Submitted to *Theoretical Economics*

Extensive Measurement in Social Choice

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Extensive measurement is the standard measurement-theoretic approach for constructing a ratio scale. It involves the comparison of objects that can be concatenated in an additively representable way. This paper studies the implications of extensively measurable welfare for social choice theory. We do this in two frameworks: an Arrovian framework with a fixed population and no interpersonal comparisons, and a generalized framework with variable populations and full interpersonal comparability. In each framework we use extensive measurement to introduce novel domain restrictions, independence conditions, and constraints on social evaluation. We prove a welfarism theorem for these domains and characterize the social welfare functions that satisfy the axioms of extensive measurement at both the individual and social levels. The main results are simple axiomatizations of strong dictatorship in the Arrovian framework and classical utilitarianism in the generalized framework.

KEYWORDS. social welfare functions, measurement theory, classical utilitarianism, variable-population ethics, Arrow's theorem.

1. INTRODUCTION

Kenneth Arrow once called himself "a kind of utilitarian manqué":

> I'd like to be utilitarian but ... I have nowhere those utilities come from. ... What are those objects we are adding up? I have no objection to adding them up if there's something to add. (Kelly and Arrow, 1987, 59)

The content of Arrow's complaint is not entirely transparent. In the orthodox economic sense of "utility," anyone who takes individuals to have numerically representable preferences certainly has somewhere "those utilities come from": a person's utility is just the numerical value of a function that represents her preferences. There is no mystery about how such utilities can be added together: they're just numbers, and we can add whatever numbers we like.

Arrow's complaint cannot be that he lacks a foundation for the numerical representation of preferences. A different complaint, which is at least inspired by Arrow's remarks, is this. A classical utilitarian believes that we should maximize *the sum of well-being*, where a person's well-being is how good things are for her. But what does it mean to "add up" people's well-beings? A person's well-being is not a number, any more than her height or weight is a number. Some properties can, intuitively, be added together: we can add together two heights, or two masses. But we cannot add heights to masses.

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And it's unclear what would be meant by the "sum" of one person's intelligence and another's, or of the hardness of two minerals. The complaint is that the classical utilitarian has not shown well-being to be the kind of thing that, like height or mass, can be added up, as opposed to the kind of thing, like intelligence or hardness, that cannot.

Extensive measurement offers a way to precisify this contrast. The idea of extensive measurement is to compare objects that can be "concatenated," or combined, to yield new objects. If the comparison of concatenated objects satisfies certain axioms (stated in section 2), it can be represented by a real-valued function with concatenation represented by the arithmetic operation of addition (Suppes, 1951, Krantz et al., 1971). A classic example is the measurement of length by stacking together rods from end to end, or of mass by stacking together objects in a weightpan.

There are various ways of trying to apply extensive measurement to well-being, which differ based on what kinds of objects are evaluated and how they are concatenated (Nebel, 2023c). Each of these methods depends on controversial assumptions about well-being. It is therefore, in my view, an open question whether or not well-being is susceptible to extensive measurement. In this paper, I want to assume that it is, and thus that well-being can be meaningfully "added up," in order to study the implications of extensive measurement for social choice and welfare theory. In particular, I want to understand what further commitments are necessary and sufficient to characterize classical utilitarianism, once it is granted that well-being is extensively measurable.

We explore the social-choice-theoretic implications of extensive measurement in two frameworks. In both frameworks, the set of alternatives is equipped with a concatenation operation. (When alternatives belong to a vector space, for example, this operation can simply be vector addition.) In section 3, we consider an Arrovian framework in which each profile is an n-tuple of individual orderings on the set of alternatives. We restrict the domain to profiles in which each individual's ordering satisfies the axioms of extensive measurement. We provide a characterization of welfarism on this domain (Theorem 1), using Pareto indifference and a suitable weakening of Arrow's Independence of Irrelevant Alternatives (IIA). This IIA condition allows for nondictatorial social welfare functions which satisfy the weak and even strong Pareto principles on our domain. However, such social welfare functions cannot be anonymous (Theorem 2), and their social preference relations are not extensively measurable (Theorem 3). These negative results motivate the use of interpersonal comparisons in our second framework, based on Hammond (1976), which is explored in section 4.

In Hammond's framework, each profile is a single relation over alternative–individual pairs. The pair (x, i) stands in this relation to (y, j) just in case alternative x is at least as good for person i as y is for person j. Interpersonal comparisons of this form are utilized and defended by Suppes (1966), Sen (1970), Arrow (1977), Harsanyi (1977), Kolm (1998), and Adler (2014). A *generalized* social welfare function, as defined by Hammond, assigns a social ordering of alternatives to each ordering of alternative–individual pairs.

We modify Hammond's framework in two ways. First, we don't assume that the population is fixed. Instead, different alternatives have potentially different populations. This generalization is crucial for evaluating choices that affect the size or composition of the population—for example, responses to climate change (Scovronick et al., 2017) or allocations of fertility-affecting resources (Pérez-Nievas et al., 2019, Córdoba and Liu, 2022). Indeed, Parfit (1984, ch. 16) argues that almost all social and economic policy choices have far-reaching effects on which people will exist in the future. Variable-population comparisons are also needed to distinguish classical (i.e., total) utilitarianism from other varieties of utilitarianism (e.g., average utilitarianism) that coincide with it in fixed-population cases. Second, we restrict the domain, as in the Arrovian case, to orderings of alternative–individual pairs that are extensively measurable. Our characterization of welfarism, in terms of Pareto indifference and an appropriate IIA condition, continues to hold on this domain (Theorem 4).

Our main result is an axiomatization of classical utilitarianism in this generalized framework. Theorem 5 shows that classical utilitarianism is the only social welfare function on our domain which satisfies the weak Pareto principle, our IIA condition, a fixed-population anonymity requirement, and the axioms of extensive measurement imposed on social preferences.

1.1 *Background*

Issues of measurement have played a central role in social choice theory since Sen (1970). In Sen's framework of social welfare functionals, a social preference ordering of alternatives is assigned to each profile of real-valued utility functions in some domain. Different views about the measurability and interpersonal comparability of welfare are captured by imposing *informational invariance* conditions on the social welfare functional. These conditions require the social ranking of alternatives to be preserved under certain classes of transformations of utilities—namely, those transformations up to which the utility representation is assumed to be unique.

The social welfare functional framework is extremely flexible. It has been used to provide axiomatic characterizations of many important theories of welfare aggregation (Roemer, 1998, d'Aspremont and Gevers, 2002, Bossert and Weymark, 2004). The informational invariance conditions lie at the core of these results. These conditions have recently been criticized, however, on the grounds that they do not really follow from the underlying measurability and comparability assumptions with which they are associated. As Sen (1977a, 1542) observes, the invariance conditions fail to distinguish between real changes in well-being (e.g., everyone becoming twice as well off) and merely representational changes in the scale on which well-being is measured (e.g., halving the unit of measurement). It is not obvious why invariance with respect to to the latter kind of transformation should require invariance with respect to the former. This criticism has been further developed by Morreau and Weymark (2016) and Nebel (2021, 2022, 2023a).

Sen's framework takes numerical scales of welfare for granted but provides no way of specifying what structures they are supposed to represent—only the class of transformations up to which they are unique. This makes it difficult to defend the invariance conditions against the criticism just mentioned. An alternative approach is to formulate our social choice problem and principles in terms of the relational structure of individual welfare, rather than (at the outset, at least) a numerical representation thereof. Let me explain.

In measurement theory, a *qualitative relational structure* is a set of objects together with one or more relations on that set (Heilmann, 2015). An example is a set X of alternatives together with an ordering $\succcurlyeq$ on that set. Another is an ordered set L of lotteries closed under an operation $\otimes : [0,1] \times L \times L \to L$, which takes any probability $\lambda \in [0,1]$ and lotteries $p, q \in L$ and returns their convex combination $\lambda p + (1-\lambda)q$. The role of this "natural operation" is explicit in von Neumann and Morgenstern (1953, 24) (see also Fishburn, 1989, Weymark, 2005).

A central business of measurement theory is to provide conditions under which qualitative relational structures can be represented by certain *numerical* relational structures. There are familiar conditions which are necessary and sufficient for an ordered set $(X, \succcurlyeq)$ to be be mapped into the numerical structure $(\mathbb{R}, \geq)$, via an order-preserving function $U : X \to \mathbb{R}$. Such a representation is unique up to strictly increasing transformation. And there are familiar conditions which are necessary and sufficient for $(L, \succcurlyeq, \otimes)$ to be mapped into the numerical structure $(\mathbb{R}, \geq, \otimes^*)$, where $\otimes^* : [0,1] \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ takes any $\lambda \in [0,1]$ and $a, b \in \mathbb{R}$ and returns their convex combination $\lambda a + (1-\lambda)b$. Such a representation is unique up to positive affine transformation.

Qualitative relational structures of these kinds are the primitive ingredients of the Arrow (1951) and Harsanyi (1955) approaches to social choice. This is in contrast to the later framework of Sen, where the primitive ingredients are numerical utility functions. Other work in the "relational" tradition includes Hammond (1976), Dhillon and Mertens (1999), Harvey (1999), Pivato (2015), Marchant (2019), Brandl and Brandt (2020), Raschka (2023), among others. None of this work, however, considers the social-choice-theoretic implications of extensive measurement.

Extensive structures are formally quite similar to the examples mentioned above. The qualitative relational structure is of the form $(X, \succcurlyeq, \circ)$, where $\circ$ concatenates each pair of alternatives in X. There are natural axioms which are necessary and sufficient for this structure to be mapped into $(\mathbb{R}, \geq, +)$, via a function $U : X \to \mathbb{R}$ which is both order-preserving and *additive*, in the sense that $U(x \circ y) = U(x) + U(y)$ for all $x, y \in X$. This representation is unique up to similarity transformation—i.e., multiplication by a positive constant. This is the characteristic uniqueness condition of a ratio scale, such as the gram scale of mass or the meter scale of length.

Our project is therefore intimately related to the study of social welfare functionals with ratio-scale measurable welfare, typically spelled out in terms of invariance to similarity transformations of utilities (Roberts, 1980, Blackorby and Donaldson, 1982, Tsui and Weymark, 1997, Nebel, 2023b). Many social welfare functionals, especially in variable-population contexts (Blackorby et al., 1999), appear to require a ratio scale, because they violate invariance conditions associated with weaker scale types. But no one in this literature has explained how such a scale can be derived. For that, we need to identify a relational structure whose numerical representation is unique up to similarity transformation. Our approach does this without requiring us to *assume* an invariance condition to capture the intended scale type. Rather, the desired invariance properties will be *derived* from conditions formulated in entirely relational terms.

1.2 *Related Literature*

There appears to be no standard axiomatic characterization of classical utilitarianism—as distinguished from other varieties of utilitarianism—in the literature. This is a striking gap, given the historical importance of this doctrine in social ethics and welfare economics. There are, of course, several axiomatizations of *fixed-population* utilitarian social welfare functionals (Maskin, 1978, d'Aspremont and Gevers, 1977, Deschamps and Gevers, 1978). But these do not discriminate between classical utilitarianism and its variants. Indeed, they rest on informational invariance conditions that, when extended to a variable-population setting, rule out classical utilitarianism (Blackorby et al., 1999). The most systematic treatment of variable-population social choice is Blackorby et al. (2005). They characterize various kinds of utilitarianism, but none that singles out classical utilitarianism in particular. Hammond (1988) derives a principle which formally resembles classical utilitarianism, but in later work he is careful to acknowledge the resemblance as "only formal" (Fleurbaey and Hammond, 2004, 1268). Xu (1990) provides axiomatic characterizations of both classical and average utilitarianism; his article appears never to have been cited.

Our characterization of classical utilitarianism is in many ways analogous to the fixed-population aggregation theorem of Harsanyi (1955). Whereas Harsanyi applies expected utility theory at both the individual and social levels, we appeal to extensive measurement. Harsanyi's aggregation theorem has been extended to variable-population cases by Blackorby et al. (1998), Broome (2004), McCarthy et al. (2020). Our contribution is especially related to that of Mongin (1994), who adapts Harsanyi's theorem to a multi-profile setting via a domain restriction and an IIA condition: he requires each individual to have a mixture-preserving utility function on some convex subset of a vector space, and requires the social ordering of two alternatives to coincide on profiles which assign the same utility vectors to those alternatives. He also considers the implications of informational invariance conditions in this setting and concludes that the interpersonal comparability of welfare is a "surprise effect" of Harsanyi's theorem (Mongin, 1994, 349). The invariance conditions derived in our framework are much weaker than their counterparts considered by Mongin. In the interpersonally noncomparable case, for example, our invariance condition avoids dictatorship where his corresponding condition implies it. And none of the invariance conditions considered by Mongin is compatible with classical utilitarianism, when extended to variable-population comparisons.

This paper is also related to the study of Arrovian social welfare functions on restricted domains. Le Breton and Weymark (2011) survey the consistency of Arrow's axioms on various domains of economic interest. Our Arrovian domain in section 3 is an example of what they (following Kalai et al., 1979) call *saturating* preference domains. Arrow's axioms are inconsistent on these domains (Le Breton and Weymark, 2011, Theorem 6). But our IIA condition is much weaker than Arrow's, in a way that makes it compatible with nondictatorial, Paretian social choice on our domain. Our Arrovian domain is also quite different than those studied by Brandl and Brandt (2020). They characterize the domains on which Arrow's axioms are consistent with one another and with an anonymity requirement; they show that such anonymous Arrovian aggregation must take a certain utilitarian form. Their domains require preferences to be continuous and convex, but not transitive. Our Arrovian domain plainly does not meet their conditions; nonetheless, our impossibility result involving anonymity (Theorem 2) does not follow from their characterization of anonymously Arrow-consistent domains, again, because of our weaker IIA condition.

2. Extensive Measurement

An extensive structure has three ingredients. There is a set X of objects to be measured: for example, rods of differing lengths. There is a binary relation $\succcurlyeq$ on that set—e.g., the *at least as long as* relation. (As usual, $\succ$ denotes the asymmetric part of $\succcurlyeq$, $\sim$ its symmetric part.) There is a binary *concatenation* operation $\circ : X \times X \to X$ which, in some sense, combines the objects together—e.g., by stacking together rods from end to end. Our set of objects is assumed to be closed under this operation, so that we can concatenate any two elements of X to form a new element of X. For any object $a \in X$, define $1a := a$ and, for any natural number $n > 1$, let $na := (n-1)a \circ a$, so that na is the concatenation of n copies of a.

The triple $(X, \succcurlyeq, \circ)$ is called an *extensive structure* iff the following five axioms are satisfied for all $a, b, c, d \in X$.

Transitivity If $a \succcurlyeq b$ and $b \succcurlyeq c$, then $a \succcurlyeq c$.
Completeness $a \succcurlyeq b$ or $b \succcurlyeq a$.
Weak Associativity $a \circ (b \circ c) \sim (a \circ b) \circ c$.
Monotonicity $a \succcurlyeq b$ iff $a \circ c \succcurlyeq b \circ c$ iff $c \circ a \succcurlyeq c \circ b$.
Archimedean If $a \succ b$, then there is some natural number n such that $na \circ c \succcurlyeq nb \circ d$.

These conditions are necessary and sufficient for a numerical representation of $\succcurlyeq$ that is additive with respect to concatenation:

Proposition 1 (Krantz et al. 1971, Theorem 3.1). *$(X, \succcurlyeq, \circ)$ is an extensive structure iff there is a function $U : X \to \mathbb{R}$ such that, for all $a, b \in X$,*

(i) *$a \succcurlyeq b$ iff $U(a) \geq U(b)$, and*

(ii) *$U(a \circ b) = U(a) + U(b)$.*

Another function U' satisfies (i) and (ii) iff $U' = kU$ for some real number $k > 0$.

