Are Rules of Inference Superfluous?
Wittgenstein vs. Frege and Russell

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RESUMEN

En el *Tractatus* 5.132 Wittgenstein argumenta que la justificación inferencial depende solo de la comprensión de las premisas y la conclusión y no está mediada por ningún otro acto adicional. Tomando lo anterior como base, Wittgenstein defiende que las reglas de inferencia de Frege y Russell "carecen de sentido" y son "superfluas". Esta línea de argumento es problemática, puesto que no está claro que pueda haber una explicación viable de la inferencia de acuerdo con la cual no haya tal mediación. Muestro que el rechazo por parte de Wittgenstein de las reglas de inferencia puede estar motivado si se tiene en cuenta su interpretación holista de la relación entre inferencia y comprensión.

PALABRAS CLAVE: *Wittgenstein, Frege, Russell, inferencia, holismo.*

ABSTRACT

In *Tractatus* 5.132 Wittgenstein argues that inferential justification depends solely on the understanding of the premises and conclusion, and is not mediated by any further act. On this basis he argues that Frege's and Russell's rules of inference are "senseless" and "superfluous". This line of argument is puzzling, since it is unclear that there could be any viable account of inference according to which no such mediation takes place. I show that Wittgenstein's rejection of rules of inference can be motivated by taking account of his holistic construal of the relation between inference and understanding.

KEYWORDS: *Wittgenstein, Frege, Russell, Inference, Holism.*

In paragraph 5.132 of the *Tractatus Logico-Philosophicus* Wittgenstein argues that inferential justification depends solely on the understanding of the premises and conclusions, and is not mediated by any further act or state [Wittgenstein (1960), henceforth *TLP*]. On this basis he goes on to reject what he calls Frege's and Russell's "laws of inference" (*Schlussgesetze*), a term which, as I will argue below, refers to the rules of inference of their systems. These, he says, "are senseless and would be superfluous". This line of argument is puzzling, however, for it is far from clear that there

could be any viable account of the justificatory nature of inference in which no such mediation takes place. What seems to make inference into a form of justification (by contrast to the mere association of thoughts) is that we draw the conclusion in light of our recognition of the goodness of the inferential connection. Indeed, Frege acknowledges this in saying that to infer is "to make a judgment because we are cognisant of other truths as providing a justification for it" [Frege (1979), p. 3] and Russell acknowledges this in saying that in inferring, the applicability of the rule "must be simply perceived." [Russell (1903), p. 41].[1]

Both Frege and Russell construe the logical mediation of premises and conclusion in terms of the application of rules of inference. Furthermore, in spelling out the role of such rules Frege and Russell are careful to distinguish it from the contribution made by adding a further premise to the inference. Frege says that rules such as *modus ponens* cannot be expressed in his logical system "because they form its basis" [Frege (1972), p. 136], and Russell says that the rule of inference "eludes formal statement" [Russell (1903), p. 34]. They thus seem to treat the rules of inference in terms of what we might nowadays call the meta-language of their systems; and the advantage in doing so is that it allows them to avoid the regress which, as pointed out by Lewis Carroll, threatens the idea that inferential justification depends on the appreciation of validity [Carroll (1895)]. To simplify somewhat, Carroll shows that if the appreciation of validity had to be codified as a premise that is added to the inference – if the inference would not be fully justified otherwise – the form of the original inference would thereby be altered, and a further demand for justification would then arise with respect to the new inferential sequence. Moreover, if the appreciation of the validity of the new inference had to be codified as a premise as well, a further, as yet unjustified inferential sequence would emerge, and there would be no stopping this regress. But since rules of inference in Frege's and Russell's accounts capture the appreciation of validity without adding such idle premises, they seem to account for inferential justification without giving rise to the regress. Indeed this is why Frege's and Russell's inexpressible rules of inference do not seem to be senseless and superfluous.

What, then, is the point of Wittgenstein's critique? As I understand him, and contrary to the existing literature on the topic, Wittgenstein does not mean to reiterate Carroll's objection, but he does mean to criticize Frege's and Russell's appeal to rules of inference, for different reasons. Wittgenstein rejects the idea that the goodness of an inference is some content which is to be cognized or a fact which must be perceived. As he sees it, Frege and Russell take the rules of inference to justify in-

ferences by bringing such substantive content to bear on the premises and conclusion — content that would not otherwise be there. But Wittgenstein rejects the idea that logic is a source of substantive content, let alone inexpressible content; moreover he rejects the idea that the components of inference – the premises and conclusion – are discretely individuated atoms of thought, which only form logical connections with one another when some such additional content is appealed to. Instead, he locates inference within the holistic context of meaning, understanding and reasoning: the appreciation of logical relations is internal to the significant use of signs, and does not need to be introduced by means of a separate act, be it the act of adding a premise or the act of applying a rule.

