"Adding Up" Reasons: Lessons for Reductive and Non-Reductive Approaches

Shyam Nair*

April 1, 2021

Abstract

How do multiple reasons combine to support a conclusion about what to do or believe? This question raises two challenges: (1) how can we represent the strength of a reason? (2) how do the strengths of multiple reasons combine?. Analogous challenges about confirmation have been answered using probabilistic tools. Can reductive and non-reductive theories of reasons use these tools to answer their challenges? Yes, or more exactly: Reductive theories can answer both challenges. Non-reductive theories, with the help of a (new?) result in confirmation theory, can answer one and there are grounds for optimism that they can answer the other.

*For comments during talks or informal conversation, I thank Louise Antony, Brad Armendt, Sara Aronowitz, Jamin Assay, Derek Baker, Nathan Biebel, Tom Blackson, Ben Blumson, Richard Bradley, Elizabeth Brake, Bruce Brower, Philip Bricker, Fabrizio Cariani, Amandine Catala, Juan Comesaña, Peter de Marneffe, Sinan Dogramaci, Kenny Easwaran, Andrew Fisher, Branden Fitelson, Jonathan Frome, Jerry Gaus, Ernesto Garcia, Michael Gil, Peter Graham, Keith Hankins, Terry Horgan, Sophie Horowitz, Mengyu Hu, Simon Huttegger, Michael Johnson, Jim Joyce, Jaemin Jung, John Hawthorne, Rachana Kamtekar, Keith Lehrer, Ben Lennertz, Joseph Levine, Paisley Livingston, Errol Lord, Yael Lowenstein, Andres Luco, Barry Maguire, Christopher Meacham, Qu Hsueh Ming, Michael Pelczar, Alejandro Pérez Carballo, Ángel Pinillos, Steven Reynolds, Dan Robins, Jon Robson, Andrea Sauchelli, David Shoemaker, Frank Saunders, Mark Schroeder, Itai Sher, Neil Sinclair, Houston Smith, Fei Song, David Sosa, Julia Staffel, Weng Hong Tang, Mark Timmons, Michael Titelbaum, Aness Webster, Jo Wolf, Christopher Woodard, Jiji Zhang as well audiences at Hong Kong University, University of Arizona, National University of Singapore, University of Massachusetts at Amherst, Arizona State University, the University of Southern California, Lingnan University, Nottingham University, the New Work on Choice and Belief conference, University of Texas at Austin (where I unfortunatel failed to note the names of several graduate students who raised important points that I benefited from), and Tulane University. For comments on a draft, I thank Brad Armendt, John Hawthrone, Bryan Leitz, Ben Lennertz, Ángel Pinillos, and Justin Snedegar. I am especially grateful to the two anonymous referees, four anonymous associate editors, and Jamie Dreier at *Ethics* for several sets of challenging comments that led to a many improvements including a revision of the thesis of this paper. Finally, I thank the Murphy Institute at Tulane and Arizona State University for support.

Introduction

A popular albeit controversial idea in moral philosophy is that what we ought to do can be explained by our reasons.[1] One challenge for this view is to provide illuminating explanations of what we ought to do in cases where multiple reasons combine to support an act. We can illustrate this by considering the following example:

> [...] there is a movie theater and a restaurant across town. And suppose that in order to get to that side of town I must cross a bridge that has a $25 toll. The toll is a reason not to cross the bridge. The movie is a reason to cross the bridge and the restaurant is also a reason to cross the bridge. It may be that if there were just the movie to see, it wouldn't be worth it to pay the toll and if there were just the restaurant, it wouldn't be worth it to pay the toll. But given that there is both the movie and the restaurant, it is worth it to pay the toll. (Nair 2016: 56)

In this case, the movie theater provides a reason to cross the bridge, the restaurant provides a reason to cross the bridge, and the toll provides a reason to not cross the bridge. Individually, the reason provided by the restaurant is worse than the reason provided by the toll and the reason provided by the movie is worse than the reason provided by the toll. But the two reasons to cross the bridge together—what is sometimes called the "accrual" of these reasons—are better than the reason provided by the toll.

Cases of this sort are ubiquitous and arise not just for action but also for belief. Here is another example:

> I am curious about what color the feathers of a certain bird are. My friend seems to remember reading in a textbook that they are black. I seem to remember seeing in a nature documentary that they are white. I also seem to remember seeing in the travel guide that they are white. It may be that my friend's memory based on the textbook is a better reason to believe that the feathers are black than my memory of the documentary or the travel guide taken individually. But it may be that together these reasons to believe that the feathers are white are better than the reason to believe that the feathers are black so that I have more reason to believe that the feathers are white. (ibid.: 57)

If we also accept the idea that what we ought to believe can be explained in terms of reasons, then we would like to understand how reasons combine in these cases as well.

My goal in this paper is to explain the challenge posed by cases of accrual and to develop some strategies for meeting this challenge. But an important

[1] This idea appears (in different terminology) at least as early as Ross 1930. Other important discussions include Dancy 2004, Hampton 1998, Nagel 1970, Parfit 2011, Raz 2002 [1975], Scanlon 1998, and Schroeder 2007.

complication arises immediately: Philosophers with very different theoretical commitments accept the idea that reasons explain what we ought to do and believe. In particular, some philosophers who accept this idea are *reductivists*—they believe reason are reducible to other normative properties or facts or to other non-normative properties or facts—while others are *non-reductivists*—they believe reasons are not reducible to any other normative or non-normative fact or property.[2,3] Accordingly, the strategies for meeting this challenge must be sensitive to these differences. Indeed, much of this paper is dedicated to this task.

Here's my plan: In reflecting on our examples above, we encountered the challenge of providing illuminating explanations of what we ought to do or believe in cases where multiple reasons combine to support an act or belief. I will show how this actually factors into two distinct, but related challenges posed by cases of accrual (§1). I then observe that analogous issues about how pieces of evidence confirm hypotheses have been fruitfully explored using probabilities. (§2). With this background in hand, the central issue of the paper is whether reductive and non-reductive theories can make use of these probabilistic tools. It turns out that both theories can but in different ways. Reductive theories can relatively straightforwardly make use of these tools to answer both challenges posed by cases of accrual (§3). But the situation is more complicated for non-reductive theories (§4). For non-reductive theories of reasons for belief, the issue turns on certain (until-now-unanswered?) questions in the probabilistic theory of confirmation. But I present results that answer these questions. For non-reductive theories of reasons for action, this same approach will not work because there are structural differences between reasons for action and probabilities. But recent work from Itai Sher (Sher 2019) develops a decision-theoretic account that can accommodate these differences. Nonetheless, both of these approaches for the non-reductivist require the assumption that the strength of reasons can be numerically represented. By contrast, this claim is a result of the reductivist account rather than an assumption it has to posit.[4]

Though the topic of this paper is obviously relevant for those who think that reasons explain what we ought to do and believe, it should also be of interest to anyone who thinks that there is some systematic theory about the interaction of

[2]For theories that reduce reasons to some non-normative notion see Finlay 2014 and Schroeder 2007. For theories that reduce reasons to some normative notion see Maguire 2016, McHugh and Way 2016, Portmore 2011, Setiya 2014, Smith 1994, and Wedgwood 2009. For theories that are non-reductive, see Dancy 2004, Scanlon 1998, and Parfit 2011.

[3]The distinction between reducibility, analyzability, fundamentality, grounding, metaphysical dependence, etc. will not matter for our purposes. So I will also call theories according to which reasons are analyzable, non-fundamental, grounded, metaphysically dependent, etc. reductive theories of reasons. On the other hand, non-reductive theories are theories according to which reasons are not analyzable, are fundamental, are not grounded, and are metaphysically independent.

[4]Though this paper is primarily concerns the prospects of probabilistic approaches, n. 55 briefly discusses qualitative accounts such as those from the default logic and argumentation theory traditions as well as Nair 2016 and Maguire and Snedegar 2021. As mentioned above, §4.2 also discusses a decision-theoretic rather than purely probabilistic account.

reasons (even if reasons do not explain 'ought's)[5] and to anyone who is interested in confirmation theory (especially §4.1 and §A) Furthermore, though our focus is on answering the two challenges posed by cases of accrual, the ideas here also have methodological implications. For instance, it turns out that though sometimes times two reasons are better than one, this is not always so. This means that even if we have some example where we know the strength of two reasons individually, there are still further questions to ask about the strengths of these reasons. Do we, as theorists, have free reign to choose whether the two reasons together are better than each individually? Do we have free reign to choose how much better? If not, what do these choices depend on?[6] The ideas developed here answer these questions.[7]

1 The Accrual of Reasons: Two Challenges

In cases like the ones from the beginning of the paper, we would like to know how the strength of the accrual is related to the strength of its members. Off hand, it seems that the strength of the two reasons together to cross the bridge is some kind of increasing function of the strength of the reasons individually. Indeed it may tempting to say that the strength of an accrual is some how the sum of the strengths of its members.

If we take talk of the "sum of strengths" at face value, it presupposes that the strength of a reason is somehow sensibly represented by a number. But since we have no pretheoretical grip on how to construct such a numerical representation, it is a pressing question whether strengths can be numerically represented and what the basis for such a representation might be.

If, on the other hand, the strengths of reasons cannot be numerically represented, it is a pressing question how to state in purely qualitative terms the

[5] Thanks to the referee who encouraged me to emphasize this. Some distinctions due to Selim Berker (Berker 2018) can make this point more vivid. Berker observes that what he had called in earlier work (Berker 2007) a "combinatorial function"—a function that maps individual reasons and their strengths to verdicts about what we ought to do—can be thought of a composition of two other functions. The first is what he calls a "aggregation function" which maps individual reasons and their strengths to results about how strongly supported by the reasons overall each act or belief is. The other is a "comparison function" that maps the outputs of the first function to verdicts about what we ought to do or believe. The issues here are most directly about Berker's aggregation function.

[6] To illustrate, suppose an agent faces a choice to do act A that saves a person x or do act B that saves person y and z. There are various theories about whether the fact that some act will save a life provides a reason. But suppose we consider a theory according to which the fact that doing A will save x is a reason to do A, the fact that doing B will save y is a reason to do B, and the fact that doing B will save z is a reason to do B. And suppose further that the theory says these reasons are each individually exactly as strong as one another. Given these assumptions, does the general theory of how reasons interact settle whether the individual reasons to do B together also provide a reason to do B? If so, does it settle how strongly the reasons together support doing B? Or, instead, do we need to make further assumptions in order to settle whether and how strongly the reasons together support doing B?

[7] The answers that follow from the accounts developed below are 'no', 'no', 'each strategy gives its own (somewhat precise) answer to which factors these choices depend on'.

relationship between the strength of an accrual and the strengths of its members. For example, it is not enough to say that the accrual of reasons to cross the bridge is stronger than the individual reasons. We must also somehow translate into qualitative terms the idea that *the extent* to which the accrual is stronger makes it so that the reasons together to cross the bridge are stronger than the reason to not cross the bridge.[8]

This, then, is the first challenge posed by cases of accrual: We must determine a suitable way of representing the strengths of reasons that allows us to understand how the strength of the accrual in certain cases is the right sort of increasing function of the strengths of its members. And we must provide some basis for such a representation.[9,10]

The second challenge concerns sorting different cases of accrual. As we have seen, there are cases where a collection of reasons to do a given act has a strength that is (strictly) greater than the strength of any individual reason. But sometimes the strength of a collection is not (strictly) greater than the strength of each individual reason.

The literature on this topic includes a variety of cases that illustrate this including putative cases where the collection is exactly as strong as an individual reason, where the collection provides a reason that is weaker than the individual reasons (and perhaps supports an incompatible act), and where the collection provides a no reason at all. It is perhaps simplest to start by illustrating this with minimal variant of the case involving reasons for action that we began the paper with:

> As before, the toll is $25 dollars, as before, there is a restaurant and a movie theater that I can access only by paying this toll. But in this case, let's suppose that the movie only has one showing and the restaurant only has one seating and they are at the same time so that I cannot attend both. Still, the movie is a reason to cross the bridge and the restaurant is a reason to cross the bridge. But the accrual of these reasons is not any stronger than these reasons individually. (Nair 2016: 66)

We can then consider the following case involving reasons for belief:

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[8] N. 55 discusses this and related concerns for theories of accrual that do not involve numerical representation. That said, the best developed views for understanding certain features of reasons (e.g, undercutting, attenuation, and intensifying) are views such Horty 2012 that do not involve numerical representation.

[9] Issues about numerical representation or measurability regarding the strength of reasons are mentioned (in different terminology) as early as Nozick 1968. See Krantz et al. 2007 for a general introduction to philosophical and formal issues related to measurability.

[10] The answers that we consider below assume a simple theory according to which reasons can be directly compared as better than or worse than or equally good as one another. This ignores a number of complications. For example, Patricia Greenspan has argued that that reason against an act and reason for refraining from doing the act must be distinguished (Greenspan 2005, Greenspan 2007, Greenspan 2010). And a variety of philosophers have argued that some reasons are incommensurable (see Chang 1997 for a classic collection on this topic). Though these are serious complications, I think it is good to approach our problem by first seeing how simple views can address it.

> You know that John and Bill are rarely found together—they dislike each other and make it a point to avoid each other. There is a party this week and you are wondering whether John or Bill but not both John and Bill will attend. In this setting finding out John will attend is a reason to believe that John or Bill but not both will attend. Similarly, finding out Bill will attend is also a reason to believe John or Bill but not both will attend. [...] But their accrual is not a reason to believe John or Bill but not both will attend. (Nair 2016: 59-60)

And finally we can consider the following sampling of cases to get a sense of the variety of examples that have been offered:

> Consider by way of example two reasons not to go jogging, viz. that it is hot and that it is raining. For a particular runner the combination of heat and rain may be less unpleasant than heat or rain alone so that the accrual is a weaker reason not to go running than the accruing reasons. And for another jogger the combination of heat and rain may be so pleasant that it is instead a reason to go jogging. (Prakken 2005: §3.1)

> Suppose, for example, that Symptom 1 is a reason for the administration of Drug A, since it suggests Disease 1, for which Drug A is appropriate, and that Symptom 2 is also a reason for the administration of Drug A, since it suggests Disease 2, for which Drug A is also appropriate; still, it might be that Symptoms 1 and 2 appearing together suggest Disease 3, for which Drug A is not appropriate. (Horty 2012: 61)

> Suppose that I have a disease. My doctor proposes a treatment, and the question I am considering is, "Should I take the treatment?" [...] Suppose that R_1 is now the proposition that the treatment would prolong my life by at least 1 year, and R_2 is the proposition that the treatment would prolong my life by at least 2 years. [...] the sum $w_1 + w_2$ of the weights of reasons R_1 and R_2 does not represent a meaningful quantity. This sum double counts the weight of the fact that the treatment will prolong my life by at least one year, as this fact is entailed by both R_1 and R_2. (Sher 2019: 104-105)

While there are ways of resisting the force of these putative cases in which the strength of the accrual is not greater than the strengths of the individual reasons, I will not discuss this here. I take it that the diversity of form and subject matter of these cases will allow different readers to find at least one to agree with. In any event, their diversity of form and subject matter make it apparent that something substantive must be said to explain the difference—whether genuine or merely apparent—between these cases and cases where the accrual of reasons has a strength that is greater than the strengths of its members. This is the second challenge posed by cases of accrual. If one has answered the first challenge by providing a suitable representation of the strength of reasons, the second challenge is to show that this representation allows "adding up" in the correct cases and deals with the range of possible cases in which "adding up" does not occur.

2 How Confirmation Theory Meets the Confirmation Analogues of These Challenges

An issue analogous to the issue of the accrual of reasons is that sometimes two pieces of evidence may confirm a theory more than one. But other times they may not. As it turns out, theories of confirmation that make use of probabilities are capable of shedding light on this phenomena.[11]

2.1 Probabilities, Confidences, and Confirmation

To start, we need to state what a probability function is. For our purposes, a probability function is any function that assigns (real) numbers to (an algebra of) propositions in a way that obeys the following axioms:

Non-Negativity: $Pr(A) \geq 0$ *for any* A

Normalization: $Pr(\top) = 1$ *where* $\top$ *is a logical truth*

Finite Additivity: $Pr(A \vee B) = Pr(A) + Pr(B)$ *for any* A, B *such that* $A \wedge B$ *is a logical falsehood*

Ratio: $Pr(A \mid B) = \frac{Pr(A \wedge B)}{Pr(B)}$ *when* $Pr(B) \neq 0$

This purely formal definition of a probability function tell us little of interest on its own.

There are however interesting philosophical arguments that purport to show that the confidences of a rational agent can be represented by a probability function. According to these arguments, a rational agent, $\mathcal{S}$, who is very confident, e.g., that it will snow tomorrow can have her confidence represented by a probability function, $Pr_{\mathcal{S}}$, according to which $Pr_{\mathcal{S}}(\textit{It will snow tomorrow})$ is some number close to one. Rational agents also have conditional confidence. So while $\mathcal{S}$ may have very little confidence that it will rain tomorrow (so $Pr_{\mathcal{S}}(\textit{It will rain tomorrow})$ is low), she may nonetheless be very confident that it will rain tomorrow conditional on the meteorologist saying that it will rain tomorrow. This can be represented by:

$$Pr_{\mathcal{S}}(\textit{It will rain tomorrow} \mid \textit{The meteorologist says that it will rain tomorrow})$$

taking some value close to one. In my stipulative usage, a *Bayesian* interpretation of a given probability function is one on which the function is understood to represent these kinds of states of an agent.

[11] Thanks Kenny Easwaran for encouraging me to pursue this approach.

