Nonsense: A Riddle Without Solution

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Abstract

Both in his earlier and in his later writings Wittgenstein grapples with the tendency of philosophers to misconstrue the nature of the difficulties they are facing. Whereas philosophers tend to assume that their problems are comparable to those that come up in the sciences, and take these problems to consist in questions the answers to which will provide them with substantive knowledge, Wittgenstein compares philosophical problems with riddles. What is characteristic of riddles is that solving them involves an alteration of the use of language, but it does not depend on the acquisition of new knowledge. In his middle and later period, Wittgenstein's highlights a related confusion in the way philosophers of mathematics understand the nature of mathematical problems. Wittgenstein rejects the realist approach according to which every mathematical theorem, whether proven or not, meaningfully specifies a possible fact which would make it true or false. By contrast, he construes unproven conjectures on the model of riddle phrases, whose meaning, prior to finding their solution, has not been determined. Moreover, Wittgenstein draws our attention to the fact that we can be caught up in an attempt to solve a riddle even when the riddle does not have a solution; the same applies, in his mind, to our engagement with philosophical and with certain mathematical problems. When it comes to philosophical riddles, Wittgenstein is convinced that they do not have a solution at all. The difficulty presented by unproven mathematical conjectures is different; in this case, we cannot tell in advance whether or not they present us with a solvable riddle. In discussing these issues, this chapter intervenes in the ongoing debates concerning the continuity of Wittgenstein's thought, his conception of nonsense, his rejection of mathematical realism, and his response to skepticism.

1. A Riddle and a Paradox

Having asserted that both the answers and the questions that philosophers put forth are in fact nonsensical, the *Tractatus* adds a curious remark:

4.003...And so it is not to be wondered at that the deepest problems are really no

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problems.1

But is this truly not to be wondered at? Wittgenstein here challenges us with a riddle, namely: What is a problem which is really no problem? And although the *Tractatus* suggests an answer to this riddle, namely, philosophical nonsense, this answer is no less puzzling than the original question, for how can something be nonsensical, and yet engage us in what seems like earnest theoretical deliberation? It is this riddle which I wish to address in this chapter.

The issues I will be concerned with pertain not only to Wittgenstein's *Tractatus* but also to his later work, where it is not only philosophical problems that are treated in this riddlesome manner, but also certain mathematical problems. Thus in the *Philosophical Remarks* Wittgenstein asserts that before they are proven, mathematical conjectures do not have a determinate sense, and he then proceeds to reformulate the Tractarian riddle in the following way:

Wouldn't all this lead to the paradox that there are no difficult problems in mathematics, since if anything is difficult it isn't a problem? (PR §151)

In framing this paradox, Wittgenstein signals his awareness of the difficult position that his conception of meaning (and hence, his conception of meaninglessness) might seem to force him into. Part of what makes Wittgenstein gravitate towards the verificationist view for which this paradox would arise is his recoil from mathematical realism, a view for which the diametrically opposed yet equally disturbing paradox arises, namely that there are no difficult problems in mathematics, since if anything is a problem, it isn't difficult. Mathematical realism gives rise to this paradox insofar as it assumes that we can say in advance, of any given problem, that its solution "must look either like *this* or like *that*" (PI §352). It thereby makes it seem as though all we need to do is open our eyes and check. According to Wittgenstein, by contrast, we may sometimes be engaged in an investigation of a problem even if, unbeknownst to us, our investigation is completely misguided, and in fact no solution is to be found; and even in those cases in which our investigation will eventually result in finding a solution, at any stage prior to our succeeding in doing that, we might not be able to make fully clear to ourselves what it is that we are searching for (PI §334, §517).

Both in calling philosophical propositions nonsense and in saying that unproven mathematical

conjectures lack a determinate sense Wittgenstein goes against the grain of how philosophers and mathematicians tend to view the subject matters of their study. In both cases, Wittgenstein thinks thinkers are misled by the analogies they implicitly draw between the problems of their respective fields and the problems of the natural sciences. In response, Wittgenstein seeks to foster in his interlocutors a sensitivity to the differences between the different kinds of intellectual difficulty that may confront us, and to the different kinds of activities that are required in order to overcome them.

Wittgenstein's remarks on one specific kind of problem – riddles – plays a pivotal role in this context. When we solve a riddle, we overcome an intellectual difficulty by means of a transformation of our method of expression; by contrast to the way we answer empirical questions, this does not require that we discover any substantial facts of which we were hitherto ignorant. By drawing attention to the differences between riddles and empirical questions and by drawing comparisons between riddles and the problems of philosophy and of mathematics, Wittgenstein seeks to weaken the hold of the misleading pictures that hold us captive, and to clear the way for an alternative understanding of our philosophical and mathematical practices. Like riddles, the solution of philosophical and (at least some) mathematical problems does not depend on our making discoveries; to overcome them, what is required is the transformation of our use of words, the result of which is the complete dissolution of those problems.

2. Kinds of Nonsense

Inquiring into these matters will allow me to address a methodological issue which has been the subject of much debate in recent scholarship, namely the shape of Wittgenstein's conception of nonsense. Some readers of Wittgenstein, which I will here call *substantial* readers, take it that nonsense can capture our interest and be involved in our earnest deliberations only if it conveys some content, even if it does this in a rather indirect manner. A substantial piece of nonsense, it is assumed, has some subject matter; for it is an expression of the attempt to say something about something, even if it ultimately fails to coherently say it. Those I will call *austere* readers of Wittgenstein reject this, and it is with their interpretation that mine will agree.² According to the austere approach all nonsense is mere nonsense, a complete failure to convey content. We therefore cannot, according to the austere readers, say what a putative piece of nonsense is about. Austere readers hold, moreover, that this approach to nonsense is constant throughout Wittgenstein's

philosophical career; their polemic against substantial readers tends to focus on the *Tractatus*, however, since it is this work which has given rise to the impression that Wittgenstein advances a more substantial conception of nonsense.

