What can our best scientific theories tell us about the modal status of mathematical objects?

Abstract

Indispensability arguments are used as a way of working out what there is: our best science tells us what things there are. Some philosophers think that indispensability arguments can be used to show that we should be committed to the existence of mathematical objects (numbers, functions, sets). Do indispensability arguments also deliver conclusions about the modal properties of these mathematical entities? Mark Colyvan (2007) and Hartry Field (1989) each suggest that a consequence of the empirical methodology of indispensability arguments is that the resulting mathematical objects can only be said to exist (or not exist) contingently. Kristie Miller has argued that this line of thought doesn't work (Miller 2012). Miller argues that indispensability arguments are in direct tension with contingentism about mathematical objects, and that they cannot tell us about the modal status of mathematical objects. I argue that Miller's argument is crucially imprecise, and that the best way of making it clearer no longer shows that the indispensability strategy collapses or is unstable if it delivers contingentist conclusions about what there is.

0 Introduction

Kristie Miller's paper 'Mathematical Contingentism' presents a series of interlocked arguments about the kinds of methods we should use when inquiring about the existence of mathematical objects. The chief target is 'post-Quinean ontology' (PQO), an approach to doing ontology that employs a naturalistic epistemology, and which says the methods of metaphysics are continuous with the methods of the natural sciences. Miller asks: assuming PQO can tell us about whether mathematical entities exist, can it also deliver a verdict as to whether mathematical objects necessarily exist? That is, can PQO tell us about their modal status? Miller's overall argument is double-pronged: PQO is self-defeating if it tells us that mathematical entities only contingently exist, and it is incoherent if it attempts to say that they necessarily exist. She recommends abandoning PQO and recommends an *a priori* methodology for investigating mathematical reality.

In this paper I will address the first prong of Miller's two-pronged argument against PQO, the argument that PQO is self-defeating if it draws contingentist conclusions. I am restricting my focus to the argument about contingentism because I take it that contingentism about mathematical objects is, at least historically, an unusual position to adopt, and therefore of some philosophical significance. The paper is structured as follows. I begin in section 1 by briefly explaining the five key '–isms' at issue: platonism, nominalism, indispensabilism, contingentism, and necessitism. I then present a target indispensability argument for platonism about mathematical objects, and articulate how it has been taken to deliver mathematical contingentism. In section 2 I present Miller's 'epistemic objection' against drawing contingentist conclusions from the indispensability argument; chiefly, she thinks that the combination of indispensabilism and contingentism together undermines indispensabilism. In section 3, I give reasons for resisting Miller's argument; I argue that it is not self-defeating or self-undermining for indispensability arguments to be used to conclude that mathematical objects exist contingently. I conclude that Miller has not shown indispensabilist contingentism to be an unstable position.

1 Terms and Context

I use the term 'indispensabilism' to refer to any positions which endorse the methodological claim that we should be committed to the existence of all and only the entities that are indispensable to our best scientific theories. *Indispensabilism* is one answer to a meta-epistemological question about ontology: 'what is the best method to use in answering the question "what exists?"'. I also follow Kristie Miller in using the label 'post-Quinean ontology' (PQO) to refer to this broad methodological approach to ontology, and consider indispensabilism to be just one application of Quine's famous dictum that we should stick to using our best scientific methods and explanations when inquiring into what there is (Quine 1969: 97).

Platonism is a metaphysical position which exemplifies a form of mathematical realism. Central to platonism is the claim that there are numbers and other mathematical objects, or (and I'll treat this claim as equivalent) that numbers and/or other mathematical objects exist. In addition, and particular to platonism, it also says that mathematical objects are abstract, rather than concrete, objects.

Platonism is contrasted with *nominalism*. I use the label 'nominalism' as a catch-all for various mathematical anti-realisms; crucially, it picks out metaphysical positions which say that there aren't any abstract mathematical objects, or (and I'll treat this claim as equivalent) that abstract mathematical objects don't exist.

Contingentism and *necessitism* are contrasting positions about the modal status of mathematical objects. *Contingentism* is the claim that whether or not mathematical objects exist, the status of their existence is modally contingent. Familiar instances of contingentist claims about the existence of other (non-mathematical) entities might include the idea that while it is true that coconuts exist, it does not seem as though they have to exist. Or consider that although it is true that there are no blue and green striped flamingoes, it nevertheless seems possible that they could exist. *Necessitism* is the claim that whether or not mathematical objects exist, the status of their existence is modally necessary. An example of a necessitist existence claim about a (non-mathematical) entity might be the idea that not only are there no flamingos which are entirely blue and entirely green all over at the same time, but that furthermore they could never exist — the existence of such things is not possible. I'll also talk about some combinations of the elements of the two distinctions introduced so far. For example: Platonist-necessitism amounts to the claim that necessarily, abstract mathematical objects exist. Nominalist-necessitism asserts that necessarily, abstract mathematical objects do not exist. [1]

The indispensabilist approach has been recruited to argue for platonism. One way of capturing the general form of this type of argument for platonism is as follows:

1. If Fs are indispensable to science then we should believe that there are Fs.
2. Numbers and other mathematical objects are indispensable to our best scientific theories.

