



Introduction

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Introduction to the Special Issue on *Lakatos' Undone Work*

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Abstract: We give an overview of Lakatos' life, his philosophy of mathematics and science, as well as of this issue. Firstly, we briefly delineate Lakatos' key contributions to philosophy: his anti-formalist philosophy of mathematics, and his methodology of scientific research programmes in the philosophy of science. Secondly, we outline the themes and structure of the masterclass *Lakatos' Undone Work – The Practical Turn and the Division of Philosophy of Mathematics and Philosophy of Science*, which gave rise to this special issue. Lastly, we provide a summary of the contributions to this issue.

Keywords: Imre Lakatos, anti-formalism, scientific research programmes

1 Introduction

Imre Lakatos (1922–1974) was a philosopher of mathematics and science. He is most famous for two main contributions to the field: first, his anti-formalist philosophy of mathematics, and second, his methodology of scientific research programmes. This special issue, and the online workshop/masterclass from which most of the contributions to this issue arose, is devoted to the philosophy of Imre Lakatos, especially his undone work on the intersection of these two topics. The masterclass as well as this SI are therefore titled 'Lakatos' Undone Work. The Practical Turn and the Division of Philosophy of Mathematics and Philosophy of Science'. As Lakatos would have celebrated his 100th birthday in 2022, this issue aims to celebrate and keep up the work on the great ideas of one of the forefathers

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of the practical turn in philosophy of mathematics, as well as a central figure in general philosophy of science.

Before introducing the contributions to this issue in more detail, we will briefly summarise Lakatos' vita, philosophy, and main contributions in this introduction. Furthermore, the specific aim and topic of the masterclass and this corresponding issue will be elaborated on in detail. Note, however, that this introduction is not intended as a comprehensive overview of Lakatos' work and life; for more complete overviews, see for instance Feyerabend (1975), Larvor (1998), Kadavy (2001), or Musgrave and Pigden (2021). A list of Lakatos' most important publications can be found in the references. Moreover, we are not discussing the political dimension of Lakatos' work, instead separating the philosophical from the political, but both dimensions are intertwined, and the latter can be of great help in understanding the former (cf. e.g., Larvor 1998).

Imre Lakatos was born in Debrecen, Hungary, as *Imre Lipsitz*, having a Jewish father. He adopted the name *Lakatos* in 1945. In 1944, he graduated from the University of Debrecen with a degree in mathematics, physics and philosophy. By 1947, Lakatos obtained a post in the Hungarian Ministry of Education. But he struggled with accepting orders from the Russian authorities, and his political views landed him in difficulties. This led to his arrest in 1950, following which Lakatos served three years in a Stalinist prison. After his release in 1953, he started working as a translator. Following the 1956 Hungarian Uprising and its suppression by Soviet tanks, Lakatos fled, first to Austria, and finally, to England (cf., Musgrave and Pigden 2021). He wrote his doctoral dissertation at the University of Cambridge, which he submitted in 1961. From 1960 until his death, he researched and taught at the London School of Economics, first as a lecturer and from 1969 onwards, as a professor. Lakatos died during a time where he had been academically very active. He had many plans for further work on both the philosophy of mathematics and the philosophy of science, as well as their intersection. The masterclass preceding and bringing about this issue dealt with topics Lakatos had to leave unfinished – the practical turn and the division of philosophy of mathematics and philosophy of science.

2 Against Formalism in Mathematics

As previously mentioned, one of Lakatos' main contributions is his anti-formalist philosophy of mathematics. Note that 'formalism' is meant to not only refer to the philosophy of Hilbert and his followers, but also to the positions of logicism and intuitionism (cf., Musgrave and Pigden 2021). Lakatos argues that the methodology and development of mathematics as a discipline is much closer to Popper's portrayal of the sciences than commonly supposed. This means that he thinks that

