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# Attempts by Avicenna and Ibn al-Nafīs to Expand the Field of the Transference of Demonstration in the Context of the Relationship Between Geometry and Medicine*

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**Abstract**

This paper aims to deal with the disputes on transferring demonstration between the various sciences in the context of the medicine-geometry relationship. According to Aristotle's *metabasis*-prohibition, these two sciences should be located in separate compartments due to the characteristics of their subject-matter. However, a thorough analysis of the critical passage in Aristotle's *Posterior Analytics* on circular wounds forces a revision of the boundaries of the interactions between sciences, since subsequently Avicenna, on the grounds of this passage, would widen the area of the transference of demonstration. Furthermore, the fact that Avicenna and Ibn al-Nafīs continued to use geometrical demonstrations in their anatomical investigations shows the need to understand kind-crossing prohibition as a reminder to take into account the present scientific infrastructure and logical rules before proceeding onto a scientific investigation instead of accepting it as a mere nominal doctrine. Therefore, whether kind-crossing was possible or not depended on the extent to which the conclusion derived at the end of the scientific investigation, using a different method after taking into account all these reminders, had contributed to the solution of a particular proposition or the achievement of an approximate truth.

**Keywords:** the transference of demonstration, *metabasis*-prohibition, Avicenna, Ibn al-Nafīs, per se, geometry, circular wounds, anatomy.

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## I. Introduction

For Peripatetic philosophers, the restrictions Aristotle set on transferring demonstration between the genera in an impermissible way must be followed meticulously. This doctrine, known as *metabasis*-prohibition,[1] formed the foundations for demonstrative investigations.[2] According to this doctrine, the investigator of each science should only examine per se propositions specific to the subject of his science. Because different sciences have different subjects, an investigator of one science should not deal with the scientific questions treated by the investigator of another science and should not use the principles used by the latter. However, when one of the two sciences falls under or is part of the other, one can use the other's principles. If no such interrelation exists between the two sciences, no possibility exists for the investigators of these sciences to have anything in common in the study of their subjects.[3] Failing this, errors such as absurdities, paradoxes, and inaccuracies would appear in the derived conclusions.[4] In order to prevent such fallacies, a requirement was put forward in the demonstrative syllogisms where the middle term should hold per se of the minor term, and the major term should hold per se of the middle term.[5] Thus, this restriction based on the concept of per

1 Metábasis eis állo génos (Greek: Μετάβασις εις άλλο γένος): "Crossing to another genus". Although the terms *metabasis*, "kind-crossing" and "transference of demonstration" are already fixed terms in the literature in the studies on philosophy and history of science, given the fact that this issue has not been sufficiently investigated, at first glance these three terms may imply three different meanings. To avoid this misunderstanding, it should be pointed out that the very term "demonstration" corresponds to the process of establishing the truth or justifying the truth of a judgment. In other words, we try to solve a proposition within the boundaries of a certain science using a method specific to another science that is close or foreign to that science. This process or method is briefly called transfer of demonstration (*naql al-burhān*) in the classical tradition of the Islamic philosophy of science. Since the subject of a science is a genus, applying the proof specific to one science to another, naturally creates the transition between the genera. In this context, the term *metabasis* can be regarded as a general one, encompassing both of those terms. Meanwhile, we should also note that studies on Aristotle's ban on kind-crossing began around the 1930s. See: Heinrich Scholz, "Die Axiomatik der Alten", *Blätter für die Deutsche Philosophie*, Band: 4 (1930): 259-278, in English: "The Ancient Axiomatic Theory", *Articles on Aristotle 1: Science*, ed. J. Barnes, M. Schofield, R. Sorabji. London: Duckworth (1975): 50-64. However, despite its significance in the history of Islamic philosophy and science, the problem of kind-crossing is perhaps one of the most neglected fields in the studies on demonstrative investigations. Given the scope of studies on Aristotle's views on transference of demonstration, why no independent study on the reception of kind-crossing prohibition in the Islamic philosophical tradition exists aside from the short sub-sections in a few doctoral dissertations is something in need of explanation. See: İhsan Fazlıoğlu, "Aristoteles'te Nicelik Sorunu" (PhD Dissertation, Istanbul University, Institute of the Social Sciences, 1998), 117-121, 151-152, Mehmet Fatih Birgül, *İbn Rüşd'de Burhân* (Istanbul: Ötüken, 2013), 232-234, Hacı Kaya, "İbn Sina'da Bilimsel Kanıtlama Teorisi" (Istanbul: Marmara University, Institute of the Social Sciences. PhD Dissertation, 2013), 392-398.

2 Fazlıoğlu, *Aristoteles'te Nicelik Sorunu*, 120.

3 Averroes, *Sharḥ al-Burhān li-Arisṭū wa-Talkhīs al-Burhān*, ed. 'Abdurrahmān Badawī (Kuwait, 1984), 333.

4 Sadık Türker, *Aristoteles, Gazzâlî ile Leibniz'de Yargı Mantığı* (Istanbul: Dergah, 2002), 97, 100.

5 Aristotle, *Posterior Analytics*, I.6-7 (75a35-39), Avicenna, *al-Shifā': al-Mantiq: al-Burhān*, ed. A. 'Afīfī.

se tightly linked the terms and premises (i.e., the parts of demonstrative syllogism) with each other without leaving any gaps.

However, the restriction which seemed theoretically perfect, encountered a number of difficulties in its implementation. In particular, empiricists since Antiquity had attributed a nominal nature to Aristotle's prohibition rule and did not always follow it.[6] Moreover, the fact that some issues are found debatable even in Aristotle's own theory of demonstration makes one requestion when demonstration can and cannot be transferred between sciences. While some argue that these question-worthy issues stem from Aristotle's uncertainty about what exactly the subject of a science is,[7] others argue that Aristotle's view of the transfer of demonstration is scattered and his basic invention (i.e., demonstrative syllogism) is a bad tool for shaping arguments in abstract sciences.[8] Others take this claim one step further, arguing that Aristotle's view of science and philosophy is messy.[9] In my opinion, the most prominent problem in Aristotle's theory of demonstration is the case of circular wounds. Historians of philosophy often avoid dealing with it, arguing this case, being an example of the medicine-geometry relationship, to be the only exception to this prohibition. However, only the higher-lower relations in the sciences may be considered as exceptions to the prohibition. If the case of circular wounds was truly an exception, addressing the prohibition would not be an issue.

Relying on the above-mentioned points, I will first make a textual analysis of the passage from Aristotle's *Posterior Analytics* (I.13$_{79a12\text{-}16}$) and argue that Aristotle actually considers many sciences in addition to medicine and geometry to be able to have a relationship where one provides the reason of fact and the other gives the reason why. When looking at the historical background of the issue starting with Hippocrates, one can see that this problem has led to various disputes among physicians since antiquity; as a result, John Philoponus combined Hippocrates' and Aristotle's accounts and tried to explain how plane geometry gives the cause of the slow healing of circular wounds. However, because this kind of explanation

(Cairo: al-Maṭba'a al-amīriyya, 1956), 154, Averroes, *Sharḥ al-Burhān*, 275.

6 Koca argues that Archimedes' physicization of mathematics by treating geometric objects as physical ones to prove his own theorems contributed to the birth of mechanical physics. See: Engin Koca, *Akıl ve Hareket* (Istanbul: Babil, 2019).

7 Steven J. Livesey, *Metabasis: The Interrelationship of the Sciences in Antiquity and the Middle Ages* (PhD Dissertation. The University of California, Los Angeles, 1982), 19.

8 R.J. Hankinson, "Aristotle on Kind-Crossing", *Philosophy and the Sciences in Antiquity*, ed. R.W. Sharples. Aldershot, Ashgate (2005): 54.

9 Lindsay Judson, "Aristotle and Crossing the Boundaries between the Sciences", *Archive für Geschichte der Philosophie* 101, no. 2 (2019): 201-202.

contradicts the rules of demonstration of fact and reason, I will examine Avicenna's and Ibn al-Nafīs' elaborations in their logic books on the fact that the physician's explanation of this case in a geometrical method would contradict the requirements the demonstrative premises and scientific questions need to meet and the reasons for this. However, given the fact that plane geometry has a certain share in this case even though it contradicts the main principles of demonstration, I will show how it forced Avicenna to make contradictory claims and expand the scope of the transference of demonstration.

Even if one continues to argue the case of circular wounds to be the only exception and should not be overly focused upon,[10] the use of geometrical and even arithmetical demonstrations in anatomical investigations, which also started with Hippocrates, necessitates quite a serious revision of the relationship between medicine and geometry. Therefore, I will discuss in the second part of the article the elaborations Avicenna and Ibn al-Nafīs made about why some parts of the human body, especially the braincase, are spherical. While Hippocrates and Galen maintain the human cranium to be spherical in shape and content themselves by just giving the reason of fact, Avicenna and Ibn al-Nafīs attempt to make a mathematical model of it and look for the reason why from geometry. Therefore, I will raise the question of whether knowing geometry is necessary for a physician in his/her capacity as a physician, and argue that the boundaries between sciences

10 The debate about whether the shape of wounds has an effect on the healing time is not restricted only to the history of philosophy of science, but is currently an ongoing issue in dermatological medical surgery. Billingham and Russell, by examining the effect of the size and shape of the open wound on the dynamics of the healing process, concluded that circular wounds heal more slowly than straight-edged wounds. See: R.E. Billingham, P.S. Russell, "Studies on Wound Healing, with Special Reference to the Phenomenon of Contracture in Experimental Wounds in Rabbits' Skin", *Annals of Surgery* 144, no. 6 (December 1956), 979. Watts argues that a circular wound does not heal at the same rate as an asymmetric wound, because the circumference of the square wound only varies between 10 and 25 percent during wound closure, while the circumference of the circular wound should contract as the wound heals. See: G.T. Watts, "Wound Shape and Tissue Tension in Healing", *The British Journal of Surgery* 47 (1960), 560. MacGrath and Simon argued that regardless of the size or shape of a wound, the constant of the rate of contraction does not change in animals matched with age and species, that different shaped wounds of the same size contract at the same rate during the healing period, and that the shape will show its effect before the onset of contraction period by defining the size of the wound entering the contraction period. See: McGrath, Mary H., Simon, Richard H., "Wound Geometry and the Kinetics of Wound Contraction", *Plastic and Reconstructive Surgery* 72, no. 1 (July 1983), 72. Zitelli suggests that the healing time depends on the diameter of the largest circle that can fit into the wound edges. See: John A. Zitelli, "Wound Healing by Second Intention", *Roenigk's Dermatologic Surgery: Current Techniques in Procedural Dermatology*. 3rd edition, ed. Randall K. Roenigk, John Louis Ratz, Henry H. Roenigk, Jr. (New York: Informa Healthcare, 2007), 503-517. Wang Jin et al. claim that the mechanisms that direct wound closure are somewhat dependent on the initial wound shape. See: Jin, Wang, Lo, Kai-Yin, Chou, Shih-En, McCue, Scott, W. and Matthew J. Simpson, "The role of initial geometry in experimental models of wound closing", *Chemical Engineering Science*, 179 (2018), 226.

become blurred when certain scientific questions and definitions that are to be proven in the sciences become the object of investigation alongside the subjects and per se accidents.