Importantly, Bayesians have arguments that explain why it is sensible to represent a state of confidence with a probability function. Unfortunately, we do not have the space to consider even the basic details of these arguments. But it suffices for now to know the general strategy behind them. The arguments work by providing a set of axioms characterizing rational confidences that are qualitative (e.g, if you are more confident in A than in B and you are more confidence in B than in C, then you are more confident in A than in C). They then show that a particular kind of numerical representation is, in a certain sense, equivalent to this qualitatively characterized notion of rational confidence. This set of qualitative axioms, then, is the sensible basis of the numerical representation.[12]

If we have a Bayesian interpretation of a given probability function, we can say something interesting about confirmation. The idea is that we can determine whether some E (e.g., that the meteorologist says that it will rain) confirms some hypothesis H (e.g., that it will rain) for you by comparing your confidence in H (it raining) with your conditional confidence in H on E (it raining given that the meteorologist says that it will rain). If you are more confident in H on E than you are in H, then plausibly E confirms H for you. If we use $Pr_{\mathcal{S}}$ to represent $\mathcal{S}$'s confidences, we can state this analysis as follows:

What it is for E to confirm H for $\mathcal{S}$ is for $Pr_{\mathcal{S}}(H \mid E) > Pr_{\mathcal{S}}(H)$

This gives us a theory of when a piece of evidence confirms a hypothesis.

But it does not tell us *how much* a piece of evidence confirms a hypothesis. As it turns out, there are different measures that have been proposed to answer this question. For example, one approach is that we just look at the difference in your confidence in H and your confidence in H on E.

But for the purposes of our discussion, it proves convenient to focus on another confirmation measure that will initially seem less straightforward:

Log Likelihood Measure: $l(H,E) = log\left(\frac{Pr(E|H)}{Pr(E|\neg H)}\right)$

There is much to be said about this measure and why it is, despite how it might seem at first, quite intuitive. And I give a brief indication of this in a note.[13]

[12] The historically most prominent arguments supporting the Bayesian view have not focused on an epistemic state of confidence. Rather they have focused on the idea that a probability function is one of pair of functions (the other being a utility function) that represents the preferences of a rational agent. Important representation theorems in this tradition include Ramsey's representation theorem (Ramsey 1931 [1923]), Savage's representation theorem (Savage 1972), Jeffrey-Bolker's representation theorem for evidential decision theory (Jeffrey 1990), and Armendt's and Gibbard's representation theorems for causal decision theory (Armendt 1986, Gibbard 1986). See Joyce 1999 (esp. ch. 7) for useful discussion.

That said, there are also results that can be understood as directly about states of confidence. These are results from the comparative probability tradition initiated by Bruno de Finetti. Though the first representation theorem in this family is Kraft, Pratt, and Seidenberg 1959, these results have come to be closely associated with Dana Scott (Scott 1964) due to Scott's elegant way of axiomatizing confidences. See for Konek 2019 for a contemporary survey that also breaks new ground.

[13] Let's start with an example. Suppose you are wondering whether it will rain and considering consulting the meteorologist. Suppose further your conditional confidence that the

But the details of how l measures confirmation are not central to this paper. This is because I am not assuming that l is the only legitimate measure of confirmation. I discuss other measures (including the one that involves taking the difference) in §B. The main text focuses on just one measure to allow for a clearer and more streamlined discussion. And I opt for l as our focus because it allows us to most easily state the results about "adding up" reasons that are our main focus.[14]

Now that we have selected a measure of confirmation to focus on, we can state the Bayesian analysis of how much confirmation a piece of evidence provides. If we write, $l_{\mathcal{S}}$ for a version of l that is defined using the probability function that represents $\mathcal{S}$'s confidences, $Pr_{\mathcal{S}}$, the idea is that:

$$\text{What it is for } E \text{ to confirm } H \text{ to degree } n \text{ for } \mathcal{S} \text{ is for } l_{\mathcal{S}}(H, E) = n$$

This analysis answers both challenges posed by the confirmation-analog of cases of accrual.

We can sensibly represent confirmation numerically: Confirmation is understood in terms of a numerical representation of confidences via the equation

meteorologist says that it will rain on it raining is the same as your conditional confidence that the meteorologist says that it will rain on it not raining. This is a kind of skepticism about the reliability of the meteorologist. So it is natural to take this to mean that you don't regard the meteorologist saying that it will rain as providing confirmation for the claim that it will rain.

But consider another set of attitudes that you might have about what the meteorologist says. You might be way more confident that the meteorologist says that it will rains on it raining than you are confident that the meteorologist says that it will rain on it not raining. Here you seem to regard the meteorologist saying that it will rain as providing confirmation of it raining. And it also seems like if you are ten times more confident in the meteorologist saying that it will rain on it raining than in the meteorologist saying that it will rain on it not raining, you regard it as pretty good evidence. On the other hand if you are only twice as confident, you regard it as good evidence but not as good evidence.

We can build on these observations to see what is plausible about l. Let E represent the claim that the meteorologist says that it will rain. Let H represent the claim that it will rain. What we have seen is comparing $Pr(E \mid H)$ to $Pr(E \mid \neg H)$ tells about how much confirmation the meteorologist says that it will rain provides for the claim that it will rain. In particular, it looked plausible to compare the ratio of $Pr(E \mid H)$ to $Pr(E \mid \neg H)$ to determine how much confirmation is provided.

Of course, l also places a *log* in front of this ratio. The purpose of this is two fold. First, it is a feature of a *log* that $log(1) = 0$. If we are using 0 to represent no confirmation either way, then this feature of a *log* is useful. Recall that what the meteorologist says provides no confirmation either way about it raining when you are equally confident that the meteorologist says that it will rain on it raining and on it not raining. When you are equally confident in this way, the relevant ratio is 1. l applies a *log* to this ratio. So it represent the degree of confirmation as 0—this correctly tells us that there is no confirmation.

Second, the *log* keeps track of how many times larger (or smaller) the top term in the ratio is than the bottom. For example, if we choose a *log* of base 2, it says that when the top term is twice as large as the bottom, we represent the confirmation as 1. We can choose whatever base for the *log* that we like (so long as it is greater than 1). Which base we choose will change exactly what numbers we use to represent the strength of confirmation. But otherwise, all the comparisons between claims about confirmation will, in a certain sense, be the same.

[14] Since we allow the *log* to take on any base (greater than 1), l defines a family of measures. As I have said, I do not assume it is the only legitimate measure but I am sympathetic to the idea that it is an especially plausible one (see Good 1983 and Fitelson 1999 for discussion).

defining the confirmation measure. We can sensibly represent confidences numerically because of the Bayesian arguments. This answers the confirmation-analog of the first challenge.

We can also answer the confirmation-analog of the second challenge. In order to state the answer, I will make use of the notion of probabilistic independence. The intuitive idea of A (e.g., the coin came up heads on the second toss) being independent of B (e.g., the coin came up heads on the first toss) is that your confidence in A wouldn't change if you learned B. So more formally, A *is independent of* B just in case $Pr(A \mid B) = Pr(A)$.[15] We can also generalize this idea to say that A *is independent of* B *conditional on* C just in case $Pr(A \mid B \wedge C) = Pr(A \mid C)$.

The challenge is answered, then, by the following result: Suppose E is independent of E' are conditional on H and conditional on $\neg H$. Then, the strength of confirmation $E \wedge E'$ provide for H is the sum of the strength of confirmation E provides for H and E' provides for H. In symbols, if the relevant independence conditions hold:

$$l(H, E \wedge E') = l(H, E) + l(H, E')$$

I leave the proof of this to a note.[16]

Indeed, there is a generalization of it (see Claim 1 in n. 16) that applies even in cases where the relevant independence conditions do not hold. We do not need to get bogged down by the exact details of the generalization. But what this illustrate is that l provides a numerical representation and model of cases in which confirmation "adds up", doesn't "add up" at all, and anything in

[15] As is illustrated here, Bayesians often take your confidence in A conditional on B to represent a commitment about how you will change your confidence in A on learning (all and only) B (for certain).

[16] It is easiest to prove this by first proving a slightly more general result. If we define $l_{|E}(H, E') = log\left(\frac{Pr(E' | H \wedge E)}{Pr(E' | \neg H \wedge E)}\right)$, then the following result is known to hold:

Claim 1. $l(H, E \wedge E') = l(H, E) + l_{|E}(H, E')$

Proof of Claim 1.

$$\begin{aligned} l(H, E) + l_{|E}(H, E') &= log\left(\frac{Pr(E \mid H)}{Pr(E \mid \neg H)}\right) + log\left(\frac{Pr(E' \mid H \wedge E)}{Pr(E' \mid \neg H \wedge E)}\right) \\ &= log\left(\frac{Pr(E \mid H) Pr(E' \mid H \wedge E)}{Pr(E \mid \neg H) Pr(Pr(E' \mid \neg H \wedge E)}\right) \\ &= log\left(\frac{Pr(E \wedge E' \mid H)}{Pr(E \wedge E' \mid \neg H)}\right) = l(H, E \wedge E') \end{aligned}$$

□

Claim 1 has the following corollary:

Corollary 1.1. $l(H, E \wedge E') = l(H, E) + l(H, E')$ *if* $l(H, E') = l_{|E}(H, E')$

Turn now to the additivity claim in the main text. Assume that the relevant independence conditions hold so that $Pr(E' \mid H \wedge E) = Pr(E' \mid H)$ and that $Pr(E' \mid \neg H \wedge E) = Pr(E' \mid \neg H)$. In this setting, $l_{|E}(H, E') = l(H, E')$. So given our corollary, the additivity claim in the main text holds.

between. As I alluded to before, similar results hold for several other measures of confirmation (§B).

Of course, there is much more that could be said about the resources of Bayesian theories of evidence to analyze different cases. And there are certain potential problems that have been raised for these theories (e.g., old evidence, logical learning, new theories). A full investigation of this subject matter is worthy of (and has been given) monograph-length treatment. But hopefully I have conveyed, at least in outline, why Bayesian confirmation theory is relevant to our topic: that theory is a model of how two pieces of evidence for given hypothesis can interact that answers the confirmation analog of both of our challenges. The question now is how this idea can be adapted to tell us about reasons.

2.2 A Bayesian Reduction of Reasons for Belief to Confidences

And it is not hard to see how we might adapt the theory to give an analysis of reasons. Suppose $Pr_{\mathcal{S}}$ is a representation of $\mathcal{S}$'s confidences, then we can say:

- What it is for P to be a reason for $\mathcal{S}$ to believe Q is for $Pr_{\mathcal{S}}(Q \mid P) > Pr_{\mathcal{S}}(Q)$
- What it is for P to be a reason for $\mathcal{S}$ to believe Q of strength n is for $l_{\mathcal{S}}(Q, P) = n$

Call this the *Bayesian Simple Theory of Reasons.* It analyzes reasons for belief in terms of the structure of an agent's rational confidences.[17,18]

This theory provides a sensible basis for numerically representing the strength of reasons by reducing this to a numerical representation of confidences that is known to be sensible (due to the Bayesian arguments). This answers the first challenge posed by cases of accrual.

[17] This analysis is not uncontroversial. As an editor at *Ethics* pointed out to me, Foley 1991 gives a putative counterexample where if one believes what the evidence supports, this changes what the evidence is. John Hawthorne has also suggested several counterexamples. The one that concerns me the most is a case where E is the proposition that H has objective chance, e.g, .4 but nonetheless E raises the probability of H because H's prior probability is lower than .4. Though I cannot discuss the issue fully, I believe that it is not devastating to bite the bullet in either case. In the first case, some comfort can be provided by ideas philosophers have developed in response to the wrong kind of reasons problems. In the second case, some comfort is provided by looking at a different feature of the force of reasons. In particular, the feature of a collection of reasons that concerns whether what is currently most supported my one's reasons is liable to change (a feature sometimes called "resilience" or "weight" in the literature on confirmation, see Joyce 2005: §3-5 for an introduction.). In any case, our focus is on clarifying the attractive features of probabilistic approaches rather than answering these objections.

[18] There are two noteworthy complications for this analysis. First, it does not require reasons to be truths or known. But arguably, reasons have these features. We can deal with this complication by adding this as an additional condition of the analysis (see, however, n. 37 below for how this issue arises for non-reductive approaches). Second, there is a general difficulty involving if and when to invoke background bodies of information in applying the Bayesian analysis of confirmation that also will arise for reasons.

Turning now to the second challenge, return to the example of John and Bill who are rarely found together. There we said that John is going to the party is a reason to believe exactly one of John or Bill will be at the party and that Bill is going to the party is a reason to believe exactly one of John or Bill will be at the party. But together these two do not provide a reason to believe that exactly one of John or Bill will be at the party. The approach we have been considering suggests that cases like this arise only when the relevant independence condition that we mentioned does not hold.

That independence condition is that the two pieces of evidence, E and E', are independent conditional on H and conditional on $\neg H$. And recall, in symbols the idea that E' is independent of E conditional on H can be written as $Pr(E' \mid E \wedge H) = Pr(E' \mid H)$. But in the example of John and Bill this independence condition does not hold. To see this, begin by noting that

$$0 = Pr_{\mathcal{S}}(Bill\ goes\ to\ the\ party \mid \\ John\ goes\ to\ the\ party\ \wedge\ Exactly\ one\ of\ John\ and\ Bill\ go\ to\ the\ party)$$

But on the other hand:

$$0 < Pr_{\mathcal{S}}(Bill\ goes\ to\ the\ party \mid \\ Exactly\ one\ of\ John\ and\ Bill\ go\ to\ the\ party)$$

Thus, our approach (correctly) does not tell us to expect that the strength of the accrual in this case is the sum of the strengths of the individual reasons.

What's more, depending on the details of how we spell the case out, it may be that the example concerning the color of the feathers of a certain bird satisfies the independence condition. And therefore, we can explain why the accrual is stronger than the individual reasons in this case.

The main problem for the *Bayesian Simple Theory of Reasons* is that it is not clear how to generalize it so that we can have an account of reasons for action.

3 Some Reductive Theories

We can do better by adopting certain kinds of reductive theories. First, I illustrate this by discussing the reductive theory of Kearns and Star 2009 as it is easy to see how their theory fits with a probabilistic approach. I then isolate the features of their theory that make it such a good fit and describe alternative theories that are also good fits with the probabilistic model. As it turns out, many theories can be regimented so that they have a probabilistic structure.

3.1 Kearns and Star's Reduction of Reasons to Evidence

Kearns and Star claim that reasons for action, belief, and other attitudes can be understood in terms of evidence. In particular, they believe a reason for action is evidence that the act ought to be done, a reason for belief is evidence that the agent ought to have the belief, and a reason for any other attitude is just evidence that the agent ought to have that attitude.

Kearns and Star's view need not be committed to the Bayesian picture of confirmation But they do believe that one of the best features of their theory is that it provides an account of the weight of reasons in terms of the weight of evidence.[19]

So let the *Bayesian Kearns and Star Theory of Reasons* be the theory that reasons are evidence of 'ought's and that evidence is to be understood in terms of Bayesian confirmation (i.e., in terms of the structure of a fully rational agents confidences). This allows us to straightforwardly apply our work from §2 to give an account of reasons for belief and action. If P is a reason for $\mathcal{S}$ to ϕ and Q is a reason for $\mathcal{S}$ to ϕ (where ϕ may be an act or an attitude), the Kearns and Star picture claims P is evidence that $\mathcal{S}$ ought to ϕ and Q is evidence that $\mathcal{S}$ ought to ϕ. The Bayesian picture tells us that if $Pr_{\mathcal{S}}(Q \mid P \wedge \mathcal{S}\ \textit{ought to}\ \phi) = Pr_{\mathcal{S}}(Q \mid \mathcal{S}\ \textit{ought to}\ \phi)$ and $Pr_{\mathcal{S}}(Q \mid P \wedge \neg\mathcal{S}\ \textit{ought to}\ \phi) = Pr_{\mathcal{S}}(Q \mid \neg\mathcal{S}\ \textit{ought to}\ \phi)$, then the strength of the accrual is the sum of the strengths of each individual reason (the picture also tell us what the strength of the individual reasons are).

Thus, by combining Kearns and Star's view and the Bayesian theory of confirmation, we get a reduction of reasons that has a probabilistic structure. This answers the twin challenges posed by cases of accrual.

One limitation of this approach is that it is not obvious whether Kearns and Star's theory is itself is compatible with the idea that what we ought to do is explained by reasons—Kearns and Star appear to explain facts about reasons in terms of prior facts about 'ought's and evidence. Thus, it may not fully vindicate the explanatory ambitions of the idea that reasons explain what we ought to do.[20,21]

3.2 The Structure of the Reduction and Other Reductive Theories

Luckily, even if Kearns and Star's approach does not get us everything that we might want, it teaches us how to find other theories that might get us what we want.[22] The *Bayesian Simple Theory of Reasons* and the *Bayesian Kearns and Star Theory of Reasons* teach us is that there are two important questions to consider in order to develop a probabilistic analysis of reasons:

[19] Kearns and Star have written many other articles that touch on our topic in addition to Kearns and Star 2009 (e.g., Kearns and Star 2008, Kearns and Star 2013). Kearns 2016 (especially his §2.2.1 and §3.2.5) explicitly discusses our topic and advocates the basic idea of the view here (even if not all of the details).

[20] The account would be adequate for those who are merely seeking a theory of the systematic interaction among reasons in these cases.

[21] Of course, Kearns and Star's view has also been subject to serious critical scrutiny. See, for example, Brunero 2009, Brunero 2018: §14.2-14.4, Hawthorne and Magidor 2018: §5.4, Schmidt 2017. I do not discuss these important objections here but instead focus on developing the attractive feature of probabilistic approaches.

[22] I thank Derek Baker for the kernel of this idea.

Q1: What does a probability function represent?[23]

Q2: What "class of hypotheses" determines what our reasons are and how do these hypotheses determine our reasons?[24]

The *Bayesian Simple Theory of Reasons* and the *Bayesian Kearns and Star Theory of Reasons* agree on their answer to *Q1*: probabilities are representations of the confidences of fully rational agents. This is what makes them both Bayesian.

The theories differ, however, on their response to *Q2*. To get the feel of what I have in mind by the "class of hypotheses", consider what each theory would make of, for example, the fact that

$$\begin{aligned}&Pr(It\ will\ rain\ tomorrow\ |\\ &\quad The\ weather\ report\ indicates\ that\ it\ rain\ will\ rain\ tomorrow)\\ &\quad > Pr(It\ will\ rain\ tomorrow)\end{aligned}$$

According to the *Bayesian Simple Theory of Reasons* this fact tells us that the weather report is a reason to believe that it will rain tomorrow. According to the *Bayesian Kearns and Star Theory of Reasons*, this fact does not immediately tell us anything about our reasons. Instead, according to this theory, we must consider

$$\begin{aligned}&Pr(You\ ought\ to\ believe\ it\ will\ rain\ tomorrow\ |\\ &\quad The\ weather\ report\ indicates\ that\ it\ will\ rain\ tomorrow)\\ &\quad > Pr(You\ ought\ to\ believe\ it\ will\ rain\ tomorrow)\end{aligned}$$

in order to determine what your reasons are.

