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Relational semantics and domain semantics for epistemic modals*

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Abstract: The standard account of modal expressions in natural language analyzes them as quantifiers over a set of possible worlds determined by the evaluation world and an accessibility relation. A number of authors – most notably Yalcin (2007) – have recently argued for an alternative account according to which modals are analyzed as quantifying over a domain of possible worlds that is specified directly in the points of evaluation. But the new approach only handles the data motivating it if it is supplemented with a non-standard account of attitude verbs and conditionals. It can be shown the the relational account handles the same data equally well if it too is supplemented with a non-standard account of such expressions.

1 Introduction

A well-known approach to modal expressions in natural language analyzes them in terms of accessibility relations between possible worlds: a sentence like *it might be raining*, for example, is true at a world w just in case there is a world accessible from w at which it is raining. This sort of *relational semantics* is familiar from modal logic, and has been elaborated in various ways in the literature on natural language semantics; a particularly influential version of this theory is due to Angelika Kratzer.[1]

Recently, however, a number of theorists have been exploring an alternative approach to the semantics of modals, according to which a modalized clause is evaluated for truth relative to a pair of a possible world and an *information state* or a *domain* (typically represented by a set of possible worlds). On this approach, which Yalcin (2007) calls *domain semantics*, a modal clause places no constraint on the world parameter, but instead quantifies directly over the worlds in the domain, so that *it might be raining* is true at a world-domain pair (w, i) just in case there is a world w' in the domain i at which it is raining.

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[1] See, for example, Kratzer (1981, 1991, 2012).

Domain semantics has been investigated in connection with epistemic modals, deontic modals, conditionals, imperatives, and the connectives *and* and *or*.[2] One of the more striking arguments on behalf of this approach concerns the behavior of epistemic modals when they occur embedded under attitude verbs and in the antecedents of indicative conditionals. Yalcin (2007, 983) observes that the following sentences are infelicitious, in the sense that more or less any utterance of them would strike us as odd:

(1) Suppose that it is raining and it might not be raining.

(2) If it is raining and it might not be raining, then...

Yalcin contends that the infelicity of these sentences can be explained if we adopt a domain semantics, but that it is less clear how these data can be accounted for given the standard relational semantics.[3]

But the problem Yalcin raises should really be seen as a problem not for relational semantics as such, but for the *combination* of a relational semantics for modals and familiar approaches to the semantics for attitude verbs and conditionals. Furthermore, the problem cannot be solved *simply* by adopting a domain semantics for modals; we need, in addition, to adopt what I shall call a *shifty* semantics for attitudes and conditionals. This raises a question: can we salvage a relational semantics for epistemic modals if we combine *it* with a suitably shifty semantics for attitude verbs and conditionals?

The primary aim of this paper is to make the case for an affirmative answer to this question. I then consider an objection to the proposed theory, and close by briefly discussing the consequences of this result for the debate between relational semantics and domain semantics.

2 Relational semantics

I will start by sketching what I take to be a fairly standard relational semantics for epistemic modals. As we shall see, I think the standard picture conflates certain distinctions, but it will help to first spell the picture out before complaining about it.

On a familiar sort of semantic theory, expressions are assigned extensions relative to a context and an *index*, the latter being a sequence of parameters that includes at least a possible world (Lewis 1980, Kaplan 1989). If we assume for the moment that indices contain only possible worlds, then on a standard

[2]Domain semantics was first discussed (though not under that name) in MacFarlane (2011), which was available in unpublished form in 2006; see Yalcin (2007, n. 10). See also Kolodny and MacFarlane (2010), Klinedinst and Rothschild (2012), and MacFarlane (2014).

Domain semantics is closely related to certain dynamic semantic theories; see Veltman (1996), Beaver (2001), and Willer (2013).

[3]It is worth noting that the infelicity of, e.g., (1) does not seem to depend on the order of the two conjuncts. Sentence (3) also appears to be infelicitous:

(3) Suppose that it might not be raining and it is raining.

relational semantics, *might* ϕ is true at a context c and a world w just in case there is a world w' compatible with the body of knowledge relevant at c at which ϕ is true. Slightly more formally:

(4) $[\![might\ \phi]\!]^{c,w} = 1$ iff there is a world w' compatible with the c-relevant information at w such that $[\![\phi]\!]^{c,w'} = 1$

(Where α is an expression, $[\![\alpha]\!]^{c,w}$ is the extension of α at c and w.) Given an arbitrary context c and world w, what is 'the c-relevant information at w' ? What the speaker of c knows at w? Or some set of propositions that is somehow calculated out of the individual bodies of knowledge of the conversational participants? Or something else altogether? This is a vexed question, which may not admit of a general answer, or may require us to re-think standard assumptions about how to assign utterances truth values.[4] I'll return to this issue shortly, but most of what I have to say will be independent of how we answer this question.

The semantics given in (4) appears to assume either that *might* is always epistemic or else that *might* is ambiguous. But there are what appear to be non-epistemic uses of *might* as when I say *I might have been born in Kansas*, even when I know that I wasn't, and (let's say) have always known this. But it seems odd to say that *might* is ambiguous, since both uses of *might* seem to have an underlying core meaning: that of some kind of possibility modal. An influential idea due to Kratzer is that, instead of positing an ambiguity here, we should posit (further) context-sensitivity in the meaning of *might*: what 'flavor' of modality (epistemic or metaphysical, for example) *might* expresses on an occasion of use is a function of the context.

A simple way to implement Kratzer's idea is to think of *might* as having as its basic meaning an abstract relational semantics, where the relation gets filled in by context. There are a number of ways to make this concrete. For example, following Kratzer (1991) and Portner (2009), we could include an accessibility relation in our indices. Alternatively, we might hypothesize that each modalized clause contains a covert variable which is assigned an accessibility relation by a variable assignment. I will eventually discuss both of these options, but it will simplify our exposition if we begin with the first one. So, for the moment, we adopt a theory according to which expressions are assigned extensions relative to a context, a world, and an accessibility relation (a binary relation on the set of worlds). The semantics for *might* would then run as follows:

(5) $[\![might\ \phi]\!]^{c,w,R} = 1$ iff $\exists w'$ ($w' \in R(w)$ and $[\![\phi]\!]^{c,w,R} = 1$)

Where R is an accessibility relation, $R(w)$ is the set of worlds w' such that wRw'. So you can think of $R(\ )$ as denoting a function from a world to a set of worlds.

