B_n) + \Pr(B_i | {\sim}(B_1 \,\&\, ... \,\&\, B_n))}.$$

Again, the denominator of this ratio is always greater than 0. Since $\Pr(B_i | B_1 \,\&\, ... \,\&\, B_n) = 1$, the least value this ratio can have is 0 – this happens if $\Pr(B_n | {\sim}(B_1 \,\&\, ... \,\&\, B_n)) = 1$.

To summarize (i) and (ii), $F(B_i, B_1 \,\&\, ... \,\&\, B_n) > 0$ and $F(B_1 \,\&\, ... \,\&\, B_n, B_i) \geq 0$. So $C_F(\mathbf{E}^*) = \text{mean}(\mathbf{S}) > 0$. Since **E** is equivalent to **E***, given (Equi-Coherence), $C_F(\mathbf{E}) > 0$, too. Thus **E** is coherent on C_F. QED.

Proof of (2). Consider any belief set $\mathbf{E} = \{B_1, ..., B_n\}$ such that $0 < \Pr(\mathbf{E}) < 1$. For any such **E**, it is always possible to construct the equivalent set $\mathbf{E}^* = \{B_1 \,\&\, ... \,\&\, B_n, B \vee {\sim}B\}$. If the set **S** defined as above is built out of **E***, $\mathbf{S} = \{F(B_1 \,\&\, ... \,\&\, B_n, B \vee {\sim}B), F(B \vee {\sim}B, B_1 \,\&\, ... \,\&\, B_n)\}$.

(i) Let us first calculate the value of $F(B_1 \,\&\, ... \,\&\, B_n, B \vee {\sim}B)$. Since $B_1 \,\&\, ... \,\&\, B_n$ is contingent and $B \vee {\sim}B$ is not a necessary falsehood, it follows from *F*'s definition that:

$$F(B_1 \,\&\, ... \,\&\, B_n, B \vee {\sim}B) = \frac{\Pr(B \vee {\sim}B | B_1 \,\&\, ... \,\&\, B_n) - \Pr(B \vee {\sim}B | {\sim}(B_1 \,\&\, ... \,\&\, B_n))}{\Pr(B \vee {\sim}B | B_1 \,\&\, ... \,\&\, B_n) + \Pr(B \vee {\sim}B | {\sim}(B_1 \,\&\, ... \,\&\, B_n))}.$$

Since Pr(B_1 & ... & B_n) > 0, the denominator of this ratio is always greater than 0, while its numerator is always equal to 0. Therefore, *F*(B_1 & ... & B_n, *B* Ú ~*B*) = 0.

(ii) Let us now calculate the value of *F*(*B* Ú ~*B*, B_1 & ... & B_n). Since *B* Ú ~*B* is a necessary truth and B_1 & ... & B_n is contingent, it follows from *F*'s definition that *F*(*B* Ú ~*B*, B_1 & ... & B_n) = 0.

To summarize (i) and (ii), *F*(B_1 & ... & B_n, *B* Ú ~*B*) = 0 and *F*(*B* Ú ~*B*, B_1 & ... & B_n) = 0. So C_F(**E***) = mean(**S**) = 0. Since **E** is equivalent to **E***, given (Equi-Coherence), C_F(**E**) = 0, too. Thus **E** is not coherent on C_F. QED.[12] (Indeed, one can derive a stronger claim that every belief set **E** is independent.)[13]

6. Olsson

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[12] Like in Shogenji's case, we can give an alternative proof of (2) that involves no tautology, using **E**** = {(B_1 & … & B_n) Ú ~B_i, B_i} such that B_i Î **E** and Pr(B_i) < 1. Since Pr((B_1 & … & B_n) Ú ~B_i|B_i) < Pr((B_1 & … & B_n) Ú ~B_i), **E**** is negatively dependent. Consequently, C_F (**E****) < 0. Thus **E**** is not coherent on C_F.