The austere approach explains the attraction of nonsense in terms of an *illusion* of sense, and it denies that in the context of a nonsensical expression words retain any of the meaning they have when they occur in senseful contexts. In so doing the austere approach takes Wittgenstein to have inherited from Frege, among other things, the insights contained in two of the three fundamental principles that Frege lists in the Introduction to *Foundations of Arithmetic*: "always to separate sharply the psychological from the logical, the subjective from the objective; never to ask for the meaning of a word in isolation, but only in the context of a proposition" (Frege 1960, p. xxii). Since a nonsensical expression is not a working context in which signs are put to significant use, from the strictly logical point of view there is no determinate meaning that can be ascribed to these signs (cf. *TLP* 3.3ff); logically speaking, nonsense is completely meaningless, and it is only its subjective effects that make it seem like a proper expression of thought (5.4733, 6.53).³

I will not seek to evaluate the entirety of the textual evidence in support of the austere reading here – an issue which has already been studied extensively. Instead, I propose to consider the merits of one line of objection that substantial readers have raised against the austere reading. I mean the objection that an austere conception of nonsense would be unable to explain the earnest intellectual engagements that, according to Wittgenstein, are infected by nonsense. I will address two specific variants of this objection. The first comes from Hans-Johann Glock. Glock is impressed by the fact that in the Tractatus 6.54 Wittgenstein says that his elucidations help his readers achieve clarity despite the fact that these elucidations are themselves nonsensical. According to Glock, in order to achieve any kind of clarity, the nonsense that Wittgenstein uses must be substantial; only so could it lead us to see that we were confused about some determinate philosophical matter. But austere readers hold that nonsense lacks all content, and so according to them there should be no inferential relations between the pieces of nonsense that Wittgenstein deploys and the pieces of nonsense that he aims to debunk. So how could Wittgenstein pretend to convince us, by means of his deployments of nonsense, to abandon any of our own philosophical nonsense (Glock 2004, p. 237)? The worry is that *mere* nonsense can only have *merely* psychological effects, and so the only way in which mere nonsense might make one stop speaking nonsense would be the way in which a punch in the gut might get one to turn silent.

The second variant of the objection I wish to consider occurs in Michael Potter's (2011) account of Wittgenstein's philosophy of mathematics in the middle period. Potter raises an objection to Wittgenstein's view, rather than to the views of his readers, thus implicitly granting that something like the austere conception of nonsense is at work in Wittgenstein's remarks concerning the relation between the meaning of mathematical theorems and their proofs. Tying meaning to proof in the strong verificationist way that the middle Wittgenstein does, Potter objects, yields an unrealistic picture of mathematical practice. Echoing the paradox that Wittgenstein himself frames for the verificationist view of mathematical meaning, Potter asks: if, as Wittgenstein has it, unproven conjectures lack all meaning until they are proven, what could explain the mathematician's interest in these conjectures, and their earnest attempts to prove them (p.129)?

In response to both objections, I will argue that it is wrong to think that the nonsensicality of an expression (understood as sheer lack of meaning) rules out its being involved in the kind of activities philosophers and mathematicians engage in, and conversely, that it is wrong to assume that the earnestness with which philosophers and mathematicians pursue such activities is a sure mark of the meaningfulness of their words. Drawing on the suggestion made by Cora Diamond (1991e) that Wittgenstein rejects realism because he does not find it *realistic* enough, I will argue that what Wittgenstein finds defective in realism is the tendency to ignore the differences between the kinds of difficulty we face in philosophy and in mathematics and the kinds of difficulty we face in the context of ordinary empirical inquiries. It is Wittgenstein's realistic approach to philosophical and mathematical problems which enables him to acknowledge that which the realist ignores, namely the dynamic dimension of our life with language, the processes through which our concepts evolve, and the activities as a result of which our understanding of the problems we face is transformed.

3. The Source of our Failure to Understand

Wittgenstein says that the origin of the problems he seeks to treat lies in the *intransparency* of our language (PI §122; cf. TLP 4.002). Indeed it is part of our difficulty, in facing such problems, that we might think we mean something by our words even when we don't, and insist that we mean it. Both in his early and in his later work, Wittgenstein's aim is to identify the roots of this insistence and thereby to help us overcome it. Thus in spelling out his conception of nonsense in the

Tractatus, Wittgenstein writes that "if [a proposition] has no sense this can only be because we have given no meaning to some of its constituent parts. (Even if we believe that we have done so.)" (TLP 5.4733) In his later work Wittgenstein similarly seeks to free us from the assumptions that we tend to make when we falsely believe that we have given meaning to our words; he frames this fallacious assumption in the following terms: "If I mean something by it, surely it must make sense" (PI §511). Against this Wittgenstein notes that when we consider an expression which (from the perspective of its grammatical form and its vocabulary) looks like a sentence we can understand, such as "I am here" or "The number of my friends is n, and $n^2+2n+2=0$ ", we cannot immediately tell whether or not it makes sense – not until we have considered whether and how it can be put to use and what role it could then play in our lives. But as Wittgenstein notes, in considering this, we might come to realize that in fact it could not have any role at all (PI §§513–514). "What I want to teach", Wittgenstein writes, is "to pass from unobvious nonsense to obvious nonsense" (PI §464); for nonsense is not always obvious, and when it is not obvious, it takes much work to overcome the illusion of sense that it involves.

But the source of the difficulties Wittgenstein is concerned with is not a superficial mix-up of words and meanings which could easily be removed; rather, the "deep disquietudes" Wittgenstein wishes to address "are as deeply rooted in us as the forms of our language" (PI §111). In his effort to overcome these difficulties Wittgenstein asks us to turn our attention from the words that we utter to our motivations for uttering them, and from our putative theses to the manner in which we frame the questions to which we thereby seek to respond – what the Preface to the Tractatus calls the Fragestellung of the philosophical problem (TLP, p. 26). For example, the philosophical Fragestellung may mislead us into thinking of propositions of logic as substantial claims, and into treating them by analogy to empirical claims, blinding us to the crucial differences. The Tractatus identifies one expression of this Fragestellung in the tendency to think of the law of excluded middle as spelling two possible properties that certain objects (namely propositions) may have, by analogy to "All roses are either yellow or red"; yet to think of the propositions of logic in this way is to completely misconstrue them (TLP 6.111).

Wittgenstein makes a related point concerning the misuse of the law of excluded middle in a later discussion (*PI* §352). In engaging with certain problems, he suggests, we are often unable to turn our eyes away from certain pictures, or models, which are suited for some, but not for all kinds of problems (to put this in relation to his earlier discussion: each such picture conveys a particular

Fragestellung). The appeal to the law of excluded middle, Wittgenstein proposes, is meant to conjure a picture of a visible domain, "the whole of which one person can survey and another can't", and within which any possible solution "must look either like *this* or like *that*." It thereby provides the realist with the false reassurance that there must be a solution to their problem, even in cases where they have no idea how to look for such a solution, and possess no method which could guide their search for it.⁵ Thus the law of excluded middle, which is nothing but a mere tautology, does not serve to *justify* anything the realist goes on to say; its only serves to blind the realist to the true nature of the difficulty they face.