[4]For discussion, see Hacking (1967), DeRose (1991), Egan *et al.* (2005), Egan (2007), Stephenson (2007), von Fintel and Gillies (2008), Dowell (2011), von Fintel and Gillies (2011), MacFarlane (2011), Schaffer (2011), Yalcin (2011), and MacFarlane (2014), among others.

The accessibility relation of the index 'gets filled in by context' in the sense that, in the definition of *truth at a context*, the value of the accessibility relation parameter will be determined by the context as follows (Kaplan 1989, 547):

(6) A sentence ϕ is true at a context c iff $[\![\phi]\!]^{c,w_c,R_c} = 1$

Here R_c might be something like the relation that w bears to w' just in case w' is compatible with what the agents relevant in c know in w.

I think that the story I have been telling so far is fairly standard, but it actually runs together two things that I think it best to keep separate. Following MacFarlane (2014), it is useful to contrast the abstract *compositional semantic* clause in (5) with the contextualist *postsemantics* embodied in the definition of truth at a context given in (6). Although the standard version of relational semantics is contextualist insofar as it assigns an important theoretical role to something like (6), it is worth noting that the compositional semantic clause given in (5) is in principle neutral on debates between contextualism, relativism, and expressivism.[5] For one could in principle combine that compositional semantic clause with an alternative postsemantics that, for example, used a *context of assessment* to initialize the accessibility relation parameter. For example, in place of (6), the relativist might offer this:

(7) A sentence ϕ is true as used at context c and as assessed at context a iff $[\![\phi]\!]^{c,w_c,R_a} = 1$

where R_a is something like the relation that w bears to w' iff w' is compatible with what the agents relevant in a know in w.

And just as a relational semanticist might adopt a relativist or expressivist postsemantics, so a domain semanticist might adopt a contextualist postsemantics, as MacFarlane (2014, 262ff.) notes.[6] Thus, the issue of relational semantics vs. domain semantics seems to cross-cut the issue of contextualism vs. relativism/expressivism, despite the tendency to conflate the two.[7]

In any case, most of my discussion will take place at the compositional semantic level, since the difference between relational semantics and domain semantics is, in the first instance, a difference at that level.

3 The problem

Recall the two sentences mentioned at the outset:

(1) Suppose that it is raining and it might not be raining.

(2) If it is raining and it might not be raining, then...

[5] For an overview of, and references to, these debates, see MacFarlane (2014, Ch. 10).

[6] Yalcin (2007, 1011) makes a similar point in discussion of what he calls the *diagonal view*, a view consistent with his compositional semantics.

[7] Dorr and Hawthorne (2013, 871, 878) appear to assume that domain semantics, for example, must be given some sort of relativist or expressivist interpretation.

What problem does the infelicity of these sentences pose for the foregoing relational approach to *might*? A simple way to see the problem is to adopt a contextualist interpretation of the relational semantics, and imagine that accessibility is simply a matter of compatibility with what the speaker knows. In that case, *it might be raining* simply means *I don't know that it is not raining.* But this hypothesis, at least in its simplest form, appears to be refuted by the fact that (8), for example, is not odd in the way (2) is:

(8) If it is raining and I don't know that it is raining, then...

To examine the problem from a slightly more formal point of view, let us consider the problem raised by (1) in more detail. Actually, rather than considering (1) itself, which is an imperative, we will consider an arbitrary supposition report that embeds an *epistemic contradiction* (Yalcin's name for sentences of the form ϕ *and might* $\neg\phi$):

(9) x supposes [or: "is supposing," if you prefer] that it is raining and it might not be raining.

This too appears to be infelicitous in much the same way (1) is.[8]

To understand the problem (9) poses we need to say something about the semantics of attitude verbs like *supposes.* In the sort of formal semantic framework we are employing, it is standard to adopt a relational semantics for attitude verbs themselves, treating *x supposes*, for example, as quantifying over the worlds compatible with what x supposes. Thus, the standard 'Hintikka semantics' for *supposes* can be stated thus (see Hintikka 1962):

(10) $[\![x \textit{ supposes } \phi]\!]^{c,w,R} = 1$ iff $\forall w'$ (if $w' \in S_x(w)$, then $[\![\phi]\!]^{c,w',R} = 1$)

Here, S_x is an accessibility relation that w bears to w' just in case w' is compatible with what x supposes at w. Thus $S_x(w)$ is the set of worlds compatible with what x supposes at w.

On this semantics, the truth-conditions of x *supposes* ϕ are given by universally quantifying in the metalanguage over the worlds compatible with what x supposes. A fairly standard assumption about natural language universal quantifiers is that they presuppose (in some sense) that their restrictors are not empty (Cooper 1983, Beaver and Geurts 2013). Thus, it is natural to suppose that, at a point (c, w, R), x *supposes* ϕ presupposes that $S_x(w)$ is not empty. We shall adopt this view in what follows.

Given the standard semantics for conjunction, it follows from the foregoing semantics for *might* and *supposes* that for any point (c, w, R):

$$[\![x \textit{ supposes } (\phi \textit{ and might } \neg\phi)]\!]^{c,w,R} = 1 \text{ iff}$$
$$\forall w' \text{ (if } w' \in S_x(w)\text{, then } ([\![\phi]\!]^{c,w',R} = 1 \text{ and } \exists w''(w'' \in R(w') \text{ and } [\![\neg\phi]\!]^{c,w'',R} = 1)))$$

[8]Though see Dorr and Hawthorne (2013, 906) for some interesting observations about the difference between uses of *suppose* in imperatives and uses in attitude reports.