## II. Aristotle's Account on Circular Wounds

In the *Posterior Analytics*, Aristotle, after explaining the functioning mechanism of the superordinate and subordinate sciences and the mathematical sciences' task of providing theorems in subordinate sciences with reason of why, claimes that medicine and geometry as well can be in such a relationship:

> Many sciences which do not fall under one another, such as geometry and medicine, are in fact related in this way; for it is for the physician to know the fact that circular wounds heal more slowly, and for the geometer to know the reason why.[11]

Aristotle says no more on this matter, nor does he explain it in detail. He probably picked up this example from Hippocrates, since Hippocrates states that wounds with circular, concave, and similar surfaces should be cut at two opposite points of the circumference in proportion to the human's height to make the wound longer.[12] Although Hippocrates does not clearly state the underlying reasons for the lengthening of circular wounds, that it was applied by doctors to form angles in circular wounds is understood from John Philoponus' comments on this passage.[13] However, the reason why Aristotle does not go into detail by giving this example is still unclear. Barnes emphasizes that the example Aristotle gave completely contradicts his thesis about essentiality in the *Posterior Analytics* I.7,[14] but his emphasis has not found much place in the scientific community. To be in complete contradiction with the thesis about essentiality means to contradict the whole theory of demonstration. From my point of view, the critical sentence that poses a danger to Aristotle's entire theory of demonstration is as follows:

> Many sciences which do not fall under one another, such as geometry and medicine, are in fact related in this way.[15]

What does Aristotle mean by this assertion? Does he suggest that other examples

11 Aristotle, *Posterior Analytics*, I.13 (79a12-16).

12 Hippocrates, "On Wounds in the Head", *Hippocrates*, tr. E.T. Withington. 3 (Massachusetts: Harvard University Press, 1959), 31.

13 Philoponus, *On Aristotle: Posterior Analytics. 1.9-18*, tr. Richard McKirahan (London: Bloomsbury Academic Publishing, 2012), 80.

14 Aristotle, *Posterior Analytics,* tr. J. Barnes (Oxford: Clarendon Press, 1993), 160.

15 Aristotle, *Posterior Analytics*, I.13 (79a12-13).

besides the relation between medicine and geometry can be found? This account from Aristotle in fact justifies arranging the relationships among the sciences as follows:

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| i) | geometry<br>↓<br>science of animals | : | Some geometrical demonstrations can be referenced to explain why animals' movements require its legs or entire body to bend.[16] |
| ii) | arithmetic<br>↓<br>science of animals | : | Arithmetical demonstrations can be used to explain why some animal species have two legs and others have four. |
| iii) | geometry<br>↓<br>cosmology | : | Some plane figures and their properties can be argued to explain the properties of the material bodies they form.[17] |
| iv) | natural science<br>↓<br>geometry | : | Just as geometry is included in explaining propositions specific to natural science, natural science can also be used to explain some scientific questions in geometry or to define geometrical figures.[18] |

16 Nussbaum claims that Aristotle in his treatise *On the Movement of Animals* violated his own *metabasis*-prohibition by using geometry as a principle in animal science. However, Kung and Angioni argue that Aristotle used geometry not as a principle to explain the movement of animals, but only as a secondary tool, so he did not violate the ban on kind-crossing. See: Aristotle, *De Motu Animalium*, tr. M.C. Nussbaum (Princeton: Princeton University Press, 1978), 112. Joan Kung, "Aristotle's *De Motu Animalium* and the Separability of the Sciences", *Journal of the History of Philosophy* 20, no. 1 (January 1982): 65-76. Lucas Angioni, "Geometrical Premises in Aristotle's *Incessu Animalium* and Kind-Crosing", *Anais de Filosofia Clássica* 12, no. 24 (2018), 66.

17 Aristotle does not think that the elements of the material world are made up of polyhedral bodies and therefore says that the use of geometry in their proof would violate the *metabasis*-prohibition. See: *De Caelo*, III.7 (306a1) - III.8 (307b18). However, Neo-Platonic philosophers such as Simplicius and Proclus regard plane figures as natural figures rather than mathematical. Moreover, even Ibn al-Ṣalāḥ, a Muslim thinker, adopts the view of geometrical atomism and criticizes Aristotle himself on this issue. See: Philipp Steinkrüger, *Aristotle's metabasis-prohibition and its Reception in Late Antiquity* (PhD Dissertation. KU Leuven, 2015), 217-218. Mubahat Türker, "İbnü'ş-Ṣalaḥ'ın *De Coelo* ve onun Şerhleri Hakkındaki Tenkitleri", *Araştırma*: Ankara Üniversitesi Dil ve Tarih-Coğrafya Fakültesi Felsefe Araştırmaları Enstitüsü Dergisi 2, 1964: 1-79.

18 Although the use of motion which is one of the characteristics of natural science in geometry, is frequently emphasized as only able to occur in the imagination, we see that neither ancient nor Islamic thinkers complied with this when they defined geometrical figures such as spheres, cones, cylinders or when examining whether some geometrical figures are equal. See: Euclid, *The Thirteen Books of Euclid's Elements* 3, tr. T.L.Heath (Cambridge: Cambridge University Press, 1908), XI.18, XI.21, 261-262. Avicenna,

According to Steinkrüger, Aristotle's purpose of giving this example is probably hidden in the fact that the circle has a larger area relative to its perimeter. Therefore, the contraction process of the circular wound (i.e., healing) takes longer than the contraction process of the incised narrow wound. A physician knows from experience that circular wounds heal more slowly than other types of wounds without knowing the theorem of the relation of the area to circumference. However, one of the reasons why this does not follow the rules as in the geometry-optics and arithmetic-music relations is probably because the relation of the wound to the circle is different from the relation of the light ray to the line. The propagation of light along a straight line is what initially allows optics to refer to geometry. Wounds as a genus, however, do not contain the form "circular" in this way. Whether the wound is elliptical, circular or rectangular is completely accidental. Therefore, the relation of the genus of wound to any geometrical figure is weaker than the relation a light ray has to the line. However, the question of how to find the reason for the slow healing of circular wounds remains unanswered. According to Aristotle's account, the physician knows only of its fact, while the geometer only knows the proposition of the relation of the area to the perimeter but is not concerned with wounds. In fact, as Aristotle himself points out, this case contains no ontological relation between the two, unlike the example of the geometry-optics relation. However, the slow healing of circular wounds compared to other types of wounds is not entirely accidental. Steinkrüger concludes that, as Aristotle himself did not explain this issue in detail, anyone who attempts to explain it on his own would have difficulty.[19]

*The Metaphysics of The Healing*, tr. Michael E. Marmura (Provo, Utah: Brigham Young University Press, 2005), 111. Omar Khayyām, "Traktaty" ("Treatises"), *Pamyatniki Literatury Narodow Wostoka. Texty. Malaya Seriya. II (Literary Monuments of the Nations of the East. Texts. Small Series. II)*, tr. B.A. Rozenfeld, Preface and comments by B.A Rozenfeld and A.P.Youshkevich (Moscow, 1961), 276.

19 Steinkrüger, *Aristotle's metabasis-prohibition*, 115-116. This explanation has led to controversies since ancient times with respect to both reason of fact and reason why. Cassius Iatrosophista (2nd or 3rd century AD), one of the ancient Greek physicians, argues that the physicians who followed the line of Herophilus (335-280 BC) explained why circular wounds were more difficult to heal than other wounds by geometrical demonstrations. Because although the circumference of circular wounds seems small to them, it is not so; on the contrary, they have greater areas than they appear. Therefore, scar tissue requires more time to form, and it turns out that such wounds hardly heal even if they seem small. Iatrosophista maintains that Asclepiades (120-40 BC) explained this case in a different way. Asclepiades suggests that if a lateral incision is made in the body where the circular wound is located and the shape of the wound becomes longer as a result of the cross incision, the scar tissue forms faster. In other words, Asclepiades, as Hippocrates, tries to explain this case relying on empirical foundations. According to Iatrosophista, this is the opposite of what Herophilus thought because if, as claimed, the size of the wound is the cause of the difficulty in treating, these wounds must become more difficult to heal when they are the same size and as a result of the crosscut. See: Herophilus, *The Art of Medicine in Early Alexandria*, ed., tr. Heinrich von Staden (Cambridge: Cambridge University Press, 1989), 411-412. Thus, following the line of Herophilus, physicians advocated the way of explaining the slow healing of circular wounds with a geometrical demonstration, and claimed that this was a function of circular

According to John Philoponus, who commented on Aristotle's passage on circular wounds, a circle contains the greatest area of any of the isoperimetric figures; for among isoperimetric figures, those with more sides always contain a greater area and the circle as the limit of polygons contains the greatest area of all figures. Philoponus claimes that this cause was not stated correctly, because when considering two non-isoperimetric wounds, a circular wound that has a smaller area than larger one with straight edges will still form a scab slowly. This is because the healthy parts are not close to each other, but far apart, and therefore nature has difficulty in bringing them together. Philoponus, following Hippocrates, claims that that is why physicians cut circular wounds to change the shape, since the distance between healthy parts is short where there are angles, nature can combine healthy parts and form a scab, because they are not far from each other. Philoponus states that, when considering medicine and astrology, another example can be given for the argument in question. According to him, the doctor says the seventh day is critical, but does not know why; however, the astrologer knows the reasons for this case, namely the quarter phases of the Moon.[20] Although Philoponus does not go into much detail, one may assume from his reasoning that astrology alongside geometry provides the reasons why for some propositions in medicine, and thus he accepts the interrelation between medicine and geometry as quite normal.