The structure of the paper is as follows. I begin by clearing up the target of Wittgenstein's critique, showing that it is the rules of inference of Frege's and Russell's logical systems that TLP 5.132 rejects. In Section II I go on to explain what precisely Wittgenstein finds to be problematic in Frege's and Russell's articulations of the role of rules of inference. In Section III I discuss Wittgenstein's approach to reasoning, and the holistic context in which he locates inference, understanding and meaning. In Section IV I consider how Wittgenstein's rejection of rules of inference reflects his general understanding of the relation between logic and thought, and in particular how it relates to his claim that inferential relations show themselves in propositions, but cannot be said by them. I argue that this claim does not commit Wittgenstein to the idea of inexpressible content which he criticizes Frege and Russell for endorsing.

I. WHAT ARE *SCHLUSSGESETZE*?

To begin with, let us consider the precise shape taken by Wittgenstein's critique of Frege's and Russell's conceptions of inference:

> If *p* follows from *q*, I can infer from *q* to *p*; derive *p* from *q*.
> The mode of inference is to be understood from the two propositions alone.
> Only they themselves can justify the inference.
> 'Laws of inference,' ['*Schlußgesetze*'] which—as in the works of Frege and Russell—are supposed to justify inferences, are senseless, and would be superfluous. [TLP 5.132, translation emended].

Wittgenstein starts from the premise that all we need in order to justify an inference is an understanding of the premises and conclusion, and proceeds to the claim that the appeal to "laws of inference" would be

"senseless" and "superfluous." But the term "Laws of inference" (*Schlußgesetze*) is not used as a technical term by either Frege or Russell. It might be taken to refer either to what Frege and Russell call the "basic laws" and "principles" (i.e. the axioms) of their systems, or to what they call the "rules" that govern proofs conducted in those systems. Russell, in particular, often fails to maintain a clear terminological distinction with respect to these two notions, and Frege is not always careful in this regard either.

    One reason to read '*Schlußgesetze*' as specifically denoting rules, however, is that the second sentence of 5.132 employs a distinctively Fregean term – 'mode of inference' (*Art des Schlusses*) – which Frege uses in *Begriffsschrift* to refer to the single rule of inference of that system, *modus ponens*. Indeed, a common reading of 5.132, which I will henceforth refer to as the standard interpretation, takes Frege's and Russell's rules of inference to be Wittgenstein's target. Many standard interpreters of 5.132 take Wittgenstein's reasons for objecting to their rules of inference to coincide with the worry raised by Lewis Carroll [e.g. Mounce (1981), p. 44f., Kenny (2006), p. 78, Baker (1988), p. 130f.] But it is, as I pointed out, far from clear that Frege's and Russell's construals of inference are vulnerable to Carroll's regress. Since Frege and Russell deny that the rules can be encoded as propositions of the language that they govern, appealing to such rules does not result in adding premises to the inference, and so it does not lead to the regress.[3] These standard interpreters thus have Wittgenstein attacking a straw man.

    This has given rise to various *non-standard* interpretations of 5.132. Ian Hacking holds that Wittgenstein would be right to criticize Russell, since the latter seems to lack a proper distinction between rules of inference and axioms and hence to be vulnerable to Carroll's regress. But such criticism would fail to gain traction against the more careful treatment of rules in Frege [Hacking (1979), p. 290]. Seeking a more plausible reading of Wittgenstein – one that would show him to be a charitable reader of Frege, and yet show his criticism to be justifiable – Thomas Ricketts suggests that at 5.132, Wittgenstein has two separate targets in view [Ricketts (1985)]. Drawing on the ambiguity of the term '*Schlussgesetze*,' Ricketts argues that 5.132 can be read as separately targeting two issues: Russell's rules of inference are criticized along the lines suggested by Carroll, and Frege is criticized for holding that an appeal to logical axioms is required in order to justify any proof.[4] Ricketts has in mind the fact that according to Frege, in order to provide a gapless proof of a conclusion q from a premise p, we must first establish the conditional p⊃q and then derive q by *modus ponens*; the conditional must be established by showing it to be

an instance of a logical theorem, which in turn can be derived from the axioms of the system. Every Fregean proof is thus grounded in the axioms. Indeed this might be the reason why Frege sometimes calls his axioms "the laws of all inferring" [Frege (1984c), pp. 283-4]; Russell similarly says of his axioms that they are the "principles of inference" [Russell (1903), p.16]. But Wittgenstein, Ricketts notes, rejects the idea that to derive one proposition from we must go all the way back to the axioms [cf. TLP 5.131, 6.127]. Ian Proops suggests a different non-standard interpretation [Proops (2001)]. As he points out, Russell's account of inference, read charitably, is no less immune to Carroll's regress than Frege's. Proops concludes that neither Frege's nor Russell's accounts of rules of inference should be taken to be the target of 5.132; in both cases, Proops holds, what is at stake must be the role played by the axioms.[5]

