Cantor on Infinity in Nature, Number, and the Divine Mind

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Abstract. The mathematician Georg Cantor strongly believed in the existence of actually infinite numbers and sets. Cantor's "actualism" went against the Aristotelian tradition in metaphysics and mathematics. Under the pressures to defend his theory, his metaphysics changed from Spinozistic monism to Leibnizian voluntarist dualism. The factor motivating this change was two-fold: the desire to avoid antinomies associated with the notion of a universal collection and the desire to avoid the heresy of necessitarian pantheism. We document the changes in Cantor's thought with reference to his main philosophical-mathematical treatise, the *Grundlagen* (1883) as well as with reference to his article, "Über die verschiedenen Standpunkte in bezug auf das aktuelle Unendliche" ("Concerning Various Perspectives on the Actual Infinite") (1885).

I.

The Philosophical Reception of Cantor's Ideas. Georg Cantor's discovery of transfinite numbers was revolutionary. Bertrand Russell described it thus:

> The mathematical theory of infinity may almost be said to begin with Cantor. The infinitesimal Calculus, though it cannot wholly dispense with infinity, has as few dealings with it as possible, and contrives to hide it away before facing the world Cantor has abandoned this cowardly policy, and has brought the skeleton out of its cupboard. He has been emboldened on this course by denying that it is a skeleton. Indeed, like many other skeletons, it was wholly dependent on its cupboard, and vanished in the light of day.[1]

[1]Bertrand Russell, *The Principles of Mathematics* (London: Routledge, 1992 [1903]), 304. For more on Cantor's influence on Russell, see Ivor Grattan-Guinness, "Georg Cantor's influence on Bertrand Russell," *History and Philosophy of Logic* 1 (1980): 61–93.

Joseph Dauben has argued Cantor's theory of transfinite numbers was a revolution, not only for mathematics, but for metaphysics. By vindicating the concept of the actual infinite, Cantor appeared to overthrow a long-standing scholastic and Aristotelian tradition.[2] Cantorian set theory requires a commitment to the existence of actual, completed infinities. Aristotle, by contrast, states that the true metaphysical concept of infinity is that of the potential (incomplete, always growing) infinity.[3] Furthermore, Aristotle claims that mathematicians only ever need to assume the existence of potential infinities in order to do their mathematics.[4] Contemporary mathematicians, for the most part, accept Cantorian set theory but do not agree on whether acceptance of set theory carries an ontological commitment to the actual infinite. So whereas Cantor was once a mathematical heretic, his mathematical views have become mainstream. The same cannot be said for Cantor's metaphysics, which remains somewhat obscure and esoteric.

One reason for the obscurity of Cantor's metaphysics is the breadth of the philosophical tradition with which Cantor engages. Cantor's first major presentation of set theory, *Grundlagen einer allgemeine Mannigfaltigkeitslehre* [Foundations of a General Theory of Manifolds] (1883) is a defense of a new theory against objections drawn from the history of philosophy.[5] It is a self-consciously metaphysical treatise, replete with extensive allusions to Plato, Leibniz, and Spinoza, among many others. With few exceptions, mathematicians run from the metaphysics and philosophers run from the mathematics.[6] However, even the exceptional studies of Hallett and Dauben do not answer all the questions regarding Cantor's engagement with philosophy. Hallett and Dauben emphasize Cantor's engagement with such canonically Christian theologians as Augustine and Constantin Gutberlet, respectively. This article, by contrast, focuses on Cantor's engagement with the philosophy of Spinoza and Leibniz.

For the purposes of this discussion, "Spinozism" refers to the metaphysics of Spinoza's *Ethics*, especially the first and second parts. The essential doctrine

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[2]Joseph W. Dauben, "Conceptual Revolutions and the History of Mathematics" in *Revolutions in Mathematics*, ed. D. Gillies (Oxford: Clarendon Press, 1992), at 59–60.

[3]Aristotle, *Physics*, trans. R. Waterfield (Oxford: Oxford University Press, 1996), III.6 206b33, at 73.

[4]*Physics* III.7: 207b27, at 76.

[5]G. Cantor, "Grundlagen einer allgemeine Mannigfaltigkeitslehre" in *Georg Cantor Gesammelte Abhandlungen*, ed. Ernst Zermelo (Berlin: Springer-Verlag, 1932/Hildesheim: Georg Olms, 1962), 165–209. English version: "Foundations of a General Theory of Manifolds: A Mathematico-Philosophical Investigation into the Theory of the Infinite," trans. William Ewald in *From Kant to Hilbert* vol. II, ed. W. Ewald (Oxford: Clarendon Press, 1996), 878–920.

[6]Joseph W. Dauben, *Georg Cantor: His Mathematics and Philosophy of the Infinite*. (Cambridge, Mass.: Harvard University Press, 1979). M. Hallett, *Cantorian Set Theory and Limitation of Size* (Oxford: Clarendon Press, 1984).

of Spinoza's metaphysics is that there is but one absolutely infinite substance, which Spinoza calls "God or nature" (*Deus sive natura*). According to Spinoza, this unique substance has two principal attributes: intellect and extension. The usual interpretation of these attributes is that they comprise an infinite divine intellect and infinite physical universe. Finally, Spinoza also holds that all things follow of necessity from the essence of the one substance. This constellation of views was identified as pantheist in the *Pantheismusstreit*.[7]

One rationale for this focus is that indirectly, Spinoza's metaphysics, through the mediation of German idealists, influenced the philosophical climate in which Cantor worked. Another rationale for the focus is a possible parallelism between the logical difficulties confronting rationalist metaphysics and set theory.[8] Finally, Spinoza is usually classified as a pantheist. Cantor's defence of the actual infinite at times looks like a pantheist position. However, by 1885 Cantor strongly protests against the classification of his position as "pantheist" by the Jesuit Cardinal Johannes Franzelin. One of the aims of the article is to determine to what extent it is appropriate to view Cantor's early metaphysics as Spinozist and pantheist in any way.

II.

Overview of Cantor's Metaphysics: Cantor's Engagement with the Metaphysics of Spinoza and Leibniz. We may divide the development of Cantor's metaphysics into two periods: an early period running from 1872 until 1883, and a later period stretching from 1886 until 1895. The years 1884–86 qualify as a transitional period. At around this time, Cantor began associating with neo-scholastic theologians willing to defend the reality of the actual infinite, such as Constantin Gutberlet.[9] By 1886, Cantor corresponded with Cardinal Johannes Franzelin concerning the theological status of his ideas.