We call U an *additive representation* of $\succcurlyeq$.[1]

Here is an example, based on Kahneman et al. (1997), of how extensive measurement might be applied to well-being. Consider a set of *hedonic episodes*. Each episode is individuated by its duration and by its felt quality—pleasure or pain—at each moment (Kahneman et al. call this "instant utility"). The concatenation of two episodes is simply an episode that starts with the first and ends with the second. These concatenable

[1] Extensive structures are closely related to *ordered semigroups* in mathematics, which play a central role in the proof of Proposition 1 (Krantz et al., 1971, ch. 2). Pivato (2013) considers scoring systems which take values in any linearly ordered (not necessarily Archimedean) abelian group.

episodes are ordered by their desirability to some agent. Narens and Skyrms (2020, ch. 12) defend the axioms of extensive measurement for this sort of structure. Other structures, which carry no commitment to hedonism about well-being, are explored by Nebel (2023c).

3. Arrovian Social Welfare Functions

Let X be a set of alternatives, which is closed under some concatenation operation $\circ : X \times X \to X$. An alternative a is *atomic* iff it is not identical to the concatenation of any alternatives (i.e., there are no $x, y \in X$ such that $x \circ y = a$). We assume there to be a subset $A \subset X$ of at least three atomic alternatives. We assume every *nonatomic* alternative $x \in X \setminus A$ to be the concatenation of some number of atomic alternatives—that is, $x = a_1 \circ \cdots \circ a_k$ for some $a_1, \ldots, a_k \in A$. We also assume that alternatives are not invertible—i.e., there are no $x, y \in X$ such that $x \circ y \circ x = x$.[2]

We assume a fixed population $N = \{1, 2, \ldots, n\}$ of individuals. An *Arrovian profile* $R = (R_1, \ldots, R_n)$ is an n-tuple of orderings on X, one for each individual in N. Our interpretation of these orderings is that xR_iy iff (according to profile R) x is at least as good for i as y. As usual, I_i denotes the symmetric part of R_i, P_i its asymmetric part. $\mathcal{R}$ is the set of all orderings on X. An *Arrovian social welfare function* (ASWF) is a function $f : \mathcal{D} \subseteq \mathcal{R}^n \to \mathcal{R}$ which assigns an overall betterness or social preference ordering to some set $\mathcal{D}$ of Arrovian profiles. For any profile $R \in \mathcal{D}$, let $\succcurlyeq_R$ denote the ordering $f(R)$.

We adopt the following domain assumption:

Extensive Domain $\mathcal{D} = \{\, R \in \mathcal{R}^n \mid (X, R_i, \circ) \text{ is an extensive structure for all } i \in N \,\}$.

By Proposition 1, every profile R in an extensive domain can be represented by a *utility profile* $U = (U_1, \ldots, U_n)$, where each U_i additively represents R_i—that is, $U_i(x) \geq U_i(y)$ iff xR_iy and $U_i(x \circ y) = U_i(x) + U_i(y)$—in which case we say that U itself additively represents R. For any Arrovian profile R, let $\mathcal{U}_R$ denote the set of all utility profiles that additively represent R, and let $\mathcal{U}_\mathcal{D} := \bigcup_{R \in \mathcal{D}} \mathcal{U}_R$.

Here is a simple example. Suppose there are $m \geq 3$ public goods. Let A be the set of standard unit vectors in $\mathbb{R}^m_+$. Each atomic alternative represents an arbitrarily small increment of a distinct public good. The concatenation operation is vector addition. Then $X = \mathbb{Z}^m_+ \setminus \{\mathbf{0}\}$ represents all possible bundles of those public goods in those increments, excluding the null bundle.[3] On this interpretation, Extensive Domain amounts to the assumption that each individual's preferences can be additively represented by a linear utility function (this of course implies nothing about their attitudes towards *risk*). Domains like this are considered by Kalai et al. (1979) and Le Breton and Weymark (2011, Example 9).

[2] This rules out the existence of an identity element. The framework can be easily generalized to accommodate identity elements, as long as atomic alternatives themselves are not invertible; we would simply require all alternatives to be either concatenations of atomic alternatives or identity elements. The proofs of Lemmas 1 and 2 would carry through unscathed.

[3] See footnote 2 on the exclusion of identity elements. The framework can also, I believe, be modified to accomodate sets of alternatives like $\mathbb{R}^m_+$ or $\mathbb{R}^m$, or any nonempty cone of a vector space, with atomic alternatives replaced by basis vectors. The proofs of Lemmas 1 and 2 should still hold.

Here is another example, based on Weymark (1981). Suppose there are $m \geq 3$ sources of income. Let A be the set of all vectors whose first k components are 1, all others 0, for all $k \in \{1, \ldots, m\}$. The unit of income can be as small as we like. The concatenation operation is again vector addition. Then $X = \{x \in \mathbb{Z}^m_+ \setminus \{\mathbf{0}\} \mid x_1 \geq x_2 \geq \cdots \geq x_m\}$ represents all income distributions, in the chosen unit, ordered from greatest to least. Each R_i, on this interpretation, represents individual i's ethical ranking of income distributions. The Monotonicity axiom of extensive measurement here corresponds to the "comonotonic independence" axiom for ranking such distributions (Weymark, 1981, Axiom 4). Extensive Domain amounts to the requirement that each individual's ethical ranking is of the "generalized Gini" form. An ASWF aggregates these generalized Gini rankings into a collective ranking.

Here is a third, more abstract example. Social welfare theorists are often interested in alternatives that are much richer than income distributions or bundles of goods. It is often supposed that an alternative is a possible *history* of the world, or of some society, over some period of time (Gibbard, 1982, 1984, Hylland, 1989, Broome, 2004, Blackorby et al., 2005, Dasgupta, 1995, 2007, 2009, Adler, 2019). Let A be a set of at least three such histories. We might imagine that histories can be concatenated into successive epochs of a single history: things proceed according to the first history, and then according to the second (much like Kahneman et al.'s concatenation of hedonic episodes). Then X is simply the closure of A under this concatenation operation. Our domain assumption then requires each individual's well-being in any history to be representable by the sum of that person's well-being across the epochs that make it up (see Nebel (2023c) for discussion).[4]

We now turn to the characterization of welfarism on our domain.

3.1 *Welfarism*

We will be interested in three standard Pareto principles:

Weak Pareto For any $x, y \in X$ and any $R \in \mathcal{D}$, if xP_iy for every $i \in N$, then $x \succ_R y$.

Pareto Indifference For any $x, y \in X$ and any $R \in \mathcal{D}$, if xI_iy for every $i \in N$, then $x \sim_R y$.

Strong Pareto For any $x, y \in X$ and any $R \in \mathcal{D}$, if xR_iy for every $i \in N$ then xRy; if, in addition, xP_iy for some $i \in N$, then $x \succ_R y$.

For any binary relation R on X and any $S \subseteq X$, let $R|_S$ denote the restriction of R to S. Arrow required, via his IIA condition, that, for any alternatives $x, y \in X$ and profiles $R, R' \in \mathcal{D}$, if $R_i|_{\{x,y\}} = R'_i|_{\{x,y\}}$ for every $i \in N$, then $x \succcurlyeq_R y$ iff $x \succcurlyeq_{R'} y$. We will instead use a weaker principle, which allows the social comparison of alternatives to depend not just on individuals' rankings of *those* alternatives, but also on their rankings of concatenations involving them. For any $S \subseteq X$, let S° denote the closure of S under $\circ$. Our weaker IIA principle is as follows:

[4] For a model that is even closer to Kahneman et al., we could let the alternatives be n-tuples of hedonic episodes. This would call for a different and more complicated domain restriction, analogous to those used when the alternatives are allocations of private goods (Le Breton and Weymark, 2011).

Ratio IIA For any $x, y \in X$ and any Arrovian profiles $R, R' \in \mathcal{D}$, if $R_i|_{\{x,y\}^\circ} = R'_i|_{\{x,y\}^\circ}$ for every $i \in N$, then $x \succsim_R y$ iff $x \succsim_{R'} y$.

The motivation for weakening Arrow's condition to Ratio IIA (and for its name) is that each $R_i|_{\{x,y\}^\circ}$ fully determines the *ratio* of $U_i(x)$ to $U_i(y)$ for any U_i that additively represents R_i. In a setting where such information is well-defined, there is no reason to exclude it as "irrelevant" to the comparison of alternatives.

For any utility profile $U \in \mathcal{U}_\mathcal{D}$ and alternative $x \in X$, the utility vector assigned by U to x is $U(x) = (U_1(x), \ldots, U_n(x))$. A social welfare function is *welfarist* iff the ordering it assigns to any profile is determined by a single *social welfare ordering* (SWO) $\succsim^*$ on the set of attainable utility vectors:

Welfarism There is a unique ordering $\succsim^*$ on $\mathbb{R}^n$ such that, for any $x, y \in X$, $R \in \mathcal{D}$, and $U \in \mathcal{U}_R$, $x \succsim_R y$ iff $U(x) \succsim^* U(y)$.

When f and $\succsim^*$ are so related, we say that $\succsim^*$ is *associated* with f.

The standard "welfarism theorem" in the framework of social welfare functionals appeals to Pareto Indifference and an IIA condition formulated in terms of numerical utilities (Bossert and Weymark, 2004, Theorem 2.2). We show (Proposition 2 in Appendix A) that Ratio IIA is equivalent to this utility-theoretic condition, given Extensive Domain. The standard welfarism theorem, however, assumes an unrestricted domain of utility profiles; it doesn't apply to the present setting because we have restricted the domain. Neither do analogous results for restricted domains due to Mongin (1994) and Weymark (1998).[5] Fortunately, we can still characterize Welfarism in terms of Pareto Indifference and Ratio IIA:

THEOREM 1. *If an ASWF f satisfies Extensive Domain, then f satisfies Pareto Indifference and Ratio IIA iff it satisfies Welfarism.*

The basic insight behind the proof of Theorem 1 is that the set of utility vectors attainable by the *atomic* alternatives is unrestricted. We are therefore able to define a SWO using only atomic alternatives, and then show how it determines the social ordering over all alternatives.

Not just any SWO is compatible with Extensive Domain, however—only those which are invariant to individual-specific similarity transformations of utilities:

Intrapersonal Ratio-Scale Invariance For any utility vectors $u, v, u', v' \in \mathbb{R}^n$, if for every $i \in N$ there is some $k_i > 0$ such that $u'_i = k_i u_i$ and $v'_i = k_i v_i$, then $u \succsim^* v$ iff $u' \succsim^* v'$.

(See Proposition 3 in Appendix A.) Intrapersonal Ratio-Scale Invariance plays a key role in the results of section 3.2.

[5]Mongin is concerned with profiles of mixture-preserving utility functions on a convex subset of a vector space; Weymark characterizes welfarism on "saturating" and "hypersaturating" *utility* domains. Our $\mathcal{U}_\mathcal{D}$ is not saturating because some pairs of nonatomic alternatives are, in Weymark's terminology, nontrivial but also not free and, thus, not connected. (This is compatible with $\mathcal{D}$ being a saturating *preference* domain in the sense of Kalai et al. (1979), Le Breton and Weymark (2011).)

A SWO $\succcurlyeq^*$ is *anonymous* iff, for every $u, v \in \mathbb{R}^n$, $u \sim^* v$ whenever there is a permutation $\sigma : N \to N$ such that $u_i = v_{\sigma(i)}$ for every $i \in N$. Given Welfarism, this condition is equivalent to the following property of social welfare functions (see Proposition 4 in Appendix A):

Anonymity For all profiles $R, R' \in \mathcal{D}$, if there is a permutation $\sigma : N \to N$ such that $R_i = R'_{\sigma(i)}$ for every $i \in N$, then $f(R) = f(R')$.

The various Pareto principles have obvious analogues in terms of the SWO as well. We do not state them separately. When we say that a SWO $\succcurlyeq^*$ violates or satisfies one of the Pareto principles, we mean that it violates or satisfies the obvious translation of that principle for $\succcurlyeq^*$.

3.2 *Possibilities and Impossibilities*

Arrow (1951) showed that if a social welfare function defined on an unrestricted domain satisfies Weak Pareto and his IIA condition, then it must be *dictatorial*: there must be some $i \in N$ such that, for any profile $R \in \mathcal{D}$ and alternatives $x, y \in X$, $x \succ_R y$ whenever xP_iy. If we weaken Arrow's domain and independence axioms to Extensive Domain and Ratio IIA, this implication is avoided, and even Strong Pareto can be satisfied. For there are nondictatorial SWOs on $\mathbb{R}^n$ which satisfy Intrapersonal Ratio-Scale Invariance and Strong Pareto. Here is a two-person example, based on a class of SWOs axiomatized by Naumova and Yanovskaya (2001); it is easily generalized to larger populations:

EXAMPLE 1. Take any $u, v \in \mathbb{R}^2$. Suppose $\mathrm{sgn}(u_i) = \mathrm{sgn}(v_i)$ for both $i \in \{1, 2\}$ (where $\mathrm{sgn}(0) = 0$). Then

$$u \succcurlyeq^* v \text{ iff } |u_1|^{\mathrm{sgn}(u_1)}|u_2|^{\mathrm{sgn}(u_2)} \geq |v_1|^{\mathrm{sgn}(v_1)}|v_2|^{\mathrm{sgn}(v_2)}.$$

If $\mathrm{sgn}(u_i) \neq \mathrm{sgn}(v_i)$ for some $i \in \{1, 2\}$, then u and v are ranked according to the following linear ordering of the quadrants and their boundaries (plus the origin):

$$(+,+) \succ (+,0) \succ (0,+) \succ (0,0) \succ (-,+) \succ (+,-) \succ (0,-) \succ (-,0) \succ (-,-).$$

This SWO satisfies Intrapersonal Ratio-Scale Invariance and Strong Pareto but is not dictatorial (see Naumova and Yanovskaya, 2001).

Orderings of the kind described in Example 1 satisfy a number of further properties. They are, *within* each quadrant or boundary, anonymous and continuous. They are also representable by a real-valued social utility function (Naumova and Yanovskaya, 2001, Corollary 4.1). They are not fully anonymous, however. Indeed, the failure of anonymity applies more generally:

THEOREM 2. *There is no ASWF that satisfies Extensive Domain, Anonymity, Ratio IIA, and Strong Pareto (or, when n is even, Weak Pareto and Pareto Indifference).*

Theorem 2 may suggest that Anonymity is too much to ask of a social welfare function in the present environment. However, the Arrovian axioms can be strengthened in a way that requires the social welfare function to be *strongly* dictatorial: there must be some $i \in N$ such that, for any $R \in \mathcal{D}$ and $x, y \in X$, $x \succcurlyeq_R y$ iff xR_iy. One way to do this is to require the SWO to be continuous. Tsui and Weymark (1997) show that a continuous SWO which satisfies Weak Pareto and Intrapersonal Ratio-Scale Invariance must be strongly dictatorial (see also Nebel, 2023b). In my view, however, the ethical content of and motivation for continuity is not obvious. It is standardly motivated by considerations regarding slight measurement errors (e.g., by d'Aspremont and Gevers, 2002, 496). But, while sensitivity to such errors may be unfortunate, it's far from obvious that the ethical ordering of utility vectors *shouldn't* be sensitive to such errors. In order to figure out which alternatives are better or worse, why shouldn't we have to identify the correct profile (as opposed to one that is merely arbitrarily "close" to the correct profile)? Especially given the distinguished role of neutral elements in an extensive structure, discontinuities when some utilities are zero in particular do not seem unreasonable. We therefore consider a different requirement which does not, by itself, entail continuity:

Extensive Social Preference For each profile $R \in \mathcal{D}$, the triple $(X, \succcurlyeq_R, \circ)$ is an extensive structure.