---

[1] An earlier version of this paper explored the relevant sense of modality involved here: should we understand necessitism / contingentism as concerning metaphysical, logical, or epistemic modality? I argued that the only interpretation on which Miller's epistemological objection (the main focus of this paper) seems to have some scope for working is if these positions are concerned with metaphysical modality, so this is how I will proceed from hereon. Reviewers for this journal sagely encouraged me to omit the bulk of that discussion, but to include a note pointing out that much of the debate between Field, Colyvan, Hale and Wright has concerned 'conceptual necessity', whereby a statement is conceptually possible if its negation is not true in virtue of its meaning (see Hale and Wright 1992, Field 1993: 285, Colyvan 2000: 88).

3. We should believe that there are mathematical objects.[2]

Here, premise 1 is a statement of a methodological norm which exemplifies indispensabilism. Premise 2 makes an empirical claim about what is in fact the case in current science, and premise 3 draws a conclusion which (by taking on board a few further assumptions) commits the indispensabilist to platonism. Hartry Field's brand of nominalism is motivated in part by rejecting this argument on the basis of denying premise 2: his book *Science without Numbers* (Field 1980) argues that our best scientific theories do not require or imply the existence of numbers and other mathematical objects, and so he thinks that we should not believe there are any mathematical objects.   And to get to nominalism, the belief that there are not any abstract mathematical objects, he notes:

> 'Admittedly, we can't have direct evidence against mathematical entities. We also can't have direct evidence against the hypothesis that there are little green people living inside electrons and that are in principle undiscoverable by human beings; but it seems to me undue epistemological caution to maintain agnosticism rather than flat out disbelief about such an idle hypothesis. [I]f ... the hypothesis is dispensable without loss ... it is natural to go beyond agnosticism and assert that mathematical entities do not exist.' (Field 1989: 44-5)

Field's position is indispensabilist as it endorses premise 1, and since he is a nominalist (for separate reasons) he denies premise 2 and in doing so finds reason to believe that mathematical objects do not exist. And going further, since Field thinks that our best scientific theories do not indispensably depend on numbers but that there is a sense in which they could have done, he is committed to the contingent non-existence of numbers.

Many philosophers have argued that Field's ambitious project to deny premise 2 is not something that can work – that at best the arguments and techniques that Field marshals might nominalise Newtonian physics, but not Einsteinian physics. Field's approach is referred to as 'the hard road to nominalism' precisely because the project undertakes the difficult task of showing that the whole of (current, best) science can be

---

[2] Here's a statement of such an indispensability argument given (but not endorsed) by Harty Field:
'[I]f our belief in electrons and neutrinos is justified by something like inference to the best explanation, isn't our belief in numbers and functions and other mathematical entities equally justified by the same methodology?' (Field 1989: 16).

reconstructed in such a way as to make no appeal to mathematical objects, that they can be dispensed with.[3]

Mark Colyvan is also committed to the indispensabilist project, which he summarises as follows:

> 'Indispensability theory tells us that mathematical knowledge is in the same epistemic boat as empirical knowledge... (Colyvan 2007: 115)'

> 'On the Quinean account, mathematical statements are known to be true by the role they play in our best scientific theories—in other words *a posteriori*...' (Colyvan 2007: 120).

But unlike Field, Colyvan thinks the road to nominalism is too hard, and that there is no easy road, and so he thinks we should be platonists. While Colyvan's position is clearly an indispensabilist platonism, going further, he appears to open the door to contingentism as well:[4]

> 'Suppose that Hartry Field has completed the nominalisation of Newtonian mechanics but that he and his successors repeatedly fail to nominalise general relativity. Let's also suppose that this failure gives us good reason to believe that general relativity cannot be nominalised. From this we conclude that mathematical entities are indispensable to general relativity, but not to Newtonian mechanics.' (Colyvan 2007: 122-3).

Had the world been Newtonian and not Einsteinian, such that the best physical theory of the world was Newtonian physics, platonism would be false, since Field's project successfully hikes us along the hard road to nominalism for Newtonian physics. As such, Colyvan permits that platonism's claims are not true necessarily. Thus, he commits to indispensabilism, platonism, and contingentism. We can spell out this argument as follows:

1. *A posteriori* discovery: the best physics of our world is Einsteinian.
2. Einsteinian physics cannot be nominalised and so the best physical theory of our world is committed to numbers.

---

[3] In contrast, the so-called 'easy road' to nominalism is to accept that our best scientific theories imply that there are mathematical objects, but to resist platonism by finding ways to show that the particular role that mathematical objects play in science is not ontologically committing. Joseph Melia's (2000) proposals for 'weaselling out of ontological commitments' is a key example of such an 'easy road' approach. See Knowles and Liggins (2015) for recent discussion of Melia's strategy.

[4] Colyvan has discussed, defended, but not quite endorsed mathematical contingentism on the basis of such an indispensability argument – in (2001) he says "Although I'm inclined to think that mathematics is contingent, it may be that indispensabilists can go either way on this issue". See Miller (2012) footnote 14.

3. There are numbers (platonism is true). [From *Indispensability*, 1 + 2]

4. But if, contra 1, the world had been Newtonian, since the best theory of our world could dispense with numbers, there wouldn't be any numbers (nominalism would be true).