As a graduate student, Cantor studied the first book of Spinoza's *Ethics* carefully. An unpublished notebook dated 1871–72 from the archives at the *Staatsbibliothek* at the University of Göttingen contains Cantor's notes in Latin commenting on Part I ("De Deo") of Spinoza's *Ethics*.[10] The notebook dates

[7]For discussion of *Pantheismusstreit*, see F. Beiser, *Fate of Reason* (Cambridge, Mass.: Harvard University Press, 1987).

[8]After the research for this article was carried out, another article discussing Cantor's engagement with Spinozism appeared: J. Ferreiros, "The Motives Behind Cantor's Set Theory—Physical, Biological, and Philosophical Questions," *Science in Context* 17(1/2): 49–83. Ferreiros discusses the influence of Romantic *Naturphilosophie* and cites the influence of Leibniz's monadology on Cantor's attempt to apply point-set theory to nature.

[9]Dauben, *Georg Cantor: His Mathematics and Philosophy of the Infinite*, 280.

[10]Cantor's short set of Latin notes on Book I of Spinoza's *Ethics* have not, to my knowledge, been published anywhere. The seven pages follow the Latin text of the definitions of part I of

from the same time that Cantor was laying the foundations for his transfinite set theory. He was working on his uniqueness proof (*Eindeutigkeitsbeweis*) for the representation of functions using trigonometric series. This work eventually led to his view that there are actually infinite point-sets, since he proved the uniqueness of the representation holds even when an infinite number of points are excepted from a continuous curve.

There is evidence from the report on his *Habilitationsschrift* oral examination of 24 November 1868 that Cantor conceived of his mathematics as a refinement of Spinoza's project of treating of metaphysics using a geometrical method.[11] The report comments:

> The philosophical work of the candidate has been to answer the question: What does Spinoza understand in his *Ethics* by the geometrical method and how is his application of it to be judged? . . . Besides an overall knowledge of the history of philosophy, he showed a unique acquaintance with Spinoza's *Ethics*.[12]

Unfortunately the text of Cantor's lecture is lost to us; we have only the report of his examiners. The connection between Cantor's "philosophical work" and Spinoza's philosophy is intriguing. In what way might Cantor's philosophical work be seen as a continuation of Spinoza's philosophy? On the face of it, it is curious that Cantor should be interested in Spinoza's metaphysics. After all, the main thesis of Part I of Spinoza's *Ethics* is that there is *only one* absolutely infinite substance, "God or Nature" (*Deus sive Natura*).[13] However, the main theme of Cantor's *Grundlagen* is that there are *multiple actual infinities*, because there is a realm of an actual, but increasable infinite known as the transfinite. Moreover, Cantor retains the traditional theological idea that *absolute* infinity pertains to God alone. This distinction between the "absolute infinite" of God and the "transfinite infinite" will be essential in differentiating Cantor's metaphysics from Spinoza's in what follows.

Spinoza's *Ethics* verbatim on the lefthand side of the notebook and contain Cantor's close summary (with paraphrases of propositions) of Spinoza's philosophy on the righthand side. There is a reference to the notebook in H. Bandmann, *Die Unendlichkeit des Seins: Cantors Unendlichkeitsidee und ihre metaphysische Wurzeln* (Frankfurt am Main, 1992), at 127.

[11]H. Bandmann, *Die Unendlichkeit des Seins: Cantors Unendlichkeitsidee und ihre metaphysische Wurzeln* (Frankfurt am Main: P. Lang, 1992).

[12]The report is quoted in W. Purkert and H. J. Illgauds, *Georg Cantor 1845–1918* (Basel: Birkhauser, 1987), 183–5 (my translation).

[13]Spinoza also recognizes that different attributes are "infinite in kind." A thing is "infinite in its kind" just in case it is not limited or bounded by another thing of the same kind. Both physical extension and thought are "infinite in kind." However, being of different types, these attributes do not amount to a multiplicity of infinities of the same type.

Cantor's belief in the existence of infinite *numbers* was contrary to received philosophical wisdom. Even philosophers such as Spinoza and Leibniz, who gave a starring role to the metaphysical conception of the infinite in their systems, were dubious about the notion of infinite *numbers*. Cantor regarded Spinoza and Leibniz as having supplied some of the best arguments against the possibility of infinite numbers. Nonetheless, Cantor regards himself as having refuted their arguments. In what follows we shall set out the positions of Spinoza and Leibniz on infinity and compare these positions with Cantor's position.

Spinoza's views on the infinite are set out in his correspondence, especially his letter to Lodewijk Meyer dated 20 April 1663. Spinoza's overarching concern was to defend his metaphysical view that the world consists of one "absolutely infinite" and "indivisible" substance.[14] Specifically, he wanted to resist the suggestion that, because substance has extension or matter, it can be divided infinitely. Descartes had claimed "the number of particles into which matter is divided is in fact indefinite, although it is beyond our power to grasp them all."[15] Spinoza maintains the distinction between "what is called infinite because it has no limits" and "that whose parts we cannot explain or equate with any number, though we know its maximum or minimum."[16] The later kind of infinite Spinoza calls "indefinite." As an example of the indefinite, Spinoza considers the space that exists between two non-concentric circles, one contained within the other, with diameters AD (the diameter of larger circle) and BC (the diameter of the smaller circle embedded in the larger circle) respectively.

Spinoza claims that contradictions result from trying to attach a number to the number of divisions we can make in each curve. He appears puzzled by the fact that one circle clearly has a greater circumference than the other, and yet it seems impossible to say that the path travelled by a particle on one circular path is more "infinitely" divisible (greater in parts) than the path travelled by particle on the other circular path.

Spinoza does not argue clearly for this conclusion in the letter to Meyer, but does explain his reasoning more clearly in his letter to Tschirnhaus in reply to a request for clarification:

> As to what I stated in my letter regarding the infinite, that it is not from the multitude of their parts that the infinity is inferred, this is evident

[14]Baruch Spinoza, *Ethics* (Book I, proposition 15). See E. Curley, ed. and trans., *The Collected Works of Spinoza* (Princeton: Princeton University Press, 1985).

[15]René Descartes, *Principles of Philosophy*, Part II: 34, in *The Philosophical Writings of Descartes* vol. I, ed. J. Cottingham, D. Murdoch, and R. Stoothoff (Cambridge: Cambridge University Press, 1985), at 239.

[16]Baruch Spinoza, Letter 12, "BDS to the very learned and expert Lodewijk Meyer, Doctor of Medicine and Philosophy" in *The Collected Works of Spinoza*, ed. and trans. E. Curley (Princeton: Princeton University Press, 1985), 200.