The axioms of extensive measurement may of course be questioned in this context. But *if* we take individual welfare to be extensively measurable, we might reasonably take the social ordering to be extensively measurable as well. For example, on the successive-epochs interpretation of $\circ$, Monotonicity can be motivated by the thought that a choice between histories $c \circ a$ and $c \circ b$ is relevantly like choosing between futures a and b after a past epoch c; what happened in previous epochs, we might think, should not matter for future evaluation except insofar as it affects people today or in the future, in which case those effects should be considered in the valuation of a and b. As in the case of individual welfare, my view is that the applicability of extensive measurement to social evaluation should be regarded as an open question, which depends on the nature of the alternatives, the interpretation of $\circ$, as well as our general ethical commitments.

Our second negative result for ASWFs is as follows:

THEOREM 3. *If a social welfare function f satisfies Extensive Domain, then f satisfies Ratio IIA, Weak Pareto, and Extensive Social Preference iff it is strongly dictatorial.*

The proof goes as follows. First, we show that Extensive Domain, Ratio IIA, Weak Pareto, and Extensive Social Preference together yield a "semistrong" Pareto principle which entails Pareto Indifference (Lemma 3). These axioms therefore entail Welfarism, by Theorem 1. Next, given Welfarism, Extensive Social Preference is equivalent to $(\mathbb{R}^n, \succcurlyeq^*, +)$ being an extensive structure (Lemma 4). Thus, by Proposition 1, $\succcurlyeq^*$ must be additively representable by a social utility function $W : \mathbb{R}^n \to \mathbb{R}$. Semistrong Pareto forces this function to be of the weighted utilitarian form—i.e., a linear combination of utilities—with nonnegative weights (Lemma 5). Finally, Weak Pareto and Intrapersonal Ratio-Scale Invariance together require exactly one person's weight to be positive; this proves the theorem. An obvious corollary of this result is that there is no ASWF that satisfies Extensive Domain, Ratio IIA, Strong Pareto, and Extensive Social Preference.

Sen (1977b, 80) influentially claims that "n-tuples of individual orderings," as in Arrow's framework, "are informationally inadequate for representing conflicts of interests." The lesson I am inclined to draw from Theorems 2 and 3 is that—at least within the confines of welfarism—Arrow's framework is still inadequate for this task even when the individual orderings are supplemented by an extensive concatenation operation. For I take Anonymity to be a fundamental requirement of impartiality. And it seems reasonable to want social preferences to have the same structure as individual welfare. We will see in section 4 that both of these desiderata can be satisfied in an informationally richer framework that allows for interpersonal comparisons of well-being.

4. Generalized Variable-Population Social Welfare Functions

We now generalize the framework of section 3 in two ways.

First, in order to distinguish classical utilitarianism from its variants, we need the population to vary between alternatives. Following Blackorby et al. (2005), let $\mathbb{N} = \{1, 2, \dots\}$ represent the set of all possible individuals. Let $\mathcal{P}$ denote the set of all finite, nonempty subsets of $\mathbb{N}$. Let X once again denote the set of alternatives. Each alternative x has a *population*, $N(x) \in \mathcal{P}$, the set of individuals who would exist (or be members of some particular society) if x obtained. For any individual $i \in \mathbb{N}$, $X_i \subseteq X$ denotes the set of alternatives in which i exists—i.e., $X_i := \{x \in X : i \in N(x)\}$. For any $N \in \mathcal{P}$, X^N denotes the set of alternatives whose populations are N—i.e., $X^N := \{x \in X : N(x) = N\}$.

We assume, as before, that X is closed under a concatenation operation $\circ : X \times X \to X$. For each $i \in \mathbb{N}$, there is a set $A^{\{i\}} \subset X^{\{i\}}$ of at least three *atomic* alternatives in which only i exists.[6] (Recall that an alternative a is atomic iff there are no $x, y \in X$ such that $x \circ y = a$.) There may also be atomic alternatives with larger populations, though we don't need there to be. We assume, as before, that all nonatomic alternatives are identical to the concatenation of some atomic alternatives, and that there are no invertible alternatives. We also require that, for all $x, y \in X$, $N(x \circ y) = N(x) \cup N(y)$. These assumptions together imply that, for each population $N \in \mathcal{P}$, there are infinitely many alternatives in X^N.

Some of our examples from section 3 can be easily adapted to this setting. Suppose again, for example, that there are $m \geq 3$ public goods. Each alternative is a pair consisting of a bundle of those goods and a population who can consume them. The atomic alternatives are the standard unit vectors of $\mathbb{R}^m_+$ paired with each singleton individual. Alternatives are concatenated by vector addition and union of populations: i.e., $(x, N) \circ (y, M) = (x + y, N \cup M)$. For any population $N \in \mathcal{P}$, X^N will contain all bundles in $\mathbb{Z}^m_+ \setminus \{\mathbf{0}\}$ paired with N, and $X = \bigcup_{N \in \mathcal{P}} X^N$. Or suppose, again, that the alternatives are possible histories of the world over some duration of time. Each history has a unique population: the set of individuals who exist at some point in that history. The atomic alternatives are histories in which only a single person exists. We imagine again

[6] Compare Blackorby et al. (2005), who assume $|X^N| \geq 3$ for all $N \in \mathcal{P}$. It is unrealistic, of course, to suppose that any individual could exist without her parents ever existing. Weymark (2019) has raised this concern for the intertemporal framework of Blackorby et al. (1995). But it also applies to standard variable-population frameworks which, like ours, lack an explicit time dimension.

that histories can be concatenated into successive epochs of a single history, with the population of the larger history being the union of the populations of its subhistories.

Second, in order to avoid the impossibilities which arose in our Arrovian framework, we include interpersonal comparisons of well-being. Such comparisons are ubiquitous in variable-population frameworks (see, e.g., Blackorby et al., 1995, Broome, 2004, Asheim and Zuber, 2014, Pivato, 2020). Indeed, such frameworks typically assume not only that interpersonal comparisons are meaningful, but also that there is a meaningful "neutral level" of welfare which divides lives that are "worth living" from those that are not. Unlike these authors, we incorporate interpersonal comparisons in entirely *relational* terms, and claims about positive, neutral, or negative well-being will be *derived* rather than assumed.

As mentioned in section 1, interpersonal comparisons can be formalized as a relation over alternative–individual pairs. For any $x \in X$ and $i \in N(x)$, I call the pair (x,i) a *life*. (By a "life," I *just* mean a pair of this form; the definition is simply meant to exclude pairs of the form (y,j) where $j \notin N(y)$.) Let $\mathcal{L} := \{ (x,i) \in X \times \mathbb{N} \mid i \in N(x) \}$ denote the set of all lives. An *interpersonal profile* R is an ordering on $\mathcal{L}$. The intended interpretation is that $(x,i)R(y,j)$ iff x is at least as good for i (according to profile R) as y is for j—or, equivalently, that i is at least as well off in x as j is in y. Such comparisons are often understood in terms of the "extended preferences" of a social observer—preferring, for one's own sake, to be one person (or to be in their "position" in some sense, having all of their tastes, values, and so on) in one alternative rather than another person in another alternative (Suppes, 1966, Sen, 1970, 1997, Arrow, 1977, Harsanyi, 1977, Suzumura, 1996, Kolm, 1998, Adler, 2014). But I do not insist on this or any other particular way of making interpersonal comparisons. My own view is that, clearly, some people are better off than others, and any plausible theory of well-being must be able to accommodate such comparisons (Scanlon, 1991, Hausman, 1995, Broome, 1999, Greaves and Lederman, 2018). (This is not to say, of course, that it is easy to explain what makes such comparisons true, or to discover which ones are true.)

Let $\mathcal{R}_{\mathcal{L}}$ denote the set of all orderings on $\mathcal{L}$, and $\mathcal{R}_X$ the set of all orderings on X. Adapting terminology from Hammond (1976), a *generalized social welfare function* (GSWF) is a mapping $f : \mathcal{D} \subseteq \mathcal{R}_{\mathcal{L}} \to \mathcal{R}_X$. For any interpersonal profile $R \in \mathcal{D}$, we write $\succcurlyeq_R$ for $f(R)$. Note that, absent further assumptions, $\succcurlyeq_R$ can wildly diverge from R. A social planner might prefer, for example, that she herself have descendants living in the future rather than someone else having descendants with better lives.

In order for our interpersonal profiles to be extensively measurable, we need a concatenation operation on the set of lives. Instead of taking such an operation as primitive, we define it here in terms of the alternative-concatenation operation $\circ$ which we already have, at the cost of two additional assumptions. The first says that for any individuals i and j and alternatives x and y in which they (respectively) exist, there is an individual k and alternatives x' and y' such that x' and y' are just as good for k as x and y are for i and j (respectively); and, in the special case where $i = j$, $x' \circ y'$ must be just as good for k as $x \circ y$ is for i:

Matching For any interpersonal profile $R \in \mathcal{D} \subseteq \mathcal{R}_{\mathcal{L}}$ and any $(x,i),(y,j) \in \mathcal{L}$, there is some $k \in \mathbb{N}$ and $x',y' \in X_k$ such that $(x',k)I(x,i)$ and $(y',k)I(y,j)$, and, for any such k, x', y', if $i = j$ then $(x \circ y, i)I(x' \circ y', k)$.

Matching lets us, for any $R \in \mathcal{D}$, define an operation $\oplus^R : \mathcal{L} \times \mathcal{L} \to \mathcal{L}$ as follows: for any $(x,i),(y,j) \in \mathcal{L}$, let $(x,i) \oplus^R (y,j) = (x' \circ y', k)$ for some k, x', y' such that $(x',k)I(x,i)$ and $(y',k)I(y,j)$. (When there are multiple such k, x', y', the choice can be arbitrary, since Matching requires all such choices to be equally good according to R.) Thus, any number of lives led by distinct individuals can, in any profile, be concatenated into a single life. This is compatible, however, with a *social* preference for the existence of the many rather than the single "utility monster" (Nozick, 1974). Further axioms are needed to rule out such a preference.[7]

The axioms of extensive measurement will tell us, for any alternatives x and y and any individual i who exists in both x and y, how to value i's life in $x \circ y$ in terms of her life in x and her life in y: $(x \circ y, i)I(x,i) \oplus^R (y,i)$. But what if i exists in x but not y? A natural hypothesis is that, since i does not even exist in y, concatenating y to x should not affect i's well-being. This is our second assumption:

Irrelevance of Nonexistence For any interpersonal profile $R \in \mathcal{D}$, $x, y \in X$, and $i \in N(x) \setminus N(y)$, $(x \circ y, i)I(x,i)$.

This seems a plausible extension of the orthodox view that nothing can be better or worse for a person who does not exist (Broome, 2004, Blackorby et al., 2005).

We can now state our domain condition:

Interpersonal Extensive Domain A profile R is in $\mathcal{D}$ iff R satisfies Matching and Irrelevance of Nonexistence, and $(\mathcal{L}, R, \oplus^R)$ is an extensive structure.

Given Interpersonal Extensive Domain, each profile $R \in \mathcal{D}$ can be additively represented by a real-valued utility function. $U : \mathcal{L} \to \mathbb{R}$ additively represents a profile $R \in \mathcal{D}$ iff, for all $(x,i),(y,j) \in \mathcal{L}$, $U(x,i) \geq U(y,j)$ iff $(x,i)R(y,j)$, and $U((x,i) \oplus^R (y,j)) = U(x,i) + U(y,j))$. As before, let $\mathcal{U}_R$ denote the set of all utility functions that additively represent R, and $\mathcal{U}_{\mathcal{D}} := \bigcup_{R \in \mathcal{D}} \mathcal{U}_R$.

4.1 *Variable-Population Welfarism*

The various Pareto conditions have the same interpretation as in section 3, so we do not state them separately here; see Appendix C.

The reformulation of Ratio IIA in this framework requires some care because our life-concatenation operation is profile-dependent. For any subset of alternatives $S \subseteq X$, let $L(S) := \bigcup_{x \in S} \{x\} \times N(x)$ denote the set of all lives led among the alternatives in S. For any such S and any profile R, let $L(S)^{\oplus^R}$ denote the closure of $L(S)$ under $\oplus^R$. Given any $S, T \subseteq X$ and any profiles R, R', a *profile isomorphism* is a bijection $\phi : L(S)^{\oplus^R} \to L(T)^{\oplus^{R'}}$ such that, for all $(x,i),(y,j) \in L(S)$:

[7] Note also that what allows us to concatenate any number of lives is not just Matching, but also the assumption that X is closed under $\circ$. Those who wish to avoid this implication might therefore prefer a version of extensive measurement in which the concatenation operation is restricted (Krantz et al., 1971, sec. 3.4). Analogous implications also hold in the standard utility-theoretic framework, where for any number of individuals and any utilities they might attain, there is some individual who can attain, in some outcome and some profile, the sum of those utilities.

(i) $(x,i)R(y,j)$ iff $\phi(x,i)R'\phi(y,j)$, and

(ii) $\phi((x,i)\oplus^R(y,j)) = \phi(x,i)\oplus^{R'}\phi(y,j)$.

Our Independence of Irrelevant Alternatives condition will be

Interpersonal Ratio IIA For all $R, R' \in \mathcal{D}$ and $x, y \in X$, if there is a profile isomorphism $\phi : L(\{x,y\})^{\oplus^R} \to L(\{x,y\})^{\oplus^{R'}}$ such that $\phi(x,i) = (x,i)$ and $\phi(y,j) = (y,j)$ for all $i \in N(x)$ and $j \in N(y)$, then $x \succcurlyeq_R y$ iff $x \succcurlyeq_{R'} y$.

As with Ratio IIA, this principle is equivalent to a more familiar condition formulated in terms of utility functions (see Proposition 5 in Appendix C).

For any utility profile $U : \mathcal{L} \to \mathbb{R}$, let $U(x,\cdot) : N(x) \to \mathbb{R}$ denote x's *utility distribution* in profile U. For any population $N \in \mathcal{P}$, $\mathbb{R}^N$ denotes the set of all utility distributions with domain N. The set of all utility distributions is $\Omega := \bigcup_{N\in\mathcal{P}} \mathbb{R}^N$. We call these "distributions" rather than "vectors" because Ω is not a vector space: we cannot add together utility distributions with different populations. The variable-population analogue of Welfarism is

Variable-Population Welfarism There is a unique SWO $\succcurlyeq^*$ on Ω such that, for any $R \in \mathcal{D}$, $U \in \mathcal{U}_R$, and $x, y \in X$, $x \succcurlyeq_R y$ iff $U(x,\cdot) \succcurlyeq^* U(y,\cdot)$.

As in section 3, the key to our welfarism theorem in this setting is that the set of attainable utility distributions for the atomic alternatives is unrestricted. We have not assumed the existence of atomic alternatives for each population, however—only for each *singleton* population. But, for any population, we can find an atomic alternative for each member of the population and concatenate them to form an alternative in which all of those individuals exist. This is the strategy behind the proof of Theorem 4 in Appendix C:

THEOREM 4 (Variable-Population Welfarism Theorem). *If a GSWF f satisfies Interpersonal Extensive Domain, then f satisfies Pareto Indifference and Interpersonal Ratio IIA iff it satisfies Variable-Population Welfarism.*

As in the fixed-population setting, Interpersonal Extensive Domain requires the SWO to be invariant to similarity transformations of individual utilities. However, the same transformation must be applied to all individuals in order to preserve interpersonal comparisons:

Interpersonal Ratio-Scale Invariance For every $u, v \in \Omega$ and positive real number k, $u \succcurlyeq^* v$ iff $ku \succcurlyeq^* kv$.

(See Proposition 6 in Appendix C.) This weaker invariance condition is what allows us to avoid the negative results of our Arrovian setting.

4.2 *A Qualitative Axiomatization of Classical Utilitarianism*

In the present framework, classical utilitarianism has a natural qualitative formulation. For any alternative $x \in X$ and profile $R \in \mathcal{D}$, let $\bigoplus^R_{i\in N(x)}(x,i)$ denote the concatenation of all the individuals' lives in x in arbitrary order.

Classical Utilitarianism For any $x, y \in X$ and $R \in \mathcal{D}$, $x \succcurlyeq_R y$ iff $\bigoplus^R_{i \in N(x)}(x,i) R \bigoplus^R_{i \in N(y)}(y,i)$.