5. Newtonian worlds are possible, *i ipso* nominalism is possible.[5]

6. So we should be committed to the contingent existence of numbers (contingentist platonism).

This argument allows us to bring out the ways in which nominalist contingentism could have been motivated (had the world been Newtonian). Suppose, instead of 1, we made the *a posteriori* discovery that the world is fully describable by Newtonian physics, but we also accepted that an Einsteinian physical world was possible, although not actually the case. Thus, while the best theory of the Newtonian world would not be committed to numbers (so: there are no numbers), platonism would remain possible, albeit contingently.[6] If this argument were sound, indispensabilist nominalists in Newtonian worlds should also be contingentists.

As we'll see in the next section, Miller does not think that this kind of indispensability argument for contingentism can work. One way of thinking about her objection is that with the addition of an extra assumption (the 'matching claim'), the combination of indispensabilism + contingentism can be shown to be self-undermining.

2 Miller's argument

Miller argues that if contingentism is true, then on most ways of understanding the link between indispensability and existence, indispensabilism undermines itself.[7] As I read it, Miller's objection to the indispensabilist argument for contingentism that we have seen in the previous section centres on the fact that indispensabilism is an

---

[5] In what follows I won't examine why indispensabilists accept the possibility of Newtonian worlds, or whether they're correct in doing so. It suffices for our argument that some indispensabilists do in fact accept this possibility, in some sense.

[6] This is not Field's argument for contingentist nominalism (see Field 1993 for a defence of his view).

[7] As I suggested in the introduction, Miller's paper isn't solely directed at contingentism (in either its platonic or nominalist forms). She has a much wider objective: to show that indispensabilism turns out to be no guide to the modal status of what exists, and so PQO should be abandoned. I won't discuss the wider objective here, instead I focus on just this component argument.

avowedly fallibilist methodology. Indispensabilism recommends that we use only the tools and results of the empirical sciences when answering the question 'what exists?', and in so doing it exposes metaphysicians to the same kind of epistemic risk that we associate with empirical science; indispensabilist metaphysicians, like natural scientists, cannot know their conclusions with certainty. As Quine puts it, such naturalistic philosophy is, like natural science, "an inquiry into reality, fallible and corrigible but not answerable to any supra-scientific tribunal", and that naturalistic philosophers "tentatively believe all of it, but believe also that some unidentified portions are wrong [and try] to improve, clarify, and understand the system from within" (Quine, 1981, p. 72).

Crucially then, indispensabilist metaphysicians should accept the possibility that their conclusions are false. Miller's objection sharpens this possibility into a pointed problem for indispensabilist contingentism and she labels it as 'the epistemic objection' precisely because it concerns these matters of certainty and fallibilism.

The epistemic objection is generated by supplementing indispensabilism with 'the matching claim', which Miller treats as a direct consequence of contingentism.

> *Matching claim*: 'For any world, w, in which mathematical objects exist and are indispensable to the best theory of w, there exists a physically indistinguishable world, w*, in which mathematical objects fail to exist.' (Miller 2012: 344)

Miller thinks that the matching claim is something that indispensabilist contingentists should accept. Her reasoning is as follows: indispensability-to-our-best-current-theory is a methodological norm that is only intended to establish an evidential relationship to ontology:[8]

> 'One might find the matching claim very plausible, since the fact that mathematical objects are indispensable to the best theory of w does not entail, cause, or constitute, their existence at w: it is just evidence that they exist. Thus one might expect there to be a world just like w, but in which mathematical objects fail to exist.' (344)

The 'epistemic objection' is this: the conjunction of indispensabilism + contingentism and the matching claim makes it hard to see why indispensabilism can be any guide to (modal) ontology whatsoever. That is, if we accept the Colyvan-style indispensabilist argument for contingentism seen in section 1, then we accept a picture of the space of

---

[8] Loosely speaking, indispensabilism is scientific 'inference to the best explanation' when it is adopted as a general methodology for doing ontology / metaphysics.

epistemic possibilities in which there are some worlds where reference to mathematical objects is indispensable to the best theory of that world ('mathematical indispensable worlds') and in other worlds where it is not ('mathematical dispensable worlds'). This is represented in figure 1.

Epistemological situations	
Mathematical indispensable worlds worlds in which the best theory of that world quantifies over mathematical objects	**Mathematical dispensable worlds** worlds in which the best theory of that world does not quantify over mathematical objects

FIGURE 1.

Commitment to contingentism represents a cross-cutting distinction about what kinds of ontological options there could be: contingentists accept that there can be worlds in which mathematical objects exist ('math worlds') and worlds in which mathematical objects do not exist ('no math worlds'). This is represented in figure 2.

Ontological Options	**Math worlds** worlds in which mathematical objects exist
	No Math worlds worlds in which mathematical objects do not exist

FIGURE 2.