The right-hand side of this biconditional is, in turn, is equivalent to:

$\forall w'$ (if $w' \in S_x(w)$, then $[\![\phi]\!]^{c,w',R} = 1$); and
$\forall w'$ (if $w' \in S_x(w)$, then $\exists w''(w'' \in R(w')$ and $[\![\neg\phi]\!]^{c,w'',R} = 1$))

Thus, on contextualist versions of the relational semantics, (9) will be true at a context c just in case x supposes that it is raining while simultaneously supposing that the agents relevant in c do not know that it is raining. Since this seems like a perfectly coherent thing for x to suppose – even if x is one of the agents relevant at c – it is hard to see how this view could predict the infelicity of (9).

Why does the foregoing approach fail in this regard? According to Yalcin, the relational semantics for *might* is to blame:

> "The problem... is the idea, practically built into a relational semantics for modals, that the evidential state relevant to the truth of an epistemic modal clause is ultimately determined as a function of the evaluation world..." (Yalcin 2007, 992)

But note that there were really two parts to the foregoing view: the relational semantics for *might* and the standard Hintikka semantics for *supposes*. I will return to this point shortly.

4 Domain semantics with domain-shifting operators

One natural thought about (9) is that it is infelicitous because the two conjuncts in the embedded clause place incompatible demands on x's state of supposition. The first conjunct requires that every world compatible with what x supposes be a *raining*-world; the second requires that some world compatible with what x supposes be a *not-raining*-world. Since no world can be both, x cannot suppose that it is raining and that it might not be raining. Yalcin's approach to the puzzle posed by (9) essentially builds on this insight.

One way to formulate domain semantics is to adopt a system broadly similar the one discussed above, with one important difference: rather than containing accessibility relations, indices now contain *domains* or *information states*. Formally, a domain or information state is simply a set of possible worlds. Within such a framework Yalcin (2007, 994) offers the following semantics for *might* :

(11) $[\![\textit{might}\ \phi]\!]^{c,w,i} = 1$ iff $\exists w'(w' \in i$ and $[\![\phi]\!]^{c,w',i} = 1)$

Now as I noted above, the 'relational package' we were considering had two parts: the relational treatment of *might* given in (5) and the Hintikka semantics for *supposes* given in (10). The first point I'd like to make is that simply taking this package and replacing the relational semantics for *might* with a domain semantics does not resolve the problem. This is not an objection to Yalcin, since he does not say otherwise. It is, nonetheless, a point worth noting.

We can see this if we *combine* the domain semantics for *might* with the above Hintikka semantics for *supposes*. For this combination does not predict the infelicity of (9) either. It yields the following truth-at-point conditions for (9):

$$[\![x \textit{ supposes } (\phi \textit{ and might } \neg\phi)]\!]^{c,w,i} = 1 \text{ iff}$$

$$\forall w' \text{ (if } w' \in S_x(w)\text{, then } ([\![\phi]\!]^{c,w',i} = 1 \text{ and } \exists w''(w'' \in i \text{ and } [\![\neg\phi]\!]^{c,w'',i} = 1)))$$

Note that the right-hand side of this biconditional is equivalent to the following conjunction:

$$\forall w' \text{ (if } w' \in S_x(w)\text{, then } [\![\phi]\!]^{c,w',i} = 1)\text{; and}$$
$$\forall w' \text{ (if } w' \in S_x(w)\text{, then } \exists w''(w'' \in i \text{ and } [\![\neg\phi]\!]^{c,w'',i} = 1))$$

Note that the second conjunct of this claim is a universally quantified conditional, but that the universal quantifier doesn't bind anything in the consequent of the embedded conditional (i.e. w' doesn't appear in the consequent of that conditional). This means that, at any point (c, w, i) at which the presupposition of (9) is satisfied (i.e. at which $S_x(w)$ is non-empty), the above conjunction will hold iff:

$$\forall w' \text{ (if } w' \in S_x(w)\text{, then } [\![\phi]\!]^{c,w',i} = 1)\text{; and}$$
$$\exists w''(w'' \in i \text{ and } [\![\neg\phi]\!]^{c,w'',i} = 1)$$

Thus, at any such point, (9) will be true iff the following sentence is also true:

(12) (x supposes that it is raining) and (it might not be raining)

But this sentence is perfectly fine. Since this semantics predicts that, whenever its presupposition is satisfied, (9) is equivalent to the felicitous (12), the semantics would seem to predict that (9) should be felicitous as well. But it is not.[9]

Although he doesn't mention this problem, Yalcin adopts a semantics for attitude verbs that avoids it. Yalcin combines the domain semantics for *might* with what we might call a *shifty* semantics for attitude verbs like *supposes*. *Shifty* is an appropriate description for this view, because on this approach, an attitude verb shifts both the world of the index and the domain of the index (Yalcin 2007, 995):

(15) $[\![x \textit{ supposes } \phi]\!]^{c,w,i} = 1$ iff $\forall w'$ (if $w' \in S_x(w)$, then $[\![\phi]\!]^{c,w',S_x(w)} = 1$)

Note that the domain parameter is now shifted to $S_x(w)$, the set of worlds compatible with what x supposes in w. Thus, on this semantics, *supposes* is a *domain-shifting operator*.

This account predicts the following truth-at-a-point conditions for (9):

[9]In addition to failing to predict the infelicity of (9), this proposal has the odd prediction that if the presupposition of (13) is satisfied at a point of evaluation e, then (13) will be true at e just in case (14) is true at e:

(13) John supposed that it might be raining.

(14) It might be raining.