Q2, then, is about which claims of the form $Pr(\,\cdot\mid E) > Pr(\cdot)$ determine what our reasons are. The *Bayesian Simple Theory of Reasons* takes any substitution for $\cdot$ to determine what our reasons are. And it takes these values to be the contents of beliefs that E gives us a reason to have.[25]

On the other hand, The *Bayesian Kearns and Star Theory of Reasons* only takes substitutions that express claims about what we ought to do determine what our reasons are. And it takes the attitude or act that is "supported"[26]

[23] Of course, this question must be understood relative to our purpose of understanding of reasons (similarly for evaluating answers to this question).

[24] Thank to a referee for helping me to see that second conjunct is also relevant.

[25] Example:

$$\begin{aligned}&Pr(It\ will\ rain\ tomorrow\ |\\ &\quad The\ weather\ report\ indicates\ that\ it\ will\ rain\ tomorrow)\\ &\quad > Pr(It\ will\ rain\ tomorrow)\end{aligned}$$

tell us there is a reason in support of believing that it will rain tomorrow.

[26] I use the language of an act or attitude "supported" by an 'ought'-claim (rather than a more precise term such as *prejacent*) as a fudge word to gloss over certain complexities related to the logical form of 'ought'.

by this 'ought'-claim to be what E gives us a reason to have.[27] Thus, the two theories answer *Q2* differently.

Seeing this, gives us two ways of generalizing our picture. One way is to answer *Q1* differently. That is, we can give up on the Bayesianism shared by both of these theories. Another way is to answer *Q2* differently. So we have a two-dimensional array of options for generalizing.

It is easy to see what some alternative answers to *Q2* might be. We may consider hypotheses involving normative notions other then 'ought'. For example, someone who is attracted to value-based views in normative theory might answer *Q2* by claiming only hypotheses about what is good or best are relevant for determining our reasons. Other views immediately come to mind as well: views that restrict the class of hypotheses to hypotheses about what is rational, what is fitting, etc. We may also consider answers to *Q2* that restrict attention to hypotheses concerning non-normative notions such as what satisfies desire, what causes pleasure, etc.

Each of these suggestions corresponds to a major tradition in moral philosophy and, therefore, there are epicycles to consider. For instance, there are a variety of desire-based or Humean views: some concern first-order desires, others higher-order desires; some concern actual desires, others hypothetical desires (either non-normatively or normatively characterized). Each of these views can be thought of as determining an answer to *Q2*.[28]

So this structure is able to accommodate many different views. This suggests that probabilistic reductions are ecumenical in a certain theoretically desirable sense. That said, this reduction is not trivial. It places constraints on how each of these views must be developed by committing them to a certain account of the strength of reasons. We have seen how this account is desirable for the purpose of giving a plausible theory of cases of accrual. But there may be other kinds of cases for which it creates problems.

For example, in order to get plausible results in certain case Schroeder 2007 (a Humean) is committed to rejecting the idea that the strength of a reason provided by a desire is determined by how strong that desire is. This commitment

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[27] Example:

$$\begin{aligned}&Pr(You\ ought\ to\ believe\ it\ will\ rain\ tomorrow\mid\\&\quad The\ weather\ report\ indicates\ that\ it\ will\ rain\ tomorrow)\\&\quad > Pr(You\ ought\ to\ believe\ it\ will\ rain\ tomorrow)\end{aligned}$$

tells us that there is a reason in support of believing that it will rain tomorrow.

[28] More broadly still, the theory, e.g., that the "class of hypotheses" concerns our first-order desires can be developed in two different ways according to *how* these hypotheses determine our reasons. Suppose for example:

$$\begin{aligned}&Pr(You\ desire\ to\ go\ to\ the\ store\mid\\&\quad There\ is\ a\ sale\ at\ the\ store)\\&\quad > Pr(You\ desire\ to\ go\ to\ the\ store)\end{aligned}$$

One way of developing the theory claims that this fact makes it the case that there is a reason for you to go to the store. Another says that it makes it the case that there is reason for you to desire to go to the store.

may not be compatible with implementing his view in the present probabilistic setting. Determining whether it is is beyond the scope of this paper, but the answer is relevant to assessing the merits of the Humean view. Conversely, if the Humean theory is otherwise sufficiently powerful but is implausible when probabilistically regimented, this would cast doubt on the reduction proposed here.

Let us turn now to *Q1*, the question of what a probability function represents. I have been adopting the Bayesian answer that probabilities are numerical representations of the confidences of fully rational agents. This idea itself is under specified. For instance, it does not tell us whether full rationality requires merely satisfy the basic axioms or whether it requires further properties as well (e.g., perhaps rational confidences must validate an appropriately formulated principle of indifference).

What's more, the idea that probabilities are numerical representations of confidences is often used as a label for a number of distinct ideas; indeed, this is how I have used it so far. Most famously, some believe that probabilities are one of a pair of numerical representations (the other representation being a utility function) of a rational agents preferences (see n. 12).

This view is distinct from the view that probabilities are numerical representations of an agent's confidences where confidences are understood to be a substantive epistemic state.[29] And it is distinct from other closely related views on which probabilities are numerical representations of evidential support or representations of plausibility relations.[30]

There are still other answers to *Q1* that are more distant from these. There are views which claim that probabilities are numerical representations of certain logical or semantic features of propositions. There are views which claim probabilities are numerical representations of frequencies or propensities. And there are views which claim probabilities are numerical representations of the notion of chance given by our best theories. All of these answers to *Q1* are historically prominent proposal about how to interpret probabilities.[31] Many of them are supported by arguments for their claim that probabilities are sensible numerical representation of the whatever quantity or ordering the view focuses on.[32]

There are additionally various applications of probabilities. For example, probabilities have been used to study the notion of promotion as part of an analysis of reasons in terms of promoting ends or values. That said, this literature has a somewhat complex relationship with the ideas in this paper that I discuss in a note.[33] Probabilities have also been used to study notion of causa-

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[29]See once again n. 12. See also Meachem and Weisberg 2011 for an important discussion related to these points.

[30]See Williamson 2000: ch. 10 for an important discussion of the "evidential" interpretation of probability. Discussion concerning the use of probabilities to represent plausibility centers around Cox's theorem (Cox 1946), see Jaynes 2003 for an important early discussion and Colyvan 2004 for an important more recent discussion.

[31]Some of these interpretations may overlap with interpretation in the previous paragraph (e.g., the frequency interpretation has connections to Cox's theorem).

[32]See Hájek 2012 for a survey.

[33]Early discussion of this proposal include Finlay 2006 and Schroeder 2007 and more recent

tion.[34]

Some of these applications are committed to reducing probabilities to rational confidences of agents. But other are non-committal and perhaps suggest that probabilities may be directly used to numerically represent promotion or causation. Similar remarks may apply to other notions such as strength of explanation or strength of motivation. This last class of example (promotion, causation, motivation, and explanation) correspond to familiar ideas in ethics. For example, consequentialism is concerned with promoting values.

Once again, we see that a variety of different views are compatible with the probabilistic reduction of reasons. But, as before, we should not overstate this point. Choices about which way to answer *Q1* are not trivial. First, it is not trivial to show that a certain mathematical function is a numerical representation of some important thing in the world. We have strong (albeit

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discussions include Coates 2013, Snedegar 2014, Sharadin 2015, Behrends and DiPaolo 2016, and Lin 2018.

This literature focuses on analyses according to which, roughly, what it is for P to be a reason to do X is for P to explain why doing X probabilistically promotes of some end (and the discussion is primarily about how to understand this probabilistic promotion). By contrast, the approached developed here does not make use of the the idea of a reason explaining a probability fact. Instead, it concerns when the reason raises the probability of some claim (e.g., doing X is good).

There are ways of bringing these approaches closer together. The main explicit proposal that I am aware is floated by (but not strictly endorsed by) Evers 2013: §4. This approach suggests providing a reason involves (among other things) the action together with background information raising the probability of an end. Evers suggests including the reason in the background information. Nonetheless, this approach does not quite fit the mold of this paper: First, it require supplementation with a notion of utility that we are not making use of here (though we do discuss this later in §4.2.2). Second, it appears to rely on a Bayesian interpretation of the probability function rather that interpreting probability raising directly in terms of promotion as suggested in the text.

But it is, in any case, worthwhile to consider probabilistic promotion approaches even if they don't fit the mold of our discussion here. What needs to be shown is that they can make use of the confirmation-theoretic tools described above to give an account of accrual. The difference between these approaches and the present approach makes it unclear whether and how they can.

Another related literature concerns the conditions under which reasons for ends transmit to reasons for means. Bedke 2013, Kolodny 2018, and Stegenga 2013 approach this question within a probabilistic framework. These approaches too does not easily fit with our discussion for many of the same reasons.

An important additional difference is that these approaches are primarily concerned with when a particular reason gives rise to another reason and the strength of that reason. There is no immediate account of how reasons combine. Bedke 2013 notes in his appendix that complications arise once we take into account how multiple reasons combine. That said, despite these differences, Stegenga 2013's approach is especially closely related in spirit to the approach discussed in this paper (due to his use of tools from confirmation theory and probabilistic approaches to causation).

Thanks to the associate editor at *Ethics* for pushing me to provide greater guidance about the relationship between the ideas in this paper and these two important topics.

[34] Probabilistic theories of causation come in two rough types: simple probability raising approaches and causal modeling approaches. Important work in the first tradition includes Suppes 1970, Cartwright 1979, and Skyrms 1980. Important work in the second tradition includes Spirtes, Glymour, and Scheines 2000 and Pearl 2009. See Hitchcock 2018 and Hitchcock 2019 for a contemporary survey.

not indubitable) arguments that probabilities can numerically represent certain things (e.g, preferences or frequencies). But for some of the proposals above we do not yet have such rigorous arguments. So these arguments must be developed in order to show that the proposal fully answers the first challenge posed by cases of accrual.

Second, it may be that a probalistically regimented theory has consequences for what reasons there are that cast doubt on a given reductive theory. Or conversely, if most plausible reductive theories have implausible commitments when regimented probabilistically, this may cast doubt on the reduction.

All of these issues require more detailed study than can be provided here. But if these theories can make peace with a probabilistic reduction of reasons, they will have a powerful account of cases of accrual. Since cases of accrual are mundane, they, in my view, are part of the core set of cases any adequate theory must account for. An important next step for one who accepts some particular theory of reasons, then, is to consider in detail whether their preferred view is plausible when probabilistically regimented.

4 Some Non-Reductive Theories

So far we have considered restrictions on the class of hypotheses (answers to *Q2*) and interpretations of probabilities (answers to *Q1*). All of the ideas that we have looked at are reductive in some way: Each analyzes what a reason is in terms of some non-reasons-based interpretation of a what a probability is (e.g., an interpretation in terms of rational confidences). Some are, in addition, reductive because they rely on a prior notion of value, desire-satisfaction, or the like.

Is it possible to use these probabilistic tools without reducing reasons to confidences or anything else from the list of options for interpreting probabilities that we have discussed?[35] In this section, we explore (and show) how this is possible for both reasons for belief and reasons for action. We begin with reasons for belief.

4.1 Probabilities for Non-Reductive Theories of Reasons for Belief

It helps to build up to things slowly. Recall the *Bayesian Simple Theory of Reasons*. Where $Pr_{\mathcal{S}}$ is a representation of a rational agent $\mathcal{S}$'s confidence, it claims:

- What it is for P to be a reason for $\mathcal{S}$ to believe Q is for $Pr_{\mathcal{S}}(Q \mid P) > Pr_{\mathcal{S}}(Q)$

[35] I thank a referee and Jamie Dreier at *Ethics* for not being satisfied with the original approach that I took to this question and pushing me to do better. Various other people also suggested that I need to improve on my original approach. Of those, I recall Richard Bradley, Ángel Pinillos, and Michael Titelbaum.

- What it is for P to be a reason for $\mathcal{S}$ to believe Q of strength n is for $l_{\mathcal{S}}(Q, P) = n$

This view analyzes reasons for belief in terms of rational confidences.

That said, if we no longer commit to this theory's claim about what probabilities represent, the following related theses are something a non-reductivist might hope to accept:

- P is a reason for $\mathcal{S}$ to believe Q iff $Pr_{\mathcal{S}}(Q \mid P) > Pr_{\mathcal{S}}(Q)$

- P is a reason for $\mathcal{S}$ to believe Q of strength n iff $l_{\mathcal{S}}(Q, P) = n$

Of course, the meaning of these claims is now unclear because we are no longer entitled to the Bayesian understanding of the probability terms on the right hand side of them.

We can make some progress toward clarifying the meaning of these claims in a non-reductivist friendly way if we take the left hand side to give us an understanding of the probability terms on the right hand side:

- What it is for $Pr_{\mathcal{S}}(Q \mid P) > Pr_{\mathcal{S}}(Q)$ is for P to be a reason for $\mathcal{S}$ to believe Q

- What it is for $l_{\mathcal{S}}(Q, P) = n$ is for P to be a reason for $\mathcal{S}$ to believe Q of strength n

And, indeed, this is the basic idea that I wish to propose on behalf of non-reductivists—non-reductivists are entitled to the full suite of probabilistic tools because probabilities can be analyzed in terms of reasons.[36,37]

[36]A different idea (suggested to me by Jiji Zhang) may be to take the strength of reasons to determine conditional probabilities directly in a way that suggests the following claims:

- What it is for $Pr_{\mathcal{S}}(Q \mid P) > \tau$ is for P to be a reason for $\mathcal{S}$ to believe Q
- What it is for $Pr_{\mathcal{S}}(Q \mid P) = \tau$ is for P to not be a reason $\mathcal{S}$ to believe Q and not a reason for $\mathcal{S}$ to believe $\neg Q$
- What it is for $Pr_{\mathcal{S}}(Q \mid P) < \tau$ is for P to be a reason $\mathcal{S}$ to believe $\neg Q$

where τ is some (perhaps contextually determined) threshold. The trouble with this idea is that a reason for belief requires that P *raise* the probability of Q relative to its prior probability rather than simply make the probability of Q higher than some threshold. The difference between this threshold account and an account that requires probability raising has been familiar at least since Carnap (see especially the "Preface to the Second Edition" in Carnap 1962) distinguished between the "firmness" notion of confirmation (the threshold account) and the "increased firmness" notion of confirmation (the probability raising account).

We can illustrate the importance of this point in different ways. Here is one: Suppose P and Q are probabilistically independent. Plausibly in a case like this P does not provide a reason to believe Q. But notice that, so long as τ is less than 1, there can always be cases where $Pr_{\mathcal{S}}(Q \mid P) = Pr_{\mathcal{S}}(Q) > \tau$.

See Titelbaum forthcoming: ch. 6 (especially §2) for a discussion of the contemporary state of play on this issue and for similar arguments.

[37]One complication is that this non-reductive approach is actually best thought of as a direct analysis of something like what Scanlon 2014 calls relation R (cf. what Horty 2012 (especially 16-7 and 42-3) calls generalizations or defeasible principles). This is a non-reductive relation between propositions that is supposed to be exactly like the reason-relation except

Of course, we must say much more in order to show that this proposal works. The remainder of §4.1 is dedicated to this task. That said, some readers may prefer to take my word for it that the proposal can be made to work. These readers are welcome to skip to the last two paragraphs before §4.2 for a summary of what this account says about the two challenges posed by cases of accrual.

4.1.1 How the Proposal Needs to Be Developed

To start, the non-reductivist is entitled to take for granted various qualitative claims such as P is (or is not) a reason for $\mathcal{S}$ to believe Q. Given this, they will be able to determine the truth of inequalities of of the form $Pr_{\mathcal{S}}(Q \mid P) > Pr_{\mathcal{S}}(Q)$. But there are many other claims about probabilities that they do not yet have an analysis of. And these other claims about probabilities play a role in an account of exactly how strong reasons are and in stating the independence conditions under which reasons "add up".

Suppose, then, that the non-reductivist also helps themselves to quantitative claims like P is a reason for $\mathcal{S}$ to believe Q of strength n. This is a strong substantive assumption. Indeed, the first challenge posed by cases of accrual was to provide a justification for a numerical representation of strengths of reasons. The assumption that we are considering simply posits that somehow the non-reductivist has provided such a justification. Nonetheless, let us simply grant this for now.

Even still, important questions remain. First, not all numerical representations will work. The non-reductivist needs a numerical representation that matches $l_{\mathcal{S}}$. This is not trivial because $l_{\mathcal{S}}$ is a function with specific properties, the properties of a *log* of a ratio of two conditional probabilities. Moreover, it is not open to the non-reductivist to overcome this difficulty by stipulating that the numerical representation of the strength of reasons is $l_{\mathcal{S}}$ as it is typically defined. $l_{\mathcal{S}}$ is typical defined in terms of probabilities. The non-reductivist, by contrast, wishes to define probabilities in terms of reasons. So they require a scheme for numerically representing the strengths of reasons that matches $l_{\mathcal{S}}$ but is defined without mentioning probabilities.

Second, even if we have such a numerical representation, we are not yet entitled to say that we have an analysis of probabilities. This is because it is not obvious that the values of $l_{\mathcal{S}}$ suffice to determine a probability function. If they are not sufficient, then even if we have a representation of the strength of reasons that matches $l_{\mathcal{S}}$, we would still not have an analysis of the probability function in terms of reasons.

As far as I know, these issues have not been discussed by confirmation theorists.[38] This is not especially surprising because confirmation theorists typically

that the thing that is the reason can fail to be true. I follow Scanlon in taking this to be an instance of the non-reductive approach to reasons (and an analysis of what are typically called reasons can be had by adding that the thing that is the reason is true). But this issue deserves further investigation.