The matching claim says that since the evidential statements which generate the epistemological situations in figure 1 do not "entail, cause, or constitute" the existence of mathematical objects, each of those epistemological situations are consistent with matching pairs of math / no math ontological options. This is represented in figure 3 by the matching (highlighted by the arrows) that holds between pairs w1 and w1*, and w2 and w2*:

If the best theory of the world is indispensably committed to mathematics, as in w1, then the indispensability argument might be employed to infer that mathematical objects exist. But if the matching claim is correct, then there is another world w1* which is physically indistinguishable from w1, in which mathematical objects do not

INDISPENSABILISM gives these situations ☞ CONTINGENTISM gives these options ☟		Epistemological situations	
		Indispensable Worlds worlds in which the best theory of that world quantifies over mathematical objects	**Dispensable Worlds** worlds in which the best theory of that world does not quantify over mathematical objects
Ontological Options	**Math worlds** worlds in which mathematical objects exist	w1	w2*
	No Math worlds worlds in which mathematical objects do not exist	w1*	w2

FIGURE 3.

exist.[9] By employing 'some sort of principle of indifference', Miller generates her epistemic objection as follows:

> *Epistemic objection*: The combination of contingentism + matching entail that '[n]o agent should give more than 50% credence to there existing mathematical objects in her world, regardless of whether it is a mathematical dispensable or indispensable world.' (2012: 344)

If the epistemic objection is correct, then we do not have reason to be committed to the existence of all and only the entities that are indispensable to the best theory of our world. Miller continues:

> 'If the epistemic objection succeeds, it undermines [the indispensabilist claim] that relative to any world, w, an agent has reason to be committed to the existence of all and only the entities that are indispensable to the best theory of w. For it undermines any reason I have to think that indispensability is a guide to ontology.' (344)

The upshot, for Miller, is this: contingentism cannot be combined with indispensabilism; contingentism undermines the central indispensabilist tenet that we have reason to be committed to the existence of all and only the entities that are indispensable to our best scientific theories. As such, the 'epistemic objection' argument aims to show that the combination of indispensabilism and contingentism is an untenable position. In the next section, I return to the 'matching claim' and attempt to defend the coherence of indispensabilist contingentism.

---

[9] The corollary case is w2; if the best theory of the world can dispense with mathematics then Field's line of argument (in §1 above) might be employed to infer that mathematical objects do not exist. But if the matching claim is correct then there is another world w2* which is physically indistinguishable from w2 in which mathematical objects do exist.

3 In defence of indispensabilist contingentism

I want to argue that against Miller's 'epistemic objection', indispensabilist contingentism is not a self-undermining position, and that it is coherent for indispensability arguments to be used to conclude that mathematical objects exist contingently.

The 'epistemic objection' pivots crucially on the 'matching claim'. As we've seen, Miller thinks that indispensabilists should endorse the matching claim because they accept that the fact something features in the best scientific theory does not "entail, cause, or constitute" the existence of that thing. But despite the surface appearance that the 'matching' claim is fairly straightforward, it is not entirely clear how we should understand the assertion that it makes or why it should be accepted. It appears to be making a claim about what is metaphysically possible, as follows:

> *Metaphysical Matching* (MM): **it is metaphysically possible that**, for any world w, in which mathematical objects exist and are indispensable to the best theory of w, there exists a physically indistinguishable world, w* in which mathematical objects fail to exist.'[10]

In what follows, I argue that there is insufficient motivation for indispensabilist contingentists to accept MM, and that it is something which they can coherently reject (§3.1). In its place I propose an alternative matching claim, MME, which does fit with the motivation that Miller has outlined, and which is sufficient to generate the epistemic objection (§3.2). But I go on to argue that if this is right and MME is the correct way to understand the matching claim, then there is no longer reason to think that there is a particularly problematic tension between indispensabilism and contingentism about mathematics (§3.3).

3.i

---

[10] In an earlier version of this paper I argued that alternative ways of understanding the modality implicit in the matching claim (in terms of epistemic possibility and logical possibility) do not generate the epistemic objection. A reviewer for this journal noted that as a consequence, neither of them are sensible interpretations of Miller's view, so I've omitted these arguments. I leave it as an exercise for the interested reader to reproduce these arguments.

The matching claim can be read as making a statement about what is metaphysically possible (MM). MM says that while our best scientific theory of the actual world might be ineliminably committed to the existence of numbers (and so there are numbers), there exists another possible world which is physically indistinguishable to our world and yet it is a world in which numbers do not exist.

MM commits the indispensabilist both to the existence of a possible world in which there are numbers and to the existence of a possible world in which there are no numbers, so MM is also a clear statement of contingentism about the existence of mathematical objects; they exist, but they don't exist in all possible worlds. And MM appears to support Miller's contention that indispensabilism will be undermined: our evidence can't help tell us whether we're in a maths world or a no-maths world, because the physical states of affairs are the same in both of these worlds.

Is MM something that indispensabilists should endorse? More precisely, since Miller thinks that Matching generates problems for indispensabilists who are also contingentists: is MM something that indispensabilist-contingentists should endorse?

To get us started, I think indispensabilist contingentists will accept the following claim:

> *MMC*: it is metaphysically possible that, for any world w in which mathematical objects are indispensable to the best theory of w (such that w is a world at which mathematical objects exist), there exists another world w* in which mathematical objects are not indispensable to the best theory of w* and do not exist.

I take it that both Colyvan's platonist version of indispensabilist contingentism and Field's nominalist version of indispensability contingentism are compatible with MMC. In understanding both Colyvan and Field as accepting something like MMC, the difference in their positions amounts to a disagreement about whether the actual world is one in which mathematical objects are indispensable to its best physics (Colyvan's platonism), or whether the actual world is one in which the best physical theory can dispense with mathematical objects (the nominalism of Field's 1980)[11]. And I take it that indispensabilist contingentists will accept MMC because MMC is really just a statement of what contingentism about mathematical objects amounts to, i.e. the claim

---

[11] This is not a totally precise statement as other differences in their positions persist. For instance, nominalists are under no obligation to endorse the view that 'in possible worlds where the best scientific theories are committed to numbers, numbers exist *and numbers are abstract objects*'.

that while mathematical objects do (/ do not) exist in w, it is also the case that they do not (/ do) exist in some other metaphysically possible world w*. So I take MMC to chiefly be a statement or articulation of (indispensabilist) contingentism itself.