[13] Fitelson has recently modified the support function *F* defined above (see fitelson.org/coherence2.pdf). The new function, however, does not affect our conclusion. Given the new function, we still can prove (1) using the set **E*** = {B_1 & ... & B_n, B_i} such that $B_i \in$ **E** and Pr(B_i) < 1. We also can prove (2) as follows: Consider any belief set **E** = {B_1, ..., B_n} such that 0 < Pr(**E**) < 1, and take any statement *B* such that Pr((B_1 & ... & B_n) Ú *B*) < 1 and Pr((B_1 & ... & B_n) Ú ~*B*) < 1. The set **E*** = {(B_1 & ... & B_n) Ú *B*, (B_1 & ... & B_n) Ú ~*B*} is equivalent to **E**. It is routine to prove that **E*** – thus **E** as well – is not coherent on C_F. Notice that Pr((B_1 & ... & B_n) Ú *B*) and Pr((B_1 & ... & B_n) Ú ~*B*), thus defined, are negatively dependent. We can then use them to produce alternative proofs of (2) for C_L and C_S that involve no tautology.

We cannot show the inconsistency of Olsson's or Bovens' and Hartmann's account similarly by constructing the equivalent sets $\{B_1 \,\&\, ... \,\&\, B_n, B_i\}$ and $\{B_1 \,\&\, ... \,\&\, B_n, B \vee {\sim}B\}$. For one thing, neither measure determines whether a belief set is coherent *tout court*; so neither (1) nor (2), which appeals to that qualitative notion, can be used for their evaluation. Yet other proofs may be given that show that both Olsson's and Bovens' and Hartmann's accounts are *trivial*, as all belief sets prove maximally coherent on them. The proofs rest on a simple application of (Equi-Coherence). At the end of the next section, we shall also prove the inconsistency of both Olsson's and Bovens' and Hartmann's accounts by exploiting this triviality result.

Olsson (2002: 249) takes coherence of a belief set to be proportional to the extent to which beliefs (or propositions) in the set agree, and offers the following measure (250):

$$\mathrm{C_O}(B_1,...,B_n) = \frac{\Pr(B_1 \,\&\, ... \,\&\, B_n)}{\Pr(B_1 \vee ... \vee B_n)}.$$

Notice that, since the probability of any finite disjunction is either equal to or greater than the probability of the conjunction of all its disjuncts, $\max(\mathrm{C_O}(B_1,..., B_n)) = 1$. Therefore, on $\mathrm{C_O}$, a belief set $\{B_1, ..., B_n\}$ is maximally coherent if and only if $\mathrm{C_O}(B_1,..., B_n) = 1$. Yet it is not difficult to show that, for *any* set $\{B_1, ..., B_n\}$, $\mathrm{C_O}(B_1,..., B_n) = 1$; that is, that *all* belief sets are maximally coherent on C_O. Olsson's measure, thus, cannot be the correct measure of coherence.[14]

Proof. Consider any belief set $\mathbf{E} = \{B_1, ..., B_n\}$ such that $\Pr(\mathbf{E}) > 0$. For any such **E**, it is always possible to construct the equivalent set $\mathbf{E}^* = \{B_1 \,\&\, ... \,\&\, B_n, B_1 \,\&\, ... \,\&\, B_n\}$. Since $(B_1 \,\&\, ... \,\&\, B_\mathrm{n}) \,\&\, (B_1 \,\&\, ... \,\&\, B_n)$ is logically equivalent to $(B_1 \,\&\, ... \,\&\, B_n)$ Ú $(B_1 \,\&\, ... \,\&\, B_n)$, the numerator and the denominator of the ratio C_O $(\mathbf{E}^*)$ are identical. Consequently, C_O $(\mathbf{E}^*)$ = 1. Since **E** is equivalent to **E***, given (Equi-Coherence), C_O $(\mathbf{E})$ = 1, too. QED.

7. Bovens and Hartmann

According to Bovens and Hartmann, "coherence is a property of information sets that plays a confidence boosting role" (Bovens and Hartmann 2003a: 603). Our degree of confidence in the joint truth of an information set is determined by the degree of prior expectance of its joint truth (or "expectance measure"), the degree of reliability of the information sources (or "reliability measure"), and the degree of coherence (or "coherence measure"). Bovens and Hartmann also assume that the information sources are independent of one another and are equally partially reliable. Take REPB_i to be the proposition that, upon consultation with the proper source, there is a report to the effect that B_i is the case.[15] The degree Pr* of confidence in the belief set $\{B_1, \ldots, B_n\}$ is the posterior joint probability of the propositions in the information set after all such reports have come in:

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[14] Bovens and Hartmann (2003b, Ch. 2) also offer apparent counterexamples to Olsson's measure.

