

# The construction of Electromagnetism

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## Abstract

We examine the construction of electromagnetism in its current form, and in an alternative form, from a point of view that combines a minimal realism with strict rational demands. We begin by discussing the requests of reason when constructing a theory and next, we follow the historical development as presented in the record of original publications, the underlying epistemology (often explained by the authors) and the mathematical constructions. The historical construction develops along socio-political disputes (mainly, the reunification of Germany and the second industrial revolution), epistemic disputes (at least two demarcations of science in conflict) and several theories of electromagnetism. Such disputes resulted in the militant adoption of the ether by some, a position that expanded in parallel with the expansion of Prussia. This way of thinking was facilitated by the earlier adoption of a standpoint that required, as a condition for understanding, the use of physical hypothesis in the form of analogies; an attitude that is antithetic to Newton's "*hypotheses non fingo*". While the material ether was finally abandoned, the epistemology survived in the form of "substantialism" and a metaphysical ether: the space. The militants of the ether attributed certainties regarding the ether to Faraday and Maxwell, when they only expressed doubts and curiosity. Thus, the official story is not the real history. This was achieved by the operation of detaching Maxwell's electromagnetism from its construction and introducing a new game of formulae and interpretations. Large and important parts of Maxwell work are today not known, as for example, the rules for the transformation of the electromagnetic potentials between moving systems. When experiments showed that all the theories based in the material ether were incorrect, a new interpretation was offered: Special Relativity (SR). At the end of the transformation period a pragmatic view of science, well adapted to the industrial society, had emerged, as well as a new protagonist: the theoretical physicist. The rival theory of delayed action at distance initiated under the influence of Gauss was forgotten in the midst of the intellectual warfare. The theory is indistinguishable in

formulae from Maxwell's and its earlier versions are the departing point of Maxwell for the construction of his equations. We show in a mathematical appendix that such (relational) theory can incorporate Lorentz' contributions as well as Maxwell's transformations and C. Neumann's action, without resource to the ether. Demarcation criteria was further changed at the end of the period making room for habits and intuitions. When these intuited criteria are examined by critical reason (seeking for the fundamentals) they can be sharpened with the use of the Non Arbitrariness Principle, which throws light over the arbitrariness in the construction of SR. Under a fully rational view SR is not acceptable, it requires to adopt a less demanding epistemology that detaches the concept from the conception, such as Einstein's own view in this respect, inherited from Hertz. In conclusion: we have shown in this relevant exercise how the reality we accept depends on earlier, irrational, decisions that are not offered for examination but rather are inherited from the culture.

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## 1 Introduction

At the beginning of the XIX century, physics was dominated by ideas that evolved from Galileo, Descartes, Newton and Leibniz, and a good number of distinguished mathematicians such as Laplace, Lagrange, Legendre, Poisson, Ampère, Gauss and Hamilton that expanded mathematics and physics at the same time. In continental European physics the space was relational (following Leibniz) although Descartes' filled space persisted as a luminiferous ether in the theory of light and had supporters in electromagnetism (EM) (Caneva, 1980). In this scheme, the only well known force, gravitational force, was described as an instantaneous action at distance producing accelerations in the direction of the line connecting interacting bodies. The tradition in Great Britain (insular Europe) was of a mixed type. In part, it followed the absolute space tradition of Newton, in the vulgar form of a frame fixed to the distant stars. At the same time, in the philosophical side, it followed the empiricist tradition of Hume. Although Hume speaks about causes and not actions (much less interactions) it appears to us that in the following paragraph causes can be exchanged by actions:

The idea, then, of causation must be deriv'd from some relation among objects; and that relation we must now endeavour to discover. I find in the first place, that whatever objects are consider'd as causes or effects, are contiguous; and that nothing can operate in a time or place, which is ever so little remov'd from those of its existence. [...] We may therefore consider the relation of contiguity as essential to that of causation; at least may suppose it such, according to the

general opinion, till we can find a more proper occasion to clear up this matter, by examining what objects are or are not susceptible of juxtaposition and conjunction.

The second relation I shall observe as essential to causes and effects, is not so universally acknowledg'd, but is liable to some controversy. 'Tis that of priority of time in the cause before the effect. Some pretend that 'tis not absolutely necessary a cause shou'd precede its effect; but that any object or action, in the very first moment of its existence, may exert its productive quality, and give rise to another object or action, perfectly co-temporary with itself. [...] The consequence of this wou'd be no less than the destruction of that succession of causes, which we observe in the world; and indeed, the utter annihilation of time. For if one cause were co-temporary with its effect, and this effect with its effect, and so on, 'tis plain there wou'd be no such thing as succession, and all objects must be co-existent. (Hume, 1896)

Action at distance and even worse, instantaneous action at distance would have been felt inappropriate to the empiricist tradition since it would imply a body causing an effect on another which is non continuous. Lord Kelvin put it in simple form:

The idea that the Sun pulls Jupiter, and Jupiter pulls back against the sun with equal force, and that the sun, earth, moon, and planets all act on one another with mutual attractions seemed to violate the supposed philosophic principle that matter cannot act where it is not. (Kelvin's preface to Hertz, 1893)

It is interesting to observe a sort of pendulum-like movement of ideas between the continent and Great Britain. Kelvin refers in (Kelvin's preface to Hertz, 1893) to an observation made by Voltaire in 1727 when he wrote:

A Frenchman who arrives in London finds a great alteration in philosophy, as in other things. He left the world full, he finds it empty. At Paris you see the Universe composed by vortices of subtile matter; at London we see nothing of the kind...

However, by the end of the XVIII century action at a distance was the dominant theory (Kelvin's preface of Hertz, 1893). The resurgence of ether is credited to Faraday. Kelvin writes:

...before his death, in 1867, he had succeeded in inspiring the raising generation of the scientific world with something approaching to faith that electric force is transmitted by a medium called ether, of which, as it had been believed by the whole scientific world for forty years, light and radiant heat are transversal vibrations.

And he continues:

...for electricity and magnetism Faraday's anticipations and Clerk-Maxwell's splendidly developed theory has been established on the sure basis of experiments by Hertz' work...

In this form goes the standard story, but, is it correct? Is it faithful to history? We will show that there are missing parts in this story probably because Hertz gave the name Maxwell's theory to all theories that agreed with Maxwell's in the final equations to be tested (Hertz, 1893). Hertz mentions Helmholtz theory as well and he indicates the existence of others. The omitted theories were those of the Göttingen group that gathered around the figure of Gauss and included rivals of Maxwell as Lorenz, Riemann, Neumann and especially Weber whose differences with Hertz' mentor, Helmholtz, are well known (Assis, 1994). Thus, personal rivalries and other social phenomena, as we will later see, might have played a role in the final (social) outcome.<sup>1</sup>

Towards the mid of the XIX century, the force of the Enlightenment was declining in Europe while the force of the British Empire and the second industrial revolution was emerging along with the struggle for the unification of Germany. Political events seldom have an influence on the development of science; rather, they influence indirectly through the "ethos of the times". However, in the construction of Electromagnetism they influenced in both forms as we will see.

Let us turn back to philosophical issues. The construction of EM must confront difficulties not present in the construction of mechanics. In particular, most EM phenomena do not appear as such, i.e., as an occurrence, perceptible to the senses, since we have constructed two covering concepts, matter and light, that naturalise most of our experience with electricity. In that time, matter and light were conceived as "external" entities on which EM phenomena occur, rather than entities whose properties are almost entirely of electromagnetic nature. Thus, the study of EM proceeds with little sensorial input in the form of experimental outcomes. Our possibilities of using our intuition and direct experience is then scarce and we must rely on reason and the process of abduction (Peirce, 1955). In turn, abduction –the process of adopting (testable) explanatory hypotheses– is often aided by analogies and habits as a form of producing the hypothesis for consideration. However, the value of the abducted hypothesis is not to be judged by such auxiliary methods but rather for its explanatory power over the facts, that must necessarily be a larger set than those that motivated the hypothesis. Peirce prevents us:

observed facts relate exclusively to the particular circumstances that happened to exist when they were observed. They do not relate to any future occasions upon which we may be in doubt how we ought to act. They, therefore, do not, in themselves, contain any

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<sup>1</sup>Helmholtz and Hertz developments occur at a particular time of German history. Helmholtz becomes head of the Berlin school of physics soon after the beginning of the first Reich (the *Kaiserreich*) (Hoffmann, 1998). In that period there was a battle against Hegel's philosophy. Helmholtz is among those struggling to abandon Hegel's idealism and methods (D'Agostino, 2004).

practical knowledge. Such knowledge must involve additions to the facts observed. The making of those additions is an operation which we can control; and it is evidently a process during which error is liable to creep in. (Peirce, 1955, p. 150)

Did these recommendations guided the construction of EM? We anticipate that the answer is no. On the contrary, we will show that analogical thinking operates as a restriction upon what can be constructed.

By the beginning of the XX century, a complete turn around of ideas in physics had been achieved. Those ideas became the consensus form of physics some years later. The changes that were operated far exceeded a scientific revolution. We intend to show in this work the epistemic transformation of physics produced by the lectures of Maxwell (rather than by Maxwell himself). The transformation of physics was not limited to the incorporation of a new domain of understanding. Rather, it was a complex development producing a new type of scientist, the theoretical physicist, that took the duties of constructing the theories, which was previously performed by mathematicians (Jungnickel and McCormach, 2017, p. 6). The new field of studies, the new social subject, came equipped with its own epistemology, a radical change of which the practitioners were probably not aware, except perhaps (and partially) two of the main protagonists: Poincaré (Poincaré, 1913a) and Einstein (Einstein, 1936, 1940).

The plan for this work is to critically follow the transformation of the ideas in Electromagnetism in the period going from Ampère (Ampère, 1825; Assis and Chaib, 2015) to the emergence of special relativity (SR) (Einstein, 1905; Schwartz, 1977a,b,c) focusing in the epistemic changes produced. We will begin by focusing in Ampère-Weber's relational approach and its social decline (a decline without refutation). Next, we move to the epistemological contributions of Maxwell that anticipate key elements in Husserl's phenomenology and the controversy regarding physical hypotheses (such as the electric fluid and the ether). The next stage consists in the abandonment of Maxwell philosophical attitude by his followers: the beginning of the new era. This crucial step explains (or perhaps just describes) how philosophy was left behind and how a new form of construction of science was born, with new elements such as interpretations and analogies being central to it. It is the epistemological change what converts a phenomenological theory as Maxwell's (in his own words) into a fundamental theory, first of the material ether and later of the philosophical ether: the space. The lax rules of the new epistemology allowed the new scientists to patch their theories, producing an era of "continuous progress", greatly facilitated by the abandonment of the idea of refutation, of the search for the fundamentals (i.e., critical thought, philosophically understood) and of the unity of reason of Kantian idealism. This is, the Gordian-knot was repeatedly cut rather than untied, thus resulting in a conceptual change of what it is meant by science: the adoption of pragmatic-realism as already described by other authors (Torretti, 2000) and substantially in agreement with the ethos of the time.

While the new scientist developed and occupied the social niche of producer of theory, the old epistemological approach did not get extinct. From time to

time, challenges to the main stream have been raised by, or re-emerged in, various scientific groups having a closer affinity with the Galilean-Newtonian heritage. Among the challenges, those emerged in the electrical engineering domain are the most interesting since this scientific community has considered discriminating experiments between a relational electromagnetism and the relativistic version of electromagnetism. Curiously enough, the situations they considered by 1956, motivated by the initial explorations of outer space have emerged in later years as an “anomaly” of the main stream electromagnetism. But what would have been trumpeted as an extraordinary predictive success of science if achieved by the dominant conceptions has gained little or no transcendence: social forces guard us from heretic knowledge as Bourdieu (Bourdieu, 1999) has taught us.

It is important to notice that reviewing the emergence of a new field of study can be done from two (extreme) sides; one side being the praiseful form and the other its opposite, the derogatory form. We began our search trying to find what at that moment we thought was a small missing link between a relativistic and a relational theory of EM. Along the way we realised that the differences cannot be bridged since there is no common epistemological ground to both approaches. A similar transition can be found in Dingle (Dingle, 1960b).

The Maxwell-Lorentz theory of electromagnetism has not evolved for at least 100 years. Yet, inconsistencies on its formulation (or at least in its most common interpretation) have emerged in recent years. Inconsistencies in the standard use of fields (Lazarovici, 2018) and in considering the Maxwell-Lorentz equations as an initial value problem (Deckert and Hartenstein, 2016) have been found. The philosophical consequences have been recently treated in (Hartenstein and Hubert, 2020). However, the analysis has been performed within the current epistemology of theoretical physics.

Using a constructivist approach we have sought to transcend the foundations of mechanics (Solari and Natiello, 2018) unearthing and giving a mathematical form to a principle of reason: the no-arbitrariness principle (NAP). It has been the philosophical guidance provided by NAP what made us to regress in time until an epoch where physics was compatible with it. We were not able to locate a period in time which was free from arbitrariness. There is e.g., arbitrariness in the Ampère-Weber theory, although it can easily be removed. In so doing, it becomes even closer to Maxwell’s theory. Maxwell himself practices the *epojé*<sup>2</sup> hence arbitrariness is put in parenthesis/suspended. This attitude differs drastically with the attitude of his followers which is the entry-point through which a full flared arbitrariness is introduced. Thus, as we adopted as starting point a commitment to reason, and a method for preserving reason, departures from the rational ideal will be highlighted. We begin by presenting requisites of reason that must be satisfied in the construction of a mathematical theory (Section 2), this is, we introduce “rational realism”. Next we review the main line of development of Electromagnetism up to and including Maxwell (Section 3). Among

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<sup>2</sup>Scepticism. The act of refraining from any conclusion for or against anything as the decisive step for the attainment of truth.

other historical information we identify in Maxwell's writings the transformations undergone by the potentials upon a Galilean change of coordinates. The ether era is discussed in Section 4. We analyse the epistemological changes that made it possible (actually, almost mandatory) to believe in the ether. The two epistemological venues are then contrasted. Subsection 4.3 and Appendices A and B explore the electromagnetism that results from applying the constructive rational rules of Section 2. Subsection 4.4 is dedicated to the contribution by Einstein, the final point of the evolution of current electromagnetism. We show that this view is not possible under the "rational realism", complementary, in Appendix B.2 we offer a relevant and example on how epistemological blindness operates. The final sections correspond to discussions and conclusions.

## 2 The scientific attitude and the inference of a theory

Scientists conceive the world as a cosmos, a harmonious totality. For them, there is nothing as fascinating as discovering this harmony. For this task they are equipped basically with two tools: reason and experience. What they call understanding is the result of the interplay of the two, for experience does not constitute knowledge if not for the intervention of reason. These ideas (and some words) are taken from Kant (Kant, 1787) and Peirce (Peirce, 1955) and do not change in their strength if, rather than considering the world as a cosmos, we change the proposition to: the goal of the scientist is to articulate a harmonic vision of the world, to make a cosmos out of the sensorial input she/he receives.

### 2.1 Some rules for the construction of a cosmos

The task of understanding the construction of EM requires some precision on what we mean by reason and the requisites for inference.

**The principle of reality** In the first place, we must indicate that the attempt of constructing a cosmos out of sensorial input implies the assumption that there is something real that reaches us through the senses, this is to say, that there is subject and object. While the truth of this statement is debatable, we can consider the dangers involved in accepting or rejecting it. Little damage is done if accepting reality were an error and it turns to be that everything is part of a unique encompassing (solipsistic) being. On the contrary, if we were in error when rejecting reality, we would become completely dysfunctional and miss one of the greatest opportunities in life. The principle is summarised in:

“Such is the method of science. Its fundamental hypothesis, restated in more familiar language, is this: There are Real things, whose characters are entirely independent of our opinions about them; those Reals affect our senses according to regular laws, and, though our sensations are as different as are our relations to the objects, yet,

by taking advantage of the laws of perception, we can ascertain by reasoning how things really and truly are; and any man, if he have sufficient experience and he reason enough about it, will be led to the one True conclusion. The new conception here involved is that of Reality.”(Peirce, 1955, p. 18)

Hence, we adopt realism as a starting point and reject conspiratorial theories, this is, we reject hypotheses which cannot be put to experimental test. We state this starting point in the form of a principle (Solari and Natiello, 2018):

**Principle 1** *There is a material world that we perceive with our senses (including experiments).*

**The no arbitrariness principle** In a recent work, we have shown that if we introduce some arbitrary decisions in the scientific discourse (be it for the sake of the argument or with the aim of facilitating an explanation), the set of possible arbitrary elements must have the internal structure of a group<sup>3</sup>, being then the set of all possible presentations of the argument a representation of the group and as such equivalents (Solari and Natiello, 2018). Further, we have shown that the facilitation of the relational concept of space due to Leibniz produced by the introduction of a privileged observer introduces a (useful) subjective element, the subjective space (the space of all elementary physics texts) along with a series of properties of this space as well as conditions that the statements regarding physical laws must satisfy if they are going to remain rational.

The set of arbitrary decisions deserves some further consideration. It has been indicated (Margenau and Mould, 1957), in consideration of their own versions of NAP, that choosing different arbitrary sets where the statement (observation) should identically hold might lead to different theories. Therefore, a clarification of the concept of arbitrariness in this context is needed. It is important to indicate that any difference that is dictated by experimental and observational methods should be considered not to be arbitrary, but there is more to it. For example, the class of (idealised) isolated systems, when we disregard their internal structure, admits a group of arbitrariness. We call this class of systems *inertial* (Solari and Natiello, 2018). They are to be distinguished from those systems that indicate as a necessity the presence of a companion one. But since isolated systems are an idealisation, the same can be said for inertial systems, thus, approximately-inertial systems must exist. In practice, then, if a system can be considered inertial or not, depends on the extent that the

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<sup>3</sup>For example: We can say that the relations in the invariant relational space are lifted into relations in the subjective spaces by arbitrary decisions, but since the subjective statements must remain equivalent, there must be a group of transformations,  $\mathcal{G}$ , that allows us to move from one presentation to the other. If we conceive now a theory as a space of statements,  $E$ , relating different concepts belonging to our subjective presentation, what is real in them is only the core that remains when we remove (mod out) the arbitrariness,  $T = E/\mathcal{G}$ , which is the result of identifying statements that only differ by the introduced arbitrariness. Thus,  $T$  is invariant while  $E$  is equivariant with respect to  $\mathcal{G}$ .



presence of other matter not being accounted for can sensibly modify the experimental outcome. This view has been held by experimentalists such as Michelson (Michelson, 1904).

This view of inertia must be contrasted with the subjective view. In the subjective view the subject tries to draw a definition from her/his own experience. She/he imagines being in a cage from which all input from the exterior has been cut, and extrapolates experiences within the cage to: “If the cage moves with constant velocity (in some subjective space she/he does not dare to mention) I won’t be able to grasp the velocity with which I am moving. On the other hand, if the cage changes direction, it comes to a stop or sets to move faster, I will know the cage is accelerated and then non-inertial, for I am likely to bump on the cage borders”. Thus, the non-inertial system is revealed as not being isotropic nor homogeneous, which are precisely the symmetries required to suppress the arbitrariness of the subject. In case our subject finds her/himself in a non-inertial system, she/he will try to find out which are the particular properties of this space, something that has to be constructed by comparison with the expectations with respect to inertial space. Such space will now be loaded with characteristics potentially varying from point to point, this is, described by a mathematical field. This is about how far the construction of reality initiated by the self-centered-child can be taken, for any step further would lead to the recognition of the observer as arbitrary (one out of many possible observers), rather than observing through the eyes of God, that sees reality the way reality is. Such a surpassing step would force our observer to leave behind some of her/his most cherished toys, such as space, or kinetic energy (in as much as it is not relational).

When constructing a theory we have to make an early decision: are we going to introduce arbitrariness or not? The decision has not much relevance if we keep track of the introduced arbitrariness, and acknowledge the necessity of (and the methods for) removing it. However, if we lose consciousness of our constructive effort, we might inadvertently enter into the realm of arbitrariness. No amount of mathematics will take us ever out of the subjectivist cage, since the necessary step is not an analytic/deductive judgement but rather a synthetic/critical one. This is, we need to understand not what the consequences of our beliefs are, but rather which is the foundation of our beliefs.

If arbitrariness is the absence of reason, the double negation in no-arbitrariness is equivalent to reason. This is, the rejection of arbitrariness is a condition put on every rational construction (Solari and Natiello, 2018).

**Principle 2 [ No Arbitrariness Principle (NAP)]** *No knowledge of Nature depends on arbitrary decisions.*

**The imagination as limit** The introduction of explanatory hypothesis, the process of abduction, is subject to the control of rationality and to the condition that the newly introduced hypothesis explains a class of problems larger than the one that motivated it, this is, that the hypothesis bears some of the main ingredients of cognitive surpass (Piaget and García, 1989) and offers itself more

openly to refutation. However, the requisites for the acceptance of explanatory hypotheses (i.e., to be able to stand in front of refutation attempts) says nothing about the method of production. There are no conditions for the process of creation of hypotheses to be tested. The most natural source would be the intuition of the sensible, *empiria*, but in the case of EM data is scarce and manifests itself as e.g., the deviation of a needle or a change in the equilibrium point of a scale, that only indirectly represent the phenomena. Hypotheses are then constructed supplementing data with imagination. Since entertaining and testing hypotheses requires considerable effort, there is a preselection of promising hypotheses. In most (all?) cases the chosen hypotheses are those produced by analogy and/or habit. Thus, physical hypothesis in the form of mechanical systems (particles, fluids, springs, ...) enter the electromagnetic scene and are erroneously placed as empirical input. Under the name of “physical reasoning”, physics instructors will train their students<sup>4</sup> to seek answers in these terms. In so doing, they advance irrational constructions in substitution of the far more difficult rational ones. The increment in the use of analogies during the second part of the XIX century is well documented in (Jungnickel and McCormach, 2017). But these irrational constructs play a fundamental role when an observational problem has to be cast into formulae and vice versa. The physical hypothesis and analogies (a constitutive part of theories seldom recognised as such since only formulae appear to have the attributes necessary for recognition) are the main nexus with the observable reality.

What we can construct as images and memories comes from what is perceived by our senses and since human beings are largely visual animals, the root “image” in “imagination” must be taken almost at a literal value. However, if we limit our explanatory hypotheses to those that can be produced by these methods, we will soon produce a sensible analogy among matters that, precisely, are of a different (immaterial) kind. For example interactions are inferred elements and do not have a material form, but our method of generating hypotheses will assign to them the characteristics of bodies, thus, they will be placed in space by analogy with waves in a material media or particles travelling from here to there. Even Lorenz, that strongly objected the ether (Lorenz, 1867) found it necessary to have a material medium (Lorenz, 1863). The following text of Lord Kelvin illustrates analogy and imagination as a limit:

I never satisfy myself until I can make a mechanical model of a thing. If I can make a mechanical model I can understand it. As long as I cannot make a mechanical model all the way through I cannot understand; and that is why I cannot get [this is probably the reporter’s Americanism for the word “accept”] the electromagnetic

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<sup>4</sup>Anthropologist Sharon Traweek observes about high-energy physicists that “Undergraduate physics students, to be successful, must display a high degree of intellectual skill, particularly in analogical thinking. The students learn from textbooks whose interpretation of physics is not to be challenged; in fact, it is not to be seen as interpretation.” (p. 74, Traweek, 1992). Also (p. 77): “Teachers show students how to recognize that a new problem is like this or that familiar problem; in this introduction to the repertoire of soluble problems to be memorized, the student is taught not induction or deduction but analogic thinking.”

theory.(Thompson, 2011, p.235)

**The continuity principle (reduction to the obvious/evident)** Arguments are constructed in such a way that they rest upon small units we consider evident or obvious. Yet, what is obvious or evident for some, may not be so for others. One of the forms in which we usually identify potentially irrational arguments is by detecting hiatus or *lacunæ* in the argumentation. The request “please, fill in the gap“ quite often reveals a belief that cannot be supported while being essential to the argument. On the contrary, the rational argumentation proceeds to fill the gaps by explaining how they consist of the concatenation of smaller pieces, iterating the process until the pieces are accepted as evident or obvious. This self-similar form corresponds to what in mathematics is called *continuity*.

**The mediation principle and the dialectical openings** We do not usually accept as reasonable that which appears out of nothingness as self-evident assertions. We normally request a new rational-belief to be derived (mediated) by acceptable argumentation from accepted beliefs. This recurrent form of reasoning cannot be pursued indefinitely. It comes to an end when we reach a point in which beliefs can no longer be derived from other accepted beliefs. At this point there seems to be only one option: Either we make explicit a layer of arbitrary assumptions (axioms) which is the opaque end that reason lets us see, or we find a set of opposing concepts and ideas that in their interplay constitute the foundation of our discussion; the dialectical openings(Solari and Natiello, 2018). The participating concepts or ideas support each other by oppositions and they exist only as dialectically opposing elements, while it is this opposition what we consider as perceived, as real. In case that we decide to introduce arbitrary hypothesis and yet remain rational, we would be forced to mod them out as explained in NAP, thus something not arbitrary would have to survive. Hence, the alternative is: irrational or dialectic.

**Logical action in front of contradictions** Whenever a chain of reasoning arrives to a contradiction, the whole chain is rejected. When the contradiction results from comparing theoretical prediction and experimental reality we speak of an experimental refutation. The logical scheme can be depicted as  $A \Rightarrow B \Rightarrow$  [further consequences]  $\Rightarrow$  False. No matter how pleasant  $B$  and other intermediate consequences are, there is no support for them. The most evident example is the hypothesis of the ether which is fundamental for the proposition of Maxwell’s displacement field. Discarding the existence of the ether (following empirical evidence) under the present principle would mean the refutation of the hypothesis as well as all of its consequences. The theory of EM would have to be retracted to the point at which it incorporated light and then reconstructed again under new hypotheses. Sustaining a thesis despite that its argumentation has been shown to be false is the case of *persistence* in Peirce’s classification of the methods of fixating beliefs (Peirce, 1955, ch. 2). It is, as such, irrational.