$[\![x \text{ } supposes \text{ } (\phi \text{ } and \text{ } might \ \neg\phi)]\!]^{c,w,i} = 1$ iff
$\forall w'$ (if $w' \in S_x(w)$, then $([\![\phi]\!]^{c,w',S_x(w)} = 1$ and $\exists w''(w'' \in S_x(w)$ and $[\![\neg\phi]\!]^{c,w'',S_x(w)} = 1)))$

Note that the right-hand side of this biconditional is equivalent to the following conjunction:

$\forall w'$ (if $w' \in S_x(w)$, then $([\![\phi]\!]^{c,w',S_x(w)} = 1))$; and
$\forall w'$ (if $w' \in S_x(w)$, then $\exists w''(w'' \in S_x(w)$ and $[\![\neg\phi]\!]^{c,w'',S_x(w)} = 1))$

Now consider any point (c, w, i) at which the presupposition of (9) is satisfied. In that case, $S_x(w)$ is not empty, which means that this conjunction reduces to:

$\forall w'$ (if $w' \in S_x(w)$, then $([\![\phi]\!]^{c,w',S_x(w)} = 1))$; and
$\exists w''(w'' \in S_x(w)$ and $[\![\neg\phi]\!]^{c,w'',S_x(w)} = 1))$

But this is impossible. For according to the second conjunct there is a $\neg\phi$-world in $S_x(w)$, which means that not every world in $S_x(w)$ is a ϕ-world, in which case the first conjunct is false.[10]

It follows that any point at which the presupposition of (9) is satisfied is one at which (9) is false. No wonder we find (9) odd: there is no way for its presupposition and its truth-conditions to be jointly satisfied.

5 Relational semantics with relation-shifting operators

Let's remind ourselves of some observations. First, attitude reports like (9) pose a problem for the combination of a relational semantics for epistemic modals and a standard Hinitkka semantics for attitude verbs. Second, Yalcin solves the problem with two innovations, not one: (i) the domain semantics for *might*, and (ii) a shifty semantics for attitude verbs. As we saw, both are needed; combining a domain semantics for modals with the Hintikka attitude semantics fails to solve the problem.

This raises a question: What if we combined the relational semantics for *might* with a suitably shifty semantics for attitude verbs? Would that solve the problem? Perhaps. But what does it means to talk of a 'shifty' semantics for attitude verbs in the context of a relational semantics for *might*?

The first thing we might try is to let *x supposes* shift the accessibility relation parameter to S_x, the relation w bears to w' just in case w' is compatible with what x supposes in w:

(16) $[\![x \text{ } supposes \text{ } \phi]\!]^{c,w,R} = 1$ iff $\forall w'$ (if $w' \in S_x(w)$, then $[\![\phi]\!]^{c,w',S_x} = 1)$

Note that the accessibility relation of the index is now S_x rather than R.

This is almost what we want, but it isn't quite right. According to this approach, (9) is predicted to have the following truth-at-a-point conditions:

[10] Although I am using ϕ as a variable over declarative English sentences in general, I will often assume (for the sake of simplicity) that when ϕ appears embedded under *might* that it is a simple, context-invariant sentence. This allows us to speak simply of a 'ϕ-world.'

> $[\![x \textit{ supposes } (\phi \textit{ and might } \neg\phi)]\!]^{c,w,R} = 1$ iff
> $\forall w'$ (if $w' \in S_x(w)$, then ($[\![\phi]\!]^{c,w',S_x} = 1$ and $\exists w''(w'' \in S_x(w')$ and $[\![\neg\phi]\!]^{c,w'',S_x} = 1$)))

So according to this view, (9) would be true if x simultaneously supposes two things: (i) that it is raining, and (ii) that x does not suppose that it is raining. But this seems perfectly coherent: I can suppose that it is raining, while simultaneously supposing that I am not supposing this. I can do this by, for example, supposing that it is raining while also supposing that I don't exist. So this doesn't predict the infelicity of (9).

To remind ourselves of what we want, recall the truth-conditions predicted by Yalcin's package:

> $[\![x \textit{ supposes } (\phi \textit{ and might } \neg\phi)]\!]^{c,w,i} = 1$ iff
> $\forall w'$ (if $w' \in S_x(w)$, then ($[\![\phi]\!]^{c,w',S_x(w)} = 1$ and $\exists w''(w'' \in S_x(w)$ and $[\![\neg\phi]\!]^{c,w'',S_x(w)} = 1$)))

Note that the two quantifiers here ($\forall w'$, $\exists w''$) are restricted by the same set, $S_x(w)$, the set of worlds compatible with what x supposes at w. On this account, when *might* is embedded under *x supposes*, it inherits its domain of quantification from the latter. This feature of Yalcin's semantics is the key to his explanation of the infelicity of (9).

Our semantics doesn't have this feature. Let's think about why this is. If we were to assess *x supposes might* ϕ for truth at a point (c, w, R) on our semantics, we would check to see whether *might* ϕ is true at all points (c, w', S_x), for all worlds $w' \in S_x(w)$. And we would do this be checking to see if there is a ϕ-world in the set that results from applying the function $S_x(\,)$ to w'. But this set, $S_x(w')$, will not in general be identical to $S_x(w)$, for the simple reason that you can suppose something without supposing that you are supposing it, and *vice-versa*. And this is the reason that our semantics diverges from Yalcin's. In the statement of the truth conditions for (9), our initial quantifier $\forall w'$ quantifies over $S_x(w)$, but the subsequent quantifier $\exists w''$ is dependent on the first quantifier, so that for each $w' \in S_x(w)$, $\exists w''$ quantifies over $S_x(w')$ instead of over $S_x(w)$.

But this suggests a solution to our problem. Rather than shifting R to S_x, *x supposes* should shift R to S_x^w, where $S_x^w(\,)$ is a *constant* function mapping each world w' to $S_x(w)$. In that case, when we are evaluating *x supposes might* ϕ for truth at (c, w, R), we will check to see whether *might* ϕ is true at all points (c, w', S_x^w), for all worlds $w' \in S_x(w)$. And we will do *this* by checking to see if there is a ϕ-world in $S_x^w(w')$. But this set $S_x^w(w')$ *will* be identical to $S_x(w)$, simply because of how we have defined S_x^w.