[15] Note that while Bovens and Hartmann use the italic 'B_i' as a variable, we use it as a constant for the sake of uniformity.

$\Pr^*(B_1, \ldots, B_n) = \Pr(B_1 \,\&\, \ldots \,\&\, B_n \mid \mathrm{REP}B_1 \,\&\, \ldots \,\&\, \mathrm{REP}B_n)$.

Then the confidence boost b given by those reports can be measured as follows:

$$b(B_1, \ldots, B_n) = \frac{\Pr^*(B_1, \ldots, B_n)}{\Pr(B_1 \,\&\, \ldots \,\&\, B_n)}.$$

Bovens and Hartmann propose to "assess the coherence [c_r] of an information set by measuring the proportion of the confidence boost b that we actually receive, relative to the confidence boost $b^{\max}$ that we would have received, *had we received this very same information in the form of maximally coherent information*" (611), that is:

$$c_r(B_1, \ldots, B_n) = \frac{b(B_1, \ldots, B_n)}{b^{\max}(B_1, \ldots, B_n)},$$

where an information set is maximally coherent if it contains only *logically equivalent* propositions.

Bovens and Hartmann show, however, that "[t]his measure is functionally dependent on the expectance measure a_0 and on the reliability measure r" (612), where $a_0 = \Pr(B_1 \,\&\, \ldots \,\&\, B_n) > 0$, and:

$$r = 1 - \frac{\Pr(\text{REP}B_i|{\sim}B_i)}{\Pr(\text{REP}B_i|B_i)}.$$

Since, in each case, all information sources are assumed to be equally partially reliable, r is constant for $i = 1, ..., n$ and $0 < r < 1$.

Precisely how $c_r(B_1, \ldots, B_n)$ functionally depends on the expectance measure a_0 and the reliability measure r is determined as follows: Consider the information set $\{B_1, \ldots, B_n\}$, and let a_i be the probability that just i propositions of $\{B_1, \ldots, B_n\}$ are false. Thus, a_1 is the probability that just one proposition of the set is false, a_2 is the probability that just two propositions of the set are false, and so on: formally, $a_1 = \Pr({\sim}B_1 \,\&\, \ldots \,\&\, B_n) + ... + \Pr(B_1 \,\&\, \ldots \,\&\, {\sim}B_n)$, $a_2 = \Pr({\sim}B_1 \,\&\, {\sim}B_2 \ldots \,\&\, B_n) + ... + \Pr(B_1 \,\&\, \ldots {\sim}B_{n-1} \,\&\, {\sim}B_n)$, and so on. Bovens and Hartmann then prove that:

$$c_r(B_1, \ldots, B_n) = \frac{a_0 + (1-a_0)(1-r)^n}{\sum_{i=0}^{n} a_i (1-r)^i}.$$

While the dependence of coherence on the expectance measure is expected, the dependence on the reliability measure is unwelcome, for, according to Bovens and Hartmann, coherence should not depend on the reliability of the sources. They thus give up on providing a coherence measure independent of the reliability measure. Still, c_r allows us to construct a quasi-ordering (i.e., incomplete ordering) of information sets independent of the reliability measure: For any two information sets **S** and **S′**, if c_r(**S**) > c_r(**S′**) for any r, then **S** is more coherent than **S′**; if c_r(**S**) = c_r(**S′**) for any r, then **S** is as coherent as **S′**; otherwise, the coherence of **S** and that of **S′** are incomparable. This completes Bovens' and Hartmann's account.

Suppose that we learn from independent, partially reliable sources that someone's pet Tweety is a bird (B), that Tweety is a ground-dweller (G), and that Tweety is a penguin (P). If we learn B and G from independent sources, we will find the set **S** = {B, G} very incoherent. But if we then learn P, the extended information set **S′** = {B, G, P} seems, intuitively, much more coherent than **S**. Bovens and Hartmann justify this intuition by means of their coherence measure.