I accept the point made by the non-standard interpretation that Wittgenstein's criticism in 5.132 cannot be charitably read as applying Carroll's original objection to Frege's or to Russell's conceptions of inference. But against the non-standard reading, I do not assume that Frege's or Russell's conception of the *rules* of inference cannot be criticized on other grounds.[6] I also do not wish to deny that Wittgenstein elsewhere objects to Frege's and Russell's idea that any logical proof ultimately depends on an appeal to logical axioms. But this is compatible with reading 5.132 as rejecting Frege's and Russell's rules of inference. Furthermore, the critique of rules of inference that I attribute to Wittgenstein is compatible with the fact that he elsewhere recognizes the practical usefulness of rule-governed formal calculi, which may facilitate the recognition of logical connections in certain cases. But this does not mean that he thinks that when we infer we apply a rule-governed calculus to our thoughts; Wittgenstein in fact denies that the process modeled by rule-governed proof is essential to logic [TLP 6.126–6.1262, and Wittgenstein (1984), p. 109]. Giving up the implicit assumption (which is shared by both standard and non-standard readings) that any critique of rules of inference must take the form of Carroll's objection opens the way for a more charitable and more plausible interpretation of 5.132, according to which it is the rules of inference that are being criticized.[7]

II. FREGE'S AND RUSSELL'S CONCEPTIONS OF RULES OF INFERENCE

Frege and Russell insist that the rules that govern their systems are not themselves expressions of those systems, and hence that appeals to

such rules do not result in adding premises; they thereby escape the Carrollian regress. But as I will argue in this section, they continue to expect these rules to function in ways that only genuine propositions can function in, namely to convey substantive content. In inference, according to Frege and Russell, otherwise unconnected beliefs are mediated by means of recognizing such inexpressible logical content. Wittgenstein, as I understand him, resolutely rejects the idea that logic is a source of content [TLP 5.61] as well as the idea that the realm of content extends beyond the limits of language [TLP, p. 26].[8] This leads him to reject the idea that significant expressions of language are logically unconnected atoms, and instead to construe the relation between inference and understanding holistically (as I will show in the next section). It is on this basis that he declares rules of inference to be senseless and superfluous.

In what sense can Frege and Russell be said to construe rules of inference in terms that involve both inexpressibility *and* contentfulness? In pursuing their logicist projects both Frege and Russell take it as their task to make explicit the logical content from which mathematical knowledge can to be derived. The inexpressibility of rules of inference counts, from this perspective, as a regrettable defect: Russell speaks of it as a "failure of the formalism" [Russell (1903), p. 34] and Frege (in a different context) speaks of language as an impediment to the full expression of the most fundamental logical distinctions [Frege (1984b), p. 193] which he takes to be founded "deep in the nature of things" [Frege (1984a), p. 156].[9] Admittedly, Frege might seem to ascribe no propositional content to the rules of inference, since he introduces them separately from the axioms of his system; the latter are propositions that state truths of absolutely general scope, and serve as the ultimate grounds of all proofs, whereas rules of inference are introduced alongside other rules for the use of the symbolism. Like these other rules, Frege says that the rules of inference cannot be properly expressed in the symbolism they govern without circularity. But in fact, Frege does use special marks to indicate which rule of inference is being used in the course of the proof, and where, and in this respect rules of inference differ from at least some the other rules for signs. For instance, Frege marks the inferential transition in *modus ponens* by means of a horizontal line drawn between the premises and the conclusion, and he deploys various other signs to signify other kinds of inferential steps, corresponding to each of the rules of his system.[10] Indeed, for Frege these marks do not officially count as referring expressions, which designate (*bedeuten*) the content of a further thought; instead, like the judgment stroke, they are said to merely indicate (*an-*

deuten) the manner in which the inference is drawn.[11] Frege could therefore continue to say, in this attenuated sense, that rules are not expressible in the notation they govern. But even so, he would not deny that the marks of inferential transition register something, namely which premises justify the conclusion and by appeal to what rule.