the development of mathematics should not be perceived as ‘a series of Euclidean deductions where the contents of the relevant concepts have been carefully specified in advance’ (cf., Musgrave and Pigden 2021). Rather than conceiving of proofs as deductions from well-defined premises, we should see them as the (temporary) end-points of an evolutionary and dialectical process. Concepts involved start out as not precisely defined and ambiguous, and then go through a dialectical process of becoming more precise. Moreover, he argues (as Popper does for the sciences; cf., Popper 1959, 1963) that mathematics advances through criticism and refutation, and that any of the accepted theories at any given time might later turn out to be false. His main work supporting this line of argument is called ‘Proofs and Refutations. The Logic of Mathematical Discovery’ (1976). It was only published after his death, with an editorial by John Worall and Elie Zahar. In addition to an earlier article (also called ‘Proofs and Refutations’), on the informal history of the Euler conjecture, it contained other parts of Lakatos’ PhD thesis. What is distinctive about Lakatos’ philosophy of mathematics is that – unlike formalists/logical positivists – he judges the so-called ‘context-of-discovery’ to be relevant and useful for the justificatory and epistemological part of mathematics (i.e., the ‘context of justification’). In other words, he conceived the context of discovery as a rational process, just like the context of justification.

His dialectical method as developed in ‘Proofs and Refutations’ (1976) starts with a conjectured theorem and a proof (where it is virtually important how the concepts involved have come to get their meaning and how one came up with the idea of that theorem). When a (heuristic or logical) counterexample appears (i.e., the presentation of a new kind of geometrical object that might be a candidate for the concept of a polyhedron to apply to), then the proof should be examined to find the assumption that is responsible for the error. Then, this ‘guilty lemma’ (to use Lakatos’ terminology) is built into the theorem as a condition (cf., Larvor 1998). This pattern is to be repeated, and ‘these conditions may accumulate to the point where they collectively define a new concept’ (cf., Ibid). This exemplifies the following: what might look like a simple proof of a theorem should be conceived of as a pattern of (dialectical) conceptual growth.

Formalism, on the other hand, is envisaged by Lakatos as embodying a kind of ‘static rationality’, which ‘refuses to treat the development of concepts as a rational process, [and] which insists that meanings remain fixed from one end of the argument to the other’ (cf., Larvor 1998). In criticising formalism, however, he does not criticise logical analysis as such. Instead, he argues that by identifying rationality with deductive logic, other important aspects of mathematics that significantly contribute to the success and the methodology of the discipline, such as the development of mathematical concepts through time, get lost.

This idea is currently further explored by a growing community working on the philosophy of mathematical practice. For an introduction into the field consult for instance: Carter (2019), Hamami and Morrison (2020), or Perez-Escobar et al. (2022). The debate about the right methodology for philosophy of mathematical practice is not settled; there is also an ongoing direct debate concerning whether there is one right methodology (cf. Rittberg 2019, Hanna et al., 2020, or Kant, Pérez-Escobar, and Sarikaya 2021). Many scholars followed Lakatos by working historically informed (cf., for instance, Mancosu 1999).

3 Lakatos' Philosophy of Science – The Methodology of Scientific Research Programmes

Lakatos' second main contribution is within the philosophy of science (for an introduction into the philosophy of science, cf. for instance, Godfrey-Smith 2003), and closely relates to the demarcation problem, i.e., the problem of demarcating science from non-science, and the criterion of rationality of the scientific enterprise. These were both central issues in Popperian philosophy of science (cf., Popper 1959, 1963), but have been discussed by many other philosophers of science, such as Hume (1739) or Kuhn (1962).

According to Popper, the 'distinctive logical fact about science is that its theories can be tested against empirical evidence' (cf., Larvor 1998), or, in other words, a theory is rational and scientific if it can be empirically falsified. Theories which, on the other hand, cannot be falsified when put to the test, should be classified as pseudo-science (Popper used Marxism and Freud's psychoanalysis as examples of pseudo-science). This is by far not the only solution to the demarcation problem put forward in the literature. Other preceding accounts were inductivism, according to which all theories that are in line with the 'inductive method' should be considered rational and scientific, as well as conventionalism, according to which proper science 'means constructing the simplest taxonomy of phenomena which saves appearances' (cf., Larvor 1998).

Lakatos does not agree with any of these putative solutions entirely, but he finds Popper's theory to be a starting point for his own account of scientific rationality.