However, as we have said, we can prove that (*) for any given r, all information sets are maximally coherent on c_r. Consequently, (independently of r) all information sets are equally coherent. This makes Bovens' and Hartmann's account trivial.

Proof of (*). Consider any information set **E** = {B_1, ..., B_n} such that Pr(**E**) > 0. For any such **E**, it is always possible to construct the equivalent set **E*** = {B_1 & ... & B_n, B_1 & ... & B_n}. Now, it is not difficult to prove that, generally, if B_1, …, B_n are logically equivalent propositions such that Pr (B_1 & … & B_n) > 0, then:

$$c_r(B_1,...,B_n) = \frac{a_0 + (1-a_0)(1-r)^n}{a_0 + (1-a_0)(1-r)^n} = 1.$$

Then, in particular, as **E*** includes only logically equivalent propositions and Pr(**E***) = Pr(**E**) > 0, c_r(**E***) = 1. Since **E** is equivalent to **E***, given (Equi-Coherence), c_r(**E**) = 1, too. QED.

Indeed, not only are Olsson's and Bovens' and Hartmann's accounts trivial, but they are incompatible with (Equi-Coherence). For the conjunction of C_O and (Equi-Coherence), as well as the conjunction c_r and (Equi-Coherence), entails both these statements:

(4) Every non-maximally coherent belief set **E** such that Pr(**E**) > 0 is more coherent than itself.

(5) Every non-maximally coherent belief set **E** such that Pr(**E**) > 0 is less coherent than itself.

Proof of (4). Consider any belief set **E** such that Pr(**E**) > 0, which is not maximally coherent, and let **E*** be a maximally coherent set equivalent to **E**. **E*** is more coherent than **E**. But **E*** is equivalent to **E**. Thus, given (Equi-Coherence), **E** is more coherent than itself. QED.

Proof of (5). Consider any belief set **E** such that Pr(**E**) > 0, which is not maximally coherent, and let **E*** be a maximally coherent set equivalent to **E**. **E** is less coherent than **E***. But **E*** is equivalent to **E**. Thus, given (Equi-Coherence), **E** is less coherent than itself. QED.[16]

8. Can Equivalent Belief Sets Have Different Degrees of Coherence?

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[16] We are grateful to Wouter Meijs for suggesting this proof.

At this point, one might think that our proofs rely on some sort of logical trick, and may be blocked in some reasonable manner. One might question, for instance, our use of tautologies in some of the proofs. However, as was suggested in Section 1, the underlying problem, the problem of belief individuation, is more general. On any of the probabilistic accounts under discussion, one and the same information set receives (sometimes drastically) different degrees of coherence (or different verdicts about its coherence *tout court*),[17] depending on how the beliefs that represent the information set are individuated. Since the fact remains that we can transform one belief set into another, equivalent belief set by purely logical reasoning, this violates (Stability) and (Equi-Coherence).[18] Our proofs are only the vehicle of dramatizing this basic fact. Much simpler examples can easily be given against each of the coherence measures examined above.

For instance, suppose, again, that we learn from independent, partially reliable sources that someone's pet Tweety is a bird (B), that Tweety is a ground-dweller (G), and that Tweety is a penguin (P). Following Bovens and Hartmann (2003a: 621), assume that $\Pr(B, {\sim}G, {\sim}P) = 0.49$, $\Pr({\sim}B, G, {\sim}P) = 0.49$, $\Pr(B, G, P) = 0.01$, and $\Pr({\sim}B, {\sim}G, {\sim}P) = 0.01$ (other combinations receive zero probabilities). Then compare the equivalent belief sets $\{B, G, P\}$ and $\{B \,\&\, G, P\}$, and consider what degrees of coherence Shogenji's C_S and Olsson's C_O give

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[17] As we have seen, this is not a totally accurate characterization of Bovens' and Hartmann's account, for their 'measure' only gives a so-called coherence quasi-ordering of various belief sets (that is., it is only a measure of *comparison*), and does not assign degrees of coherence. However, its underlying problem is essentially the same.