> from the fact that if infinity were inferred from the multitude of parts, it would be impossible for us to conceive a greater multitude of parts, but this multitude of parts ought to be greater than any given number. This is untrue, for in the total space between the two non-concentric circles, we conceive twice as many parts as in half that space, and yet the number of parts in both the half-space and the whole space exceeds any assignable number.[17]

The same concern with infinite quantities greater or lesser than one another appears in condensed form in the letter to Meyer: "Lastly, there are things called infinite, or if you prefer indefinite, because they cannot be accurately expressed by any number, while yet being conceivable as greater or less."[18] Spinoza is puzzled by the fact that two sets of points can have the same number of members (continuum-many points) and yet compose areas that differ in extent.[19] Rather than deem the infinite sets to be the same size (have the same cardinal number), Spinoza avoids attaching cardinal numbers at all to infinite sets.

Cantor refers to Spinoza's letter to Meyer in the *Grundlagen*, which he describes as "highly important" and "rich in content" and promises to provide later "a detailed and thorough discussion."[20] In the absence of Cantor's promised discussion, we can only conjecture based on knowledge of Cantor's point-set topology how Cantor would have solved Spinoza's problem of the two circles. Cantor would have equated the *number* of points in the two areas. Similarly, he would have identified the *number* of points in the circumferences of each of the two circles. In each case the reasoning is the same: a one-to-one correspondence obtains between the set of all points in one geometrical entity and the other. Let's consider the simplest case of the two circumferences. To see the correspondence, note that the radius of the larger circle can sweep through every point on the circumference of the smaller circle and on its own circumference. This is in fact a simple case of the more general result that any continuous space (of any dimension) is equipollent with, i.e., has the same cardinality as, the linear (one-dimensional) continuum, a surprising result in topology known as "the invariance of dimension."[21] Cantor proved a limited version of the invariance of dimension in his early paper of 1874. In his

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[17]Baruch Spinoza, Letter 81, to Tschirnhaus, in *The Ethics, Treatise on the Emendation of the Intellect and Selected Letters*, trans. S. Shirley (Indianapolis, Ind.: Hackett, 1981), 291–2.

[18]Spinoza, Letter 12, in *The Collected Works of Spinoza*, 200.

[19]In the letter to Meyer, Spinoza seems concerned about the circumferences of the circles; in the letter to Tschirnhaus, he seems concerned about the areas between the two circles.

[20]G. Cantor, *Gesammelte Abhandlungen*, 175; English version *From Kant to Hilbert*, vol. II, 890.

[21]D. M. Johnson, "The Problem of the Invariance of Dimension in the Growth of Modern Topology, Part I," *Archive for History of Exact Sciences* 20(2): 91–188.

famous 1877 letter to Dedekind, Cantor showed that there was a one-to-one mapping between the points on a line segment of unit length [0,1] and the points on a square with sides of unit length [0,1] × [0,1]. We may conjecture that one reason Spinoza's letter to Meyer would have interested Cantor so much would have been its perceived relevance to fundamental questions in basic topology.

Despite his careful study of Spinoza's metaphysics and his letters, in the end Cantor's own metaphysics (especially his views on the infinite) differed a great deal from Spinoza's metaphysics. In some ways, Cantor's views are actually closer to Leibniz than Spinoza. Leibniz is generally regarded as a "friend of the infinite," although his tripartite position is highly nuanced. First, Leibniz believes there are infinitely many individuals in nature. Second, he sharply distinguishes between God's absolute infinity and the infinity of individuals in nature. Third, he denies that there are infinite *numbers*. Thus, Leibniz's position bears some affinity to Spinoza's position (in denying infinite numbers) but is also more orthodox (in distinguishing between the infinity of God and infinity in nature). As we shall see, Cantor agrees with Leibniz's views on infinity in nature and God, but disagrees with Leibniz's position on infinite numbers.

In his "Letter to Simon Foucher" (1692), Leibniz advocated the existence of an actual infinite in nature. Cantor quoted with approval this part of Leibniz's letter in the *Grundlagen*:

> I am so much in favour of an actual infinite that instead of admitting that nature abhors it, as is commonly said, I hold that it affects nature everywhere in order to indicate the perfections of its Author. So I believe that every part of matter is, I do not say divisible, but actually divided, and consequently the smallest particle should be considered as a world full of an infinity of creatures.[22]

However, in contrast to his acceptance of infinity in nature, Leibniz rejects infinity in number. In his *New Essays on Human Understanding*, Leibniz claims that a paradox would result from countenancing the idea of an infinite number.[23] There can be no greatest number. The notion of an "infinite number" suggested to Leibniz a number surpassing all other numbers. He rightly rejects such a notion as incoherent. Cantor's transfinite numbers are not supreme maxima: each trans-

[22]Cantor, *Gesammelte Abhandlungen*, (*Grundlagen*, §7), 179. Cantor quotes Leibniz in French from the Erdmann edition of Leibniz's letters.

[23]For a discussion of Leibniz on infinite number, see Jonathan Bennett, *Kant's Dialectic* (Cambridge: Cambridge University Press, 1974), 130. For citation of original Leibniz sources, see B. Russell, "The Labyrinth of the Continuum" in *The Philosophy of Leibniz*, (London: Routledge, 1992), 109–10.

finite number can be superceded by a yet greater transfinite number. Transfinite numbers share with finite numbers the property of being increasable.

In Cantor's view, once we have infinite collections, we must have infinite (transfinite) numbers. He says "Infinite number and set are indissolubly bound up with each other; if we give up one of them, we no longer have the right to the other."[24] Cantor's essential innovation was to suggest that an infinity can be mathematically determinate while still retaining its character as truly infinite.[25] He insists, of course, that the true infinite is an actual (completed and determinate) infinite. This rehabilitation of the notion of the actual infinite is something Cantor shares with Spinoza and Leibniz. However, Cantor's view is that the actual infinite is not merely a metaphysical concept, but also a mathematical concept embodied in the form of his transfinite numbers.

Cantor's references to the metaphysics of Spinoza and Leibniz are not all critical; some of his references are positive and demonstrate that he accepted a philosophy of nature (*Naturphilosophie*) that reflected aspects of the metaphysics of both thinkers. In his *Grundlagen*, in section 5, Cantor states that

> [A]n organic explanation [of nature] can, I believe, only be approached through a resumption and advancement of Spinoza's and Leibniz's work and endeavours.[26]

The context of the quotation shows that Cantor views the metaphysics of Spinoza and Leibniz as an antidote to the mechanical explanation of natural phenomena found elsewhere in natural philosophy. Moreover, Cantor views his mathematical work as having applications that will advance the project of giving a non-mechanical explanation of natural phenomena.[27] The applications are not really laid bare in the *Grundlagen* but are evident in Cantor's correspondence.