To get a better grip on Wittgenstein's conception of what being caught up in a philosophical difficulty consists in and what overcoming such problems requires, it would be useful to recall a paragraph from Hertz's *Principles of Mechanics* with which Wittgenstein was acquainted from early on, and which in numerous methodological discussions he presents as summing up his approach to philosophy:

...we have accumulated around the terms 'force' and 'electricity' more relations than can be completely reconciled amongst themselves. We have an obscure feeling of this and want to have things cleared up. Our confused wish finds expression in the confused question as to the nature of force and electricity. But the answer which we want is not really an answer to this question. It is not by finding out more and fresh relations and connections that it can be answered; but by removing the contradictions existing between those already known, and thus perhaps by reducing their number. When these painful contradictions are removed, the question as to the nature of force will not have been answered; but our minds, no longer vexed, will cease to ask illegitimate questions. (Hertz 1956, pp. 7–8)

Wittgenstein's approach to philosophical problems, both in his earlier and in his later work, is deeply informed by these Hertzian insights into the nature of the difficulties faced by his contemporaries and into the nature of the progress that one makes by removing such confusion.⁶ Two points made by Hertz are particularly relevant for my purposes. First, Hertz diagnoses the confusion of his contemporaries in terms of their imposing an excess of conflicting demands on the key terms of their physical theories; the result is ambiguity and indeterminacy of meaning, of which these theoreticians are not fully aware. Second, he proposes that to overcome such confusions, what is needed is not to provide answers to the questions that these thinkers take themselves to have successfully framed, but rather to effect a thoroughgoing transformation of their use of language, which would lead to replacing their confused questions with different

questions altogether. Progress, as Hertz conceives it, is not cumulative and linear, but rather revolutionary and transformative.⁷

In Wittgenstein's view, the difficulties which underly philosophical and mathematical problems resemble the difficulty Hertz observes in his field, in the following sense. Like the problems Hertz addresses, philosophical and (at least some) mathematical problems arise from conceptual confusions, and overcoming them requires a transformation of the use of language; as a result of such a transformation, the appearance that these problems amount to questions that can be answered will be dissolved:

The characteristic feature of this is that a confusion is expressed in the form of a question that doesn't acknowledge the confusion, and that what *releases* the questioner from his problem is a particular alteration of his method of expression. (PG p. 193).

That an alteration of the method of expression can remove an intellectual difficulty is the point of Wittgenstein's comparisons of philosophical and mathematical problems with riddles, to which I now turn.

4. Riddles and Carroll's Riddle

Wittgenstein draws analogies between riddles and a range of distinct intellectual difficulties, and it would be rash to assume that each of the analogies he draws serves to make the same point. In the *Tractatus* he explicitly appeals to riddles in two contexts. In the context of his discussion of the domain of the ethical, Wittgenstein refers in 6.4312 to what he calls "the riddle of life"; and in 6.5 he discusses the idea of there being an unanswerable question, which he there calls "the riddle". 6.5 and its relevance for our understanding of Wittgenstein's conception of nonsense will form the topic of the next section; and while a detailed discussion of the analogy between ethical problems and riddles lies beyond the scope of the present paper, I will occasionally draw attention to the relevant contrasts between Wittgenstein's discussions of ethical problems and his discussion of philosophical problems.⁸ It is in texts from the middle and later period that Wittgenstein deploys the analogy between mathematical problems and riddles, which I discuss in sections 6 through 8.

To begin to see the point of these various analogies, let us recall what riddles are and how they differ from ordinary questions. Take for example the following riddle, which originates from the

Italian Renaissance:

My veiled face is my face; unveiled, I am annulled.9

In composing a riddle such as this, the riddler extends the use of familiar phrases to an unfamiliar context, thereby generating misleading analogies that conceal the intended solution. The ambiguities exploited by the riddle mislead us into looking for the solution in the wrong place; in this particular case, being guided (or misguided) by the familiar meanings of the words "veiled face", we might think that they signify the covering of the anatomical part of a living creature. As long as we are thus misled, it is impossible for us to solve the riddle. Once the riddle is solved, however, a new way of using words is discovered, and determinate meanings are assigned to the words that make up the phrase. The riddle thereby collapses into a trivial, empirical question, a matter of finding out which object fits a certain description that we already understand. Thus by being solved, the riddle ceases to be a riddle, and it is in this sense that by being unveiled, a riddle is annulled. This is a feature of riddles that Wittgenstein is keenly aware of, and which is highlighted in the *Tractatus* in paragraph 6.521: "The solution of the problem of life is seen in the vanishing of this problem". 10

Until our understanding of the riddle undergoes the transformation that allows us to solve it, it does not function as a fully meaningful question. That we do not fully understand what a riddle asks us, at this initial stage, can be seen from the fact that even if the actual solution to the riddle was right in front of our eyes (which, in a sense, is the situation we are in with respect to the Renaissance riddle) we might fail to recognize it as the solution – for as Wittgenstein puts it in 6.4321, "The facts all belong only to the task and not to its solution". To fully solve the riddle, we neither need to discover new facts, nor to draw inferences from what we already know; rather, we need to discover a new way of using words. In this sense, solving the riddle does not consist in making any substantial claims. The kind of difficulty we face when confronted with the riddle is not that of having to look for an object in some domain, but the difficulty of not knowing our way about, i.e. of having no method of seeking any object; once we find the method, however, locating the solution becomes a trivial task. This holds, at any rate, for what ordinarily counts as lawful riddles, though as we shall soon see, the norms of riddle-making are often subject to abuse.