Moreover, despite their inexpressibility, Frege's rules are not thought of as devoid of content. I here wish to argue against the very common tendency to assimilate Frege's construal of rules to the one found in modern thinkers such as Carnap and Tarski; even van Heijenoort makes this assimilation in saying that Frege's rules of inference are purely syntactic, and are "void of any intuitive logic" [van Heijenoort (1967), p. 326]. Admittedly, Frege groups the rules of inference with the rules for the formation of expressions of his system. But Frege nonetheless takes the rules of inference to encode substantive content. Consider for example the way he spells out the relation between the rules and axioms of his system:

> We have already introduced in the first chapter several principles of thought in order to transform them into rules for the application of our symbols.
> …In this way, we obtain a small number of laws in which (if we add the laws *contained* in the rules) is included, though in embryonic form, the *content* of all of them. …Perhaps there is yet another series of judgements from which (with the addition of those *contained* in the rules) all the laws of thought can be derived. [Frege (1972), p. 136. My emphases]

On Frege's view, in designing a logical system, we may choose to transform some of the axioms into rules and vice versa. Case in point, whereas *Begriffsschrift* employs only one rule and a larger number of axioms, the system of *Grundgesetze* employs three rules of inference and fewer axioms. This tradeoff, Frege holds, does not affect the overall content of each of the systems, measured by their capacity to form an adequate basis for arithmetic [Frege (1972) p. 107, (2013) p. 26]. But it does show that for Frege, the content of the logical system resides not only in the axioms but also in the rules. So *pace* van Heijenoort, Frege's rules of inference, though inexpressible in his systems, *are* meant to convey content. For Frege, these rules are thus neither senseless nor superfluous.

In *Principles of Mathematics*, Russell argues that the inexpressibility of the rules of inference guarantees that Carroll's regress does not arise [Russell (1903), p. 34; cf. Russell (1910), p. 94]. But like Frege, this does not prevent him from construing the role of the rules of inference by

analogy to that of contentful propositions. Indeed like Frege, Russell proposes in *Principia* that any axiom can be transformed into a rule and vice versa [Russell (1910), p. 106]. Moreover, in considering the question whether applying a rule of inference requires further justification, Russell declares that "the fact that our rule does imply the said implication", i.e. the fact that the inference accords with the rule, is something that must be "simply perceived" [Russell (1903), p. 41]. Russell here both speaks of rules as something which "implies" other things, and takes inference to depend on the appreciation of a further "fact", beyond those expressed by the premises and conclusion. He thus takes the rules to function in the way contentful, general truths do — in this sense, Russell's rules are not senseless. At the same time, he treats the propositions to which rules apply as inert and disconnected atoms whose relations of implication can only be appreciated by means of the further act of applying the rule — in this sense, his rules are not superfluous. Wittgenstein's critique, as I will now argue, reflects his rejection of this understanding of the relation between logical form and the content of thought.

III. INFERENCE AND UNDERSTANDING

To fully appreciate Wittgenstein's critique of Frege's and Russell's accounts of inference we must consider the alternative he seeks to offer, and in particular the way Wittgenstein construes the relation between inference and understanding. To understand a proposition, according to Wittgenstein, is to put ourselves in a relation to the way the world is, that is, to come to see how the proposition is answerable to the world: "To understand a proposition means to know what is the case if it is true" [TLP 4.024]. But for a proposition to be a truth-evaluable claim requires that its sense be determinate [TLP 5.156c]; and this means that in understanding it we must understand not only what is affirmed, but also what is excluded by it [TLP 3.144; cf. Wittgenstein (1984), p. 95 and p. 102]. Accordingly, to affirm any proposition, at least some of its inferential relations to other propositions must already be appreciated: for instance, we must be able to reject its negation. Logical complexity is in this sense prefigured in propositions, including atomic propositions, and is not some foreign element that is added on to them, at a second stage [TLP 5.47].[12]

This point is captured in Wittgenstein's claim that propositions occupy positions in a "logical space" [TLP 3.4]. The relations between propositions in this logical space, he holds, are the "scaffolding" which

constitutes their sense [TLP 3.42]. But "logical space" is a somewhat misleading metaphor, since the logical relations between propositions in logical space are not external, but *internal* relations. Being an internal property or relation means that the identity of the object it belongs to is constituted by those features: "A property is internal if it is unthinkable that its object does not possess it" [TLP 4.123]. Accordingly, the proposition, its location in logical space, and its relations to other locations in this space are one and the same thing:

> If the truth of one proposition follows from the truth of others, this expresses itself in relations in which the forms of these propositions stand to one another… these relations are internal, and exist as soon as, and by the very fact that, the propositions exist. [TLP 5.131]

For Wittgenstein the significant symbol – the sign in use – has priority over the merely perceptible aspect of it, the mere sign [TLP 3.326]. The logical identity of an expression – what makes various appearances of it into appearances of a single symbol – consists in its being used to make the same logical contribution in every context [TLP 3.311]. Thus, for example, to properly understand a name, to "recognize the symbol in the sign", one must not be misled by ambiguous, but logically distinct uses of it; and in the case of properly understanding two synonymous names, one must be able to discern the logical relations between sentences in which they appear:

> Can we understand two names without knowing whether they signify the same thing or two different things? Can we understand a proposition in which two names occur, without knowing if they mean the same or different things?
> If I know the meaning of an English and a synonymous German word, it is *impossible* for me not to know that they are synonymous, it is *impossible* for me not to be able to translate them into one another. [TLP 4.243, my emphasis].