The part of MM that makes it significant for Miller's argument, and stronger than MMC, is the inclusion of the idea that for any maths-world w there exists a "physically indistinguishable" no-maths world w*.[12]

It is no part of Field's or Colyvan's contingentism that there might have existed a physically-indistinguishable alternative world to our world. Recall that Colyvan puts the point about contingentism like this: the world could have turned out to be Newtonian rather than Einsteinian, in which case the best physical theory of the world could have been nominalised.[13] But in saying this he is not committed to the idea that the counterfactual world, being Newtonian, is also physically indistinguishable from the actual (Einsteinian) world. Newtonian worlds are physically distinguishable from Einsteinian worlds. So MM doesn't follow from contingentism alone.

Since the principle difference between MMC and MM is the idea that the counterfactual possible world will be 'physically indistinguishable' from the actual world, and since I've argued that MMC is something that indispensabilist contingentists, *qua* contingentists, should endorse, the question becomes why Miller thinks that indispensabilist contingentists will accept the 'physically indistinguishable' stipulation that is an additional component of MM? What part of the combination of indispensabilism + contingentism might motivate endorsing this claim?

Recall what Miller says to motivate the idea that indispensabilist contingentists might endorse the Matching claim:

> 'One might find the matching claim very plausible, since the fact that mathematical objects are indispensable to the best theory of w does not *entail, cause, or constitute* their existence at w: it is just evidence that they exist. Thus one might expect there to be a world just like w, but in which mathematical objects fail to exist.' (344, my emphasis)

---

[12] In saying that MM makes a stronger claim than MMC, I just mean that MM entails MMC, but MMC does not entail MM.

[13] In s1 above, this claim was presented as Premise 5: 'Newtonian worlds are possible, *i ipso* nominalism is possible'.

I think there are at least a couple of unclear components about Miller's reasoning here, and I'll try to unpack them one by one.

I think it's a correct understanding of indispensabilism to say that the 'fact that mathematical objects are indispensable to the best theory of w does not entail, cause or constitute their existence at w'. I take it that the indispensability of Fs to the best theory of w does not bring it about (causally, logically, metaphysically) that Fs exist at w. And it's also correct to say that, to the indispensabilist, the indispensability of Fs to the best theory of w is evidence that Fs exist at w. So indispensabilists accept that evidence for the existence of Fs at w does not bring it about (causally, logically, metaphysically) that Fs exist at w. To be clear about the provenance of this view, I want to stress that I take it that Miller is reporting a pretty standard and widespread claim about the nature of evidence, rather than one that is unique to indispensabilism. That our evidence that p does not entail, cause or constitute the truth of p is common ground among most epistemologies.[14]

But the inference that Miller draws from this does not follow. She says "Thus one might expect there to be a world just like w, but in which mathematical objects fail to exist." I take Miller to be using "a world just like w" to mean "a world that's physically indistinguishable from w", since her purpose is to motivate the Matching claim, which talks about w and w* being 'physically indistinguishable'. To be precise, the inference from

>*Evidence*: the evidence for the existence of Fs at w does not cause, entail or constitute the existence of Fs at w

plus

>*Contingentism*: there is a possible world w where Fs exist and there is a possible world w* where Fs do not exist

to

>*Indistinguishable*: there is a possible world w at which Fs exist that is physically indistinguishable from a possible world w* at which Fs do not exist (a version of MM)

does not work. To see why, consider that talk of Fs here is referring to electrons. Many of us, indispensabilist or not, might accept the *Evidence* claim that the best evidence for the existence of electrons in our world does not cause, entail or constitute the

---

[14] The epistemology of self-warranting beliefs might count as a kind of exception which proves the rule.

existence of electrons in our world. Again, indispensabilist or not, we might accept the *Contingentism* claim that there are counterfactual possible worlds in which electrons do not exist. But I take it that we would be right to resist the *Indistinguishable* conclusion that the worlds in which electrons do not exist are physically indistinguishable from our world (where they do). We would rightly insist that the worlds in which electrons do not exist can be physically distinguished from our world in light of the fact that they have no electrons in them: that they differ precisely in respect of the physical presence and absence of electrons. To draw the parallel even closer to home: suppose you accept that the world might have been Newtonian (that worlds with Newtonian physics are metaphysically possible); it is hard to see why you thereby need to accept that such a possible Newtonian world would have been physically indistinguishable from the actual (Einsteinian) world; indeed, you should think they differ precisely in their physics. With these kinds of cases in mind it is hard to see where the *Indistinguishable* claim about 'physical indistinguishability' is coming from – it seems to have nothing to do with *Evidence,* nor to do with *Contingentism.*

So the way that Miller has motivated MM for indispensabilist contingentists doesn't appear to work, but that doesn't mean that it MM is entirely unmotivated. We might supplement Miller's reasoning here with a further explicit assumption: mathematical objects are abstract, not physically-realised things. So unlike electrons, mathematical entities are non-spatiotemporal causally-inefficacious things. On this view, there can be maths-worlds and no-maths-worlds which are physically indistinguishable, since if all they differ in is the presence or absence of mathematical objects, they don't differ in respect of anything spatiotemporal or causally-efficacious.