Implicit in Wittgenstein's observations on the various intellectual difficulties that resemble riddles is a distinction between two perspectives. To view a riddle from the external perspective is to consider it as a problem posed by someone who already possesses the solution; to view it from the internal perspective is to consider it from the point of view of the one to whom it is posed, before she solves it. When viewed from within, it might not be clear to the riddled person whether what she faces is a riddle, rather than a question. Moreover, to try to explain the attraction of the riddle, before it is solved, in terms of a concern with what it is about, is to confuse the internal and the external perspectives. For instance, the Sphinx's riddle, before it is solved, and viewed from within, cannot be said to be about that which has four legs in the morning, for at that stage we simply do not know what these words mean. 12 Crucially, given that Wittgenstein explains nonsense in terms of the absence of a determination of meaning (TLP 5.4733, and, implicitly, 4.5 and 6.53), what the riddle phrase is, from the internal perspective, and before it is solved, is nonsense, though this might not be evident to us so long as we occupy the internal perspective. And it is furthermore crucial to note that this distinction between internal and external does not coincide with the Fregean distinction between the logical and the merely psychological; the internal is, no less than the external, a domain of language that has its own logic – albeit a defective one.

The process of riddle-solving can be further clarified by drawing on the terms with which Hertz describes the shape of the difficulties that he seeks to address and the non-linear conception of progress that he presents. When we assume that the Renaissance riddle is about some living creature's face, whose veil cannot be removed without annihilating their bearer, we place an excess of conflicting demands on the words we utter. Viewed from within, we may not notice this; our attempt to make progress with the riddle would lead us to treat these words as though their meanings were determinate, and to take them to justify us in drawing further inferences. Even if all we have to go on, at this stage, is a seeming-understanding, and even if our reasoning consists in seeming-inferences, this may suffice for getting us to realize that the demands we placed on our words conflict, and that what we thought we were thinking about did not make sense. Once we have rejected our initial assumption, we might find a way to assign the words new meanings. If we do that, the question we initially thought we are faced with (e.g. concerning a living creature that has such and such features) would not be answered, but it would no longer vex us. Thus riddle-solving, too, is not a linear and cumulative process of reasoning, but a revolutionary one: it transforms our language and the logical space of possibilities that it opens up, thus leading us from

one logical space to another. And this is anything but a merely psychological effect.

The analogies with riddles that Wittgenstein draws in his inquiries into the nature of our intellectual difficulties highlight certain features which are present in common riddles, yet Wittgenstein is also aware of the differences, and of the ultimately limited explanatory power of such analogies. Indeed, it is very often clear that it is not to common riddles that Wittgenstein wishes to draw our attention, but to riddles of a different sort. Consider the following remark, made in Wittgenstein's 1922 letter to C.K. Ogden, which concerns the proper English translation of the word "*Rätsel*" as it appears in the *Tractatus*:

6.4312 ... There is another difficulty about this prop[osition] still. "Rätsel" has been translated with "riddle". Now I don't know if this is right. Possibly it is. The word "Rätsel" in German has two meanings. Such e.g. "why is a raven like a writing desk" we call a Rätsel. But we also talk of, say, the "Rätsel des menschlichen Lebens" or of the "Rätsel der Existenz der Welt" etc. and here the word "Rätsel" has a different meaning, it has a higher meaning. Now do you use "riddle" in this second sense also? IF SO, IT IS QUITE RIGHT. Only I don't w<h>ish that there should be anything ridiculous or profane or frivolous in the word when used in the connection "riddle of life" etc.

6.5 "The riddle". Here "The" must be printed in italics too. It means as much as "the riddle 'par excellence'". As to the word "riddle" see above. (Wittgenstein 1973, pp. 35–37)

Wittgenstein first notes the ambiguity of the term "Rätsel", as he himself uses it, and warns Ogden against choosing a translation which would eliminate one of its meanings, thereby making the riddles which are at issue in *TLP* 6.4312 seem like a "ridiculous" thing. Engaging with the ethical riddle of life is attending to a "higher meaning", that is, the kind of ethical meaning that no proposition can properly capture (6.4 and 6.432), and whether or not it ultimately makes sense, this is a concern which, as he puts it later on, Wittgenstein "cannot help respecting deeply", adding "I would not for my life *ridicule* it" (*LOE* p. 12, my emphasis).

That Wittgenstein insists on using the same ambiguous term, "*Rätsel*," in the translation of 6.5, where his concern shifts from the problems of ethics to the problems of philosophy, might be taken

to mean that such problems, too, express an aspiration to address what is "higher," which, given that he also says that philosophical propositions are nonsensical, might be taken to speak in favor of the idea of substantial nonsense. But in fact, it is far from clear that this is Wittgenstein's intent.¹³ Indeed as I will argue in the next section, what the expression "*The riddle*" in 6.5 is meant to capture is the way philosophers *mis*construe their problems, not the way Wittgenstein himself understands these problems. As he understands the problems of philosophy, there is in fact something "ridiculous" in them; consider for instance what he says about his achievement in the Preface to the book:

I am, therefore, of the opinion that the problems have in essentials been finally solved. And if I am not mistaken in this, then the value of this work secondly consists in the fact that it shows *how little* has been done when these problems have been solved. (*TLP*, p. 27; my emphasis)

When looked at more closely, Wittgenstein's remark to Ogden is much more enigmatic than it might initially seem. For Wittgenstein contrasts the use of "Rätsel" when it denotes the ethical riddle of life with its use when it denotes an ordinary riddle by appealing to what is only apparently such an ordinary riddle, namely the riddle posed by the Hatter in Lewis Carroll's *Alice in Wonderland. Prima facie*, Wittgenstein's primary intention seems to be to draw a sharp distinction between the ethical riddle and the everyday, frivolous practice of amusing ourselves by telling riddles, a practice which Carroll can be taken to have brought to perfection. What is remarkable in the example he chooses for the frivolous kind of riddle, however, is that it is not an ordinary riddle at all. The Hatter himself confesses to Alice that he does not know the solution to his own riddle, and Carroll has Alice remark in a wry, almost Tractarian manner: "I think you might do something better with the time... than wasting it in asking riddles that have no answers" (Carroll 2000, p. 72).