There is another instance of the same logical scheme which is not usually considered, namely when the contradiction stems from the logical structure of the theory (e.g., inconsistent postulates). Assume  $A$  is True, then  $\neg A$  is False. The construction  $A \Rightarrow B \Rightarrow [\text{further consequences}] \Rightarrow (\neg A)$  discloses an internal contradiction of the theory and, as above, it forces us to reject the full chain. It is observed that some authors are tempted to change the sequence into: Axiom  $B$  is true,  $B \Rightarrow [\text{further consequences}] \Rightarrow (\neg A)$  detaching the statements from its production process keeping (without an alternative argumentation) the desired result. Informally, we call the process “cutting the branch we are sitting on“, we fall in this case into a violation of the continuity principle and the mediation principle. This situation relates directly to observations made by Dingle (1960b) in which the author finds that in the construction of Special Relativity the formulae are detached of their physical meaning.

**On reason** What is reason then? How can we reason about what reason is? Reason can only be conceived in front of no-reason, of arbitrariness. But arbitrary is the statement that introduces a belief without admitting doubt about it (perhaps by conviction/persistence, or authority). Hence, to discuss reason and arbitrariness we need to discuss belief and doubt, the forms of fixation of beliefs, and the relations between these elements. Further, since what is imposed one way or another (as in arbitrariness) cannot be said to be harmonious, we find that reason is directly related to harmony. Perhaps at this point it is better to consider our fundamental insight: we feel in peace only with a rational world, a world with harmony, a cosmos, and not a world that is the result of power and arbitrariness. Hence, our choice of reason for the organisation of the world emerges directly from our feelings. These feelings can be made universal, for it is possible to conceive a humanity of free persons that consistently entertain doubts and peacefully cooperate to progress in their common beliefs, while it is not possible to conceive a peaceful coexistence of arbitrary beliefs, except in the final state of uniformity (death). Rather than coexisting peacefully, arbitrary beliefs tend to replace each other by forceful imposition. Thus, our choice for rationality is not rational, it emerges from our feelings, and it is perhaps because of them that reason will ever re-emerge.

## 2.2 Vulgar pragmatism

Would a scientist agree on the need of being reasonable and on the meaning of reason? We expect not to find a scientist rejecting the idea of being reasonable. Furthermore, we cannot conceive a scientist admitting not being reasonable without at the same time withdrawing her/his argument. But self-evaluation of our own reasonability is usually poor in self-criticism, for reasoning is often this thing that I am doing, while I have no access to the totality of the processes of reasoning in others. Some schools of thought (we will call them sophists) regard reasoning as just one form (among several others) of disputing and convincing. Philosophers have in turn indicated that reasoning is not always at the core

of scientific research (Feyerabend, 1987) and that only instrumental reason is actually being used (Horkheimer, 1947).

One of the cornerstones of reasonability is to reject logical inconsistency. The attitude we take in front of error is decisive: the scientific person yields to reason and to empirical evidence. If a theory is found to be inconsistent, and also if it produces predictions against empirical evidence, its reasonability collapses, we are no longer satisfied by its explanatory power and it is therefore rejected, as Peirce explicitly requested<sup>5</sup> and later Popper will make central to his philosophy of science (Popper, 1959).

At first sight, this vision is probably shared by most scientists. On the contrary, the attitude of persistence in beliefs (tenacity (ch. 2, Peirce, 1955), in Peirce's denomination) despite empirical evidence against them or despite logical inconsistencies, is one form of being non-scientific. Note that our objection is not to the results or predictions that a non-scientific theory could produce, but rather to its construction. Considering success in prediction ability as the only measure of scientific correctness amounts to replacing the attitude of the scientist with some form of vulgar pragmatism. Peirce (Fixation of belief, Peirce, 1955) regards human actions as the outcome of the struggle between doubt and belief. The former provokes tension and the urge for resolution, the latter is a state of peace of mind towards which we strive. He further identifies reason as the only way of fixating beliefs that is free of arbitrariness, quite in line with the concept of critical pragmatism (Pennycook, 1997) or –as we prefer to call it– scientific pragmatism. Any method of fixating beliefs other than reason, e.g., the argument by confirmation or verification that a belief is useful for any purpose other than reasonability (personal benefit, success, the imposition of an idea, etc.) will be called vulgar pragmatism, in line with (Pennycook, 1997).

The problems presented by the incorporation of light into EM had become so frustrating by the beginning of the XX century that Poincaré was willing to accept a radical change in the scientific attitude, this is, the acceptance of “persistence in beliefs“, the unwillingness of dropping our theories when they are contradicted by experiments (a form of vulgar pragmatism already identified by Peirce (Peirce, 1955, ch. 2) by 1887). But perhaps most remarkable, is the degree of collective blindness that developed, since shortly after Maxwell's landmark paper, read in December 1864 (Maxwell, 1865), Lorenz presented his extension of Weber's theory incorporating light without ether (Lorenz, 1867), a relational theory that as such predicted way in advance the outcome of the famous Michelson-Morley experiment and that was found to be identical to Maxwell's theory when later applied to Hertz' experiments. We look to this situation with perplexity. By 1881 there were two theories (Lorenz' and Maxwell's)

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<sup>5</sup>See e.g., “It is a great mistake to suppose that the mind of the active scientist is filled with propositions which, if not proved beyond all reasonable cavil, are at least extremely probable. On the contrary, he entertains hypotheses which are almost wildly incredible, and treats them with respect for the time being. Why does he do this? Simply because any scientific proposition whatever is always liable to be refuted and dropped at short notice.“ (p.36) “But the scientific spirit requires a man to be at all times ready to dump his whole cartload of beliefs, the moment experience is against them.“ (p.46) (Peirce, 1955)

and one discriminant experiment (Michelson's). Despite the correspondence of the experimental results with only one of the theories, we persist in patching the failed theory using conspiratorial thoughts?

This was not an isolated fact. By the turn of the century the change in epistemological attitude away from the scientific approach and towards verificationism became increasingly evident. Let us consider the proposition:

Supposons que, dans une nuit, toutes les dimensions de l'univers deviennent mille fois plus grandes: le monde sera resté semblable à lui-même, en donnant au mot de similitude le même sens qu'au 3<sup>e</sup> livre de géométrie. Seulement, ce qui avait un mètre de long mesurera désormais un kilomètre, ce qui était long d'un millimètre deviendra long d'un mètre. Le lit où je suis couché et mon corps lui-même se seront agrandis dans la même proportion. Quand je me réveillerai le lendemain matin, quel sentiment éprouverai-je en présence d'une aussi étonnante transformation? Eh bien, je ne m'apercevrai de rien du tout. Les mesures le plus précises seront incapables de me rien révéler de cet immense bouleversement puisque les mètres dont je me ser virai auront varié précisément dans les mêmes proportions que les objets que je chercherai à mesurer... D'après Lorentz et Fitzgerald tous les corps entraînés dans le mouvement de la Terre subissent une déformation. Cette déformation est à la vérité très faible, puisque toutes les dimensions parallèles au mouvement de la Terre diminueraient d'un cent millionième, tandis que les dimensions perpendiculaires à ce mouvement ne seraient pas altérées. (Poincaré, 1906)

This argumentation has several problems. The proposed contraction is imperceptible, nothing changes in our relation to the world if we accept it or reject it except probably the issue that it intends to explain. As an explanatory hypothesis it does not meet the requisite of explaining more than the observation that suggested it. The explanation proceeds through something otherwise unverifiable and of no practical consequences whatsoever (as the excerpt claims). There are no grounds to prefer this explanation to other alternatives. It only serves the purpose of fixating our belief that there is no mistake in the considerations that led us to the conclusion that the speed of light should depend on the velocity of Earth relative to the ether. The idea of having to accept such a conspiratorial hypothesis to save a theory of its wreckage is indignant. With higher dignity, Kelvin rejects the vibrations of the ether as an explanation of light since: "I firmly believe in an electromagnetic theory of light, and that when we understand electricity and magnetism and light we shall see them all together as parts of a whole. But I want to understand light as well as I can, without introducing things that we understand even less of." (Thompson, 2011, pp. 835–836)

It should be noticed on passing that Poincaré's attitudes towards Lorentz' theories had been fluctuating. In a homage to Lorentz, Poincaré writes:

It would no doubt seem strange that in a monument raised to the glory of Lorentz I would review the considerations which I presented previously as an objection to his theory. I could say that the pages which follow are rather in the nature of an attenuation rather than a magnification of that objection. But I disdain that excuse, because I have one which is 100 times better: Good theories are flexible. Those which have a rigid form and which can not change that form without collapsing really have too little vitality. But if a theory is solid, then it can be cast in diverse forms, it resists all attacks, and its essential meaning remains unaffected. (Poincaré, 1900)

It appears from this text that a theory is not “vital” enough if it “collapses” in front of contradictory empirical evidence.

What is, then, the goal of an explanatory hypothesis? In the views of scientific pragmatism it is “to lead to the avoidance of all surprise and to the establishment of an habit of positive expectation that shall not be disappointed” (p. 267, Peirce, 1955), through subjection to the test of experiment. In the absence of any special reasons for the contrary, any hypothesis, therefore, may be admissible provided it being capable of experimental verification, and only in so far as it is capable of such verification. Peirce again: “An explanatory hypothesis, that is to say, a conception which does not limit its purpose to enabling the mind to grasp into one a variety of facts, but which seeks to connect those facts with our general conceptions of the universe, ought, in one sense, to be verifiable; that is to say, it ought to be little more than a ligament of numberless possible predictions concerning future experience, so that if they fail, it fails” (p. 267, Peirce, 1955, (our emphasis)).

Other indications of the advance of different forms of vulgar pragmatism arose along the XX century:

Science is the attempt to make the chaotic diversity of our sense experience a logically uniform system of thought. (Einstein, 1940)

The word “uniform” has taken the place of “harmony”. But harmony is the quality of producing a single and pleasant totality, while uniformity implies the suppression of differences, something that many of us find utterly unpleasant since we regard life as a harmonious diversity and death as uniformity (remaining the same in all cases and at all times). The search for the unity of reason is then replaced by the search of a uniform symbolic manipulation of all forces, see for example (Smolin and Harnad, 2008), being the later a goal akin to industrialisation rather than to Enlightenment. The unity of reason requires to transcend the oppositions/differences (at the same time preserving them) or unifying them in their differences by surpassing them (Piaget and García, 1989), achieving a more abstract and encompassing vision.

According to Einstein:

“Physics constitutes a logical system of thought which is in a state of evolution, and whose basis cannot be obtained through distillation

by any inductive method from the experiences lived through, but which can only be attained by free invention. The justification (truth content) of the system rests in the proof of usefulness of the resulting theorems on the basis of sense experiences, where the relations of the latter to the former can only be comprehended intuitively. Evolution is going on in the direction of increasing simplicity of the logical basis“. (Einstein, 1940)

It appears from this statement that physics has no roots, it floats in the air as a free invention without ties. Since the ties that enter the construction of a theory rule the use of the theory, the reverse link between theoretical results and sensorial experiences rests upon some sort of “intuition“. It is not intuition with its usual philosophical meaning (a central idea in (Husserl, 1983) where intuition mediates in the process of ideation, the construction of the empirical input into ideas and later theories). The word “intuition“ in the above quotation must be understood as involving elements removed from rational control. The “physics“ is thus bracketed between nothingness and irrationality since the ties of philosophical intuition as well as the rational ties of the continuity and mediation principles have been removed, constructing then a new epistemology that resembles vulgar pragmatism: it is true because it is useful.

### 2.3 Maxwell’s epistemic position

James Clerk Maxwell’s contributions to electromagnetism culminated in his *Treatise* (Maxwell, 1873) where he extends his foundational paper (Maxwell, 1865) into a comprehensive work encompassing old and new mathematics and physics developed along several decades. In both works, Maxwell introduces considerations that correspond better to epistemology than to mathematics or physics.

From the beginning, his goal is to describe electromagnetic phenomena by means of local interactions with the surrounding medium (the ether), as opposed to the action at a distance theories, more developed outside England. Maxwell begins by celebrating the continental tradition of electromagnetism (Maxwell, 1865):

The most obvious mechanical phenomenon in electrical and magnetical experiments is the mutual action by which bodies in certain states set each other in motion while still at a sensible distance from each other. The first step, therefore, in reducing this phenomena into scientific form is to ascertain the magnitude and direction of the force acting between the bodies [...]

In this way, mathematical theories of statical electricity, of magnetism, of the mechanical action between conductors carrying currents and of the induction of currents have been formed [...]

These theories assume, more or less explicitly, the existence of substances the particles of which have the property of acting on one



another at a distance by attraction or repulsion. The most complete development of a theory of this kind is that of M. W. (Weber, 1846) who made the same theory include both electrostatic and electromagnetic phenomena.

In doing so, however, he has found it necessary to assume that the force between two electric particles depend on their relative velocity, as well as on their distance.

This theory, as developed by M. W. Weber and C. Neumann, is exceedingly ingenious, and wonderfully comprehensive in its application to phenomena of statical electricity, electromagnetic attractions, induction of currents and diamagnetic phenomena; and it comes to us with the more authority as it has served to guide the speculations of one who has made so great an advance in the practical part of electric science, both by introducing a consistent systems of units in electrical measurement, and by actually determining electrical quantities with an accuracy hitherto unknown<sup>6</sup>.

In other parts of the Treatise, Maxwell points out that his theory and Weber's arrive to the same descriptions of different electromagnetic phenomena. See e.g., ([644], [855] Maxwell, 1873) among similar comments scattered along the work. However, he finds Weber's approach too difficult (Maxwell, 1865), arguing as follows for his point of view:

(2) The mechanical difficulties, however, which are involved in the assumption of particles acting at distance with forces that depend on their velocities are much as to prevent me from considering this theory as an ultimate one, though it might have been and may yet be useful in leading to the coordination of phenomena.

I have therefore preferred to seek an explanation of the fact in another direction by supposing them to be produced by actions which go on the surrounding medium as well as in the excited bodies, and endeavouring to explain the action between distant bodies without assuming the existence of forces capable of acting directly at sensible distances.

(3) The theory I propose may be therefore be called a theory of the **Electromagnetic Field**, because it has to do with the space in the neighbourhood of the electric or magnetic bodies, and may be called a **Dynamical Theory**, because it assumes that in that space there is matter in motion, by which the observed electromagnetic phenomena are produced.

In his Treatise, Maxwell explains further ([529] Maxwell, 1873):

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<sup>6</sup>Maxwell tries to confine Weber within the status of somebody concerned with the "practical part", since the great advance indicated is precisely Weber's work.

We are accustomed to consider the universe as made up of parts, and mathematicians usually begin by considering a single particle, and then conceiving its relation to another particle and so on. This has generally been supposed the most natural method. To conceive of a particle, however, requires a process of abstraction, since all our perceptions are related to extended bodies, so that the idea of the all that is in our consciousness at a given instant is perhaps as primitive an idea as that of any individual thing. Hence there may be a mathematical method in which we proceed from the whole to the parts instead of from the parts to the whole. For example, Euclid, in his first book conceives a line as traced out by a point, a surface as swept out by a line and a solid as generated by a surface. But he also defines a surface as the boundary of a solid, a line as the edge of a surface and a point as the extremity of a line.

Both in the main text of these works and also in the Preface of the Treatise, Maxwell claims that his views are based on the approach of Faraday to electromagnetic phenomena. However, while Faraday entertained doubts about the necessity of the ether and indeed advanced an alternative explanation, Maxwell misinterprets Faraday writings (in text and spirit) as already pointed out clearly in Dingle (1960b, p. 250-52). In the work cited by Maxwell, Faraday writes, in the manuscript entitled “Thoughts on ray vibrations“ (Faraday, 1855, p. 447),

The point intended to be set forth for consideration of the hearers was, whether it was not possible that the vibrations that in certain theory are assumed to account for radiation and radiant phenomena cannot occur in the lines of force which connect particles, and consequently masses of matter together; a motion that, as far as it is admitted, will dispense with the æther, which in another view is supposed to be the medium in which this vibrations take place.

You are aware of the speculation which I some times since uttered respecting that view of the nature of matter that considers its ultimate atoms as centres of force, and not as so many little bodies surrounded by forces, the bodies being considered in the abstract as independent of the forces and capable of existing without them. In the later view, this little atoms have a definite form and a certain limited size; in the former view such is not the case, for that which represents size may be considered as extending to any distance to which the lines of force of the particle extends: the particle indeed is supposed to exist only by these forces, and where they are it is.

Faraday’s speculation about matter is “A speculation touching Electric Conduction and the Nature of Matter“ (Faraday, 1844, p. 284) and rests upon ideas of Boscovich. The speculation is not an opinion, Faraday philosophically proceeds to show that current ideas on matter (of that time, 1844) were incompatible with electricity before adopting Boscovich’s view. The view basically consists

in a duality<sup>7</sup>. We only know about matter because of its actions (forces) and we locate matter where the centre of this action lies. If the action extends through the universe, the atoms are “where they act“, this is in all the universe. Matter and force cannot be separated, they are two aspects of an unity. Maxwell actually resents such ideas and tries to amend Faraday:

[speaking of Faraday] ...He even speaks of the lines of force belonging to a body as in some sense part of itself, so that in its action to distant bodies it cannot be said to act where it is not. This, however, is not a dominant idea with Faraday. I think he would rather have said that the field of space is full of lines of force, whose arrangement depends on that of the bodies in the field, and that the mechanical and electrical action on each body is determined by the lines which abut on it ([529] Maxwell, 1873).

From a rational perspective, Maxwell’s argumentation is disappointing. To sustain that Faraday should have said what he did not say amounts to the vulgar strategy of substituting adverse observations for desired observations. If the idea was dominant or not with Faraday is beside the point. The beliefs of Faraday are of no importance, what matters are his arguments and Maxwell chooses not to confront them but to hold to his own beliefs. This way of avoiding adverse ideas contrasts with the otherwise careful and open-minded attitude of Maxwell both as a philosopher and even more as a mathematician <sup>8</sup>. We will return to Maxwell’s philosophical views in Section 4.1, contrasting them with Hertz’ ideas.

The tradition of natural philosophy to which Faraday adhered comes to an abrupt end in Maxwell. For the sake of completeness it must be noted that Faraday never criticised nor put down his idea on ray vibrations. It is advanced as a hypothesis open to criticism, and there it stands, still today. It appears that he never returned to the subject.

The foundation of Maxwell’s beliefs can be found in ([866] Maxwell, 1873) where he discusses action at distance and writes

Now we are unable to conceive propagation in time, except either as the flight of a material substance through space, or as the propagation of a condition of motion or stress in a medium already existing in space.[...] If something is transmitted from one particle to another at a distance, what is its condition after it has left the one particle and before it has reached the other?

The belief (provisional in Maxwell) in the ether is clearly stated as the result of a limitation by reduction to ideas that are part of our experience with matter, but applied to inferred ideas such as (inter)actions. It represents a fear of abstraction (at the end a fear to reason) which is present for example in (Berkeley,

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<sup>7</sup>We will return to this issue in Section 3

<sup>8</sup>Recall for example the discussion on Ampère’s force ([509],[510], [526] Maxwell, 1873), the derivation of the electromagnetic force inside a conductor ([598] Maxwell, 1873) or his remarks about the limits of the fluid analogy ([574] Maxwell, 1873).

2017, 10 intro). This is the main hidden lemma of theoretical physics which is of epistemic or philosophical character and consists in the rejection, *in limine*, of ideas that can not be imagined (constructed with mental images of material experiences). Faraday’s “vibrating rays“ could be easily ridiculed if taken literally. The rays are only a form of illustration of some aspects of an interaction; imagining a vibrating illustration is a valid pedagogical tool, but it can hardly be considered part of the Real (except as illustration). But the image facilitates communication (and then acceptance) reaching those, as Kelvin for example, that need a material support for their thoughts. The obstacle is not Physics, or the Universe or the Cosmos, the obstacles are our own capabilities for reasoning and our confidence in reason. Such a problem is not present in the construction of classical mechanics.

Last, but not least, Maxwell’s introduces a new form of exploring possible organisations of the Cosmos. The introduction of a Lagrangian for electromagnetic phenomena (the starting point of Maxwell’s research) is compared Maxwell ([554] 1873) with the use of the same mathematical object by Lagrange “the end of Lagrange was to bring dynamics under the power of the calculus[...] our aim, on the other hand, is to cultivate our dynamical ideas“. Thus, Maxwell introduces mathematical analogies. If the idea of current, as explained by Faraday, implies the movement of electricity, and we have an expression (due to F. Neumann ([542-543] Maxwell, 1873)) for the energy involved in the interaction between two circuits with electrical currents, then we can construct the associated momentum (the electro-tonic state of Faraday (Par. 60 Faraday, 1839)) of the current as the derivative of the “kinetic energy“ (the energy involved in the movement) with respect to the current. This fundamental quantity is what in today’s language is known as the vector potential,  $A$ , and correspondingly, its time derivative is a “force“, the electromagnetic force<sup>9</sup>,  $fem = -\frac{dA}{dt}$ . We will come back to the important details of this matter.

## 2.4 Poincaré’s proposed reorganisation of science

Poincaré’s attitude towards Electrodynamics contrasts with his important contributions to Mechanics. He argues for the ether hypothesis from a conservative standpoint (without it the description of phenomena would be more complicated than what we are familiar with). Also, his mathematical arguments are incorrect in several points:

And does our ether really exist? We know the origin of our belief in the ether. If light reaches us from a distant star, during several years it was no longer on the star and not yet on the earth; it must then be somewhere and sustained, so to speak, by some material support.

The same idea may be expressed under a more mathematical and more abstract form. What we ascertain are the changes undergone by material molecules; we see, for instance, that our photographic

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<sup>9</sup>Note that the time derivative here is a convective, or “total“, derivative and not a partial derivative.

plate feels the consequences of phenomena of which the incandescent mass of the star was the theater several years before. Now, in ordinary mechanics the state of the system studied depends only on its state at an instant immediately anterior; therefore the system satisfies differential equations. On the contrary, if we should not believe in the ether, the state of the material universe would depend not only on the state immediately preceding, but on states much older; the system would satisfy equations of finite differences. It is to escape this derogation of the general laws of mechanics that we have invented the ether.

That would still only oblige us to fill up, with the ether, the interplanetary void, but not to make it penetrate the bosom of the material media themselves. Fizeau's experiment goes further. By the interference of rays which have traversed air or water in motion, it seems to show us two different media interpenetrating and yet changing place one with regard to the other.

We seem to touch the ether with the finger.

Yet experiments may be conceived which would make us touch it still more nearly. Suppose Newton's principle, of the equality of action and reaction, no longer true if applied to matter alone, and that we have established it. The geometric sum of all the forces applied to all the material molecules would no longer be null. It would be necessary then, if we did not wish to change all mechanics, to introduce the ether, in order that this action which matter appeared to experience should be counterbalanced by the reaction of matter on something.

Or again, suppose we discover that optical and electrical phenomena are influenced by the motion of the earth. We should be led to conclude that these phenomena might reveal to us not only the relative motions of material bodies, but what would seem to be their absolute motions. Again, an ether would be necessary, that these so-called absolute motions should not be their displacements with regard to a void space, but their displacements with regard to something concrete. (Poincaré, 1913a, p. 147)<sup>10</sup>

But Poincaré realises that the ether is an intuition: "We seem to touch the ether with the finger" is the English translation of the French expression: "On croit toucher l'éther du doigt" which means to understand intuitively.<sup>11</sup>

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<sup>10</sup>Torretti (2007) quotes his own translation of the first two paragraphs and comments that the mathematical insight of the second paragraph is wrong. We add that it was known to be wrong to Gauss and the school of Göttingen as well as to Maxwell.

<sup>11</sup>There are at least two digital versions of this book. We quote the translation by Halsted which is faithful to the French original version in this point as well as to printed versions. There is another version translated by W.J.G. (Poincaré, 1913b) which reads: "The ether is all but in our grasp"

The first paragraph almost reproduces Maxwell's argument (quoted above). It is interesting to realise that when the ether was at last derogated, the operation was performed by switching to the other option given by Maxwell in his alternative (the flight of a material substance), this is, by creating the photon that, as Poincaré understands, must make some elements of the old physics (as Poincaré calls it) persistent.

(Poincaré, 1913a, pp. 300–301) proposes a reorganisation of physics in terms of “principles“ and he lists them:

1. Conservation of energy (Mayer's principle)
2. Degradation of energy (Carnot's)
3. Equality of action and reaction (Newton's)
4. Relativity, according to which the laws of physical phenomena must be the same for an stationary observer or for an observer carried along in a uniform form of translation; so that we have not and cannot have any means of discerning whether or not we are carried along in such motion.
5. Conservation of mass (Lavoisier's)
6. (I will add) the principle of least action.

The application of these five or six general principles to the different physical phenomena is sufficient for our learning of them all that we could reasonably hope to know of them. The most remarkable example of this new mathematical physics is, beyond question, Maxwell's electromagnetic theory of light.[...]

This principles are results of experiments boldly generalized; but they seem to derive from their very generality a high degree of certainty. In fact, the more general they are, the more frequent are the opportunities to check them, and the verifications multiplying, taking the most varied, the most unexpected forms, end by no longer leaving place for doubt. (Poincaré, 1913a, Original in French of 1905)

Poincaré establishes his principles (beliefs) from experience as he says. But the experiences here implied are of two different kinds. On one side 1, 2, 5 and perhaps 3 can be the subject of experimental tests, they come from *empiria*. Actually, Lavoisier's law was a falling belief at the time of his writings. In contrast, the relativity and the least action principles come from the experience of our practices, they are constructive principles (Margenau and Mould, 1957) that come from *habitus*. Poincaré does not seem to be aware of the difference between the empirical input (the observation) and our ideation.