Hankinson argues that this type of medicine-geometry relation does not violate the ban on kind-crossing. According to him, the explanation that gives the reason why for the slow healing of circular wounds has a geometrical content, but as in the subordinate sciences such as optics, this content is only formal. The premise is particularized by the subject, and the formal principle of the ratio between perimeter and area becomes peculiar to medical science; in other words, only circular wounds heal in that way. Their slow healing is not a necessary situation but an empirical fact

wounds having the smallest perimeter/surface area ratio of any planar shape, and this is probably the explanation Aristotle envisioned. Hankinson points out that the geometrical fact that the circle itself has the smallest perimeter ratio is insufficient to make any medical explanation and needs more guidelines to show why it is relevant to the state of healing. The healing rate is proportional to the perimeter/area because wounds heal with the growth of the material at the edges, which will be a function to fill the edge, the perimeter, and the amount of the area. All subsequent explanations will be proper to medical science. See: Hankinson, *Aristotle on Kind-Crossing*, 50-51.

20 Philoponus, *On Aristotle: Posterior Analytics. 1.9-18*, 80. Hankinson doubts whether Philoponus' explanation is authentic, not taking into account that the slow healing of circular wounds was first suggested by Hippocrates and that Philoponus used Hippocrates' explanations to comment on Aristotle's passage. Nevertheless, Hankinson, moving from Philoponus' way of reasoning, says that we need another explanation because wounds are not two-dimensional planar surfaces unlike skin lesions, but this is in itself enough to suggest that the slow healing of circular wounds is not directly related to plane geometry. See: Hankinson, *Aristotle on Kind-Crossing*, 50.

obtained through experience and observations. Hankinson tries to explain this in a question-answer fashion by emphasizing how the explanation that gives the reason why works here in two separate stages, one medically appropriate and one medically inappropriate: Why do circular wounds heal slowly? Because the circle has the smallest perimeter-to-area ratio. Why do circles have the smallest perimeter-to-area ratio? Since it is a geometrical fact demonstratable on circles. This is the sense in which the superordinate discipline provides the reason why for a scientific question in the subordinate discipline. According to Hankinson, this does not mean that geometry gives the cause of all propositions within the real context of medical science, and so medicine is really an independent science subject to Aristotle's own theoretical restrictions. Thus, Hankinson concludes that even this medical case does not violate Aristotle's own restrictions on the illicit transfer of demonstration.[21]

## III. Contradictory Claims by Avicenna and Ibn al-Nafīs

### *a) The Syllogism Formed from the Premises Belonging to Medicine and Geometry is not Demonstrative*

Because Aristotle himself does not explain in detail the issue of circular wounds' slow healing, Avicenna falls into a difficult situation that forces him to make contradictory claims in *al-Burhān*. He says that in order for the premises in the demonstrative

21 Hankinson, *Aristotle on kind-Crossing*, 51. At this point, however, if the use of the geometrical demonstration to explain the slow healing of circular wounds means that it does not violate the kind-crossing prohibition, one might ask what the rules of restriction actually mean then. Crossing between the genera as prohibited by Aristotle is not possible, but it seems that this type of transition is actually characterized as something difficult to do. According to Hankinson, this is not exactly true, but what matters is the conditions under which the content derived from other sciences is tied. Bryson's mistake, for instance, when he thought that he proved something only through a geometrical principle, is that he used what was derived quite generally from the science of quantity. Bryson was unaware of the precise status of the principle in question. Of course, the physician does not have to investigate the geometry of the circle in his own science, but it is sufficient to know it only in terms of fact, although he should know it anyway. Hankinson believes that the physician's use of the perimeter/area theorem as a reason of fact can also be used to provide the reason why about wounds, so that the explanation of a geometrical truth can be reduced to geometry quite neatly. So according to him, the explanation for the slow healing of circular wounds by the geometer does not mean that there is a hierarchical relationship between medicine and geometry. While it is possible to allow medicine to benefit from physiology, biology, biochemistry, molecular biology, chemistry, and perhaps even physics, it is not possible to talk about the existence of such a thing as geometrical medicine similar to geometrical optics. Hankinson says this is probably because the case is exceptional, meaning that medicine generally does not use geometry to make explanations within the boundaries of its field. Because there is no reasonable geometric medical science, that is, in medical science, unlike optics and mechanics, there is no part that only applies geometrical reasoning. There is only a probable situation here, where it is claimed that geometry gives the physician the reason why circular wounds heal slower than any other shape of wounds. See: Hankinson, *Aristotle on kind-Crossing*, 52, 50.

syllogism to be really demonstrative, they must be related to the conclusion and therefore be included in the integrity of the knowledge where the conclusion is found.[22] He understands the expression "the appropriateness of a premise to the conclusion" as "one or both of the premises in the syllogism not being from a foreign science."[23] Given that premises are also made up of terms, the necessity of the predicates for these terms to be per se reveals the main structure of the demonstrative syllogism as a unity consisting only of per se accidents.[24] In such unity, the premises naturally become related to the conclusion and cause of the conclusion. Therefore, when trying to explain the reason circular wounds heal slowly using the properties of the circular figure, a geometrical premise unrelated to the conclusion is included in the syllogism of medical science:

($P_1$) The circle has the most encompassing shape.

($P_2$) Wounds with the most encompassing shape heal slowly.

Therefore,

(C) Circular wounds heal slowly.

Even though a syllogism like this is problematic and cannot be qualified as a real demonstration, the correct conclusion is derived.[25] However, if the correct premises are unrelated to the genus under study and are added externally, then even if speaking of a certainty is possible, it would not explain anything in terms of scientific certainty. These correct premises do not attest to the causes as the causes must be compatible with the object under study. Because investigation in any discipline is always an examination of causes and hence obtaining a necessary certainty is impossible unless the causes are understood.[26] Avicenna argues that the truth of their premises does not provide necessity or reason, but only truth itself, whereas not every truth is related or specific unless it is necessary.[27] Apart from this, the fact that the premises are unrelated to the conclusion also means a direct

22 Avicenna, *al-Burhān*, 106.

23 Avicenna, *Kitāb al-Najāt fī al-ḥikma al-manṭiqiyya wa-l-ṭabīʿiyya wa-l-ilāhiyya*, ed. Majid Fakhri (Beirut: Dār al-Āfāq al-Jadīda, 1985), 106.

24 For detailed studies on Avicenna's views on per se, see: Riccardo Strobino, "Per Se, Inseparability, Containment and Implication. Bridging the Gap between Avicenna's Theory of Demonstration and Logic of the Predicables", *Oriens* 44, 2016, 181-266, Fedor Benevich, *Essentialität und Notwendigkeit: Avicenna und die Aristotelische Tradition* (Leiden-Boston: Brill, 2018), Ömer Türker, *İbn Sînâ'da Metafizik Bilginin İmkânı* (Ankara: TDV, 2019), 293-325.

25 Avicenna, *al-Burhān*, 106.

26 Ömer Türker, *İslam'da Metafizik Düşünce: Kindî ve Fârâbî* (Istanbul: Klasik, 2019), 138.

27 Avicenna, *al-Burhān*, 151.

violation of the rule that the middle term, which plays a major role in the permissible transfer of demonstration, must hold per se of the minor term and the major term must hold per se of the middle term, despite the correct conclusion being derived.

Ibn al-Nafīs, in two sections of *Commentary on the Leaflets on Logic* (*Sharḥ al-Wurayqāt fī al-manṭiq*), his book on logic in which he describes the premises and theorems of demonstration, also states that explaining the reasons for the slow healing of circular wounds using plane geometry is incorrect. In order to clarify why this kind of explanation is wrong, he first gives the general definition of demonstration, defining it as "the argument of the certainty of the conclusion" (*ḥujjat yaqīniyyat al-intāj*), which undoubtedly is syllogistic in form. Since the purpose of demonstration is to express truth precisely, its premises must be absolutely true. For the doubts on the correctness of any premise of a demonstrative syllogism will naturally raise doubts on the accuracy of the conclusion. In addition, the premises must be known before the conclusion, as the information obtained by the conclusion is derived from them. Something that is known beyond a certainty is surely certain because all of its premises must be true. In this case, one of the two premises should be fixed. Designing the premise of a demonstration in such a way that it confirms the two opposites at the same time is not possible, as the premises are the cause of the knowledge obtained through the conclusion, and the cause is prior to what is caused. Therefore, the premises of a demonstration should come first with respect to knowledge and time. In the demonstration of reason, the middle term becomes the cause of the conclusion per se (*fī nafsihā*). Therefore, while it should be prior in terms of knowledge and time, it should also be prior per se. Given the fact that the middle term in the demonstration of reason is appropriate to the cause and the cause is appropriate to what is caused; the demonstration of reason will hence have to be appropriate. The appropriate demonstration (*al-burhān al-munāsib*) is a demonstration whose middle term holds per se of the minor term, the minor term is the subject of the conclusion, and the subject of the conclusion is either the subject of the discipline or the attributes of its subject. In this case, the middle term must be from the same science or from a science common to the subject. Ibn al-Nafīs defines any demonstration whose structure is not as described above as an inappropriate demonstration. For example, when the doctor says that circular wounds heal slowly because the circle is the largest figure, he leaves the medical discipline and starts doing geometry.[28]

28 Ibn al-Nafīs, *Sharḥ al-Wurayqāt fī al-manṭiq*, ed. ʿAmmār Ṭālibī, Farīd Zaydānī, Fuad Malit (Tunis: Dār al-Gharb al-Islāmī, 2009), 270-271.