In a letter to Mittag-Leffler of 16 November 1884, Cantor specifies how he would apply his mathematics to nature. In that letter we learn that Cantor hoped to apply his set theory to nature in a way that would result in non-mechanistic explanations of phenomena, especially consciousness. The outlines of how this project is to proceed are exceedingly unclear. Cantor does tell us that he will use his concept of cardinal numbers to gauge the abundance and concentration of different kinds of matter in nature. He proposes that there are as many "corporeal

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[24]Cantor, *Gesammelte Abhandlungen*, 394.

[25]However, finitists would argue that Cantor's transfinite numbers are *too determinate, too similar to finite numbers*, to be truly infinite. Finitists agree with Aristotle that the proper conception of infinity is that of something that is endless and essentially incomplete and indeterminate.

[26]Cantor, *Gesammelte Abhandlungen*, 177 (translation mine).

[27]For a discussion, see Joseph Dauben, *Georg Cantor*, 291–3. Dauben covers both the letter to Mittag-Leffler and points out the affinity with Leibniz's philosophy.

monads" as there are natural numbers and as many "aetherial monads" as there are points in the continuum. Leibniz's influence is evident in Cantor's decision to characterize extensionless points of mass as "monads." It is unclear how committed to this view Cantor was, as it was produced in response to a request for concrete applications of his set theory.[28] Cantor did not think that mathematics required applications for its justification: "the essence of mathematics lies in its freedom" was his motto in the *Grundlagen*.[29] Nonetheless, we find nothing to indicate that the Leibnizian metaphysics was not sincerely adopted, and we find references to Leibnizian metaphysics even in his correspondence with Cardinal Franzelin as late as 1886.

Cantor's *Grundlagen* presents a distinctive metaphysics and conception of the objects of mathematics. In particular, Cantor distinguishes between the *immanent* and *transeunt* reality of ideas.[30] The immanent reality of an idea consists in its internal coherence and consistency. Mathematics is concerned properly only with immanent reality of ideas. The task of metaphysics (according to Cantor in the *Grundlagen*) is to determine the transeunt reality of ideas, by examining internally consistent ideas and determining whether any objects correspond to them in the natural, physical universe. Mathematics is "free" in the specific sense that it is and ought to be free from metaphysical disputes about the transeunt reality of ideas.

In endnote 5 of the *Grundlagen*, Cantor explains his notion of the "immanent reality" of ideas with reference to Spinoza's definition of an "adequate idea," viz. an idea that considered in itself has all the marks of a true idea (internal consistency and coherence).[31] Cantor's belief that the mathematical universe conforms to Spinoza's metaphysics is strikingly evident in his claim that the two types of reality attributed to ideas are co-extensive:

> Given the thoroughly realist—simultaneously, however, no less idealist—foundations of my investigations, there is no doubt in my mind that these two types of reality will also be found together, in the sense that a concept [idea] to be regarded as existent in the first respect [immanently real] will always in certain, even in infinitely many ways, possess a transeunt reality as well. [Here Cantor refers the reader to his sixth endnote.] This coherence of the two realities has its true foundation in the unity of the All to which we ourselves belong as well.[32]

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[28] *Georg Cantor*, 291, 359: Dauben cites Cantor's Letter to Mittag-Leffler of November 6, 1884, reproduced in Ivor Grattan-Guinness, "An Unpublished Paper by Georg Cantor: 'Principien einer Theorie der Ordungstypen erste Mitteilung,'" *Acta Mathematica* 124 (1970): 65–107, at 78–80.

[29] Cantor, *Gesammelte Abhandlungen*, 182.

[30] Ibid., 175.

[31] Ibid., 206 (endnote 5 to the *Grundlagen*).

[32] Ibid., 181–2.

In the sixth endnote that accompanies the passage from Cantor quoted above, there is an explicit reference to Spinoza's metaphysics, especially his doctrine of the parallelism between thoughts and extension. According to Spinoza's doctrine of parallelism, "the order and connection among ideas is the same as the order and connection of things."[33] It is possible that Cantor thought of his isomorphism between immanently real and transeuntly real ideas as a version of Spinoza's parallelism.

Hallett describes Cantor's allusion to 'the unity of the All' in the passage above as "mysterious" and "mystical."[34] He is not sure how "the unity of the All" is supposed to explain the coincidence of the immanent and transeunt reality of ideas according to Cantor. Hallett interprets Cantor's metaphysics as relying on an existential maximal principle: "as many things as are possible exist." Hallett correctly locates this principle as pertaining to a certain kind of metaphysics, on which all things are conceived to have existence in the divine intellect. Hallett is also correct in suggesting that such metaphysics is Augustinian. However, Hallett does not remark that this principle of plenitude is also characteristic of the metaphysical systems of Spinoza and other figures Cantor admires, such as Bruno and Leibniz.[35]

Despite their differences, Cantor does share with Spinoza the *principle of plenitude*, according to which as much as possible is created. Such plenitude is implied by Cantor's principle in the *Grundlagen* that all immanently real ideas will have a corresponding transeunt reality. For simplicity, let's suppose that if *x* is an "immanently real" idea then it is a possibility, and if *x* is a "transeuntly real" idea, then it is an actuality. Then Cantor's principle says that to every possibility there corresponds an actuality. That principle is not equivalent to a global plenitude principle. From the fact that every possible idea is actual, it does not follow that all ideas are conjointly possible and actual. Indeed, the "immanent reality" of an idea might seem to concern only its internal consistency not its consistency with an existing body of knowledge. A truly global plenitude principle would say that as much as possible (as is jointly consistent) must exist. Cantor's plenitude principle says that *if* an idea is inherently possible (internally consistent, "immanently real") then it will be actual (transeuntly real). In fact these alternative renderings of the plenitude principle are not always sharply distinguished in Cantor's writing.

In Spinoza's *Ethics*, the principle of plenitude is found in Book I, proposition 33: "From the necessity of the divine nature there must follow infinite things

[33]Spinoza, *Ethics*, Book II, proposition 7.

[34]Hallett, *Cantorian Set Theory*, 20.