Given Interpersonal Extensive Domain, Classical Utilitarianism is equivalent to the claim that, for any $x, y \in X$, $R \in \mathcal{D}$, and $U \in \mathcal{U}_R$, $x \succcurlyeq_R y$ iff $\sum_{i \in N(x)} U(x,i) \geq \sum_{i \in N(y)} U(y,i)$. For each $U \in \mathcal{U}_R$ additively represents R, so $U(\bigoplus^R_{i \in N(x)}(x,i)) = \sum_{i \in N(x)} U(x,i)$ and $U(\bigoplus^R_{i \in N(y)}(y,i)) = \sum_{i \in N(y)} U(y,i)$.

Our axiomatization of Classical Utilitarianism appeals to Weak Pareto, Interpersonal Ratio IIA, and two further conditions. First, we require the restriction of the social ordering to the alternatives facing a *fixed* population to be invariant to permutations on that fixed set of individuals:

Fixed-Population Anonymity For any $N \in \mathcal{P}$ and $R, R' \in \mathcal{D}$, if there is a permutation $\sigma : N \to N$ and a profile isomorphism $\phi : L(X^N)^{\oplus^R} \to L(X^N)^{\oplus^{R'}}$ such that $\phi(x,i) = (x, \sigma(i))$ for all $(x,i) \in L(X^N)$, then for all $x, y \in X^N$, $x \succcurlyeq_R y$ iff $x \succcurlyeq_{R'} y$.

(See Appendix D for the utility-theoretic analogue of this condition.) Our second principle has much the same interpretation as in subsection 3.2:

Extensive Social Preference For all $R \in \mathcal{D}$, $(X, \succcurlyeq_R, \circ)$ is an extensive structure.

Analogues of these two principles led to our negative results for ASWFs in section 3. It is therefore noteworthy that they are not just compatible with our other axioms in the generalized framework; they lead, in conjunction with the other axioms, to Classical Utilitarianism:

THEOREM 5. *If a GSWF f satisfies Interpersonal Extensive Domain, then f satisfies Interpersonal Ratio IIA, Weak Pareto, Fixed-Population Anonymity, and Extensive Social Preference iff f satisfies Classical Utilitarianism.*

The strategy behind the proof is this. We first show that, given our axioms, adding an individual with "zero" utility to a population is always a matter of indifference (Proposition 8). We are therefore able to strengthen Variable-Population Welfarism by constructing an "extended" SWO on the space $\mathbb{R}^\infty$ of all infinite sequences with finite support (Lemma 8). Fixed-Population Anonymity then requires this extended SWO to be fully anonymous (Lemma 9). By Extensive Social Preference and Proposition 1, the extended ordering can be additively represented by a real-valued social utility function. The proof of Theorem 5 then amounts to showing that this additive representation is of the weighted utilitarian form and that all weights must be equal.

The reason why our axioms lead to such a different result in this framework is the presence of interpersonal comparisons. The richer informational basis provided by interpersonal comparability leads to a considerably weaker invariance condition, which avoids the impossibilities that arose in the Arrovian framework.

A great deal of the work in proving Theorem 5 is done by Extensive Social Preference. Clearly this is a very strong condition. We might therefore want to know how Classical Utilitarianism might be derived in this framework without assuming Extensive

Social Preference. An answer is provided in Appendix E. Theorem 6 there characterizes Classical Utilitarianism in terms of Interpersonal Extensive Domain, Strong Pareto, Interpersonal Ratio IIA, and five principles imposed directly on the SWO. This theorem illustrates how the classical utilitarian is committed to a large number of independent principles, some of which lack an obvious ethical motivation or qualitative interpretation. One thing we learn from Theorem 5 is how many of these commitments can be weakened, unified, and subsumed in a simple way via Extensive Social Preference.

5. Conclusion

Extensive measurement gives rise to natural weakenings of Arrow's conditions which are jointly consistent even when welfare is not interpersonally comparable. But, while there are nondictatorial ASWFs which satisfy Extensive Domain, Ratio IIA, and Strong Pareto, there are none which also satisfy Anonymity or Extensive Social Preference. In the generalized framework, by contrast, analogues of these conditions are not only consistent; together, they uniquely characterize Classical Utilitarianism.

Extensive measurement, as we have seen, does much more for the classical utilitarian than just giving "meaning to the utilities to be added" (Arrow, 1973, 255). It is, when applied at the social level, what distinguishes classical utilitarianism from other anonymously welfarist approaches to social choice, including other versions of utilitarianism.

Appendix A: Proofs for Section 3.1

We first show that Ratio IIA is equivalent, on our domain, to the following familiar condition:

Utility IIA For any $x, y \in X$, $R, R' \in \mathcal{D}$, and $U \in \mathcal{U}_R, U' \in \mathcal{U}_{R'}$, if $U_i(x) = U_i'(x)$ and $U_i(y) = U_i'(y)$ for every $i \in N$, then $x \succcurlyeq_R y$ iff $x \succcurlyeq_{R'} y$.

Proposition 2. *If an ASWF f satisfies Extensive Domain, then f satisfies Ratio IIA iff it satisfies Utility IIA.*

Proof. Suppose that f satisfies Extensive Domain and Ratio IIA. Take some $x, y \in X$, $R, R' \in \mathcal{D}$, and $U \in \mathcal{U}_R, U' \in \mathcal{U}_{R'}$ such that $U_i(x) = U_i'(x)$ and $U_i(y) = U_i'(y)$ for every $i \in N$. For each $z \in \{x, y\}^\circ$, there must be nonnegative integers n and m such that $U_i(z) = nU_i(x) + mU_i(y)$ and $U_i'(z) = nU_i'(x) + mU_i'(y)$ for every $i \in N$. Thus $U_i(z) = U_i'(z)$ for all $z \in \{x, y\}^\circ$. We must therefore have $R_i|_{\{x,y\}^\circ} = R_i'|_{\{x,y\}^\circ}$ for every $i \in N$, so $x \succcurlyeq_R y$ iff $x \succcurlyeq_{R'} y$ by Ratio IIA, and Utility IIA is therefore satisfied.

For the other direction, suppose that f satisfies Extensive Domain and Utility IIA. Take some $x, y \in X$ and $R, R' \in \mathcal{D}$ such that $R_i|_{\{x,y\}^\circ} = R_i'|_{\{x,y\}^\circ}$ for every $i \in N$. Take some $U \in \mathcal{U}_R$ and $V \in \mathcal{U}_{R'}$. For any $w, z \in \{x, y\}^\circ$ and $i \in N$, we have wR_iz iff $wR_i'z$ iff $V_i(w) \geq V_i(z)$, and $V_i(w \circ z) = V_i(w) + V_i(z)$. It follows that each $V_i|_{\{x,y\}^\circ}$ additively represents $R_i|_{\{x,y\}^\circ}$. Since $U \in \mathcal{U}_R$, $U_i|_{\{x,y\}^\circ}$ also additively represents $R_i|_{\{x,y\}^\circ}$. Thus, by the uniqueness component of Proposition 1, for each $i \in N$ there must be some $k_i > 0$ such that $V_i = k_iU_i$. Now let $U_i' = (1/k_i)V_i$ for every $i \in N$, so that $U' =$

$(U'_1,\ldots,U'_n)\in\mathcal{U}_{R'}$ and $U'_i|_{\{x,y\}^\circ}=U_i|_{\{x,y\}^\circ}$. We have $U_i(x)=U'_i(x)$ and $U_i(y)=U'_i(y)$ for every $i\in N$, so $x\succcurlyeq_R y$ iff $x\succcurlyeq_{R'} y$ by Utility IIA, and Ratio IIA is therefore satisfied. (Indeed, since $U_i|_{\{x,y\}^\circ}=U'_i|_{\{x,y\}^\circ}$, we also have the stronger consequence that $\succcurlyeq_R|_{\{x,y\}^\circ}=\succcurlyeq_{R'}|_{\{x,y\}^\circ}$.)

□

The following lemma plays a key role in the proof of Theorem 1; it appeals crucially to our assumption that there are at least three atomic alternatives:

LEMMA 1. *If an ASWF f satisfies Extensive Domain, then for any alternatives $x,y\in X$, utility profile $U\in\mathcal{U}_{\mathcal{D}}$, and any utility vector $w\in\mathbb{R}^n$, there is an atomic alternative $a\in A\subset X$ and some profile $V\in\mathcal{U}_{\mathcal{D}}$ such that $V(x)=U(x)$, $V(y)=U(y)$, and $V(a)=w$.*

PROOF. Any alternatives x and y are either atomic or concatenations of atomic alternatives. Thus there are atomic alternatives $a_1,\ldots,a_k\in A$ and nonnegative integers $m_1,\ldots,m_k$ (at least one of which is positive) and $m'_1,\ldots,m'_k$ (at least one of which is positive), where either m_i or m'_i is positive for every $i\in\{1,\ldots,k\}$, such that for any profile $U\in\mathcal{U}_{\mathcal{D}}$, $U(x)=\sum_{i=1}^k m_iU(a_i)$ and $U(y)=\sum_{i=1}^k m'_iU(a_i)$, by Extensive Domain.

Fix a particular profile U. If $k<3$, the proof is trivial: since there are at least three atomic alternatives, simply let $V(a_i)=w$ for some $a_i\in A\setminus\{a_1,a_2\}$ and $V(a_j)=U(a_j)$ for all $j\neq i$. This obviously preserves $V(x)=U(x)$ and $V(y)=U(y)$. Suppose instead, then, that $k\geq 3$.

We know that the following system is satisfied:

$$\begin{pmatrix} m_1 & \cdots & m_k\\ m'_1 & \cdots & m'_k\end{pmatrix}\begin{pmatrix} U_1(a_1) & U_2(a_1) & \cdots & U_n(a_1)\\ \vdots & \vdots & \ddots & \vdots\\ U_1(a_k) & U_2(a_k) & \cdots & U_n(a_k)\end{pmatrix}=\begin{pmatrix} U_1(x) & \cdots & U_n(x)\\ U_1(y) & \cdots & U_n(y)\end{pmatrix}.$$

Write the above system as $\mathbf{MA}=\mathbf{U}$, and pick any vector $w\in\mathbb{R}^n$.

Since we have $\mathbf{MA}=\mathbf{U}$, we know (by the Rouché-Capelli theorem) that $\operatorname{rank}(\mathbf{M})=\operatorname{rank}(\mathbf{M}\mid\mathbf{U})$. And since $k\geq 3>\operatorname{rank}(\mathbf{M})$, there must be some $2\times(k-1)$ submatrix $\hat{\mathbf{M}}$ of $\mathbf{M}$ such that $\operatorname{rank}(\hat{\mathbf{M}})=\operatorname{rank}(\mathbf{M})$. (Just find some 2×2 submatrix of $\mathbf{M}$ with $\operatorname{rank}(\mathbf{M})$-many linearly independent columns—there must be at least one—and delete a column not in that submatrix.)

Without loss of generality let

$$\hat{\mathbf{M}}=\begin{pmatrix} m_1 & \cdots & m_{k-1}\\ m'_1 & \cdots & m'_{k-1}\end{pmatrix},\ \hat{\mathbf{U}}=\begin{pmatrix} U_1(x)-m_kw_1 & \cdots & U_n(x)-m_kw_n\\ U_1(y)-m'_kw_1 & \cdots & U_n(y)-m'_kw_n\end{pmatrix}.$$

It is not difficult to see that $\operatorname{rank}(\hat{\mathbf{M}}\mid\hat{\mathbf{U}})=\operatorname{rank}(\mathbf{M}\mid\mathbf{U})$, since $\hat{\mathbf{U}}=\mathbf{U}-\binom{m_k}{m'_k}w$ and $\operatorname{rank}(\mathbf{M}\mid\mathbf{U})=\operatorname{rank}(\mathbf{M})=\operatorname{rank}(\hat{\mathbf{M}})$. Thus, $\operatorname{rank}(\hat{\mathbf{M}}\mid\hat{\mathbf{U}})=\operatorname{rank}(\hat{\mathbf{M}})$. So (by Rouché-Capelli again) there is a $(k-1)\times n$ matrix $\mathbf{B}$ such that $\hat{\mathbf{M}}\mathbf{B}=\hat{\mathbf{U}}$. If we simply let $V(a_j)$ equal the jth row of $\mathbf{B}$ for all $j\in\{1,\ldots,k-1\}$, and $V(a_k)=w$, we have $V(x)=U(x)$ and $V(y)=U(y)$, as desired. □

Next we derive a multi-profile "neutrality" property for the atomic alternatives:

LEMMA 2. *If an ASWF f satisfies Extensive Domain, Pareto Indifference, and Utility IIA, then for any $a,b,a',b' \in A$, $R,R' \in \mathcal{D}$, and $U \in \mathcal{U}_R, U' \in \mathcal{U}_{R'}$, if $U'(a') = U(a)$ and $U'(b') = U(b)$, then $a \succcurlyeq_R b$ iff $a' \succcurlyeq_{R'} b'$.*

PROOF. Take any $a,b,a',b' \in A$, $R,R' \in \mathcal{D}$, and $U \in \mathcal{U}_R, U' \in \mathcal{U}_{R'}$. Suppose $U'(a') = U(a) = u$ and $U'(b') = U(b) = v$.

Given Extensive Domain, the domain of utility profiles is unrestricted with respect to atomic alternatives. And we have assumed there to be at least three atomic alternatives. So there must be some $c \in A \setminus \{b,b'\}$, $R^1,R^2,R^3 \in \mathcal{D}$, and $U^1 \in \mathcal{U}_{R^1}, U^2 \in \mathcal{U}_{R^2}, U^3 \in \mathcal{U}_{R^3}$ such that

1. $U^1(a) = U^1(c) = u$ and $U^1(b) = v$,
2. $U^2(c) = u$ and $U^2(b) = U^2(b') = v$, and
3. $U^3(a') = U^3(c) = u$ and $U^3(b') = v$.

These profile assignments are displayed in Table 1.

	a	a'	b	b'	c
U	u		v		
U^1	u		v		u
U^2			v	v	u
U^3		u		v	u
U'		u		v	

TABLE 1. $a \succcurlyeq_R b$ iff $a' \succcurlyeq_{R'} b'$

Utility IIA and Pareto Indifference (plus transitivity) imply, in alternating order, that aRb iff aR^1b iff cR^1b iff cR^2b iff cR^2b' iff cR^3b' iff $a'R^3b'$ iff $a'R'b$. Thus aRb iff $a'R'b'$.

□

PROOF OF THEOREM 1. Suppose that f satisfies Extensive Domain, Pareto Indifference, and Ratio IIA. By Proposition 2, f also satisfies Utility IIA. Define a SWO $\succcurlyeq^*$ on $\mathbb{R}^n$ as follows: for any $u,v \in \mathbb{R}^n$, $u \succcurlyeq^* v$ iff for some atomic $a,b \in A$, $R \in \mathcal{D}$, and $U \in \mathcal{U}_R$, $U(a) = u$, $U(b) = v$, and $a \succcurlyeq_R b$.

For any $u,v \in \mathbb{R}^n$, there are $a,b \in A$, $R \in \mathcal{D}$, and $U \in \mathcal{U}_R$ such that $U(a) = u$ and $U(b) = v$. So, by the completeness of $\succcurlyeq_R$, either $u \succcurlyeq^* v$ or $v \succcurlyeq^* u$.

Now take any $x,y \in X$, $R \in \mathcal{D}$, and $U \in \mathcal{U}_R$. We show that $x \succcurlyeq_R y$ if and only if $U(x) \succcurlyeq^* U(y)$. Suppose without loss of generality that $U(x) = u$ and $U(y) = v$.