Ibn al-Nafīs maintains that explaining the reason for the slow healing of circular wounds in that way contradicts not only the building style of the premises in the main structure of the demonstrative syllogism, but also the way of setting up the demonstrative propositions. He starts by giving a general definition of the demonstrative proposition as he did in the premises above. Ibn al-Nafīs states that the things demanded in the sciences are called propositions (*maṭālib*) and are specific to each science that requires proofs due to the possibility of doubt. Each scientific question is a specific proposition divided into either a predicated, contiguous conditional or discrete conditional proposition. Each predicated proposition has a certain subject and predicate. Subject of the question (*maṭlab*) is called data, the thing that is assumed because exactly what the subject of the proposition is; *maṭlab* is assumed first and its predicate is required. The predicate of the proposition is called the required thing (*maṭlūb*) because it is required to be either proven or rejected. The data on each proposition consists of either the sum of the subjects of the discipline or the sum of its per se attributes. Otherwise, just as the physician requires that the sphere to be the largest figure, the proposition will be distant and unrelated to the discipline and becomes unusable even as a proposition.[29]

### *b) The Distinction between Mathematics and Medicine*

The case of circular wounds again raises the question of exactly what principles medicine uses. Avicenna clearly maintains that the real principles of medicine should be based on are the rules of natural science:

> Physician in his capacity as a physician should imagine some of these things only in quiddity as scientific conceptions and confirm their existence by the fact that they are postulated for his acceptance by experts in natural science; physician is obliged to demonstrate other things within his discipline. Speaking about those things that have similar principles, the physician must affirm their existence unconditionally, for the principles of the particular sciences are indisputable and are proven and explained in the other sciences that are prior to them. This is how it goes, on and on until the principles of all sciences rise to the first philosophy, which is called the science of metaphysics. Some of those who pretend to be a physician is mistaken when he attempts to prove the elements, temperaments, and similar things that follow them which are actually the subject of natural science, because he includes in the medicine that which is not from the medical discipline. He is also mistaken regarding his belief that he explained something, while he has explained nothing at all.[30]

29 Ibn al-Nafīs, *Sharḥ al-Wurayqāt fī al-manṭiq*, 278.

30 Therefore, according to Avicenna, Galen tried to demonstrate the theoretical part of medicine as a philosopher speaking about natural science, not in the capacity of a physician. In the same way, a jurist, trying to substantiate why it is necessary to follow the unanimous decision of the authorities, can do

The discussion of seasons clearly shows that the real principles that medicine should be based on are actually the principles of natural science, not mathematics. This is because a scientific investigator needs to make appropriate a particular attribute and turn it into a per se accident specific to his own science; for example, the physician examines the effects of seasons on human health differently than the mathematician astronomer. In other words, seasons are not same for physicians and astronomers. As for astronomers, the four seasons are periods of the sun's successive transition from one quarter of the zodiac to another, starting from the vernal equinox. Meanwhile, physicians define spring as a period when no need exists for a significant amount of heat due to the cold, as a significant amount of coolness due to the temperature in temperate countries, and as the time when trees start to grow. Astronomers define it as the period between the vernal equinox, which may start a little early or late, and the point where the sun enters halfway into Taurus. Consequently, spring as defined by astronomers is of no use to physicians with respect to the effect it has on human health; as such, the physician has to define this season in a way that is useful to the physician's discipline. Likewise, the entire period of heat is called summer, and the entire period of cold is called winter. Thus, the duration of spring and autumn is shorter than summer and winter, according to physicians' calculations.[31]

As a result, the knowledge of seasons' durations obtained through mathematical calculations is of no use in natural science, especially in its sub-division, medicine. Accordingly, the natural scientist or physician defines summer as the hot season when wetness evaporates due to the severity of the heat, when the substance of the air

this not as a jurist, but as a theologian. However, a physician in the capacity of a physician and a jurist in the capacity of a jurist cannot prove the principles of their own sciences, since this will create a vicious circle. See: Avicenna, *al-Qānūn fī al-ṭibb*, I, ed. Muḥammad Amīn al-Zanāwī (Beirut: Dār al-Kutub al-ʿIlmiyya, 1999), 15.

31 Avicenna argues that the days being hot and dry due to the closeness of the sun to the zenith is postulated as astronomical definition of the summer, and the strength of the rays emanating from it, which in summer appear to be reflected either at a very acute angle, or returning back along the same lines along which they fell. In this case, the rays seem to thicken. However, according to Avicenna, the real explanation for this is that the place where the sun rays fall is like a cone with a cylindrical axis, and the cone itself emerges from the center of the sun by falling on the opposite thing. Sometimes, the place where the sun's rays fall is like a surface, circle or circle-like figure. The strength of the sun's rays is more intense on the axis because their effect is directed towards the axis from all sides, while those that fall on the sides of the cone are weaker. Avicenna maintains that we are on or near the axis of the cone in summer and that this period lasts longer for people living in northern latitude, and that we are on the edges in winter. Therefore, the sun's lights are brighter in summer, although the distance between our location and the sun closer to its climax is longer. Avicenna says that this kind of distance and proximity is treated in the mathematical astronomy section of philosophy, and the increase in temperature due to the intensity of the sun's rays is investigated in natural science section. See: Avicenna, *al-Qānūn fī al-ṭibb*, I, 114-115.

begins to dilute and resemble the nature of fire, and when condensation and rain are less common. On the other hand, winter is defined as the cold and wet season with characteristics opposite that of summer. Another reason why the astronomer and the physician define the four seasons differently is that different types of diseases emerge in seasonal changes in each climate, so knowing the characteristics of each season is necessary in order to ensure the physician takes the required precautions and determines treatment methods based on precise knowledge.[32]

Another reason why medicine and geometry are included in the list of sciences that do not fall under one another is that mathematics and natural science study objects from different perspectives. Natural science deals with things that also fall under the scope of mathematical examination but with respect to their being perceptible. For all the things that mathematics investigates contain the nature of the kinds of categories required in them. For this reason, mathematics investigates them with respect to being abstracted from all sensible objects, while natural science investigates with respect to these being kinds of things that can be sensed. Regarding the causes of sensible objects, mathematics should thus only content with what each thing is, whereas natural science gives all the causes for all things within the scope of its study. In other words, while natural science examines the answers to the questions of *what is it? why?* and *how?* in each of the objects within the scope of its study, mathematics does not address objects that exist outside of the categories in the question *what is it?* with respect to each of the objects where mathematics provides its quiddity. Since mathematical objects consist only of forms, searching for answers apart from the question *what is it?* is impossible for mathematicians.[33]

Avicenna's method of explaining some issues in medicine using geometrical demonstrations, which he acquired from Galen, directly contradicts the basic distinction between mathematics and natural science. Moreover, this type of method goes against Avicenna's own words in his work *Physics*, where he states, "Pure geometry does not share in common anything with natural science in scientific theorems."[34] One might argue, saying "Even if geometry has nothing in common with natural science in theorems, natural science can have a relationship with geometry in propositions because medicine is a part of natural science." However, neither medicine nor its sub-branch anatomy nor dermatological surgery (which deals with circular wounds) is included in the list of the subordinate sciences containing an

32 Avicenna, *al-Qānūn fī al-ṭibb*, I, 115-117.

33 al-Fārābī, *Kitāb al-Ḥurūf*, ed. Muḥsin Mahdī (Beirut: Dār al-Mashriq, 1986), 67-70.

34 Avicenna, *The Physics of The Healing*. Book 1, tr. Jon McGinnis (Provo, Utah: Brigham Young University Press, 2009), 54.

interrelation of geometry (or arithmetic) and natural science. As was mentioned, some of the principles of subordinate sciences contain principles of natural science while others contain principles of geometry or arithmetic. However, this does not mean that all subordinate sciences share something in common with natural science in regard to scientific questions. This is because the pure mathematical thing is abstract and definitely not material, and what is natural is the place where this abstract thing takes place over a specific matter. Another feature of subordinate sciences is that they have common propositions that both mathematicians and natural scientists deal with, but the way these two prove these common propositions is different.[35] However, the fact Avicenna's geometrical explanations used in human anatomy must be regarded as invalid because they themselves attribute a quality to planar geometrical figures that are constituted of forms and do not have a specific purpose makes accepting dermatological surgery or anatomy as a subordinate science consisting of an interrelation of medicine and geometry impossible. Moreover, the case of the healing of circular wounds constituting a single exceptional example does not mean that dermatological surgery should be considered as a mixed science of natural science and geometry.

Avicenna says that when the mind imagines the magnitude of itself, it does not need to imagine it in matter because it does not need matter to imagine and define magnitude, which is a subject for geometry. For this reason, when something is demonstrated in arithmetic and geometry, no need exists to turn to natural matter or to take the premises that refer to matter in any way.[36] However, although Avicenna considers the use of matter impossible in the abstract sciences, he also criticizes those who consider putting aside matter and dealing with form in natural science to be necessary; Avicenna maintains that their views are wrong.[37] Thus, although

35 Avicenna, *Physics I*, 55-56.

36 Avicenna, *Physics I*, 57. However, even Avicenna himself contradicts his arguments above when he defines some geometrical figures and demonstrates some theorems. For example, he defines cylinder as follows: "When a circle is moved, a straight line with one end of which is the center of the circle in the initial position is linearly necessary in this movement," and "the cone is formed when you move a right-angled triangle around one of its two perpendicular sides, maintaining the circle's center with the end of that side and moving the other side along the circumference of a circle". See: Avicenna, *Metaphysics*, 111. Cf. Euclid, *The Thirteen Books of Euclid's Elements*, XI.18, XI.21, 261-262. While Avicenna gives the generic definitions of cylinder and cone, he avoids defining the sphere in such way. In addition, Avicenna adopts the method of superposition, which is interpreted as a kinematic process, that is, the rigid movement of figures, in Euclid's proposition I.4 in the *Elements* to prove that two triangles whose sides and angles encompassing these sides are equal to each other, will also have equal bases and angles. See: Avicenna, *al-Shifā': Uṣūl al-handasa*, ed. Abdulhamid Sabra and Abdulhamid Luṭfī (Qum: Maktaba Samāha Ayātullah al-'Uzma al-Mar'āshī al-Najafī al-Kubrā, 2012), 25-26.

37 Avicenna, *Physics I*, 60.

matter is not completely excluded from investigation, the view that investigating form in natural science or its sub-divisions (i.e., to examine why the matter under study is in a certain form by including arithmetic or geometry) does not fall within the scope of the study of natural science.[38]

### *c) The Syllogism Constructed from Premises Belonging to Medicine and Geometry is Demonstrative*

Avicenna in fact could have contented himself with expressing a syllogism that includes a geometrical premise to be an "unreal demonstration, for its premises are unrelated to the conclusion." However, based on Aristotle's account that "Many of the sciences that do not fall within the scope of one another are related in this way," it is thought to be a real demonstration. In addition, Avicenna does not directly criticize Aristotle for not explaining this case in detail. As a result, Avicenna had to remedy the situation in the following pages of *al-Burhān* by synthesizing natural science and geometry in some way in order for geometry to provide the reason for why circular wounds heal slowly, completely contradictory to his opinion from the previous pages. Avicenna claims that this kind of synthesis corresponds to the third form of interrelations between sciences; in this case, not a whole or a certain part of a science, but only a specific issue (*mas'ala*) will fall under another science. Thus, the subject of science becomes proper by conjoining a distant accident; then the per se accident will be required to have the following:

> Taking into account that the healing motion is generally toward the center, by giving a component cause based on natural science and geometry, when an angle is present, the direction of the motion is determined and thus meeting becomes easier; when there is no angle, the movement spreads to the whole edges, the parts resist and healing slows down.[39]

We can express this in a syllogistic chain:

($P_1$) Circle as being the most encompassing figure has no angles.

($P_2$) Because the healing motion is toward the center, healing becomes easier when an angle exists.

38 Avicenna presents as an example the sphere that meets a flat surface at a point and emphasizes that it would be wrong to claim that this could occur in natural reality, since this makes the laws of nature dependent upon certain mathematical abstractions of the estimative faculty, which is not right. Because beyond going outside the discipline of natural science, that argument does not entail what was wanted to be proven. The continuity of the meeting of sphere with a flat surface is impossible only in natural cases other than estimations, and it is not impossible to comprehend it in imagination. See: *Physics*, 2009, 456.