According to Popper, the best and most rational theories are the ones that are easily falsifiable. But, as Lakatos observes, if one consults the history of science, it becomes clear that most scientists, in fact, do not give up a theory immediately after it has been falsified by empirical evidence. Thus, it becomes clear that real scientists were and are much more dogmatic than Popper postulates a good scientist ought to

be. This raises the question whether the work of scientists who hold on to their theory beyond falsification should be considered irrational and unscientific. But this is not the right call according to Lakatos. He argues that ‘every great scientific theory in history has been engulfed in an “ocean of anomalies” from the moment of its formulation’ (cf., Larvor 1998). And, furthermore, that ‘[i]f a criterion of proper (i.e., rational) science sorts the history into rational and non-rational episodes in a way which brands many of our most cherished scientific successes as irrational, then there is probably something wrong with it’ (cf., Ibid). Thus, according to Lakatos, a desirable solution to the demarcation problem must respect the history of science, which, according to him, neither inductivism, nor conventionalism, nor falsificationism do. This belief causes his solution to be far less normative than the preceding ones and much more descriptive, respecting the evidence that the history of science provides.

The methodology of scientific research programmes is Lakatos’ proposal for solving the demarcation problem. The first way in which he departs from Popper’s account is that the appropriate entity to be judged regarding their rationality, should not be a single theory (or even proposition). Rather, it should be sequences of historically related theories, which he calls ‘research programmes’. This is because a family of theories over time can maintain its identity as the theories change. In other words, even if some aspect of the research programme must be changed in the light of falsifying evidence, the research programme is, in principle, able to maintain its identity. Thus, it does not have to be abandoned as a whole and substituted by a new programme

Within a research programme, Lakatos distinguishes between a ‘hard core’ and a ‘protective belt’. The hard core contains a set of commitments which cannot be abandoned without abandoning the entire programme altogether. And, using Popper’s terminology, one might say that this hard core is – at least to some degree – unfalsifiable. The role of the protective belt is to protect the hard core. It contains auxiliary hypotheses which are supposed to shield the hard core from falsification. This means that, if an experiment would falsify the programme (from Popper’s point of view), the distinction between hard core and protective belt allows the scientist to choose to conclude that some element of the protective belt has caused the falsification. Therefore, the hard core does not have to be abandoned. In this sense, Lakatos provides a ‘solution’ to the Duhem–Quine problem, according to which, if a theory is tested, it is never tested in isolation, but in conjunction with auxiliary assumptions. And then, when the theory is falsified, it remains unclear which of the conjuncts of this conjunction has been falsified, some auxiliary assumption or one of the central hypotheses of the theory (for an overview over the problem of underdetermination, cf., Stanford 2021). His solution is to ascribe the hard core epistemic superiority. In other words, when the research programme seems falsified, then one should first abandon or alter the elements of the protective belt without touching the

hard core of the programme. Therefore, given that the programme is constantly tested against empirical evidence, the protective belt will be ever-changing.

A further and important contrast Lakatos develops is the distinction between progressive and degenerative research programmes. Progressive ones are of a rational and desirable type, whilst the degenerative ones are of an irrational and unscientific type. Degenerative research programmes are characterised by never-ending and ad-hoc modifications of the protective belt while the hard core is not solving new problems anymore, and rather, encounters substantive problems itself. A progressive programme on the other hand is characterised by developing and refining the programme's central idea (i.e. the hard core) using the problem-solving techniques specific to the respective research programme (Lakatos calls this set of techniques the 'heuristic' of the programme).

4 Lakatos' Undone Work

The overarching theme of this masterclass was an analysis of a 'practical turn' in the philosophy of mathematics. Is this turn analogous to the practical turn in the philosophy of science(s)? Should it be? What are the exact roles of the actual empirical studies of mathematical practice (e.g., in mathematics education, sociology of mathematics, etc.)? How should those interact with the philosophy of mathematics?

We wanted to encourage work looking at the following areas. First, we wanted to draw attention to the seminal work of Imre Lakatos: a) he was a key figure in developing early themes after the practical turn in the philosophy of physics through his idea of 'research programmes'; b) his work in the philosophy of mathematics was a turning point of the discipline. However, he unfortunately did not have the time to develop his ideas due to his early death. He particularly aimed at transferring his insights from his work on the philosophy of science to the philosophy of mathematics.