As I've noted above, the view that mathematical objects are abstracta is distinctive to *platonism* – that not only are mathematical objects real, but that they are non-spatiotemporal acausal entities. While it might seem initially plausible that a mathematical platonist who is also a contingentist might accept something like MM on the basis of the kind of reasons that Miller gives, in fact this will depend on what precisely motivates their contingentism; the dialectical sequence by which they arrive at a contingentist position is significant. Indispensabilism itself does not entail contingentism, and contingentism can be adopted for a number of reasons; which reasons are employed makes a difference as to whether MM is a reasonable further claim to endorse. The case in point is Colyvan's position, whose contingentism is predicated on the idea that although the actual world is Einsteinian it is nevertheless metaphysically possible that it could have been Newtonian. To accept that the world

could have been Newtonian is precisely not an instance of accepting MM and the metaphysical possibility that the world could have been physically indistinguishable but for the absence of mathematical entities. To grant that the world could have been Newtonian is to accept the metaphysical possibility that the world could have been very different physically. So here we have a paradigm case in which the combination of indispensabilism + platonism + contingentism involves rejecting MM's 'physically indistinguishable' clause. And absent further motivation, it is hard to see how Miller's epistemic objection gets started.

3.ii

Is there a related sense in which indispensabilist contingentists should accept MM? Drawing only on indispensabilism + contingentism we can motivate something that is relevantly close to MM. For the sake of argument I will use a simplified illustration.

Consider: there was a lengthy period when Newtonian physics was the best theory of the physics of the world. Theories of light in the nineteenth century continued a programme of research presented a century earlier in Newton's *Opticks* which maintained that there is a medium through which light transmits vibrations, a luminiferous ether. While competing claims about the properties of the luminiferous ether started to be developed and challenged over the course of the century, its existence was still presupposed by the best scientific theories. If we bookend this period with the Michelson–Morley experiment of 1887, we can pick an earlier date—say 1830—with which to work. An indispensabilist seems committed to the following: that the best scientific theory of the world in 1830 is indispensably committed to the existence of a propagating luminiferous ether, and so the right thing to say in 1830 is that the luminiferous ether exists. We now know that this is false, and was also false in 1830: the ether theory entails that the speed of light must be variable, but the best contemporary theory says that it is constant is a vacuum, so there is no luminiferous ether. But the indispensabilist should accept that in 1830 the correct thing to say is that the best available evidence supports the worldly existence of ether—the theory's success or bestness is grounds to infer its truth—and also that the theory is false, since ether does not exist.

Now suppose that there is a possible world w* in which Newtonian physics is true, a propagating luminiferous ether exists, and the history of scientific discovery in w* is identical to its history in our world (w). Here we have a scenario in which luminiferous ether is indispensable to the best theory of w* in 1830, just as it was in the actual world w in 1830. The metaphysical possibility of w* captures the kind of contingentism that Colyvan expresses: the world might have been Newtonian. Let us agree that in neither w* or the actual world w does the fact that luminiferous ether is indispensable to the best theory of that world entail, cause, or constitute the existence of ether. In both the actual world in 1830 and w* in 1830, the best scientific theorising has precisely the same evidence to hand. On the basis of this scenario one might find the following matching claim plausible: that it's metaphysically possible that in w* in which luminiferous ether exists and is indispensable to the best theory of w*, there exists an evidentially indistinguishable world w, in which luminiferous ether fails to exist.

A generalised version of this matching claim is as follows:

> *MME:* it is metaphysically possible that, for a world w*, in which Fs exists and are indispensable to the best theory of w*, there exists an evidentially indistinguishable world, w in which Fs do not exist.

Unlike MM, MME does not say that w and w* are physically indistinguishable, rather it says that they are evidentially indistinguishable. So far we have seen that MM is unmotivated, but in contrast we have just seen that MME seems to follow from the combination of indispensabilism + contingentism (plus Miller's plausible assumption that the evidence that there are Fs does not 'cause, entail or constitute' that there are Fs). So MME is a good way to make sense of Miller's matching claim.

While there are various ways to understand the notion of 'evidentially indistinguishable' (or 'evidentially equivalent'), for our purposes the strength of the claim made in MME is just related to our earlier supposition that the precise same evidence is available to scientists in 1830 in both w and w*. And since that available evidence is unable to tell those scientists whether they are in w (the actual Einsteinian world) or w* (a Newtonian world), MME seems able to generate something like the self-defeat objection that Miller has been drawing attention to. A version of this self-defeat argument would go like this:

1. *Indispensabilism* says that the best theories tells us what exist.

2. *Contingentism* says that its metaphysically possible that, unlike the actual world (w), the world might have been Newtonian (w*).

3. *MME:* in 1830, w and w* are evidentially indistinguishable.

4. In 1830, in both w and w*, the best theory is committed to luminiferous ether.

5. In 1830, in w the best theory is false, but in w* the best theory is true.

6. From (5), it follows that the best theory is no guide to truth.[15]

7. From (6), indispensabilism (1) is false. So the combination of indispensabilism + contingentism is self-undermining.

I've given this argument to show that MME is sufficient to generate the kind of self-undermining-ness that Miller says follows from the Matching claim. Since we've seen that MM is not motivated, and since MME seems to follow from indispensabilism + contingentism and is also able to generate the same objection, MME seems like a better interpretation for understanding Miller's matching claim.