39 Avicenna, *al-Burhān*, 208. Turgut Akyüz, *Fahreddîn er-Râzî'nin el-Mantıku'l-Kebîr'inin Tahkik ve İncelemesi* (Marmara University Institute of Social Sciences. PhD Dissertation. Istanbul, 2017), 973.

Therefore,

(C) The healing is not facilitated in circular wounds.

It should be noted that the middle term in the above syllogism (i.e., the presence/absence of an angle) is not actually a cause compounded from natural science and geometry as Avicenna claims, but rather it is a completely geometrical cause; even $P_1$ is a complete geometrical premise.[40] Avicenna tries to construct the syllogism as follows: An accident beyond the subject of medicine (i.e., circularity of a wound) may occur. This in turn will require a per se accident (i.e., difficulty of healing). According to Avicenna, the subject of the wound thereby becomes appropriate and requires difficult healing by being conjoined with circularity; in this case the reason why comes not from medicine, but from the science circularity is based upon: geometry.[41] I think, Avicenna's argument that the subject of a discipline becomes appropriate by being conjoined with a distant accident and becomes appropriate for acquiring the per se accident is perhaps the most critical argument. Because, relying on this passage, making demonstrative explanations at the level of scientific propositions by transferring the premise of the demonstrative syllogism specific to a particular discipline to the science that does not fall within the scope of that discipline becomes possible. Therefore, one can advocate that some arguments that had been criticized for violating the ban on kind-crossing

40 The anonymous referee claims that we do not correctly express Avicenna's syllogism. According to him, "the movement of healing is towards the center" is the natural cause, "the presence of an angle" is the geometrical cause and thus Avicenna derives this cause from both natural science and geometry. The anonymous referee says that the syllogism should actually be expressed as follows:
($P_1$) When the healing movement is towards the center and the wound does not have an angle, the healing slows down.
($P_2$) In the circular wound, the healing movement is towards the center and there is no angle.
Therefore,
(C) Circular wound heals slowly.
The anonymous referee says that this syllogism is a conditional syllogism, that the minor premise is the affirmation of the antecedent, and the conclusion is the affirmation of the subsequent. He also maintains that the pure geometrical premise in the major premise, "the circle is the most encompassing figure and does not have an angle", is used by being appropriated to medical science and that it provides the reason why for the case (difficult healing of the wound) in medicine by its appropriated form, since the premise, in any case, does not enter into the syllogism in its absolute form. However, it should be noted that the natural cause that the referee points to, that is, the "healing movement towards the center" of any wound is actually a demonstration of fact. The main issue discussed here is why circular wounds heal slowly. One of the per se properties of the circle, "the absence of an angle" gives the real reason, that is, the cause of the slow healing of circular wounds, and serves as the middle term at the same time. Therefore, Avicenna's statement "by giving a reason compounded from natural science and geometry" should not mislead us. Although a kind of medicine-geometry relation is mentioned here in order to explain a certain case in a satisfactory way at least, it cannot be said that a reason can be formed from the combination of nature and geometry. As I pointed out above, the purpose of Avicenna's statement is, in fact, an effort to "save" the complicated situation that occurred due to the obscurity of Aristotle's statements.

41 Avicenna, *al-Burhān*, 208.

(e.g., the method of superposition Euclid used to prove the propositions I.4, I.8, III.24, and III.26 in the *Elements*) do not actually violate it.[42]

So, what does "not the whole or a certain part of a science, but a specific issue falls under another science," which Avicenna includes in the case of slow healing of circular wounds, actually mean? Suppose that science A has subject $A_1$; namely, in science A, the per se accidents of $A_1$ are examined. So, what does the issue (*mas'ala*) refer to? The issue is a proposition whose predicates are per se accidents of $A_1$ or its types.[43] In simpler terms, the issue falls within the scope of $A_1$[44] and, according to al-Fārābī, is proven in each science.[45] But whatever the definition of question is, they certainly must not necessarily exceed the boundaries of $A_1$. In other words, investigating the things encompassed by $A_1$ by that which falls within the realm of $B_1$ in science B is not actually possible. Therefore, the case of wounds healing, whether circular or otherwise, should be explained only by means of the per se accidents of medicine; if this argument is constructed as a syllogistic chain, the parts of the syllogism (i.e., the premises and terms) should only consist of elements that fall within the scope of the subject of medicine.

Thus, why healing motion is toward the center is a question that needs to be investigated only in medical science. Moreover, the reasons for the slowdown or acceleration of this healing should also be sought in medicine. While the absence of an angle in only circular wounds or any wound for that matter may be the reason for their slower or faster healing, this is a distant rather than a near cause; or perhaps no reason exists at all. Hence, the geometrical premise "Circle, being the most encompassing figure, has no angles" which does not belong to medicine at all, is transferred to medicine. The purpose of maintaining the case of circular wounds, which seems to completely contradict the views put forward by Avicenna in *al-Burhān*, *Najāt*, and *Physics*, to correspond to a third form of interrelations between the sciences is probably Avicenna's attempt to expand the field of the transfer and use of demonstrations.

Undoubtedly, Avicenna could not ignore this old case of medicine in *The Canon of Medicine*. He argues that a circular wound as well as any wound that occurs in the

42 For medieval debates on whether Euclid's method of superposition in the proof of those propositions is proper to plane geometry, see: Angela Axworthy, "The debate between Peletier and Clavius on superposition", *Historia Mathematica* 45, 2018, 1-38.

43 Avicenna, *al-Burhān*, 155.

44 Avicenna, *Dânişnâme-i Alâî: Alâî Hikmet Kitabı*, tr. Murat Demirkol (İstanbul: Türkiye Yazma Eserler Kurumu Başkanlığı, 2013), 114.

45 al-Fārābī, Abū Naṣr, *al-Manṭiqiyyāt li-l-Fārābī* (vol. 1), ed. M. Taqī Dānish'pazhūh (Qum: Manshūrāt maktaba Āyat Allah al-'Uẓmā al-Mar'ashī al-Najafī, 2012), 307.

absence of flesh is not able to rapidly heal,[46] and thus carries the discussion he started in *al-Burhān* to *the Canon*. Furthermore, according to Avicenna, wounds that heal with difficulty (e.g., such as circular or similarly shaped wounds) are fatal for children, because they cannot withstand the severe pain and difficult treatment caused by such hard-healing wounds.[47] Therefore, although Avicenna does not go into much detail, as can be seen from his elaboration here, a circular wound is understood to heal slowly due to the per se attributes of the circle.

Averroes, another commentator on the *Posterior Analytics*, follows Aristotle, Philoponus, and Avicenna's line of thinking in explaining the slow healing of circular wounds; however, he does not go into much detail. According to him, the case that forced Aristotle to claim that sciences also exist with no superordinate-subordinate relationship (e.g., medicine-geometry) was for clarifying what the real reason for the slow healing of circular wounds is. While the physician knows the reason of fact that circular wounds heal slowly, the geometer knows the reason why: because of the absence of angles, which facilitate healing in the circular wound, or because of the size of the spherical body. For geometers provide the proof that the spherical body is a more encompassing body than non-spherical bodies of equal perimeter.[48]

## IV. Why is the Human Braincase Round?

### *a) Explanations by Hippocrates and Galen*

Another example of the medicine-geometry relation and close to circular wounds in a sense is the explanations for the shape of some parts of the human body, especially the braincase. Hippocrates states the entire human braincase to look like a sponge, apart from the upper and lower parts (most likely the lower and upper jawbone, i.e., facial bones).[49] Although Hippocrates does not say that roundness is the natural shape of the human braincase, in a discussion about a society with the tradition of lengthening their braincases, he says that this long-headed society forced the heads of newborns to grow elongated by depriving them of roundness with the help of bandages and various tools as infants' braincases are softer.[50] Therefore, Hippocrates

46 Avicenna, *al-Qānūn fī al-ṭibb*, I, 308.

47 Avicenna, *al-Qānūn fī al-ṭibb*, III, 226.

48 Averroes, *Sharḥ al-Burhān*, 372-373, *İkinci Analitiklerin Orta Şerhi*, ed., tr., Hacı Kaya (Istanbul: Klasik Yayınları, 2015), 72.

49 Hippocrates, "On Wounds in the Head", *Hippocrates*, III, tr. E.T. Withington (Massachesetts: Harvard University Press, 1959), 9.

50 Hippocrates, "On Airs, Waters and Places", *Hippocrates*, I, tr. W.H.S. Jones (Massachusetts: Harvard University Press, 1957), 111.

also admits roundness to be the natural shape of the braincase, and those with non-round braincases to be the unnatural ones. However, unlike Avicenna, Hippocrates does not dwell on the question of why the braincase is round.

For Galen, the true shape of the human head is like a waxy sphere that is slightly flattened from all sides. In addition, the front and back sides of such a head must be slightly protruding, unlike the sphere, and the sides must be flatter.[51] Moreover, if an animal's braincase is round, the shape of the brain must also be round, and if it is elongated, the brain must naturally be elongated.[52] Considering the anatomy of the brain, Galen indicates the dura mater to be externally covered with a helmet called a braincase,[53] but says nothing about why the braincase looks like a helmet. However, like Hippocrates, he adopts the analogy of comparing the shape of the braincase to a sponge,[54] but does not explain it with a geometrical method. As can be seen, although he argues the human braincase to be round, he does not directly question why this is so.[55] Even so, he explains using a geometrical proposition why the fornix is round or rather spherical. According to him, the fornix corresponds to a part of the curvature of the sphere. He regards the sphere itself as the best geometrical figure for any part of the human body, or the best structural component with the most robust capacity. The sphere ensures an even distribution of weight, and Galen extends the analogy to explain the shapes of veins, channels, ventricles, and cavities containing any other substance. Galen takes the analogy one step further and states that ventricles can be imagined as exactly spherical when considered as a whole and stripped of their surrounding adnexa.[56]

### *b) Avicenna's Account*

Avicenna appears to have also adopted the Galenian style of explanation by using geometrical premises in the anatomy of the brain, because when discussing the anatomy of the brain in *the Canon*, he maintains that its beginning is covered with an arch-like spherical roof. According to him, this arch functions as a passage and

51 Galen, "Meditsinskoye Iskusstvo" (Ars Medica), tr. Irina Prolygina, *Intellektualnyye Traditsii w Proshlom i Nastoyashshem (Intellectual Traditions in the Past and Present)*, ed. M.S. Petrova, no. 2. (Moscow: Akwilion, 2014), VI.4, 102.