[38]Thanks to Kenny Easwaran, Branden Fitelson, and Michael Titelbaum for helping me with the research that has made me more confident (though still not certain) that this question had not been answered in the literature.

take the notion of probability to be more basic than the notion of confirmation. But in the context of developing a non-reductive account of reasons for belief, this is not an option. So we must face up to these questions.

Thankfully, both of these questions can be answered: We can axiomatically define a reasons weighing function without mentioning probability and prove that this reasons weighing function matches $l_{\mathcal{S}}$. We can also use the reasons weighing function to define another function and prove that this function is a probability function—indeed, it is the very probability function involved in the $l_{\mathcal{S}}$ that matches our reasons weighing function.

The full development of these ideas is some what technical and, as of this time, the proofs that I have are not compact. So I confine them to §A. But what I wish to do next is sketch at least the basic approach. As mentioned earlier, readers who would prefer to take my word for it may skip to the last two paragraphs before §4.2.

4.1.2 Sketch of the Proposal

The approach is this: *Bayesian Simple Theory of Reasons* entails various claims about the relations among the strengths of reasons. Instead of taking these claims to be a consequence of accepting this reductive theory, the non-reductivist will take these claims to be axioms in a non-reductive theory of reasons. Since the reductivist is committed to these claims, they cannot directly object to the non-reductivist's axioms.[39]

More exactly, we will show that any reductivist who accepts the *Log* likelihood measure of confirmation, l, in a form that claims that $l(Q, P)$ tells us how strong of a reason P is to believe Q will be committed to accepting the claims that we take as axioms below. Of course, some reductivists might reject this particular measure of confirmation. Though I believe that similar results can be established for alternative measures, we do not have the space here to discuss this issue.[40] So though I will speak more loosely at times below, the key idea of the approach is that the non-reductivist takes as axioms claims that are endorsed by this *particular group* of reductivists.

Of course, the trick is to identify a set of axioms that suffice to get the non-reductivist what they want. Let's look at how we might do this. We begin by simplify things a bit. Let us keep reference to the agent whose reasons we are discussing implicit. So we now write Pr and l instead of $Pr_{\mathcal{S}}$ and $l_{\mathcal{S}}$. And let

[39] Of course, they might still object indirectly on the grounds of simplicity, elegance, etc. Or, as I point out later, on the grounds that there is no qualitative structure that has been shown to provide a basis for this numerical representation. This, as I emphasize later, is an important unresolved challenge for the non-reductivist.

[40] More exactly, I believe, based on some work in progress, that similar results can be established for the measures that exhibit the kind of conditional additivity discussed in §B. I have not explored whether similar results can be shown for the measures that do not exhibit this kind of conditional additivity. And of course, some reductivists answer $Q2$ so it is the degree to which P confirms, e.g., the claim that you ought to believe Q that determines the strength of reason P provides to believe Q. I do not see how to develop a non-reductivist approach that mirrors these approaches.

us assume that we are only considering probability functions that are *regular* in the sense that if P is not a contradiction, then $Pr(P) \neq 0$. This is an important limitation but we will work within this more restricted setting in what follows.[41]

Next recall how l is defined:

$$l(H, E) = log\left(\frac{Pr(E \mid H)}{Pr(E \mid \neg H)}\right)$$

This definition leaves implicit the fact that a *log* has a certain base. But for our current purposes, we will need to be explicit about this:

$$l_b(H, E) = log_b\left(\frac{Pr(E \mid H)}{Pr(E \mid \neg H)}\right)$$

Accordingly, the reasons weighing function to be defined will strictly speaking be a function that is relativized to a base.[42] So we will write this function as $\mathbf{r}_b$. We will discuss only three of the axioms defining $\mathbf{r}_b$ here. But this will be enough to give a sense of the general approach.[43,44]

The axioms that we will discuss primarily concern only certain pairs of propositions, (H, E). Let us say a pair (H, E) is *extreme* just in case E entails H or E entails $\neg H$. The first axiom concerns those pairs (H, E) where E is the tautology, $\top$, and (H, E) is *not* extreme. In this setting, it can be shown that:

$$l_b(H, \top) = log_b(1) = 0$$

This just means that $\top$ is not a reason for believing H and not a reason against believing H (in cases where $(H, \top)$ is not extreme).

Our strategy, then, tells us that the non-reductivist should take this claim to be an axiom about reasons rather than a consequence of a reductive account:

> No Reason: if $(H, \top)$ is not extreme,
>
> $$\mathbf{r}_b(H, \top) = log_b(1) = 0$$

The next two axioms concerns cases where (H, E) is not extreme and E is not the tautology. We say these are cases where (H, E) is *not trivial.*

[41] This limitation is discussed in a bit more detail in §A.2 and §A.4. But nothing in this paper confronts the important philosophical issues about regularity (see Easwaran 2014 for discussion).

[42] Note, however, that we can map measures that differ only in choice of base onto each other using the "change of base" formula:

$$log_b(a) = \frac{log_d(a)}{log_d(b)}$$

[43] There are in total nine axioms that define $\mathbf{r}_b$. The footnotes mention some of the axioms not discussed in the main text of §4.1.2

[44] The first axiom governing this function is about this base:

> Base Propriety: $b > 1$

This claim (which is implicitly assumed by those who accept l_b) ensures that as the term inside the *log* grows so does l_b.

Indeed, we will focus on cases where (H, E) is not trivial and is such that H entails E. It can be shown for such (H, E) that:

$$l_b(H, E) > log_b(1) = 0$$

This just means that if H entails E (and (H, E) is not trivial), then E is a reason for believing H.

Our strategy, then, tells us that the non-reductivist should take this claim to be an axiom about reasons rather than a consequence of a reductive account:

Entailed Reason: if (H, E) is not trivial and H entails E,

$$\mathbf{r}_b(H, E) > log_b(1) = 0$$

Since we have said that $\mathbf{r}_b(H, \top) = log_b(1) = 0$, another way to think of this idea is that it is saying when H entails E and (H, E) is not trivial, E is a better reason to believe H than $\top$ is a reason to believe H.

The last axiom that we will discuss in §4.1.2 is more complex. Seeing how we arrive at this more complex axiom will reveal a key idea involved in finding the other more subtle axioms described in §A.1.

We begin by noting the following fact (Lemma 1.4.1 which I prove in §A.3.2) about cases where (H, E) is not trivial and H entails E:

$$l_b(H, E) = log_b\left(\frac{Pr(\neg E)}{Pr(E \wedge \neg H)} + 1\right)$$

In the context of the previous claims, what this tell us is that the extent to which E provides a better reason for H than $\top$ provides a reason for H is a function of the ratio $\frac{Pr(\neg E)}{Pr(E \wedge \neg H)}$.

We can make use of this fact together with basic facts about the mathematical relations among ratios to discover other connections among reasons. For example, as $\frac{a}{b}$ grows larger, $\frac{b}{a}$ grows smaller and vice-versa. So if we can find a reason that is related to:

$$\frac{Pr(E \wedge \neg H)}{Pr(\neg E)}$$

as $\mathbf{r}_b(H, E)$ is related to

$$\frac{Pr(\neg E)}{Pr(E \wedge \neg H)}$$

we will have discovered an interesting connection between two reason. And it turns out, $\mathbf{r}_b(H, \neg E \vee H)$ is such a reason (in cases where (H, E) is not trivial and H entails E). In particular, once we recall that $b^{log_b(x)} = x$, with a little work it can be shown that:

$$l_b(H, \neg E \vee H) = log_b\left(\frac{b^{l_b(H,E)}}{b^{l_b(H,E)} - 1}\right)$$

What this describes is a particular negative correlation between two reasons. And, indeed, the strength of the reason E provides to believe H is intuitively negatively correlated with the strength of the reason $\neg E \vee H$ provides to believe H. In any case, whether immediately intuitively plausible or not, this is a generalization entailed by the reductive approach.

So the strategy we are pursuing tell us to take this as an axiom:

> Negatively Correlated Reasons: if (H, E) is not trivial and H entails E,
> $$\mathbf{r}_b(H, \neg E \vee H) = log_b\left(\frac{b^{\mathbf{r}_b(H,E)}}{b^{\mathbf{r}_b(H,E)} - 1}\right)$$

Obviously, this axioms is more complicated and imposes stronger constraints on what a reasons weighing function is like.[45] But recall that the non-reductivist has a standing defense of their axioms: the axioms are claims that reductivist must accept. The reductivist and non-reductivist only differ about whether this claim is taken to be axiomatic or to be a consequence of a reduction.

What I do in §A is develop a different notation for discussing cases where (H, E) is non-trivial that lets us quickly discover further correlations among the strengths of reasons.[46] We then take the claims that describe these correlations to be axioms.[47] In addition to these axioms, we need one straightforward axiom to cover the cases where (H, E) is extreme. Overall, the axioms vary in complexity from very simple to even more complicated than Negatively Correlated Reasons. But each one is, on reflection, plausible and, in any case, is a claim that the non-reductivist's opponent is committed to.

My hope is that this gives a sense of how the idea of defining a reasons weighing function without mentioning probabilities can work. The precise and complete statement of all the axioms that define a reasons weighing function, $\mathbf{r}_b$, is given in Definition 1 in §A.1. Though I will not discuss the details here, I also show there how to define (Definition 2) a function, $f_{\mathbf{r}_b}$, based on $\mathbf{r}_b$ and prove that it is a probability function.[48] The main result, then, that we prove

[45] Those who read §A will notice that many of the axioms include the term -1 and $+1$. We saw that for (H, E) that is not trivial and is such that H entails E, E is a reason to believe H of strength $n > \mathbf{r}_b(H, \top) = log_b(1)$. Conceptually then, these $+1$ and -1 terms function to isolate the extent to which a given reason is better than the reason provided by $\top$ for H. And the axioms work by relating the extent to which one reason is better than $\top$ with the extent to which another reason is better than $\top$.

[46] Accordingly, some of axioms and claims above are give in different notation.

[47] Negatively Correlated Reasons relies on the relationship between $\frac{a}{b}$ and $\frac{b}{a}$. We discover a further axiom (Positively Correlated Reasons) that claims there is a positive correlation between reasons based on the fact that $\frac{a}{c} = \frac{a}{b}\frac{b}{c}$. Another axiom (Aggregative Reasons) claims there is a summation like correlation between reasons based on the fact that $\frac{a_1+a_2+\cdots+a_n}{b} = \frac{a_1}{b} + \frac{a_2}{b} + \cdots + \frac{a_n}{b}$. A third axiom (Factored Reasons) claims there is a complex correlation among reasons based on the fact that:

$$\frac{\frac{a}{b}}{\frac{c}{b}} = \frac{\frac{a}{d}\frac{d}{b}}{\frac{c}{d'}\frac{d'}{b}}$$

A fourth axiom (Complimentary Reasons) relies on a property of $logs$'s and ratios together, namely that $log(\frac{a}{b}) = -log(\frac{b}{a})$.

[48] The basic idea, once again, makes use of the fact that the value of $b^{\mathbf{r}_b(H,E)}$ for non-trivial (H, E) such that H entails E is 1 plus the ratio of two probabilities. In turns out that we can

in detail in §A is the following:

> Theorem 1: For any reason weighing function, $\mathbf{r}_b$, (i) $f_{\mathbf{r}_b}$ is a probability function and (ii) for any propositions H, E either
>
> $$\mathbf{r}_b(H,E) = log_b\left(\frac{f_{\mathbf{r}_b}(E \mid H)}{f_{\mathbf{r}_b}(E \mid \neg H)}\right)$$
>
> or $\mathbf{r}_b(H,E)$ and $log_b\left(\frac{f_{\mathbf{r}_b}(E|H)}{f_{\mathbf{r}_b}(E|\neg H)}\right)$ are both undefined.

This proof in turn contains the materials to show this second important result:

> Theorem 2: For any *regular* probability function, Pr, there is a reasons weighing function, $\mathbf{r}_b$, such that (i) for any proposition P, $Pr(P) = f_{\mathbf{r}_b}(P)$ and (ii) for any propositions H, E, either
>
> $$log_b\left(\frac{Pr(E \mid H)}{Pr(E \mid \neg H)}\right) = \mathbf{r}_b(H,E)$$
>
> or $log_b\left(\frac{Pr(E|H)}{Pr(E|\neg H)}\right)$ and $\mathbf{r}_b(H,E)$ are both undefined.

These results demonstrate how non-reductive theories of reasons can earn the right to make use of probabilistic tools.

This allows the non-reductivist to answer the second challenge posed by cases of accural—the challenge of showing that given a numerical representation of the strengths of reasons, this representation allows us to distinguish cases where reasons "add up" from cases where they don't "add up" at all and everything in between. That said, we have not fully responded to the first challenge because we have simply assumed that the strength of reasons can be numerically represented. While we have seen why this particular numerical representation is plausible and that reductivists cannot deny the claims that we take as axioms, we have not provided a full justification for it. To do this, we must provide a set of plausible qualitative axioms and show that the reasons weighing function is a numerical representation of these qualitative features. While I am optimistic that the relevant qualitative axioms can be discovered, this is a non-trivial task.

I conclude therefore that while non-reductive accounts of reasons for belief answer the second challenge posed by cases of accrual, they have yet to answer the first challenge. In this respect, Bayesian and some (but not all) other reductive approaches to reasons have an advantage as of now.

4.2 Probabilities for Non-Reductive Theories of Reasons for Action

Given our success in the case of reasons for belief, it is reasonable to hope that an analogous approach to reasons for action will succeed. But there is obstacle to this approach.

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use $b^{\mathbf{r}_b(H,E)} - 1$ to fix the ratios of the probabilities of all the maximally specific propositions. Since the probability of the maximally specific propositions sums to 1, we can then take $f_{\mathbf{r}_b}$ to be that function which respects these ratios and sums to 1.

4.2.1 Symmetry Properties of Reasons for Action and Confirmation

To see the obstacle, consider the following observation:

> THE ASYMMETRY OF REASONS FOR ACTION: F can be reason for action in favor of A without it being true that A is reason for action in favor of F.

Examples make this clear. Plausibly, the fact that I promised to help Callie move is a reason for me to help her move. Now consider the following question: Does my helping Callie move provide a reason for action for the claim that I promised to help her move? This question is perhaps simply incoherent so cannot be answered. Or if it can be answered, the answer is 'no'. This is what THE ASYMMETRY OF REASONS FOR ACTION says.

Compare this to the following fact about confirmation:

> THE SYMMETRY OF CONFIRMATION: P confirms Q iff Q confirms P

This result holds for every confirmation measure that we have discussed in this paper because these measures satisfy the qualitative condition that P confirms Q iff $Pr(Q \mid P) > Pr(Q)$.[49]

This tell us that the analog of the approach that we developed for reasons for belief will not work for reasons for action. The approach for reasons for belief claimed P being a reason to believe Q is structurally equivalent to P confirming Q.[50] So the analogous approach for reasons for action (claiming P being a reason

[49]To show this, it suffices prove that if $Pr(Q \mid P) > Pr(Q)$, then $Pr(P \mid Q) > Pr(P)$. Bayes' Theorem tells us that:

$$Pr(Q \mid P) = \frac{Pr(P \mid Q)Pr(Q)}{Pr(P)}$$

Thus, we can rewrite $Pr(Q \mid P) > Pr(Q)$ and reason as follows:

$$\begin{gathered}\frac{Pr(P \mid Q)Pr(Q)}{Pr(P)} > Pr(Q) \\ Pr(P \mid Q)Pr(Q) > Pr(Q)Pr(P) \\ Pr(P \mid Q) > Pr(P)\end{gathered}$$

[50]This suggest that reasons for belief obey:

> THE SYMMETRY OF REASONS FOR BELIEF: P is a reason to believe Q iff Q is a reason to believe P

While perhaps initially surprising, this is no more controversial than the same principle concerning evidence.

Of course, one might worry that arguments such as those in n. 17 pull reasons for belief and evidence apart in a way that will yield counterexamples to this principle for reasons for belief. I do not believe those (putative) counterexamples yield a problem for the above symmetry thesis. But I admit once the connection between reasons for belief and evidence is broken, matters become more complicated. Thanks to the editor at *Ethics* for bringing this concern to my attention.

for action supporting A is structurally equivalent to P confirming A) cannot work because of the difference in symmetry between these two notions.[51,52]

4.2.2 Sher's Reduction of Probabilities and Utilities to Reasons for Action

That said, there is an approach developed in the groundbreaking paper Sher 2019 that is promising for the non-reductivist.[53] Sher's account is not purely probabilistic. Instead, it is structurally similar to decision theory in which one

[51] Reductive views like *Bayesian Kearns and Star Theory of Reasons* allow for the truth of The Asymmetry of Reasons for Action. Suppose E is a not an 'ought'-claim but confirms some 'ought'-claim. *Bayesian Kearns and Star Theory of Reasons* says that E is a reason. This entails that the 'ought'-claim confirms E. But this does not, on it own, tell us that the 'ought'-claim is a reason. Similar remarks apply about other reductive views with this structure. That said, Eva Schmidt in an insightful paper (Schmidt 2017) shows that there are special contexts where symmetry worries may recur.

[52] Since $Pr(P \mid Q) > \tau$ does not entail $Pr(Q \mid P) > \tau$ it might seem that a threshold approach like the one in n. 36 for reasons for action will avoid this problem. Unfortunately a similar problem arises here due to the asymmetry of reasons for action. For example, for suitable values of P, Q, and ϕ, the following can all be true:

1. P is a reason for $\mathcal{S}$ to ϕ
2. Q is not a reason for $\mathcal{S}$ to ϕ
3. ϕ-ing is not a reason supporting P or supporting $\neg P$
4. ϕ-ing is not a reason supporting supporting Q or supporting $\neg Q$
5. $\top$ is not a reason supporting P or supporting $\neg P$
6. $\top$ is not a reason supporting Q or supporting $\neg Q$

But the threshold approach entails that they are inconsistent.