### 2.4.1 The principle of relativity

In their epistemological appraisal of special relativity, Margeneau and Mould (Margeneau and Mould, 1957) analyse the principle of relativity from a philosophical perspective. For these authors it is a constructive principle, and we agree. They introduce two versions of the principle: the new and the old. Referring to the old they say: “Historically, relativity is associated with problems arising out of the need to provide a reference for particle motion. The question which philosophers have asked is: Should quantities like particle position, velocity and acceleration be referred to an absolute, primitively given spatial frame of reference; or should the kinematic notions which fix the state of the particle have meaning relative only to other particles?” and later, they state “The modern form of the principle is not concerned with the status (primitive or defined) of the concepts it treats, but rather with the extensibility of the axioms over the range of individuals included in the axiomatic structure. It requires the elimination of special or preferred individuals. ‘Individual’ here refers to membership in any given or generated collection of constructs of a specified kind.”

The principle of relativity appears in (Poincaré, 1913a; Margeneau and Mould, 1957; Dingle, 1960b) as emerging from irrationality, this is, it is not mediated by other beliefs but it appears as true to the elite that has been previously trained in Newtonian mechanics, for it extrapolates the *habitus* of the Newtonian construction. It is only very recently that we have surpassed the direct irrational acceptance showing that it is a requirement of reason and of the principle of reality (a bolder belief). If there exists something real which is the concern of science, the real cannot depend on our decisions, as stated in Peirce’s paragraph quoted above. By considering that the objective must be intersubjective we arrived to NAP (Solari and Natiello, 2018). But NAP requires a mathematical structure relating arbitrariness, the structure of a mathematical group.<sup>12</sup>

Although NAP was thought in terms of the description of the real with empirical foundation, its significance overpasses it. If we consider a fundamental theory as real (although not directly related to observations) the theory must be free of arbitrariness as well, and since the consecutive applications of arbitrary actions is an arbitrariness as well, the set of arbitrary elements must be related by operations pertaining to a group. Poincaré’s relativity principle can be viewed then as the requirement that the relation between symbols be invariant (indirectly, that the concepts ideated from the empiria and mediated by the rules of correspondence have invariant relations).

Margeneau and Mould seek to escape from the problems of Special relativity by enlarging the set of arbitrariness the way Einstein did in the General Theory of relativity Margeneau and Mould (1957). The problem is then: Do (approximately) inertial systems exist? If the idea of inertial systems were an

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<sup>12</sup>Margeneau and Mould use the word group Margeneau and Mould (1957) in at least two forms: as equivalent to set (as in “this group includes many teachers”) and with the mathematical meaning used in NAP (Solari and Natiello, 2018) (as in “transformation group” or the non-existent but frequently mentioned “Lorentz group”).

arbitrariness, then we should go along with Einstein. Otherwise, we should restrain to label as arbitrary what is in fact an idealisation emerging from an empirical result. If we conceive the motion of a reference system in relation to absolute space or the ether, it is clear that the notion falls when we reject both ideas. But as we have shown (Solari and Natiello, 2018) the idea of inertial motion emerges from the concept of isolation, and absolute isolation is an idealisation of empirical observations accessible to trained and untrained eyes. Non-inertial motion can be measured. A key experiment related to the present issues was performed using a ring interferometer and a rotating table (Sagnac, 1913). Michelson showed the effect of rotation of the Earth in the propagation of light (Michelson, 1925; Michelson and Gale, 1925) and discussed why the experiment could not account for the movement of Earth around the sun (Michelson, 1904). Sagnac (ring) interferometers are used regularly as inertometers in aerial navigation (Post, 1967).<sup>13</sup>Hence, approximately inertial is measurable and inertial systems are an idealised category. Margenau and Mould have no right to mix inertial frames and non-inertial frames in the same arbitrariness class, a tradition that goes back (at least) to Mach Mach (1919, (II.VI.5, p. 232)) and was criticised by Poincaré (Poincaré, 1913a, Ch. VII) using precisely the idea of isolation.

### 3 Matter and electricity

From the earlier investigations on static electricity it was clear that there were two kinds of materials: conductors and insulators. While in conductors the static electricity was able to move, no such macroscopic displacement was possible in insulators. Electricity was therefore conceived as matter inside matter. This is, either as a fluid (or two fluids) or as particles moving inside matter. In the second conception, the particles were positive and negative, and it was later assumed that they would move in opposite directions with equal velocities, an image known as Fechner hypothesis (Assis, 1994, p. 52). These early images still lurk behind some debates that oppose Lorentz' force to Ampère's force, despite their difference being clarified by Maxwell (see below). Since electricity was bound to matter, the force exerted within the electrical substance would be transmitted to the wire that was constraining it. The concepts arising from this matter-inside-matter picture led to the distinctions ponderable matter vs. imponderable matter and mechanical force vs. electromotive force.

However, the movement of electricity (so conceived) within some materials challenged the existing ideas of matter. It was Faraday who, at an early stage, clearly understood the problem:

The view of the atomic constitution of matter which I think is most

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<sup>13</sup>Ironically, Sagnac's experiment is used by relativists (for example (Malykin, 2000)) as well as by defenders of the ether (e.g., (Silvertooth, 1989)) to confirm their theories, and there is no contradiction in that because only pre-existing beliefs can be confirmed. The fact is that the formula produced by "extending" special relativity to slowly rotating systems and by retaining the ether are just the same expression.



prevalent, is that which considers the atom as a something material having a certain volume, upon which those powers were impressed at the creation, which have given it, from that time to the present, the capability of constituting, when many atoms are congregated together into groups, the different substances whose effects and properties we observe. These, though grouped and held together by their powers, do not touch each other, but have intervening space, otherwise pressure or cold could not make a body contract into a smaller bulk, nor heat or tension make it larger; in liquids these atoms or particles are free to move about one another, and in vapours or gases they are also present, but removed very much further apart, though still related to each other by their powers. [...]

**But it is always safe and philosophic to distinguish, as much as is in our power, fact from theory; the experience of past ages is sufficient to show us the wisdom of such a course; and considering the constant tendency of the mind to rest on an assumption, and, when it answers every present purpose, to forget that it is an assumption, we ought to remember that it, in such cases, becomes a prejudice, and inevitably interferes, more or less, with a clear-sighted judgment [...]**

If the view of the constitution of matter already referred to be assumed to be correct, and I may be allowed to speak of the particles of matter and of the space between them (in water, or in the vapour of water for instance) as two different things, then space must be taken as the only continuous part, for the particles are considered as separated by space from each other. Space will permeate all masses of matter in every direction like a net, except that in place of meshes it will form cells, isolating each atom from its neighbours, and itself only being continuous (Faraday, 1844, p. 284–286)(emphasis added).

Faraday proceeds then to show that space cannot be conceived neither as an insulator nor a conductor, hence the conception of matter was incompatible with experimental phenomena. He then continues his exposition:

If we must assume at all, as indeed in a branch of knowledge like the present we can hardly help it, then the safest course appears to be to assume as little as possible, and in that respect the atoms of Boscovich appear to me to have a great advantage over the more usual notion. His atoms, if I understand aright, are mere centres of forces or powers, not particles of matter, in which the powers themselves reside. If, in the ordinary view of atoms, we call the particle of matter away from the powers  $a$ , and the system of powers or forces in and around it  $m$ , then in Boscovich's theory  $a$  disappears, or is a mere mathematical point, whilst in the usual notion it is a little unchangeable, impenetrable piece of matter, and  $m$  is an

atmosphere of force grouped around it. (Faraday, 1844, p. 289–290)

Notice that the already quoted article by Maxwell ([529] Maxwell, 1873) precisely tries to amend Faraday adhering to the criticised theory. Faraday proceeds further with this idea:

To my mind, therefore, the *a* or nucleus vanishes, and the substance consists of the powers or *m*; and indeed what notion can we form of the nucleus independent of its powers? all our perception and knowledge of the atom, and even our fancy, is limited to ideas of its powers: what thought remains on which to hang the imagination of an *a* independent of the acknowledged forces? A mind just entering on the subject may consider it difficult to think of the powers of matter independent of a separate something to be called *the matter*, but it is certainly far more difficult, and indeed impossible, to think of or imagine that *matter* independent of the powers. **Now the powers we know and recognize in every phenomenon of the creation, the abstract matter in none; why then assume the existence of that of which we are ignorant, which we cannot conceive, and for which there is no philosophical necessity?** [...] (emphasis added)

In the view of matter now sustained as the lesser assumption, matter and the atoms of matter would be mutually penetrable. As regards the mutual penetrability of matter, one would think that the facts respecting potassium and its compounds, already described, would be enough to prove that point to a mind which accepts a fact for a fact, and is not obstructed in its judgement by preconceived notions. With respect to the mutual penetrability of the atoms, it seems to me to present in many points of view a more beautiful, yet equally probable and philosophic idea of the constitution of bodies than the other hypotheses, especially in the case of chemical combination. If we suppose an atom of oxygen and an atom of potassium about to combine and produce potash, the hypothesis of solid unchangeable impenetrable atoms places these two particles side by side in a position easily, because mechanically, imagined, and not infrequently represented; but if these two atoms be centres of power they will mutually penetrate to the very centres, thus forming one atom or molecule with powers, either uniformly around it or arranged as the resultant of the powers of the two constituent atoms; and the manner in which two or many centres of force may in this way combine, [...] (Faraday, 1844, p. 290–293)(emphasis added).

At a moment in time when one or two fluids and imponderable matter were being discussed, Faraday's views are marvellously bold. He regards as accidental such concepts as shape, extension, etc., pertaining to the traditional view. For him, the identity of matter relates instead to its "powers", i.e., the different possibilities of interaction and their associated fields. As stated before (see

(Faraday, 1855, p. 447) above), Faraday soon realised that this view allowed to get rid of the ether.

Faraday's views developed over a long period of time. By 1821 he had written:

Those who consider electricity as a fluid, or as two fluids, conceive that a current or currents of electricity are passing through the wire during the whole time it forms the connection between the poles of an active [voltaic] apparatus. There are many arguments in favour of the materiality of electricity, and but few against it; but still it is only a supposition; and it will be as well to remember, while pursuing the subject of electro-magnetism, that we have no proof of the materiality of electricity, or of the existence of any current through the wire. (p. 212–213 Assis and Chaib, 2015, quote from Faraday)

We cannot avoid to underline the philosophical attitude of Faraday, who is making all possible efforts to avoid prejudice and to preserve understanding. Faraday is a philosopher and refers to himself as such. To our knowledge, he is the last natural scientist to regard himself as a philosopher.

### 3.1 Following Ampère's footprints

While the work by Coulomb considering electrostatic forces did not rise controversies and was later incorporated in all theories of EM, the work by Ampère did raise controversies, while it also was fundamental in the progression of EM theories. A recent work (Assis and Chaib, 2015) informs us how research and controversies about currents and their forces developed around 1820-1830, involving colleagues/adversaries (producers of alternative views) all across Europe. Scientists such as Ørsted, Faraday, Biot-Savart and Grassmann criticised Ampère's force between current elements. Most of the criticisms revolve around different analogies with material entities (fluids or particles) that were favoured by one or the other scientist. Grassmann pointed out some degree of arbitrariness in Ampère's assumption that forces between current elements were central (along the line joining the elements). Most noticeably, Faraday only presented words of caution, since he wisely entertained doubts.

Finally, Maxwell ([518-527] Maxwell, 1873) showed that a complete experimental deduction of the force between current elements starting from the forces between closed circuits was not possible. We add that the current element is a mental segmentation of real current-carrying wires, and it is closer to metaphysics than to physics since it cannot be physically realised. According to Assis' research, Ampère was criticised both for not considering electric particles moving inside the conductor (Ørsted) and for considering them (Biot-Savart). Actually, the core of the discussion was about which is the "correct" analogy with matter, be it moving charges, one or two magnetic fluids, atomic magnets and so on. It is remarkable though, that the subtitle of Ampère's contribution

was “Theory of Electrodynamical Phenomena, Uniquely Deduced from Experience“ an expression that illustrates how our hypotheses are hidden for our own observation.

Despite the controversy with respect to the force between current elements, the integrated force between closed circuits that carry currents presented by Ampère still stands without objections. Indeed, the other alternatives to this force (Grassman, Biot-Savart) yield the same result as Ampère’s ([518-527] Maxwell, 1873).

### 3.2 The relational point of view

Work on the relational view was centred in Göttingen and followed the lead of Carl F. Gauss who left his guidance in two short communications (bd.5 p. 602-626 and 627-629, Gauss, 1870). This view rests on the assumption that electromagnetism is an interaction analogous to e.g., gravity, in that it can be fully described as the mutual influence between charged particles, depending on intrinsic and relative properties. This description is expressed by a “force” (although of a different nature than e.g., the gravitational force) between pairs of particles, depending on the individual charges, relative distance, relative velocity and relative acceleration. In particular, the dependence on velocity is discussed in Maxwell’s Treatise, where it says in [851] that Gauss in July 1835 interpreted as a fundamental law of electrical action, that: “Two elements of electricity in a state of relative motion attract or repel one another, but not in the same way as if they are in a state of relative rest”. Gauss suspended his work on EM by 1836 (his own account in (bd.5 p. 627-629, Gauss, 1870))<sup>14</sup>. Ten years later Gauss will express the suspicion that the interaction does not reflect instantaneous action at a distance but rather a delayed action at a distance (bd.5 p. 627-629, Gauss, 1870).

Some of these relational efforts are reviewed by Maxwell in (v.2 Ch.XXIII Maxwell, 1873) where forces advanced by Weber, Gauss, Riemann, Clausius and Betti are discussed).

#### 3.2.1 Weber’s force

Among the attempts just mentioned, Weber’s force  $F_{21}$  between charged particles Weber (1846) is the one that has deserved most attention, being still discussed in our days Assis (1994):

$$F_{21} = \frac{q_1 q_2}{4\pi\epsilon_0} \frac{\vec{r}_{12}}{r_{12}^3} \left( 1 - \frac{\dot{r}_{12}^2}{2C^2} + \frac{r_{12} \ddot{r}_{12}}{C^2} \right)$$

Here,  $F_{21}$  is the force on particle 2 due to particle 1,  $q_1, q_2$  are the electric charges,  $\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$  is the relative position vector between the particles,

<sup>14</sup>Soon later, Wilhelm Weber, his friend and main experimental mind in the research, was expelled out of Göttingen following the protests in favour of the (old) liberal constitution. Weber was one of the “Seven of Göttingen“. He lost then his laboratory and the daily contact with Gauss.

with length  $r_{12}$ . The force involves the derivatives of this length. Weber’s work settled the choice of units by relating this force to other known forces also measurable with a dynamometer. The constant  $\epsilon_0$  relates to static electricity (where the derivatives of the distance are zero) and the constant  $\mu_0$  relates to magnetic forces. The ratio of these two forces relates to the quantity  $C^2 = (\mu_0\epsilon_0)^{-1}$ , later known as the speed of light and omnipresent in electromagnetism.

Maxwell has mixed opinions about Weber’s approach. On one hand, we will see in Section 3.3 that he criticises instantaneous action at a distance in favour of the “propagating medium” theory (we defer the discussion of this to Section 4, advancing only that the criticism is not conclusive but expresses just that two possible approaches could be pursued, and Maxwell pursues one of them). On the other hand, Maxwell recognises in [552] that this approach leads to conceptions that are “as beautiful as they are bold“, while in [856] he notes that “Weber’s law, with the various assumptions about the nature of electric currents which it involves, leads by mathematical transformations to the formula of Ampère. [...] Weber’s law is also consistent with the principle of the conservation of energy” and “Weber’s law will explain the induction of electric currents.“

Let us consider in detail the insights behind Weber’s force. The force expresses instantaneous action at a distance. Further, it is a central force obeying Newton’s principle of action and reaction. These two assumptions were current in those times (although Gauss, Riemann, Betti and Lorenz suspected otherwise) and were not put to test. Also, for zero relative velocity Weber force reduces to Coulomb’s force between charged particles.

One of the goals of Weber was to incorporate the forces between currents as studied by Ampère<sup>15</sup> and Faraday. Weber assumed that the current within a conductor responds to the fact that positively and negatively charged (point-like) particles move within the conductor with certain relative velocity. Originally, positive and negative charges were supposed to have opposite velocities of equal size –what is called the Fechner hypothesis (p. 52, Assis, 1994)–, but Weber’s framework does not depend on this hypothesis (Ch. 4.2, Assis, 1994) to describe Ampère’s force or Faraday’s induction law. In short, Weber’s force is the simplest relational central force of instantaneous action that satisfies Coulomb’s, Ampère’s and Faraday’s laws under the assumption that current consists of electrically charged (point like) particles in relative motion<sup>16</sup>. Weber’s electrical particles were supposed to exist within matter and to be imponderable (weightless), as opposed to usual (ponderable) matter.

Weber’s mechanistic view of electricity was the current theory around 1850. It came to coexist with Maxwell’s theory some 15 years later, while after 1885 Maxwell’s theory (as understood by his followers) was dominating. A serious difficulty with Weber’s approach was the impossibility to unify light and electric-

<sup>15</sup>Actually, action and reaction could not be put to test, since it cannot be deduced from Ampère’s force, as noted by Maxwell in the Treatise, [527].

<sup>16</sup>The idea of conduction current as the movement of charged particles will return in Section 4.2, where we discuss that it is more restricted than the view of Maxwell and it enters in conflict with experimental data.

ity, something that Maxwell's theory was able to achieve (v.2 Ch.XX, Maxwell, 1873). However, electrical waves were first advanced within Weber's framework.

### 3.2.2 Franz Neumann's contribution

Ampère's force was a fundamental expression for further developments by the EM community. In its most symmetric form it reads

$$\begin{aligned} F &= -\frac{\mu_0}{4\pi} I_1 I_2 \oint_1 \oint_2 \frac{x_1(l_1) - x_2(l_2)}{|x_1(l_1) - x_2(l_2)|^3} (dl_1 \cdot dl_2) \\ &= \frac{\mu_0}{4\pi} I_1 I_2 \oint_1 \oint_2 \left( \nabla_x \frac{1}{|x|} \right)_{x=x_1(l_1)-x_2(l_2)} (dl_1 \cdot dl_2) \end{aligned}$$

where  $I_1, I_2$  are the currents through conductors 1 and 2. The second line suggests that there is an energy involved in producing the configuration of

$$P = \frac{\mu_0}{4\pi} I_1 I_2 \oint_1 \oint_2 \frac{1}{|x_1(l_1) - x_2(l_2)|} (dl_1 \cdot dl_2) \quad (1)$$

This form, introduced by F. Neumann (Neumann, 1846), was one of the starting points in Maxwell's theory. In contrast to Weber, Neumann did not rest on a material image, but rather abducted the expression because it served to organise and integrate Faraday's induction with Ampère force. The integrated expression, eq.(1), is a safer starting point than Weber's force since it requires a lesser number of hypothesis (usually analogies) about the unknown.

Eq.(1) admits at least two readings. The energy can be regarded as the potential energy that results from starting in a situation where the circuits do not interact and subsequently bringing one circuit to the proximity of the other, preserving the currents in each of them (in other words it is, a mechanical energy). Also, the expression can be considered to be the kinetic energy associated to the electrical currents. In formulas,

$$\begin{aligned} P &= \oint_2 I_2 A(x_2(l_2)) \cdot dl_2 \\ A(x_2) &= \frac{\mu_0}{4\pi} I_1 \oint_1 \frac{1}{|x_1(l_1) - x_2|} dl_1 \end{aligned} \quad (2)$$

F. Neumann is credited with the first introduction of what today is called the vector potential,  $A$ .

It is worth to keep in mind that these developments use the space as an auxiliary element since all expressions are relational. The same can be said of Weber's force.

### 3.2.3 Kirchhoff and electrical waves

In 1857 Kirchhoff(Kirchhoff, 1857) advanced a model for the propagation of electricity in wires, combining existing and new elements. He started by computing

the electromotive force associated to charges and currents within a wire inspired in Weber's theories. Further, he combined these results with Ohm's law (namely that current in a wire is proportional to electromotive force) and the continuity equation (i.e., that spatial variation in current corresponds to time-variation in charge, also called the conservation of charge), thus obtaining a wave-like equation for the current (again, current here is identified with charges in motion). Apparently, Weber was simultaneously working on the same lines at the time (Assis, 2000, 2003), although his results were published some years later. The resulting wave-like equation came to be called the "Telegraph equation".

As much as this wave behaviour was encouraging, its differences with light were only too large. A wire is filled with electrically active particles (this is the current view about wires still today), both positive and negative and in roughly the same amount if the wire is to be electrically neutral as a whole, thus offering a tangible electrical medium for the waves to propagate (more or less like the pressure waves originated in a string musical instrument). Light, on the other hand, was expected to exist in vacuum where there is no tangible medium at all. A possible escape way at the time was to endow the vacuum with electrical properties, by resorting to the ether (see Section 4), an idea that was already circulating in different forms.

### 3.2.4 Lorenz and Delayed Action at a Distance (DAD)

The danish physicist Ludvig Lorenz presented a series of works on light propagation (Lorenz, 1861, 1863, 1867) in the 1860's. Especially in his memory of 1867 he expresses discomfort with the ether hypothesis, which had only been useful to "furnish a basis for our imagination" (p. 287). He sets up to develop an ether-free theory of light, something that he achieves by introducing delays. Electrodynamics could be an action at a distance theory, but not an instantaneous one. This was a dramatic novelty, that is still not completely grasped. Retarded action was being discussed at the time. Indeed, in Maxwell's Treatise (Ch. XXIII) the theories of C. Neumann, Betti and Riemann (all published in 1867-68) are briefly discussed, along with a criticism by Clausius. Curiously, Lorenz far-reaching contribution is not mentioned in that part of the Treatise. It appears in Art. [805] as a novel theory of light propagation, without mention of the delayed action at a distance. Maxwell ends his recollection by noting that Lorenz' conclusions are "similar to those of this chapter, though obtained by an entirely different method" (Maxwell also points out that his own theory was published earlier, in 1865).

The proposal of delayed action opens up for different possibilities regarding how this delay could take place. Within an emission theory, light generates at the source, it travels through space and reaches the detector, somewhat like sound waves, or even a projectile. This idea goes back at least to Huygens and Newton and it was supported by Maxwell, Clausius and almost all of Maxwell's followers. It rests strongly in the assumption of a propagating medium. Consequently, this view attempts to describe delays with information about the location of the source at the departing time and that of the detector at the

(later) detection time. However, this quantity is not universal, it takes different values for different observers and it cannot be used “as is” to gain understanding about light propagation (see below). Outside the emission theory, other possibilities open up. The Göttingen school advocated a different view, better adapted to Faraday’s intuitions (Faraday, 1855, p. 447) based on an objective measure of the delay involved in the interaction, as it will be presently exemplified. A more detailed discussion about delayed action at a distance is given in Appendix A.

### 3.2.5 C. Neumann, Betti, Riemann and Clausius’ criticism

Three more attempts were made to incorporate delayed action at a distance by Gauss’ followers Betti (Betti, 1867), Riemann (Riemann, 1867) and Carl Neumann (Neumann, 1868). These scientists tried to justify in different forms the use of an objective form of the delay, aiming to formulate the propagation law guessed by Gauss (the wave equation) and mathematically enunciated by Maxwell (1865) and Lorenz (1867).<sup>17</sup>

The fate of Neumann’s theory is better known as a consequence of the polemic with Clausius discussed by Archibald and Assis (Archibald, 1986; Assis, 1994). Archibald observes that “These exchanges illuminate some of the points that are at issue when an instantaneous action at a distance model is replaced by one where the action is propagated with finite velocity. In so doing, they illustrate two views of the role of mathematics in physical science, showing that the twentieth century is not the first time that mathematical model offering good results have posed problems for those who seek a more intuitive picture”.

Neumann explicitly proposes a potential “travelling” from source to detector. The locations and times of the interaction must satisfy the relation

$$|x_d(t) - x_s(t)| = C(t - t_0) \quad (3)$$

( $d$  for detector and  $s$  for source) being  $t_0$  the time for the electromagnetic disturbance in the source. This relation, common to Lorenz and the whole Göttingen school, is universal (independent of observers or choices of reference frames); hence, all observers compute the same “velocity”,  $C = \frac{1}{(\mu_0\epsilon_0)}$ , which is Weber’s measured relation between static and dynamic electricity, a quantity for which it makes no sense to consider a frame of measurement. However, Neumann’s focus is in the action of a delayed potential. He insists that the travel is similar but not identical to that of light (Neumann, 1869).

Clausius (Clausius, 1869) criticism of Neumann (and also of Riemann in this particular topic) is based on an emission theory. Clausius claims that the

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<sup>17</sup>Lorenz’ and Maxwell’s propagation of electromagnetic disturbance arrive to the same final equation, the difference being in the foundations. Maxwell uses the displacement current based on the ether to obtain what is now called Ampère-Maxwell’s law,  $\Delta A = -\mu_0 j + \frac{1}{C^2} \frac{\partial^2 A}{\partial t^2}$  while Lorenz obtains  $\Delta A - \frac{1}{C^2} \frac{\partial^2 A}{\partial t^2} = -\mu_0 j$  using the delays, explicitly avoiding to invoke any ether ( $j$  is here the current density, i.e., the current per unit volume).



“distance” between source and detector must satisfy

$$|x_d(t_d) - x_s(t_s)| = C(t_d - t_s).$$

This expression does not have the universal character of eq.(3). In the terms of NAP, Clausius proposal equates a subjective quantity  $|x_d(t_d) - x_s(t_s)|$  to an objective one (Solari and Natiello, 2018). The laws of physics are thus made to depend on the arbitrariness of the selection of a reference frame since for an arbitrary change from a frame to another moving with relative velocity  $v$  we would have  $|x_d(t_d) - x_s(t_s)| \mapsto |x_d(t_d) - x_s(t_s) + v(t_d - t_s)|$ . This view cannot be sustained unless an absolute reference frame is introduced: the ether or absolute space. The substantialist view<sup>18</sup> at the basis of the emission theory of light has never been questioned, but it persists beyond any doubts about its appropriateness.

Further, it was later realised that this view forces to deform our conceptions of space and time in order to compensate for the arbitrariness. Clausius’ criticism does not go further than that: Neumann’s programme is regarded as inadequate since it does not conform to Clausius’ views. It suggests without proof that the ether-based emission theory would be appropriate and final. Rather than a search for the foundations and an attempt to refine or improve the criticised work, the criticism dismissed the proposal on improper grounds.