Let us approach things more slowly. We begin by adopting the following definition:

(17) Definition.

> For any accessibility relation R and world w:
>
> the *rigidification of* R *at* w is an accessibility relation R^w such that for all w', w'', $w'R^w w''$ iff wRw''.

Note that $R^w(\ )$ is a function mapping a world w' to the set of worlds w'' such that $w'R^w w''$. Since $w'R^w w''$ iff wRw'', $R^w(\ )$ is a function mapping a world w' to $R(w)$. Thus, for every world w', $R^w(w') = R(w)$, which means that $R^w(\ )$ is a constant function.

Rigidification is appropriate here, since what 'superscript ^{w}' does to "$R(\ )$" is similar to what the addition of *actual* does to an ordinary one-place predicate relative to a world w. $R(\ )$ has a semantic similarity to the intensions of ordinary one-place predicates like *is a rich person*, insofar as both are *non-constant* functions from worlds to sets of something (individuals in one case, worlds in the other). $R^w(\ )$ on the other hand corresponds to the intension of a one-place predicate that is 'rigidified' at a world w by the insertion of an actuality operator, e.g. *is an actual rich person.* Both of these are *constant* functions from worlds to sets of some kind.

Since S_x is an accessibility relation, S_x^w is well-defined, and $S_x^w(\ )$ is a function mapping a world w' to $S_x(w)$. Thus, at a point (c, w, R), *x supposes* should shift the accessibility relation parameter not to S_x, but to its rigidification at w, S_x^w:

(18) $[\![x \textit{ supposes } \phi]\!]^{c,w,R} = 1$ iff $\forall w'$ (if $w' \in S_x(w)$, then $[\![\phi]\!]^{c,w',S_x^w} = 1$)

We can verify that this approach predicts the infelicity of (9). Note that:

$$[\![x \textit{ supposes } (\phi \textit{ and might } \neg p)]\!]^{c,w,R} = 1 \text{ iff}$$
$$\forall w' \text{ (if } w' \in S_x(w) \text{, then } ([\![\phi]\!]^{c,w',S_x^w} = 1 \text{ and } \exists w''(w'' \in S_x^w(w') \text{ and } [\![\neg\phi]\!]^{c,w'',S_x^w} = 1)))$$

As we noted above, for all worlds w', $S_x^w(w') = S_x(w)$. Thus, the right-hand side of the above conditional is equivalent to the following:

$$\forall w' \text{ (if } w' \in S_x(w) \text{, then } ([\![\phi]\!]^{c,w',S_x^w} = 1 \text{ and } \exists w''(w'' \in S_x(w) \text{ and } [\![\neg\phi]\!]^{c,w'',S_x^w} = 1)))$$

This is essentially equivalent to the truth-conditions predicted by Yalcin's approach, and so the same account of the infelicity of (9) can be given: any point of evaluation at which the presupposition of (9) is satisfied is a point at which the sentence is false.

When it comes to (9), a relational semantics with appropriate relation-shifting operators can do more or less precisely what a domain semantics with domain-shifting operators can do.

6 Conditionals

Recall that conditionals with epistemic contradictions in their antecedents are typically infelicitous:

(2) If it is not raining and it might be raining, then...

Let's begin by looking at how domain semantics accounts for the infelicity of sentences like (2). To appreciate that explanation, we need to state an account of indicative conditionals in the context of domain semantics. To state that account, it will help to adopt the following definition (Yalcin 2007, 1004):

(19) Definition.

For any sentence ϕ, context c, and domain i:

i *accepts* ϕ at c iff: $\forall w$ (if $w \in i$, then $[\![\phi]\!]^{c,w,i} = 1$).

We can use this notion to define another (Kolodny and MacFarlane 2010, 136):

(20) Definition.

For any sentence ϕ, context c, and domains i, i':

i' is a *maximal ϕ-subset* of i at c iff:

(i) $i' \subseteq i$,

(ii) i' accepts ϕ at c, and

(iii) there is no domain i'' such that i'' accepts ϕ at c, and $i' \subset i'' \subseteq i$.

We can then state the relevant semantics for indicatives as follows:[11]

(21) $[\![\phi \rightarrow \psi]\!]^{c,w,i} = 1$ iff: $\forall i'$ (if i' is a maximal ϕ-subset of i at c, then i' accepts ψ at c)

For reasons discussed earlier, we can suppose that, at a point (c, w, i), an indicative conditional $(\phi \rightarrow \psi)$ presupposes that there is a non-empty maximal ϕ-subset of i at c.

To understand how this proposal predicts the infelicity of (2), it will help to consider the following claim (Dorr and Hawthorne 2013, 870):

(22) Fact 1.

For any information state i and context c:

i accepts (ϕ *and might* $\neg\phi$) at c iff i is empty.

To see that this is true, first note that the right-to-left direction is trivial, since, for any sentence ϕ and context c, the empty set trivially accepts ϕ at c. Now let us suppose that i accepts (ϕ *and might* $\neg\phi$) at c. We want to show that i is empty. So suppose, for *reductio*, that i is not empty. Note that if a domain accepts a conjunction at a context, it accepts both conjuncts at that context. Since i accepts (ϕ *and might* $\neg\phi$) at c, this means that:

$\forall w$ (if $w \in i$, then $[\![\phi]\!]^{c,w,i} = 1$); and

$\forall w$ (if $w \in i$, then $[\![\textit{might } \neg\phi]\!]^{c,w,i} = 1$)

Let w' be a world in i. Then by the second conjunct of the above conjunction, $[\![\textit{might } \neg\phi]\!]^{c,w',i} = 1$. So $\exists w''(w'' \in i$ and $[\![\neg\phi]\!]^{c,w'',i} = 1)$. So some world in i is a $\neg\phi$-world. So not every world in i is a ϕ-world. But according to the first

[11]This semantics for the indicative conditional is closer to the semantics presented in MacFarlane (2014, 270) than it is to the one presented in Yalcin (2007, 998). Yalcin's semantics presumes that there is a unique maximal ϕ-subset of i at c, which isn't always the case (Kolodny and MacFarlane 2010, 136; Dorr and Hawthorne 2013, n. 4).

conjunct of the above conjunction, every world in i *is* a ϕ-world. Contradiction. So i is empty after all.