To see this, notice that 1-6 amount to the following claims according to the threshold approach:

1. $Pr_{\mathcal{S}}(\mathcal{S}\ does\ \phi \mid P) > \tau$
2. $Pr_{\mathcal{S}}(\mathcal{S}\ does\ \phi \mid Q) \leq \tau$
3. $Pr_{\mathcal{S}}(P \mid \mathcal{S}\ does\ \phi) = \tau$
4. $Pr_{\mathcal{S}}(Q \mid \mathcal{S}\ does\ \phi) = \tau$
5. $Pr_{\mathcal{S}}(P \mid \top) = \tau$
6. $Pr_{\mathcal{S}}(Q \mid \top) = \tau$

Bayes theorem applied to 1 and 2 gets us:

$$Pr_{\mathcal{S}}(\mathcal{S}\ does\ \phi \mid P) = \frac{Pr_{\mathcal{S}}(P \mid \mathcal{S}\ does\ \phi)Pr_{\mathcal{S}}(\mathcal{S}\ does\ \phi)}{Pr_{\mathcal{S}}(P)} > \tau$$

$$Pr_{\mathcal{S}}(\mathcal{S}\ does\ \phi \mid Q) = \frac{Pr_{\mathcal{S}}(Q \mid \mathcal{S}\ does\ \phi)Pr_{\mathcal{S}}(\mathcal{S}\ does\ \phi)}{Pr_{\mathcal{S}}(Q)} \leq \tau$$

Since 3 and 4 tell us that $Pr_{\mathcal{S}}(P \mid \mathcal{S}\ does\ \phi) = Pr_{\mathcal{S}}(Q \mid \mathcal{S}\ does\ \phi) = \tau$, all the terms in these equations must have the same value except $Pr_{\mathcal{S}}(P)$ and $Pr_{\mathcal{S}}(Q)$. Since the value of the first equation is greater than the value of the second, it follows that $Pr_{\mathcal{S}}(P) < Pr_{\mathcal{S}}(Q)$. But 5 an 6 tell us that $Pr_{\mathcal{S}}(P) = Pr_{\mathcal{S}}(P \mid \top) = \tau$ and that $Pr_{\mathcal{S}}(Q) = Pr(Q \mid \top) = \tau$. So $Pr_{\mathcal{S}}(P) \not< Pr_{\mathcal{S}}(Q)$ which shows 1-6 are inconsistent according to the threshold analysis.

[53] Thanks to Itai Sher for correspondence that helped me to better understand this paper and its merits (though I do not discuss it in nearly the detail it deserves here).

has both a probability function and a utility function. Interestingly for the non-reductivist, Sher shows one need not take the probability function and utility function as basic and define the weight of reasons in terms of them. Instead, one can take the weight of reasons for action to be basic and define a probability and utility function. In this framework, one can give a precise account of accrual for reasons for action. The model he gives and the theorem that he proves to show this are well-worth detailed study. But I omit discussion of the proof and leave the statement of Sher's assumptions to a note in order to highlight some basic conceptual points.[54]

Sher's result is similar to the results that I have presented for reasons for belief: It assumes from the start that reasons for action can be numerically represented and shows that this representation is of the right sort to model the dynamics of reasons "adding up". So the two non-reductive approaches answer the second challenge posed by cases of accrual. But they both fail to answer the first challenge because they do not provide grounds (e.g., a set of plausible qualitative axioms) that show this numerical representations is sensible. I am optimistic about the prospects of finding qualitative axioms to ground these numerical representations. But my optimism is not based on any concrete proposal so this is an important open question for non-reductivists.

I close with two points. First, our non-reductive approaches give two distinct pictures of reasons for belief and action rather than a single unified one. I do not know whether this a serious cost. But I suspect it is not. Second, there are

[54] Sher assumes that we have a function w that maps actions and and propositions into the real numbers. Sher imposes (in slightly different notation) the following constraints on this function:

Simplification: *For all pairs of disjoint reasons, R_1 and R_2,*

$$w_a(R_1) \neq w_a(R_2)$$

No Information:

$$w_a(\top) = 0$$

Averaging: *For all pairs of disjoint non-trivial reasons, R_1 and R_2*

if $w_a(R_1) < w_a(R_2)$, then $w_a(R_1) < w_a(R_1 \vee R_2) < w_a(R_2)$

Chain Rule: *For all reasons R, S, and T such that S is consistent, R entails S, and S entails T,*

$$\rho_a^{R|T} = \rho_a^{R|S}\rho_a^{S|T}$$

where we define for any R and S such that R entails S and S is consistent

$$\rho_a^{R|S} = \frac{w_a(S) - w_a(S - R)}{w_a(R) - w_a(S - R)}$$

Common Ratios: *If $w_a(R) \neq 0$ and $w_b(R) \neq 0$, then*

$$\frac{w_a(R)}{w_a(\neg R)} = \frac{w_b(R)}{w_b(\neg R)}$$

Sher proves the fascinating result that if w satisfies these conditions, one can define a probability and utility function based on w such that when one acts for most reason one maximizes the expectation given by this probability and utility function.

some approaches to accrual that are more distant from the probabilistic ones that have been our focus. I discuss them in two notes.[55,56]

5 Conclusion

The question that we have asked is whether and how reductive and non-reductive theories can make use probabilistic tools to understand the accrual of reasons.

We saw that a variety of reductive theories (though not every reductive theory) can make use of probabilistic tools both to provide a basis for the numerical representation of the strengths of reasons and to model the different ways reasons can "add up". But we should not overstate what has been shown. Since probabilities have precise features, this constrains how we can develop particular reductive theories. These constraints will make clear what predictions the theories make. The result may be that some reductive views are more promising than others. Conversely, if most plausible reductive views look unappealing when regimented in a probabilistic setting, this may cast doubt on the reduction advocated here.

[55] Let me briefly discuss some purely qualitative approaches. One approach claims that the the relationship between individual reasons and their accrual is a brute one. It is hard to know what to say in response to someone who adopts this kind of quietism. So I simply report my feelings: Quietism about a phenomena may be reasonable if there is little evidence that any going theory can explain it. It is much less reasonable, if there is evidence that a variety of theories can give a detailed account of the phenomena in question. (Nair 2016: §6 makes a similar point but also follows the lead of Prakken 2005: §3 in observing that there are certain generalizations about accrual that require explanation and the brute approach fails to provide an explanation of these generalizations.)

Next a number of qualitative approaches in the default logic tradition (Reiter 1980) and in the argumentation theory tradition (Dung 1995) have been developed to model accrual (Delgrande and Schaub 2004, Gómez Lucero, Chesñevar, and Simari 2009, Gómez Lucero, Chesñevar, and Simari 2013, Modgil and Bench-Capon 2010, Prakken 2005, Verheij 1995, and Wassell 2014). These qualitative accounts do provide conditions under which, for example, the accrual of the reasons provided by the movie and the restaurant is stronger than these reason individually. But they are unable to tell us under what conditions the accrual is stronger than the individual reasons to an extent that makes it so that there is more reason to cross the bridge than to not cross the bridge.

Finally, there are approaches such as those inspired by (though not full endorsed by) Nair 2016 and Maguire and Snedegar 2021 that make use of the distinction between derivative/non-derivative reasons (where this distinction is understood to be influenced by work of Korsgaard 1983 on goodness) or perhaps something akin to this distinction. And Johnson King 2019 discusses some related ideas as part of her solutions to problems for buck passing accounts of goodness. While there are certain aspects of the ideas in §A of this paper that are suggestive of such a distinction (see n. 61), nothing in the work of advocates of these views provides enough detail about how this distinction is drawn to replicate the rich quantitative structure that probabilities and utilities have. That said, Maguire and Snedegar 2021 and Johnson King 2019 have slightly different targets in mind so this criticism may not be a problem for their core projects. Thanks to Barry Maguire and Justin Snedegar for discussion of this issue.

[56] Close to when this paper was accepted, I learned of Ralph Wedgwood's recent work (Wedgwood forthcoming) adapting Harsanyni's theorem to give an account of the accrual of reasons for action. I think this is a promising approach to explaining the accrual of reasons (it also makes use of broadly decision-theoretic tools like Sher's approach). But I do not believe that it helps the non-reductivist about reasons because Wedgwood's approach is naturally understood as reducing reasons to values.

We also saw that non-reductive views can make use of probabilistic or decision-theoretic tools to model the variety of ways reasons can "add up". They do this, however, by assuming that the strength of reasons can be numerically represented rather than by providing a basis for this numerical representation. This is a remaining challenge for non-reductive approaches. I am optimistic that with some additional work, a sensible basis—in the form of plausible qualitative axioms about the strengths of reasons—can be provided. But others may disagree.

Finally, like particular versions of reductive theories, it remains to be seen exactly what predictions particular non-reductive theories make when regimented by the constraints required to make use of probabilistic or decision-theoretic tools. It also remains to be seen whether those predictions are plausible. Conversely, if most plausible non-reductive theories look unappealing when regimented in a way that allows them to make use of probabilities, this may cast doubt on the approach advocated here.

However these matters turn out, we have seen that probabilistic frameworks are surprisingly rich and ecumenical: They can provide a detailed treatment of cases of accrual. They can accommodate a variety of reductive theories. And they can be accommodated by a variety of non-reductive theories.

My hope is that these frameworks will be fruitful for those interested in in confirmation theory, those interested in the systematic interaction among reasons, and especially those interested in how reasons can explain what we ought to do and believe.

A Probabilities Are Reducible to Reasons

In this appendix, we prove the results in §4.1.[57] We assume that propositions are elements of an algebra based on a partition $U = \{A_1, A_2, \ldots, A_n\}$ where the A_i's are the cells of the partition and $n \geq 3$. So a proposition is a (possibly empty) set of cells of the partition. We adopt some shorthand for designating particular propositions: $\top = U, \bot = \emptyset$. If P, Q are propositions, we will use the following notation when it is convenient: $\neg P = \top - P, P \vee Q = P \cup Q, P \wedge Q = P \cap Q$. We will frequently omit the braces around propositions that are singletons so we will write $\{A_i\}$ as A_i. Finally we say P entails Q exactly if $P \subseteq Q$.

We begin by defining the reasons weighing function as a function on propositions from this background structure.

A.1 Definitions

It helps to start by introducing some terminology to describe certain pairs of propositions:

[57] Thanks to Kenny Easwaran for comments on an initial sketch of these ideas. Thanks to both Kenny Easwaran and Branden Fitelson for encouragement and for helping me to see what issues need to be addressed. Unfortunately, many of these issues will have to be dealt with elsewhere.

(H, E) is *extreme* exactly if E entails H or E entails $\neg H$.

(H, E) is *vacuous* exactly if (H, E) is not extreme and $E = \top$.

(H, E) is *trivial* exactly if (H, E) is extreme or vacuous.

(P, Q) is *a non-trivial determiner* exactly if $P \neq \bot, Q \neq \bot$, $P \vee Q \neq \top$, and $P \wedge Q = \bot$

The letters used in the first three definitions indicate that we are interested in (H, E) as a pair where the first element is the hypothesis (the thing supported by the reason) and the second element is the evidence (the reason). The letters used in the fourth definition, by contrast, suggest that we are not primarily interested in (P, Q) as a pair consisting of a hypothesis and evidence. Instead, these pairs can be used to determine other pairs of propositions that are hypotheses and evidence which have properties that are of interest to us. The following fact explains this more precisely:

Notational Variants: *If (H, E) is not trivial and H entails E, then there is exactly one (P, Q) such that (P, Q) is a non-trivial determiner and $H = \neg P \wedge \neg Q$ and $E = \neg Q$. And if (P, Q) is a non-trivial determiner, then $(\neg P \wedge \neg Q, \neg Q)$ is not trivial and $\neg P \wedge \neg Q$ entails $\neg Q$.*

The proof of this fact is left to a note.[58] Notational Variants tells us, then, about a particular way we can characterize (H, E)'s that are not trivial and

[58]We prove the two claims in Notational Variants seperately.

Notational Variants 1: *If (H, E) is not trivial and H entails E, then there is exactly one (P, Q) such that (P, Q) is a non-trivial determiner and $H = \neg P \wedge \neg Q$ and $E = \neg Q$*

Proof of Notational Variants 1. Suppose (H, E) is not trivial and H entails E. Since (H, E) is not extreme, E does not entail H so $H \subset E$. Since (H, E) is not extreme E also does not entail $\neg H$ so $H \neq \bot$ and $\bot \subset H \subset E$. Since (H, E) is not vacuous $E \neq \top$ so $\bot \subset H \subset E \subset \top$.

Let us first establish that there is at least one (P, Q) such that (P, Q) is a non-trivial determiner and $H = \neg P \wedge \neg Q$ and $E = \neg Q$. We show this for the following particular choice of P and Q:

$$Q = \top - E = \neg E$$
$$P = E - H = E \wedge \neg H$$

This (P, Q) is a non-trivial determiner (which recall means that $P \neq \bot, Q \neq \bot$, $P \vee Q \neq \top$, and $P \wedge Q = \bot$): First notice that since $E \neq \top$, $Q = \neg E \neq \bot$. Second, notice that $H \subset E$, $P = E - H \neq \bot$. Third, consider that $P \vee Q = (E \wedge \neg H) \vee \neg E = (\neg E \vee E) \wedge (\neg E \vee \neg H) = \neg E \vee \neg H$. Since H entails E, $\neg E$ entails $\neg H$ so $\neg E \vee \neg H = \neg H$. Since $H \neq \bot$, $P \vee Q = \neg H \neq \top$. Fourth and finally, notice that $P \wedge Q = (E \wedge \neg H) \wedge \neg E = \bot$.

And this (P, Q) is such that $H = \neg P \wedge \neg Q$ and $E = \neg Q$. To begin, since $Q = \neg E$,

$$\neg Q = \neg\neg E = E$$

. Next since $P = E \wedge \neg H$,

$$\neg P = \neg(E \wedge \neg H) = \neg E \vee H$$

Finally since $H \subset E$, we know $H \wedge E = H$ therefore:

$$\neg P \wedge \neg Q = (\neg E \vee H) \wedge E = (E \wedge \neg E) \vee (E \wedge H) = E \wedge H = H$$

Thus, there is at least one (P, Q) such that (P, Q) is a non-trivial determiner and $H = \neg P \wedge \neg Q$ and $E = \neg Q$.

such that H entails E. As we will see, this is useful for structuring some of the proofs. It also turns out that the term $\mathbf{r}_b\left(\neg P \wedge \neg Q, \neg Q\right)$ is closely related (in the way described by (Lemma 1.4.1) below) to the term $\frac{Pr(Q)}{Pr(P)}$. This notation allows us easily keep track of this connection.

We now define a (class of) function(s) that is intended to represent the strength of reasons.

Definition 1. *A function from pairs of propositions from the algebra based on U to the interval* $(-\infty, \infty)$, $\mathbf{r}_b$, *is a* reasons weighing function *exactly if it satisfies the following axioms:*

BASE PROPRIETY: $b > 1$

UNDEFINED REASONS: *if* (H, E) *is extreme,* $\mathbf{r}_b(H, E)$ *is undefined*

NO REASON: *if* (H, E) *is vacuous,*

$$\mathbf{r}_b(H, E) = log(1) = 0$$

COMPLIMENTARY REASONS: *if* (H, E) *is not extreme,*

$$\mathbf{r}_b(\neg H, E) = -\mathbf{r}_b(H, E)$$

ENTAILED REASON: *if* (H, E) *is not trivial and* H *entails* E,

$$\mathbf{r}_b(H, E) > log_b(1) = 0$$

NEGATIVELY CORRELATED REASONS: *if* (P, Q) *is a non-trivial determiner,*

$$\mathbf{r}_b(\neg P \wedge \neg Q, \neg P) = log_b\left(\frac{b^{\mathbf{r}_b(\neg P \wedge \neg Q, \neg Q)}}{b^{\mathbf{r}_b(\neg P \wedge \neg Q, \neg Q)} - 1}\right)$$

To complete the proof, we still must show that there is no more than one (P, Q) with these two features. To show this, suppose for *reductio* that it is false. So there is some non-trivial determiner (P', Q') such that either $P \neq P'$ or $Q \neq Q'$ and $H = \neg P \wedge \neg Q = \neg P' \wedge \neg Q'$ and $E = \neg Q = \neg Q'$. It is immediate that $Q = Q'$. So $P \neq P'$ and therefore there is an x such that either $x \in P, \notin P'$ or $x \notin P, \in P'$. Suppose $x \in P, \notin P'$ and therefore $x \notin \neg P, \in \neg P'$. Since $x \in P$, $x \notin Q = Q'$ and $x \in \neg Q = \neg Q'$. Therefore $x \notin \neg P \wedge \neg Q$ but $x \in \neg P' \wedge \neg Q'$. Thus $H = \neg P \wedge \neg Q \neq \neg P' \wedge \neg Q'$. Suppose then instead, $x \notin P, \in P'$. By analogous reasoning we established $x \in \neg P \wedge \neg Q$ but $x \notin \neg P' \wedge \neg Q'$. Thus (P, Q) are unique which completes the proof of the left-to-right direction. □

Notational Variants 2: *If* (P, Q) *is a non-trivial determiner, then* $(\neg P \wedge \neg Q, \neg Q)$ *is not trivial and* $\neg P \wedge \neg Q$ *entails* $\neg Q$.