When we have an understanding of a name, Wittgenstein here says, it is "impossible" for us not to know whether or not it is logically related to other names that we understand. But this is not meant as a substantive claim — the necessity at issue is neither a metaphysical nor a psychological matter. Rather, Wittgenstein's point is that behaving otherwise – neglecting to draw the relevant inferences or drawing inferences that conflict with such understanding – would reflect a failure to use the sign determinately,

i.e. that contrary to our hypothesis, it would not amount to having a proper understanding of the symbol conveyed by that sign. Importantly, indeterminacy in our use of signs, which may indeed occur without our noticing it, is also the source of what Wittgenstein calls nonsense: an apparently meaningful use of signs that ultimately fails to express anything [TLP 5.4733]. The relevance of this point will emerge in a moment.

As with the understanding of names, a proper understanding of propositions is such that she who understands them cannot fail to discern the logical, internal relations that constitute them. But the sense in which it would be "impossible" for her to truly understand propositions and yet to behave in ways that conflict with their internal logical relations is once again not a substantive one. Rather, it is simply this: given the constitutive role of internal relations in individuating signs as determinate expressions of significant propositions, it makes no sense to represent any thinker as being in such a situation — to assume that she *can* have a determinate grasp of those propositions and yet not recognize their relations.

In purporting to represent such a situation as possible we would inevitably be equivocating in our own use of the terms "grasp", "proposition," "understand", "infer", etc. In other words, it would be *we* who would then be uttering nonsense.[13] It is precisely by obscuring this point that Carroll makes a mystery of the possibility of inference, and it is by dispelling Carroll's nonsense, rather than by introducing rules of inference and declaring them to be inexpressible, that Wittgenstein's *Tractatus* would respond to the apparent regress. Carroll misleads his reader by portraying a rational being who putatively understands the premises and conclusion of a valid inference, and indeed purports to "accept" the premises, while at the same time pretending not to be able to take them to form reasons for drawing the conclusion. But such a creature would fail to meet the minimal bar below which no understanding (let alone "acceptance") could genuinely be attributed to anyone. Carroll thus tempts us to treat as intelligible what is no more than a piece of philosophical nonsense.

IV. INFERENCE AND LOGICAL FORM

The interconnectedness of what we understand as competent users of a language is not, according to Wittgenstein, some additional content that we come to believe or represent. Indeed, Wittgenstein denies that logical form is something that can be represented by means of proposi-

tions, and thereby provide the content for belief; he says that logical form shows itself, but cannot be said [TLP 4.1212]. Wittgenstein's distinction between saying and showing is notoriously elusive, and this is not the place to attempt an interpretation of it. It is clear, however, that if we are to make sense of Wittgenstein's claim that the justification of inference depends solely on our understanding of the premises and conclusion, and if we are to make sense of his rejection of rules of inference, we must avoid assimilating the manner in which for Wittgenstein inferential relations show themselves in propositions but cannot be said by them [TLP 4.122] to the manner in which for Frege and Russell inexpressible rules justify inferences.

On the account Wittgenstein *rejects*, for logical form to show itself in inference is for us to grasp what a logical rule would say, were it not for the fact that it is inexpressible, and this is assumed to be something which stands apart from and is not yet conveyed by the premises and conclusions themselves. Indeed, Wittgenstein himself often seems to draw dangerously close to affirming this manner of putting things, and many of his interpreters have read him in precisely this way.[14] Consider, for example, the following two passages:

> …If two propositions contradict one another, this is shown by their structure; similarly if one follows from another, etc. [TLP 4.1211]

> That the truth of one proposition follows from the truth of other propositions, we perceive from the structure of the propositions. [TLP 5.13]

As we have seen, Russell, too, speaks of "perceiving" that the rule is applicable to a given inference, and Frege speaks of inferring in terms of being "cognisant" of the support that the truth of the premises provides to the conclusion. As I construe Frege's and Russell's views, it is substantive content that they think is thereby being grasped. In what sense, then, is Wittgenstein's use of "perceiving" and "showing" any different? In other words, how is his talk of showing to be understood, such that his critique of Russell's and Frege's accounts of rules in 5.132 could even seem to get off the ground?