[18] Henceforth we shall sometimes ignore the difference between the two versions of these principles, and omit the *-versions for the sake of simplicity.

to each set. While $C_S(B, G, P) = 0.08$ and $C_O(B, G, P) \approx 0.01$, $C_S(B \& G, P) = 100$ and $C_O(B \& G, P) = 1$. This violates (Equi-Coherence).[19]

We have argued that several probabilistic measures of coherence thus far proposed clash with (Stability) and (Equi-Coherence) and suffer from the problem of belief individuation, that is, the problem that, on those measures, one and the same information set can have drastically different degrees of coherence depending on how beliefs that represent the set are individuated. One may wonder, however, whether it is really plausible to assume that belief individuation ought not to affect the degree of coherence. Indeed, Bovens and Hartmann, at least at one point of their paper, answer this question negatively:

> One might object that on our analysis, the coherence of an information set is dependent on how we partition the information. Consider the information set $\mathbf{S}' = \{B, G, P\}$. Suppose that we partition the information as follows: $\mathbf{S}'' = \{B \& G, P\}$. Since given the background information $B \& G$ and P are equivalent propositions, it is easy to show that $\mathbf{S}''$ is a more coherent set than $\mathbf{S}'$. But how is this possible, since the conjunction of the propositions in $\mathbf{S}'$ entails the conjunction of the propositions in $\mathbf{S}''$ and vice versa? (622)

Bovens and Hartmann respond to this charge by contending that "the coherence of an information set is subject to how the information is partitioned and information sets with conjunctions of propositions that are equivalent may display different degrees of coherence" (622).

[19] This species of the problem of belief individuation is called the *problem of conjunction* in Akiba (2000).

They give the following example in support of their contention:

> If a small percentage of men are unmarried and a small percentage of unmarried people are men in the population, then reports that the culprit is a man [*M*] and that the culprit is unmarried [*U*] bring a certain tension to the story. How can that be, we ask ourselves? Aren't most men married and aren't most unmarried people women? The story does not seem to fit together. But if we hear straightaway that the culprit is a bachelor, then this tension is lost. The information that the culprit is a bachelor may be unexpected, since there are so few bachelors. But reporting that the culprit is a bachelor brings no tension to the story. (622)

Thus, their suggestion, apparently, is that whether we obtain a single belief, *M* & *U* (taking 'bachelor' to be equivalent to 'unmarried man'), from a single source or two beliefs, *M* and *U*, from different sources may affect the coherence degree of an otherwise identical belief set.

In a similar vein, Shogenji, in reply to Akiba (2000), suggests that for the evaluation of coherence, beliefs ought to be individuated by their sources rather than contents:

> If we wish to relate the concept of coherence to epistemic justification, … we must individuate beliefs by their sources, and not by their contents. … When beliefs have different sources, we cannot treat them as a single conjunctive belief to claim that they are neither [sic] coherent nor incoherent. (Shogenji 2001: 150)

Thus, Shogenji insists that the belief set {[The fossil was deposited 64-to-66 million years ago], [The fossil was deposited 63-to-67 million years ago]} has different degrees of coherence depending on whether the second proposition is obtained by an independent measurement or is derived logically from the first proposition. (Henceforth we shall express the origin-indexed belief sets in question as {[The fossil was deposited 64-to-66 million years ago]^{M1}, [The fossil was deposited 63-to-67 million years ago]^{M2}} and {[The fossil was deposited 64-to-66 million years ago]^{M1}, [The fossil was deposited 63-to-67 million years ago]^{M1}}, where '*M*1' and '*M*2' are short for 'Measurement One' and 'Measurement Two'.)[20]

In concluding this paper, we shall briefly consider the above suggestion that belief individuation ought to affect the degree of coherence, or, more specifically, that the notion of coherence is to be applied not to propositions or beliefs identified as propositions, but to *origin-indexed* beliefs. If that is the case, then the plausibility of (Stability) may be called into question. In response to this argument, we shall make the following two points: First and most important, the validity of (Stability) and (Equi-Coherence) would remain unaffected even if the notion of coherence were to be applied to origin-indexed beliefs: even