3.iii

In this section I want to make two observations about the preceding self-defeat argument, in order to suggest that if we understand the matching claim as I think we should, as MME, then Miller's argumentative strategy no longer works.

The first is is to do with the fact that the argument at the end of the previous section concerns luminiferous ether, as in doing so it shows that if Miller indeed intended something like MME rather than MM, the form of argument that she is relying upon is not restricted to indispensabilism + contingentism about mathematical objects. As the argument suggests, the self-undermining-ness that is supposed to be a consequence of indispensabilism + contingentism + MME looks as though it can be generated just in virtue of endorsing both indispensabilism about whether Fs exist and in thinking that Fs only contingently exist, regardless of whether Fs are concrete or abstract objects, or if they're mathematical or non-mathematical arguments. So this self-undermining objection can also be deployed in all of those cases where we use IBE to infer the existence (or non-existence) of contingent non-mathematical objects such as ether or electrons. In so far as Miller's objective in the paper is to suggest that there is something self-defeating about the combination of indispensabilism + contingentism about mathematical objects in particular, the idea that it might also impugn an

---

[15] Miller spells out the 'Epistemic Objection' as an extra step which I have skipped over here.

indispensabilist approach to more familiarly contingently-existing-entities suggests that something has gone wrong. MME might well do the work that Miller needs for her argument about mathematical objects, but its application is too general or too liberal to show that there is a particular problem associated with indispensabilism + contingentism about mathematical objects.

The second observation is related to the first, but makes the concern even more general. For although the self-defeat argument given above relies on premises expressing both indispensabilism and contingentism to generate its conclusion, the contingentist premise is essentially redundant. If MME is indeed the correct way to make sense of Miller's matching claim, given the motivation she outlines, contingentism plays no critical role in generating the objection. The contingentist idea that 'the world could have been Newtonian' certainly appeared to play a pivotal role in the argument we have been evaluating so far, as it asserts that the physics (and thus the mathematics) of this world are only contingently the case, but in fact we can arrive at MME without relying on it at all.

To see this, we need only recall that indispensabilism (and IBE more generally) is a fallibilist methodology: indispensabilist metaphysicians accept the possibility that their conclusions might be false.[16] That is, indispensabilists will accept MME because they recognise that despite having evidence for thinking that the actual world is Einsteinian, they also accept that this evidence does not guarantee that their world is Einsteinian. As fallibilists, they accept that it is possible for all of the evidence to be just the same ('evidentially indistinguishable') as that which suggests an Einsteinian physics even while the physics and ontology of that world turn out to be entirely different.

As we saw at the start of §2, it is uncontroversial that the (scientifically inspired) indispensabilist methodology is fallibilist; that the knowledge or beliefs we acquire in this way are not certain. One way of expressing the idea that scientific knowledge falls short of certainty is to say that there can be very strong evidence for the results of a scientific inquiry but that nevertheless the evidence does not rule out other possibilities, including the possibility that the conclusion is wrong.[17] For instance, on the basis of

---

[16] It is perhaps useful to clearly distinguish the two claims here in a footnote. *Fallibilism* asserts that it is possible that the evidence E supports p and yet not-p. *Contingentism* says that it is possible that p and it is possible that not-p. Fallibilism does not entail contingentism.

[17] This kind of fallibilism is not the same as (academic) skepticism, which proceeds by undermining the available evidence for p and thereby undermines any knowledge that p. Rather, it concedes that not-p is consistent with the best evidence adduced in support of p.

our best scientific investigations —paleoglaciological inversion of the glacial landform record, sedimentary carbon-dating— we know that the last British-Irish Ice Sheet retreated from the Glens of Antrim before it retreated from the Lake District (Hughes, Clark and Jordan, 2014). But even though these reconstructed ice-sheet retreat patterns represent the best evidence for former ice sheet extent and chronology, we can nevertheless express the thought that the methods we use do not deliver certainty. We do this by saying that despite our best evidence it remains possible that the retreat occurred in the Lakes before it occurred in Antrim. And I take it that it is this fallibilism that is the relevant component of Miller's stated motivation for thinking indispensabilists will endorse MME, since she notes that indispensabilists accept that the evidence that p does not itself entail p.

Such fallibilism is sufficient to generate MME. The pivotal component of this matching claim, as we have seen, is the thesis that ontologically different states of affairs might be 'evidentially indistinguishable'. Such a claim follows from fallibilism, since if our best evidence tells us that there are Fs, and if evidence is understood to be fallible, then despite the evidence it remains metaphysically possible that there are no Fs.

Since indispensabilism is a fallibilist epistemology, and since fallibilism is sufficient for MME, contingentism plays no crucial role in the argument for thinking that there is something self-undermining going on here. The reason this is significant is that it shows that Miller's argument turns on an objection that can be applied against any kind of fallibilist epistemology. And indeed, it has been; fallibilism in general has been criticised for involving precisely this kind of epistemic self-undoing, by which we assert that we know that p on the basis of e, but that at the same time we grant that it is consistent with e that not-p.[18] So I want to emphasise that Miller's argument does not show that the combination of indispensabilism + contingentism about mathematical objects is somehow an especially or uniquely unstable or self-undermining position. If her argument hinges on MME, as I've argued that it does, then the objection at stake is really a disguised version of a very general concern about using fallibilist epistemologies.