52 Galen, *On Anatomical Procedures*, tr. Charles Singer (Oxford: Oxford University Press, 1956), 2.

53 Galen, *On the Usefulness of the Parts of the Body*, tr. Margaret Tallmadge May (Ithaca: Cornell University Press, 1968), 411.

54 Galen, *On the Usefulness of the Parts*, 426.

55 In fact, Galen's statement that the circular shape is more suitable for resisting various types of damage because it does not have external angles that can be broken or split means that he accepts sphericity as the most suitable shape for the skull. See: Galen, *On the Usefulness of the Parts*, 82.

56 As was mentioned, for Galen, the spherical body being the same in all its points makes it the least vulnerable and the largest figure among all figures of equal circumference. In addition, the round body has a higher capacity and is more suitable for lifting a load. See: *On the Usefulness of the Parts*, 415.

is less easily damaged due to its circular shape. It is also strong enough to carry the folded membrane covering it.[57] If this argument to be constructed in a syllogistic chain, one may find that Galen and Avicenna are implying that geometry gives the reason why the beginning of the middle ventricle is formed like this:

($P_1$) Circular figure is less vulnerable to damage.

($P_2$) The beginning of the middle ventricle is covered with a circular (or spherical) roof.

Therefore,

(C) The beginning of the middle ventricle is less susceptible to damage.

However, ($P_1$) cannot be defined as a geometrical premise. Which principles Galen relied on to define the circular or spherical figure as the figure that is less easily damaged than other figures are unclear. Consequently, why Avicenna openly follows Galen's geometrical form of demonstration in some anatomical investigations is equally unclear, for as we have argued above, Avicenna opposes the combination of medicine and geometry. In my opinion, the reasons that led Galen and Avicenna to this kind of explanation can be attempted to be understood within the framework of the basic distinction between linearity and circularity. As is known, a straight line is an abstract one-dimensional figure. When even a slight pressure is applied to a straight line, it ceases being a straight line and becomes an obtuse angle consisting of two lines. However, if such an application is made to a circular figure or a spherical body, this figure and body will continue to resemble circularity and sphericity in many ways, though not completely. The same problem is again encountered when investigating the reasons for the roundness of the braincase, as will be examined in the example given by Avicenna below.

Avicenna claims the natural shape of the human braincase bones to be round due to two conditions and two benefits. One of the benefits belongs to the inside of the braincase. He argues that the circle as a figure encompasses a larger area compared to figures consisting of straight lines when both their perimeters are equal. Another benefit is with respect to the outer part of the braincase. Here again, Avicenna, following Galen, states the circular figure to be less easily affected by damage, unlike angled figure.[58] Thus by expanding the geometrical method used by Galen in anatomy, Avicenna explains that geometry also gives the reason why

57 Avicenna, *al-Qānūn fī al-ṭibb*, II, 7-8.

58 Avicenna, *al-Qānūn fī al-ṭibb*, I, 43.

the braincase, in addition to ventricles, is created in a cyclical and spherical form. Moreover, according to him, roundness provides benefits to the braincase with respect to both its interior and exterior; however, he does not say anything about how circularity benefits the inner part of the braincase. In the part of the anatomy of the brain examined above, apart from the ventricles, he says nothing about the benefit of the roundness of the braincase to the way the brain functions in general. However, Avicenna argues that the braincase is created elongated as well as round because the places where the nerves of the brain originate are longitudinal to prevent the nerves from being pressed. This is probably why the braincase is spherical. If we construct this argument in a syllogistic chain, we can express it as follows:

($P_1$) Any part of the body with a circular shape prevents nerves from being pressed.

($P_2$) The braincase has a circular shape.

Therefore,

(C) The braincase prevents nerves from being pressed.

Premise $P_1$ is a geometrical premise, and gives the reason why to an anatomical problem. Geometry also gives the reason why for the benefits of the outer part of the braincase:

($P_1$) Circular-shaped parts of the body are less susceptible to damage than angular-shaped ones.

($P_2$) The outer part of the braincase has a circular shape.

Therefore,

(C) The outer part of the braincase is less susceptible to damage.[59]

The geometrical method Galen followed in the anatomy of the brain is also made out to have influenced Avicenna in the construction of his argument.

### *c) Ibn al-Nafis' Commentary*

The influence of Galen's geometrical method is seen in the words from Ibn al-Nafīs' anatomical investigations, who commented on Avicenna's mathematical modeling of the human braincase, at the beginning of his *Commentary on the Anatomical Part of the Canon*. Ibn al-Nafīs maintains his inability to directly apply anatomical

59 I would like to express my gratitude to the anonymous referee who shared his views on the correct construction of this and the previous syllogism.

observations due to the constraints of the Sharīʿa and the morality of compassion and therefore he would rely on those who'd implemented such practices to study the shape of internal organs, especially Galen's books, because according to Ibn al-Nafīs, they are some of the most perfect books on anatomy to have survived until his time.[60]

Ibn al-Nafīs says that anatomy benefits the physician with respect to the theory, practice, and deduction (*istidlāl*). Its benefit with respect to a deduction is in terms of prioritizing theoretical knowledge and other aspects. He explains at length the benefits anatomy provides to the physician with respect to a deduction. The part regarding our subject is as follows: According to Ibn al-Nafīs, the shape of a certain part of the body is its per se (*fī nafsihā*) accident, namely its accident in itself. As such, the physician proves the shape, color, and size of the organ as its per se accidents. Ibn al-Nafīs states just observing organs to be insufficient, but rather reasoning and deduction are also needed for learning their benefits. Deduction examines the existence or absence of a part in the human body. Therefore, according to Ibn al-Nafīs, the physician has to clarify a certain per se accident of a certain organ using deduction. Given that the shape of the human head is a per se accident of the head, the physician proves reason of the circularity of the head by saying that the circular one is less susceptible to damage and has the largest volume.[61]

In the discussion of the proof of the benefits of organs, Ibn al-Nafīs attempts to explain why organs have certain geometric shapes while criticizing the group that denies the benefits of organs. According to him, God gives every creature the best of substance, quantity, quality, and other characteristics, because of His care for this world. When it comes to knowing which properties exist for a particular organ, the creation of that organ in that way is then found to be the most appropriate for that organ. However, after clarifying the ways in which the organ was certainly created, the purpose of the creation of that organ was thought to be the same. Ibn al-Nafīs maintains the purpose of the creation of a particular part of the body to be different from what one considers as its benefit, that a hidden reason exists. For example, our

60 Ibn al-Nafīs, *Sharḥ Tashrīḥ al-Qānūn*, ed. Salmān Qatāya (Cairo: al-Hay'a al-Miṣriyya al-ʿĀmma lil-Kitāb, 1988), 17.

61 Ibn al-Nafīs, *Sharḥ Tashrīḥ al-Qānūn*, 23, 27-28. As the anonymous referee points out, if the roundness of the braincase is indeed its essential feature, in this example the roundness, unlike the example of the circular wounds, is a geometrical per se accident of the anatomical object. In other words, it is the near cause, not the distant cause or distant attribute. Because the wound does not always have to be round, but every human braincase must be round, that is, not linear, not square or rectangular. The anonymous referee wonders whether there is a direct relationship between anatomy and geometry not via the distant cause, but via the near cause.

argument that "the human head is created in a circular shape because this makes it less vulnerable to damage" means that the creation of the head in exactly this way is appropriate for the purpose, but that does not mean that the head was created in a circular shape only for this reason. Consequently, Ibn al-Nafīs says that there can be several benefits of something at the same time, and the benefit hidden from us among them can be the sum of all of those benefits.[62]

Following Avicenna, Ibn al-Nafīs argues that the braincase has unnatural shapes apart from its natural longitudinal shape and the circular, slightly elongated shape like a sphere filled with something on both sides is the natural shape of the braincase. According to him, the first of Avicenna's arguments where the human braincase is circular because of two benefits (i.e., when comparing a sphere with another shape of equal volume, the sphere becomes the figure with the smallest surface area able to encompass the brain and other organs) is undemonstratable in the anatomy book.[63] Most likely, Ibn al-Nafīs is aware that using this kind of geometrical demonstration in an anatomical problem is incorrect. Hence, he decides to put forth a presumptive form of clarification that appears close to the human mind. According to Ibn al-Nafīs, the volume of the spherical cone is smaller than the volume of the cube of equal surface area, and the volume of the cube is smaller than the area of the dodecahedron consisting of pentagons. Likewise, the volume of the dodecahedron is smaller than the volume of the polygon consisting of hexagons of equal surface area.[64] Thus, the volume of any object whose shape approaches a sphere will undoubtedly be greater than the volume of the previous object. The sphere has the largest volume of all bodies with equal surface area. Ibn al-Nafīs maintains that testing this in a two-dimensional space is possible as well: For instance, a triangle fit within a fixed space (with respect to the perimeter) has less surface area than a quadrilateral fitted the same way, while a quadrilateral has less surface area than a pentagon. Thus, the surface area of figures with more sides will be larger than the preceding ones as they approach the circle. In this case, the circle has the largest surface areas and is the most encompassing of the planar figures; this is the benefit related to what the braincase contains. The second benefit of the braincase being spherical is that, unlike angled figures, it is less susceptible to external damage, because angles lack anything that strengthen their resistance to external damage. For this reason, in objects with angles fractures occur first at the vertex of the angle. The spherical body,

62 Ibn al-Nafīs, *Sharḥ Tashrīḥ al-Qānūn*, 25-26.

63 Ibn al-Nafīs, *Sharḥ Tashrīḥ al-Qānūn*, 57.

64 Since it is impossible for the object to be constructed only from hexagons, Ibn al-Nafīs' expression "a figure made up of hexagons" is incorrect.

meanwhile, is not susceptible to damage at vertexes because all sides are equal. Here such a feature benefits the bone.[65]

The benefit of the braincase being slightly longitudinal is that the brain nerves are located longitudinally, that is, each pair of nerves is located posteriorly of the other pair. Seven pairs of cerebral nerves exist, and when examined for orientation, all seven pairs are seen to be longitudinally oriented with only two nerves being located transversely. Therefore, the shape of the braincase, aside from being spherical, should also be longitudinal due to the large number of nerves. Ibn al-Nafīs states another reason to exist as to why the braincase is created in such a way: the main aim of the braincase is to protect the brain like a helmet. This can only be achieved by surrounding the brain from all sides. The shape of the brain, as has been said, is longitudinally circular and thus the shape of the object surrounding it must be like this; otherwise, some areas will have more protection than is needed or other areas will have a deficiency that can cause the brain to wedge into.[66]