Use Lemma 1 to find an $R^1 \in \mathcal{D}$, $U^1 \in \mathcal{U}_{R^1}$, and atomic $a \in A$ such that $U^1(a) = U^1(x) = u$ and $U^1(y) = v$, and then another $R^2 \in \mathcal{D}$, $U^2 \in \mathcal{U}_{R^2}$, and atomic $b \in A$ such that $U^2(a) = u$ and $U^2(b) = U^2(y) = v$. (See Table 2.) Utility IIA and Pareto Indifference (given transitivity) imply, in alternating order, that $x \succcurlyeq_R y$ iff $x \succcurlyeq_{R^1} y$ iff $a \succcurlyeq_{R^1} y$ iff $a \succcurlyeq_{R^2} y$ iff

$a \succsim_{R^2} b$. Lemma 2 then implies that for any $a', b' \in A$, $R' \in \mathcal{D}$, and $U' \in \mathcal{U}_{R'}$ such that $U'(a') = u$ and $U'(b') = v$, $a' \succsim_{R'} b'$ iff $a \succsim_R b$. It follows that $x \succsim_R y$ iff $U(x) \succsim^* U(y)$, as desired.

	x	y	a	b	a'	b'
U	u	v				
U^1	u	v	u			
U^2		v	u	v		
U'					u	v

TABLE 2. $x \succsim_R y$ iff $a \succsim_{R^2} b$ iff $a' \succsim_{R'} b'$.

To show that $\succsim^*$ is transitive, suppose that $u \succsim^* v$ and $v \succsim^* w$. There must be some $R \in \mathcal{D}$, $U \in \mathcal{U}_R$, and $a, b, c \in A$ such that $U(a) = u$, $U(b) = v$, and $U(c) = w$. Given what we just showed above, we must have $a \succsim_R b \succsim_R c$, and thus $a \succsim_R c$ by the transitivity of $\succsim_R$. Thus $u \succsim^* w$.

It is easy to see that Welfarism implies Pareto Indifference and Utility IIA and thus, given Extensive Domain and Proposition 2, Ratio IIA. □

The next two results are used in the proofs of Theorems 2 and 3.

PROPOSITION 3. *If an ASWF f satisfies Extensive Domain and Welfarism, then the SWO associated with f must satisfy Intrapersonal Ratio-Scale Invariance.*

PROOF. Suppose that f satisfies Extensive Domain and Welfarism. Take any utility vectors $u, v, u', v' \in \mathbb{R}^n$ for which, for every $i \in N$, there is some $k_i > 0$ such that $u'_i = k_i u_i$ and $v'_i = k_i v_i$. Suppose that $u \succsim^* v$, where $\succsim^*$ is the SWO associated with f. Then for any $R \in \mathcal{D}$, $U \in \mathcal{U}_R$, and $x, y \in X$ such that $U(x) = u$ and $U(y) = v$, $x \succsim_R y$. For any such R and U, the profile $U' = (k_1 U_1, \ldots, k_n U_n)$ additively represents R as well, by the uniqueness component of Proposition 1. So by Welfarism, $u' \succsim^* v'$ as well. □

The following condition is equivalent to Anonymity on our domain:

Utility Anonymity For all $R, R' \in \mathcal{D}$, $U \in \mathcal{U}_R$, and $U' \in \mathcal{U}_{R'}$, if there is a permutation $\sigma : N \to N$ such that $U_i = U'_{\sigma(i)}$ for every $i \in N$, then $f(R) = f(R')$.

PROPOSITION 4. *If an ASWF f satisfies Extensive Domain, then f satisfies Anonymity iff f satisfies Utility Anonymity. If, in addition, f satisfies Welfarism, then f satisfies Anonymity or Utility Anonymity iff $\succsim^*$ is anonymous.*

PROOF. Suppose that f satisfies Extensive Domain and Anonymity. Take any $R, R' \in \mathcal{D}$, $U \in \mathcal{U}_R$, and $U' \in \mathcal{U}_{R'}$, and permutation $\sigma : N \to N$ such that $U_i = U'_{\sigma(i)}$ for every $i \in N$. Since U_i and $U'_{\sigma(i)}$ additively represent R_i and $R'_{\sigma(i)}$ respectively, this implies $R_i = R'_{\sigma(i)}$ for all $i \in N$. So $f(R) = f(R')$ by Anonymity and Utility Anonymity is satisfied.

Suppose next that f satisfies Extensive Domain and Utility Anonymity. Take any $R, R' \in \mathcal{D}$ and $\sigma : N \to N$ such that $R_i = R'_{\sigma(i)}$ for every $i \in N$. Fix a profile $U \in \mathcal{U}_R$.

Let $U' = (U_{\sigma(1)}, \ldots, U_{\sigma(n)})$. Clearly $U' \in \mathcal{U}_{R'}$. So $f(R) = f(R')$ by Utility Anonymity and Anonymity is satisfied.

Now suppose that f satisfies Extensive Domain, Welfarism, and Anonymity and therefore Utility Anonymity. The anonymity of $\succsim^*$ follows from the proofs of d'Aspremont and Gevers (1977, Lemmas 4 and 5). It is easy to see that if $\succsim^*$ is anonymous, then f must satisfy Utility Anonymity and therefore Anonymity. □

Appendix B: Proofs for Section 3.2

Proof of Theorem 2. Take a social welfare function f that satisfies Extensive Domain, Ratio IIA, and either Strong Pareto or the conjunction of Pareto Indifference and Weak Pareto. By Theorem 1 and Proposition 3, f satisfies Welfarism and its associated SWO $\succsim^*$ satisfies Intrapersonal Ratio-Scale Invariance. By Proposition 4, f satisfies Anonymity iff its associated SWO is anonymous. We show that $\succsim^*$ cannot be anonymous given Strong Pareto or, when n is even, Weak Pareto.

First assume Strong Pareto. Let $a > b > 0$. By the anonymity of $\succsim^*$ and Strong Pareto, $(a, 0, \ldots, 0) \sim (0, \ldots, 0, a) \succ (0, \ldots, 0, b)$, so $(a, 0, \ldots, 0) \succ (0, \ldots, 0, b)$. By the same reasoning, $(0, \ldots, 0, a) \succ (b, 0, \ldots, 0)$. But Intrapersonal Ratio-Scale Invariance implies that $(a, 0, \ldots, 0) \succ (0, \ldots, 0, b)$ iff $(b, 0, \ldots, 0) \succ (0, \ldots, 0, a)$, by multiplying person 1's utilities in both vectors by b/a and person n's by a/b.

Next assume Weak Pareto and suppose that n is even. For any $x, y \in \mathbb{R}$, let $(\mathbf{x}, \mathbf{y})$ denote the vector in $\mathbb{R}^n$ the first half of whose components equal x and whose second half equals y. As before, assume that $a > b > 0$. By the anonymity of $\succsim^*$ and Weak Pareto, $(\mathbf{a}, -\mathbf{b}) \sim (-\mathbf{b}, \mathbf{a}) \succ (-\mathbf{a}, \mathbf{b})$, so $(\mathbf{a}, -\mathbf{b}) \succ (-\mathbf{a}, \mathbf{b})$. By the same reasoning, $(\mathbf{b}, -\mathbf{a}) \sim (-\mathbf{a}, \mathbf{b}) \prec (-\mathbf{b}, \mathbf{a})$, so $(\mathbf{b}, -\mathbf{a}) \prec (-\mathbf{b}, \mathbf{a})$. But these are inconsistent with Intrapersonal Ratio-Scale Invariance, which implies that $(\mathbf{a}, -\mathbf{b}) \succ (-\mathbf{a}, \mathbf{b})$ iff $(\mathbf{b}, -\mathbf{a}) \succ (-\mathbf{b}, \mathbf{a})$. □

We now lay out three results concerning Extensive Social Preference; these lead to the proof of Theorem 3.

First, we derive the following Pareto condition from Extensive Domain, Ratio IIA, Weak Pareto, and Extensive Social Preference:

Semistrong Pareto For any $x, y \in X$ and any Arrovian profile $R \in \mathcal{D}$, if xR_iy for every $i \in N$, then $x \succsim_R y$.

Semistrong Pareto is, like Strong Pareto and unlike Weak Pareto, a strengthening of Pareto Indifference; it was named and distinguished by Weymark (1991, 1993).

Lemma 3. *If an ASWF f satisfies Extensive Domain, Ratio IIA, Weak Pareto, and Extensive Social Preference, then it must also satisfy Semistrong Pareto.*

Proof. Suppose that f satisfies Extensive Domain, Ratio IIA, Weak Pareto, and Extensive Social Preference. Suppose for reductio that, for some $x, y \in X$ and $R \in \mathcal{D}$, xR_iy for all $i \in N$ but $y \succ_R x$. Take some $U \in \mathcal{U}_R$ and use Lemma 1 to find an $R' \in \mathcal{D}$, $V \in \mathcal{U}_{R'}$, and $z \in X$ such that $V(x) = U(x), V(y) = U(y), V(z) = U(y) - (1, \ldots, 1)$. By

Ratio IIA and Proposition 2, $y \succ_{R'} x$. This implies, by the Archimedean property, that for some natural number n, $ny \circ z \succcurlyeq_{R'} nx \circ x$. By Extensive Domain, $V(ny \circ z) = V(ny) + V(z) = (n+1)V(y) - (1, \ldots, 1)$, and $V(nx \circ x) = V(nx) + V(x) = (n+1)V(x)$. But since $V_i(x) \geq V_i(y)$ for every $i \in N$, $(n+1)V_i(x) > (n+1)V_i(y) - 1$ for every $i \in N$ and natural number n. Thus we cannot have $ny \circ z \succcurlyeq_{R'} nx \circ x$ by Weak Pareto. □

LEMMA 4. *If a social welfare function f satisfies Extensive Domain and Welfarism, then f satisfies Extensive Social Preference iff its associated SWO $\succcurlyeq^*$ satisfies Extensive SWO:*

Extensive SWO *The triple $(\mathbb{R}^n, \succcurlyeq^*, +)$ is an extensive structure.*

PROOF. Suppose that f satisfies Extensive Domain and Welfarism. Transitivity and Completeness are built into the definitions of $\succcurlyeq_R$ and $\succcurlyeq^*$. Vector addition is associative, and Weak Associativity of $\circ$ with respect to $\sim_R$ follows from Extensive Domain and Pareto Indifference, which is implied by Welfarism. So it remains to show that $(X, \succcurlyeq_R, \circ)$ satisfies Monotonicity and Archimedean iff $(\mathbb{R}^n, \succcurlyeq^*, +)$ does.

For Monotonicity, take any $u, v, w \in \mathbb{R}^n$, and any $R \in \mathcal{D}$, $U \in \mathcal{U}_R$, and $x, y, z \in X$ such that $U(x) = u$, $U(y) = v$, and $U(z) = w$. Welfarism implies that $u \succcurlyeq^* v$ iff $x \succcurlyeq_R y$, and $x \circ z \succcurlyeq_R y \circ z$ iff $u + w \succcurlyeq^* v + w$. Extensive Social Preference implies that $x \succcurlyeq_R y$ iff $x \circ z \succcurlyeq_R y \circ z$; Extensive SWO implies $u \succcurlyeq^* v$ iff $u + w \succcurlyeq^* v + w$. Whichever we assume, the other follows. The proof for the Archimedean axiom is analogous. □

LEMMA 5. *If a SWO $\succcurlyeq^*$ satisfies Extensive SWO and Semistrong Pareto, then it is additively represented by a social utility function $W : \mathbb{R}^n \to \mathbb{R}$ of the following form: for some $c_1, \ldots, c_n \geq 0$,*

$$W(u) = \sum_{i \in N} c_i u_i \text{ for all } u \in \mathbb{R}^n. \tag{1}$$

PROOF. By Extensive SWO and Proposition 1, $\succcurlyeq^*$ is representable by some $W : \mathbb{R}^n \to \mathbb{R}$ which satisfy's Cauchy's functional equation (2):

$$W(u + v) = W(u) + W(v) \text{ for all } u, v \in \mathbb{R}^n. \tag{2}$$

The general solution to such an equation is of the following form (Aczél and Dhombres, 1989, p. 35):

$$W(u) = \sum_{i=1}^{n} W_i(u_i), \tag{3}$$

where each $W_i : \mathbb{R} \to \mathbb{R}$ satisfies equation (4):

$$W_i(x + y) = W_i(x) + W_i(y) \text{ for all } x, y \in \mathbb{R}. \tag{4}$$

In order to satisfy Semistrong Pareto, each W_i must be nondecreasing. Thus, by Aczél and Dhombres (1989, Corollary 2.5, p. 15), for each W_i there must be a constant $c_i \geq 0$ such that

$$W_i(x) = c_i x \text{ for all } x \in \mathbb{R}. \tag{5}$$

Putting equations (3) and (5) together, we get (1). □

PROOF OF THEOREM 3. Suppose that f satisfies Extensive Domain, Ratio IIA, Weak Pareto, and Extensive Social Preference. By Lemma 3, f also satisfies Semistrong Pareto and thus Pareto Indifference. So, by Theorem 1, Proposition 3, and Lemma 4, f satisfies Welfarism and the associated SWO $\succsim^*$ satisfies Intrapersonal Ratio-Scale Invariance and Extensive SWO. Lemma 5 then implies that $\succsim^*$ must be additively representable by a $W : \mathbb{R}^n \to \mathbb{R}$ which satisfies equation (1) with nonnegative weights.

In order to satisfy Weak Pareto, there must be some $i \in N$ such that $c_i > 0$. We then show that, for any $j \in N \setminus \{i\}$, $c_j = 0$. Suppose for reductio that, for some distinct $i, j \in N$, $c_i > 0$ and $c_j > 0$. Consider the unit vectors $\mathbf{e}_i, \mathbf{e}_j \in \mathbb{R}^n$ with all components equal to 0 except the ith (resp., jth) which equals 1. We have $W(\mathbf{e}_i) = c_i$ and $W(\mathbf{e}_j) = c_j$ by equation (1). If c_i and c_j are both positive, then there must be some natural numbers n and m such that $nc_i > c_j$ and $mc_j > c_i$ by the Archimedean property of the real numbers. Since $W(n\mathbf{e}_i) = nc_i$ and $W(m\mathbf{e}_j) = mc_j$, this implies that $n\mathbf{e}_i \succ^* \mathbf{e}_j$ and $m\mathbf{e}_j \succ^* \mathbf{e}_i$. But, by Intrapersonal Ratio-Scale Invariance, $n\mathbf{e}_i \succ^* \mathbf{e}_j$ implies $\mathbf{e}_i \succ^* m\mathbf{e}_j$.

We have shown there to be exactly one $i \in N$ such that $c_i > 0$; for all other $j \in N$, $c_j = 0$. Thus, $W(u) = c_i u_i$ for all $u \in \mathbb{R}^n$, so the social welfare function must be strongly dictatorial. It is easy to see that if f satisfies Extensive Domain and is strongly dictatorial, it must also satisfy Ratio IIA, Weak Pareto, and Extensive Social Preference. □

APPENDIX C: PROOFS FOR SECTION 4.1

We first reformulate our Pareto and utility-theoretic IIA conditions in the generalized framework:

Weak Pareto For any $N \in \mathcal{P}$, $x, y \in X^N$, and $R \in \mathcal{D}$, if $(x,i)P(y,i)$ for every $i \in N$, then $x \succ_R y$.

Pareto Indifference For any $N \in \mathcal{P}$, $x, y \in X^N$, and $R \in \mathcal{D}$, if $(x,i)I(y,i)$ for every $i \in N$, then $x \sim_R y$.

Semistrong Pareto For any $N \in \mathcal{P}$, $x, y \in X^N$, and $R \in \mathcal{D}$, if $(x,i)R(y,i)$ for every $i \in N$, then $x \succsim_R y$.

Strong Pareto For any $N \in \mathcal{P}$, $x, y \in X^N$, and $R \in \mathcal{D}$, if $(x,i)R(y,i)$ for every $i \in N$ then $x \succsim_R y$; if, in addition, $(x,i)P(y,i)$ for some $i \in N$, then $x \succ_R y$.