To recap, Miller's objection is that contingentism undermines the indispensabilist's methodology, a methodology which says that on the basis of our best scientific

---

[18] It is no coincidence that Miller's epistemic objection has this family resemblance to Laudan's pessimistic meta-induction argument against IBE; both involve the same kind of 'self-defeat', and indispensabilism is a variety of IBE.

evidence we can come to know about the (mathematical) contents of our world. The matching claim (MME) is supposed to show that if an indispensabilist accepts that it is possible that their evidence is consistent with a completely different ontology, then they cannot come to know how things stand ontologically on the basis of that evidence. But contingentism plays no role in this argument; fallibilism is sufficient to generate MME which is then used to undermine any claim to know that the world is nominalist (or platonist).

Going further, I think the sense that there is something self-defeating going on is not a tenable criticism of indispensable or PQO in general. This kind of fallibilism is an entirely normal part of scientific inquiry, and is recognised as such by anyone who thinks that science involves some kind of non-deductive, ampliative reasoning, like abduction or inference to the best explanation.[19] The challenge of providing a general justification for ampliative reasoning is closely connected to Hume's problem of induction, and so a version of this problem will show up wherever ampliative methods are employed, just as it does here where indispensabilists are doing precisely that. Since I think it highly unlikely that metaphysical inquiry can be conducted without making ampliative inferences, this kind of objection seems all but inevitable. But just because there is this well-known problem for ampliative inferences in general does not mean it constitutes an objection to adopting the combination of indispensabilism + contingentism about mathematical objects in particular.

Conclusion

Miller's argument against the combination of indispensabilism + contingentism about mathematical objects hinges on the matching claim, but if the matching claim is understood as MM it is unmotivated, and is something that indispensabilists can resist. I've suggested that perhaps Miller's intended matching claim is MME, a claim about 'evidential indistinguishability', as it seems to be capable of being applied in Miller's argument, and appears to fit with the way that she motivates the matching claim. But if the epistemic objection depends on MME, Miller's argument fails to show that indispensabilism and contingentism about mathematical objects are in tension. Going further, if MME is the right way to understand this component of her argument, then

---

[19] I note Alexander Bird argues that a key component of scientific method involves a form of inference to the best explanation that he calls 'Holmesian eliminative abduction', or 'inference to the 'only' explanation.' Bird argues that Holmesian inferences are non-ampliative and infallibilist. But he does not think that this is the only form of reasoning employed in science. See Bird (2005) and (2010).

Miller's argument is effectively an instance of a much wider concern about the role of ampliative methods in metaphysics.

I do not take the foregoing to have shown that indispensabilist contingentism (whether nominalist or platonist) is correct. But I have defended the idea that an indispensabilist methodological approach does not undermine itself in virtue of taking seriously the idea that mathematical entities might exist (or not exist) only contingently.[20]

References

Bird A. (2005a). "Abductive knowledge and Holmesian inference". In Gendler T. S., Hawthorne J. (eds) *Oxford studies in epistemology*. (Oxford University Press, Oxford), pp. 1–31.

Bird, A. (2010). "The epistemology of science—a bird's-eye view". *Synthese* 175, pp. 5–16.

Colyvan, Mark (2000). "Conceptual contingency and abstract existence." *The Philosophical Quarterly* 50 (198), pp.87-91.

Colyvan, M. (2001). *The Indispensability of mathematics*. (Oxford: Oxford University Press).

Colyvan, M. (2007). "Mathematical recreation versus mathematical knowledge". In M. Leng, A. Paseau, & M. Potter (eds.), *Mathematical knowledge* (Oxford: OUP), pp. 109-122.

Field, H. (1980). *Science without numbers*. (New York: Blackwell).

Field, H. (1989). *Realism, mathematics and modality*. (Oxford: Blackwell).

Field, H. (1993). "The conceptual contingency of mathematical objects". *Mind*, 102, pp. 285-299.

Hale, R. and Wright, C. (1992): "Nominalism and the contingency of abstract objects". *Journal of Philosophy* 89, pp. 111-135.

Hughes, A., Clark, C. and Jordan, C. (2014). "Flow-pattern evolution of the last British Ice Sheet." *Quaternary Science Reviews*, 89, pp. 148-168.

Knowles, R. and Liggins, D. (2015). "Good weasel hunting". *Synthese*, 192.10, pp. 3397-3412.

Laudan, L. (1981)). "A confutation of convergent realism". *Philosophy of Science*, 48: pp. 19-49.

Melia, J. (2000). "Weaseling away the indispensability argument". *Mind*, 109, pp. 455–479.

Miller, K. (2012). "Mathematical contingentism". *Erkenntnis*, 77 (3), pp. 335-359.

Quine, W.V. (1969). "Existence and quantification" in Quine, W.V. *Ontological relativity and other essays* (New York: Columbia University Press), pp. 91-113.

---

[20] There are some people whose help I'd like to acknowledge here.

Quine, W.V. (1981). "Five milestones of empiricism" in Quine, W.V. *Theories and things* (Cambridge, Mass.: Harvard University Press), pp. 67-72.