The Hatter's unsolvable riddle has been a cultural sensation ever since the publication of *Alice in Wonderland*, provoking readers to propose various solutions of their own (a practice which continues to our day).¹⁴ It is possible, though not very probable, that when Wittgenstein appealed to it in his letter to Ogden in 1922 he failed to recognize that the riddle was not meant to have a solution. What is certain, however, is that he was keenly aware of this fact when he returned to this riddle in a later text (composed around 1932–33) where the frivolity of the riddle is not at all what is at issue:

Mathematical Problem. Think about the activity of solving riddles. In particular about solving a riddle, of which one does not know whether it has a solution: Lewis Carroll's "why is a raven like a writing desk" (compare what he writes about it.) It is by the way remarkable that the essence of riddles is not taken seriously in logic. (Wittgenstein 2009b, p. 59r. My translation)

Explaining what this comment is about – how mathematical problems can be illuminated by comparison with riddles such as Carroll's – will be the task of sections 6 through 8 of this chapter. For now, let us consider the remark about the Hatter's riddle which Carroll made in retrospect, to which Wittgenstein here refers:

Enquiries have been so often addressed to me, as to whether any answer to the Hatter's Riddle can be imagined, that I may as well put on record here what seems to me to be a fairly appropriate answer, viz: "Because it can produce a few notes, tho they are *very* flat; and it is nevar [*sic*] put with the wrong end in front!" This, however, is merely an afterthought: the Riddle, as originally invented, had no answer at all. (Carroll 2000, p. 71 fn. 5)

What we have here is a fascinating testimony to the processes of thought that a master-riddler engages in in the attempt to solve a riddle – extending the use of words, exploiting their ambiguities, and so on. Thus the phrase "produce a few notes" is ambiguous between making noises and writing, and while the notes produced in writing are flat in the literal sense of the word, a raven's "notes" are flat both because they are not particularly pleasant and because flat notes are played by the black keys of a piano. The word "nevar", which might seem to be a typographical error, is in fact the result of putting the word "raven" "with the wrong end in front" – whereas the writing desk Carroll has in mind must be the kind which only allows its user to comfortably sit on one side. But for all that, Carroll's belated attempts to render the riddle solvable do not change the fact that he initially posed the riddle without intending it to have any solution; that is, the fact that by means of it Carroll intended to deceive his readers into undergoing an experience of searching for a solution to no avail.

Carroll's riddle brings into focus the fact that all riddles, including the ordinary, solvable ones,

involve deception. We may distinguish three different forms of deceptiveness that riddles may employ. For first, they deceive us into taking words to mean one thing whereas in fact they must mean something else if the solution is to be found. Secondly, they are deceitful to the extent that they present themselves to us as genuine, substantial questions, that is, as instructions to search for something that either we or the one posing the riddle do not already know. And thirdly, as we have seen in Carroll's case, some riddles may deceive us into thinking that they possess a solution even when in fact they do not.

The deceptiveness of riddles takes a particularly sinister turn with this third form, where what presents itself as a riddle is really no riddle at all. In a wide range of folkloric traditions, riddlers are often depicted as resorting to such unlawful riddles in order to extricate themselves from unfair situations or in order to achieve an unfair advantage; instead of posing a legitimate riddle, they either pose a riddle that has no solution (at least so far as they themselves know) or they pose an empirically answerable question, disguised as a riddle, whose solution is known to them alone. 15 Apart from the Hatter's riddle, Wittgenstein draws our attention to at least three other examples of illegitimate riddles of this sort. The first occurs in the German fairytale in which a prince seeks to take advantage of a smith by challenging him to bring him a "Klamank" - a word which, as Wittgenstein presents the riddle, lacks meaning. 16 The second example occurs in the Grimm Brothers' tale about the peasant's wise daughter, to which the king poses a riddle: come neither naked nor dressed, neither walking nor riding, neither on the road nor off it (she came wearing a fishnet, dragged by a donkey, with only one toe touching the ground). Of this riddle Wittgenstein notes that in posing it, the king "did not really know what he wanted her to do" (AWL pp. 185–6). In both cases Wittgenstein goes on to compare these riddles with the kind of difficulty we encounter when we face an unproven conjecture in mathematics, suggesting that it might be unprovable (a point I will return to below). The third example stems from a recollection by Fania Pascal of an occasion on which Wittgenstein picked up a volume of the Grimm Brothers's tales containing the fable of Rumpelstiltskin and read from it, "with awe in his voice", the line "Ach, wie gut ist, daß niemand weiß, daß ich Rumpelstilzchen heiß."17 Upon reading it Wittgenstein remarked "Profound, profound" and then went into "a state of hushed, silent awe" (Pascal 1979, p. 30). It is not implausible to assume that at least part of what Wittgenstein was struck by was Rumpelstiltskin's sinister abuse of the norms of riddling. For Rumpelstiltskin's challenge to the queen could only be met by someone who possesses a particular piece of empirical knowledge, to which Rumpelstiltskin alone had access (until he inadvertently revealed it). 18 Like the prince and like the king in the fairytales that Wittgenstein appeals to, Rumpelstiltskin pretends to offer his victim a fair deal by posing a seemingly solvable task, yet none of them has the intention of being fair, and their riddles are solely meant to deceive.

The association of deceptive, unsolvable riddles with moral depravity should be kept in mind as we go on to consider Wittgenstein's comparisons between riddles and the problems of philosophy. Since it is we ourselves who pose the problems of philosophy, and since like unsolvable riddles, these problems involve deception, it is the failure of self-deception that philosophical problems confront us with. As Wittgenstein sees it, overcoming such self-inflicted evil is not just a theoretical, but also an ethical challenge.

5. The Riddle

The point of Wittgenstein's comparison of philosophical problems and riddles has already begun to emerge. Like philosophical problems, what a riddle phrase amounts to, prior to its being solved, and so long as the meanings of the words which make it up have not been determined, is nonsense. Philosophical problems, like riddles, are expressions that involve ambiguities and indeterminacies of meaning (*TLP* 3.323–24, 4.003–0031); hence they too are not really about what they initially seem to be about. Furthermore, as we have seen, a solution to an ordinary riddle does not consist in any substantial claim, even if the riddle initially presents itself as a substantial question; similarly, what the philosopher takes there to lie behind philosophical expressions is some substantial piece of knowledge, but as Wittgenstein seeks to show, there is nothing that such expressions could really mean. An important point of difference is that riddles ordinarily do have a solution, whereas it is characteristic of philosophical problems, according to Wittgenstein, that they do *not* have one. What philosophical nonsense resembles (and what at least some mathematical problems resemble) are riddles without solution, which involve all three forms of deception distinguished above.

The comparison of philosophical problems and riddles is the topic of the prominently placed paragraph 6.5, which precedes and introduces Wittgenstein's discussion of his own method in philosophy:

6.5 For an answer which cannot be expressed the question too cannot be expressed.

The riddle does not exist.