Maxwell presented his views on C. Neumann’s theory in articles [863] and [866] of the Treatise:

Besides this, the velocity of transmission of the potential is not, like that of light, constant relative to the aether or to space, but rather like that of a projectile, constant relative to the velocity of the emitting particle at the instant of emission.

It appears, therefore, that in order to understand the theory of Neumann, we must form a very different representation of the process of the transmission of potential from that to which we have been accustomed in considering the propagation of light. Whether it can ever be accepted as the ‘construirbar Vorstellung’<sup>19</sup> of the process of transmission, which appeared necessary to Gauss, I cannot say, but I have not myself been able to construct a consistent mental representation of Neumann’s theory. (art [863])

In the theory of Neumann, the mathematical conception called Potential, which we are unable to conceive as a material substance, is supposed to be projected from one particle to another, in a manner which is quite independent of a medium, and which, as Neumann has

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<sup>18</sup>Substantialism: The doctrine that behind phenomena there are substantial realities (Oxford dictionary). In Newton the space is the place of objects. Now it is taken further since it becomes the place of impending actions as well. See for example <https://plato.stanford.edu/entries/spacetime-theories/#5.2>

<sup>19</sup>Maxwell quotes from Gauss’ letter to Weber (bd.5 p. 627-629, Gauss, 1870) an expression of difficult translation: “construirbare Vorstellung” (possibly meaning “constructible representation”, or simply “a representation that can be carried out”).

himself pointed out, is extremely different from that of the propagation of light. In the theories of Riemann and Betti it would appear that the action is supposed to be propagated in a manner somewhat more similar to that of light. (art [866])

Apparently, Maxwell tries to regard C. Neumann's potential as a material substance, without success. As we have previously seen (Section 2.3), Maxwell's epistemic view blinds him when considering Faraday's ray theory which is precisely what Neumann's has built in formulae.

Riemann was closer to Lorenz in that he considered only necessary to introduce the wave-propagation<sup>20</sup> without resting on analogies or images of matter or the space. Remarkably, he offered some possibilities in which his hypothesis could be thought in material terms

This supposition can be fulfilled in various ways. Let us assume, for instance, that the conductors are crystalline in their smallest particles, so that the same relative distribution of the electricity is periodically repeated at definite distances which are infinitely small compared with the dimensions of the conductors

Unlike Lorenz', Riemann's theory did not incorporate light polarisation.

Clausius criticises Riemann in two points, the first of them being the same as his critic of Neumann. The other is a procedural issue. Riemann's calculations hold only if the delay is characterised by a relation such as eq.(3), but it is unclear what Riemann meant since he uses various notations to describe distances along the work. Betti's work attempted to provide a different framework for Riemann's ideas. Clausius' criticism of Betti's work focuses on general formal properties of series expansions, but he did not discuss if such criticism was relevant for the issue in question.

The work of Lorenz mentioned above also considers a delay compatible with eq.(3). Remarkably, his work appeared contiguously after Riemann's work in the same issue of the *Poggendorff Annalen* (1867)<sup>21</sup> but it was not criticised at that time, nor afterwards. It is surprising that neither Lorenz' nor other non canonical view about light propagation has ever been considered as an alternative.

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<sup>20</sup>To be fair, we should note that the proposal is earlier than Maxwell's, although it was published at a later time. In the Treatise, article [862], Maxwell dismissed Riemann's approach referring to Clausius' criticism.

<sup>21</sup>Riemann's manuscript was presented in 1858 but published in 1867 (after his death). Betti comments on this as "...supponendo che la corrente consista nel movimento delle due elettricità positiva e negativa che vanno contemporaneamente nel filo in direzioni opposte, [...] Questo concetto della corrente elettrica tutto ideale è poco in armonia con ciò che si conosce di essa, e pare che Riemann non ne fosse soddisfatto, avendo ritirato l'articolo dalla Segreteria dell'Accademia, ed essendosi astenuto dal pubblicarlo", namely that Riemann possibly was unsatisfied with his hypothesis about the nature of electric current. Clausius (referring to the alleged "error" he discussed) conveys his own version as "I believe that Riemann subsequently convinced himself of this error, and that this was the reason he withdrew his paper", thus ignoring both Lorenz' work (sustaining basically the same result than the criticised works) and the reasons offered by Betti (in the same paper that Clausius also criticises), friend and collaborator of Riemann in this matter.

The odd circumstance of three papers around the same idea and a weak attempt of refutation coming soon after, move us to consider the historic environment of these events. Carl Neumann was the son of Franz Neumann and had the greatest admiration for Riemann (Jungnickel and McCormach, 2017). In turn, Riemann became a friend of Betti during a visit of the latter to Göttingen. Later Riemann visited Betti in Italy. Riemann was an assistant to Weber during 18 months around 1849 and Weber is one of his recognised influences<sup>22</sup>. By the year 1866, Hannover and Prussia (“Göttingen and Berlin”) enter in war. Riemann fled Göttingen for Italy because of the war, and he died there the same year. The war lasted until 1868 when Hanover was annexed as a province of the new empire. Clausius was born in Prussia, studied in Berlin, taught physics at the Royal Artillery and Engineering School in Berlin and was Privatdozent in Berlin University. In 1870 he organised an ambulance corps during the Franco-Prussian war and was wounded in battle. It can be said that the battle of ideas was held under the emotional atmosphere of the battles for the reunification of Germany. By 1871 the German Empire was proclaimed.

### 3.2.6 Revisions

The activity on a relationist view of electromagnetism ceased by 1868 except for a reprint of C. Neumann’s work and the immediate reaction of Clausius. More than a half century later, Moore and Spencer (and collaborators) researched the subject in a series of articles (Moon and Spencer, 1954, 1956, 1960; Moon et al, 1989b,a, 1991, 1994) in which they considered the main experiments that support the currently accepted theory. Their 1960 book (Moon and Spencer, 1960) presents the systematic revision and comparison. The result of the revision is that, at the time (1960), there was no experimental evidence to prefer the Maxwell-Lorentz-Einstein views to the Delayed Action at Distance theory. In the same terms of Dingle (Dingle, 1960b) they find that the constancy of the speed of light remains a conjecture since the moment it was proposed by Einstein (p. 257, Moon and Spencer, 1960).

In front of possible criticism from an intuitive point of view they state: “But we can hardly expect intuitive ideas to hold for light. Light is not a wave in a medium and is not a particle: it is a unique phenomena unlike anything else in nature. To expect to visualise this unique phenomenon in terms of mechanistic pictures of water waves and bullets is indeed naive” (p.256).

In page 251 Moon and Spencer write an example of synchronisation of clocks using light assuming the Galilean formulation. They write: “Thus we have an operational method of establishing an universal time, subject of course to the assumptions that the space is euclidean and that ‘velocity of light’ is a meaningful expression. In the advent of space ships and the establishment of colonies in other planets, such a synchronisation of clocks would be highly desirable” they proceed then to consider the differences between reflected and re-emitted light. They also easily show that, within their “postulational basis”

<sup>22</sup>See <https://www-history.mcs.st-andrews.ac.uk/Biographies/Riemann.html>

for the discussion it is not possible to synchronise clocks in terms of special relativity.

It is then relevant to notice the so called “flyby anomaly” (of the Pioneer and other spacecrafts) (Bilbao et al, 2014; Bilbao, 2016), where two tracking systems were used: one with a re-emitting device and the other with a reflector. The systems disagree in values associated to the relativistic correction of time. These observations are anomalies for the standard electromagnetism but regularities (actually, predictions) for the relational electromagnetism. Certainly, there are other *a posteriori* explanations for the anomalies, but the *ad hoc* explanations cannot account for the fine detail as the relational view does. Needless to say, these articles have received little or no attention from the physics community.

### 3.3 Maxwell

In Maxwell, as well as in all other developers of EM, a basic assumption about the behaviour of electrodynamics systems is that the electrodynamic forces exerted on matter behave in the same way as the previously known mechanical forces. In particular, whatever external force that is required to keep a system at rest is taken to be equal and of opposite sign to the internal force generated by electrodynamic interactions (and that, if not compensated, would alter the static equilibrium). In this way, by looking at external forces only, it is possible to learn something about the electrodynamic system.

Today it is believed that “external“ forces such as mechanical resistance to deformations or frictional forces in the “contact“<sup>23</sup> between two bodies also have an electromagnetic origin, but this topic was less clear two centuries ago. In any case, it was expected that isolated electrodynamic systems should have some sort of conserved electrodynamic energy and electrodynamic forces should be associated to the time-variation of some “momentum“.

We may identify a series of abductions that increasingly organised the existent knowledge, starting from F. Neumann who realised that the force between closed circuits had an associated energy. Helmholtz and Thomson (Thomson, 1851) associated this energy to the mechanical energy (the ability of generating mechanical work). Indeed, Helmholtz states,

The entire electromotive force of the induced current, generated by a change of **position of a magnet relative to a closed conductor**, is equal to the change which thereby takes place in the potential of the magnet towards the conductor, when the latter is traversed by the current (p. 157, Helmholtz, 1853)(emphasis added)

In formulae, the electromotive force,  $\mathcal{E}$ , is

$$\mathcal{E} = -\frac{d}{dt} \left( \frac{\mu_0}{4\pi} I_1 \oint_1 \frac{1}{|x_1(l_1, t) - x_2(l_2, t)|} dl_1 \right) \quad (4)$$

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<sup>23</sup>These quoted words from the usual language hide assumptions that we are barely aware of. We speak freely of contact between bodies, we may detect a difference between contact and no contact, but still today it is not a trivial question to relate the detected difference to the atomic constituents of each body and their interactions.

This was the starting point that Maxwell adopted to develop his contributions to EM.

Despite the stories usually told in physics textbooks, Maxwell did not consider electricity as a fluid, except for illustration purposes. Following Faraday, Maxwell was certain that electricity (whatever it is) had the ability of moving when associated to conductors, but he restrained from further hypothesis ([552], Maxwell, 1873). Then he proceeded to conceptualise  $P$  (2) no longer as a potential energy but rather as the electrokinetic energy, assimilating the currents  $I_i$  with velocities. Then, by analogy with Lagrange's formulation of mechanics, he would write

$$A_i = \frac{\delta P}{\delta I_i}$$

The variation  $\delta I_i$  included the possibility of changing the locus of the cable (additionally, he formally extended the expressions to conductors not necessarily produced as cables). The quantity  $A_i$  is then the electrical momentum following the standard wording in Lagrange's mechanics. He then tried to determine experimentally whether the electrical momentum had an associated mechanical momentum as well, concluding that they were completely decoupled ([574-575], Maxwell, 1873). Thus, the energy (and then the Hamiltonian), corresponding to two pieces of matter is the addition of the energy of the ponderable matter and the electrokinetic energy ([571], Maxwell, 1873), being the electrokinetic energy independent of the motion of the ponderable matter. The electromotive force (4) is then the external force required to balance the internal force that is necessary to maintain the current ([576], Maxwell, 1873).

It is important to realise at this point that physical hypotheses play two different roles. Some of them, like the fluid hypothesis in Maxwell, are pedagogical devices directed to facilitate the perception and explanation of abducted relations. These kind of hypothesis can be changed without changing the abduction. For example, we can think of massless electrical particles instead of fluids, as Weber did, and nothing is changed in our expressions. Other kind of hypotheses force us to modify the equations involved in the theory. In the present situation, the decoupling of the kinetic energies (the electrokinetic one and the one arising from the movement of matter) results in that some energy terms have been considered to be identically zero<sup>24</sup>. Such a hypothesis cannot be dropped or modified without a complete reworking of the results. In particular, currents cannot be read as  $qv$ , being  $v$  a subjective velocity (a velocity that changes with our choice of reference frame), maintaining Maxwell's ideation.

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<sup>24</sup>When current is involved, Maxwell's belief is that "electricity" is moving, some way or the other. Following the Lagrangian formulation of mechanics, in [578] he calls  $\dot{y}$  the variables associated to currents, while  $y$  is the conjugated variable (other authors will relate  $y$  to the transport of net charges). Further, he realises that the mechanical variables  $x, \dot{x}$  corresponding to ordinary matter are not coupled to  $\dot{y}$  and that the energy associated to currents (the electrokinetic energy) is an homogeneous quadratic function of the currents. Further, in [632-8] the relationship between magnetic and electrokinetic energy is presented, thus closing the circle with C. Neumann's formulation, since electrokinetic energy relates to the vector potential  $A$  and the magnetic field. Hence, the whole electrodynamic energy of a material system is obtained as the sum of electrostatic and electrokinetic energy.

The second, and best known, contribution of Maxwell derives from his conviction that forces of action at distance were not really needed in order to describe EM phenomena ([59-60] [552], Maxwell, 1873). In practice, Maxwell never went that far, large portions of his work assume instantaneous action at a distance. Early in his treatise he indicates that one of his goals is to pursue the hypothesis "... that electric action is not a direct action between bodies at a distance, but is exerted by means of the medium between the bodies..." ([60], Maxwell, 1873).

Using Gauss formula, and the generalised expression corresponding to  $A$  (2) Maxwell produces

$$\begin{aligned} A(x, t) &= \frac{\mu_0}{4\pi} \int_V \left( \frac{j_1(x_1(y, t), t)}{|x_1(y, t) - x|} \right) \left[ \frac{\partial x_1}{\partial y} \right] d^3y \\ \Delta A(x, t) &= -\mu_0 j_1(x, t) \end{aligned} \quad (5)$$

here,  $x_1(y, t)$  stands for the location of a small volume,  $d^3y$ , of ponderable matter at time  $t$  with initial condition  $x_1(y, 0) = y$ , and  $\Delta$  is Laplace's operator<sup>25</sup>. Maxwell proposed that a current corresponding to the variation of the polarisation of the dielectric should be added to the equation, so that  $j_1 = j_1^g + j_1^d$  would be <sup>26</sup> the sum of the galvanic current,  $j_1^g$ , and a current arising from the variation of the induced polarisation. In the case of observer (detector) and source at relative rest,  $j_1^d = K\epsilon_0 \frac{\partial E}{\partial t}$ . Here  $K$  is a dimensionless constant characteristic of the dielectric, the specific inductive capacity, that takes the minimum value of  $K = 1$  in vacuum and it is also 1 (or slightly above) in air ([52], [60],[75] Maxwell, 1873). Further  $E = -\frac{\partial A}{\partial t} - \nabla V$  (also for the case of relative rest) is the the electromotive force ( $V$  stands for an undetermined potential function in (eq. B, [598], Maxwell, 1873), with  $v = 0$ ). It is interesting to notice that Maxwell focused his attention on the differential (derived) form  $\Delta A(x, t) = -\mu_0 (j_1(x, t) - \epsilon_0 K \frac{\partial}{\partial t} (\frac{\partial A}{\partial t} + \nabla V))$  which is not entirely equivalent to the integral form. The equivalence holds only if the (unstated) boundary conditions are properly satisfied. If we, alternatively, fix our attention on the integral form, we realise that for the convergence of the integral expression it is required that the current decays at infinity faster than  $\frac{1}{|x|^2}$ , a condition that  $\frac{\partial A}{\partial t}$  does not satisfy. This means that the ideation is slightly inconsistent, it is actually an improper form of proposing that the equation governing the vector potential is

$$\square A = -\mu_0 (j_1 + \epsilon_0 K \nabla V) \quad (6)$$

<sup>25</sup>Actually, Maxwell did not write the integral form. He moved intuitively from the expression in cables (wires) to the expression in three-dimensional matter arrangements. The expression of the electromagnetic momentum can be transformed by a coordinate change to

$$A(x, t) = \frac{\mu_0}{4\pi} \int_{V(t)} \frac{j_1(y, t)}{|y - x|} d^3y$$

we have preferred the formulae (5) because it displays more readily the transformation properties of  $A$ .

<sup>26</sup>In standard textbooks this is called the **Ampère-Maxwell law**. However, the contribution from  $j_1^g$  is not from Ampère, but rather from F. Neumann who inspired his developments in Ampère's force.

where  $\square = \Delta - (K\mu_0\epsilon_0)\frac{\partial^2}{\partial t^2}$  is the D'Alembert operator. Later, Maxwell would seek solutions to this equation.

### 3.3.1 Maxwell on reference systems

Maxwell's Treatise on Electromagnetism (Maxwell, 1873) is a complex work. In particular, the equations are not self-contained. The reader is expected to follow and understand the derivations. Quite often the notation is simplified in excess, resulting in expressions that do not apply to the general case. For example, dropping a gradient contribution in the electromotive force renders several final equations only apt for the use in the case of closed circuits. The handling of the space is not transparent either. Partial derivatives with respect to time occur and, unfortunately, they are indicated with the same symbol  $\frac{d}{dt}$  as the "total" (convective, following the body) derivative.

In Art. [600] Maxwell discusses the invariance in form of the electromotive force with respect to the choice of reference system. The problems of notation are particularly acute in [598-600]. The general setup is that  $A(x, t)$  is allowed to have a more general (and undefined) form than in eq.(2). The location of the secondary cable  $x_2(l, t)$  is allowed to change in time and so is the location and current of the primary, but since there is no explicit expression for  $A$  this will be reflected as the partial derivative in time  $\frac{\partial A(x, t)}{\partial t}$  that collects all the changes in time due to changes in the primary circuit. Maxwell centred his attention in how  $\frac{\partial A(x, t)}{\partial t}$  must change under a change of reference system (that could comprise rotations and translations with respect to the original system). To avoid complexities, and since this is sufficient for our discussion, we will address only the translational case (i.e., where the coordinate axes in a given inertial system are parallel to the corresponding axes in another system). Instead of stating Maxwell's result for  $\frac{\partial A(x, t)}{\partial t}$ , we collect all the hypotheses introduced in [600] and rewrite his result in terms of invariance of form.

**Maxwell's invariance theorem** Let  $A(x, t)$  be the electromagnetic momentum, in terms of the position  $x$  with respect to a coordinate system  $L$ , and  $\mathcal{E}$  the electromotive force (introduced in [597]). Maxwell computes in (eq.B, [598], Maxwell, 1873)

$$\mathcal{E} = v \times (\nabla \times A) - \frac{\partial A}{\partial t} - \nabla\psi \quad (7)$$

where  $v = \frac{dx}{dt}$  is the velocity of a point on the (moving) secondary circuit with respect to  $L$  and  $\psi(x, t)$  is an undetermined scalar function. Maxwell explains that "We shall find, however, that when we know all the circumstances of the problem, we can assign a definite value to  $\psi$ , and that it represents, according

to a certain definition, the electric potential..<sup>27</sup>

Consider coordinates  $x'$  on a system with motion given by  $\bar{x}(t)$  relative to a system with coordinates  $x$ , so that  $x' = x - \bar{x}$ <sup>28</sup>. Then, the following result holds:

**Theorem (Maxwell's invariance theorem):**

Let  $x = x' + \bar{x}(t)$ , and correspondingly  $v = v' + \dot{\bar{x}}$ . Define  $A'(x', t) \equiv A(x, t)$ <sup>29</sup>, then the value of the electromotive force at a point  $x$  does not depend on the choice of reference system if and only if  $\psi(x, t)$  transforms as  $\psi'(x', t) \equiv \psi(x' - \bar{x}, t) - \dot{\bar{x}} \cdot A'(x', t)$ . In formulae,  $\mathcal{E}'(x', t) = \mathcal{E}(x, t)$ , where

$$\mathcal{E}'(x', t) = v' \times (\nabla \times A'(x', t)) - \frac{\partial A'(x', t)}{\partial t} - \nabla \psi'(x', t).$$

**Proof:** First, according to the definition, we have  $A'(x', t) = A(x' + \bar{x}, t)$ .

Next, we note that by straightforward vector calculus identities, Maxwell's electromotive force (eq. 7) can be restated as

$$\mathcal{E}(x, t) = -\frac{\partial A(x, t)}{\partial t} - (v \cdot \nabla) A(x, t) - \nabla (\psi(x, t) - v \cdot A(x, t))$$

In the new coordinate system we compute:

$$\begin{aligned} \mathcal{E}'(x', t) &= v' \times (\nabla \times A'(x', t)) - \frac{\partial A'(x', t)}{\partial t} - \nabla \psi'(x', t) \\ &= -\frac{\partial A'}{\partial t}(x', t) - (v' \cdot \nabla) A' - \nabla (\psi'(x', t) - v' \cdot A'(x', t)). \end{aligned}$$

Subsequently, under the present assumption  $A'(x', t) \equiv A(x, t)$  we may rewrite

$$\frac{\partial A'(x', t)}{\partial t} = \frac{\partial A(x, t)}{\partial t} \Big|_{x'+\bar{x}} + (\dot{\bar{x}} \cdot \nabla) A$$

leading to

$$\begin{aligned} \mathcal{E}'(x', t) &= -\frac{\partial A(x, t)}{\partial t} - (\dot{\bar{x}} \cdot \nabla) A - (v' \cdot \nabla) A' - \nabla (\psi'(x', t) - v' \cdot A'(x', t)) \\ &= -\frac{\partial A(x, t)}{\partial t} - (v \cdot \nabla) A - \nabla (\psi'(x', t) - v' \cdot A'(x', t)) \\ &= -\frac{\partial A(x, t)}{\partial t} - (v \cdot \nabla) A - \nabla (\psi(x, t) - \dot{\bar{x}} \cdot A(x, t) - v' \cdot A'(x', t)) \\ &\quad - \frac{\partial A(x, t)}{\partial t} - (v \cdot \nabla) A - \nabla (\psi(x, t) - v \cdot A(x, t)) = \mathcal{E}(x, t) \end{aligned}$$

<sup>27</sup>In ([70], Maxwell, 1873)Maxwell defines: "The Potential at a Point is the work which would be done on a unit of positive electricity by the electric forces if it were placed at that point without disturbing the electric distribution, and carried from that point to an infinite distance"

<sup>28</sup>Maxwell writes "Let  $x', y', z'$  be the coordinates of a point referred to a system of rectangular axes moving in space, and let  $x, y, z$  be the coordinates of the same point referred to fixed axes", what makes evident he is considering absolute space, since "at rest" does not refer to any particular reference. However, the result is valid for any pair of systems where the hypotheses are fulfilled.

<sup>29</sup>Maxwell refers to this expression as: "the theory of the motion of a body of invariable form". For any property of matter, this relation is immediate.



where we used the condition  $\psi'(x', t) \equiv \psi(x, t) - \dot{\hat{x}} \cdot A(x, t)$ , to proceed from the second line to the third.  $\square$

Maxwell's Art. [601] states "It appears from this that the electromotive intensity is expressed by a formula of the same type, whether the motions of the conductors be referred to fixed axes or to axes moving in space, the only difference between the formulae being that in the case of moving axes the electric potential  $\psi$  must be changed into  $\psi + \psi'$ ." (here being  $\psi' = -A' \cdot \dot{\hat{x}}$ ).  $\psi$ 's numeric value depends on the choice of reference system. In the appendix to Chapter IX, J. J. Thomson –the curator of the third edition– indicates: "...It does not appear legitimate to assume that  $\psi$  in equations (B) represents the electrostatic potential when the conductors are moving, for in deducing those equations Maxwell leaves out a term  $-\frac{d}{ds}(A \cdot v)$  since it vanishes when integrated round a closed circuit...".

While Thomson's concern is reasonable and follows from the calculations on the Treatise in [598], Maxwell did not pursue the issue in depth, only suggesting (in [598] and [630]) that  $\psi$  is to be identified with  $V$ . We will see in the next Section that Lorentz adopts the same identification, thus leading to the standard formulae for the electromagnetic force found in textbooks. The exact nature of the potential  $\psi$  is an issue to be decided through experiments.

### 3.3.2 Electromagnetic Energy

The Treatise considers in Art. [85] the potential energy associated to building up a given arrangement of charges, relating it to the electrostatic potential. It returns to this topic in [630-31], restating it as properties belonging to the electric field. An important difference is that the computation in [85] was intended to be performed only in the regions of space occupied by charged matter. It can be formally extended to all space, assuming that the density of charge outside the (bounded) region of charged matter is zero. However, the computation in [630-31] must extend to all space, otherwise a compensating term on the boundary of the integration region is required. In formulae and using modern notation

$$\begin{aligned} \frac{1}{2} \int_U \rho(x) V(x) d^3x &= \frac{1}{2} \int_U \epsilon_0 (\nabla \cdot E) V d^3x \\ &= \frac{1}{2} \int_U \epsilon_0 (\nabla \cdot (EV) + |E|^2) d^3x \\ &= \frac{1}{2} \int_U \epsilon_0 |E|^2 d^3x + \frac{1}{2} \int_U \epsilon_0 \nabla \cdot (EV) d^3x \\ &= \frac{1}{2} \int_U \epsilon_0 |E|^2 d^3x + \frac{1}{2} \int_{\partial U} \epsilon_0 (EV) \cdot \hat{n} d^2s \end{aligned}$$

where  $U$  is the region of space where the charge density  $\rho$  is supported. We have used the relation  $\nabla \cdot E = \frac{\rho}{\epsilon_0}$ . Using Gauss' theorem, the last term in the third equation is restated as a surface integral over the boundary  $\partial U$  of  $U$ . Only when this surface integral is zero, we can identify the  $\rho V$  integral with the

$|E|^2$  integral. For bounded charge distributions the potential and field behave in such a way that the surface integral vanishes at infinity.

Maxwell extends in Art. [631] the validity of this result to any electric field. Thomson indicates in a footnote (v.2 p.271, Maxwell, 1873) that the deduction holds only in the electrostatic case. He adds that in the general case the energy should be considered as that contained in the polarisation of all the dielectric (ether included). In such a form it extends to all sources of polarisation.

Other forms of energy such as the magnetic energy are put by Maxwell in the same form, as integrals of the square of the fields over all space. A step that would have pleased Faraday.