[35]On Cantor's affinity with Bruno, see Marie Luise Heusler-Kessler, "Georg Cantors transfinite Zahlen und Giordano Brunos Unendlichkeitsidee," *Selbstorganisation* 2 (1991): 221–44. Cantor's primary references to Bruno are at *Gesammelte Abhandlungen*, 205.

in infinite modes, (i.e., everything which can fall under an infinite intellect)."[36] Spinoza claims that God creates everything that can fall within the scope of an infinite intellect, viz. infinite things in infinite ways. It is not surprising that the principle of plenitude is found tucked away in a proposition which purports to demonstrate the *necessity* with which everything flows from the divine essence, for it is this principle which helps legitimate this view. If there is no surplus of possibilities for the world, it follows that everything that happens in it is part of a necessary plan.

In writings after the *Grundlagen*, Cantor clarified his principle of plenitude. He now made it clear that the reality of an idea is to be based on its conceivability by the *divine*, rather than human, intellect. This is evident in his letter to Ebhard Illigens of 21 May1886:

> If I have recognised the inner consistency of a concept which points to a being, then the idea of God's omnipotence impels me to think of the being as expressed by the concept as in some way actually realizable.[37]

For which kind of reality, immanent or transeunt, did Cantor think consistency of an idea was sufficient? The answer to this question divides his earlier and later metaphysics. By the time of his later metaphysics he abandoned the view that both transeunt and immanent reality could be determined by the consistency of a concept, and settled for the more modest claim that *mere* immanent reality resulted. Consequently, not all ideas, though possible in themselves, are realized in the external world. This qualification is already evident is his letter to Illigens, quoted above, which goes on to emphasize that:

> Consequently I call the being concerned a "possible" being. By this is not meant that the being somewhere, somehow and sometime exists in actuality (*Wirklichkeit*), since that depends on further factors, but only that it can exist. Thus for me the two concepts "suited for existence, i.e., for being created" and "possibility" coincide.[38]

This position stands in opposition to Cantor's uncritical principle in the *Grundlagen* that the two types of reality, possible and actual existence, will "doubtless always be found together." In short, it represents a change from Spinozistic pantheism and monism to a more orthodox, indeterminist dualism. What was the reason that led to this change?

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[36]Spinoza, *Ethics*, Book I, proposition 16, 424.
[37]As quoted in Hallett, *Cantorian Set Theory*, 20.
[38]Hallett, *Cantorian Set Theory*, 20; Cantor's *Nachlass* 16, 52–3.

III.

Avoiding the Antinomies. It might be thought that Cantor's principle of plenitude led to the set-theoretic antinomies (or "paradoxes"). In this way, his change of metaphysics could be motivated by a desire to avoid antinomies. In this section, we argue that this supposition is mistaken. The set-theoretic antinomies did not threaten Cantor's set theory, nor did his awareness of the antinomies play any significant role in motivating him to change metaphysical views.[39]

One might think that the principle of plenitude licenses treating the entire set-theoretic universe as a set. There is no obvious reason why all the sets in the universe are not jointly compossible and hence such that they can be gathered into a single set.

Yet it is evident that Cantor regards the set-theoretic universe as a whole as something that cannot be considered a set. In particular, in his letter to Hilbert of 15 November 1899, he explains why it is not even possible to gather up all the cardinal numbers together into a set. The attempt to do so would result in "an inconsistent multiplicity."

For such a class of all cardinals would have to have a cardinal number itself, which would contradict the hypothesis that it contained all numbers.[40]

Cantor informs us in his letter to Hilbert of 15 November 1899 that his knowledge of the set-theoretic antinomy was "clearly though intentionally hidden in my 1883 *Grundlagen*, specially in the endnotes."[41] The passage Cantor alludes to does indeed show an understanding of the problem:

> I have no doubt that, as we pursue this path [of generating numbers] even further, we shall never reach a boundary that cannot be crossed; but that we shall also never achieve even an approximate conception of the absolute. *The absolute can only be acknowledged [anerkannt] but never known [erkannt]—and not even approximately known*. For just as in the number class (I) every finite number, however great, always has the same power of the finite numbers greater than it, so every supra-finite number, however great, of any of the higher number-classes (II) or (III), etc., is followed by an aggregate of numbers and number-classes whose

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[39]For the role of Zermelo in creating the impression that Cantorian set theory is naive, see Hallett, *Cantorian Set Theory*, 238–9. See also Russell's treatment of "Cantor's Paradox," in B. Russell, *The Principles of Mathematics* (London: Routledge, 1992 [first edition 1903]), 362.

[40]Cf. Ignacio Jané, "The Role of the Absolute Infinite in Cantor's Conception of Set," *Erkenntnis* 42(3): 375–402. Jané discusses the letter to Hilbert in depth as well as going into more detail on the principles for generating numbers in the *Grundlagen*.

[41]Walter Purkert, "Cantor's Views on the Foundations of Mathematics" in *The History of Modern Mathematics* I, ed. D. Rowe and J. McCleary (Poughkeepsie, N.Y.: Academic Press, 1989), 49–65.

> power is not in the slightest reduced compared to the entire absolutely infinite aggregate of numbers, starting with 1. As Albrecht von Haller says of eternity: "I attain to the enormous number, but you, O eternity, lie always ahead of me." *The absolutely infinite sequence of numbers thus seems to me to be an appropriate symbol of the absolute*; in contrast, the infinity of the first number-class (I) . . . seems to me to dwindle into nothingness by comparison.[42]

In this paragraph Cantor shows that he has a metaphysical picture that prevents the antinomy from arising at all. He considers all the number-classes to form a scale or ladder. On each step of the ladder sit all the ordinal types with the same size (associated with the same "power" or cardinal number). The entire ladder—the sequence of all the number-classes—is "absolutely infinite." In general, "absolute infinity" pertains to God and is not mathematically determinate. It is clear that being "absolutely infinite," the class of all number-classes does not itself have a number. This suffices to block Cantor's Paradox. What is less clear is the explanation for why the class of all number-classes lacks a number.[43] We can give two kinds of explanation: metaphysical and mathematical.

Let's consider the metaphysical explanation first. According to the traditional conception of God as an *ens realissimum*, God is the most perfect and real being, and possesses all attributes or perfections. This conception of God is evident in Spinoza's *Ethics*, where God is defined as "an absolutely infinite being."[44] However, given such an infinity of attributes, it might be assumed to follow that God cannot be adequately conceived by a finite mind.[45] In the same way, the collection of absolutely all the numbers is beyond adequate comprehension by a finite mind. In the quotation above Cantor considers such a number sequence as "an appropriate symbol of the absolute." Each number itself may be considered as an idea in the divine mind. But the entire number sequence, like the notion of all God's ideas, is not an object that can be represented and comprehended by human mathematical thought.[46] For Cantor, the collection

[42]Cantor, *Gesammelte Abhandlungen* (*Grundlagen* endnote 2, 205; *From Kant to Hilbert* II, 916–7 [emphasis mine]).