If a question can be put at all, then it *can* also be answered.

Wittgenstein here seeks to challenge the philosopher's understanding of the nature of philosophical problems. In order to show philosophers that their problems are distinct from ordinary questions, Wittgenstein observes that ordinary questions *can* be solved. Wittgenstein implies that the propositions which philosophers take to form answers to their questions are not really answers at all. Of course this is not something philosophers would agree with, so one thing we need to look into is why Wittgenstein thinks he can appeal to this observation at this point. Assuming he could get the philosopher to acknowledge this, he then imagines the philosopher making a rejoinder of the following form: "while my attempts to answer the question fail to make sense, the question itself might; even if it is unanswerable, it is still a valid question". Though this is only a first approximation to the shape of the philosopher's rejoinder, it is this, or something close to it, that Wittgenstein signals the rejection of by saying "The riddle does not exist".

Let us first consider the claim that philosophical propositions fail to amount to answers. The overarching strategy of the *Tractatus* consists in spelling out the general form of the proposition, and thereby marking the limits of sense "from within" (cf. 4.114, and 5.61). The general form of the proposition "foresees" (4.5) the range of all the propositions that can have sense, and shows these to be all the propositions that can be built up from elementary propositions through truth-functional construction by means of the operator of joint-negation (4.52, 6.002). In this way Wittgenstein proposes to draw a limit to what may count as a proposition, and hence to what may count as an answer to any meaningful question. And he goes on to argue that no senseful proposition corresponds to the expressions with which philosophers aim to frame such answers (4.003, 6.53).

Recall however that the philosopher's use of language, according to Wittgenstein, tends to involve lexical ambiguities and ambiguities of logical form, which mislead philosophers into treating expressions in which no determinate meaning has been assigned to signs as though they were meaningful. What is crucial is that this fact – that in the context of the philosophical expression the signs have not been assigned a determinate meaning – is something which the philosophers themselves might not be aware of (5.4733), and indeed, it is something that they are likely to deny. Their condition is one of being under an illusion, which consists in their insisting that they must

mean something even when their words lack sense. The reason why philosophers are drawn to such expressions in the first place, and why they tend to insist that they must make sense, is not a matter of mere psychology. It has to do with the deep desires and temptations that are fostered by our modern, science-dominated culture and guide (or misguide) the *Fragestellung* of philosophy—for instance, the desire for simplicity (5.4541), or the desire to provide an ultimate foundation for our forms of thought (6.1271, 6.371). Since the source of the philosophical problem is the philosopher's own tendencies, it is these tendencies that must be treated. Legislating that only such propositions may count as senseful which result from the application of the joint-negation operator to the elementary propositions would not be sufficient for rooting out these tendencies. It is the entire person (or even the entire culture within which such a personality has emerged) that needs to be addressed, not the abstract objects instantiated by the person's statements.¹⁹

As I indicated above, philosophical riddles, like all riddles, are deceitful; but with the riddles of philosophy, it is we who are both the ones posing the riddle and the ones attempting to solve it. In so far as in philosophy we pose to ourselves problems that have no solution, what we are dealing with is therefore a matter of self-deceit. In this sense, the endeavor to overcome the riddles of philosophy can be seen as an engagement in an *ethical* struggle. As Wittgenstein puts it later on,

What makes a subject difficult to understand – if it is significant, important – is not that some special instruction about abstruse things is necessary to understand it. Rather it is the contrast between the understanding of the subject and what most people want to see. Because of this the very things that are most obvious can become the most difficult to understand. What has to be overcome is not a difficulty of the intellect, but of the will. Work on philosophy is ... actually more of a ... work on oneself. On one's own conception. On the way one sees things. (And what one demands of them). (*PO* pp. 161–163).

My suggestion is that the challenge Wittgenstein takes himself to face, already in the *Tractatus*, has precisely this form, the form of working on ourselves so as to free ourselves from the distorting effect of what we (confusedly) want to see, that is, so as to help us overcome the deceptive riddles we pose to ourselves. In a 1919 letter to Ludwig von Ficker, Wittgenstein famously states that "the point of the book is an ethical one." I believe this remark can be taken to point in the same direction, namely that it is not solely as a theoretical problem that Wittgenstein proposes to approach the self-deceiving riddle of philosophical nonsense, but also as an ethical one.²⁰

This can also be seen in the way in which Wittgenstein describes the "only strictly correct method of philosophy" in 6.53, as well as in the way in which he describes his own method at 6.54. In 6.53 Wittgenstein proposes to help his interlocutor overcome their illusions of sense without uttering nonsense at all, that is, without entering the internal perspective of the riddle solver. Instead, Wittgenstein puts forth the idea, or ideal, of an engagement with philosophical riddles solely from the external perspective:

6.53 The right method of philosophy would be this: To say nothing except what can be said, i.e. the propositions of natural science, i.e. something that has nothing to do with philosophy: and then always, when someone else wished to say something metaphysical, to demonstrate to him that he had given no meaning to certain signs in his propositions. This method would be unsatisfying to the other – he would not have the feeling that we were teaching him philosophy – but it would be the only strictly correct method.

"The right method of philosophy" is described here as an open-ended, dialogical procedure of seeking to help the philosopher assign determinate meaning to their words, by going through all the possible meaningful propositions that could serve as adequate translations of them; the philosopher would be brought to recognize the nonsensicality of their expressions once they acknowledge that nothing in the realm of sense corresponds to them. It is obvious that this can only be an ideal, and that in practice, one cannot actually go through all the possible senseful propositions one by one, and rule out that any of them corresponds to the philosophical thesis. As Cora Diamond suggests, one may think of the introduction of the general form of the proposition as providing a substitute for this infinite task, since it makes perspicuous what sorts of propositions with sense one would get if one started with a set of contingent, elementary propositions and proceeded to apply the operator of joint negation to them (Diamond 2011, pp. 244ff). It is important, however, that Wittgenstein envisions the possibility that his philosophical interlocutor would be dissatisfied with the method proposed in 6.53, even if it were practically possible to continue it indefinitely. He thereby indicates that the illusion of sense might persist even after every possible effort has been made to demonstrate that no proposition with sense corresponds to the philosophical thesis.