With FACT 1 in hand, we can explain the infelicity of (2) as follows. Let (c, w, i) be an arbitrary point of evaluation. The sentence $((\phi$ *and might* $\neg\phi) \rightarrow \psi)$ presupposes at (c, w, i) that there is a non-empty maximal $(\phi$ *and might* $\neg\phi)$-subset of i at c. But if i' is a maximal $(\phi$ *and might* $\neg\phi)$-subset of i at c, i' accepts $(\phi$ *and might* $\neg\phi)$ at c. But given FACT 1, if i' accepts $(\phi$ *and might* $\neg\phi)$ at c, i' is empty. Thus, the presupposition of $((\phi$ *and might* $\neg\phi) \rightarrow \psi)$ is not satisfied at (c, w, i). Since (c, w, i) was an arbitrary point of evaluation, it follows that there is no point (c, w, i) at which the presupposition of $((\phi$ *and might* $\neg\phi) \rightarrow \psi)$ is satisfied. This should suffice to explain the infelicity of indicative conditionals with epistemic contradictions in their antecedents.

Can the advocate of a relational semantics offer a similar explanation of the infelicity of (2)? That is: can the relational semanticist construct a semantics for the indicative conditionals that yields a similar result?

To see the case for an affirmative answer to this question, suppose that, in the context of relational semantics, we define *acceptance* as follows:

(23) DEFINITION.

For any sentence ϕ, context c, world w, and accessibility relation R:

$R(w)$ *accepts* ϕ at c iff $\forall w'$ (if $w' \in R(w)$, then $[\![\phi]\!]^{c,w',R^w} = 1$).

Using this notion, we can define another:

(24) DEFINITION.

For any sentence ϕ, context c, world w, and accessibility relations R, R':

$R'(w)$ is a *maximal ϕ-subset* of $R(w)$ at c iff:

(i) $R'(w) \subseteq R(w)$,
(ii) $R'(w)$ accepts ϕ at c, and
(iii) there is no accessibility relation R'' such that $R''(w)$ accepts ϕ at c, and $R'(w) \subset R''(w) \subseteq R(w)$.

We can then state the relevant semantics for indicatives as follows:

(25) $[\![\phi \rightarrow \psi]\!]^{c,w,R} = 1$ iff: $\forall R'$ (if $R'(w)$ is a maximal ϕ-subset of $R(w)$ at c, then $R'(w)$ accepts ψ at c)

And we add a similar presupposition to this story: at a point (c, w, R), $(\phi \rightarrow \psi)$ presupposes that there is an accessibility relation R' such that $R'(w)$ is a non-empty maximal ϕ-subset of $R(w)$ at c.

But, again, there will be no point (c, w, R) at which the presupposition of $((\phi$ *and might* $\neg\phi) \rightarrow \psi)$ is satisfied. The presupposition of $((\phi$ *and might* $\neg\phi) \rightarrow \psi)$ is satisfied at a point (c, w, R) iff there is an R' such that $R'(w)$ is a non-empty maximal $(\phi$ *and might* $\neg\phi)$-subset of $R(w)$ at c. But $R'(w)$ is a maximal $(\phi$ *and might* $\neg\phi)$-subset of $R(w)$ at c only if $R'(w)$ accepts $(\phi$ *and might* $\neg\phi)$ at c. But we can show the following:

(26) Fact 2.

For any accessibility relation R, context c, and world w:

$R(w)$ accepts (ϕ *and might* $\neg\phi$) at c iff $R(w)$ is empty.

Again, the right-to-left direction is trivial, since if $R(w)$ is empty, $R(w)$ accepts every sentence at c. So suppose now that $R(w)$ accepts (ϕ *and might* $\neg\phi$) at c. And suppose, for *reductio*, that $R(w)$ is not empty. Note that if $R(w)$ accepts a conjunction at a context, it accepts both conjuncts at that context. This means that we have:

$\forall w'$ (if $w' \in R(w)$, then $[\![\phi]\!]^{c,w,R^w} = 1$; and

$\forall w'$ (if $w' \in R(w)$, then $[\![\textit{might } \neg\phi]\!]^{c,w',R^w} = 1$)

Let w' be a world in $R(w)$. Then by the second conjunct of the above conjunction, there is a world w'' such that $w'' \in R^w(w')$ and $[\![\neg\phi]\!]^{c,w'',R^w} = 1$. Since $R^w(w'') = R(w)$, this means that that there is a world w'' such that $w'' \in R(w)$ and $[\![\neg\phi]\!]^{c,w'',R^w} = 1$. So there is a $\neg\phi$-world in $R(w)$. So not every world in $R(w)$ is a ϕ-world. But according to the first conjunct of the above conjunction, every world in $R(w)$ *is* a ϕ-world. Contradiction. So $R(w)$ is empty after all.

Thus, combining the relational semantics for *might* with the foregoing semantics for the indicative conditional yields an explanation of the infelicity of (2) that is broadly analogous to the one offered by domain semantics.

7 Hidden variabilism

Consider the following objection:

> *One might question whether what you have been calling a 'relational semantics' for "might" really ought to be called such. For on the present version of the relational semantics, indices contain accessibility relations. This means that the compositional semantic value of a sentence at a context is not a proposition in the ordinary sense (function from worlds to truth values), but a more complex entity, viz. a function from world-accessibility relation pairs to truth values. Perhaps it is no surprise that a semantics that is non-standard in this manner can predict Yalcin's data. Both this account and Yalcin's are alike insofar as they employ non-standard semantic values for sentences.*[12]

As I noted earlier, Kratzer (1991) and Portner (2009, 52) formulate a relational semantics for modals by employing indices that contain accessibility relations,[13] and Yalcin (2007, 990, n. 9) himself notes that this is one way of

[12]Thanks to Jeremy Goodman for raising an objection along these lines.