65 Ibn al-Nafīs, *Sharḥ Tashrīḥ al-Qānūn*, 57-58. From a purely geometrical perspective, the circle (or sphere) is the most appropriate figure (body) with respect to the ratio of the area of any figure to the length of its edge (in the case of a sphere, the ratio of its surface area to its interior volume). When we draw a circle on the paper and a square with the same area next to it and measure the perimeters with a thread, the square will have a longer perimeter than the circle. However, we will face great difficulties here. As a result, we will have to solve the problem of squaring the circle, which has no solution, to draw a square whose area is equal to the area of the circle by compass and the straightedge. But we can create a presumptive picture to clarify what Avicenna and Ibn al-Nafīs mean by the phrase "if their areas are equal". If we take into account that the number $\pi$ is equal to 3.14, the area of the circle with a radius of 2 cm will be 12.56 $cm^2$ and the circumference will be 12.56 cm. By equalizing the area of the square and the circle and solving the equation, we understand that the side of the square must be 3.54 cm. In this case, the area of the square will be 12.53 $cm^2$, which is very close to the area of the circle. However, the length of the sides of the square (circumference equal to 14.16 cm) will be significantly longer than the circle. Likewise, we can calculate the side of the triangle with this method. That is, the circle has less contact surface with the external surroundings than square and triangle. The same rule applies to three-dimensional objects. The sphere has a smaller surface with the outer perimeter than a cube or pyramid. Therefore, such a figure or body is energetically more favorable to nature: less effort is required to preserve it, all points of the surface are equidistant from the center, and access to them is easier. In addition, the sphere is much more resistant to pressure than a cube. Due to its proper shape, the strikes on the sphere will slip and be much weaker. This means that if the braincase is spherical, it protects what is inside better.

66 Ibn al-Nafīs, *Sharḥ Tashrīḥ al-Qānūn*, 57-58. Whether sphericity has a role in the strength of the braincase, as in the example of circular wounds, has been widely debated up to now. Demes claims that in mammals, a correlation exists between the shape of the braincase and the thickness of its walls, and the more closely the outlines approach the shape of a sphere, the thinner the walls. The fact that spherical domes possess the greatest strength among shell structures, (cylindrical shells, on the contrary, are less resistant) is seen in the effect of transferring the stresses created by joint forces in the cranial joint to the domes of the braincase through the cranial walls. This case requires the basic shape of the braincase base to be oval. In this way, less pressure is applied to the less mechanically resistant areas of the braincase base, namely the middle part of the braincase cavity where many small holes are spread, and the thin roofs of the nose and eye cavities and the anterior braincase cavity. Von Bergmann and von Bruns demonstrated the elasticity of the braincase by describing the mode of movement of a braincase thrown

### *d) Ibn al-Nafīs' Geometrical Analysis of the Sutures on the Human Braincase*

The geometrical analysis of the human head is not restricted to the braincase. It should be noted that while Avicenna is content with giving only the forms of the braincase sutures, Ibn al-Nafīs explains this in such a way as their forms give the reason why. Avicenna says that the braincase has two protrusions directed to the back and anterior side in order to protect the nerves descending from both sides of the brain, while the braincase with this shape has three real (*durūz ḥaqīqiyya)* and two false sutures (*darzān kādhibān*). One of the real sutures is curved and joins the forehead in a shape ∩ and is called the coronal suture. Also, the suture that runs through the middle of the head, called the arrowhead-shaped (sagittal) suture, is a real suture type, and the place this suture adjoins the coronal one is called the spit-shaped suture. Its shape is similar to a vertical line constructed in the middle of an arc. The third suture is angular shaped one and joins the back and base of the skull, adjoining the arrowhead-shaped suture at one end. This suture is called lambdoid suture because it resembles the Greek letter lambda (λ). The false sutures run longitudinally on both sides of the head parallel to the arrowhead-shaped suture. They are called false sutures because they do not go deep into the braincase bone.[67]

Ibn al-Nafīs does not content with just the geometrical demonstration of the sphericity of the braincase; he also explains the shape of the sutures in the braincase

on the ground, that is, the braincase bounced up and down several times like a ball before it became stationary. They concluded that this elastic movement of the braincase is a result of its spherical shape. See: Brigitte Demes, "Biomechanics of the Primate Skull Base", *Advances in Anatomy, Embryology and Cell Biology* 94 (Springer-Verlag, 1985), 7, 44. Thus, at the center of Demes' argument of the biomechanical properties of the skull dome lies the claim that a skull with curved bones (a more spherical skull) is more resistant to pressure than skulls exhibiting flatter bones (a narrow skull). Because curved bones can disperse pressure more effectively, they are thinner and require less thickness for protection.

67 Avicenna describes the braincase designed exactly this way as the natural shape of the head. In addition, there are also unnatural shapes, and they come in three types. In the first, the head lacks the frontal protrusion and in this case coronal suture is absent. In the second, the head lacks the posterior protrusion, and in this case lambdoid suture is absent. In the third, the head lacks two protrusions and the head takes the shape of a sphere and is equal in both length and width. It should be noted that Avicenna describes sphericity as the unnatural shape of the head. When he defines the braincase, he uses a two-dimensional abstract circular shape (*al-shakl al-mustadīr*). Referring to Galen, Avicenna says that in such a head shape, the distances are equal, so the division of the sutures should be fairly equal. However, in the natural shape of the skull, the sutures are divided into one longitudinal and two transverse sutures. The unnatural braincase bone has both a longitudinal and a transverse suture; moreover, the transverse suture extends transversely from one ear to the other in the middle of the braincase in a shape X. The longitudinal suture also extends longitudinally in the center of the braincase. Also, Galen considers it impossible to have the unnatural fourth shape of the braincase in which the length is shorter than the width, because the reduction of the inner part of the braincase containing the brain is not possible as this is contrary to life and the correct formation of the brain structure. Consequently, Avicenna claims that Galen also confirmed Hippocrates' words, who determined the four shapes of the human skull. See: Avicenna, *al-Qānūn fī al-ṭibb*, I, 44.

by means of geometrical premises. Using the method of analogy, Ibn al-Nafīs first provides the general image of the sutures and emphasizes the reasons for their formation. According to him, the sutures form as a result of the interference of each of the two bones of the braincase in many places, as if the teeth of one of the two saws enter between the teeth of the other, or as if the one short finger enters between the other two short fingers in front of it. However, the difference in the case of two saws or fingers is that the wide base of the saw's tooth is like a triangle narrowing toward the tip, whereas the width of the fingers is equal. The ends of these three real sutures (although it is not correct to say that the ends of all sutures are like this) are wider than their bases. Ibn al-Nafīs argues that the sutures are formed in this way in order to ensure the structure of the braincase is stronger and more arranged; the length of the suture is much longer and thus the number of steam holes that need to be spread is many. Ibn al-Nafīs states the two false sutures to not actually be sutures but to in a sense resemble glue; thus some have called the "glue" (*lizāq*) and others "two false sutures" (*darzān kādhibān*).[68]

Up to that point, Ibn al-Nafīs gave the general image of the braincase sutures within the boundaries of the science of anatomy (i.e., the work anatomists have to do within their field of research). However, when he attempted to explain the way each of the sutures is positioned in the braincase, Ibn al-Nafīs is seen to have followed the geometrical method. The upper part of the first real sutures is the coronal suture that surrounds the top of the braincase and connects the uppermost bone with the bone at the top of the skull. According to Ibn al-Nafīs, the reason why it is called the coronal suture is that the part of the head that is crowned is located at the last line of this suture, and the vapors in the anterior ventricle spread exactly from this suture. Thus, this suture formed this way in order to facilitate the diffusion of the vapors because the diffusion passes around the ventricle. The reason for the crown shape of this suture is that the protrusion on the front of the head is large and acquires a spherical shape with the other parts of the head. If one cuts the sphere equilaterally, the cut surface becomes a circle and this suture is like the circumference of the circle segment.[69]

The second suture passes under the veins in a linear fashion, and most of the vapors in the head, especially in the middle part, emanate from this suture. Ibn al-Nafīs argues that this suture is created in a linear fashion in order to cover the entire top parts of the head. Thereby, linearity makes the structure of the braincase

68 Ibn al-Nafīs, *Sharḥ Tashrīḥ al-Qānūn*, 59-60.

69 Ibn al-Nafīs, *Sharḥ Tashrīḥ al-Qānūn*, 60.

more regular and stronger and causes the diffusion of vapors to be easier and more frequent, which happens as follows:

1. The reason why the linearity causes the vapors to spread much more is that if this suture, from the topmost part of the head tilts to its side, this part of the uppermost part would prevent the spread of vapors.

2. The reason why linearity makes the braincase's form stronger is that the straight line is the shortest line among the lines. Therefore, the separation is minimal. If it was curvilinear, the opposite would happen due to the elongation of the line. Moreover, if it had not been created in a linear fashion and was not under the veins, it would emit less vapors because in this case the vapors that end up at the top of the head have no place to go. Given that this suture separates the head at top, its anterior end must necessarily abut on the crown of the coronal suture. Because any equal surface or straight line divides the sphere in two, any circle is divided in half by right angles, and the resulted section of the circle becomes part of the sections of these circles.[70]

The upper part of the third of the real sutures is adjacent to the upper part of the braincase and the fourth wall. If the protrusion on the back of the braincase were small, the head would not be spherical, but rather less wide. The upper part of this suture is narrow because the lower part needs width for the spinal cord to pass comfortably. Therefore, the shape of this suture becomes like the part of the cone that is cut from its top to the base, with a surface equal to the cone's surface. Ibn al-Nafīs also likens this suture to the isosceles triangle, for it is not possible to bend either side of the head and this part starts from the top of the head and continues towards the bottom. Therefore, both sides must meet at the end of the straight suture. Ibn al-Nafīs maintains a benefit and protection to be present in both sides of this suture being linear. The benefit is that the damage is less and the form of the braincase is stronger. The protection, on the other hand, reduces the retention of the vapors in the rear ventricle due to its downward direction.[71]

70 Ibn al-Nafīs, *Sharḥ Tashrīḥ al-Qānūn*, 60.

71 Ibn al-Nafīs, *Sharḥ Tashrīḥ al-Qānūn*, 61. Avicenna and Ibn al-Nafīs' use of geometrical demonstrations in anatomical investigations is not limited to the organs we have examined above. In addition, they claim that the spine, arm bone, trachea, stomach, finger bones, and the tip of the fingernails were created in a circular shape to reduce damage. See: Avicenna, *al-Qānūn fī al-ṭibb*, I, 52, 54, 57, II, 301, 402. Ibn al-Nafīs, *Sharḥ Tashrīḥ al-Qānūn*, 127, 138-139, 224, 400. Furthermore, it is interesting that Avicenna uses the geometric explanation form in discussing the shape diseases (*amrāḍ al-shakl*), which is a type of structural diseases. According to Avicenna, this disease means that a certain organ changes its natural shape and this change affects the movement of organs negatively. For example, curving the linearity, rectifying the curved, squaring the circle, and rounding the square constitute such diseases. Avicenna's expressions like these enable us to better understand the distinction between linearity and circularity using the analogy method. These different properties of line are its natural characteristics