This helps explains why, in 6.5, Wittgenstein first assumes that he could bring the philosopher to admit that her putative answers are not answers at all, and then goes on to imagine the philosopher's entering her rejoinder. In being dissatisfied by Wittgenstein's efforts to demonstrate the nonsensicality of her theses, the philosopher would reveal herself to be the one who poses requirements on what may count as a proper solution to her problem – requirements which exclude, in advance, any solutions that consist in ordinary, expressible propositions that have sense. In so far as she excludes such answers in advance, the philosopher may be brought to admit that (at least in our sense of "answer") her problems form unanswerable questions, and yet to insist that they are nonetheless problems, of a different, queer sort.²¹

We can now reformulate the philosopher's rejoinder, envisioned in 6.5, in the following terms: "even if my question has no senseful answer, it may still count as a legitimate question, and even if, like a riddle, the question itself has not been given a determinate sense, this does not disqualify it from counting as a question; after all, it encourages me to look for something that I wish to know, though I cannot tell you what that thing is". This seems to be the idea that the term "*The riddle*" is supposed to capture, and it is this idea which Wittgenstein rejects as incoherent, for it runs together the distinct features of riddles and of questions, and collapses the distinction between the internal and the external perspective. An unsolvable riddle is an expression whose attraction is merely illusory; not only is it not a question, it is, in the last account, not even a riddle.²²

In 6.54 Wittgenstein admits that in order to help reveal the nonsensicality of his interlocutor's words, he himself is using nonsensical expressions of his own. This is so since in the context of the method Wittgenstein describes here, he proposes to take the internal, rather than the external perspective on the riddles of philosophy. In order to address his interlocutor's philosophical desires Wittgenstein proposes to use expressions that create the appearance that he indulges in similar desires. His goal is to thereby help the interlocutor overcome the tendencies that lead them to take nonsense for sense:

6.54 My propositions are elucidatory in this way: he who understands me finally recognizes them as nonsensical, when he has climbed out through them, on them, over them. (He must so to speak throw away the ladder, after he has climbed up on it.)

Offering for our consideration claims that upon reflection would be revealed to make no sense is the signature gesture of the *Tractatus*. To mention a few prominent examples of this self-cancelling gesture, this is how the *Tractatus* treats the notion of drawing a limit for thought, which is revealed to be incoherent since "in order to be able to draw a limit to thought, we should have to find both sides of the limit thinkable (i.e. we should have to be able to think what cannot be thought)" (*TLP*, p. 26; cf. 5.61). This gesture is also at work in the way the *Tractatus* addresses the philosophical attempt to explain our understanding of logic in terms of claims about the states of affairs that must obtain for such understanding to be possible; as he puts it in 5.552, "The 'experience' that we need in order to understand logic is not that something or other is the state of things, but that something *is*: that, however, is *not* an experience". And as we have seen the gesture is also manifested in the manner in which Wittgenstein treats the notion of the "deepest problems" of philosophy, which turn out not to be problems at all (4.003). 6.5, too, can be taken to display this signature gesture, insofar as it proposes for our consideration something it calls "*the riddle*" – the unanswerable question *par excellence* – only to tell us that such a thing does not exist.

When Glock objects to the austere approach to Wittgenstein's conception of nonsense, he suggests that Wittgenstein's self-cancelling gestures can only be effective if what underlies them are arguments. To flesh out his understanding of what Wittgenstein does with nonsense in the *Tractatus*, Glock draws a connection to arguments in Wittgenstein's later philosophy, which Glock construes as arguments by *reductio ad absurdum*; these arguments, too, draw on nonsense in order to lead the interlocutor to recognize that they themselves speak nonsense. But the way such arguments do that, according to Glock, is by exploiting the inferential relations that hold between pieces of nonsense. The austere conception of nonsense, Glock argues, would fail to explain how Wittgenstein's arguments can do that, since if, as the austere reading has it, nonsense is mere nonsense, then there wouldn't be such inferential relations to operate on. So how can Wittgenstein hope to convince us? (Glock 2004, p. 237)

Glock's insistence that nonsense must be substantial if it is to help us achieve clarity rests on two misconceptions, both of which reflect a neglect of the insights that we have gathered from consideration of the analogy between philosophical problems and riddles. For first, he collapses the distinction between the external and the internal perspectives, and second, he assumes a conception of philosophical progress which is alien to Wittgenstein.

When viewed from without, that is, from a more clarified logical point of view, the nonsensical expressions of philosophy lack content, and there are indeed no proper inferential relations that they can be said to have. But when viewed from within, from the perspective of the philosopher who is under the spell of an illusion of sense, and does not notice the ambiguities and indeterminacies of her words, the philosopher's expressions do not manifest an absence of inferential relations, but rather an excess, and it is this excess that nonsensical elucidations aim to exploit. To follow the method proposed in 6.54 is to enter into the philosopher's perspective, and to tinker with it from within. The kinds of inferences that would seem to be involved in this elucidatory activity would not be valid in the logical space of the external logical point of view; but from the philosopher's point of view, they might seem compelling, and it is enough that as a result, a contradiction would seem to the interlocutor to arise, for this might break the illusion of sense they are under, thereby leading them to revise the meaning of their indeterminate words and transform their language, revising the logical space that that language opens up. In this connection, Cora Diamond has made the helpful suggestion that the Tractarian deployment of nonsensical propositions consists in a merely "transitional" use of words (Diamond 1991d, p. 183). For the elucidatory propositions of the Tractatus are meant to create the mere semblance of sense, and once we identify the indeterminacy that underlies these nonsensical expressions and proceed to alter our use of them, we should no longer be prone to treat them as sensical claims, and so we can throw them away.

Viewed from the external perspective, all nonsense is sheer nonsense, and it is wrong to treat nonsensical elucidations as forming arguments that refute *views* which Wittgenstein finds problematic, as Glock proposes. To speak of a *reductio* argument taking place by means of nonsensical elucidations can only mean that from the internal perspective, the *appearance* of a contradiction would arise; but the philosopher to whom the elucidations are addressed is not expected to stop there, but to go on and realize that both his initial thesis and the elucidations themselves involve an indeterminate use of words.²³ It is not proper *views* which are thereby refuted, but mere confusions which are removed; and the clarity which transpires when confusion disappears consists neither in the correction of false beliefs, nor in the increase of knowledge. The progress towards clarity, as Wittgenstein conceives it, is not linear and cumulative, but transformative – just as Hertz conceived of the progress which is required in his own field. Moreover, sheer nonsense can play a salutary role and lead us to such clarity precisely because what it is tasked with is not to inform, but to transform us. Indeed, since it is a radical

transformation of our use of words that the successful elucidation is meant to bring about, it makes little sense to insist, as Glock does, that the content expressed by nonsense (be it the philosopher's nonsense or Wittgenstein's elucidatory nonsense) must coincide to some degree with content expressible by us, in our language. But there need not be anything in common to the internal and the external perspective, apart from the mere signs that make up the riddle.