Generalized Utility IIA For any $R, R' \in \mathcal{D}$, $U \in \mathcal{U}_R$, $U' \in \mathcal{U}_{R'}$ and $x, y \in X$, if for all $i \in N(x), j \in N(y)$, $U(x,i) = U'(x,i)$ and $U(y,j) = U'(y,j)$, then $x \succsim_R y$ iff $x \succsim_{R'} y$.

PROPOSITION 5. *If a GSWF f satisfies Interpersonal Extensive Domain, then f satisfies Interpersonal Ratio IIA iff f satisfies Generalized Utility IIA.*

PROOF OF PROPOSITION 5. Suppose first that f satisfies Interpersonal Extensive Domain and Interpersonal Ratio IIA, and that for some $R, R' \in \mathcal{D}$, $U \in \mathcal{U}_R$, $U' \in \mathcal{U}_{R'}$ and $x, y \in X$, $U(x,i) = U'(x,i)$ and $U(y,j) = U'(y,j)$ for all $i \in N(x), j \in N(y)$. Define a bijection $\phi : L(\{x,y\})^{\oplus R} \to L(\{x,y\})^{\oplus^{R'}}$ as follows. If $s \in L(\{x,y\})$, let $\phi(s) = s$. If $s \in L(\{x,y\})^{\oplus R} \setminus L(\{x,y\})$, there must be some $s_1, \dots, s_k \in L(\{x,y\})$ with $k \geq 2$ such that $s = s_1 \oplus^R \cdots \oplus^R s_k$; let $\phi(s) = s_1 \oplus^{R'} \cdots \oplus^{R'} s_k$. Clearly $U(s) = U'(\phi(s))$ for all $s \in L(\{x,y\})$; and for all $s = s_1 \oplus^R \cdots \oplus^R s_k \in L(\{x,y\})^{\oplus R} \setminus L(\{x,y\})$, we have

$$\begin{aligned} U(s) &= U(s_1 \oplus^R \cdots \oplus^R s_k) \\ &= U(s_1) + \cdots + U(s_k) \\ &= U'(s_1) + \cdots + U'(s_k) \\ &= U'(s_1 \oplus^{R'} \cdots \oplus^{R'} s_k) \\ &= U'(\phi(s)). \end{aligned}$$

Thus, $U(s) = U'(\phi(s))$ for all $s \in L(\{x,y\})^{\oplus R}$. So for any $s,t \in L(\{x,y\})^{\oplus R}$, $U(s) \geq U(t)$ iff $U'(\phi(s)) \geq U'(\phi(t))$, so sRt iff $\phi(s)R'\phi(t)$; and, by construction, $\phi(s \oplus^R t) = \phi(s) \oplus^{R'} \phi(t)$. Thus ϕ is a profile isomorphism, so by Interpersonal Ratio IIA, $x \succcurlyeq_R y$ iff $x \succcurlyeq_{R'} y$ and Generalized Utility IIA is satisfied.

Suppose next that f satisfies Interpersonal Extensive Domain and Generalized Utility IIA, and that for some $R, R' \in \mathcal{D}$ and $x, y \in X$, there is a profile isomorphism $\phi : L(\{x,y\})^{\oplus^R} \to L(\{x,y\})^{\oplus^{R'}}$ such that $\phi(x,i) = (x,i)$ and $\phi(y,j) = (y,j)$ for all $i \in N(x)$ and $j \in N(y)$. Pick a $U \in \mathcal{U}_R$ and $U' \in \mathcal{U}_{R'}$. For any $s,t \in L(\{x,y\})^{\oplus^R}$, we have sRt iff $\phi(s)R'\phi(t)$ iff $U'(\phi(s)) \geq U'(\phi(t))$, and $U'(\phi(s \oplus^R t)) = U'(\phi(s) \oplus^{R'} \phi(t)) = U'(\phi(s)) + U'(\phi(t))$. Let $V : L(\{x,y\})^{\oplus^R} \to \mathbb{R}$ denote the composition of $U'|_{L(\{x,y\})^{\oplus R'}}$ with ϕ. We've just seen that V additively represents $R|_{L(\{x,y\})^{\oplus R}}$: for any $s,t \in L(\{x,y\})^{\oplus^R}$, sRt iff $V(s) \geq V(t)$ iff $U'(\phi(s)) \geq U'(\phi(t))$, and $V(s \oplus^R t) = V(s) + V(t) = U'(\phi(s)) + U'(\phi(t))$. Since $U \in \mathcal{U}_R$, $U|_{L(\{x,y\})^{\oplus R}}$ also additively represents $R|_{L(\{x,y\})^{\oplus R}}$. Thus, by the uniqueness component of Proposition 1, there must be some $k > 0$ such that $V(s) = kU(s)$ for all $s \in L(\{x,y\})^{\oplus^R}$. Now let $V' = (1/k)U'$, so that $V' \in \mathcal{U}_{R'}$ and $V'(\phi(s)) = U(s)$ for all $s \in L(\{x,y\})^{\oplus^R}$. Remember that $\phi(s) = s$ for all $s \in L(\{x,y\})$. So $V'(x,i) = U(x,i)$ and $V'(y,j) = U(y,j)$ for all $i \in N(x)$ and $j \in N(y)$. Therefore, by Generalized Utility IIA, $x \succcurlyeq_R y$ iff $x \succcurlyeq_{R'} y$, and Interpersonal Ratio IIA is satisfied.

□

For any population $N \in \mathcal{P}$, let $\{a_i\}_{i \in N}$ be a set of atomic alternatives with $N(a_i) = \{i\}$ for each a_i. Let $\bigcirc_{i \in N} a_i$ denote the concatenation of all these alternatives in arbitrary order, so that $N(\bigcirc_{i \in N} a_i) = N$. Let A^N denote the set of all such concatenations of one-person alternatives involving the members of N. For any populations $M, N \in \mathcal{P}$ and $x \in A^M$ and $y \in A^N$, where $x = \bigcirc_{i \in M} a_i$ and $y = \bigcirc_{i \in N} b_i$, say that x and y are *nonoverlapping* iff $\{a_i\}_{i \in M} \cap \{b_i\}_{i \in N} = \emptyset$. We have the following lemma:

Lemma 6. *If a GSWF f satisfies Interpersonal Extensive Domain, then for any populations $M, N, O \in \mathcal{P}$, there are nonoverlapping alternatives $x \in A^M$, $y \in A^N$, and $z \in A^O$. And, for any such x, y, z, and any utility distributions $u \in \mathbb{R}^M, v \in \mathbb{R}^N, w \in \mathbb{R}^O$, there is a utility profile $U \in \mathcal{U}_{\mathcal{D}}$ such that $U(x,\cdot) = u, U(y,\cdot) = v, U(z,\cdot) = w$.*

PROOF. Recall that we have assumed, for each individual, the existence of at least three atomic alternatives in which only that individual exists. So we can find disjoint sets of atomic alternatives $\{a_i\}_{i\in M}$, $\{b_j\}_{j\in N}$, and $\{c_k\}_{k\in O}$. Let $x=\bigcirc_{i\in M}a_i$, $y=\bigcirc_{j\in N}b_j$, $z=\bigcirc_{k\in O}c_k$, so that $x\in A^M$, $y\in A^N$, and $z\in A^O$ are nonoverlapping concatenations of single-person atomic alternatives. For any $u\in\mathbb{R}^M, v\in\mathbb{R}^N, w\in\mathbb{R}^O$, we can find some $U\in\mathcal{U}_\mathcal{D}$ such that $U(a_i,i)=u_i$, $U(b_j,j)=v_j$, and $U(c_k,k)=w_k$ for all $i\in M, j\in N, k\in O$. By the Irrelevance of Nonexistence condition of Interpersonal Extensive Domain, $(x,i)I(a_i,i)$, $(y,j)I(b_j,j)$, and $(z,k)I(c_k,k)$ for all $i\in M, j\in N, k\in O$. So $U(x,i)=u_i$, $U(y,j)=v_j$, and $U(z,k)=w_k$ for every $i\in M, j\in N, k\in O$. Thus $U(x,\cdot)=u$, $U(y,\cdot)=v$, and $U(z,\cdot)=w$, as desired. □

Lemma 6 provides us with a set of *free triples* in the sense of Weymark (1998)—i.e., a set of three alternatives for which the domain of attainable utility distributions is unrestricted. We also have the following analogue of Lemma 1:

LEMMA 7. *If f satisfies Interpersonal Extensive Domain, then for any populations $M,N,O\in\mathcal{P}$, alternatives $x\in X^M$ and $y\in X^N$, any utility profile $U\in\mathcal{U}_\mathcal{D}$, and utility distribution $w\in\mathbb{R}^O$, there is a $z\in A^O$ and $V\in\mathcal{U}_\mathcal{D}$ such that $V(x,\cdot)=U(x,\cdot)$, $V(y,\cdot)=U(y,\cdot)$, and $V(z,\cdot)=w$.*

PROOF. The proof is analogous to that of Lemma 1, except that we choose an atomic alternative $a_i\in A^{\{i\}}$ for each $i\in O$ and let z be the concatenation of all these alternatives. This is trivial for $i\notin O\cap M\cap N$. For $i\in O\cap M\cap N$, we can use the same strategy as the one used in the proof of Lemma 1 to find an $a_i\in A^{\{i\}}$ and a $V\in\mathcal{U}_\mathcal{D}$ such that $V(a_i,i)=w_i$ while preserving $V(x,i)=U(x,i)$ and $V(y,i)=U(y,i)$. We then let z be the concatenation of all these atomic, one-person alternatives, so that $V(z,\cdot)=w_i$ for every $i\in O$ while preserving $V(x,\cdot)=U(x,\cdot)$ and $V(y,\cdot)=U(y,\cdot)$, as desired. □

We can then use Lemmas 6 and 7 to define our SWO on Ω:

PROOF OF THEOREM 4. Suppose that f satisfies Interpersonal Extensive Domain, Pareto Indifference, and Interpersonal Ratio IIA. Define the SWO as follows: for any $M,N\in\mathcal{P}$, $u\in\mathbb{R}^M$, and $v\in\mathbb{R}^N$, $u\succcurlyeq^* v$ iff, for some nonoverlapping $a\in A^M$ and $b\in A^N$, $R\in\mathcal{D}$ and $U\in\mathcal{U}_R$ such that $U(a,\cdot)=u$ and $U(b,\cdot)=v$, $a\succcurlyeq_R b$.

By Lemma 6, for any $u\in\mathbb{R}^M$, and $v\in\mathbb{R}^N$, there must be some nonoverlapping $a\in A^M$ and $b\in A^N$, $R\in\mathcal{D}$, and $U\in\mathcal{U}_R$ such that $u=U(a,\cdot)$ and $v=U(b,\cdot)$. Since $\succcurlyeq_R$ is complete, we have either $a\succcurlyeq_R b$ or $b\succcurlyeq_R a$, which implies either $u\succcurlyeq^* v$ or $v\succcurlyeq^* u$ respectively. Thus $\succcurlyeq^*$ is complete.

We claim that, for any $x,y\in X$, $R\in\mathcal{D}$, and $U\in\mathcal{U}_R$, $x\succcurlyeq_R y$ if and only if $U(x,\cdot)\succcurlyeq^* U(y,\cdot)$. The proof of this claim is exactly analogous to that of the corresponding claim on page 19 (including proof of Lemma 2), and is therefore omitted.

To show that $\succcurlyeq^*$ is transitive, take any $M,N,O\in\mathcal{P}$ and $u\in\mathbb{R}^M, v\in\mathbb{R}^N, w\in\mathbb{R}^O$ such that $u\succcurlyeq^* v\succcurlyeq^* w$. By Lemma 6, there must be some nonoverlapping $a\in A^M, b\in A^N, c\in A^O$, $R\in\mathcal{D}$, and $U\in\mathcal{U}_R$ such that $U(x,\cdot)=u, U(y,\cdot)=v, U(z,\cdot)=w$. We have just shown above that $x\succcurlyeq_R y\succcurlyeq_R z$ and thus $x\succcurlyeq_R z$ by the transitivity of $\succcurlyeq_R$, so $u\succcurlyeq^* v$, as required.

It is easy to see that $\succsim^*$ is unique and that Variable-Population Welfarism implies Pareto Indifference and Generalized Utility IIA and therefore Interpersonal Ratio IIA. □

PROPOSITION 6. *If a GSWF f satisfies Interpersonal Extensive Domain and Variable-Population Welfarism, then the SWO associated with f must satisfy Interpersonal Ratio-Scale Invariance.*

The proof of Proposition 6 is exactly similar to that of Proposition 3 and is therefore omitted.

APPENDIX D: PROOFS FOR SECTION 4.2

In terms of numerical utilities, Fixed-Population Anonymity amounts to the following:

Fixed-Population Utility Anonymity For any $R, R' \in \mathcal{D}$, $U \in \mathcal{U}_R$, $U' \in \mathcal{U}_{R'}$ and $N \in \mathcal{P}$, if there is a permutation $\sigma : N \to N$ such that $U(x,i) = U'(x,\sigma(i))$ for all $\mathcal{L} \in X^N \times N$, then for all $x, y \in X^N$, $x \succsim_R y$ iff $x \succsim_{R'} y$.

PROPOSITION 7. *If a GSWF f satisfies Interpersonal Extensive Domain, then f satisfies Fixed-Population Anonymity iff f satisfies Fixed-Population Utility Anonymity.*

PROOF OF PROPOSITION 7. Suppose that f satisfies Interpersonal Extensive Domain and Fixed-Population Anonymity. Take some $R, R' \in \mathcal{D}$, $U \in \mathcal{U}_R$, $U' \in \mathcal{U}_{R'}$, $N \in \mathcal{P}$, and permutation $\sigma : N \to N$ such that $U(x,i) = U'(x,\sigma(i))$ for all $\mathcal{L} \in X^N \times N$. Define $\phi : L(X^N)^{\oplus^R} \to L(X^N)^{\oplus^{R'}}$ as follows. For all $(x,i) \in L(X^N)$, let $\phi(x,i) = (x,\sigma(i))$; if $s \in L(X^N)^{\oplus R} \setminus L(X^N)$, there must be some $s_1, \ldots, s_k \in L(X^N)$ with $k \geq 2$ such that $s = s_1 \oplus^R \cdots \oplus^R s_k$, so let $\phi(s) = \phi(s_1) \oplus^{R'} \cdots \oplus^{R'} \phi(s_k)$. By reasoning analogous to that in the first paragraph of the proof of Proposition 5, ϕ is a profile isomorphism. Therefore, for all $x, y \in X^N$, $x \succsim_R y$ iff $x \succsim_{R'} y$, so Fixed-Population Utility Anonymity is satisfied.

Suppose next that f satisfies Interpersonal Extensive Domain and Fixed-Population Utility Anonymity. Take some $R, R' \in \mathcal{D}$, $N \in \mathcal{P}$, permutation $\sigma : N \to N$, and profile isomorphism $\phi : L(X^N)^{\oplus^R} \to L(X^N)^{\oplus^{R'}}$ such that $\phi(x,i) = (x,\sigma(i))$ for all $(x,i) \in X^N \times N$. By reasoning analogous to that in the second paragraph of the proof of Proposition 5, there exist $U \in \mathcal{U}_R$, $U' \in \mathcal{U}_{R'}$ such that $U(x,i) = U'(x,\sigma(i))$ for all $(x,i) \in X^N \times N$. So, by Fixed-Population Utility Anonymity, $\succsim_R |_{X^N} = \succsim_{R'} |_{X^N}$, and Fixed-Population Anonymity is satisfied. □

One especially powerful implication of Extensive Social Preference in the variable-population setting is that, in the presence of Interpersonal Extensive Domain and Pareto Indifference, it implies that the addition of "null" lives to a population is always a matter of social indifference. An alternative $z \in X$ is *null for individual* $i \in N(z)$, relative to a profile R, iff $(z,i) \oplus^R (z,i) I (z,i)$. An alternative z is *universally null*, relative to R, iff z is null for all $i \in N(z)$. In any given profile, there may or may not be universally null alternatives. But if there are, the following condition says that their concatenation to an alternative is always a matter of indifference:

Null Critical Levels For any $R \in \mathcal{D}$ and any $z \in X$ that is universally null in R, $x \circ z \sim_R x$ for all $x \in X$.