[43]Both Hallett, *Cantorian Set Theory*, and Shaughan Lavine, *Understanding the Infinite* (Cambridge, Mass.: Harvard University Press 1994), discuss this important passage from Cantor's *Grundlagen*. Moreover, both support the contention that Cantorian set theory was always free from paradox.

[44]Spinoza, *Ethics*, Book I, proposition 8, scholium 1.

[45]To be sure, it is fallacious to claim that, *because* a mind is finite, therefore it cannot conceive of infinitely many items. Cantor identifies this fallacy in the *Grundlagen* §5, [4] (*Gesammelte Abhandlungen*, 176; *From Kant to Hilbert* vol. II, 891).

[46]Elsewhere (in his "Mitteilungen" [1886]) Cantor entertains Augustine's view that all the numbers exist as ideas in the divine mind *Gesammelte Abhandlungen*, 401–3, footnote 3. For discussion of aspects of Augustine's views in Cantor, see Hallett, *Cantorian Set Theory*, 35–7.

of everything mathematizable, which is "absolutely infinite," is of a radically different type from ordinary mathematical objects, including the transfinite numbers.[47] It is in this context that we should consider the qualification in Cantor's motto: "Omnia seu finita seu infinita *definita* sunt et excepto Deo ab intellectu determinari possunt" (All things, whether finite or infinite are definite and, with the exception of God, can be determined by the intellect).[48] God, or the absolute infinite, cannot be determined by the intellect: that is, it cannot be rendered an object of mathematical knowledge.

It might be thought that Cantor still lacked a precisely formulated mathematical rationale for blocking the antinomy. However, on closer inspection, there are already mathematical principles in place in the *Grundlagen* that ensure that Cantorian set theory is not vulnerable to the set-theoretic antinomies.[49] We give an abbreviated exposition of the matter, as our emphasis is on Cantor's metaphysics, not the details of his early set theory. The main such principle is Cantor's belief that every set is well-ordered. On Cantor's definition, a set is well-ordered just in case it is a well-defined set whose elements are bound together by a definite given law of succession, where the succession makes it clear which element is the first element of the set, and for every element, which element follows.[50] In the crucial passage quoted above, Cantor implies that the class of all numbers cannot be numbered. Lavine points out that the notions of enumeration and countability in the *Grundlagen* are used in a non-standard sense: "countable" for Cantor means capable of being counted in principle, i.e., well-ordered.[51] So when Cantor says that the class of all transfinite numbers "cannot be numbered," he is stating that this class cannot be well-ordered. Yet in the *Grundlagen* all sets are well-ordered. Even though the Axiom of Choice—known to be equivalent to the principle that all sets can be well-ordered—was not officially codified until Zermelo's work in 1908, the key idea is already present in Cantor's *Grundlagen*:

> The concept of a *well-ordered* set is thereby shown to be fundamental for the whole of set theory. That it is always possible to bring every *well-defined* set into the form of a *well-ordered* set seems to me to be a law of thought (*Denkgesetz*) rich in consequences and especially remarkable for its general validity.[52]

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[47]Hallett, *Cantorian Set Theory*, 43.

[48]Cantor, *Gesammelte Abhandlungen*, 176; *From Kant to Hilbert*, vol. II, 891.

[49]Lavine, *Understanding the Infinite*, 53. Lavine acknowledges the influence of Hallett, *Cantorian Set Theory*, in his interpretation.

[50]Cantor, *Gesammelte Abhandlungen*, 168.

[51]*Understanding the Infinite*, 53.

[52]Cantor, *Gesammelte Abhandlungen* (*Grundlagen* §3), 169 (my translation). The passage is also translated in Hallett, *Cantorian Set Theory*, 155.

Consequently, if the class of all numbers cannot be well-ordered, and if all Cantorian sets can be well-ordered, it follows that the class of all numbers is not a Cantorian set. Thus Cantorian set theory is quite different from the naive set theory that would place no restrictions on what could count as a set.

Cantor, then, had a conception of the set-theoretic universe that prevented the antinomies from arising. Yet in his philosophical asides and comments, Cantor sometimes adopts metaphysical principles, such as the *principle of plenitude*, that threaten to bring on the set-theoretic paradoxes. Why would Cantor say in the *Grundlagen* that every immanently real (logically possible) idea corresponds to a transeuntly real (actual, embodied) idea? If applied to the set-theoretic universe as a whole, this principle would lead to antinomies. Nonetheless, Cantor maintained the principle in restricted form. The reason he maintained it is that he needed as loose a criterion for the reality of ideas as logical possibility to prove that his transfinite numbers were as real as finite numbers. Whereas initially he did think that his transfinite numbers were mere empty symbols (not pointing to a concrete reality beyond the symbols), by the time of the *Grundlagen* (1883), he was convinced that they had as much right to be considered "concrete numbers of real significance" as the finite numbers.[53] Cantor's early metaphysical picture with its strong existential assumptions provided philosophical support for his belief in the reality of his transfinite numbers. Moreover, we have seen that Cantor's early metaphysics was free from contradictions. It therefore seems plausible to assume that Cantor's motivation for changing his metaphysics was not logical or mathematical. The evidence does in fact point to one such alternative motive: the desire to avoid heresy.

IV.

Cantor and the Pantheism Controversy. Spinoza is traditionally regarded as a pantheist, because he identifies God with the whole of nature and its immanent cause.[54] Cantor is clearly not a pantheist in this sense, because he did not reject the idea of a transcendent God who stands outside of his creation. Nonetheless, as we have seen, at the time of the *Grundlagen*, he shared with Spinoza the principle that the immanent reality suffices for the transeunt reality of an idea. This principle amounts to the collapse of the possible into the actual. The coincidence of the possible and the actual is strictly independent of pantheism. However, once the divine essence is conceived of as the realm of all possibilities, the coincidence of possibility and actuality follows from the pantheist (and

[53]Cantor, *Gesammelte Abhandlungen*, *Grundlagen*, §1, 166; *From Kant to Hilbert*, vol. II, 883: "The infinite real integers . . . (which I discovered many years ago, without becoming clearly aware that they are concrete numbers of real significance)."

[54]Spinoza, *Ethics*, Book I, proposition 18, 428.

neo-Platonist) doctrine of the necessary emanation of things from the divine essence. So pantheism, necessitarianism, and the modal principle of plenitude ("everything possible is actual") are a set of closely allied doctrines.