[13]Strictly speaking, Kratzer and Portner have indices that contain *conversational backgrounds* rather than accessibility relations, but this difference is irrelevant in the present context.

implementing the general idea behind relational semantics. So I'm not sure it is wrong to call the foregoing view a 'relational semantics.'

Furthermore, it is not clear that on anyone's considered view, the compositional semantic value of a sentence at a context is a proposition (function from worlds to truth values), for the simple reason that quantifiers (or other variable-binding operators) would appear to require us to take the compositional semantic value of a sentence to be something that varies in truth value over variable assignments.[14]

In connection with this last observation, we can respond more directly to the objection by showing that our results do not in fact depend on the inclusion of accessibility relations in our indices. An alternative to our formulation of relational semantics involves positing a distinguished covert variable in the syntax of modal claims, an element that is assigned an accessibility relation by a variable assignment (e.g. von Fintel and Heim 2011, 38). On this view, the syntax of a sentence like *it might be raining* might be represented as follows:

$$\textit{might}_r\ (\textit{it's raining})$$

where r represents the distinguished covert variable over accessibility relations.

Thus, on this approach, we might state the semantics for *might* as follows:

(27) $[\![\textit{might}_r\ \phi]\!]^{c,w,g} = 1$ iff $\exists w'(w' \in g(r)(w)$ and $[\![\phi]\!]^{c,w',g} = 1)$

(Note that the index here now contains the variable assignment g.) Can our strategy for predicting the infelicity of (2) and (9) be implemented using this version of the relational semantics?

Indeed it can, so long as we adopt the right sort of semantics for attitude verbs and conditionals. In the case of *supposes*, the key is allow the verb to shift the variable assignment as follows:

(28) $[\![x\ \textit{supposes}\ \phi]\!]^{c,w,g} = 1$ iff $\forall w'$ (if $w' \in S_x(w)$, then $[\![\phi]\!]^{c,w,g'} = 1$), where $g' = g[r \mapsto S_x^w]$[15]

It is straightforward to verify that this semantics predicts the infelicity of (9) in more or less the same manner as the relational semantics of §5 (assume again that x *supposes* ϕ presupposes at (c, w, g) that $S_x(w)$ is not empty).

A similar move can also be made in the case of indicative conditionals. We need to posit that our covert variable r appears in the syntax of indicative conditionals, so that *if* ϕ, ψ is represented as follows:

$$\phi \rightarrow_r \psi$$

We then revise the definitions of *acceptance* and *maximal ϕ-subset* as follows:

(29) Definition.

[14] For relevant discussion, see Ninan (2010, 2012), Rabern (2012, 2013), and Yalcin (2014).

[15] The variable assignment $g[r \mapsto S_x^w]$ is the variable assignment just like g with the possible exception that $g[r \mapsto S_x^w]$ maps r to S_x^w.

For any sentence ϕ, context c, world w, assignment g, and accessibility relation R:

$R(w)$ *accepts* ϕ at c and g iff $\forall w'$ (if $w' \in R(w)$, then $[\![\phi]\!]^{c,w',g'} = 1$), where $g' = g[r \mapsto R^w]$

(30) DEFINITION.

For any sentence ϕ, context c, world w, assignment g, and accessibility relations R, R':

$R'(w)$ is a *maximal ϕ-subset* of $R(w)$ at c and g iff:

(i) $R'(w) \subseteq R(w)$,

(ii) $R'(w)$ accepts ϕ at c and g, and

(iii) there is no accessibility relation R'' such that $R''(w)$ accepts ϕ at c and g and $R'(w) \subset R''(w) \subseteq R(w)$.

We can then offer the following semantics for indicatives:

(31) $[\![\phi \rightarrow_r \psi]\!]^{c,w,g} = 1$ iff: $\forall R'$ (if $R'(w)$ is a maximal ϕ-subset of $g(r)(w)$ at c and g, then $R'(w)$ accepts ψ at c and g)

And we assume that, at (c, w, g), $(\phi \rightarrow_r \psi)$ presupposes that that there is an accessibility relation R' such that $R'(w)$ is a non-empty maximal ϕ-subset of $g(r)$ at c and g. As the reader can verify, the presupposition of $((\phi \textit{ and } \textit{might}_r \neg\phi) \rightarrow_r \psi)$ will fail at every point. The resulting view will thus predict the infelicity of (2) in more of the less the same manner as the relational semantics of §6.

8 Conclusion

As should be clear, the 'relational package' formulated in this paper is very similar to the 'domain package' offered by Yalcin. Are there any differences? How should we choose between these two approaches? These are difficult questions, and I cannot pretend to survey all the considerations that might bear on this choice. I shall close by mentioning two.

If we focus solely on the case of epistemic modals and their interactions with attitude verbs and indicative conditionals, then simplicity considerations might seem to favor domain semantics. The version of relational semantics discussed in this paper does seem more complex than the domain semantics we've been considering. Our relational semantics mimics domain semantics by using 'rigidified' accessibility relations to do the job that might more naturally be done by simple sets of worlds, i.e. by domains. So even if the theories turned out to be 'empirically equivalent' in some sense (something that has not been established), one might think domain semantics does a better job of carving linguistic nature at its joints.

But if we widen our gaze to include non-epistemic modals, the situation might look rather different. Suppose that at least some modals (or some readings of modals) are best understood as having a relational semantics of some kind. Then one of the main Kratzerian ambitions – to derive different readings of modals via context-sensitivity – might speak in favor of a unified relational approach to all modals.

In order to evaluate this last consideration, we would need to examine two issues. First, is our supposition correct: *are* there considerations that suggest that at least some modals must be given a relational semantics? Second, even supposing the answer to the first question is *yes*, we might still wonder whether or not we really ought to seek a unified modal semantics in the manner suggested by Kratzer. For if there really are deep differences between the semantics of epistemic modals and that of non-epistemic modals, then a technical unification might simply obscure important underlying differences.