PROPOSITION 8. *If a GSWF f satisfies Interpersonal Extensive Domain, Pareto Indifference, and Extensive Social Preference, then it satisfies Null Critical Levels.*

PROOF OF PROPOSITION 8. Take any profile R and $z \in X$ such that $(z,i) \oplus^R (z,i) I(z,i)$ for all $i \in N(z)$. By the Matching condition of Interpersonal Extensive Domain, $(z,i) \oplus^R (z,i) I(z \circ z, i)$ for all $i \in N(z)$. Thus, by Pareto Indifference, $z \circ z \sim_R z$. So, by the Monotonicity condition of Extensive Social Preference, $x \circ (z \circ z) \sim_R x \circ z$; by Weak Associativity, $x \circ (z \circ z) \sim_R (x \circ z) \circ z$, so $(x \circ z) \circ z \sim_R x \circ z$ by Transitivity; by Monotonicity again, $x \circ z \sim_R x$. □

As mentioned in section 4, the field Ω of the SWO $\succsim^*$ is not a vector space: we cannot add together utility distributions with different domains. This can be rectified by strengthening Variable-Population Welfarism in the following way. Let $\mathbb{R}^\infty$ denote the set of all infinite sequences with finite support—i.e., $\mathbb{R}^\infty := \{u : \mathbb{N} \to \mathbb{R} \mid u_i \neq 0$ for finitely many i Unlike Ω, $\mathbb{R}^\infty$ is a vector space: for any $u, v \in \mathbb{R}^\infty$, $(u+v)_i = u_i + v_i$ for every $i \in \mathbb{N}$. For any population $N \in \mathcal{P}$, let $\iota_N : \mathbb{R}^N \hookrightarrow \mathbb{R}^\infty$ denote canonical inclusion such that for each $u \in \mathbb{R}^N$, $\iota_N(u)_i = u_i$ for all $i \in N$ and $\iota_N(u)_j = 0$ for all $j \in \mathbb{N} \setminus N$. Let $\iota : \Omega \hookrightarrow \mathbb{R}^\infty$ (no subscript) denote the union of all these inclusions. We call an ordering $\succsim^\infty$ on $\mathbb{R}^\infty$ an *extended* SWO.

Extended Welfarism There is a unique SWO $\succsim^\infty$ on $\mathbb{R}^\infty$ such that, for any profile $R \in \mathcal{D}$, any $U \in \mathcal{U}_R$, and any alternatives $x, y \in X$, $x \succsim_R y$ iff $\iota(U(x,\cdot)) \succsim^\infty \iota(U(y,\cdot))$.

LEMMA 8. *If a GSWF f satisfies Interpersonal Extensive Domain, then f satisfies Variable-Population Welfarism and Null Critical Levels iff it satisfies Extended Welfarism.*

PROOF. Take any $M, N \in \mathcal{P}$, $u \in \mathbb{R}^M$, and $v \in \mathbb{R}^N$. Suppose $\iota_M(u) = \iota_N(v)$. We show that $u \sim^* v$. This is obvious if $M = N$, since then $u = v$. So suppose $M \neq N$. Let $u \frown v$ denote the utility distribution in $\mathbb{R}^{M \cup N}$ such that, for all $i \in M \cup N$, $(u \frown v)_i = u_i = v_i$ if $i \in M \cap N$ and $(u \frown v)_i = 0$ otherwise. We show that $u \sim^* (u \frown v) \sim^* v$.

By Lemma 6, there must be some $x \in X^M, z \in X^N$, $R \in \mathcal{D}$, and $U : \mathcal{L} \to \mathbb{R}$ which additively represents R such that $U(x,\cdot) = u$ and $U(z,i) = 0$ for all $i \in N$. It follows from Proposition 1 that z is universally null. So by Proposition 8, $x \circ z \sim_R x$. Notice, however, that $U(x \circ z, \cdot) = u \frown v$, so by Variable-Population Welfarism $u \sim^* (u \frown v)$. An exactly similar argument shows $v \sim^* (u \frown v)$. Thus $u \sim^* v$.

We now define $\succsim^\infty$ as follows: for all $u, v \in \mathbb{R}^\infty$, $u \succsim^\infty v$ iff, for some $M, N \in \mathcal{P}$ and $u' \in \mathbb{R}^M, v' \in \mathbb{R}^N$ such that $\iota(u') = u$ and $\iota(v') = v$, $u' \succsim^* v'$. For any such $u, v \in \mathbb{R}^\infty$, there exist $M, N \in \mathcal{P}$ and $u' \in \mathbb{R}^M, v' \in \mathbb{R}^N$ such that $\iota(u') = u$ and $\iota(v') = v$, so $\succsim^\infty$ inherits completeness from $\succsim^*$. And we've just seen that for any $M', N' \in \mathcal{P}, u^* \in \mathbb{R}^{M'}, v^* \in \mathbb{R}^{N'}$ such that $\iota(u^*) = \iota(u') = u$ and $\iota(v^*) = \iota(v') = v$, $u' \sim^* u^*$ and $v' \sim^* v^*$, so $u^* \succsim^* v^*$ iff $u \succsim^\infty v$. It's easy to see that $\succsim^\infty$ must also be transitive and is unique.

For the other direction, suppose that f satisfies Extended Welfarism. Then we define the SWO $\succsim^*$ as follows: for all $u, v \in \Omega$, $u \succsim^* v$ iff $\iota(u) \succsim^\infty \iota(v)$. It's clear that $\succsim^*$ is an ordering and that, by Extended Welfarism, for any $x, y \in X$, $R \in \mathcal{D}$, and $U \in \mathcal{U}_R$, $x \succsim_R y$ iff $U(x,\cdot) \succsim^* U(y,\cdot)$. Finally, to see that Extended Welfarism implies Null Critical Levels, suppose that z is universally null in a profile R. Then for any $U \in \mathcal{U}_R$, $U(z,i) = 0$ for all $i \in N(z)$. For any $x \in X$, $\iota(U(x \circ z, \cdot)) = \iota(U(x,\cdot))$, so by Extended Welfarism, $x \circ z I x$. $\square$

An extended SWO is *fully anonymous* iff, for any permutation $\sigma : \mathbb{N} \to \mathbb{N}$ and $u, v \in \mathbb{R}^\infty$ such that $u_i = v_{\sigma(i)}$ for every $i \in \mathbb{N}$, $u \sim^\infty v$.

LEMMA 9. *If a GSWF f satisfies Interpersonal Extensive Domain and Extended Welfarism, then f satisfies Fixed-Population Anonymity iff its associated extended SWO $\succsim^\infty$ is fully anonymous.*

PROOF. Suppose f satisfies Interpersonal Extensive Domain and Extended Welfarism. Clearly if $\succsim^\infty$ is fully anonymous, then Fixed-Population Anonymity must be satisfied. For the other direction, suppose that f satisfies Fixed-Population Anonymity and thus Fixed-Population Utility Anonymity (by Proposition 7). Take any $u, v \in \mathbb{R}^\infty$ such that, for some permutation $\sigma : \mathbb{N} \to \mathbb{N}$, $u_i = v_{\sigma(i)}$ for every $i \in \mathbb{N}$. Let $N = \{\, i \in \mathbb{N} \mid u_i \neq v_i \,\}$. Since u and v have finite support, N must be finite even if σ itself has infinite support. Consider the distributions $u^*, v^* \in \mathbb{R}^N$ such that $\iota(u^*) = u$ and $\iota(v^*) = v$. There is a permutation $\sigma^* : N \to N$ such that $u^*_i = v^*_{\sigma^*(i)}$ for every $i \in N$. By Fixed-Population Utility Anonymity and Proposition 7, $u^* \sim^* v^*$. Thus, by Extended Welfarism, $u \sim^\infty v$, as desired. $\square$

PROOF OF THEOREM 5. Suppose that f satisfies Interpersonal Extensive Domain, Interpersonal Ratio IIA, Weak Pareto, Fixed-Population Anonymity, and Extensive Social Preference. By Lemma 3, f must also satisfy Semistrong Pareto and thus Pareto Indifference. So by Theorem 4 and Proposition 8, f satisfies Variable-Population Welfarism and Null Critical Levels and thus, by Lemma 8, Extended Welfarism. By Fixed-Population Anonymity and Lemma 9, the extended SWO $\succsim^\infty$ is fully anonymous.

The proof of Lemma 4 can be easily adapted to show that $(\mathbb{R}^\infty, \succsim^\infty, +)$ is an extensive structure. So $\succsim^\infty$ is additively representable by a social utility function $W : \mathbb{R}^\infty \to \mathbb{R}$ which satisfies Cauchy's functional equation (6):

$$W(u+v) = W(u) + W(v) \text{ for all } u, v \in \mathbb{R}^\infty. \tag{6}$$

For each $i \in \mathbb{N}$, define $W_i : \mathbb{R} \to \mathbb{R}$ so that $W_i(x) = W(\iota_{\{\,i\,\}}(i \mapsto x))$ for all $x \in \mathbb{R}$.

For any $u \in \mathbb{R}^\infty$, there must be some $k \in \mathbb{N}$ such that $u_i = 0$ for all $i > k$. (Otherwise u would have infinite support.) Thus, for any $u \in \mathbb{R}^\infty$, there is a $k \in \mathbb{N}$ such that

$$u = (u_1, 0, 0, \dots) + (0, u_2, 0, 0, \dots) + \cdots + (0, \dots, 0, u_k, 0, 0, \dots) + (0, 0, \dots). \tag{7}$$

This implies, by equation (6),

$$W(u) = W(u_1, 0, 0, \dots) + W(0, u_2, 0, 0, \dots) + \cdots + W(0, \dots, 0, u_k, 0, 0, \dots) + W(0, 0, \dots). \tag{8}$$

Since $W(0,0,\dots)=0$, this simplifies to

$$W(u)=(u_1,0,0,\dots)+(0,u_2,0,0,\dots)+\cdots+(0,\dots,0,u_k,0,0,\dots). \tag{9}$$

So, by equation (6) and the definition of W_i, we have that for any $u\in\mathbb{R}^\infty$, there is a $k\in\mathbb{N}$ such that

$$W(u)=\sum_{i=1}^{k}W_i(u_i). \tag{10}$$

For each $i\in\mathbb{N}$, we must have:

$$W_i(x+y)=W_i(x)+W_i(y) \text{ for all } x,y\in\mathbb{R}. \tag{11}$$

Thus $W_i(0)=0$ for all $i\in\mathbb{N}$, so

$$W(u)=\sum_{i=1}^{\infty}W_i(u_i) \text{ for all } u\in\mathbb{R}^\infty. \tag{12}$$

Each W_i must be nondecreasing in order to satisfy Semistrong Pareto. So by Aczél and Dhombres (1989, Corollary 2.5, p. 15), for each W_i there must be a constant $c_i\geq 0$ such that

$$W_i(x)=c_i x \text{ for all } x\in\mathbb{R}. \tag{13}$$

In order to satisfy Weak Pareto and the full anonymity of $\succsim_\infty$, there must be some $c>0$ such that $c_i=c$ for all $i\in\mathbb{N}$. So

$$W(u)=\sum_{i=1}^{\infty}c(u_i)=c\sum_{i=1}^{\infty}u_i \text{ for all } u\in\mathbb{R}^\infty. \tag{14}$$

For any such c, and any $x,y\in X$, $R\in\mathcal{D}$, and $U\in\mathcal{U}_R$, $W(\iota(U(x,\cdot)))\geq W(\iota(U(y,\cdot)))$ iff $\sum_{i\in N(x)}U(x,i)\geq\sum_{i\in N(y)}U(y,i)$.

It is straightforward to verify that, given Interpersonal Extensive Domain, Classical Utilitarianism satisfies Interpersonal Ratio IIA, Weak Pareto, Fixed-Population Anonymity, and Extensive Social Preference. □

Appendix E: An Alternative Characterization of Classical Utilitarianism

Blackorby et al. (2005, Theorem 6.24) characterize the "classical means of order r." These are SWOs which compare utility distributions as follows: there exist $\beta,r\in\mathbb{R}_{++}$ such that for any $M,N\in\mathcal{P}$, $u\in\mathbb{R}^M$, and $v\in\mathbb{R}^N$, $u\succsim^* v$ iff

$$\sum_{i\in M:u_i\geq 0}u_i^r-\beta\sum_{i\in M:u_i<0}(-u_i)^r\geq\sum_{i\in N:v_i\geq 0}v_i^r-\beta\sum_{i\in N:v_i<0}(-v_i)^r \tag{15}$$

To characterize these orderings we introduce three new conditions:

Variable-Population Continuity For all $N,M\in\mathcal{P}$ and $u\in\mathbb{R}^M$, the sets $\{v\in\mathbb{R}^N\mid v\succsim^* u\}$ and $\{v\in\mathbb{R}^N\mid u\succsim^* v\}$ are closed in $\mathbb{R}^N$.

Weak Existence of Critical Levels For some $N \in \mathcal{P}$, $u \in \mathbb{R}^N$, $i \in \mathbb{N} \setminus N$, and $v \in \mathbb{R}^{N \cup \{i\}}$ such that $v_j = u_j$ for all $j \in N$, $u \sim^* v$.

Existence Independence For all $u, v, w \in \Omega$ such that $u \cup w, v \cup w \in \Omega$, $u \succcurlyeq^* v$ iff $u \cup w \succcurlyeq^* v \cup w$.

PROPOSITION 9. *If a GSWF f satisfies Interpersonal Extensive Domain, then f is associated with a classical mean of order r iff f satisfies Interpersonal Extensive Domain, Interpersonal Ratio IIA, Strong Pareto, and Fixed-Population Anonymity and its associated SWO satisfies Variable-Population Continuity, Weak Existence of Critical Levels, and Existence Independence.*

PROOF. Weak Existence of Critical Levels, Existence Independence, and Strong Pareto together imply that there is a single critical level for all alternatives (Blackorby et al., 2005, Theorem 6.9). Interpersonal Ratio-Scale Invariance then implies Null Critical Levels (Blackorby et al., 2005, Theorem 6.23), so the social welfare function satisfies Extended Welfarism (Lemma 8). So, by Fixed-Population Anonymity and Lemma 9, the SWO must be fully anonymous. The axioms of Blackorby et al. (2005, Theorem 6.24) are therefore satisfied, so $\succcurlyeq^*$ must be a classical mean of order r. □

The classical utilitarian SWO is the classical mean of order r with $\beta, r = 1$. As Blackorby and Donaldson (1982, Theorem 4) show, we can force $r = 1$ and $\beta \geq 1$ by requiring the SWO to be weakly averse to inequality, in the following sense. A *Pigou-Dalton transfer* is a non-leaky, non-rank-switching transfer in utility from a better-off to a worse-off person, with no one else affected. For any $N \in \mathcal{P}$ and distributions $u, v \in \mathbb{R}^N$, u is *unambiguously at least as equal as* v iff either u is a permutation of v or u is obtainable from v via finitely many Pigou-Dalton utility transfers (Blackorby et al., 2005, 93).

Weak Inequality Aversion For any $N \in \mathcal{P}$ and $u, v \in \mathbb{R}^N$, if u is unambiguously at least as equal as v, then $u \succcurlyeq^* v$.