Pantheism was both fashionable and heretical in nineteenth century Germany.[55] In 1861, Pope Pius IX felt the need to issue a formal prohibition against pantheism, a sure sign that the doctrine was popular as well as threatening. Given the moral and intellectual support that Cantor received from Catholic neo-Scholastic theologians (such as Constantin Gutberlet) in the face of heavy criticism from mathematicians, it is likely that Cantor would have been motivated to avoid (at least the appearance of) heresy.[56] Cantor was aware that his opinions contradicted those of Aquinas, whose philosophy was fast becoming the favoured philosophy of the Church. Some theologians feared his transfinite numbers might undermine the doctrine of creation by making a return to the notion of an eternal world possible.[57] As Cantor came to realize in the early months of 1886 through his correspondence with Cardinal Johannes Franzelin, by far the biggest challenge orthodoxy posed against his belief in the transfinite was to show that it did not lead to pantheism.

Cantor sent the Cardinal an essay he had written in 1885, later published in the *Zeitschrift für Philosophie*, vol. 88, as "Über die verschiedenen Standpunkte in bezug auf das aktuelle Unendliche" (Concerning Various Perspectives on the Actual Infinite). He asked the Cardinal to submit the essay to examination for any possible errors, especially those which might pose a danger to religion. In this essay, Cantor distinguishes between three kinds of actual infinities:

(i) *The Absolute infinite*—which pertains to God alone;

(ii) *The Concrete infinite*—found in nature ("the Transfinitum");

(iii) *The Abstract infinite*—found in mathematics, including the transfinite numbers and the forms of ordinal types.

Cantor claims that he is the first philosopher to affirm the reality of the infinite in the abstract as well as in the concrete. The infinite in the concrete (or transfinite) he identified with Spinoza's "natura naturata," the created universe; the infinite in the abstract with transfinite order types and cardinals; and the infinite in God with "natura naturans," the creator of the universe.

In this 1885 letter, Cantor claims that pantheism results from the failure to distinguish between the two forms of the actual infinite, the transfinite and

[55]Much of the background history for this section can be found in Joseph Dauben, "Georg Cantor and Pope Leo XIII: Mathematics, Theology, and the Infinite," *Journal of the History of Ideas* 38 (1977): 85–108.

[56]Dauben, *Georg Cantor*, 145.

[57]See for example Cantor's correspondence with Heman, at the end of his first letter book in the *Nachlass*, Cod. Ms. G. Cantor 16.

the absolute. Only if one held that God's absolute infinity were fully realised in nature would one truly be a pantheist:

> Another common confusion is seen with the two forms of the actual infinite, in which namely the transfinite is mixed up with the absolute, while these concepts are strongly distinguished, insofar as the former is infinite, but can still be added to, while the later is essentially such that it cannot be added to, and therefore is not be determinable by mathematical thought; we encounter this failure, for example, in pantheism, and it constitutes the Achilles' heel of Spinoza's *Ethics*, of which in fact F.H. Jacobi remarked, that it cannot be contradicted by principles of reason (*Vernunftgründen*).[58]

Despite Cantor's attempts to differentiate his position from pantheism, the Cardinal seems to have recognised the Spinozist, necessitarian consequences of Cantor's doctrine of the identity of possibility and actuality. In his response to Cantor, he warns him of the pantheist nature of his argument, and its necessitarian consequences:

> I confess, however, that in my opinion, what the author calls "transfinitum in natura naturatum" cannot be defended, and in a certain sense, which the author appears not to have given, would contain the error of pantheism.[59]

Cantor's reply to Franzelin illustrates simultaneously his desire to avoid pantheism and the appeal it held for him. In each attempt where Cantor technically disassociated his philosophy from pantheism, he reveals his sympathy with it. In his letter to Franzelin of 22 January 1886, he attempts again to distinguish his position from pantheism by drawing attention to his distinction between the transfinite and the absolute. He tries to explain the special sense he gives to Spinoza's expressions, *natura naturata* and *natura naturans*:

> I use the following expressions "*natura naturans*" and "*natura naturata*" in my short essay "Concerning Various Perspectives on the Infinite" in the same sense which the Thomists have given them, so that the former expresses God as the creator and preserver of substances produced from Him out of nothing, while the later designates the world created by Him. Accordingly, I distinguish between an "*Infinitum aeternum sive Absolutum*," that refers to God and his attributes and an "*Infinitum creatum sive Transfinitum*" that above all testifies to where in *natura creata* an

[58]Cantor, *Gesammelte Abhandlungen*, 375, translation mine.

[59]H. Meschowski, and W. Nilson, eds., *Georg Cantor, Briefe* (Berlin: Springer, 1991), 253.

> actual infinite must be acknowledged, as for example, according to my firm conviction, in the actual infinite number of created individuals in the entire universe and also on our earth, and in all probability, in every smallest part of extended space itself, a matter on which I completely agree with Leibniz.[60]

Unlike pantheists, then, Cantor does not think that God's infinity is exhausted in nature. However, one is hardly reassured by his enthusiastic endorsement of Leibniz's views on the actual infinite. Leibniz's theory of infinitely many monads has sometimes been characterized as pantheism overlaid with a few concessions to orthodox religion.[61] One finds traces, too, of his allegiance to pantheism, in his use of a kind of principle of plenitude:

> One proof [of the reality of the Transfinite] proceeds from the concept of God and infers from the greatest perfection of God's essence the possibility of the creation of a transfinite order, from his supreme goodness to the necessity that there should actually follow a Transfinite.[62]

Yet Cantor's letter ends with an explicit repudiation of pantheism:

> no system leads further away from my chief convictions than pantheism, when I foresee its consequence of materialism, with which I have absolutely no association.[63]

Cardinal Franzelin's response to Cantor's attempt to distance his position from pantheism was more vigorous and drawn out than his initial remarks on Cantor's essay. Thus it must have been with some consternation that Cantor read:

> On the assumption that your actual Transfinitum contains no contradiction in itself, your conclusion of the *possibility of the creation* of a Transfinitum from the concept of God's omnipotence is almost right. Only to my regret, you go further and end "out of his absolute goodness (*Allgüte*) and glory (*Herrlichkeit*) follows *of necessity* the actual creation of the Transfinitum." Even though God is in himself absolute infinite goodness and glory, qualities which cannot wax or wane, the necessity of creation would always be as it were a contradiction of the freedom of creation.[64]

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[60]Ibid., 254, translation mine.