## 4 The ether

At the very end of the Treatise, in the last Article, [866], and after discussing action at a distance very much following Clausius, Maxwell states:

But in all of these theories the question naturally occurs : –If something is transmitted from one particle to another at a distance, what is its condition after it has left the one particle and before it has reached the other? If this something is the potential energy of the two particles, as in Neumann’s theory, how are we to conceive this energy as existing in a point of space, coinciding neither with the one particle nor with the other? In fact, whenever energy is transmitted from one body to another in time, there must be a medium or substance in which the energy exists after it leaves one body and before it reaches the other, for energy, as Torricelli remarked, ‘ is a quintessence of so subtile a nature that it cannot be contained in any vessel except the inmost substance of material things.’ Hence all these theories lead to the conception of a medium in which the propagation takes place, and if we admit this medium as an hypothesis, I think it ought to occupy a prominent place in our investigations, and that we ought to endeavour to construct a mental representation of all the details of its action, and this has been my constant aim in this treatise.

Far from proving the existence of the ether, the paragraph establishes a promising program of study that rests on Maxwell’s substantialism, for energy must leave one body to reach the other as if it were matter, which indeed, appears to be almost the case if we are to believe that energy is contained in matter, this is, localised in the space occupied by matter. This conception of energy and matter is a far cry from Faraday’s leading vision, but it must be conceded to Maxwell that his intuition deserved to be studied.

The arguments in favour of the ether or against action at distance never went further than those by Maxwell. These arguments are the consequence of an axiom of the proposed ideation: we must conceive the unknown by means of

analogies with matter accessible to the intuition. Cf. with Kelvin's mechanical intuition (Thompson, 2011, p.235) discussed in Subsection 2.1.

In all, Maxwell's equations are supported by action at a distance (present in Faraday's induction law and Coulomb's electrostatic force), F. Neumann's energy of a system of currents (which also rests in action at a distance) and in the polarisation current, an ingredient that operates even in empty space, where there are no charge or current carriers to support it. It is only here that the ether enters in Maxwell's theory, quite early in the Treatise and basically because Maxwell could not conceive an alternative explanation. Even when Faraday opened to alternative possibilities, Maxwell felt forced to rectify him (Cf. Subsection 2.3). In the next Subsection we will see how Hertz tried to rectify what he regarded as faulty in Maxwell's conception.

#### 4.1 Hertz as a theoretician

While Hertz' most famous contributions come from his experiments, he wrote two papers where he exposed his views on Maxwell equations. In (pp- 19-21 Hertz, 1893) he explains his motivations (referring to Maxwell's theory) as follows:

(p. 19) "Casting now a glance backwards we see that by the experiments above sketched the propagation in time of a supposed action-at-a-distance is for the first time proved. This fact forms the philosophic result of the experiments and, indeed, in a certain sense the most important result. The proof includes a recognition of the fact that the electric forces; can disentangle themselves from material bodies, and can continue to subsist as conditions or changes in the state of space. The details of the experiments further prove that the particular manner in which the electric force is propagated exhibits the closest analogy with the propagation of light; indeed, that it corresponds almost completely to it. [...] Since the year 1861 science has been in possession of a theory which Maxwell constructed upon Faraday's views, and which we therefore call the Faraday-Maxwell theory. This theory affirms the possibility of the class of phenomena here discovered just as positively as the remaining electrical theories are compelled to deny it."

(p. 20) "I have not always felt quite certain myself of having grasped the physical significance of his statements. Hence it was not possible for me to be guided in my experiments directly by Maxwell's book. I have rather been guided by Helmholtz's work, as indeed may plainly be seen from the manner in which the experiments are set forth. But unfortunately, in the special limiting case of Helmholtz's theory which leads to Maxwell's equations, and to which the experiments pointed, the physical basis of Helmholtz's theory disappears, as indeed it always does, as soon as action-at-a-distance is disregarded. I therefore endeavoured to form for myself in a consistent

manner the necessary physical conceptions, starting from Maxwell's equations, but otherwise simplifying Maxwell's theory as far as possible by eliminating or simply leaving out of consideration those portions which could be dispensed with inasmuch as they could not affect any possible phenomena. This explains how the two theoretical papers (forming the conclusion of this collection) came to be written."

(p. 21) "To the question: 'What is Maxwell's theory?'. I know of no shorter or more definite answer than the following:— Maxwell's theory is Maxwell's system of equations. Every theory which leads to the same system of equations, and therefore comprises the same possible phenomena, I would consider as being a form or special case of Maxwell's theory;[...]"

Hertz' operation is of an epistemological character. Actually, his reading of Maxwell-Faraday contradicts Faraday's writing, for in Faraday there was a duality between action and the (naive) view of matter while in Hertz actions disentangle themselves from the bodies to meet the requirements of Hertz epistemology. Also, the last quoted sentence from p. 19 implies to ignore Lorenz' theory (Lorenz, 1867) (which is mentioned by Maxwell in ([805] Maxwell, 1873)).

Hertz confesses not to understand the physical significance of Maxwell's statements. This raises the question about Hertz' idea of understanding which is greatly influenced by Helmholtz (who was his Ph.D. advisor and life-long protector). Hertz needed to see "pictures" of the real based upon the elements he has learned to trust: "a mechanistic conception within which natural phenomena were to be explained by the action of mechanically moved matter" (Schiemann, 1998). This view represents an extraordinary departure from Faraday's philosophical attitude, since Faraday was willing to change his most basic building blocks if this were necessary, while in Hertz we see the opposite attitude: the persistence of beliefs.

The general idea that transpires from the quoted pages is that it is possible (p. 21) to separate the process of construction of a theory, what we have called the "rules of correspondence" used in the construction, from the theory's mathematical content. Instead, the theory has to be provided with what Hertz called interpretation, a prescription about how to apply the theory to experiments. Along this lines, Hertz offers four interpretations of Maxwell's equations, including Maxwell's, Helmholtz' and his own and preferred one, the most consistent with the idea of the ether. Indeed, he attempts to recast Maxwell's work in terms of his views about the ether.

Hertz' departure from the traditional scientific epistemology is illustrated in Figure 1. Let  $\Pi$  be a projection of the observable into a symbolic language (say mathematics),  $\phi$  the logical elaboration and  $\Gamma$  the interpretation. The left panel describes the traditional scientific construction, where it is required that  $\Pi \circ \Gamma = Id$  and  $\Gamma \circ \Pi = Id'$ , in other words that no distortions on both theory and observations is introduced by the interpretation  $\Gamma$  with respect to the correspondence  $\Pi$  applied in the construction. The thin red arrows show

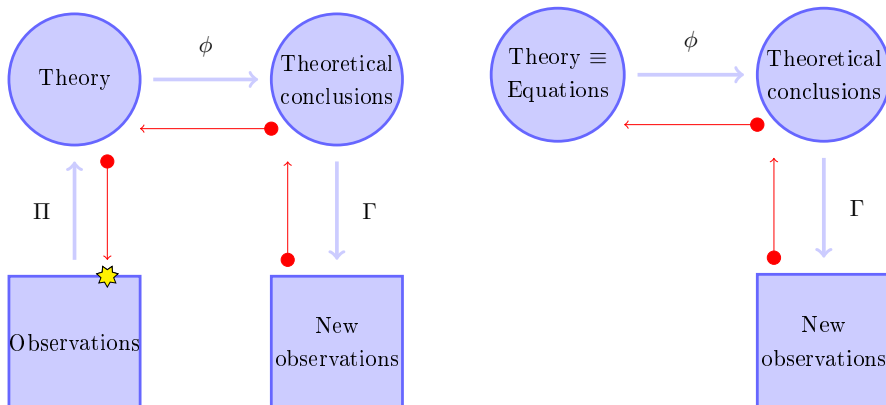


Figure 1: The epistemological change introduced by Hertz.

the flow of falsity (Lakatos, 1978a). In the traditional epistemology of science (say that of Faraday), inconsistencies between predictions and new observations flow back all the way through the diagram triggering an improvement in the construction: Falsity can force us to change our view of the world. The right panel describes Hertz' view, where the flux of falsity cannot affect the ideation since the ideation is suppressed. This conception represents a weakening of the conditions for a theory to be acceptable, allowing the scientist to change interpretations constrained only by their predictive success.

Hertz' deep epistemological change has been highlighted by D'Agostino: "...by separating the mathematical structure of a theory from its modes of representation he [Hertz] has profoundly challenged the conception of a physical theory as an indivisible unity of the two – a conception accepted by Maxwell and other nineteenth century mathematical physicists." (D'Agostino, 1968; D'Agostino, 2004).

Maxwell exposed his programme to address Faraday's ideas and experiments in the first few pages of (Maxwell, 1856), also reviewed in (D'Agostino, 1968). He advocates care in formulating models, so as not to fall in "...that blindness to facts and rashness in assumption which a partial explanation encourages." (p. 155-156, Maxwell, 1856). Further, Maxwell introduces the idea of physical analogies, (p. 156), similar to what we may call "working hypotheses", finally declaring his goal: "...to present the mathematical ideas to the mind in an embodied form" (p. 187), namely integrating the mathematics with its physical significance.

The discussion about the four interpretations of Maxwell's equations aims to retain the equations while removing their foundations. Action at a distance was regarded as an obstacle. Since Maxwell's and Helmholtz' views are supported by it, a new view was required. The equations (which were good enough since they led to the wave equation) had to be detached from the scaffolding used in their construction. Hertz believed that Maxwell started from mechanics and ended

with the ether (actually an oversimplification), and sought a fully ether-based interpretation instead.

For Hertz, action at a distance was not to be taken into account as a possibility and hence the potentials  $A, V$  were undesirable. Instead, he regarded the electric and magnetic fields ( $E, B$ ) as the state of the ether (p. 251), therefore deserving a central importance in physics. Moreover, in Hertz' interpretation Maxwell did not consider bodies in movement (p. 247) (despite any average reader of Maxwell's Treatise can verify that this statement is false) and proceeds to incorporate the movement under his famous hypothesis that the ponderable matter drags the ether (p. 242). Forcing his system to correspond with the traditional (Galilean) view, he incorporates terms representing this effect (eq. 3, p. 251). The equations obtained with the new interpretation are regarded as new, but they are actually identical to what is called equation (B) in (Maxwell, 1865, 1873), now under a new interpretation.<sup>30</sup>

## 4.2 H. A. Lorentz

The dutch physicist Hendrik Lorentz introduced a number of ideas that have propagated to the present day. In a work from 1895 (p. 2, Lorentz, 1895), he explains part of his scientific trajectory, that first developed along action at a distance for some time. He informs us that "This I have shown in a previous paper [13], in which I admittedly have derived the equations of motion from actions at a distance, and not, what I now consider to be much easier, from Maxwell's expressions." And later (p.3) he states "The influence that was suffered by a particle B due to the vicinity of a second one A, indeed depends on the motion of the latter, but not on its instantaneous motion. Much more relevant is the motion of A some time earlier, and the adopted law corresponds to the requirement for the theory of electrodynamics, that was presented by Gauss in 1845 in his known letter to Weber [18]"<sup>31</sup>.

The views of Lorentz constitute a progression. In 1892 Lorentz considered the advantages of Hertz approach:

M. Hertz ne s'occupe guère d'un rapprochement entre les actions électromagnétiques et les lois de la mécanique ordinaire. Il se contente d'une description succincte et claire, indépendante de toute idée préconçue sur ce qui se passe dans le champ électromagnétique.

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<sup>30</sup>It is remarkable that this mistake has persisted for centuries in standard textbooks. It might have originated in the fact that Heaviside was the first to write Maxwell's equations in their modern form (around 1885, (p. 429, Heaviside, 2011)), for the special case of bodies at rest. However, the most likely origin is Maxwell himself who wrote under the title "The propagation of electromagnetic disturbance": "Let us next determine the conditions of the propagation of an electromagnetic disturbance through a uniform medium, which we shall suppose to be at rest, that is, to have no motion except that which may be involved in electromagnetic disturbances."([783] Maxwell, 1873). Is Maxwell thinking in the possibility of motion with respect to absolute space? He is certainly considering a non moving ether, where the reference for the movement remained in his brain.

<sup>31</sup>Lorentz' reference [18] corresponds to (bd.5 p. 627-629, Gauss, 1870).

Inutile de dire que cette méthode a ses avantages.(p. 368, Lorentz, 1892)

In this work he considers both the case of the ether being dragged by the ponderable bodies (Chapter II) and the case in which the bodies move in the ether without dragging it (Chapter IV). He also considers the hypothesis of the electric fluid (§31). Following Hertz and Fizeau, Lorentz assumes (p. 1, Lorentz, 1895):

...that ponderable matter is absolutely permeable, namely that at the location of an atom, also the aether exists at the same time, which would be understandable if we were allowed to see the atoms as local modifications of the aether.

However, Lorentz conceives an ether at “rest“. He later adds:

That we cannot speak about an absolute rest of the aether, is self-evident; this expression would not even make sense. When I say for the sake of brevity, that the aether would be at rest, then this only means that one part of this medium does not move against the other one and that all perceptible motions are relative motions of the celestial bodies in relation to the aether.(p. 2, Lorentz, 1895)

We have to observe that the notion of “perceptible motion“ is not reconcilable with an imperceptible ether. The struggle to conciliate the concept of space with electromagnetism continues in Lorentz. It has been asserted that Lorentz’ ether is some form of (absolute) space (see e.g., (p. 172, Ritz, 1908)). Lorentz’ ether hypothesis contradicts that of Hertz (and also that of Stokes (Stokes, 1845)) in which the ether was dragged by ponderable bodies. Indeed, in (Lorentz, 1887) Lorentz criticises different alternatives of ether drag.

While he admits that the Michelson and Morley experiment (Michelson and Morley, 1887) (failing to detect a velocity of the earth relative to the ether) represents an objection to his view, he proceeds to reformulate his view incorporating the now famous contraction in the direction of movement (§89–§92, Lorentz, 1895). Lorentz attitude corresponds to the “persistence of belief“ in the terms of Peirce: The existence of the ether is never questioned, the propagation of light cannot be conceived without an emission theory and a propagating medium. In this respect, Lorentz retains the view of Maxwell and Hertz and in so doing he opposes Faraday’s view of vibrating rays. However, Lorentz’ ether cannot be reconciled with an ether that is dragged by matter. In this issue concerning the ether “at rest“ or not at rest, Lorentz remains close to Maxwell’s Art. [783] of the Treatise, while Hertz advanced a different view.

In his version of Maxwell’s theory Lorentz leaves behind Maxwell’s practice of the *epojé*. Thus, where Maxwell sustained the doubts resulting from several undecidable possibilities, Lorentz adopts the hypotheses.

The writing of Lorentz presents an evolution towards an authoritative form in contrast with that of Ampère, Faraday, Maxwell and Hertz, to mention just a few. Lorentz late writing is much closer to current textbooks where a doctrine

is being transmitted without providing clues on how the author has established her/his results.<sup>32</sup>

#### 4.2.1 The current

As a central issue in his career, (Lorentz, 1892, 1895, 1899), Lorentz adopted the early view of electrical particles. It is remarkable that this idea of Gauss relates to the ether-free formulation and is central to, for example, Weber's theory (Weber, 1846), considered in Section 3.2. In paragraphs §75 to §80 in (Lorentz, 1892), he considers the force on a ponderable body that moves through the ether with velocity  $v$  relative to it (for the precedents and history of the force see (p. 143, Assis, 1994)). In §75 the total current is introduced,

$$J = j + \frac{\partial D}{\partial t} \quad (8)$$

(in modern notation), being  $\frac{\partial D}{\partial t}$  Maxwell's displacement current (in vacuum  $D = \epsilon_0 E$ ). Lorentz proposes  $j = \rho v$ , this is the product of the density of charge times the velocity relative to the ether, an expression present in the current theory of electromagnetism and now denominated Lorentz' current. He then proceeds to find the force following Maxwell's idea regarding the variation of the energy with respect to a virtual displacement of the particle. Because of the relevance of the expression of the Lorentz' force we will address its derivation in a dedicated subsection.

The Lorentz current is introduced in all textbooks of physics that we know about. However, it deserves to be questioned from an experimental point of view. A bounded density of charge implies that charges can be obtained in infinitesimally small amounts (just by integrating this density in appropriate small domains), this is, in an amount smaller than any given one, for example, smaller than the charge of the electron. Nevertheless, the work by Millikan (Millikan, 1913) persuaded physicists that the charge of an electron is the minimum value for electric charge, the quantum of electricity. If the charge density must integrate in all domains to a multiple of the electron charge, it cannot be continuous. Following this line of thought to arbitrarily small distances, it follows that charges and the electron in particular must be point-like objects and that charge density is unbounded. That charged particles are not point-like is an experimental observation that is in itself inconsistent with the assumptions made by Lorentz in his deduction, but furthermore, Lorentz' construction is inconsistent with our understanding of conductors and electron states in conductors after quantum theory. As Einstein perceived (Einstein, 1940), present field theories and quantum theory are not compatible, the efforts made to forcibly put them together added insurmountable difficulties that can only be dealt with by destroying the mathematical building (Natiello and Solari., 2015) on which they rest.

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<sup>32</sup>Actually, Maxwell would disagree with us, since this is Maxwell's criticism of Ampère([528], Maxwell, 1873), but in comparison to Lorentz, Ampère is transparent.



### 4.2.2 Lorentz' force

The changes introduced by Hertz, Lorentz and others to Maxwell equations<sup>33</sup> (despite still naming the modified equations after Maxwell) make Maxwell's (modified) equations unsuited to describe cases of induction that appear in the foundational experimental background of the theory. Experiments such as Arago's disk ([81], Faraday, 1839) or the rotating magnet of Faraday ([217-], Faraday, 1839; Assis, 1994; Munley, 2004) cannot be understood using Maxwell's (modified) equations. This matter constitutes today the so-called "exceptions to the flux rule" (referring to the Faraday-Maxwell induction law) (Ch. 17-2, Vol. ii, Feynman et al, 1965). We are instructed then that "the correct physics is always given by the two basic laws ..." being the first of the two "basic laws" the expression of Lorentz force. The resolution of this misunderstanding has been given in (Munley, 2004). Using Faraday-Maxwell's flux law and following Faraday's and Maxwell's original insight, rather than the modified view proposed by Lorentz to accommodate for the ether, Munley accounts for the rotating magnet and shows how the discrepancy was built when the signification of the original flux law was altered to its present status.

The force experienced by a charged point-like particle by the influence of the electric and magnetic fields still plays today a central role in electrodynamics. However, its origin is seldom discussed (cf., e.g., (Jackson, 1962), where the Lorentz' force is mentioned on p. 3, 260, 553, etc. (3rd. edition), but it is never derived from basic principles). It comes as a piece of received knowledge without a sound insertion in the theory.

The force is presented in (§74-§80 (Ch. IV), Lorentz, 1892) under the following assumptions:

- The state of the ether is defined by the associated fields created by matter. There is a distinguished body on which the force will be computed. The body is a rigid solid. The charge density is nonzero only at the location of each body and it adds up to a smooth function  $\rho$ , with its associated Lorentz current in the case of the moving body. No other current in the distinguished body is considered. The ether acts through the fields  $E$  and  $B$ , satisfying Maxwell's equations.
- It is assumed that each point that takes part in the electromagnetic movements is known and determined by our cognition of all the particles of the system and the electric field in all the points of the space (§75 f. (Ch. IV), Lorentz, 1892).
- The force with which the ether acts on the distinguished body (with charge density  $\rho_0$ ) within this electrically active matter is computed by an appli-

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<sup>33</sup>The modification consists in accepting Maxwell equations in the cases of bodies without relative motion and then proposing different extensions based upon different ideas regarding the ether for the case of bodies in motion (no longer relative motion, but with respect to the ether). Motion is then considered with respect to the ether although no experimental basis exists for it. All the motions considered in Faraday's experiments and Maxwell formulae are relative motions.

cation of the variational method (with roots in Lagrange and D'Alembert), inspired in Maxwell. A **virtual displacement** with respect to the ether is proposed for the particle, leaving the rest of the matter without modification (hence, the displacement can be thought of both as relative to the ether as well as relative to the rest of the bodies). The variation is not completely calculated mathematically (despite it being possible). Rather, some steps are circumvented with arguments pertaining to the ether, as e.g., the restricted variation in §78 and its subsequent effects in §80 b. The final result reads

$$F_L = \int d^3x \rho_0 (E + v \times B)$$

There are a number of epistemological issues around this result. First, Lorentz departs from Maxwell in the proposal of a current. While Maxwell leaves as an undecided question the fundamental nature of the current, Lorentz postulates that the only possible current for a charged particle is given by its velocity relative to the ether at rest. Second, Lagrange's method rests in allowing arbitrary variations compatible with the boundary conditions. The peculiar variation proposed by Lorentz, automatically restricts its validity. The force can be such only in a situation where the proposed variation would be the most general possible. Indeed, the mechanical force computed by Maxwell (eq. C, [619], Maxwell, 1873) is more general than Lorentz' expression. But perhaps the more striking innovation is to put an end to the separation between guessing and conjectures and mathematics. In Newton, Maxwell and all the precedent physics, there is a time for synthetic propositions, usually inspired in experimental observations, while the theory develops further through analytic propositions performed with the correctness and consistency of mathematics. Not only Lorentz engages in interpretations as Hertz but he goes beyond them by substituting mathematical steps with his imagination on the behaviour of the ether, see (§80, Lorentz, 1892)

Lorentz Lagrangian reads

$$\begin{aligned} \mathcal{L} &= \frac{1}{2} \int \left( \frac{1}{\mu_0} |B|^2 - \epsilon_0 |E|^2 \right) d^3x \\ &= \frac{1}{2} \int [A \cdot J] d^3x - \frac{1}{2} \int \rho V d^3x \end{aligned}$$

where  $(A, V)$  are the potentials,  $(j, \rho)$  the measurable current and charge densities,  $J$  as in eq.(8), while  $(E, B)$  stand for the electric and magnetic fields. The integral extends over the region occupied by the system. These quantities are

related by Maxwell's modified equations

$$\begin{aligned}
\nabla \cdot B &= 0 \\
\nabla \cdot E &= \frac{\rho}{\epsilon_0} \\
\nabla \times B &= \mu_0 \left( j + \frac{\partial E}{\partial t} \right) \\
\nabla \times E &= -\frac{\partial B}{\partial t} \\
\nabla \times A &= B \\
-\frac{\partial A}{\partial t} - \nabla V &= E
\end{aligned} \tag{9}$$

The fourth and sixth equations differ from Maxwell's original formulation in that the present version holds only for stationary problems and rigid circuits. In this way, the equations can no longer describe Arago's problem and the like (see above). From the expression of the electromagnetic Lagrangian an action,  $\mathcal{A}$  can be formally written as

$$\begin{aligned}
\mathcal{A} &= \int dt \mathcal{L} = \frac{1}{2} \int dt \left[ \int \left( \frac{1}{\mu_0} |B|^2 - \epsilon_0 |E|^2 \right) d^3x \right] \\
&= \frac{1}{2} \int dt \left[ \int d^3x [A \cdot J] - \int d^3x \rho V \right]
\end{aligned}$$

where in all cases the limits of integration have been absorbed in the support of the integrands. We defer to Appendix B a more technical derivation of the electromagnetic force from the Lagrangian stated above, rendering explicit the restrictions that lead to the Lorentz' force. In short, Lorentz proposal leads to

$$F_L = \int d^3x [(\rho v) \times B + \rho E]$$

if we consider the material current to be

$$j = \rho v$$

where  $v$  is a velocity relative to the ether and we use by habit the Lagrangian formulation of mechanics.

In Lorentz' original context the Lagrangian is not invariant under changes of reference systems and the velocity involved is the velocity with respect to the frame of reference or the ether. We can derive by Lorentz' method the force exerted by the ether on all the system as

$$F_L = \int d^3x [(\rho v) \times B + \rho E]$$

and for the force exerted by the ether on the particle 0

$$F_L^0 = \int d^3x [(\rho_0 v_0) \times B + \rho_0 E]$$

Thus, in Lorentz, the ether (or the space, after suppressing the material ether) exerts a force on every electric body which adds up to some non necessarily zero amount. It is also well known that Lorentz' force does not conform the action-reaction principle (II 26-2, Feynman et al, 1965)(Ch. 6, Assis, 1994).

Lorentz will later (eqs. Ia-Va, Lorentz, 1899) summarise electrodynamics with five equations, namely the four Maxwell's equations described from a reference system fixed to the ether at rest plus Lorentz' force. Basically, this idea rests on the assumption that the electromagnetic fields are given: they are properties of the ether that suffer no influence from the particle subject to  $F_L$ .

The action above, expressed as  $\mathcal{A} = \frac{1}{2} \int dt [\int d^3x [A \cdot J] - \int d^3x \rho V]$  has an interesting symmetry related to the now famous Lorentz transformations. Let  $X = (a, b)$  (with  $a \in R^3$  and  $b \in R$ ) be a four vector such as  $(A, \frac{V}{C^2})$  or  $(j, \rho)$  or  $(x, t)$ , where  $C$  is Weber's constant (later known as the "speed of light"). Further, let  $T$  be the transformation

$$T_u X = \left( \gamma(a - uCb) + (1 - \gamma)\hat{u} \times (a \times \hat{u}), \gamma(b - \frac{u \cdot a}{C}) \right) \quad (10)$$

where  $u$  is a dimensionless parameter with  $|u| < 1$ ,  $\gamma = \frac{1}{\sqrt{1 - u^2}}$  and  $\hat{u} = \frac{u}{|u|}$  a unit vector. The Lorentz transformation  $TL_u$  of a four vector valued function  $F$  of the four vector space-time,  $X = (x, t)$ , reads

$$\begin{aligned} TL_u F &= T_u F(\xi, \tau) \\ (\xi, \tau) &= T_{-u} X \end{aligned}$$

and leaves invariant the action  $\mathcal{A}$ . Hence, it represents a symmetry of the action. The discussion of this symmetry and its possible meaning corresponds to the contribution of Einstein. Further, notice that if  $j$  would represent the current associated to a particle at rest in some reference frame, (which is zero in Lorentz' conception) its representation in another frame as given by a Lorentz transformation, would be

$$\begin{aligned} TL_u(j, \rho) &= T_u(j, \rho)(\xi, \tau) \\ (\xi, \tau) &= T_{-u} X \end{aligned}$$

which disagrees with the Lorentz current (there is a factor  $\gamma$  in excess). In other words, the construction is inconsistent with special relativity and only acceptable under the new epistemology that has no place for constructive consistence.