Proof of Notational Variants 2. Consider then (P, Q) that is a non-trivial determiner. (And recall once again that for (P, Q) to be a non-trivial determiner is for the following to hold: $P \neq \bot, Q \neq \bot$, $P \vee Q \neq \top$, and $P \wedge Q = \bot$.) It is immediate the $\neg P \wedge \neg Q$ entails $\neg Q$. Next since $P \neq \bot$, $P \not\subseteq \neg P \wedge \neg Q$. Since $P \wedge Q = \bot$, $P \subset \neg Q$. Thus, $\neg Q \not\subseteq \neg P \wedge \neg Q$, so $\neg Q$ does not entail $\neg P \wedge \neg Q$. Next notice that since $\neg P \wedge \neg Q$ entails $\neg Q$, $\neg Q$ entails $\neg(\neg P \wedge \neg Q)$ only if $\neg Q = \bot$. But since $P \vee Q \neq \top$, $Q \neq \top$ so $\neg Q \neq \bot$. So $\neg Q$ does not entails $\neg(\neg P \wedge \neg Q)$. So $(\neg P \wedge \neg Q, \neg Q)$ is not extreme. Since $Q \neq \bot$, $\neg Q \neq \top$ so $(\neg P \wedge \neg Q, \neg Q)$ is not vacuous. So as desired $(\neg P \wedge Q, \neg Q)$ is not trivial and $\neg P \wedge Q$ entails $\neg Q$ □

Positively Correlated Reasons: *if (P,Q), (Q,R), and (P,R) are non-trivial determiners,*

$$\mathbf{r}_b(\neg P \wedge \neg R, \neg R) = log_b\left(\left(b^{\mathbf{r}_b(\neg Q \wedge \neg R, \neg R)} - 1\right)\left(b^{\mathbf{r}_b(\neg P \wedge \neg Q, \neg Q)} - 1\right) + 1\right)$$

Aggregative Reasons: *if (P,Q) is non-trivial determiner,*

$$\mathbf{r}_b\left(\neg P \wedge \neg Q, \neg Q\right) = log_b\left(\left(\sum_{Q_i \in Q} b^{\mathbf{r}_b(\neg P \wedge \neg Q_i, \neg Q_i)} - 1\right) + 1\right)$$

Factored Reasons: *if (H,E) is not trivial, H does not entail E, and $\neg H$ does not entail E, then for any D, D' such that (H,D) and $(\neg H, D')$ are non-trivial determiners,*

$$\mathbf{r}_b(H,E) = log_b\left(\frac{\left(b^{\mathbf{r}_b(\neg D \wedge \neg(H \wedge E), \neg(H \wedge E))} - 1\right)\left(b^{\mathbf{r}_b(\neg H \wedge \neg D, \neg D)} - 1\right)}{\left(b^{\mathbf{r}_b(\neg D' \wedge \neg(\neg H \wedge E), \neg(\neg H \wedge E))} - 1\right)\left(b^{\mathbf{r}_b(H \wedge \neg D', \neg D')} - 1\right)}\right)$$

The relationship between the axioms and Theorem 1 (restated below) will emerge in the course of the proofs. But two points to note here. First, in light of Notational Variants, Negatively Correlated Reasons–Aggregative Reasons are axioms concerning cases where (H,E) is not trivial and H entails E. Second, I will not explicitly mention Base Propriety. But it is relied on implicitly to ensure the relevant *log*'s are defined and are the right kind increasing function of their arguments.

Now we may define a second function:

Definition 2. *A function from propositions from the algebra based U to the interval $(-\infty, \infty)$, $f_{\mathbf{r}_b}$,* is the prior based on $\mathbf{r}_b$ function *exactly if it satisfies the following axioms:*[59]

Ratios of Cells: *If $U = \{A_1, A_2, \cdots A_n\}$ then,*

$$\begin{aligned}
1 &= f_{\mathbf{r}_b}(A_1) + f_{\mathbf{r}_b}(A_2) + \cdots + f_{\mathbf{r}_b}(A_n) \\
f_{\mathbf{r}_b}(A_2) &= (b^{\mathbf{r}_b(\neg A_1 \wedge \neg A_2, \neg A_2)} - 1) f_{\mathbf{r}_b}(A_1) \\
f_{\mathbf{r}_b}(A_3) &= (b^{\mathbf{r}_b(\neg A_1 \wedge \neg A_3, \neg A_3)} - 1) f_{\mathbf{r}_b}(A_1) \\
&\vdots \\
f_{\mathbf{r}_b}(A_n) &= (b^{\mathbf{r}_b(\neg A_1 \wedge \neg A_n, \neg A_n)} - 1) f_{\mathbf{r}_b}(A_1)
\end{aligned}$$

Sum of Cells: *For any proposition P,*

[59] It is possible to more explicitly albeit less intuitively define $f_{\mathbf{r}_b}(A_i)$. We explicitly define:

$$f_{\mathbf{r}_b}(A_1) = \frac{1}{1 + (b^{\mathbf{r}_b(\neg A_1 \wedge \neg A_2, \neg A_2)} - 1) + (b^{\mathbf{r}_b(\neg A_1 \wedge \neg A_3, \neg A_3)} - 1) + \cdots + (b^{\mathbf{r}_b(\neg A_1 \wedge \neg A_n, \neg A_n)} - 1)}$$

For any other, A_i, $f_{\mathbf{r}_b}(A_i)$ is this same fraction except replacing the 1 in the numerator with $b^{\mathbf{r}_b(\neg A_1 \wedge \neg A_i, \neg A_i)} - 1$.

- *if* $P = \emptyset$, $f_{\mathbf{r}_b}(P) = 0$
- *if* $P \neq \emptyset$, $f_{\mathbf{r}_b}(P) = \sum_{A_i \in P} f_{\mathbf{r}_b}(A_i)$

Given a particular reasons weighing function $\mathbf{r}_b$, $f_{\mathbf{r}_b}$ is uniquely determined. Our main aim is to prove the following claim about these functions:

Theorem 1. *For any reason weighing function,* $\mathbf{r}_b$*, (i)* $f_{\mathbf{r}_b}$ *is a probability function and (ii) for any propositions* H, E *either*

$$\mathbf{r}_b(H, E) = log_b \left(\frac{f_{\mathbf{r}_b}(E \mid H)}{f_{\mathbf{r}_b}(E \mid \neg H)} \right)$$

or $\mathbf{r}_b(H, E)$ *and* $log_b \left(\frac{f_{\mathbf{r}_b}(E|H)}{f_{\mathbf{r}_b}(E|\neg H)} \right)$ *are both undefined.*

This theorem shows that the reasons weighing function that we defined (i) determines a probability function and (ii) is equivalent to the *log* likelihood confirmation measure based on that probability function.

A.2 $f_{\mathbf{r}_b}$ is a Probability Function

Here we show (i) in Theorem 1:

Proposition 1.1. $f_{\mathbf{r}_b}$ *is a probability function.*

Proof of Proposition 1.1. It suffices to show that $f_{\mathbf{r}_b}$ satisfies the following conditions:

Non-Negativity: $f_{\mathbf{r}_b}(P) \geq 0$ for any proposition P

Normalization: $f_{\mathbf{r}_b}(\top) = 1$

Finite Additivity: $f_{\mathbf{r}_b}(P \vee Q) = f_{\mathbf{r}_b}(P) + f_{\mathbf{r}_b}(Q)$ when $P \wedge Q = \bot$

Ratio: $f_{\mathbf{r}_b}(P \mid Q) = \frac{f_{\mathbf{r}_b}(P \wedge Q)}{f_{\mathbf{r}_b}(Q)}$ when $f_{\mathbf{r}_b}(Q) \neq 0$

Begin with Normalization. By Sum of Cells, $f_{\mathbf{r}_b}(\top) = f_{\mathbf{r}_b}(A_1) + f_{\mathbf{r}_b}(A_2) + \cdots + f_{\mathbf{r}_b}(A_n)$. Next, by the first equation in Ratios of Cells, $f_{\mathbf{r}_b}(A_1) + f_{\mathbf{r}_b}(A_2) + \cdots + f_{\mathbf{r}_b}(A_n) = 1$, Thus, $f_{\mathbf{r}_b}(\top) = 1$.

Next turn to Finite Additivity. Assume one of P or Q are empty. Without loss of generality suppose it is P, then by Sum of Cells $f_{\mathbf{r}_b}(P) = 0$ and $P \vee Q = Q$. Thus $f_{\mathbf{r}_b}(P \vee Q) = f_{\mathbf{r}_b}(Q) + 0 = f_{\mathbf{r}_b}(Q) + f_{\mathbf{r}_b}(P)$ so Finite Additivity holds. Suppose instead that P and Q are both non-empty and that $P \wedge Q = \bot$. Let $P = \{A_{P_1}, A_{P_2}, \ldots A_{P_n}\}$ and $Q = \{A_{Q_1}, A_{Q_2}, \ldots A_{Q_n}\}$. Since $P \wedge Q = \bot$, $P \vee Q = \{A_{P_1}, A_{P_2}, \ldots A_{P_n}, A_{Q_1}, A_{Q_2}, \ldots A_{Q_n}\}$ where this specification doesn't list the same cell twice. By Sum of Cells, we know that:

$$\begin{aligned}
f_{\mathbf{r}_b}(P) &= f_{\mathbf{r}_b}(A_{P_1}) + f_{\mathbf{r}_b}(A_{P_2}) + \cdots + f_{\mathbf{r}_b}(A_{P_n}) \\
f_{\mathbf{r}_b}(Q) &= f_{\mathbf{r}_b}(A_{Q_1}) + f_{\mathbf{r}_b}(A_{Q_2}) + \cdots + f_{\mathbf{r}_b}(A_{Q_n}) \\
f_{\mathbf{r}_b}(P \vee Q) &= f_{\mathbf{r}_b}(A_{P_1}) + f_{\mathbf{r}_b}(A_{P_2}) + \cdots + f_{\mathbf{r}_b}(A_{P_n}) + \\
&\quad f_{\mathbf{r}_b}(A_{Q_1}) + f_{\mathbf{r}_b}(A_{Q_2}) + \cdots + f_{\mathbf{r}_b}(A_{Q_n})
\end{aligned}$$

Thus, $f_{\mathbf{r}_b}(P \vee Q) = f_{\mathbf{r}_b}(P) + f_{\mathbf{r}_b}(Q)$.

Now turn to Non-Negativity. Every proposition, P, is a (possibly empty) set of cells. Suppose P is empty, then Sum of Cells says $f_{\mathbf{r}_b}(P) = 0$ so Non-Negativity holds. Suppose P is non-empty so, by Sum of Cells, $f_{\mathbf{r}_b}(P) = \sum_{A_i \in P} f_{\mathbf{r}_b}(A_i)$. If we can prove that $f_{\mathbf{r}_b}(A_i) \geq 0$ for all $A_i \in U$, this will suffice to establish Non-Negativity. To show $f_{\mathbf{r}_b}(A_i) \geq 0$ for all $A_i \in U$, recall Ratios of Cells:

$$
\begin{aligned}
1 &= f_{\mathbf{r}_b}(A_1) + f_{\mathbf{r}_b}(A_2) + \cdots + f_{\mathbf{r}_b}(A_n) \\
f_{\mathbf{r}_b}(A_2) &= (b^{\mathbf{r}_b(\neg A_1 \wedge \neg A_2, \neg A_2)} - 1) f_{\mathbf{r}_b}(A_1) \\
f_{\mathbf{r}_b}(A_3) &= (b^{\mathbf{r}_b(\neg A_1 \wedge \neg A_3, \neg A_3)} - 1) f_{\mathbf{r}_b}(A_1) \\
&\vdots \\
f_{\mathbf{r}_b}(A_n) &= (b^{\mathbf{r}_b(\neg A_1 \wedge \neg A_n, \neg A_n)} - 1) f_{\mathbf{r}_b}(A_1)
\end{aligned}
$$

We now reason by cases of the value of $f_{\mathbf{r}_b}(A_1)$.

Begin by supposing $f_{\mathbf{r}_b}(A_1) = 0$. This entails that $f_{\mathbf{r}_b}(A_2) = 0$ and similarly for the other cells. This is incompatible with $f_{\mathbf{r}_b}(A_1) + f_{\mathbf{r}_b}(A_2) + \cdots + f_{\mathbf{r}_b}(A_n) = 1$.

Suppose next then, that, $f_{\mathbf{r}_b}(A_1) < 0$. Since $\neg A_1 \wedge \neg A_2$ entails $\neg A_2$ and since $(\neg A_1 \wedge \neg A_2, \neg A_2)$ is not trivial[60], Entailed Reason says that $\mathbf{r}_b(\neg A_1 \wedge \neg A_2, \neg A_2) > log_b(1)$. Thus, $b^{\mathbf{r}_b(\neg A_1 \wedge \neg A_2, \neg A_2)} - 1 > 0$. So $f_{\mathbf{r}_b}(A_2)$ is negative. Similarly for the other cells. This is incompatible with $f_{\mathbf{r}_b}(A_1) + f_{\mathbf{r}_b}(A_2) + \cdots + f_{\mathbf{r}_b}(A_n) = 1$.

Thus, $f_{\mathbf{r}_b}(A_1) > 0$. Since $b^{\mathbf{r}_b(\neg A_1 \wedge \neg A_2, \neg A_2)} - 1 > 0$, $f_{\mathbf{r}_b}(A_2) > 0$. Similarly for the other cells. Thus, $f_{\mathbf{r}_b}(A_i) > 0$ for all A_i. So Non-Negativity holds.

Finally, we force Ratio by defining $f_{\mathbf{r}_b}(P \mid Q)$ to be $\frac{f_{\mathbf{r}_b}(P \wedge Q)}{f_{\mathbf{r}_b}(Q)}$ when $f_{\mathbf{r}_b}(Q) \neq 0$. □

We have in fact shown the stronger claim that $f_{\mathbf{r}_b}$ is a *regular* probability function in the sense that for any $P \neq \perp, f_{\mathbf{r}_b}(P) \neq 0$. We have shown this because our proof established that for all $A_i \in U$, $f_{\mathbf{r}_b}(A_i) > 0$. Since (by Sum of Cells) every proposition except $\perp$ is the sum of the A_i's, it follows that any proposition that is not $\perp$ is assigned a number greater than 0. I discuss this fact a bit more in §A.4.

Having established that $f_{\mathbf{r}_b}$ is a regular probability function, we will freely make use of this below.[61]

[60] This is ensured by the fact that $|U| \geq 3$.

[61] It is also worth noting in passing that this proof essentially shows that for U of n-elements, $n-1$ values of $\mathbf{r}_b$ suffice to determine a probability function. Similarly, the proof below shows that if we fix $n-1$ values of $\mathbf{r}_b$ (for example, we could use the same $n-1$ claims used for $f_{\mathbf{r}_b}$ and fix the values for CASE 1 in the proof of Proposition 1.4), we can use the axioms to fix the remaining values. This perhaps suggests that there may be $n-1$ "non-derivative" reasons that determine the much larger total set of claims about reasons and probabilities. That said, the result itself only tells us that there is an entailment from these $n-1$ claims to all the claims about reasons; it does not establish that there is a determination relation. Indeed, the particular $n-1$ claims we choose are somewhat arbitrary. What I suggested is that we make use of $n-1$ claims of the form $\mathbf{r}_b(\neg A_1 \wedge \neg A_i, \neg A_i)$ for $i \neq 1$. But how we enumerate the partition is arbitrary so we could have started with a different set of $n-1$ claims.

A.3 $\mathbf{r}_b = l_{f_{\mathbf{r}_b}}$

Here we show (ii) of Theorem 1. We prove this in piecemeal fashion starting with (H, E) that are trivial.[62]

A.3.1 Trivial (H, E)

Trivial (H, E) are either extreme or vacuous. Begin with the extreme case.

Proposition 1.2. *For any H, E such that (H, E) is extreme, $\mathbf{r}_b(H, E)$ and $log_b\left(\frac{f_{\mathbf{r}_b}(E|H)}{f_{\mathbf{r}_b}(E|\neg H)}\right)$ are both undefined.*

Proof of Proposition 1.2. Since (H, E) is extreme, E entails H or E entails $\neg H$ In either of these cases, Undefined Reasons tells us that $\mathbf{r}_b(H, E)$ is undefined. To see that $log_b\left(\frac{f_{\mathbf{r}_b}(E|H)}{f_{\mathbf{r}_b}(E|\neg H)}\right)$ is also undefined begin by supposing E entails H. In this setting,

$$0 = f_{\mathbf{r}_b}(E \wedge \neg H) = \frac{f_{\mathbf{r}_b}(E \wedge \neg H)}{f_{\mathbf{r}_b}(\neg H)} = f_{\mathbf{r}_b}(E \mid \neg H)$$

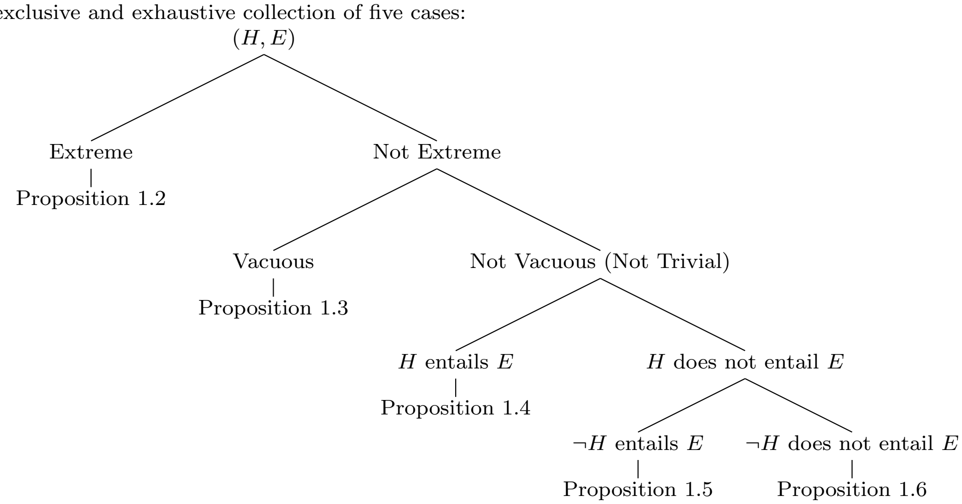
so $log_b\left(\frac{f_{\mathbf{r}_b}(E|H)}{f_{\mathbf{r}_b}(E|\neg H)}\right)$ is undefined because the term inside the log involves division by 0. Suppose instead E entails $\neg H$. In this setting,

$$0 = f_{\mathbf{r}_b}(E \wedge H) = \frac{f_{\mathbf{r}_b}(E \wedge H)}{f_{\mathbf{r}_b}(H)} = f_{\mathbf{r}_b}(E \mid H) = \frac{f_{\mathbf{r}_b}(E \mid H)}{f_{\mathbf{r}_b}(E \mid \neg H)}$$

so $log_b\left(\frac{f_{\mathbf{r}_b}(E|H)}{f_{\mathbf{r}_b}(E|\neg H)}\right)$ is undefined because $log(0)$ is undefined. $\square$

Next we consider vacuous (H, E).