[20] It might be debatable whether the source of the last belief ought to be just 'Measurement One' or 'Measurement One plus a logical inference'; that is, generally, it might be debatable whether logical inferences ought to count as an additional source or not. However, we shall assume in this paper that they do not. Intuitively, it seems odd to say that an empirical belief and another belief logically derived from it cannot come from the same source. Furthermore, neither Bovens' and Hartmann's nor Shogenji's example suggests that belief sets same in content but different in *logical* origin may have different degrees of coherence; at best, they only suggest that belief sets same in content but different in *empirical* origin may.

in that case, all the accounts of coherence examined in this paper would still face the problem of belief individuation. Second, it is not clear whether the notion of coherence is indeed to be applied to origin-indexed beliefs.

First, note that in our definition of equivalent belief sets given in Section 2, two belief sets are considered equivalent if and only if the believer holding the beliefs of one set can obtain the beliefs of the other set by simply deducing the propositional contents of the latter beliefs from the propositional contents of the former beliefs. In Bovens' and Hartmann's bachelor culprit case, this leaves open the question whether $\{M^A, U^B\}$ is equivalent to $\{M \,\&\, U^C\}$, where propositions M and U are obtained from different sources A and B, and $M \,\&\, U$ is obtained from a single source C. If beliefs are to be individuated only by propositional contents, then these belief sets are derivable from each other by deduction, and are thus equivalent. But if beliefs are to be individuated not only by contents but also by sources, as Bovens and Hartmann seem to maintain in their example, then the above belief sets are not derivable from each other, and are thus not equivalent. We may accept this.

What we are committed to saying is that the believer can obtain from the belief set $\{M \,\&\, U^C, K^D\}$[21] the belief set $\{M^C, U^C, K^D\}$, and *vice versa*, by deduction; therefore, these two belief sets are equivalent. However, their degrees of coherence (or the verdicts as to whether they are coherent *tout court*) can be different, according to each of the accounts we have examined, and this violates (Stability) and (Equi-Coherence). Bovens' and Hartmann's example suggests that two belief sets identical in content but different in origin may have different degrees of coherence. But our arguments show that two belief sets identical both in

[21] An arbitrary belief K^D is added to avoid the singleton problem (see note 6).

content and in origin should have the same degree of coherence, while on each of the coherence measures we have examined, they do not. Bovens' and Hartmann's example simply does not address this problem.[22] As was said at the outset of this paper, the problem of belief individuation arises once logical inferences get into the picture. We cannot solve this problem simply by considering beliefs obtained non-inferentially.[23]

The same thing can be said about Shogenji's example. Suppose that Diane obtains the belief set **B** = {[The fossil was deposited 64-to-66 million years ago]^{M1}, [The fossil was deposited 63-to-67 million years ago]^{M2}}, and that Eric obtains the belief set **B′** = {[The fossil was deposited 64-to-66 million years ago]^{M1}, [The fossil was deposited 63-to-67 million years ago]^{M1}}. Then we are not committed to saying that **B** and **B′** are equivalent. What we are committed to saying is that if Diane, who holds the beliefs in **B**, can obtain the beliefs in another set, say **B″**, by simply deducing the propositional contents of the latter beliefs from those of the former beliefs, and if she can transform the beliefs in **B″** into the beliefs in **B** in the same way, then **B** and **B″** are equivalent, and, thus, they ought to be equally coherent.

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22 Bovens' and Hartmann's account applies only if all members of the relevant set have independent sources. As our example shows, however, one of the equivalent belief sets in question may have members whose sources are *dependent* on one another; indeed, in {M^C, U^C, K^D}, M^C and U^C have an *identical* source. On Bovens' and Hartmann's account, such a set cannot have any degree of coherence, while the other set equivalent to it may. This, again, is a violation of (Equi-Coherence).

23 A referee of this paper has suggested another interpretation of Bovens' and Hartmann's and Shogenji's contentions, according to which they are simply proposing to restrict their accounts to beliefs obtained non-inferentially. If that interpretation is correct, then their accounts simply will not help achieve our goal of obtaining a general probabilistic account of coherence.