Ignoring what Wittgenstein says about the holism of inference and understanding, and assuming that propositional signs can be individuated independently of the context of significant use, might encourage one to take TLP 4.1211 and 5.13 to support the idea that logical form is something which we perceive separately from our grasp of the propositions, and on the basis of which we then ground our inferences. But there is a

different way in which Wittgenstein's claims can be construed: logical
form and relations of logical structure show themselves in propositions
insofar as these are truly propositions, that is, insofar as they belong to
the inferential nexus of the language of a competent speaker. We could
get a better sense of this by considering TLP 5.1311:

> When we conclude from p ∨ q and ∼p to q the relation between the forms
> of the propositions "p ∨ q" and "∼p" is here concealed by the method of
> symbolizing. But if we write, e.g. instead of "p ∨ q" "p|q.|.p|q" and instead
> of "∼p" "p|p" (p|q = neither p nor q), then the inner connexion becomes
> obvious. [TLP 5.1311]

Acquiring mastery of the sheffer-stroke notation renders certain modes
of inference trivial, which in other notations might seem frustratingly
complicated. But Wittgenstein does not mean this as a psychological ob-
servation. Rather, what he takes this to illustrate is that all inferences, so
far as they involve propositions that we actually understand, and regard-
less of the notation in which they are expressed, are just as simple and
immediate as the transition, in Russell's notation, from *p.q* to *p*.[15] Indeed
in a notation that we do not master, inferences might seem to require an
appeal to logical rules such as Disjunction Elimination in order to reveal
the underlying logical connections. But this apparent need for rules of in-
ference is a symptom of our lack of mastery of the specific notation.
Once we acquire such mastery, we acquire the capacity "to recognize the
symbol in the sign" [TLP 3.326], and thereby to use that notation as our
language, and that means, to reason by means of it. It is thus not before
and apart from, but within the significant use of language – within the
activities of inferring in a language that we master – that we genuinely
deal with propositions, rather than mere propositional signs, and it is on-
ly in such context that the logical interrelatedness of propositions can be
said to show itself. Indeed, what is shown and what does the showing
within the competent use of language are in a sense one and the same,
namely the clarity of our thought.[16]

A competent user of language is not someone who uses indetermi-
nate signs, but someone who operates with determinate, meaningful
symbols, whose relations to each other are not obfuscated by the pecul-
iarities of the notation. For such a thinker, understanding a proposition is
not the passive intake of isolated content, which is as inert as a mere
sign; rather, understanding a proposition consists in acquiring the ability
to reason with it, informing the entire logical space of the thinker. The

ability to infer is constitutive of what this proposition is for the thinker, and is what renders her use of the propositional sign determinate, allowing it to convey a symbol.

If the thinker seems to deviate in her reasoning from what her prior understanding dictates, however, that would inevitably also involve her failing to use signs in the determinate way she so far has. Importantly, her failure would then be located at the level of understanding, not at the level of reasoning. This is what Wittgenstein points to when he says that "In a certain sense we cannot make mistakes in logic" [TLP 5.473]. For what might seem like a mistake in reasoning would reduce, on Wittgenstein's holistic account, to a confusion concerning the logical identity of the propositional signs the thinker purports to infer to and from. A thinker who takes herself to infer where, by the lights of her own understanding, no inference can be made, would therefore be unwittingly uttering nonsense. It is for this reason, too, that appeals to rules of inference would be superfluous. Such rules cannot serve to ensure that we only infer validly, just as they cannot ensure that we only use our signs determinately. Indeed the application of rules presupposes the determinacy of the signs they are meant to apply to. But insofar as our use of signs has such determinacy, we already use them in a way that reflects their logical form, and hence we already have everything that the rule might seem to provide.

V. CONCLUSION

Frege and Russell, I have argued, extend the realm of content beyond the realm of expression. I suggested that it is this idea that is targeted in 5.132 — the idea that the significant use of language depends on the appreciation of logical content which lies beyond the limits of language. For Wittgenstein, by contrast, logical form resides in the determinacy of our representation of content; it is not some kind of content that we appreciate in addition to what our propositions represent. The rejection of rules of inference in 5.132 thus marks an important difference between Wittgenstein and his predecessors: whereas they take the logical relations of premises and conclusion to be appreciated by means of the application of rules to propositions, Wittgenstein thinks that without the appreciation of these logical relations, we do not yet have the propositions in view, and conversely, that to understand propositions is already to appreciate their logical relations. In the opening of this paper I raised

the worry that by rejecting the rules of inference Wittgenstein is prevented from accounting for the justificatory nature of inferring. But if Wittgenstein is correct, there is no need to assume that inferential justification depends on anything beyond the understanding of the propositions involved in the inference.