[62] The following diagram indicates the way in which we prove the result by considering an exclusive and exhaustive collection of five cases:



Proposition 1.3. *For any H, E such that (H, E) is vacuous*

$$\mathbf{r}_b(H,E) = log_b\left(\frac{f_{\mathbf{r}_b}(E \mid H)}{f_{\mathbf{r}_b}(E \mid \neg H)}\right)$$

Proof of Proposition 1.3. Since (H, E) is vacuous, No Reason tells us that $\mathbf{r}_b(H, E) = 0$. Since (H, E) is vacuous, $E = \top$ and $H \neq \top, \bot$.[63] So $f_{\mathbf{r}_b}(H) \neq 0$, $f_{\mathbf{r}_b}(\neg H) \neq 0$, $E \wedge H = H$, and $E \wedge \neg H = \neg H$. Thus:

$$f_{\mathbf{r}_b}(E \mid H) = \frac{f_{\mathbf{r}_b}(E \wedge H)}{f_{\mathbf{r}_b}(H)} = \frac{f_{\mathbf{r}_b}(H)}{f_{\mathbf{r}_b}(H)} = 1$$

and

$$f_{\mathbf{r}_b}(E \mid \neg H) = \frac{f_{\mathbf{r}_b}(E \wedge \neg H)}{f_{\mathbf{r}_b}(\neg H)} = \frac{f_{\mathbf{r}_b}(\neg H)}{f_{\mathbf{r}_b}(\neg H)} = 1$$

Therefore as desired:

$$log_b\left(\frac{f_{\mathbf{r}_b}(E \mid H)}{f_{\mathbf{r}_b}(E \mid \neg H)}\right) = log_b\left(\frac{1}{1}\right) = log_b(1) = 0$$

$\square$

The cases that are not trivial (i.e, neither extreme nor vacuous) take more work.

A.3.2 H non-trivially entails E

We begin with the cases where (H, E) is not trivial and H entails E. Given Notational Variants, the result that we wish to establish is this:

Proposition 1.4. *For any (P, Q) that is a non-trivial determiner,*

$$\mathbf{r}_b\left(\neg P \wedge \neg Q, \neg Q\right) = log_b\left(\frac{f_{\mathbf{r}_b}\left(\neg Q \mid \neg P \wedge \neg Q\right)}{f_{\mathbf{r}_b}\left(\neg Q \mid \neg(\neg P \wedge \neg Q)\right)}\right)$$

It helps to begin with a lemma.

Lemma 1.4.1. *For any (P, Q) that is a non-trivial determiner,*

$$log_b\left(\frac{f_{\mathbf{r}_b}(\neg Q \mid \neg P \wedge \neg Q)}{f_{\mathbf{r}_b}(\neg Q \mid \neg(\neg P \wedge \neg Q))}\right) = log_b\left(\frac{f_{\mathbf{r}_b}(Q)}{f_{\mathbf{r}_b}(P)} + 1\right)$$

Proof of Lemma 1.4.1. Since $\neg P \wedge \neg Q$ entails $\neg Q$, we know:

$$log_b\left(\frac{f_{\mathbf{r}_b}(\neg Q \mid \neg P \wedge \neg Q)}{f_{\mathbf{r}_b}(\neg Q \mid \neg(\neg P \wedge \neg Q))}\right) = log_b\left(\frac{1}{f_{\mathbf{r}_b}(\neg Q \mid \neg(\neg P \wedge \neg Q))}\right)$$

[63] If $H = \top$, then E entails H so (H, E) is extreme and hence not vacuous. If $H = \bot$, E entails $\neg H$ so (H, E) is extreme and hence not vacuous.

The denominator of the term inside the *log* can be simplified (here the transition to the third equality from the second relies (twice) on the assumption that $P \wedge Q = \perp$):

$$\begin{aligned} f_{\mathbf{r}_b}(\neg Q \mid \neg(\neg P \wedge \neg Q)) &= \frac{f_{\mathbf{r}_b}(\neg Q \wedge \neg(\neg P \wedge \neg Q))}{f_{\mathbf{r}_b}(\neg(\neg P \wedge \neg Q))} \\ &= \frac{f_{\mathbf{r}_b}(\neg Q \wedge (P \vee Q))}{f_{\mathbf{r}_b}(P \vee Q)} \\ &= \frac{f_{\mathbf{r}_b}(P)}{f_{\mathbf{r}_b}(P) + f_{\mathbf{r}_b}(Q)} \end{aligned}$$

We then reason with the whole term inside the *log* as follows:

$$\begin{aligned} \frac{f_{\mathbf{r}_b}(\neg Q \mid \neg P \wedge \neg Q)}{f_{\mathbf{r}_b}(\neg Q \mid \neg(\neg P \wedge \neg Q))} &= \frac{1}{\frac{f_{\mathbf{r}_b}(P)}{f_{\mathbf{r}_b}(P)+f_{\mathbf{r}_b}(Q)}} \\ &= \frac{f_{\mathbf{r}_b}(P) + f_{\mathbf{r}_b}(Q)}{f_{\mathbf{r}_b}(P)} \\ \left(\frac{f_{\mathbf{r}_b}(\neg Q \mid \neg P \wedge \neg Q)}{f_{\mathbf{r}_b}(\neg Q \mid \neg(\neg P \wedge \neg Q))}\right) f_{\mathbf{r}_b}(P) - f_{\mathbf{r}_b}(P) &= f_{\mathbf{r}_b}(Q) \\ \left(\frac{f_{\mathbf{r}_b}(\neg Q \mid \neg P \wedge \neg Q)}{f_{\mathbf{r}_b}(\neg Q \mid \neg(\neg P \wedge \neg Q))} - 1\right) f_{\mathbf{r}_b}(P) &= f_{\mathbf{r}_b}(Q) \\ \frac{f_{\mathbf{r}_b}(\neg Q \mid \neg P \wedge \neg Q)}{f_{\mathbf{r}_b}(\neg Q \mid \neg(\neg P \wedge \neg Q))} &= \frac{f_{\mathbf{r}_b}(Q)}{f_{\mathbf{r}_b}(P)} + 1 \end{aligned}$$

Thus:

$$log_b\left(\frac{f_{\mathbf{r}_b}(\neg Q \mid \neg P \wedge \neg Q)}{f_{\mathbf{r}_b}(\neg Q \mid \neg(\neg P \wedge \neg Q))}\right) = log_b\left(\frac{f_{\mathbf{r}_b}(Q)}{f_{\mathbf{r}_b}(P)} + 1\right)$$

□

We now turn to our main task.

Proof of Proposition 1.4. We can place any (P, Q) that is a non-trivial determiners into one of six exclusive and exhaustive cases. We prove our result for each one of these cases.[64]

Case 1: $P = A_1$ and $Q = A_i$ for $i \neq 1$. Given Definition 2 (and Ratios of Cells in particular), we know:

$$\begin{aligned} f_{\mathbf{r}_b}(A_i) &= (b^{\mathbf{r}_b(\neg A_1 \wedge \neg A_i, \neg A_i)} - 1) f_{\mathbf{r}_b}(A_1) \\ \frac{f_{\mathbf{r}_b}(A_i)}{f_{\mathbf{r}_b}(A_1)} + 1 &= b^{\mathbf{r}_b(\neg A_1 \wedge \neg A_i, \neg A_i)} \end{aligned}$$

Since $\mathbf{r}_b(\neg A_1 \wedge \neg A_i, \neg A_i) = log_b\left(b^{\mathbf{r}_b(\neg A_1 \wedge \neg A_i, \neg A_i)}\right)$, we have:

$$\mathbf{r}_b(\neg A_1 \wedge \neg A_i, \neg A_i) = log_b\left(\frac{f_{\mathbf{r}_b}(A_i)}{f_{\mathbf{r}_b}(A_1)} + 1\right)$$

[64] The following diagram indicates the six exclusive an exhaustive cases involving non-trivial determiners:

So by Lemma 1.4.1, we have our desired result:

$$\mathbf{r}_b\left(\neg A_1 \wedge \neg A_i, \neg A_i\right) = log_b\left(\frac{f_{\mathbf{r}_b}(\neg A_i \mid \neg A_1 \wedge \neg A_i)}{f_{\mathbf{r}_b}(\neg A_i \mid \neg(\neg A_1 \wedge \neg A_i))}\right)$$

Case 2: $P = A_i$ for $i \neq 1$ and $Q = A_1$. Obviously $A_i \neq \bot$, $A_1 \neq \bot$, and $A_i \wedge A_1 = \bot$. And, since $|U| \geq 3$, $A_i \vee A_1 \neq \top$. (A_1, A_i) is a non-trivial determiner and therefore Negatively Correlated Reasons tells us that:

$$\mathbf{r}_b\left(\neg A_i \wedge \neg A_1, \neg A_1\right) = \mathbf{r}_b\left(\neg A_1 \wedge \neg A_i, \neg A_1\right) = log_b\left(\frac{b^{\mathbf{r}_b(\neg A_1 \wedge \neg A_i, \neg A_i)}}{b^{\mathbf{r}_b(\neg A_1 \wedge \neg A_i, \neg A_i)} - 1}\right)$$

We know from Case 1 that:

$$b^{\mathbf{r}_b(\neg A_1 \wedge \neg A_i, \neg A_i)} = \frac{f_{\mathbf{r}_b}(A_i)}{f_{\mathbf{r}_b}(A_1)} + 1$$

So:

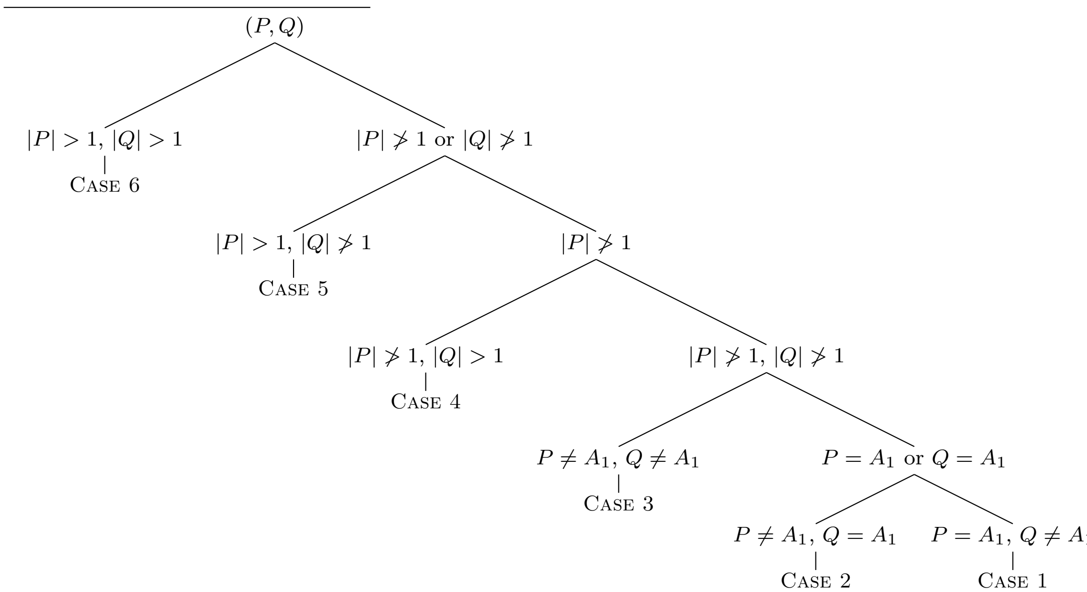
$$\mathbf{r}_b\left(\neg A_i \wedge \neg A_1, \neg A_1\right) = log_b\left(\frac{\frac{f_{\mathbf{r}_b}(A_i)}{f_{\mathbf{r}_b}(A_1)} + 1}{\frac{f_{\mathbf{r}_b}(A_i)}{f_{\mathbf{r}_b}(A_1)}}\right)$$

The term inside the *log* then can be simplified as follows:

$$\frac{\frac{f_{\mathbf{r}_b}(A_i)}{f_{\mathbf{r}_b}(A_1)} + 1}{\frac{f_{\mathbf{r}_b}(A_i)}{f_{\mathbf{r}_b}(A_1)}} = \frac{f_{\mathbf{r}_b}(A_i) f_{\mathbf{r}_b}(A_1)}{f_{\mathbf{r}_b}(A_1) f_{\mathbf{r}_b}(A_i)} + \frac{f_{\mathbf{r}_b}(A_1)}{f_{\mathbf{r}_b}(A_i)} = 1 + \frac{f_{\mathbf{r}_b}(A_1)}{f_{\mathbf{r}_b}(A_i)}$$

So:

$$\mathbf{r}_b\left(\neg A_i \wedge \neg A_1, \neg A_1\right) = log_b\left(1 + \frac{f_{\mathbf{r}_b}(A_1)}{f_{\mathbf{r}_b}(A_i)}\right)$$



Thus, by Lemma 1.4.1, we have our desired result:

$$\mathbf{r}_b\left(\neg A_i \wedge \neg A_1, \neg A_1\right) = \log_b\left(\frac{f_{\mathbf{r}_b}(\neg A_1 \mid \neg A_i \wedge \neg A_1)}{f_{\mathbf{r}_b}(\neg A_1 \mid \neg(\neg A_i \wedge \neg A_1))}\right)$$

Case 3: $P = A_i$ for $i \neq 1$ and $Q = A_j$ for $j \neq 1$ and $i \neq j$. Obviously $A_i \neq \bot$, $A_1 \neq \bot$, $A_j \neq \bot$, $A_i \wedge A_1 = A_1 \wedge A_j = A_i \wedge A_j = \bot$. And, since $|U| \geq 3$, $A_i \vee A_1 \neq \top$, $A_1 \vee A_j \neq \top$, and $A_i \vee A_j \neq \top$. So (A_1, A_j) and (A_i, A_1) are non-trivial determiners and therefore Positively Correlated Reasons tells us that:

$$\mathbf{r}_b\left(\neg A_i \wedge \neg A_j, \neg A_j\right) = log_b\left(\left(b^{\mathbf{r}_b(\neg A_1 \wedge \neg A_j, \neg A_j)} - 1\right)\left(b^{\mathbf{r}_b(\neg A_i \wedge \neg A_1, \neg A_1)} - 1\right) + 1\right)$$

We know from Case 1 that:

$$b^{\mathbf{r}_b(\neg A_1 \wedge \neg A_j, \neg A_j)} = \frac{f_{\mathbf{r}_b}(A_j)}{f_{\mathbf{r}_b}(A_1)} + 1$$

and from Case 2 that:

$$b^{\mathbf{r}_b(\neg A_i \wedge \neg A_1, \neg A_1)} = \frac{f_{\mathbf{r}_b}(A_1)}{f_{\mathbf{r}_b}(A_i)} + 1$$

So:

$$\mathbf{r}_b\left(\neg A_i \wedge \neg A_j, \neg A_j\right) = log_b\left(\left(\frac{f_{\mathbf{r}_b}(A_j)}{f_{\mathbf{r}_b}(A_1)}\right)\left(\frac{f_{\mathbf{r}_b}(A_1)}{f_{\mathbf{r}_b}(A_i)}\right) + 1\right) = log_b\left(\frac{f_{\mathbf{r}_b}(A_j)}{f_{\mathbf{r}_b}(A_i)} + 1\right)$$

Thus, by Lemma 1.4.1, we have our desired result:

$$\mathbf{r}_b\left(\neg A_i \wedge \neg A_j, \neg A_j\right) = log_b\left(\frac{f_{\mathbf{r}_b}(\neg A_j \mid \neg A_i \wedge \neg A_j)}{f_{\mathbf{r}_b}(\neg A_j \mid \neg(\neg A_i \wedge \neg A_j))}\right)$$

Case 4: $P = A_j$ and $Q = \{Q_1, Q_2, \ldots, Q_n\}$ where $|Q| > 1$, $P \vee Q \neq \top$, and $P \wedge Q = \bot$. Given this, Aggregative Reasons applies and tells us:

$$\mathbf{r}_b\left(\neg A_j \wedge \neg Q, \neg Q\right) = log_b\left(\left(\sum_{Q_i \in Q} b^{\mathbf{r}_b(\neg A_j \wedge \neg Q_i, \neg Q_i)} - 1\right) + 1\right)$$

We know from Case 1–Case 3:

$$b^{\mathbf{r}_b(\neg A_j \wedge \neg Q_i, \neg Q_i)} - 1 = \left(\frac{f_{\mathbf{r}_b}(Q_i)}{f_{\mathbf{r}_b}(A_j)} + 1\right) - 1 = \frac{f_{\mathbf{r}_b}(Q_i)}{f_{\mathbf{r}_b}(A_j)}$$

So:

$$\sum_{Q_i \in Q} b^{\mathbf{r}_b(\neg A_j \wedge \neg Q_i, \neg Q_i)} - 1 = \frac{f_{\mathbf{r}_b}(Q_1)}{f_{\mathbf{r}_b}(A_j)} + \frac{f_{\mathbf{r}_b}(Q_2)}{f_{\mathbf{r}_b}(A_j)} + \cdots + \frac{f_{\mathbf{r}_b}(Q_n)}{f_{\mathbf{r}_b}(A_j)} = \frac{f_{\mathbf{r}_b}(Q)}{f_{\mathbf{r}_b}(A_j)}$$

Therefore:

$$\mathbf{r}_b\left(\neg A_j \wedge \neg Q, \neg Q\right) = log_b\left(\left(\left(\sum_{Q_i \in Q} b^{\mathbf{r}_b(\neg A_j \wedge \neg Q_i, \neg Q_i)} - 1\right) + 1\right) = log_b\left(\frac{f_{\mathbf{r}_b}(Q)}{f_{\mathbf{r}_b}(A_j)} + 1\right)$$

Thus, by Lemma 1.4.1, we have our desired result:

$$\mathbf{r}_b\left(\neg A_j \wedge \neg Q, \neg Q\right) = log_b\left(\frac{f_{\mathbf{r}_b}(\neg Q \mid \neg A_j \wedge \neg Q)}{f_{\mathbf{r}_b}(\neg Q \mid \neg(\neg A_j \wedge \neg Q))}\right)$$

CASE 5: $P = \{P_1, P_2, \ldots, P_n\}$ and $Q = A_j$ where $|P| > 1$, $P \vee Q \neq \top$, and $P \wedge Q = \bot$. The proof proceeds analogously to CASE 2 but relying on the results of CASE 4.