### 4.3 Mathematical picture of DAD

Having reached the point of experimental refutation of the idea of the ether, we feel obliged to collapse what originates from this idea. Thus, from the competing theories of propagation, ether and DAD, we must turn to the latter again and see whether Lorentz' current and force belong to it. Especially in the case of the force since it was "derived" assuming at the same time displacements

with respect to the complement of the probe in the universe and with respect to the ether. The work is imminently mathematical and is presented in Appendix B. We show in the Appendix that the following elements fit harmoniously in unity: Faraday’s insight regarding field interactions, Gauss’ insight on retarded action, Maxwell’s Lagrangian approach, equations and transformation, Lorentz’s delays, C. Neumann’s minimal action approach, Lorentz’ Lagrangian and force, the principle of action and reaction and Newton’s equations. But above all, in the epistemological side, Newton’s approach and motto: *hypotheses non fingo* appears as superior, for the production of a theory than Helmholtz-Hertz pictorial method which needs the support of physical hypotheses.

#### 4.4 Einstein contribution and his view of the ether

Einstein’s 1905 work (Einstein, 1905) represents the final metamorphosis of the electromagnetism initiated by Faraday and Maxwell. As Dingle (Dingle, 1960b) observes, (accepted) electromagnetism was not revised later and continues to be the same since then. Einstein’s equations for electromagnetism are directly taken from Lorentz (Lorentz, 1904) although the velocity participating in the equations will change its meaning to “velocity with respect to a frame of reference“. In so doing he adheres to the epistemological view of Hertz: to keep the equations while producing a new interpretation. The motivation for a new interpretation is clearly written as:

It is known that Maxwell’s electrodynamics—as usually understood at the present time—when applied to moving bodies, leads to asymmetries which do not appear to be inherent in the phenomena. Take, for example, the reciprocal electrodynamic action of a magnet and a conductor. The observable phenomenon here depends only on the relative motion of the conductor and the magnet, whereas the customary view draws a sharp distinction between the two cases in which either the one or the other of these bodies is in motion. For if the magnet is in motion and the conductor at rest, there arises in the neighbourhood of the magnet an electric field with a certain definite energy, producing a current at the places where parts of the conductor are situated. But if the magnet is stationary and the conductor in motion, no electric field arises in the neighbourhood of the magnet. In the conductor, however, we find an electromotive force, to which in itself there is no corresponding energy, but which gives rise—assuming equality of relative motion in the two cases discussed—to electric currents of the same path and intensity as those produced by the electric forces in the former case.

which is the opening paragraph of the 1905 work<sup>34</sup>. In the second paragraph, after rejecting absolute motion and the idea of the ether as reference, he will

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<sup>34</sup>Einstein’s discussion of the symmetry is not clear. First, we cannot know what it is meant by “electrodynamics—as usually understood at the present time—“, but most likely it refers to the electrodynamics as described by Lorentz. Certainly, there is no asymmetry in Faraday’s

introduce “The principle of relativity“ (without mentioning Poincaré), a conjecture that is raised to the level of principle. He also introduces the second (explicit) postulate of the theory: “that light is always propagated in empty space with a definite velocity  $c$ <sup>35</sup> which is independent of the state of motion of the emitting body“.

As it frequently occurs in physics, Einstein’s explicit postulates have to be complemented with implicit postulates, that are taken for granted without even mentioning them. Among them we count in this case “there exists something real we call space“, “there is something real that we call relative velocity“ –this one in particular after the opening paragraph– and “light is something emitted“ (and in this respect it is body-like). These three postulates are by 1905 hidden in the habits and other forms of irrational acting of scientists. We have already explained (Solari and Natiello, 2018) that space emerges from the construction of the child and it is an auxiliary concept in mechanics. Much later, Einstein will become more explicit about his view of the ether

When we speak here of aether, we are, of course, not referring to the corporeal aether of mechanical wave-theory that underlies Newtonian mechanics, whose individual points each have a velocity assigned to them. This theoretical construct has, in my opinion, been superseded by the special theory of relativity. Rather the discussion concerns, much more generally, those things thought of as physically real which, besides ponderable matter consisting of electrical elementary particles, play a role in the causal nexus of physics. Instead of ‘aether’, one could equally well speak of ‘the physical qualities of space’. Now, it might be claimed that this concept covers all objects of physics, for according to consistent field theory, even ponderable matter, or its constituent elementary particles, are to be understood as fields of some kind or particular ‘states of space’.  
(Einstein, 1924)

Rather than description tools, the fields become entities of “real” existence, particular ‘states of space’.

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experimental work and in Maxwell’s original formulation. The movement they considered is relative motion and the results were checked experimentally. However, a description of the situation using subjective coordinates with reference to either the magnet or the conductor and breaking down the interaction by dissociating the source from the detector (despite the fact that there can be no induction without both conductor and magnet) may have inspired the expectation by Einstein of a “field arising [or not, depending on the choice of subjective reference] in the neighbourhood of the magnet“ although there is no reason for it since the terms of the relation are not equivalent. Einstein will not come back to this matter in the paper but only to the issue of relative motion. In Appendix B we show that the sort of symmetry of form Einstein was seeking is present in the DAD, i.e., in electromagnetism after suppressing the ether. Further, also the assertion “there is no corresponding energy“ cannot make sense outside the subjective description since the phenomenon is rooted in the relative motion and the energy involved in the electromotive force and induced current is always there, regardless of the observer and his/her viewpoint.

<sup>35</sup>In this work we use  $C$  throughout for the associated electromagnetic quantity, i.e.,  $C^2 = (\mu_0\epsilon_0)^{-1}$ .

This is a remarkable paragraph since it goes back to Hertz' idea of electromagnetic fields as the "state of the ether", just changing the word ether by space. Actually, despite the usual claims about Special Relativity –for example (p.354, Torretti, 2007)–, only the materiality of the ether has been suppressed, Poincaré's vision of the ether, already presented (Section 2.4), lives in this paragraph. It is the fundamental need, introduced by the new epistemology, of sustaining a mechanistic (material) view of actions what leads to consider the space as an immaterial ether.

Einstein postulates that the Lorentz transformations (named in honour of Lorentz but introduced as such by Einstein), which leave the electromagnetic action invariant (and correspondingly, are associated to equivariant transformations of Maxwell equations), correspond to changes of reference frames in relative motion. Since neither Einstein nor Poincaré discussed the fundamentals of the Relativity principle (there is no critical motion towards the fundamentals, the principle is a conjecture, an intuition in them) there are no restrictions imposed to the transformations. Einstein will then postulate the Lorentz transformation as a proper substitute for the Galileo transformations.

Let us now study Einstein postulates from the point of view of the No Arbitrariness Principle.

#### 4.4.1 A metaphysical symmetry

We shall first make explicit the equivariance (too often called covariance) in Maxwell's equation. To simplify the exposition we will introduce the vector and scalar potential in the Lorentz gauge

$$\begin{aligned} E &= -\frac{\partial A}{\partial t} - \nabla V \\ B &= \nabla \times A \\ \square(A, \frac{V}{C^2}) &= -\mu_0(j, \rho) \end{aligned}$$

Where  $\square$  is D'Alembert operator, defined in eq.(6). The third equation establishes a relation between two four-vectors and since

$$TL_v \circ \square = \square \circ TL_v$$

the four-vector  $(A, \frac{V}{C^2})$  changes under the Lorentz transformation in the same form than the four vector  $(j, \rho)$ , hence  $(A, \frac{V}{C^2})$  and  $(j, \rho)$  are **equivariant** under a Lorentz transformation<sup>36</sup>. This is the sort of invariance of form sought by Einstein. We shall recall that a change of reference frame corresponds to a frame

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<sup>36</sup>We notice that the wave equation is an incomplete statement of a Physical law. To be complete, a law expressed by differential equations needs (generalised) boundary conditions to be expressed. Then, the equivariance requires the differential equation to be used with boundary conditions that preserve the form under the change of coordinates proposed. This matter is completely de-emphasised in physics textbooks.

that moves relative to the original one. Let us now suppose there is a neutral current (no accumulation of static charges is present) going through a cable, and the cable is not moving in the frame where we have measured the current (let's say that the galvanometer is part of the circuit, as it usually is). Consider now a second observer in motion with respect to the galvanometer and the circuit, with some velocity  $v$ . According to Einstein's view, the charge and current corresponding to the second observer can be obtained by transforming the original four vector  $(j, 0)$  with a Lorentz transformation. However, no method of measurement has ever been offered for moving observers (independent of the fields). How do we measure a current while flying by? We are forced to conclude at this time that the charge and current so obtained are metaphysical. The symmetry does not exist except in our construction. They are only tautologies obtained through formulae in the theory that cannot be physically verified. On the contrary, DAD is not metaphysical, it relates measurable quantities in two systems (source and detector, even if in relative motion) among each other.

#### 4.4.2 Collapsing Einstein's Special Relativity theory.

In section 2 we have discussed the collapse of a reasoning that is produced when a contradiction is found. The idea is present in the form of *reductio ad absurdum* in mathematics. As we have seen, some of the postulates of Einstein are not explicit. We will focus our attention in the concept of "relative velocity" which is mentioned on intuitive grounds at several places of the 1905 work. The concept makes sense in relational mechanics as a result of the invariance of relative position and time differences (Solari and Natiello, 2018), however, if we change the structure of space-time, we open for the possibility of destroying the concept. The real character of the relative velocity is expressed in mathematical terms by its invariance with respect to reference frames. As Peirce would have put it: the real does not depend on opinions. This is to say, that there should be an operation " $\ominus$ " that produces the relative velocity  $v_{ab}$  from the observed velocities of the two members (say  $v_a^s$  and  $v_b^s$ ). This is

$$v_{ab} = v_a^s \ominus v_b^s$$

where the superscript  $s$  runs over the equivalent observers. This is, the relative velocity,  $v_{ab}$ , is invariant under changes of observers (reference frames). It corresponds to the foundation of the concept of group of transformations that all those operations that leave an object invariant (be it equation, figure or any other) form a group.

The concept of relative velocity is assumed and discussed as a part of Special Relativity. However, the Lorentz transformations do not form a group and the velocity addition by Einstein does not form a group either.<sup>37</sup> Thus, the theory collapses since there is no conceivable invariant quantity  $v_{ab}$  in it (thus

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<sup>37</sup>A quick demonstration is as follows. The generators in the Lie algebra of the Lorentz transformation do not close an algebra but rather have Lie-products in the algebra of rotations. Equivalently, the Lorentz transformations correspond with the cosets in the Poincaré-Lorentz,  $SO(3, 1)$ , group where the set of rotations,  $SO(3)$ , is a subgroup but not a normal subgroup.



contradicting one of the implicit assumptions) but there are at most opinions about it depending on the choice of reference frame. We offer more mathematical details in Appendix C. The problem with special relativity is then that it does not speak about reality (as otherwise understood) since it fails to satisfy the first postulate of logical realism: there exists a reality independent of the observers.

Many objections to Special Relativity have appeared throughout the decades, all of them meeting the same compact rejection despite being intrinsically different. In 1974 Thomas E Phipps published a manuscript discussing whether the Lorentz contraction had any real existence (Phipps, 1974). The answer was negative, after setting up a simple measurement device consisting of a rotating a disc with radial scratches. The relevant issue is that about 70 years had been necessary to set up an experiment to refute (or not) a basic prediction of the theory in the standard terms of Natural Science theories. On the contrary, many article pages and textbooks had been written in the mean time discussing the consequences of Lorentz contractions from the point of view of “thought experiments”.<sup>38</sup>

## 5 Discussion

In physics, the XIX century was signed by the quest of understanding problems such as: What is light and how does it propagate? How does light propagation relate to the propagation of electricity in a conductor?

As stated before, Maxwell admits in the Treatise that he cannot conceive light propagation outside the emission theory, with a source that emits light, a medium on which light propagates and eventually a detector. Moreover, the emitted light should behave like a wave. Kirchhoff (Kirchhoff, 1857) had already advanced a wave-like model for electricity propagation in wires. Unfortunately, F. Neumann’s energy suggests a form for the vector potential  $A$  that cannot host waves. Starting from the persuasion that light is an electromagnetic phenomenon as suggested by the experiments of Faraday, Lorenz and others and from the abundant experimental evidence of light as a wave phenomenon, the quest becomes how to incorporate it to the electromagnetic phenomena evidenced by experiments with circuits and magnets. This is the point of divergence. For Gauss, Lorenz and later C. Neumann and others a delay in the propagation of electromagnetic disturbances is what is needed and, especially in Lorenz, no other justification needs to be added. In other terms, this is inference or abduction in its pure form. The reformulation is possible because it does not contradict experiments and achieves a higher level of conceptual unity.

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Therefore, there is no “quotient group” for  $SO(3,1)$  relative to  $SO(3)$ , but just a quotient manifold. The addition in the coset manifold corresponds to Einstein’s velocity addition and therefore it does not form a group. The structure of the Poincaré group in relation with its subgroups and cosets is well presented in (Gilmore, 1974).

<sup>38</sup>Thought experiments cannot challenge beliefs. At most, they can check consistency issues between conflicting beliefs. As such, they can be used as a method for indoctrination, to let go “false” beliefs retaining the “proper” ones.

Pure reason appears not to be sufficiently convincing for others, including Maxwell. Maxwell introduces an equivalent term in a differential equation sustaining his view of the (possible) existence of the ether, a matter worth to investigate. Thus, Maxwell introduces the displacement current. The next step in this quest is here represented by Hertz, who advanced in the physical properties of the ether and adjusted Maxwell's equations to them. If Maxwell's equations and the ether were weakly linked in the original, now they became a unity. Further, Hertz opens for the possibility of separating the equations from the ideas that originated them. The next turn by Lorentz was to conceive an ether at rest, adapting likewise Maxwell's equations to the situation.

Along this process, the explanatory power of Maxwell's theory (restricted to the four equations that are in use still today) was reduced to particular cases, leaving out important foundational experiments. Electromagnetic systems such as Arago's disk and Faraday's unipolar generator where the intervening parts are in relative motion were instead addressed with the introduction of the Lorentz' force. It is interesting to consider the introduction of Lorentz' force in this context. In Maxwell, the electromotive force *e.m.f.* is the force that moves electricity (be it fluid, particles or any other thing) and reads  $e.m.f. \equiv \phi = v \times B - \nabla\psi - \frac{\partial A}{\partial t}$  (eq. B, [619], Maxwell, 1873), where  $\psi$  is undetermined. Lorentz' force is then Maxwell's electromotive force exerted over a free-to-move charge. We get the following correspondence of symbols

$$E = -\frac{\partial A}{\partial t} - \nabla\psi$$

with the additional differences that Lorentz interprets the symbol  $v$  (originally a relational velocity) as the velocity with respect to the ether and  $\psi$  is identified with  $V$ .

Taking the rotor in the last equation we get the "Maxwell equation" (not present in Maxwell's Treatise)

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

In this way, Faraday's induction law has been split into two parts and the same can be said for the electromotive force. The operation reveals why the "new" Faraday law (the equation above) no longer explains some experiments by Faraday, that now require the Lorentz' force. Thus, Lorentz goes in the reverse direction than Maxwell: while the former seeks unity the later separates equivalent electrical effects. To confuse things further, names and symbols are kept but their mathematical meaning is changed.

At last, the contradictions and refutations related to the ether being dragged or not with the movement of the earth became insurmountable. It is clear in Lorentz that the ether is just an instrument, the relevant dynamical quantities are the fields, expressing the state of the ether. It was a small step then to conceive the fields as properties of space, thus getting rid of the ether in the wording of the theory, while in practice Lorentz' formulation is used still today,

with the additions of Einstein's special relativity. Before Einstein the electromagnetic theory of light was untenable in view of its internal conflicts. After Einstein the problem shifted to a more subtle one, since it is based on concepts that the theory cannot sustain (such as relative velocity and the group property of coordinate transformations) and on assumptions (the constancy of the velocity of light) that were not put to experimental test for many decades.

As previously discussed in Section 2, only reason is the supporting pillar of scientific theories, in particular the absence of internal contradictions and the ability to honestly resist refutation attempts. On the contrary, a characteristic of the construction of EM has been that failures, drawbacks, contradictions and refutations had little effect. Since there appeared to be no alternative to the emission/propagation theory of light (once the alternative lines of thought were suppressed or ignored), no mishap whatsoever could weaken this substantialist belief. However, alternatives did exist, only that they were never sufficiently studied, and still aren't.

Thus, the historic construction of current electromagnetism reveals a much larger and decisive underlying process: it represents a breaking point that changes the goals of science from seeking the harmony of the cosmos to the much limited search for some sort of usefulness or success. The scientific goal becomes vague or poorly defined and it substantially broadens the meaning of science. When Einstein states "The justification (truth content) of the system rests in the proof of usefulness of the resulting theorems on the basis of sense experiences", we do not know the service or purpose that has been stated for science: what does useful mean in this context? The meaning will soon be provided by the society at large, true will then become "useful for production", and *Scientia* will be spelled *techné*.

When science is being redefined, the scientist producing successful predictions will later be appreciated as a "seer" (Oxford dictionary: "A person of supposed supernatural insight who sees visions of the future."). Thus, prediction ("vision") is what is to be commended and not the consistency implicit in harmony. It is no longer a surprise that authors as L. Smolin and J. Harnad contrast "seers" and "craftsmen" (in their assessment of string theory (Ch. 18; Smolin and Harnad, 2008)) as the theoretical physicist subtypes. For Smolin and Harnad, Einstein is the prototype of the seer.

The new wisdom may have ancestral roots, since sorcerers were the guardians of accumulated knowledge in most primitive cultures and technology is a fundamental element in defining a culture. We observe that the new scientist, the theoretical physicist, occupied an empty niche in the society emerging from the second industrial revolution and the decline of the Enlightenment culture. The new scientists become the priests that interpret the oracle of "science" relevant for technology.

Departing from the philosophical and mathematical realm that was the place of physics until the middle of the XIX century, the new epistemic view puts intuition above reason when they come in conflict. But since "We have no power of Intuition, but every cognition is determined logically by previous cognitions" (pp. 230; Peirce, 1955), intuition sets us back to the constructs of the child

and in particular, the notion of space, time and of reality (Piaget, 1999)<sup>39</sup>. The construction of the child is elevated to a dogma: to understand is to put the observed in terms of the real (as constructed by the child). The transition goes from a philosophical ideation to simple ideation (Husserl, 1983). The inferred or abducted is rejected when it does not conform the intuitive form: Not only instantaneous action at distance is rejected, but delayed action at distance is rejected as well. It is not the case that Hertz cannot understand Maxwell, it is a conscious decision of enforcing his own view of what it means to understand but taking benefit of the previously produced equations. This means to give no consideration to the inferences of Faraday and Maxwell (and of a large list of mathematicians); they are rejected *in limine*. A free game of interpretations is then entertained, the equations become runes that have to be deciphered, interpreted. In a last move Einstein observed that a symmetry that must be present is not present in the inherited wisdom. The symmetry (see Appendix B) certainly was present in Faraday's perceptions and experiments as well as in Maxwell's original equations, what was needed was (is) to restore what had been destroyed by free interpretation. Instead, we get a new interpretation, being Einstein immersed in the new culture of theoretical physics.

The new approach adopted by theoretical physics has reproductive advantages as well since it builds on top of students' intuitions. Physics textbooks do not discuss notions of space or time, they just construct physics supported on the children's intuitions for such concepts and build new "intuitions" in the form of habits by repetitive exercising. In contrast, the critical approach requires the development of confidence in the own forces of the student, it rests upon the *bildung* (Sorkin, 1983) including its tense relation with teaching as it emerges from W. von Humboldt words: "Whatever man is inclined to, without the free exercise of his own choice, or whatever only implies instruction and guidance, does not enter into his very being, but still remains alien to his true nature, and is, indeed, effected by him, not so much with human agency, as with the mere exactness of mechanical routine"(von Humboldt, 1792 (1854 Edited)).

## 6 Conclusions

For the construction of mechanics we have an ample experience of the notions of body, space, time and motion, that were produced in our infancy (Solari and Natiello, 2018). Thanks to this early organisation of our perceptions, mechanics can be built with two pillars: simple intuition (experience) and reason. In contrast, our direct relation with electromagnetism is not part of our early intuitive construction. Experience is scarce, it is provided by experiments limited in several forms and is subject to a conscious interpretation in terms of our pre-existent beliefs. Ultimately, experiments challenged beliefs such as: the principle of action and reaction, the material existence of the ether and what is matter and how matter acts onto (relates with) matter.

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<sup>39</sup>Apparently Science and scientists are unaware of this epistemic operation, that occurs completely behind the scenes, guided by habit and training.

Ideally, electromagnetism can be constructed with these two pillars, acting both as requisites on equal footing, and that was Maxwell's original attempt. In replacement of an insufficient experience he (and others) introduced analogies, although he (following Faraday's teachings and Newton's tradition) was very reluctant to formulate physical hypothesis and kept them to a minimum. In contrast, the Göttingen school appears as not needing intuition, or at least, as keeping always intuition under rational supervision ("philosophical intuition" in the terminology of Husserl), intuition was not on an equal footing than reason but just an aid.

While Maxwell's epistemology is rooted in British empiricism, in continental Europe the dictum of Hegel "What is rational is real and what is real is rational" appears to have influenced the scientific minds. Thus, reality is the rational organisation of the electromagnetic phenomena that gives unity to the observed but exists only in relation with the particular (observed) realisations. Much later, Jean-Paul Sartre would beautifully write: "an electrical current does not have a secret reverse side; it is nothing but the totality of the physical-chemical actions which manifest it (electrolysis, the incandescence of a carbon filament, the displacement of the needle of a galvanometer, etc.). No one of this actions alone is sufficient to reveal it. But no action indicates anything that is behind itself; it indicates itself and the principle of the series." (Introduction, Sartre, 1966). Both lines of thought, the continental and the insular, had a broad coincidence when it comes to the final mathematical expressions, their divergence lying in the foundations sustaining the actual development.

The epistemological grounds of Maxwell's approach were laid in 1856 (Maxwell, 1856). In replacement of an insufficient experience he (and others) introduced analogies, although Maxwell (following Faraday's teaching and Newton's tradition) was cautious and regarded his analogies as tentative. Maxwell in his treatise as well as in his 1865 paper, follows a construction method that parallels the method of Lagrangian mechanics. It can be said then that Lagrangian formulations transcend (or surpass in Piaget-García's language) mechanics becoming a method for the construction of mathematical physics. The beliefs obtained by this method are accepted only because they organise the experience in electromagnetism, i.e., the theory is abduced. As Lorenz insightfully indicates, the ether in Maxwell, and to a lesser degree in Faraday, "had only been useful to furnish a basis for our imagination".

However, in one point Maxwell's insight came short of Faraday's, namely in conceiving light as anything else than a wave emitted and propagated through some propagation medium. He even attempted to "correct" Faraday when the latter advanced an alternative view<sup>40</sup>. We recognise that Maxwell was in front of a deep difficulty: how to understand a reasoning that requires to change what we consider understanding? If understanding that one is (or may be) wrong about something is not difficult, understanding that what we call "to understand" may not be correct appears almost impossible.

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<sup>40</sup>It is remarkable that Faraday's vibrating ray theory is compatible with (and probably was influential to) the developments of the Göttingen School at those times.

Maxwell complained referring to the Göttingen school: “There appears to be, in the minds of these eminent men, some prejudice or *à-priori* objection, against the hypothesis of a medium in which the phenomena of radiation of light and heat and the electric actions at a distance take place. [...] Hence the undulatory theory of light has met with much opposition, directed not against its failure to explain the phenomena, but against its assumption of the existence of a medium in which light is propagated.” ([865], Maxwell, 1873). It should be noted that the “undulatory theory of light” did not meet any opposition from the Göttingen school. They also provided a wave equation and attempted different motivations for it, based in Gauss’ suggestion of delayed action at a distance. The opposition focused in the hypothesis of a propagating medium and in the peculiar and contradictory properties that were required for this medium.

In fact, in the same article [865] Maxwell acknowledges that the Göttingen school had good reasons to be sceptical about the ether: “...It is true that at one time those who speculated as to the causes of physical phenomena were in the habit of accounting for each kind of action at a distance by means of a special æthereal fluid, whose function and property it was to produce these actions. They filled all space three and four times over with æthers of different kinds, the properties of which were invented merely to ‘save appearances,’ so that more rational enquirers were willing rather to accept not only Newton’s definite law of attraction at a distance, but even the dogma of Cotes, that action at a distance is one of the primary properties of matter, and that no explanation can be more intelligible than this fact...” Nevertheless, if Maxwell’s form of knowing (his epistemology) was correct, the ether must be real, and hence it becomes necessary to know more about it, he indeed closes the Treatise (last paragraph) stating that “all these theories lead to the conception of a medium in which the propagation takes place, and if we admit this medium as an hypothesis, I think it ought to occupy a prominent place in our investigations, and that we ought to endeavour to construct a mental representation of all the details of its action, and this has been my constant aim in this treatise.”

Some years later a new epistemology spread through continental Europe emanating from Berlin and in coincidence with the German unification and the construction of the first Reich: the *bild* conception (D’Agostino, 2004). This new epistemology required for understanding the construction of mental images of the real. Whether this real was accessible to the senses or not, it did not matter. Hence, under the new epistemology, the ether became real without carrying the research proposed by Maxwell.