In assuming that inference requires a mediating act that is separate from the acts of affirming the premises and conclusion, Frege and Russell treat these propositions as inert atoms; merely understanding each of them does not yet entitle us to take any conclusion to be justified.[17] This is an assumption which Wittgenstein continues to criticize in his later work, in denying that propositions are lifeless objects, into which we must breathe life by means of further acts, such as the act of applying rules to them [Wittgenstein (1969), p. 3]. Since this assumption is still widely endorsed, I believe there is much that contemporary philosophers can learn from Wittgenstein's critique.

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NOTES

[1] Boghossian (2014) influentially argues that any account of inference must conform with what he calls the Taking Condition, namely that "Inferring necessarily involves the thinker *taking* his premises to support his conclusion and drawing his conclusion *because* of that fact". The way I see it, Wittgenstein denies that such taking is either an act or a state the inferrer must add to the premises and conclusion in order for her inference to be justified.

[2] Evidence for the ambiguity in Russell's use of the term 'rule', 'law' and 'principle' is recorded by Proops (2001) and Kremer (2001), p. 67, fn. 17. Proops shows that on some occasions Frege, too, fails to uphold a terminological distinction between rules and laws.

[3] See Frege (1972), p. 136, Russell (1903), p. 16, p. 35 and p. 41.

[4] There are other passages in the *Tractatus* that require such a double-barrelled reading, where Wittgenstein cannot be charitably taken to advance the same line of argument against Frege that he does against Russell, e.g. TLP 5.521; cf. Diamond (2014), p. 23.

[5] Proops (2001) reads the rejection of *Schlussgesetze* as targeting the idea, which Proops finds both in Frege and in Russell, that the relation of entailment between two propositions depends on the axioms. He nonetheless agrees with

Ricketts that 5.132 might be taken to pursue two separate targets at once, and thus to also target Russell's rules of inference.

⁶ There is a later text from the 1930s in which Wittgenstein motivates the conclusions of 5.132 by appeal to a regress argument: "What justifies the inference is seeing the internal relation. No rule of inference is needed to justify the inference, since if it were I would need another rule to justify the rule and that would lead to an infinite regress" [Wittgenstein (1982), p. 56]. However, the regress Wittgenstein describes here is different from the one described by Carroll, since it involves an endless series of rule-applications rather than an endless series of inferences whose number of premises keeps growing. A version of the worry that inference might be subject to a regress of rules is addressed by Russell, who attributes it to Bradley [Russell (1903) p. 41]. But there is no sign for this in the *Tractatus*.

⁷ A further apparent reason to think that '*Schlussgesetze*' denotes axioms, not rules, is the fact that they are rejected not only for being "superfluous" but also for being "senseless," and senselessness is paradigmatically applied, in the *Tractatus*, to logical propositions. But on closer look, the *Tractatus* sometimes uses the term 'senseless' in a wider sense (e.g. in 5.1362), so this does not follow. On my interpretation, 'senseless' in 5.132 is used to point out that contrary to Frege's and Russell's view, rules could not convey any content.

⁸ In this I concur with Diamond 1991b, against readers such as Hacker (1986) and Geach (1976).

⁹ And see the discussion in Geach (1976).

¹⁰ In *Begriffsschrift* there is only one rule of inference, and every inference is marked by a horizontal line between premises and conclusion [Frege (1972), p. 119]. *Grundgesetze*, by contrast, employs three basic rules of inference, each of which is marked by a different sign [Frege (2013), p. 22ff.].

¹¹ Frege presents the distinction between indicating (*andeutende*) and referring (*bedeutende*) expressions in discussing the signification of the latin-letter variables [Frege (2013), p. 11 and p. 31; see also Frege (1984c), p. 313]. The judgment stroke is described as indicating in Frege (1984a), p. 149, fn. 7; the inferential stroke is described as indicating in Frege (2013), p. vi and p. 43. Wittgenstein criticizes Frege's judgment stroke for being logically meaningless [TLP 4.442], and Frege's inferential signs seem open to similar objections.

¹² Anscombe (1959) lays particular emphasis on this point, which leads her to reject the idea that the Tractarian theory of sense has two parts — a picture theory for elementary propositions and a truth-functional theory for molecular ones.

¹³ On this conception of nonsense and on why a failure to determine the meaning of signs is not the indication of any substantive impossibility, see Diamond (1991a).

¹⁴ See e.g. Anscombe (1959), Geach (1976) and Hacker (1986). Against these, Diamond (1991b) argues that Wittgenstein's distinction between saying and showing prevents the assimilation of what is shown to what can be said.

[15] The argument is closely related to the one in TLP 5.441, with which Wittgenstein seeks to justify what he calls his *Grundgedanke* — the thesis that the logical constants do not contribute any *content* to representation [TLP 4.0312]. He there argues that the fact that in a different notation we can eliminate some of the logical constants in favor of others shows that there is nothing to which the constants refer.