## V. Arithmetic-Anatomy Relation: Phalanges of Fingers

Not only geometrical but arithmetical demonstrations as well are seen to have been used in anatomical investigations. According to Galen, if the phalanges were formed of a single bone in a straight line, they would only be able to acquire the shape of straight lines and could never have a fully spherical shape. Also, he argues that no more than three or fewer phalanges are needed for each finger: A larger number of phalanges would certainly not contribute in any way regardless of their function, but rather, prevent them from fully bending by making them less reinforced than they are now, since organs made up of many parts bend more easily than those with fewer parts. However, if fingers were made of less than three phalanges, the fingers would not have acquired as many intermediate positions. Galen, therefore, considers the finger to be composed of three phalanges sufficient to prevent unnecessary bending while ensuring a variety of movement.[72]

Following Galen, Avicenna says that the fingers' phalanges were not created from one bone in order to prevent their movement from being difficult, like people suffering from convulsions. Therefore, the phalanges are limited to three, because if their number were more than this, the number of movements of the phalanges would increase, which would naturally cause the thing which should be held tight to be loose and weak. Similarly, if the phalanges were created as two for instance, the strength of the fingers would be more than required and less mobile than necessary. However, instead of extreme strength, fingers need freeness to perform various movements.[73] According to Avicenna, sufficient strength for holding something firmly is the per se attribute of fingers. But in order for the fingers to do this, a specific number of phalanges are required. In fact, Avicenna might not have gone far enough by simply defining a finger as "a part of the human body consisted of three bones." However, on the basis of experience and observations, he states the

with respect to being opposite (*mutaqābil*) per se attributes. Therefore, due to the kind-crossing prohibition, it is impossible to transform them into one another. Despite this, if one claims that they can be transformed into each other, this explanation will not be a demonstrative, but rather, it will cause certain fallacies in science. Likewise, changing the natural shapes of the organs causes various types of damage in the human body. This kind of damage leads to the deterioration of the balance of the human body, because the transformation of the natural forms of the basic organs causes the balance between them to be lost; one organ acquires a different temperament and the other organ adopts a completely different temperament. After all, the disproportionate nature of the human body means that it is also bad in terms of intelligence. See: Avicenna, *al-Qānūn fī al-ṭibb*, I, 105, 164. Therefore, it is made out that the protection of the natural forms of human organs is important with respect to the balance of health and temperament, just as the balance of per se accidents specific to each discipline ensures that this organic structure is solid and its functioning mechanism works completely.

72 Galen, *On the Usefulness of the Parts*, 83, 85.

73 Avicenna, *al-Qānūn fī al-ṭibb*, I, 56.

phalanges to have been created in three parts, and so tries to explain why they are created in three by involving arithmetic in the demonstration. Likewise, Ibn al-Nafīs, following Galen and Avicenna, says if the phalanges of finger were made of a single phalanx, the fingers would lack skill in directing movement and conversely if fingers consisted of more than three phalanges, the movements of the fingers would be more relaxed than required. Therefore, the fact that the fingers consist of three phalanges is considered sufficient in terms of strength, direction of movement, and dexterity.[74] However, al-Fārābī states that for instance, when the definition "a two-legged animal" is proven for "human", the middle term used in it should be "a mortal rational animal." Hence, the definition is composed as follows:

($P_1$) Every human is a mortal rational animal.

($P_2$) Every mortal rational animal is a two-legged walking animal.

Therefore,

(C) Every human is a two-legged walking animal.

Thus, al-Fārābī maintains that the middle term should also be the definition of the object whose definition is required. This is one of the requirements of the absolute demonstration, in which case the proven definition must necessarily be the definition of the object with another definition that precedes this proven definition.[75] It should be noted that while al-Fārābī defines human as a two-legged animal, he does not mention why human has exactly two legs. Moreover, no philosopher has ever addressed why a human is a two-legged animal, rather than a one, three, or four legged one.[76] If Avicenna's chain of reasoning is constructed as a repetitive syllogism (*qiyās*

74 Ibn al-Nafīs, *Sharḥ Tashrīḥ al-Qānūn*, 138.

75 al-Fārābī, *al-Burhān*, 88.

76 The anonymous referee points out that in the passage of al-Fārābī where he argues that the middle term, which is also the definition of the object, cannot go to infinity, so there should be a preceding definition that cannot be proved by another preceding definition. In other words, according to the anonymous referee, if we define human as a two-legged animal, we prove it with another definition that is the cause for this definition and is also the definition of the object, that is, the rational mortal animal, and this syllogism is the reason why human is a two-legged animal. So the middle term used in the absolute demonstration must be the first and foremost cause, and since this cause also gives the nature of the object, it must be the primary and preceding definition, that is, the true definition. What is essential in the absolute demonstration is that the middle term comes before the first term, that is, the major term, both in nature and knowledge. Although the major term is the definition that is intended to be proven here, it is not the definition that states the quiddity due to the primary definition. The anonymous referee claims that al-Fārābī's use of the term *ḥadd* for this should not mislead us, he only uses it to express that the definition cannot go to infinity and that a primary and preceding definition must be found, since in logic the *ḥadd* is used to express the real definition. While I accept all these arguments from the anonymous referee, I would like to emphasize that my aim, by giving the relevant passage of al-Fārābī as an example, is only to draw a parallel with Avicenna's arguments.

*istithnāʾī*), he turns out to include the number three in the definition of the phalanges of the fingers and makes the same number three as the middle term; consequently, the number three gives the reason why the phalanges are sufficiently stable:

($P_1$) If the number of phalanges in the finger of the human hand were one, two, or four instead of three, the fingers would not be sufficiently strong and would be unable to perform a variety of movements.

($P_2$) However, the number of phalanges in the finger of the human hand is three.

Therefore,

(C) Human fingers are sufficiently robust and capable of various movements.

However, unlike fingers, to be able to hold something tightly and perform various movements are not per se attributes of the toe phalanges; Avicenna defines them differently than the finger phalanges, although the number of phalanges toes have is also three.[77]

## VI. Conclusion

Finding the most appropriate balance between observation and theory has been an important issue in epistemological debates that have lasted for centuries on the position of medicine. For the successful diagnosis and treatment of disease at that time, as well as today, practical skills and experience as well as an appropriate conceptual framework have been required.[78] For this reason, the case of the slow healing of circular wounds has been a problem for both physicians and philosophers since ancient times and continues to be one. This extraordinary example of inter-science relations, which is not included in *Posterior Analytics*' definition of scientific questions, forced especially Avicenna to make contradictory claims in *al-Burhān*. While he emphasized that the premises of the demonstration must be related to each other on the basis of the concept of per se, the fact that the geometrical properties of the circle actually have a certain share in the slow healing of circular wounds also forced him to open up a new field for the transference of demonstration.[79] Therefore,

77 Avicenna, *al-Qānūn fī al-ṭibb*, I, 60.

78 Hardy Grant, "Geometry and Medicine: Mathematics in the Thought of Galen of Pergamum", *Philosophia Mathematica*, II.4, sy. 1 (1989): 29-30.

79 According to Türker, it is necessary to take into account that investigations on any subject do not always take place in per se accidents with their defined meanings, and that it is not possible to avoid the broad meaning of "being related." In addition, the construction of demonstration on any subject does not guarantee that the subject is fully known even with respect to the construction of demonstration,

Avicenna's reduction of the interrelations of the sciences which does not follow the rules of the *Posterior Analytics* to the level of questions (*masā'il*) reveals that many issues which were alleged to violate the ban on kind-crossing in the history of philosophy and science did not in fact violate it. Hence, Avicenna's and Ibn al-Nafīs' use of geometrical demonstrations in human anatomy, which they adopted from Galen, can be accepted and tolerated. The geometrical demonstrations, which had largely spread to medicine under the influence of Galen, continued to have an effect on Avicenna and Ibn al-Nafīs' medical investigations. However, Avicenna's and Ibn al-Nafīs' ascriptions of sphere to being the least susceptible to damage because it has the smallest surface area among all objects of equal volume, has the largest volume among all objects with equal surface area, and most importantly because it is a figure that has no angles is actually still a matter for debate. This is because these kinds of spherical properties do not make the braincase's shape the least susceptible to damage with respect to the principles of natural science. Likewise, explaining the linear creation of sutures in the braincase through the properties of the straight line is also open to debate. Another point worth mentioning is that in *the Canon*, Avicenna is content with only giving the reason of fact of the slow healing of circular wounds. However, nothing prevented him from claiming that circular wounds also heal slowly for the same reason as he used while explaining the reason why a shape of a certain part of the human body is circular in anatomical investigations (i.e., saying that the circular shape is the widest figure).

Unlike in *al-Burhān*, Avicenna comfortably used geometrical demonstrations in anatomical investigations in *the Canon*; however, he clearly avoided using such a method when explaining the case of circular wounds in the same work. Even though geometrical demonstrations are accepted as having to be involved in one dermatological and a few anatomical cases; as Hankinson suggests, this does not mean that a separate or sub-discipline exists like geometrical medicine. For this reason, Avicenna's interpretation of Aristotle's words "many sciences which do not fall under one another are in fact related in this way" as "not the whole or a certain part of a science, but a certain proposition falls under another science" allows for a demonstration to be transferred from one science to another when a particular problem needs to be solved. However, all these points do not necessarily mean

because the scope of demonstration is much wider to the extent that it includes "related" things, and there is no certainty in this large field, which is highly dependent on perceptible data. See: Türker, *İbn Sînâ'da Metafizik Bilginin İmkânı*, 315, 331. According to Türker, Avicenna therefore, in two parts of the *al-Burhān* criticizes those who claim that demonstration can only be constructed from per se attributes and says that the concept of demonstration should not be overestimated. See: Avicenna, *al-Burhān*, 130, 266.

that the ban on kind-crossing is just a nominal doctrine, because the prohibition is just like a reminder reminding that the researcher should take into account the existing scientific infrastructure and logical rules when attempting to any scientific investigation. Therefore, what is important is whether the conclusion derived at the end of an investigation using a different method, after taking all these reminders into consideration, contributes to the solution of a particular problem or to reaching an approximate truth.

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