This still leaves the possibility that $\beta > 1$, in which case negative utilities are weighted more heavily than positive ones. This can be ruled out by imposing

Reflection Anti-Invariance For any $u, v \in \Omega$ and $k < 0$, $u \succcurlyeq^* v$ iff $kv \succcurlyeq^* ku$.[8]

Putting all this together, we have:

THEOREM 6. *If a GSWF f satisfies Interpersonal Extensive Domain, then f satisfies Classical Utilitarianism iff f satisfies Strong Pareto and Ratio IIA and its associated SWO satisfies Variable-Population Continuity, Weak Existence of Critical Levels, Existence Independence, Weak Inequality Aversion, and Reflection Anti-Invariance.*

PROOF OF THEOREM 6. Suppose that f satisfies Interpersonal Extensive Domain, Strong Pareto, and Ratio IIA. By Theorem 4 and Proposition 6, f satisfies Variable-Population Welfarism and the associated SWO satisfies Interpersonal Ratio-Scale Invariance. Weak

[8]The decision-theoretic analogue of this principle is introduced and defended by Goodsell (2023).

Inequality Aversion implies Fixed-Population Anonymity so, by Proposition 9, $\succcurlyeq^*$ is a classical mean of order r. Weak Inequality Aversion then implies (by Blackorby and Donaldson, 1982, Theorem 4) that $r = 1$. It is easy to see that Reflection Anti-Invariance can then be satisfied only if $\beta = 1$ as well. □

References

Aczél, János and Jean G. Dhombres (1989), *Functional Equations in Several Variables*. Cambridge University Press, Cambridge; New York. [22, 29]

Adler, Matthew D. (2014), "Extended Preferences and Interpersonal Comparisons: A New Account." *Economics & Philosophy*, 30(2), 123–162. [2, 13]

Adler, Matthew D. (2019), *Measuring Social Welfare: An Introduction*. Oxford University Press, Oxford. [8]

Arrow, Kenneth J. (1951), *Social Choice and Individual Values*. New York: Wiley. [4, 10]

Arrow, Kenneth J. (1973), "Some Ordinalist-Utilitarian Notes on Rawls's Theory of Justice." *The Journal of Philosophy*, 70(9), 245–263. [17]

Arrow, Kenneth J. (1977), "Extended Sympathy and the Possibility of Social Choice." *The American Economic Review*, 67(1), 219–225. [2, 13]

Asheim, Geir B. and Stéphane Zuber (2014), "Escaping the Repugnant Conclusion: Rank-Discounted Utilitarianism with Variable Population." *Theoretical Economics*, 9(3), 629–650. [13]

Blackorby, Charles, Walter Bossert, and David Donaldson (1995), "Intertemporal Population Ethics: Critical-Level Utilitarian Principles." *Econometrica*, 63(6), 1303–1320. [12, 13]

Blackorby, Charles, Walter Bossert, and David Donaldson (1998), "Uncertainty and Critical-Level Population Principles." *Journal of Population Economics*, 11(1), 1–20. [5]

Blackorby, Charles, Walter Bossert, and David Donaldson (1999), "Information Invariance in Variable-Population Social-Choice Problems." *International Economic Review*, 40(2), 403–422. [4, 5]

Blackorby, Charles, Walter Bossert, and David Donaldson (2005), *Population Issues in Social Choice Theory, Welfare Economics, and Ethics*. Cambridge University Press, New York. [5, 8, 12, 14, 29, 30]

Blackorby, Charles and David Donaldson (1982), "Ratio-Scale and Translation-Scale Full Interpersonal Comparability without Domain Restrictions: Admissible Social-Evaluation Functions." *International Economic Review*, 23(2), 249–268. [4, 30, 31]

Bossert, Walter and John A. Weymark (2004), "Utility in Social Choice." In *Handbook of Utility Theory: Volume 2: Extensions* (Salvador Barberà, Peter J. Hammond, and Christian Seidl, eds.), 1099–1177, Kluwer Academic Publishers, Boston, MA. [3, 9]

Brandl, Florian and Felix Brandt (2020), “Arrovian Aggregation of Convex Preferences.” *Econometrica*, 88(2), 799–844. [4, 5]

Broome, John (1999), *Ethics Out of Economics*. Cambridge University Press, Cambridge; New York. [13]

Broome, John (2004), *Weighing Lives*. Oxford University Press, Oxford; New York. [5, 8, 13, 14]

Córdoba, Juan Carlos and Xiying Liu (2022), “Malthusian Stagnation is Efficient.” *Theoretical Economics*, 17(1), 415–460. [3]

Dasgupta, Partha (1995), *An Inquiry into Well-Being and Destitution*. Clarendon Press, Oxford. [8]

Dasgupta, Partha (2007), “Reply to Putnam and Walsh.” *Economics & Philosophy*, 23(3), 365–372. [8]

Dasgupta, Partha (2009), “Facts and Values in Modern Economics.” In *The Oxford Handbook of Philosophy of Economics* (Don Ross and Harold Kincaid, eds.), 580–640, Oxford University Press, Oxford. [8]

d’Aspremont, Claude and Louis Gevers (1977), “Equity and the Informational Basis of Collective Choice.” *The Review of Economic Studies*, 44(2), 199–209. [5, 21]

d’Aspremont, Claude and Louis Gevers (2002), “Social Welfare Functionals and Interpersonal Comparability.” In *Handbook of Social Choice and Welfare: Volume 1* (Kenneth J. Arrow, Amartya K. Sen, and Kotaro Suzumura, eds.), 459–541, Elsevier, Amsterdam. [3, 11]

Deschamps, Robert and Louis Gevers (1978), “Leximin and Utilitarian Rules: A Joint Characterization.” *Journal of Economic Theory*, 17(2), 143–163. [5]

Dhillon, Amrita and Jean-François Mertens (1999), “Relative Utilitarianism.” *Econometrica*, 67(3), 471–498. [4]

Fishburn, Peter C. (1989), “Retrospective on the Utility Theory of von Neumann and Morgenstern.” *Journal of Risk and Uncertainty*, 2(2), 127–157. [4]

Fleurbaey, Marc and Peter J. Hammond (2004), “Interpersonally Comparable Utility.” In *Handbook of Utility Theory: Volume 2 Extensions* (Salvador Barberà, Peter J. Hammond, and Christian Seidl, eds.), 1179–1285, Kluwer Academic Publishers, Boston, MA. [5]

Gibbard, Allan (1982), “Rights and the Theory of Social Choice.” In *Logic, Methodology and Philosophy of Science VI* (L. Jonathan Cohen, Jerzy Łoś, Helmut Pfeiffer, and Klaus-Peter Podewski, eds.), 595–605, Elsevier, Amsterdam. [8]

Gibbard, Allan (1984), “Health Care and the Prospective Pareto Principle.” *Ethics*, 94(2), 261–282. [8]

Goodsell, Zachary (2023), *Unbounded Utility*. Ph.D. thesis, University of Southern California. [30]

Greaves, Hilary and Harvey Lederman (2018), “Extended Preferences and Interpersonal Comparisons of Well-being.” *Philosophy and Phenomenological Research*, 96(3), 636–667. [13]

Hammond, Peter J. (1976), “Equity, Arrow’s Conditions, and Rawls’ Difference Principle.” *Econometrica*, 44(4), 793–804. [2, 4, 13]

Hammond, Peter J. (1988), “Consequentialist Demographic Norms and Parenting Rights.” *Social Choice and Welfare*, 5(2-3), 127–145. [5]

Harsanyi, John C. (1955), “Cardinal Welfare, Individualistic Ethics, and Interpersonal Comparisons of Utility.” *Journal of Political Economy*, 63(4), 309–321. [4, 5]

Harsanyi, John C. (1977), *Rational Behavior and Bargaining Equilibrium in Games and Social Situations*. Cambridge University Press, Cambridge; New York. [2, 13]

Harvey, Charles M. (1999), “Aggregation of Individuals’ Preference Intensities into Social Preference Intensity.” *Social Choice and Welfare*, 16(1), 65–79. [4]

Hausman, Daniel M. (1995), “The Impossibility of Interpersonal Utility Comparisons.” *Mind*, 104(415), 473–490. [13]

Heilmann, Conrad (2015), “A New Interpretation of the Representational Theory of Measurement.” *Philosophy of Science*, 82(5), 787–797. [4]

Hylland, Aanund (1989), “The Purpose and Significance of Social Choice Theory: Some General Remarks and an Application to the ‘Lady Chatterly Problem,’.” In *Foundations of Social Choice Theory* (Jon Elster and Aanund Hylland, eds.), 45–74, Cambridge University Press, Cambridge. [8]

Kahneman, Daniel, Peter P. Wakker, and Rakesh Sarin (1997), “Back to Bentham? Explorations of Experienced Utility.” *The Quarterly Journal of Economics*, 112(2), 375–405. [6]

Kalai, Ehud, Eitan Muller, and Mark A. Satterthwaite (1979), “Social Welfare Functions When Preferences Are Convex, Strictly Monotonic, and Continuous.” *Public Choice*, 34(1), 87–97. [5, 7, 9]

Kelly, Jerry S. and Kenneth J. Arrow (1987), “An Interview with Kenneth J. Arrow.” *Social Choice and Welfare*, 4(1), 43–62. [1]

Kolm, Serge-Christophe (1998), *Justice and Equity*. MIT Press, Cambridge, MA. [2, 13]

Krantz, David H., R. Duncan Luce, Patrick Suppes, and Amos Tversky (1971), *Foundations of Measurement: Vol. I*. Academic Press, New York. [2, 6, 14]

Le Breton, Michel and John A. Weymark (2011), “Arrovian Social Choice Theory on Economic Domains.” In *Handbook of Social Choice and Welfare* (Kenneth J. Arrow, Amartya Sen, and Kotaro Suzumura, eds.), volume 2, 191–299, Elsevier, Amsterdam. [5, 7, 8, 9]

Marchant, Thierry (2019), “Utilitarianism Without Individual Utilities.” *Social Choice and Welfare*, 53(1), 1–19. [4]

Maskin, E. (1978), "A Theorem on Utilitarianism." *The Review of Economic Studies*, 45(1), 93–96. [5]

McCarthy, David, Kalle Mikkola, and Teruji Thomas (2020), "Utilitarianism With and Without Expected Utility." *Journal of Mathematical Economics*, 87, 77–113. [5]

Mongin, Philippe (1994), "Harsanyi's Aggregation Theorem: Multi-Profile Version and Unsettled Questions." *Social Choice and Welfare*, 11(4), 331–354. [5, 9]

Morreau, Michael and John A. Weymark (2016), "Measurement Scales and Welfarist Social Choice." *Journal of Mathematical Psychology*, 75, 127–136. [3]

Narens, Louis and Brian Skyrms (2020), *The Pursuit of Happiness: Philosophical and Psychological Foundations of Utility*. Oxford University Press, Oxford. [7]

Naumova, Natalia and Elena Yanovskaya (2001), "Nash Social Welfare Orderings." *Mathematical Social Sciences*, 42(3), 203–231. [10]

Nebel, Jacob M. (2021), "Utils and Shmutils." *Ethics*, 131(3), 571–599. [3]

Nebel, Jacob M. (2022), "Aggregation Without Interpersonal Comparisons of Well-Being." *Philosophy and Phenomenological Research*, 105(1), 18–41. [3]

Nebel, Jacob M. (2023a), "Ethics Without Numbers." *Philosophy and Phenomenological Research*, forthcoming. [3]

Nebel, Jacob M. (2023b), "Strong Dictatorship via Ratio-Scale Measurable Utilities: A Simpler Proof." *Economic Theory Bulletin*, 11(1), 101–106. [4, 11]

Nebel, Jacob M. (2023c), "The Sum of Well-Being." *Mind*, doi: 10.1093/mind/fzad019. [2, 7, 8]

Nozick, Robert (1974), *Anarchy, State, and Utopia*. Basic Books, New York. [14]

Parfit, Derek (1984), *Reasons and Persons*. Clarendon Press, Oxford. [3]

Pérez-Nievas, Mikel, José I. Conde-Ruiz, and Eduardo L. Giménez (2019), "Efficiency and Endogenous Fertility." *Theoretical Economics*, 14(2), 475–512. [3]

Pivato, Marcus (2013), "Variable-Population Voting Rules." *Journal of Mathematical Economics*, 49(3), 210–221. [6]

Pivato, Marcus (2015), "Social Choice with Approximate Interpersonal Comparison of Welfare Gains." *Theory and Decision*, 79(2), 181–216. [4]

Pivato, Marcus (2020), "Rank-Additive Population Ethics." *Economic Theory*, 69(4), 861–918. [13]

Raschka, Robert (2023), "A Single Relation Theory of Welfarist Social Evaluation." Unpublished mansucript, Department of Economics, University of Hamburg. [4]

Roberts, Kevin (1980), "Interpersonal Comparability and Social Choice Theory." *The Review of Economic Studies*, 47(2), 421–439. [4]

Roemer, John E. (1998), *Theories of Distributive Justice*. Harvard University Press, Cambridge, MA. [3]

Scanlon, Thomas M. (1991), "The Moral Basis of Interpersonal Comparisons." In *Interpersonal Comparisons of Well-Being* (John E. Roemer and Jon Elster, eds.), Studies in Rationality and Social Change, 17–44, Cambridge University Press, Cambridge. [13]

Scovronick, Noah, Mark B. Budolfson, Francis Dennig, Marc Fleurbaey, Asher Siebert, Robert H. Socolow, Dean Spears, and Fabian Wagner (2017), "Impact of Population Growth and Population Ethics on Climate Change Mitigation Policy." *Proceedings of the National Academy of Sciences*, 114(46), 12338–12343. [2]

Sen, Amartya (1970), *Collective Choice and Social Welfare*. Holden-Day, San Francisco. [2, 3, 13]

Sen, Amartya (1977a), "On Weights and Measures: Informational Constraints in Social Welfare Analysis." *Econometrica*, 45(7), 1539–1572. [3]

Sen, Amartya (1977b), "Social Choice Theory: A Re-Examination." *Econometrica*, 45(1), 53–89. [11]

Sen, Amartya (1997), *On Economic Inequality*, enlarged edition. Oxford University Press, Oxford. [13]

Suppes, Patrick (1951), "A Set of Independent Axioms for Extensive Quantities." *Portugaliae Mathematica*, 10(4), 163–172. [2]

Suppes, Patrick (1966), "Some Formal Models of Grading Principles." *Synthese*, 16(3/4), 284–306. [2, 13]

Suzumura, Kotaro (1996), "Interpersonal Comparisons of the Extended Sympathy Type and the Possibility of Social Choice." In *Social Choice Re-Examined* (Kenneth J. Arrow, Amartya Sen, and Kotaro Suzumura, eds.), 202–229, Palgrave Macmillan UK, Basingstoke. [13]

Tsui, Kai-yuen and John A. Weymark (1997), "Social Welfare Orderings for Ratio-Scale Measurable Utilities." *Economic Theory*, 10(2), 241–256. [4, 11]

von Neumann, John and O. Morgenstern (1953), *Theory of Games and Economic Behavior*, third edition. Princeton University Press, Princeton, NJ. [4]

Weymark, John A. (1981), "Generalized Gini Inequality Indices." *Mathematical Social Sciences*, 1(4), 409–430. [8]

Weymark, John A. (1991), "A Reconsideration of the Harsanyi–Sen Debate on Utilitarianism." In *Interpersonal Comparisons of Well-Being* (Jon Elster and John E. Roemer, eds.), 255–320, Cambridge University Press, Cambridge. [21]

Weymark, John A. (1993), "Harsanyi's Social Aggregation Theorem and the Weak Pareto Principle." *Social Choice and Welfare*, 10(3), 209–221. [21]

Weymark, John A. (1998), "Welfarism on Economic Domains." *Mathematical Social Sciences*, 36(3), 251–268. [9, 25]

Weymark, John A. (2005), "Measurement Theory and the Foundations of Utilitarianism." *Social Choice and Welfare*, 25(2), 527–555. [4]

Weymark, John A. (2019), "Demographic Issues in Formal Models of Population Axiologies." Presentation at the Oxford Workshop in Global Priorities Research. Global Priorities Institute, University of Oxford. [12]

Xu, Yongsheng (1990), "Population Evaluation Functions: Axiomatizations of Two Utilitarian Principles." *Mathematical Social Sciences*, 20(1), 91–98. [5]