CASE 6: $P = \{A_{P_1}, A_{P_2}, \ldots, A_{P_n}\}$ and $Q = \{A_{Q_1}, A_{Q_2}, \ldots, A_{Q_m}\}$ where $|P| > 1$, $|Q| > 1$, $P \vee Q \neq \top$, and $P \wedge Q = \bot$. The proof proceeds analogously to CASE 3 but relying on the results of CASE 4 and CASE 5. $\square$

A.3.3 The Remaining Non-Trivial Cases

Now that we have establish Proposition 1.4, we can extend it to other cases. Our first extension covers values of $\mathbf{r}_b(H, E)$ when $\neg H$ entails E:

Proposition 1.5. *For any H, E such that (H, E) is non-trivial and $\neg H$ entails E*

$$\mathbf{r}_b(H, E) = log_b\left(\frac{f_{\mathbf{r}_b}(E \mid H)}{f_{\mathbf{r}_b}(E \mid \neg H)}\right)$$

Proof of Proposition 1.5. We know from Complimentary Reasons:

$$\mathbf{r}_b(H, E) = -\mathbf{r}_b(\neg H, E)$$

From Proposition 1.4 and the fact that $\neg H$ non-trivially entails E[65], we know:

$$\mathbf{r}_b(\neg H, E) = log_b\left(\frac{f_{\mathbf{r}_b}(E \mid \neg H)}{f_{\mathbf{r}_b}(E \mid H)}\right)$$

So:

$$\mathbf{r}_b(H, E) = -log_b\left(\frac{f_{\mathbf{r}_b}(E \mid \neg H)}{f_{\mathbf{r}_b}(E \mid H)}\right)$$

Since $log(\frac{a}{b}) = -log(\frac{b}{a})$, we have our desired result:

$$\mathbf{r}_b(H, E) = log_b\left(\frac{f_{\mathbf{r}_b}(E \mid H)}{f_{\mathbf{r}_b}(E \mid \neg H)}\right)$$

$\square$

[65] This relies on claim that if (H, E) is not trivial, then $(\neg H, E)$ is not trivial. Here's a proof: Since (H, E) is not trivial, E does not entail H, E does not entail $\neg H$, and $E \neq \top$. It follows from this that E does not entail $\neg H$, E does not entail $\neg\neg H$, and $E \neq \top$. So $(\neg H, E)$ is not trivial.

Our final cases is one where neither H nor $\neg H$ entail E.

Proposition 1.6. *For any H, E such that (H, E) is not trivial and H does not entail E and $\neg H$ does not entail E,*

$$\mathbf{r}_b(H, E) = log_b \left(\frac{f_{\mathbf{r}_b}(E \mid H)}{f_{\mathbf{r}_b}(E \mid \neg H)} \right)$$

To establish this proposition, it helps to begin with the following Lemma.

Lemma 1.6.1. *If (H, E) is not trivial and H does not entail E and $\neg H$ does not entail E, then there are D, D' such that (H, D) and $(\neg H, D')$ are non-trivial determiners.*

Proof of Lemma 1.6.1. Suppose (H, E) is not trivial and H does not entail E and $\neg H$ does not entail E. Thus, $H \neq \top$ and so there is a $D \neq \bot$ such that $D \wedge H = \bot$. But suppose for *reductio* there is no such D that is also such that $D \vee H \neq \top$. For this to be the case, it must be that there is exactly one $A^* \in U$ such that $A^* \notin H$.[66] Since (H, E) is not trivial, E does not entail H. So there is an $E_i \in E$ such that $E_i \notin H$. Thus, $A^* = E_i \subseteq E$. Thus, A^* entails E. But $A^* = \neg H$ so this contradicts our assumption that $\neg H$ does not entail E. Thus, there must be a $D \neq \bot$ such that $D \wedge H = \bot$ and $D \vee H \neq \top$.

By analogous reasoning but relying on the fact that H does not entail E (and that (H, E) is not trivial and hence $(\neg H, E)$ is not trivial), it follows that there is a $D' \neq \bot$ such that $D' \wedge \neg H = \bot$ and $D' \vee \neg H \neq \top$. □

We now turn to the main proof.

Proof of Proposition 1.6. Consider then any H, E such that (H, E) is not trivial and H does not entail E and $\neg H$ does not entail E. We know by Lemma 1.6.1 that there are a D, D' such that (H, D) and $(\neg H, D')$ are non-trivial determiners. So Factored Reasons tells us that:

$$\mathbf{r}_b(H, E) = log_b \left(\frac{(b^{\mathbf{r}_b(\neg D \wedge \neg(H \wedge E), \neg(H \wedge E))} - 1)(b^{\mathbf{r}_b(\neg H \wedge \neg D, \neg D)} - 1)}{(b^{\mathbf{r}_b(\neg D' \wedge \neg(\neg H \wedge E), \neg(\neg H \wedge E))} - 1)(b^{\mathbf{r}_b(H \wedge \neg D', \neg D')} - 1)} \right)$$

Given that (H, D) is a non-trivial determiner, we know from Proposition 1.4:

$$\mathbf{r}_b \left(\neg H \wedge \neg D, \neg D \right) = log_b \left(\frac{f_{\mathbf{r}_b}(D)}{f_{\mathbf{r}_b}(H)} + 1 \right)$$

Since it follows from $H \vee D \neq \top$ that $D \vee (H \wedge E) \neq \top$, it follows from $(H \wedge E) = \bot$ that $D \wedge (H \wedge E) = \bot$, and it follows (H, E) being not trivial that $H \wedge E \neq \bot$, we also know that $(D, H \wedge E)$ is a non-trivial determiner. So Proposition 1.4 tells us:

$$\mathbf{r}_b \left(\neg D \wedge \neg(H \wedge E), \neg(H \wedge E) \right) = log_b \left(\frac{f_{\mathbf{r}_b}(H \wedge E)}{f_{\mathbf{r}_b}(D)} + 1 \right)$$

[66] If there is no such A^*, $H = \top$ which contradicts out assumption that (H, E) is not trivial. If there is a A^*, $A^{**} \in U$ such that $A^* \neq A^{**}$ and $A^*, A^{**} \notin H$, then A^* is a D such that $D \neq \bot$, $D \wedge H = \bot$ and $D \vee H \neq \top$.

We can then simplify the numerator as follows:

$$(b^{\mathbf{r}_b(\neg D\wedge\neg(H\wedge E),\neg(H\wedge E))}-1)(b^{\mathbf{r}_b(\neg H\wedge\neg D,\neg D)}-1)=\left(\frac{f_{\mathbf{r}_b}(H\wedge E)}{f_{\mathbf{r}_b}(D)}\right)\left(\frac{f_{\mathbf{r}_b}(D)}{f_{\mathbf{r}_b}(H)}\right)$$
$$=\frac{f_{\mathbf{r}_b}(H\wedge E)}{f_{\mathbf{r}_b}(H)}=f_{\mathbf{r}_b}(E\mid H)$$

For analogous reasons, we also know from Proposition 1.4 that:

$$\mathbf{r}_b\left(\neg D'\wedge\neg(\neg H\wedge E),\neg(\neg H\cap E)\right)=log_b\left(\frac{f_{\mathbf{r}_b}(\neg H\wedge E)}{f_{\mathbf{r}_b}(D')}+1\right)$$
$$\mathbf{r}_b\left(H\wedge\neg D',\neg D'\right)=log_b\left(\frac{f_{\mathbf{r}_b}(D')}{f_{\mathbf{r}_b}(\neg H)}+1\right)$$

And therefore by similar reasoning the denominator can be simplified:

$$(b^{\mathbf{r}_b\left(\neg D'\wedge\neg(\neg H\wedge E),\neg(\neg H\wedge E)\right)}-1)(b^{\mathbf{r}_b\left(H\wedge\neg D',\neg D'\right)}-1)=f_{\mathbf{r}_b}(E\mid\neg H)$$

Thus we have our desired result:

$$\mathbf{r}_b(H,E)=log_b\left(\frac{f_{\mathbf{r}_b}(E\mid H)}{f_{\mathbf{r}_b}(E\mid\neg H)}\right)$$

□

A.4 Further Issues

Let us close by discussing a few related issues.

One issue worth discussing is whether roughly the converse of Theorem 1 holds:

> For any probability function, Pr, there is a reasons weighing function, $\mathbf{r}_b$, such that (i) for any proposition P, $Pr(P)=f_{\mathbf{r}_b}(P)$ and (ii) for any propositions H,E, either
>
> $$log_b\left(\frac{Pr(E\mid H)}{Pr(E\mid\neg H)}\right)=\mathbf{r}_b(H,E)$$
>
> or $log_b\left(\frac{Pr(E|H)}{Pr(E|\neg H)}\right)$ and $\mathbf{r}_b(H,E)$ are both undefined.

As it turns out, this claim does not quite hold. Instead the following weaker claim holds:

Theorem 2. *For any* regular *probability function, Pr, there is a reasons weighing function, $\mathbf{r}_b$, such that (i) for any proposition P, $Pr(P)=f_{\mathbf{r}_b}(P)$ and (ii) for any propositions H,E, either*

$$log_b\left(\frac{Pr(E\mid H)}{Pr(E\mid\neg H)}\right)=\mathbf{r}_b(H,E)$$

or $log_b\left(\frac{Pr(E|H)}{Pr(E|\neg H)}\right)$ and $\mathbf{r}_b(H,E)$ are both undefined.

This is not surprising given that our proof of (i) of Theorem 1 showed that the $f_{\mathbf{r}_b}$ is a regular probability function (§A.2). I omit the proof of Theorem 2 because it relies primarily on techniques that we have already used. In a supplement, I present the proof as well as provide more background.[67] It is an interesting question how to modify Definition 1 so that we can remove the restriction to regular probability functions.[68,69]

There are a variety of other important issues that are worthy of more discussion than I can provide here. First, since the axiomatization and proofs in this paper are rather inelegant, it would be good to search for a more elegant version of our results. Second, it would be good to explore whether similar results can be established for other confirmation measures and to compare the different axioms defining these measures.[70]

But, as emphasized in the main text, the most pressing issue is to identify a set of *qualitative* axioms to characterize when one reason is better than another reason and prove that our quantitatively defined reasons weighing function can be understood as a numerical representation of this underlying qualitative structure.

B Some Other Confirmation Measures

In this appendix, I discuss three confirmations measures that have properties analogous to l and comment on two other measures.

We begin with perhaps the most well-known measure:

Difference Measure: $d(H, E) = Pr(H \mid E) - Pr(H)$

Earman 1992, among others, advocates d. If we define $d_{|E}(H, E') = Pr(H \mid E \wedge E') - Pr(H \mid E)$, it is known that:

[67] More exactly, the first five axioms correspond to well-known features of l. The remaining axioms can be shown by using Lemma 1.4.1 and performing some cancelling of terms that we have already seen in the proof of Theorem 1. For example to prove l satisfies Negatively Correlated Reasons, one can begin by making use of Lemma 1.4.1, then expand the term inside the *log* in the reverse of the way done in the proof of Case 2 of Proposition 1.4, and then apply Lemma 1.4.1 once more in the other direction. Similar methods work for Positively Correlated Reasons (relying on the proof of Case 3 of Proposition 1.4), Aggregative Reasons (relying on the proof of Case 4 of Proposition 1.4), and Factored Reasons (relying on the proof of Proposition 1.6).

[68] The first thing to do is to change Entailed Reason so that if (H, E) is not trivial and H entails E, $\mathbf{r}_b(H, E) \geq 0$ rather than strictly greater than 0. From here some other modifications to the axioms and proofs are needed to accommodate cases where relevant values of $\mathbf{r}_b$ are undefined now because of certain propositions having probability 0.

[69] Our discussion is also limited to functions defined over an algebra of propositions generated from a finite partition. A good question is whether our results can be generalized to other ways of representing propositions.

[70] We also have only defined an unconditional reasons weighing function but we might wish to have a notion of such a function conditional on some proposition. This is easy to do: $\mathbf{r}_{b_{|E}}$ is the reasons weighing function conditional E and defined so that for all (H, E'), $\mathbf{r}_{b_{|E}}(H, E') = \mathbf{r}_b(H, E \cap E') - \mathbf{r}_b(H, E)$. An interesting question is how to proceed if we take the notion of a conditional reasons weighing function as basic.

Claim 2. $d(H, E \wedge E') = d(H, E) + d_{|E}(H, E')$

Proof of Claim 2.

$$\begin{aligned} d(H,E) + d_{|E}(H,E') &= Pr(H \mid E) - Pr(H) + Pr(H \mid E \wedge E') - Pr(H \mid E) \\ &= Pr(H \mid E \wedge E') - Pr(H) = d(H, E \wedge E') \end{aligned}$$

□

Claim 2 has the following corollary:

Corollary 2.1. $d(H, E \wedge E') = d(H, E) + d(H, E')$ *if* $d(H, E') = d_{|E}(H, E')$.

Next consider the following measure:

Log Ratio Measure: $r(H, E) = log\left(\frac{Pr(H|E)}{Pr(H)}\right)$

Milne 1996, among others, advocates r. If we define $r_{|E}(H, E') = log\left(\frac{Pr(H|E \wedge E')}{Pr(H|E)}\right)$, it is known that:

Claim 3. $r(H, E \wedge E') = r(H, E) + r_{|E}(H, E')$

Proof of Claim 3.

$$\begin{aligned} r(H,E) + r_{|E}(H,E') &= log\left(\frac{Pr(H \mid E)}{Pr(H)}\right) + log\left(\frac{Pr(H \mid E \wedge E')}{Pr(H \mid E)}\right) \\ &= log\left(\frac{Pr(H \mid E)Pr(H \mid E \wedge E')}{Pr(H)Pr(H \mid E')}\right) \\ &= log\left(\frac{Pr(H \mid E \wedge E')}{Pr(H)}\right) = r(H, E \wedge E') \end{aligned}$$

□

Claim 3 has the following corollary:

Corollary 3.1. $r(H, E \wedge E') = r(H, E) + r(H, E')$ *if* $r(H, E') = r_{|E}(H, E')$.

The third measure for which we can establish similar results is the follow:

Z Measure: if $Pr(H \mid E) \geq Pr(H)$, then $z(H, E) = \frac{Pr(H|E) - Pr(H)}{1 - Pr(H)}$
if $Pr(H \mid E) < Pr(H)$, then $z(H, E) = \frac{Pr(H|E) - Pr(H)}{Pr(H)}$

Crupi, Tentori, and Gonzalez 2007 are the most prominent advocates of z. Unfortunately, I cannot provide the result for this measure that is exactly analogous to the ones that I have provided for the other measures. But I can provide a less general result. Let us say that E, E', and $E \wedge E'$ *point the same direction with respect to* H exactly if either $Pr(H \mid E) \geq Pr(H)$, $Pr(H \mid E') \geq Pr(H)$, and $Pr(H \mid E \wedge E') \geq Pr(H)$ or if $Pr(H \mid E) < Pr(H)$, $Pr(H \mid E') < Pr(H)$, and $Pr(H \mid E \wedge E') < Pr(H)$. Next let us define a term $z_{/E}(H,E')$ slightly differently than the other conditionalized measures that we have discussed:

$$z_{/E}(H,E') = \frac{Pr(H \mid E \wedge E') - Pr(H \wedge E)}{\alpha}$$

where $\alpha = 1 - Pr(H)$ if $Pr(H \mid E') \geq Pr(H)$ and where $\alpha = Pr(H)$ if $Pr(H \mid E') < Pr(H)$. We can then show that:

Claim 4. *$z(H, E \wedge E') = z(H,E) + z_{/E}(H,E')$ if E, E', and $E \wedge E'$ point the same direction with respect to H.*

In the following proof, the assumption that the evidence points the same direction ensures the α terms in the denominators are the same:

Proof of Claim 4.

$$\begin{aligned} z(H,E) + z_{/E}(H,E') &= \frac{Pr(H \mid E) - Pr(H)}{\alpha} + \frac{Pr(H \mid E \wedge E') - Pr(H \mid E)}{\alpha} \\ &= \frac{Pr(H \mid E \wedge E') - Pr(H)}{\alpha} = z(H, E \wedge E') \end{aligned}$$

□

Claim 4 has the following corollary:

Corollary 4.1. *$z(H, E \wedge E') = z(H,E) + z(H,E')$ if E, E', and $E \wedge E'$ point the same direction with respect to H and $z(H,E') = z_{/E}(H,E')$.*

Thus, our results for z are more limited than those for other measures but still useful.[71]

There are, however, certain measures for which I cannot provide any useful results. Two prominent ones are the following:

> Normalized Difference Measure: $s(H,E) = Pr(H \mid E) - Pr(H \mid \neg E)$[72]
>
> Carnap's Measure: $c(H,E) = Pr(E)(Pr(H \wedge E) - Pr(H))$

Fitelson 2001 has shown that it does not generally hold that:

$$s(H, E \wedge E') = s(H,E) + s_{|E}(H,E')$$

where $s_{|E}(H,E') = Pr(H \mid E \wedge E') - Pr(H \mid E \wedge \neg E')$.[73] I am not aware of results about c of this sort, but there very well may be such results.

This does not fully settle the issue of whether there is a useful condition for assessing issues related to additivity. There may be such conditions using some kind of non-standard conditional measure like the one described for z. I do not know whether such techniques will yield results for s or c.

[71] A recent note from Branden Fitelson (Fitelson forthcoming) provides a result that makes this limitation vivid: according to z, it cannot both be that two pieces of evidence point different direction and that they are independent of one another in a way that allows for the kind of additive results that we have for the measures above.

[72] This measure is advocated by, among others, Joyce 1999 and Christensen 1999.

[73] See Eells and Fitelson 2000 for discussion of the arguments in Christensen 1999 and of the properties of normalized measures more generally.

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