In any case, I must leave these as topics for future inquiry.[16]

References

Beaver, D. I. (2001). *Presupposition and assertion in dynamic semantics*. CSLI Publications, Stanford.

Beaver, D. I. and Geurts, B. (2013). Presupposition. In E. N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Fall 2013 edition.

Cooper, R. (1983). *Quantification and Syntactic Theory*. Reidel, Dordrecht.

DeRose, K. (1991). Epistemic possibilities. *Philosophical Review*, **100**(4), 581–605.

Dorr, C. and Hawthorne, J. (2013). Embedding epistemic modals. *Mind*, **122**(488), 867–913.

Dowell, J. (2011). A flexible contextualist account of epistemic modals. *Philosophers' Imprint*, **14**, 1–25.

Egan, A. (2007). Epistemic modals, relativism, and assertion. *Philosophical Studies*, **133**(1), 1–22.

Egan, A., Hawthorne, J., and Weatherson, B. (2005). Epistemic modals in context. In G. Preyer and G. Peter, editors, *Contextualism in Philosophy*, pages 131–168. Oxford University Press, Oxford.

von Fintel, K. and Gillies, A. (2008). CIA leaks. *Philosophical Review*, **117**(1), 77–98.

von Fintel, K. and Gillies, A. (2011). "Might" made right. In A. Egan and B. Weatherson, editors, *Epistemic Modality*, pages 108–130. Oxford University Press, Oxford.

von Fintel, K. and Heim, I. (2011). Intensional semantics lecture notes. Notes

[16] I should also note that I have not attempted to answer various objections to domain semantics which might also constitute objections to the proposed version of relational semantics. For some important objections, see Dorr and Hawthorne (2013), Moss (2015), and Schroeder (2015). It may be that, in light of these objections, the correct theory of these matters takes a rather different shape than either of the theories considered here.

for class taught at MIT. Available at: http://mit.edu/fintel/fintel-heim-intensional.pdf.

Hacking, I. (1967). Possibility. *Philosophical Review*, **76**(2), 143–168.

Hintikka, J. (1962). *Knowledge and Belief*. Cornell University Press, Ithaca, N.Y.

Kaplan, D. (1989). Demonstratives. In J. Almog, J. Perry, and H. Wettstein, editors, *Themes from Kaplan*, pages 481–563. Oxford University Press, New York.

Klinedinst, N. and Rothschild, D. (2012). Connectives without truth tables. *Natural Language Semantics*, **20**, 137–175.

Kolodny, N. and MacFarlane, J. (2010). Ifs and oughts. *Journal of Philosophy*, **107**(3), 115–143.

Kratzer, A. (1981). The notional category of modality. In H.-J. Eikmeyer and H. Rieser, editors, *Words, Worlds, and Contexts*, pages 38–74. Walter de Gruyter and Co., Berlin and New York.

Kratzer, A. (1991). Modality. In A. von Stechow and D. Wunderlich, editors, *Semantics: An International Handbook of Contemporary Research*, pages 639–650. Walter de Gruyter, Berlin.

Kratzer, A. (2012). *Modals and Conditionals*. Oxford University Press, Oxford.

Lewis, D. K. (1980). Index, context, and content. In S. Kanger and S. Öhman, editors, *Philosophy and Grammar*, pages 79–100. D. Reidel Publishing Company, Dordrecht. Reprinted in Lewis 1998, 21–44. Page references are to the 1998 reprint.

Lewis, D. K. (1998). *Papers in Philosophical Logic*. Cambridge University Press, Cambridge, UK.

MacFarlane, J. (2011). Epistemic modals are assessment-sensitive. In A. Egan and B. Weatherson, editors, *Epistemic Modals*, pages 144–178. Oxford University Press, Oxford.

MacFarlane, J. (2014). *Assessment-Sensitivity: Relative Truth and its Applications*. Oxford University Press, Oxford.

Moss, S. (2015). On the semantics and pragmatics of epistemic vocabulary. *Semantics and Pragmatics*, **8**(5), 1–81.

Ninan, D. (2010). Semantics and the objects of assertion. *Linguistics and Philosophy*, **33**(5), 355–380.

Ninan, D. (2012). Propositions, semantic values, and rigidity. *Philosophical Studies*, **158**(3), 401–413.

Portner, P. (2009). *Modality*. Oxford University Press, New York.

Rabern, B. (2012). Propositions and multiple indexing. *Thought*, **1**(2), 116–124.

Rabern, B. (2013). Monsters in Kaplan's logic of demonstratives. *Philosophical Studies*, **164**(2). 393–404.

Schaffer, J. (2011). Perspective in taste predicates and epistemic modals. In A. Egan and B. Weatherson, editors, *Epistemic Modality*, pages 179–226.

Oxford University Press, Oxford.

Schroeder, M. (2015). Attitudes and epistemics. In *Expressing Our Attitudes: Explanation and Expression in Ethics, Volume 2*, pages 225–256. Oxford University Press.

Stephenson, T. (2007). Judge dependence, epistemic modals, and predicates of personal taste. *Linguistics and Philosophy*, **30**, 487 – 525.

Veltman, F. (1996). Defaults in update semantics. *Journal of Philosophical Logic*, **25**(3), 221–261.

Willer, M. (2013). Dynamics of epistemic modality. *Philosophical Review*, **122**(1), 45–92.

Yalcin, S. (2007). Epistemic modals. *Mind*, **116**(464), 983–1026.

Yalcin, S. (2011). Nonfactualism about epistemic modality. In A. Egan and B. Weatherson, editors, *Epistemic Modality*, pages 295–332. Oxford University Press, Oxford.

Yalcin, S. (2014). Semantics and metasemantics in the context of generative grammar. In A. Burgess and B. Sherman, editors, *Metasemantics*, pages 17–54. Oxford University Press, Oxford.