Hertz’ *bild* conception appears as a militant epistemology for it suppressed large portions of Maxwell’s treatise arguing that it reflected a mechanistic approach incompatible with the ether. Nevertheless, he conveniently kept Maxwell’s equations but without offering an alternative, “non mechanistic”, derivation. In the same movement it achieved three goals: first, to ignore Faraday-Maxwell’s epistemology; second, to suppress any mention of the old German electromagnetism in its evolved form, probably feeling justified by the argument that the equations were “the same” than those in Maxwell; and third, to suppress the Lagrangian basis of the organisation of electromagnetism. The meaning of sci-

ence was changed by detaching the symbolic relations produced, disregarding the construction and allowing for a free interpretation of the equations. This movement began the transition from modern science to techno-science, freeing the later of the rigidities imposed by reason. Such an idea goes completely against Hegel's conceptions as well, for Hegel "Everything, other than the reality which is established by the conception, is transient, surface existence, external attribute, opinion, appearance void of essence, untruth, delusion, and so forth. Through the actual shape [Gestaltung], which it takes upon itself in actuality, is the conception itself understood. This shape is the other essential element of the idea, and is to be distinguished from the form [Form], which exists only as conception [Begriff]." (Hegel, 2001) <sup>41</sup>

Schematically, Maxwell requirements to accept a belief as a (temporary) truth were: to mathematically organise the subject (rationality) and to be compatible with a substantialist (intuitive) view. The Göttingen school put rationality over intuition, while the Berlin school put intuition over rationality. Hertz must be recognised as the one who understood that, unlike the experience with mechanics, in electromagnetism intuition and reason had come to some degree of incompatibility. Maxwell's goal was impossible to achieve.

Later experiments in the search of the ether showed that none of the conceptions of the ether was compatible with experiences. The well known Michelson-Morley experiment shows that if light is considered as moving through space the way bodies move (i.e., if it can be regarded in analogy with material bodies or perturbations of matter) then space-time cannot be conceived in Galilean terms. When substantialism fails to reach conscious state<sup>42</sup> and in so doing avoids the inspection of reason, the antecedent is never questioned. This amounts to putting intuition above reason when they come in conflict. Hence, the accepted conclusion of the experiment became that the world was not as Galileo and Leibniz conceived it. Thus, substantialism is not an interpretation of physics, it pre-exist physics and forges it. It is difficult to put substantialism under examination because we tend to believe (or we are indoctrinated to believe) that we (the scientists) understand a world whose rules pre-exist our understanding effort. The constructivist thinks/knows otherwise and he/she is willing to offer his form of understanding for philosophical/rational examination, this is, to criticism.

The historical construction of electromagnetism cannot be separated from its place in history. The goals behind the attack against the school of Göttingen carried out by Clausius do not appear to be only academic. During the second part of the XIX century, Germany, which was very active in science, was emerging to industrialisation on the economic side and was reunified on the political side. The growth of the universities (in terms of number of students) and a new emphasis in research resulted in the installation of a second professor in physics, this is, there was an important expansion in the number of physicists working in

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<sup>41</sup>Notice that the paragraph is more meaningful if we translate *Gestaltung* as: shaping, forming, manifesting, ...

<sup>42</sup>We may indeed speak of "hidden substantialism", as in Maxwell (and almost all of his followers) that could not conceive light propagation otherwise.

the academia. The “second physicist” is the origin of the “theoretical physicist” as we learn from (Jungnickel and McCormach, 2017). The first “theoretical physicist” they recognise as such is Rudolph Clausius. In this social and political context, the expansion of the academic positions should have resulted in the dissemination of the theories and epistemology dominating in Berlin.<sup>43</sup>

Prisoner of his own doctrine<sup>44</sup> and of his time, Einstein tried to restore some rationality to the electromagnetism of his time, but he resorted to the same epistemological approach that had left it devoid from reason: a new interpretation of the equations separated from their conception. The principle of relativity presented by Poincaré and used by Einstein impress us as true because it builds upon the habits acquired in mechanics and it has a reminiscence of the principle of no-arbitrariness. The absence of a process to attain its conception undermines its truth; actually, in a Hegelian view it is untrue. The principle is based on the hidden proposition: every real, objective, interaction can be presented in subjective form. However, this proposition is not necessarily true and perpetuates Newton’s concept of force. After assuming that electromagnetism could be presented in subjective form, the new proposal failed to incorporate Lorentz’ space-time as a real concept and not just as mere opinion. For a concept to be real, it must be intersubjective, which in terms of mathematics demands the existence of a group structure. The substantialist view of light and electromagnetic interactions clashes with the inherited structure of the space-time. Since –as for Maxwell before– an alternative to this substantialist view could not be conceived (or it passed unnoticed), the remaining option was an attempt to change space-time. It is worth to emphasise that for the relational point of view, the Michelson-Morley experiment of 1881 only gives the expected results, and actually, one would have considered a waste of time to perform it<sup>45</sup>.

The construction of Special Relativity begins by assuming the existence of a relational velocity (real, unique, not just an opinion) that at the end will find no place in the theory. This happens on the mathematical side because of the lack of an appropriate group structure and on the construction side because intuitions such as relative velocity belong to a different conception of space-time. The solution to this nightmare was a new epistemological reform, where theories arise from “free thinking” and are only vaguely related to the observed/measured reality. This is, by insisting in that concepts have a life and a reality by themselves and not in relation to the conceptualisation, science becomes mere appearance and lacks reality (always thinking in Hegel’s key). It is only opinion, for every reference system in special relativity is entitled to its

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<sup>43</sup>For example, Albert Einstein was born in the German Empire, Kingdom of Württemberg, and his first advisor in Zürich was Heinrich Friedrich Weber (not to be confused with Wilhelm Eduard Weber) who was for some time the first assistant of Helmholtz at Berlin.

<sup>44</sup>A quote of Chuang Tse in Ortega y Gasset’s “Mission of the university” reads: “How will I be able to speak about the sea with the frog when she has never gone out of her pond? How will I be able to speak about the ice with the summer bird when he is anchored in his season? How will I be able to speak with the savant about Life when he is prisoner of his doctrine?” (Ortega y Gasset, 2004-2010) (translated by the authors).

<sup>45</sup>On the contrary, Michelson’s experiment from 1925 in the relational view reveals the relative velocity between source and detector.



opinion, but there is no common point or consistent equivalence between the opinions, hence, there is no reality.

The absence of the real, however, is no longer a problem when the measure of truth is usefulness (Einstein, 1940). This completes the epistemological voyage towards vulgar pragmatism, transforming science from understanding nature (an in so doing adapting to it and empathising with it) into being a platform for technology, sharing its goals, equating truth with success and allowing for technology (science) to dominate nature, then playing the role wanted by the society at large: the industrial society.

The accepted version of electromagnetic theory (as discussed e.g., in textbooks) is the outcome of both electromagnetism as a scientific problem and of a series of epistemic decisions that impelled the reformulation of science in the second half of the XIX century. The Enlightenment era with Rousseau's social contract and Kant's critical reason was declining and giving way to a new era signed by industrial development and a return to imperial thinking (and acting).

Electromagnetism develops symbiotically with the new form of "savant" adapted to the epoch, the theoretical physicist. Indeed, theoretical physicists will adopt electromagnetism as a model of scientific construction. The equations constitute "the theory" and as such will be placed above any criticism, as occurred with Maxwell's equations and is the case also for Relativity. Underneath the equation level a broad interpretation game develops. Critical thinking, including its role of challenging the foundations of theories upon mismatches with experiment, is suppressed. Where the theory could be in fault it is patched with new *ad hoc* substantialist forms. In this way elements such as neutrinos, dark matter or dark energy emerge. Their universal form is prescribed by the epistemology and only the particular form depends on empirical data.

S. Traweek expresses the "common sense" of normal (in the sense of Kuhn (Kuhn, 1962)) physicists in the terms "it must be true because it works", reflecting the decisive motto of vulgar pragmatism. She perceives that "high energy physicists construct their world and represent it to themselves as free of their own agency, a description, as thick as I could make it, of an extreme culture of objectivity: a culture of no culture" (Epilogue, Traweek, 1992). As we have seen, much of the physicist's conception of the world comes from themselves, not from their knowledge, but rather from the ignorance about their own limitations.

This state of things was mediated by changes in the society at large. The foundations of physical science have ceased to be available to the general reasoning and can only be handled by specialists. Like medieval guilds, specialists become the guardians of a way of doing things within a community. Such elitist practice avoids exposing the basic dogma of substantialism and lesser practices such as "mathematical fetishism" (the adoration of mathematical formulae) to the inspection by the philosopher. Philosophy of science more often than not adopts the form of "praise of the scientist", considering only issues that science has left open, but laymen are not admitted in the discussion of space, time, or the universe, no matter how philosophically solid they could be.<sup>46</sup> It is often

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<sup>46</sup>We here agree with the critics of elitism in science made by I. Lakatos (Lakatos, 1978b) He

said that the theoretical physicist has rescued cosmology from religion but in so doing he/she has become dogmatic.

Enlightenment was signed by the supremacy of reason and the social contract. The present postmodernism (understood as reason's loss of supremacy and the abandonment of critical reason), that emerged in science before pervading culture in general, it has now to be walked through and negated (in Hegel's meaning). Our time is the epoch of global warming, the fires consuming the Amazonas, the mass extinction of species, etc., the milestones of an era we cannot escape and that is in need of a second Enlightenment, a reconstruction of reason. For this, a new social and environmental contract is required, regaining the plenitude of reason and its right to express itself in all issues, recovering its critical strengths, the search for foundations and the unity of reason. We beg the reader to allow us to believe that the elaboration of this manuscript, being possible now and not in earlier times, is a sign that the transit to a new epoch has already begun.

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writes: "In the demarcationist tradition, philosophy of science is a watchdog of scientific standards.". He asserts as well: "Among scientists the most influential tradition in the approach to scientific theories is elitism."

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# Appendices

## A Delayed action at a distance

The origins of delayed action at a distance have been reproduced in several works along the years. The initial impetus was apparently given by Gauss around 1845, as mentioned in his letter to Weber (bd.5 p. 627-629, Gauss, 1870). The idea of delays connect with Faraday’s “ray vibrations” (Faraday, 1855, p. 447), from 1846 as well. The first explicit mathematical formulation was given by Riemann (Riemann, 1867) (presented in 1858 and published in 1867) and L. Lorenz (Lorenz, 1867) in adjacent articles in the same issue of *Annalen der Physik*. The idea was discussed in detail by Carl Neumann (Neumann, 1868) soon after. Eventually, the idea of retarded effects entered the textbooks. In more recent times, the issue has been reconsidered in various contexts by Moon&Spencer (Moon et al, 1989b,a, 1994), and Bilbao (Bilbao, 2016; Bilbao et al, 2014) among others.

For the sake of the issue about the propagation of electromagnetic effects, the common grounds are the use of electric and magnetic potentials. Basically, eq.(5) for the magnetic vector potential with no delays is re-proposed as  $A(x, t) = \frac{\mu_0}{4\pi} \int_U \left( \frac{j(y, t)}{|x - y|} \right) d^3y$ . Technically, the potentials are obtained by a convolution, i.e., the integral of the material support of the potential (a current source  $j$  occupying the region  $U$ ) with the kernel

$$\mathbf{W}(x - y, t - s) = \frac{\mu_0}{4\pi} \frac{\delta(t - s)}{|x - y|}$$

in the following way:

$$A(x, t) = \int ds \int_U d^3y \mathbf{W}(x - y, t - s) j(y, s)$$

where  $\delta$  is the delta distribution.

Under the prerequisites of the No Arbitrariness Principle (Solari and Natiello, 2018) the introduction of the delays reconsiders the form of the vector potential for a current source  $j$  occupying the region  $U$  acting on a particle at position  $x$ , time  $t$  in a situation of relative rest between source and target. The delayed potential is

$$A(x, t) = \frac{\mu_0}{4\pi} \int_U \left( \frac{j(y, t - \frac{1}{c}|x - y|)}{|x - y|} \right) d^3y \quad (11)$$

A similar expression for the electric potential, generalising Poisson’s law reads

$$V(x, t) = \frac{1}{4\pi\epsilon_0} \int_U \frac{\rho(y, t - \frac{1}{c}|x - y|)}{|x - y|} d^3y = \frac{\mu_0 C^2}{4\pi} \int_U \frac{\rho(y, t - \frac{1}{c}|x - y|)}{|x - y|} d^3y \quad (12)$$

Note that  $A$  and  $\frac{V}{C^2}$  satisfy the same constitutive equation in relation to  $j$  and  $\rho$  and that these two quantities are assumed to be a property of matter, i.e., they are identically zero outside the region  $U$  where matter exists (in the sequel we skip indicating  $U$  to lighten the notation). In convolution terms the delayed kernel

$$\mathbf{W}^{\mathbf{d}}(x-y, t-s) = \frac{\mu_0}{4\pi} \frac{\delta(t-s - \frac{1}{C}|x-y|)}{|x-y|}$$

is now used.

This formulation follows the ideas of C. Neumann and Lorenz as expressed in eq.(3). All involved quantities are relational, independent of the coordinate system and therefore universal. Different subjective representations of the resulting interaction are equivalent, being the Galilean transformations the underlying group.

Without delays, the vector potential from eq.(5) satisfies Poisson's equation  $\Delta A = -\mu_0 j$ . Maxwell's path to the wave equation was to incorporate the required time-derivatives in his instantaneous action theory through the introduction of the displacement current  $j_D$ , to be added to the galvanic current  $j$ . Basically,  $-\mu_0 j_D = \frac{1}{C^2} \frac{\partial^2 A}{\partial t^2}$ . This "current" was to be present even in the absence of matter and Maxwell related it to the ether.

The delayed potentials, on the other hand, satisfy a wave-equation automatically. Indeed, starting from eq.(11) or, correspondingly from eq.(12), we have (we do not write the arguments of  $j_i$  when it helps to simplify the notation, we also write  $r$  for  $|x-y|$ ):

$$\begin{aligned} \nabla_x A_i &= \frac{\mu_0}{4\pi} \int d^3 y \left( \frac{-\frac{\partial}{\partial t} j_i \nabla_x \frac{r}{C}}{r} + j_i \nabla_x \frac{1}{r} \right) \\ \Delta A_i &= \nabla_x \cdot \nabla_x A_i \\ &= \frac{\mu_0}{4\pi} \int d^3 y \left( j_i \Delta \frac{1}{r} - 2 \nabla_x \frac{1}{r} \cdot j_{i,t} \nabla_x \frac{r}{C} - \frac{\frac{\partial}{\partial t} j_i \Delta \frac{r}{C}}{r} + \frac{\frac{\partial^2}{\partial t^2} j_i}{r} \left| \nabla_x \frac{r}{C} \right|^2 \right) \end{aligned}$$

Moreover, standard operations of vector calculus give

$$\begin{aligned} j_{i,t} \left( 2 \nabla_x \frac{1}{r} \cdot \nabla_x \frac{r}{C} + \frac{\Delta \frac{r}{C}}{r} \right) &= 0 \\ \left| \nabla_x \frac{r}{C} \right|^2 &= \frac{1}{C^2} \end{aligned}$$

and therefore

$$\Delta A_i(x, t) = \frac{\mu_0}{4\pi} \int d^3 y j_i(y, t - \frac{r}{C}) \Delta \left( \frac{1}{r} \right) + \left( \frac{1}{C^2} \right) \frac{\mu_0}{4\pi} \int d^3 y \frac{\partial^2}{\partial t^2} \frac{j_i(y, t - \frac{r}{C})}{r}$$

The time derivative in the last term can be extracted outside the integral, thus

yielding,

$$\begin{aligned}
\Box A_i(x, t) &\stackrel{def}{=} \Delta A_i(x, t) - \left(\frac{1}{C^2}\right) \frac{\mu_0}{4\pi} \int d^3y \frac{\partial^2 j_i(y, t - \frac{r}{C})}{\partial t^2} \\
&= \Delta A_i(x, t) - \left(\frac{1}{C^2}\right) \frac{\partial^2 A_i(x, t)}{\partial t^2} \\
&= \frac{\mu_0}{4\pi} \int d^3y j_i(y, t - \frac{|x-y|}{C}) \Delta \left(\frac{1}{r}\right) \\
&= -\mu_0 j_i(x, t)
\end{aligned}$$

Maxwell's and Lorenz' alternatives are the only two options starting from static potentials satisfying Poisson equation. In order to arrive to an inhomogeneous wave equation, either one adds a "correction" to an instantaneous current  $j(y, t)$  (or charge  $\rho(y, t)$ ) or one builds up the sources following the general solutions of the wave equation. In other words, the option is ether (or dependence of charge and current with the frame of reference) or delays.

## A.1 Relative motion

The assumption of delayed action opens up new possibilities that were precluded by instantaneous action. The question arises whether two systems in relative motion will have the same sort of interaction as systems at relative rest whenever their relative distance at the time  $t$  in consideration is the same in both cases. For the sake of clarity consider two identical sources, in relative motion that for a given time  $t$  "coincide" (ideally) in space, along with a detector that is at rest relative to one of the sources. Instantaneous action at time  $t$  as given by eq.(5) cannot distinguish between the sources, having both the same current (or charge) and being at the same distance relative to the detector at that given time. However, delayed action conveys a clear distinction since the current (or charge) of the source at an earlier time enters the description, and at that earlier time it was possible to distinguish the sources: their distances to the detector were not the same.

Recasted under the hypothesis of delayed action, the phenomenon of induction originating in the relative motion of source and detector reveals itself as a natural consequence. The delay hypothesis provides the wave equation and it also reorganises previous knowledge in a more integrated manner.

Let  $(\Phi, \zeta)$  represent any of the pairs  $(A_i, j_i)$  or  $(\frac{V}{C^2}, \rho)$  connected to each other by the same convolution kernel. Consider further that source and detector move relative to each other with constant relative velocity  $v$ . If  $x$  represents a local coordinate of the detector, we are now interested in computing the potentials  $A$  and  $V$  at a point  $x' = x + vt$  relative to the source. Without the delay hypothesis a natural choice is:

$$\Phi_v(x, t) = \Phi(x + vt, t) = \frac{\mu_0}{4\pi} \int \frac{\zeta(y, t)}{|x + vt - y|} d^3y \quad (13)$$

where  $\Phi_v$  expresses the value of the potential at the point  $x$  of the detector. This corresponds to the convolution kernel

$$\mathbf{W}_v(x - y, t - s) = \frac{\mu_0}{4\pi} \frac{\delta(t - s)}{|x + vt - y|}.$$

For zero relative velocity, eq.(5) is recovered. In this expression motion is of no consequence other than that of altering the relative distances.

We cannot expect the above equation to hold for low velocities, since already the delay is missing in it. When taking into account the delay hypothesis, whatever proposed generalisation of  $\mathbf{W}$  must have the correct limit for low velocities, but also satisfy the known experimental behaviour of electromagnetic phenomena, namely the Doppler effect (Dingle, 1960a; Mandelberg and Witten, 1962; Kaivola et al, 1985). This effect expresses the observed fact that the frequencies associated to electromagnetic phenomena perceived by a detector in motion relative to a source are different from the frequencies of the source in a precisely determined way. However,  $\mathbf{W}_v$  does not comply with Doppler.

An option that has not been previously investigated is

$$\mathbf{W}_v^d(x - y, t - s) = \frac{\mu_0}{4\pi} \frac{\delta(t - s - \frac{1}{g(v)C}|x - y + vg(v)(t - s)|)}{|x - y + vg(v)(t - s)|}$$

where  $g(v)$  is a monotonically increasing function of relative velocity such that  $g(0) = 1$ . This function has to be chosen in such a way that the delayed kernel  $\mathbf{W}_v^d$  gives the correct expression for the Doppler effect. Further,

$$\begin{aligned} \Phi_v(x, t) &= \frac{\mu_0}{4\pi} \int ds \int d^3y \zeta(y, s) \frac{\delta(t - s - \frac{1}{g(v)C}|x - y + vg(v)(t - s)|)}{|x - y + vg(v)(t - s)|} \\ &= \frac{\mu_0}{4\pi} \int ds \int d^3z \zeta(z + g(v)v(t - s), s) \frac{\delta(t - s - \frac{1}{g(v)C}|x - z|)}{|x - z|} \\ \Phi_v(x, t) &= \frac{\mu_0}{4\pi} \int d^3z \frac{\zeta(z + \frac{v}{C}|x - z|, t - \frac{1}{g(v)C}|x - z|)}{|x - z|} \end{aligned}$$

The second line arises from the variable substitution  $z = y - vg(v)(t - s)$  and the last line from time-integration.

Relative motion influences also the perception of charge and current. For low velocities the equation of continuity requires that  $j_v = j - \rho v$ . In the general case, from the second line above, with  $\rho$  in place of  $\zeta$  and for constant velocity, we obtain

$$\frac{\partial \rho}{\partial t} = \nabla \rho \cdot g(v)v = \nabla \cdot (g(v)\rho v)$$

whence  $j_v = j - g(v)\rho v$ .

Turning to the Doppler effect, we attempt to compute the Fourier transform of the kernel. For a situation of relative rest between source and detector, we have

$$\begin{aligned}\mathcal{F}(\mathbf{W}^{\mathbf{d}})(k, w) &= \frac{\mu_0}{4\pi} \int d\tau \int d^3z \exp(-ikz - iw\tau) \frac{\delta(\tau - \frac{1}{C}|z|)}{|z|} \\ &= \frac{\mu_0}{4\pi} \int d^3z \frac{\exp(-ikz - i\frac{w}{C}|z|)}{|z|} = \frac{\mu_0}{|k|^2 - \left(\frac{w}{C}\right)^2} \quad (14)\end{aligned}$$

where the substitutions  $z = x - y$  and  $\tau = t - s$  were used. Alternative to the present computation by integration in the distributional sense, we may consider the Fourier transform of  $\Phi(x, t)$  recovering the same result:

$$\begin{aligned}\mathcal{F}(\Phi) &= \mathcal{F}(\mathbf{W}^{\mathbf{d}}) \mathcal{F}(\zeta) = \mathcal{F}(\mathbf{W}^{\mathbf{d}}) \left(-\frac{1}{\mu_0}\right) \mathcal{F}(\square\Phi) \\ &= \mathcal{F}(\mathbf{W}^{\mathbf{d}}) \left(-\frac{1}{\mu_0}\right) \left(-|k|^2 + \left(\frac{w}{C}\right)^2\right) \mathcal{F}(\Phi)\end{aligned}$$

Repeating the process and substitutions for the case of relative motion, we obtain a similar integral:

$$\mathcal{F}(\mathbf{W}_{\mathbf{v}}^{\mathbf{d}})(k, w) = \int d\tau \int d^3z \exp(-ikz - iw\tau) \frac{\delta(\tau - \frac{1}{g(v)C}|z + g(v)v\tau|)}{|z + g(v)v\tau|}$$

Letting  $u = z + g(v)v\tau$  and  $\sigma = g(v)\tau$ , it follows that

$$\begin{aligned}\mathcal{F}(\mathbf{W}_{\mathbf{v}}^{\mathbf{d}})(k, w) &= \int \frac{d\sigma}{g(v)} \int d^3u \exp(-iku - i\frac{w}{g(v)}\sigma) \exp(ik \cdot v\sigma) \frac{\delta(\sigma - \frac{1}{C}|u|)}{|u|} \\ &= \frac{1}{g(v)} \int d^3u \frac{\exp(-iku - i\frac{w}{g(v)C}|u|) \exp(ik \cdot \frac{v}{C}|u|)}{|u|} \\ &= \frac{1}{g(v)} \frac{\mu_0}{|k|^2 - \left(\frac{w}{g(v)C} - k \cdot \frac{v}{C}\right)^2}\end{aligned}$$

The observed Doppler effect is recovered letting

$$g(v) = \frac{1}{\sqrt{1 - \left(\frac{v}{C}\right)^2}}$$

since for  $k \cdot v = |k| |v|$  the frequency shifts to  $w = g(v)(vk \pm kC) = g(v)k(v \pm C)$  and for  $k \cdot v = 0$  it gives  $w = \pm g(v)|kC|$ .

Doppler experiments are accurate up to  $O(\frac{v}{C})^2$ , so experimental observations are compatible with any delay factor such that  $g(v) = 1 + \frac{1}{2}(\frac{v}{C})^2 + O(\frac{v}{C})^3$ .

### A.1.1 An alternative view

Let us consider the change of variables  $u = g(v)v$ , for  $g(v)^{-1} = \sqrt{1 - \left(\frac{v}{C}\right)^2}$ . Then we may recast  $\mathbf{W}_v^d$  and the previous developments in terms of  $u$ , using that

$$v = \frac{u}{\sqrt{1 + \left(\frac{u}{C}\right)^2}} \quad \text{and} \quad g(v) = \sqrt{1 + \left(\frac{u}{C}\right)^2}.$$

Letting then  $\gamma = \sqrt{1 + \left(\frac{u}{C}\right)^2}$ , we find that

$$\begin{aligned} \mathbf{W}_u^d(x-y, t-s) &= \frac{\mu_0}{4\pi} \frac{\delta\left(t-s - \frac{1}{\gamma C}|x-y+u(t-s)|\right)}{|x-y+u(t-s)|} \\ \Phi_u(x, t) &= \frac{\mu_0}{4\pi} \int ds \int d^3y \zeta(y, s) \frac{\delta\left(t-s - \frac{1}{\gamma C}|x-y+u(t-s)|\right)}{|x-y+u(t-s)|} \\ &= \frac{\mu_0}{4\pi} \int d^3z \frac{\zeta\left(z + \frac{u}{C}|x-y|, t - \frac{1}{\gamma C}|x-z|\right)}{|x-z|} \end{aligned}$$

This suggests a different connection between mechanics and electrodynamics. We may regard  $u$  as the mechanical velocity (the one that is determined with rods and chronometers) and  $v$  as a sort of “electromagnetic velocity”, bounded by  $C$ <sup>47</sup>. This electromagnetic velocity usually is computed in indirect form, by energy measurements in order to determine  $\gamma = g(v)$  and thereafter obtain  $v$ . For  $v \ll C$ , the difference between  $u$  and  $v$  is negligible while for comparatively large velocities  $u$  cannot be measured independently.