[61]There is a discussion of this issue in Robert Merrihew Adams, *Leibniz: Determinist, Theist, Idealist* (Oxford: Oxford University Press, 1994): "The most serious question about the theistic orthodoxy of Leibniz's 1676 conception of divine perfection is indeed whether it is pantheistic in a Spinozistic way" (124). Adams attributes the view that Leibniz is a Spinozist to Wolfgang Janke.

[62]*Georg Cantor, Briefe*, 255, translation mine.

[63]Ibid., 255, translation mine.

[64]Cantor, *Gesammelte Abhandlungen*, 386 and Notebook Cod. Ms. 16.

> In my opinion the necessity of creation will be a great hindrance to you in your fight, which is so commendable, against pantheism, and at the least, weakens the power of your proof. I have dwelled on this point for so long, because I fervently wish that your great sagacity (*Scharfsinn*) will free itself from so fatal an error, which many other great minds have lapsed into taking for orthodox (*rechtglaubig*).[65]

Cantor's next letter to Franzelin, dated 29 January 1886, contains a significant retreat from his previous position. He no longer tries to maintain the content of his philosophy while arguing its separability from pantheism. Instead he modifies that system to meet the demands of orthodoxy.[66] Cantor now claims in his letter that the creation of the finite as well as the transfinite realm has a merely "subjective necessity for us" that follows from our contemplation of the divine nature:

> It was not my intention, in the passage in question, to speak of an objective, metaphysical necessity in the creative act, a necessity that would subjugate God's absolute freedom. I meant, rather, to point to a kind of subjective necessity for us (not issuing from God [*nicht a parte Dei zu erfolgende*]) of inferring (*folgern*), from God's absolute goodness and glory, the fact of an actually accomplished creation—not merely of a finite system, but of a transfinite system.[67]

Cantor's notion of "purely subjective necessity" could be a contortion designed to bend his views into orthodox shape. But it is clear that he cannot have it both ways. Either his transfinite numbers are necessary, and God does not create freely, or the numbers are contingent beings and God creates freely. In the end, Cantor's desire to uphold the thesis of divine freedom won out over his desire to find a metaphysical argument for the existence of the transfinite numbers. In his later letters,[68] he is careful to emphasize God's freedom, and

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[65] *Nachlass* (note 49) and Notebook Cod. Ms. 41.

[66] At the end of his work *Das Unendliche* (1878), Constantin Gutberlet observes that it is characteristic of pantheism to postulate the necessity of the existence of the actual order of nature. Gutberlet was a student of Cardinal Franzelin, and a comparison of his book with Franzelin's letter shows that there was a consensus among neo-Scholastic theologians that the thesis of God's free creation of the world was the only orthodox option.

[67] " . . . war es an der betreffenden Stelle nicht meine Meinung, von einer objective, metaphysischen Nothwendigkeit zum Schöpfungsact, welcher Gott der absolute Freieunterworfen gewesen ware, zu sprechen, sondern ich wollte auf eine gewisse subjective Nothwendigkeit für uns hindeuten, aus Gottes Allgüte und Herrlichkeit auf eine thatsächlich erfolgte (nicht a parte Dei zu erfolgende) Schöpfung, nicht bloss eines Finitum ordinatum, sondern auch eines Transfinitum ordinatum zu folgern" *Georg Cantor, Briefe*, 258, translation mine.

[68] See, e.g., his letters to Pater Jeiler in Meschkowski, "Auf den Briefbüchern Georg Cantors," *Archive for the History of the Exact Sciences* 2, (1965), 503–19, and his letter to Pater Esser in *Georg*

cites Franzelin's final approval of his revised system several times. In his letter to Ignatius Jeiler of 8 June 1888, Cantor goes so far as to say that he thinks his letter to the Cardinal of 22 January 1886 will actually prove fatal to pantheism and positivism.

V.

Orthodoxy, Heresy, and Sincerity. How sincere was Cantor about the modification of his doctrine? An unpublished letter of 6 February 1887 to his friend and fellow believer in the actual infinite, the theologian Constantin Gutberlet, suggests Cantor was ambivalent. He continued to affirm the correctness of the Cardinal's opinion and his modified doctrine, but nonetheless characterised the Cardinal's criticism as a polemic:

> Now however follows a polemic against such a creation [of the transfinite] as the necessary consequence of God's goodness and glory, in which the Cardinal is thoroughly correct.[69]

There cannot be any doubt of Cantor's respect for the Cardinal, whom he praises profusely upon notifying Gutberlet of his former teacher's death (11 December 1887). Cantor was proud to have the Cardinal's endorsement and frequently quoted it to correspondents. He regarded himself as providing a service to the Church, enabling it to develop a correct theory of mathematical infinity.[70]

Belief in an actual infinity realized apart from God has always been dangerous. Giordano Bruno was burned at the stake for believing in an infinity of worlds. Spinoza was reviled as an atheist for two centuries after his work, his reputation following the opinion of his early critic, Pierre Bayle.[71] With the birth of German romanticism, Spinoza's reputation was partly rehabilitated. A Spinoza Renaissance flourished in Germany, and Spinoza's monism influenced German thinkers such as Hegel, Schelling, and Goethe. As we have shown, the Spinoza Renaissance reached the outpost of the University of Halle, where Cantor spent most of his academic career. Cantor's belief in the existence of actual

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Cantor, Briefe. The letter to Jeiler of 8 June 1888 exists in a typescript for the doctoral thesis of Bernhard Jaegar, *Das actuale and das absolute Unendliche. Zum Dialog zwischen Georg Cantor und Theologen seiner Zeit*, Eberhards Karls Univ. Tübingen 1992. Readers should also consult Christian Tapp, *Kardinalität und Kardinäle* (Stuttgart: Franz Steiner Verlag, 2005).

[69]The letter is mentioned in *Das actuale and das absolute Unendliche*, 50. The whole letter exists in Cantor's *Nachlass*, Cod. Ms. G. Cantor 16, 94. Readers can also consult *Kardinalität und Kardinäle* as above.

[70]Joseph Dauben, "Georg Cantor and Pope Leo XIII: Mathematics, Theology, and the Infinite," *Journal of the History of Ideas* 38 (1977): 85–108.

[71]P. Bayle, *Dictionnaire historique et critique* (1697). English version: *Historical and Critical Dictionary*, trans. Richard Popkin (Indianapolis: Hackett, 1991).

infinities was—like that of Bruno and Spinoza—potentially heretical. However, by modifying his doctrine, Cantor was able, as was Leibniz, to sharply distinguish between the infinity of the world, of number, and of God. In this way, Cantor narrowly avoided heresy and contradiction.

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