In addition to that, we wanted to encourage work on the interplay of the philosophy of mathematics and the philosophy of science. We aimed to tackle questions such as: Should philosophy of science conferences have tracks for philosophy of mathematics? What was the role of Tarski in the split of the two disciplines? How close should philosophers of mathematics be to logicians regarding their respective fields? How does this question relate to the growing community of philosophers of mathematical practice and the cooperation with other disciplines (such as ethno-mathematical studies, mathematical education, cognitive sciences, etc.)? Another possible string of investigation was the analysis of new scientific methodologies in the philosophy of mathematics. How far are

they philosophically significant? How can they inform a philosopher of mathematics? Similar problems already arise in philosophy of science in connection to science studies.

5 Structure of the Masterclass

The masterclass consisted of 14 individual mentoring schemes between mentors and mentees varying from weekly meetings to one-off feedback. The aim of this mentoring relation was orientation for the mentee, insights into the academic world in general, and feedback on mentees' paper projects. Some of those projects were submitted to this special issue, others were submitted to other journals. The masterclass also encompassed a two-day workshop, where the mentees were able to present their work and receive feedback both from mentors and other mentees. This workshop is documented here: <https://lakatosundonework.weebly.com/>. All contributions in this special issue went through double-blind review with at least two reviewers.

6 Overview of the Contributions

In *Degeneration and Entropy*, **Eugene Y. S. Chua** analyses Jaynes' 'Information Theory and Statistical Mechanics' in Lakatosian terms. Chua's analysis particularly centres around the notions of 'superfluity' and 'authoritarianism', which were introduced in Lakatos' seminal 'Proofs and Refutations'. He concludes that an analysis through a Lakatosian lense leads to critiques of Jaynes's account, and that we must classify Jaynes's theory as degenerative.

José Antonio Perez-Escobar's *Showing Mathematical Flies the Way Out of Foundational Bottles: The Later Wittgenstein as a Forerunner of Lakatos and the Philosophy of Mathematical Practice* argues that the later Wittgenstein should be seen as a forethinker for philosophy of mathematical practice. Perez-Escobar works on questions of rule-bending and offers a Wittgensteinian reconstruction of Lakatos' famous analysis of the Polyhedra Theorem. The article, furthermore, offers an introduction to the philosophy of the later Wittgenstein and Lakatos' philosophy of mathematics as well as the historical developments leading to the flourishing community of philosophers of mathematical practice.

HYPHER-REF: A General Model of Reference for First-Order Logic and First-Order Arithmetic by **Pablo Rivas-Robledo** offers a project in Lakatosian spirit. His idea is to enrich our logical tools with tools from the philosophy of language, specifically, the notion of reference. A key feature of Rivas-Robledo's account is its

hyperintensionality, which stands in contrast to the extensionality of classical logic. He argues that using the latter to model ‘consequence’, as used in mathematical practice, might cause problems. The mathematician’s notion of consequence is – in practice – not always the ‘logical’ one, for instance, when they make assertions such as ‘Theorem B follows from Theorem A’. Imagine, someone claiming that Fermat’s Conjecture follows from the Pythagorean Theorem. This is logically true by the semantics of logical implication; however, a mathematician would typically demand a proof of the former from the latter to accept this claim.

The Quasi-Empirical Epistemology of Mathematics by **Ellen Yunjie Shi** offers an exposition of Lakatos’ ‘A Renaissance of Empiricism in the Recent Philosophy of Mathematics’. They analyse a key idea of Lakatos’, namely that mathematics is quasi-empirical. This analysis acknowledges the role of extrinsic justification for axioms but ends up arguing against a quasi-empirical epistemology of mathematics. Shi concludes with an exposition of mathematical pluralism, which – so the article – is the adequate picture of mathematics.

In *Lakatos’ Quasi-Empiricism Revisited*, **Wei Zeng** offers a Wittgensteinian solution to the problem that Lakatos, while focusing on actual practices, failed to account for the inductive reasons mathematicians feature while forming their conjectures. To achieve this, she argues, we need to turn to Steiner’s reading of Wittgenstein. He rooted the regularity of mathematical behaviour in physical regularities and rules for their description, but not in mere sociological regularities, as previously championed by Bloor. Zeng contends that this improves Lakatos’ quasi-empiricism about mathematics.

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