Hence, we may accept the origin-based[24] belief individuation, and still our proofs show the same problem for the coherence measures we have examined. Second, however, it is questionable whether the origin-based belief individuation is really more plausible than the content-based individuation. Shogenji appeals to our intuition that Diane's belief set is more coherent than Eric's because of its superior pedigree. This intuition, however, can easily be neutralized by the following alternative account of the apparent difference. When Diane and Eric obtain the beliefs of the set **B*** = {[The fossil was deposited 64-to-66 million years ago], [The fossil was deposited 63-to-67 million years ago]}, not only do they obtain those two beliefs, they also obtain beliefs about the *sources* of the beliefs. That is, what Diane actually obtains is not **B***, but $\mathbf{B_D}$ = {[The fossil was deposited 64-to-66 million years ago], [The fossil was deposited 63-to-67 million years ago], [The first proposition came from Measurement One], [The second proposition came from Measurement Two]}. Similarly, what Eric obtains is not **B*** either, but $\mathbf{B_E}$ = {[The fossil was deposited 64-to-66 million years ago], [The fossil was deposited 63-to-67 million years ago], [The first proposition came from Measurement One], [The second proposition was logically derived from the first proposition]}. The coherence degrees of these (different) belief sets may differ. And so long as Diane and Eric retain these beliefs, their action and reasoning may well be affected by the difference between $\mathbf{B_D}$ and $\mathbf{B_E}$. What affects the degree of coherence is not the sources of beliefs, but *beliefs* about the sources.

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[24] More precisely, *partially* origin-based, as propositional contents still matter. But we omit this qualification for the sake of simplicity.

This alternative account actually seems more plausible than the origin-based account, for imagine when Diane and Eric forget where their beliefs about the fossil have come from, that is, when the last two beliefs in $\mathbf{B}_{\mathbf{D}}$ and $\mathbf{B}_{\mathbf{E}}$ are lost, and what is left for both Diane and Eric is $\mathbf{B}^*$. Many people seem inclined to say that, at that point, Diane's belief set is not more coherent than Eric's anymore, despite its superior pedigree (though, admittedly, this inclination is not universal). Evidently, at that point Diane cannot act any better or derive more beliefs than Eric anymore, and that is explained by the sameness of their belief sets. In contrast, on the origin-based account, Diane and Eric still retain the different belief sets, {[The fossil was deposited 64-to-66 million years ago]^{M1}, [The fossil was deposited 63-to-67 million years ago]^{M2}} and {[The fossil was deposited 64-to-66 million years ago]^{M1}, [The fossil was deposited 63-to-67 million years ago]^{M1}}, and Diane's belief set is still more coherent than Eric's because of its superior pedigree, even though that difference will not be reflected in Diane's action or reasoning.[25] So the origin-based account seems to lack a genuine explanatory advantage.[26] It is thus not entirely clear whether we really must employ the origin-based belief individuation. Needless to say, if we employ the content-based individuation, we can more easily get equivalent belief sets with different degrees of coherence.

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[25] An analogous analysis can be given to Bovens' and Hartmann's bachelor culprit case.

[26] One could say, however, that Diane's belief set in the last example is more *justified* than Eric's, even though that difference will not be reflected in their action or reasoning. This seems analogous to the reliabilist's claim that a belief produced by a reliable mechanism is more justified than another belief with the same content that is produced by an unreliable mechanism (though the claim is false on an internalist notion of justification). We thus must admit that our contention here is hardly decisive.

In conclusion, the problem of belief individuation haunts all the measures of coherence thus far proposed. In reaction to this disappointing fact, one might insist that we still may find in the future a more sophisticated measure of coherence that solves the problem; after all, the project of giving a probabilistic account of coherence is still in its infancy. However, the very idea of explaining coherence in terms of the probability calculus appears doomed from the outset. The idea is that of determining coherence of an information set on the basis of probabilistic correlations among the beliefs that represent the set. However, as we have argued repeatedly, the same information set can be represented by different belief sets, and probabilistic correlations vary depending on how the beliefs are individuated. Thus, very plausibly, no matter what measure we select, there will be cases in which the coherence degree of the same information set given by that measure changes as the believer formally transforms her relevant beliefs. This will be a violation of the natural and reasonable constraint expressed in (Stability) and (Equi-Coherence).

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