[16] For a related construal of the Tractarian notion of showing, see Narboux (2014). For the idea that the correlate of showing is not content, but the mastery of the language in which content is deployed, see Kremer (2002).

[17] The discussion instigated by Boghossian (2014) seems to me to be premised on this assumption; a recent account of inference that avoids making this assumption is found in Marcus (2019).

REFERENCES

ANSCOMBE, G.E.M. (1959), *An Introduction to Wittgenstein's Tractatus*; London, Hutchinson University Library.
BAKER, G. (1988), *Wittgenstein, Frege and the Vienna Circle*; Oxford, Basil Blackwell.
BOGHOSSIAN, P. (2014), 'What is Inference'; *Philosophical Studies*, vol. 169 (1), pp. 1–18.
CARROLL, L. (1895), 'What the Tortoise said to Achilles'; *Mind*, vol. 4 (14), pp. 278–280.
DIAMOND, C. (1991a), 'What Nonsense Might Be'; *The Realistic Spirit*, Boston, MIT Press, pp. 95–114.
— (1991b), 'Throwing Away the Ladder'; *The Realistic Spirit*, pp. 179–204.
— (2014), 'Addressing Russell Resolutely'; *Philosophical Topics*, vol. 42 (2), pp. 13–43.
FREGE G. (1972), *Conceptual Notation and Related Articles*; translated and edited by T. W. Bynum, Oxford, Clarendon Press.
— (1979), 'Logic'; in *Posthumous Writings*, edited by H. Hermes et al., Chicago, University of Chicago Press, pp. 1–9.
— (1984a), 'Function and Concept'; in *Collected Papers on Mathematics, Logic, and Philosophy,* edited by B. McGuinness, translated by P. Geach, Oxford, Basil Blackwell, pp. 137–156
— (1984b), 'On Concept and Object'; in *Collected Papers*, translated by P. Geach, pp. 182–194.
— (1984c), 'On the Foundations of Geometry, Second Series'; in *Collected Papers*, translated by E.-H. W. Kluge, pp. 293–340.
— (2013), *Frege: Basic Laws of Arithmetic*; edited and translated by P. A. Ebert and M. Rossberg, Oxford University Press.
GEACH, P. T. (1976), 'Saying and Showing in Frege and Wittgenstein'; *Acta Philosophica* Fennica, vol. 28, pp. 54–70.
HACKING, I. (1979), 'What is Logic'; *Journal of Philosophy*, vol. 76 (6), pp. 285–319.
HACKER, P. (1986), *Insight and Illusion: Themes in the Philosophy of Wittgenstein*; second edition. Oxford, Clarendon Press.

KENNY, A. (2006), *Wittgenstein*; Oxford, Blackwell.

KREMER, M. (2001), 'The Purpose of Tractarian Nonsense'; *Noûs*, vol. 35 (1), pp. 39–73.

— (2002), 'Mathematics and Meaning in the *Tractatus*'; *Philosophical Investigations*, vol. 25 (3), pp. 272– 303.

MARCUS, E. (2020), 'Inference as Consciousness of Necessity'; *Analytic Philosophy*, vol. 61 (4), pp. 304–322.

MOUNCE, H. (1981), *Wittgenstein's Tractatus: An Introduction*; University of Chicago Press.

NARBOUX, J. P. (2014), 'Showing, the Medium Voice, and the Unity of the Tractatus'; *Philosophical Topics*, vol. 42 (2), pp. 201-262.

PROOPS, I. (2001), 'Tractatus on Inference and Entailment'; in Erich Reck (ed.), *From Frege to Wittgenstein: Essays on Early Analytic Philosophy*, Oxford University Press, pp. 283–307.

RICKETTS, T. (1985), 'Frege, the Tractatus, and the Logocentric Predicament'; *Noûs*, vol. 19 (1), pp. 3–15.

RUSSELL, B. (1903), *The Principles of Mathematics*; Cambridge University Press.

— (1910) (with Alfred North Whitehead), *Principia Mathematica*, vol 1; Cambridge University Press.

VAN HEIJENOORT, J. (1967), 'Logic as Calculus and Logic as Language'; *Synthese*, vol. 17 (1), pp. 324–330.

WITTGENSTEIN, L. (1960), *Tractatus Logico-Philosophicus*; translated by C. K. Ogden, London: Routledge & Kegan Paul. [Abbreviated TLP].

— (1969), *The Blue and Brown Book*; Oxford, Blackwell.

— (1982), *Wittgenstein's Lectures: Cambridge, 1930–1932*; ed. by D. Lee, University of Chicago Press.

— (1984), *Notebooks 1914-1916*, ed. by G.H. von Wright and G.E.M. Anscombe, tr. by G.E.M. Anscombe, 2nd ed, University of Chicago Press.