Under this assumptions,  $\mathbf{W}_v^d$  differs from  $\mathbf{W}_v$  only in one  $\gamma$  factor. The “role” of the factor is to adjust the delay time. Electromagnetic interactions evolve “more slowly” when source and detector are in relative motion as compared with the corresponding interactions at relative rest. It is not space-time that undergoes deformations but only the properties of electromagnetic interactions.

## A.2 Relationism and the speed of light

The idea that a relational view is necessarily associated to a “speed of light” of the form  $(C \pm v) = \left|\frac{w}{|k|}\right|$  has been so deeply inculcated that it needs to be addressed. The question we want to raise is: Are there relational kernels that have as low velocity limit Maxwell’s electromagnetism and also provide the

<sup>47</sup>This connects nicely with the fact that  $u$  is an unbounded quantity that can be associated to an additive group (the usual Galilean group connected to mechanical velocities), with a nonlinear counterpart for  $v$ , compatible with  $v < C$  (Solari and Natiello, 2018, Theorem 2).



correct Doppler shift with  $C = \frac{w}{|k|}$ ? The answer is affirmative. We provide here an example. We first introduce the vector decomposition:

$$\begin{aligned} a &= a_{\parallel} + a_{\perp} \\ a_{\parallel} &= (a \cdot \hat{v}) \hat{v} \\ a_{\perp} &= a - a_{\parallel} = \hat{v} \times (a \times \hat{v}) \end{aligned}$$

Next, consider the convolution kernel

$$\begin{aligned} \mathbf{K}(\tau, X) &= \frac{\mu_0}{4\pi} \delta(\tau' - \frac{1}{C}|X'|) \frac{1}{|X'|} \\ \tau' &= \gamma(\tau + \frac{v}{C^2} X_{\parallel}) \\ X' &= \gamma(X_{\parallel} + v\tau) + X_{\perp} \end{aligned}$$

where  $\gamma = \frac{1}{\sqrt{1 - (\frac{v}{C})^2}}$  and  $v$  is the relative velocity between source and detector.

Further, let the propagation of an electromagnetic current or charge density be of the same form as above, namely

$$\Psi_v(x, t) = \int ds \int d^3y \mathbf{K}(t - s, x - y) \zeta(y, s)$$

$\mathbf{K}$  is a relational kernel that does not depend on the choice of origin of time or the reference for the space and coincides with  $\mathbf{W}^d$  for  $v = 0$ . The Fourier transform,  $\mathcal{F}$ , of the potential  $\Psi_v$  is now

$$\mathcal{F}(\Psi_v) = \mathcal{F}(\mathbf{K})\mathcal{F}(\zeta)$$

where

$$\mathcal{F}(\mathbf{K}) = \frac{\mu_0}{4\pi} \int d^3X \int d\tau \exp(-i(k \cdot X + w\tau)) \delta(\tau' - \frac{1}{C}|X'|) \frac{1}{|X'|}$$

Letting  $L_v$  be the Lorentz' transformation, we have

$$\begin{aligned} (k \cdot X + w\tau) &= (k, w)^\dagger (L_{-v} L_v)(X, \tau) \\ &= (L_{-v}(k, w))^\dagger (L_v(X, \tau)) \\ &= (L_{-v}(k, w))^\dagger (X', \tau') \end{aligned}$$

after a change of variables in the integral we obtain the expression already considered in eq.(14), the result being

$$\begin{aligned} \mathcal{F}(\Psi_v) &= \mathcal{F}(\rho) \left( \frac{\mu_0}{|k'|^2 - \left(\frac{w'}{C}\right)^2} \right) \\ k' &= k_{\perp} + \gamma(k_{\parallel} - \frac{v}{C^2} w) \\ w' &= \gamma(w - k_{\parallel} v) \end{aligned}$$

It can now be verified that if  $\mathcal{F}(\rho) = \delta(w - w_0)$  the observed frequency  $w'$  presents the correct Doppler shift and that  $\frac{|w'|}{|k'|} = C = \frac{|w|}{|k|}$  is the relation for the emerging light.

## B Electromagnetic Force and Lorentz' Lagrangian

Maxwell introduced Lagrangian methods in electrodynamics transcending (or surpassing) their mechanical origin. We compute here the electromagnetic force  $F_{em}$  exerted on a probe system by electric and magnetic fields  $E, B$  following Maxwell's method. We advance that specialising Lorentz' hypothesis about current, the Lorentz force is recovered.

From the electric and magnetic potentials the corresponding fields can be obtained via  $B = \nabla \times A$  and  $E = -\nabla V - \frac{\partial A}{\partial t}$ . Maxwell's equations read

$$\begin{aligned}\nabla \cdot B &= 0 \\ \nabla \cdot E &= \frac{\rho}{\epsilon_0} \\ \nabla \times B &= \mu_0 \left( j + \epsilon_0 \frac{\partial E}{\partial t} \right) \\ \nabla \times E &= -\frac{\partial B}{\partial t}\end{aligned}$$

Where the fields (and the equations) are given from a reference system at rest with the source. Maxwell's total current reads  $J = j + \frac{\partial E}{\partial t}$ . The Lagrangian introduced by Lorentz (Chapter I and IV; Lorentz, 1892) reads

$$\mathcal{L} = \frac{1}{2} \int dt \int \left( \frac{1}{\mu_0} |B|^2 - \epsilon_0 |E|^2 \right) d^3x$$

and the variation to be considered is that of the probe, namely  $\delta E = \delta E_2$ ,  $\delta B = \delta B_2$ . The subscript 2 stands for "secondary" electrical body.

Before proceeding further, we need to introduce the action, following an insight that can be found in C. Neumann (Neumann, 1868), thus departing from Lorentz's approach that is based on a non-rigorous "physical argumentation" supported upon his idea of a material ether. Let the electromagnetic contribution to the mechanical action be

$$\mathcal{A} = \int dt \mathcal{L}$$

and accept, following Hamilton's principle, that  $\delta E$  and  $\delta B$  are zero in the extremes of the interval of time-integration. This mechanical action may be recast in different ways by performing partial integrations along with Gauss' relation

$$\int_U d^3x (\nabla \cdot F) = \int_{\partial U} d\sigma \cdot F$$

and expressions derived from it for some electromagnetic field  $F$ . Further, we accept Maxwell's hypothesis that the fields decay at infinity in a such a form that the surface integral can be neglected. Maxwell's equations were obtained under these conditions, which means that they cannot be used if this restriction is lifted.<sup>48</sup> After introducing the vector potential,  $A$ , and the scalar potential,  $V$  with the relations

$$\begin{aligned} E &= -\left(\frac{\partial A}{\partial t} + \nabla V\right) \\ B &= \nabla \times A \end{aligned}$$

and "integrating by parts" (e.g. [631] p. 270, Maxwell, 1873) in space and time, the following correspondence is found

$$\delta\mathcal{A} = \int dt \int d^3x [A \cdot \delta j - V \delta \rho] \quad (15)$$

In what follows, we will consider only the low velocity case, setting  $g(v) = 1$ , which is the case considered by Maxwell and Lorentz. We need to introduce more details in the calculation. Let  $\bar{x}(t)$  be the relative position between the primary and secondary electrical bodies (the primary being everything else in consideration except the probe). We define currents and potentials through

$$\begin{aligned} \bar{\rho}_2(x, t) &= \rho_2(x - \bar{x}(t), t) \\ \bar{j}_2(x, t) &= j_2(x - \bar{x}(t), t) + \dot{\bar{x}}\rho_2(x - \bar{x}(t), t) \\ \square A_2 &= -\mu_0 \bar{j}_2 \\ \square V_2 &= -\frac{1}{\epsilon_0} \bar{\rho}_2 \end{aligned} \quad (16)$$

The potentials can be found using eq.(13), but this latter property will not used in the present discussion.

In such conditions, the variation of the current and of the charge distribution due to the motion of the probe are:

$$\begin{aligned} \delta \bar{j}_2 &= -(\delta \bar{x} \cdot \nabla) \bar{j}_2 + \bar{\rho}_2 \delta \dot{\bar{x}} \\ \delta \bar{\rho}_2 &= -(\delta \bar{x} \cdot \nabla) \bar{\rho}_2 \end{aligned}$$

We have then

$$\begin{aligned} \delta\mathcal{A} &= \delta \int dt \mathcal{L} = \int dt \int d^3x [A \cdot \delta \bar{j}_2 - V \delta \bar{\rho}_2] \\ &= \int dt \int d^3x \left[ \bar{j}_2 \cdot (\delta \bar{x} \cdot \nabla) A - \bar{\rho}_2 (\delta \bar{x} \cdot \nabla) V - \delta \bar{x} \cdot \frac{\partial}{\partial t} (A \bar{\rho}_2) \right] \quad (17) \end{aligned}$$

---

<sup>48</sup>Notice that this implies that electromagnetism cannot be used to describe an infinite universe. Only a finite universe is compatible with our tools, an infinite universe is beyond our capabilities of explanation. The finite universe of cosmology is an hypothesis required by our limitations and not a conclusion reached from our knowledge.

(the second line after some integrations by parts and the use of the wave equation for the vector potential). Further transformation with mathematical identities allows us to write

$$\int dt \int d^3x \delta x \cdot [\bar{j}_2 \times B + \bar{\rho}_2 E].$$

This is, following the standard use of Hamilton's principle in mechanics we arrive to an electromagnetic contribution to the force on the probe

$$F_{em} = \bar{j}_2 \times B + \bar{\rho}_2 E$$

Lorentz considered only the case  $j_2 = 0$ , hence  $\bar{j}_2 = \bar{\rho}_2 \dot{\bar{x}}$ . We have then derived from Hamilton's principle and the Lagrangian base in Lorentz (which is actually based upon C. Neumann's Lagrangian and action) the Lorentz' force, after assuming Lorentz' current. We must emphasise however that the velocity in our deduction is relational and in Lorentz' work is relative to the ether. The second difference is that our presentation is fully mathematical while Lorentz' one contains hand-waving arguments. The third difference is perhaps more striking. Since  $\bar{x}$  is a relational position the simultaneous motion of primary and secondary with the same motion with respect to a reference frame does not change the interaction, in other terms, the principle of action and reaction holds and the conservation of the total linear moment is assured by Noether's theorem.

Maxwell's derivation of the mechanical force (eq. C on the Treatise, art. [619] p. 258), arrives to a similar result. For Maxwell,  $j_2$  is the "total" current, i.e., the sum of the galvanic current and the displacement current. Remarkably, Maxwell's derivation of this force is performed for the galvanic current only and therefore it coincides with the present one. The displacement current was added to the galvanic one by analogical thinking, without further justification.

## B.1 Maxwell's transformation and the Lorentz's force

Whenever we have an integral expression like eq.(15) it is possible to change the spatial variable of integration without affecting the result. We intend to change from  $x$  to  $z$ , with  $x = z + \bar{x}(t)$ , the integration variable. But instead of performing the change in eq.(15) we will save effort and perform it in eq.(17), prior to the partial integration in time, namely

$$\delta \mathcal{A} = \int dt \int d^3x [\bar{j}_2 \cdot (\delta \bar{x} \cdot \nabla) A - \bar{\rho}_2 (\delta \bar{x} \cdot \nabla) V + \delta \dot{\bar{x}} \cdot (A \bar{\rho}_2)]$$

(with  $\bar{j}_2(x, t)$ ,  $\bar{\rho}_2(x, t)$  given by eq.16). We introduce the following notation

$$\begin{aligned} z &= x - \bar{x}(t) \\ \underline{V}(z, t) &= V(z + \bar{x}(t), t) \\ \underline{A}(z, t) &= A(z + \bar{x}(t), t) \end{aligned} \tag{18}$$

Hence, the variation reads now

$$\begin{aligned}\delta \int \mathcal{L} dt &= \int dt \int d^3 z [(j_2(z, t) + \dot{\bar{x}} \rho_2(z, t)) \cdot (\delta \bar{x} \cdot \nabla) \underline{A} - \rho_2(z, t) (\delta \bar{x} \cdot \nabla) \underline{V}] \\ &\quad + \int dt \int d^3 z [\delta \dot{\bar{x}} \cdot (\underline{A} \rho_2(z, t))]\end{aligned}$$

integrating by parts in time the last term and using the relations

$$\begin{aligned}\int dt [\delta \dot{\bar{x}} \cdot (\underline{A} \rho_2)] &= \int dt \left[ -\rho_2 \delta \bar{x} \cdot \left[ \frac{\partial \underline{A}}{\partial t} \right] - \delta \bar{x} \cdot \underline{A} \frac{\partial \rho_2}{\partial t} \right] \\ \left[ \frac{\partial \underline{A}}{\partial t} \right] &\equiv \frac{\partial}{\partial t} A(z + \bar{x}(t), t) = \frac{\partial}{\partial t} A(x, t) \Big|_{x=z+\bar{x}(t)} + (\dot{\bar{x}} \cdot \nabla) A(z + \bar{x}(t), t) \\ -\nabla \underline{V} - \left[ \frac{\partial \underline{A}}{\partial t} \right] &= -\nabla \underline{V} - \frac{\partial}{\partial t} A(x, t) \Big|_{x=z+\bar{x}(t)} - (\dot{\bar{x}} \cdot \nabla) \underline{A} \\ \frac{\partial \rho}{\partial t} + \nabla \cdot j &= 0\end{aligned}$$

(the last equation is the continuity equation, valid in all setups), we arrive after some algebra to

$$\begin{aligned}\delta \int \mathcal{L} dt &= \int dt \int d^3 z [j_2 \cdot (\delta \bar{x} \cdot \nabla) \underline{A} - j_2 \cdot \nabla (\delta \bar{x} \cdot \underline{A})] \\ &\quad + \int dt \int d^3 z \left[ (\rho_2 \dot{\bar{x}}) \cdot (\delta \bar{x} \cdot \nabla) \underline{A} + \rho_2 \delta \bar{x} \cdot (-\nabla \underline{V} - \frac{\partial \underline{A}}{\partial t}) \right]\end{aligned}$$

Making use of the following relations,

$$\begin{aligned}\dot{\bar{x}} \cdot (\delta \bar{x} \cdot \nabla) \underline{A} &= \delta \bar{x} \cdot \nabla (\dot{\bar{x}} \cdot \underline{A}) \\ (\delta \bar{x} \cdot \nabla) \Phi(x, t) &= -\delta \bar{x} \times (\nabla \times \Phi) + \nabla (\delta \bar{x} \cdot \Phi) \\ j_2 \cdot (\delta \bar{x} \cdot \nabla) \underline{A} - j_2 \cdot \nabla (\delta \bar{x} \cdot \underline{A}) &= j_2 \cdot (-\delta \bar{x} \times (\nabla \times \underline{A}))\end{aligned}$$

the result for the electromagnetic force is:

$$\begin{aligned}\delta \int \mathcal{L} dt &= \int dt \int d^3 z [j_2 \cdot (-\delta \bar{x} \times (\nabla \times \underline{A}))] \\ &\quad + \int dt \int d^3 z \left[ \delta \bar{x} \cdot \rho_2 \left( - \left[ \frac{\partial \underline{A}}{\partial t} \right] - \nabla (\underline{V} - \dot{\bar{x}} \cdot \underline{A}) \right) \right] \\ &= \int dt \int d^3 z \delta \bar{x} \cdot \left[ j_2 \times \underline{B} + \rho_2 \left( - \left[ \frac{\partial \underline{A}}{\partial t} \right] - \nabla (\underline{V} - \dot{\bar{x}} \cdot \underline{A}) \right) \right]\end{aligned}$$

Hence we have two expressions for the mechanical contribution of the electromagnetic force: The one obtained from eq.(17) above and the present one,

i.e.,

$$F_{em} = \int d^3x [\bar{j}_2 \times B + \bar{\rho}_2 E] = \int d^3z \left[ j_2 \times \underline{B} + \rho_2 \left( - \left[ \frac{\partial \underline{A}}{\partial t} \right] - \nabla(V - \dot{\underline{x}} \cdot \underline{A}) \right) \right]$$

(recall the relation among functions defined in 16 and 18). This corresponds to the transformation proposed by Maxwell and discussed in Theorem 3.3.1.

## B.2 The symmetry Einstein failed to see.

Let us consider a general quantity of the form

$$(\zeta_1 | \zeta_2) = \frac{\mu_0}{4\pi} \int ds \int dt \int d^3x \int d^3y \zeta_1(y, s) \zeta_2(x, t) \frac{\delta(t - s - \frac{1}{C}|x - y + u(t - s)|)}{|x - y + u(t - s)|}$$

e.g., the mechanical action corresponding to the electromagnetic interaction of  $\zeta_1$  and  $\zeta_2$ . We write a first form (using  $\tau = t - s$  and also  $z = x + ut$ )

$$\begin{aligned} (\zeta_1 | \zeta_2) &= \frac{\mu_0}{4\pi} \int ds \int d\tau \int d^3x \int d^3y \zeta_1(y, s) \zeta_2(x, \tau + s) \frac{\delta(\tau - \frac{1}{C}|x - y + u\tau|)}{|x - y + u\tau|} \\ &= \frac{\mu_0}{4\pi} \int ds \int d^3z \int d^3y [\zeta_1(y, s) \zeta_2(z - u\tau, \tau + s)] \frac{1}{\tau = \frac{1}{C}|z - y|} \frac{1}{|z - y|} \end{aligned}$$

And a second (equivalent) form now using  $z = y - u\tau$

$$\begin{aligned} (\zeta_1 | \zeta_2) &= \frac{\mu_0}{4\pi} \int dt \int d\tau \int d^3x \int d^3y \zeta_1(y, t - \tau) \zeta_2(x, t) \frac{\delta(\tau - \frac{1}{C}|x - y + u\tau|)}{|x - y + u\tau|} \\ &= \frac{\mu_0}{4\pi} \int dt \int d^3z \int d^3x [\zeta_1(z + u\tau, t - \tau) \zeta_2(x, t)] \frac{1}{\tau = \frac{1}{C}|x - z|} \frac{1}{|x - z|} \end{aligned}$$

The first form uses a forward potential and second a backward potential, the different accounts for the difference between source and detector. But except for this difference, forward fields can be defined following the same relations that backward fields. Despite the well-known objection of Einstein (Einstein, 1905, first paragraph) it is possible to make the reading in terms of the “electric field arising in the neighbourhood of the magnet”, although we are not used to think in these terms.

## C Is there a relative velocity in special relativity?

Let us examine the standard view in special relativity (SR) that we take from an authoritative source, the Feynman lectures of physics (Ch. 16, Vol. i, Feynman et al, 1965). It starts stating that the **correct** transformations between systems

moving with relative velocity  $v$  are Lorentz transformations (emphasis added). We stated its general expression in eq.(10), letting  $u = \frac{v}{C}$  (we drop the index  $C$  in  $T_{\frac{v}{C}}$  for simplicity):

$$\begin{aligned} T_v X &= \left( \gamma_v(x - vt) + (1 - \gamma_v)\hat{v} \times (x \times \hat{v}), \gamma_v\left(t - \frac{v \cdot x}{C^2}\right) \right) \\ &= \left( \gamma_v((x \cdot \hat{v})\hat{v} - vt) + \hat{v} \times (x \times \hat{v}), \gamma_v\left(t - \frac{v \cdot x}{C^2}\right) \right) \end{aligned}$$

where  $x \in \mathbf{R}^3$  is a spatial coordinate and  $\gamma_v = (1 - \frac{v^2}{C^2})^{-\frac{1}{2}}$ . As in Appendix (A.2), we use that  $x = (x \cdot \hat{v})\hat{v} + \hat{v} \times (x \times \hat{v})$ . The book presents only the special case where the velocity  $v$  between systems is parallel to the  $\hat{x}_1$ -axis of the  $S_1$  system, in components:  $(x'_1, x'_2, x'_3, t') = T_v(x_1, x_2, x_3, t) = (\gamma_v(x_1 - vt), x_2, x_3, \gamma_v(t - \frac{vx_1}{C^2}))$ , arguing that the general case is “rather complicated”. Here  $(x'_1, x'_2, x'_3, t') \equiv (x', t')$  is the position and time in a system  $S_2$ . That the inverse transformation corresponds to  $T_{-v}$  is explicitly highlighted, for otherwise “we would have a real cause to worry!”.

Further, Einstein’s velocity transformation is presented by transforming the line  $(x, t) = (ut, t)$  with  $t \in [-\infty, \infty]$  as points using  $T_v$ , resulting in the set

$$(x', t') = t \left( \gamma_v((u \cdot \hat{v})\hat{v} - v) + \hat{v} \times (u \times \hat{v}), \gamma_v\left(1 - \frac{v \cdot u}{C^2}\right) \right)$$

If  $u$  is the (constant) velocity of a third system  $S_3$  moving with respect to  $S_1$ , we would like to describe its velocity as seen by  $S_2$  as  $u' = \frac{x'}{t'}$ . We note that the line  $(x, t) = (vt, t)$  depicting the trajectory of  $S_2$  according to  $S_1$  transforms as  $(x', t') = t(0, \frac{1}{\gamma})$  indicating that  $S_2$  does not move according to  $S_2$ , as it is always the case with a subjective vision. However, this reasoning is arbitrary (it selects a preferred system  $S_1$  for no reason) unless the same calculation interchanging  $S_2$  with  $S_3$  could give the relative velocity  $-u'$ , and this for **any** choice of system  $S_1$ . Only in this way, relative velocity between  $S_2$  and  $S_3$  could be free from dependencies on arbitrary choices. Computing the trajectory of  $S_2$  as seen from  $S_3$  as above, we obtain

$$(x'', t'') = t \left( \gamma_u((\hat{u} \cdot v)\hat{u} - u) + \hat{u} \times (v \times \hat{u}), \gamma_u\left(1 - \frac{v \cdot u}{C^2}\right) \right)$$

The two resulting trajectories,  $(x'', t'')$  and  $(x', t')$  are not parallel unless  $u \parallel v$ , and similarly  $\frac{x'}{t'} \neq -\frac{x''}{t''}$ , or equivalently  $\frac{x'}{t'} + \frac{x''}{t''} \neq 0$ . Therefore, the relative velocity depends on the arbitrary form we choose to calculate it, i.e., there is no genuine concept of relative velocity, it is a mere opinion that depends on arbitrary decisions. The reader may want to verify the statement using  $v = |v|\hat{x}_1$  and  $u = |u|\hat{x}_2$ . In short, we have:

**Theorem C.1:** The Lorentz transformation does not define an equivalence relation.

**Proof:** The reflexive and symmetric properties of equivalences are satisfied, but the transitive property is not.  $\square$

**Corollary C.1:** It is not possible to define the inertial class in terms of the Lorentz transformation.

This problem emerges because of the lack of a group structure in Einstein's velocity addition (the same problem arises for the Lorentz transformations that by themselves do not constitute a group). In the velocity transformations, the associative property is missing. If we have an operation that we call velocity addition and symbolise it by  $\oplus$ , having the basic property that  $v \oplus (-v) = 0$ , then we would expect the relative velocity to be

$$u \oplus (-v)$$

and to satisfy the no arbitrariness relation

$$u \oplus (-v) = -(v \oplus (-u))$$

or what is the same, that the law of transformation of reference systems is transitive. If the operation  $\oplus$  were to define a group, we could apply the associative property of the group in the form

$$[v \oplus (-u)] \oplus [u \oplus (-v)] = v \oplus [((-u) \oplus u) \oplus (-v)] = v \oplus (-v) = 0$$

but this is not the case for relativistic velocity addition, since the lhs of this equation corresponds to  $\frac{x'}{t'} + \frac{x''}{t''}$ . All sorts of contradictions can be obtained in SR when three reference systems moving without restrictions are considered.

If points are transformed from  $S_1$  to  $S_2$  ( $S_3$ ) as

$$\begin{aligned} (x_2, t_2) &= T_v(x_1, t_1) \\ (x_3, t_3) &= T_u(x_1, t_1) \end{aligned}$$

for any set of points  $B = \{(x_1, t_1)\}$  it follows that the transformation from  $S_2$  to  $S_3$  should be

$$(x_3, t_3) = T_u(x_1, t_1) = (T_u T_{-v})(x_2, t_2)$$

However, since the Lorentz transformations do not commute in general, and they are symmetric,  $T_u T_{-v}$  is not a Lorentz transformation but rather an element in the Poincaré group. Any element  $P$  of the Poincaré group is written as:

$$P = T_w R$$

where  $R$  is a rotation. Thus,  $T_u T_{-v} = T_w R$ . If we further care to apply this transformation to  $B' = \{(0, s), s \in [-\infty, \infty]\}$ , being  $R(0, s) = (0, s)$  we get  $T_u T_{-v}(0, s) = T_w(0, s) = \gamma_v s T_u(-v, 1)$  which is Einstein's velocity addition law  $u \oplus (-v) = w$  in its general form. This law is neither associative nor commutative.

The rotation  $R$  is around the vector  $u \times v$  and is known in SR as Thomas' rotation. It is a spurious rotation that appears as a result of operating with two



successive Lorentz transformations in different directions and it has nothing to do with normal rotations. It is worth noticing that if  $u = 0$  or  $v = 0$  the rotation is the identity. Thus, the angle of rotation depends not only on the directions of  $u, v$  but also on their absolute value. Hence, it depends on the intermediate system  $S_1$  chosen to perform the transformation. It follows that the transformation from  $S_2$  into  $S_3$  is not unique, it depends on which system we have privileged to transform into others with  $R = Id$ . Arbitrariness has a price: SR has no room for objective relations, despite every observer being entitled to her/his own opinion. In the case of electromagnetism the natural choice would be the system where the electromagnetic disturbance takes place, but at this point, the pretension of all systems being equivalent is completely lost. In other words, we have proven the following

**Theorem C.2:** Given a system  $S$  and two bodies,  $\{a, b\}$ , moving uniformly following a straight line through the origin, for the general case in which the respective velocities are not parallel, it is not possible to define consistently a relative velocity.

**Proof:**  $v_a \ominus v_b \neq -(v_b \ominus v_a) \square$

Putting together Theorems C.1 and C.2, the inertial class that corresponds to isolated systems does not exist in SR.