The interpretation of the 6.5s that I proposed here thus reveals a sense in which the substantial conception of nonsense is not only not embraced, but rather put under pressure in the *Tractatus*. For what *the riddle* purports to be (that is, what it seems, from the internal perspective, to be) is nothing other than a piece of substantial nonsense: a nonsensical expression which is nonetheless taken to be *about* some ineffable thing. Yet as Wittgenstein has it, *the riddle* – that which philosophers purport to pursue, when they take nonsense to be substantial – does not exist. So to truly overcome the riddles of philosophy we must also overcome the deception of the idea of substantial nonsense.

One important point of difference between the kind of riddle-solving activity that Wittgenstein wants philosophers to engage in, on their way to greater clarity, and the kind of riddle-solving activity that Hertz recommends, is that in the philosophical case the result of the transformation of our language is that we no longer find there to be any substantive philosophical problems that need to be resolved; there are no correct philosophical theories with which Wittgenstein aims to replace the old, confused philosophical expressions. And as we will see in the following sections, in this respect philosophical riddles also differ from mathematical riddles, as Wittgenstein construes them.

6. The Riddles of Mathematics

In the opening of this chapter I posed a riddle about whether and how nonsensical expressions can form the object of earnest intellectual engagement, if being nonsensical, as the austere reading contends, amounts to lacking all meaning. I proposed that it would be helpful to compare what Wittgenstein says in the *Tractatus* about philosophical nonsense with what he says in later writings, particularly in the early 1930s, about the meaning of mathematical conjectures, and how such meaning depends on whether the expressions do or do not have a proof. Like philosophical theses whose sense has not been made determinate, mathematical conjectures, before we possess a method for proving them, do not yet possess a sense; our engagement with the problem of

establishing such claims, like our engagement with philosophical problems, must be clearly distinguished from our engagement with empirical questions, and this can be done by observing their resemblance to riddles. Like philosophical problems, mathematical problems assume the appearance of genuine questions when we fail to notice that in framing them, we extend the use of words to contexts in which they have not yet received a determinate meaning. In this context, too, Wittgenstein alludes to unsolvable riddles, and compares some cases of mathematical searching to the searching of riddle-solvers who engage with riddles such as Carroll's. A crucial difference between philosophical and mathematical problems, as Wittgenstein sees them, is that whereas the former lack any solution whatsoever, the latter might. What Wittgenstein's discussion of mathematical problems brings out, however, is the indistinguishability, from the internal perspective of the riddle-solver, between cases in which the riddle *has not* been solved and cases in which the riddle *cannot* be solved. A dynamic dimension of language thereby comes into view: our life with language is an open-ended process of enlightenment, a scene of struggle against our own fallibility and our own tendency to succumb to illusions of understanding.

In order to draw our attention to the shape of the difficulties we are facing in mathematical inquiry, Wittgenstein offers in the early 1930s a striking reformulation of the argument of *Tractatus* 6.5, putting mathematical problems in the place previously occupied by the problems of philosophy:

I said: Where you can't look for an answer, you can't ask either, and that means: Where there's no logical method for finding a solution, the question doesn't make sense either.

Only where there's a method of solution is there a problem (of course that doesn't mean 'Only when the solution has been found is there a problem'). (*PR* §149)

It is noteworthy that Wittgenstein stops short of adding a remark paralleling the second line of 6.5, "The riddle does not exist", and this indicates an important difference between the way he conceives of philosophical problems and the way he conceives of mathematical problems. To see this, note that when it comes to our engagement with mathematical problems we may distinguish four kinds of cases. The first two cases are comparable to the different stages of our engagement with ordinary, meaningful questions: the case in which we possess both a method of solution and the solution itself, and the case in which we possess a method of solution but no solution has yet been found. Wittgenstein contrasts these two kinds of cases with cases in which we do not possess

a method of solution, and which therefore cannot be seen as proper questions. Among this latter group of cases we may draw a further distinction. Thus there are cases where no method has been found yet, even though it might eventually be found – I will call these cases of the third kind – and cases in which no method could possibly be found – I will call these cases of the fourth kind. It is to cases of the fourth kind that the problems of philosophy, whose unsolvability is irreparable, most resemble, and Wittgenstein wants us to see that this also holds for at least some mathematical problems. Wittgenstein groups these cases with cases of the third kind, in which a method might one day be found, even though, as we shall see below, he is not ignorant of the difference between them.

In the last quoted passage, where mathematical problems are implicitly compared with philosophical problems, Wittgenstein ignores the distinction between cases of the third and the fourth kind. What could explain this? The reason seems to be that from *within* the practices in which the appearance of a solvable problem arises, the distinction between the third and the fourth kinds of cases cannot be drawn. In what follows I will try to bring out the significance of this point: it is the way things look from within this internal perspective that we must attend to if we seek a *realistic* understanding of mathematics, yet this is precisely what the *realist* fails to attend to.

Here is one example Wittgenstein provides for the way in which mathematical riddles emerge. Recall Wittgenstein's claim that the realist's appeal to the law of excluded middle serves to disguise the true nature of mathematical difficulty, by making the investigation of the domain of the infinite seem like a matter of finding an object whose visibility is obstructed by some contingent factor. Under the grip of this picture, Wittgenstein proposes, the mathematician takes words she is familiar with from the context of finite arithmetical calculation to retain their meaning despite the radical change of context. And this Wittgenstein finds to be problematic:

Consider the fact that in leading a dog, the longer the leash the more freedom for the dog. Now suppose I say the leash is infinitely long. Then I might as well say I do not lead him at all. Analogously, if I ask, "Are there three 7's in this infinite series?", I might as well say the question cancels itself. Its grammar is such that it is not a question. (*AWL* p.196)

In shifting from the finite to the infinite dog leash, we are no longer speaking of leading a dog at

all; the result of the unnoticed shift in context is the emergence of a mere illusion of sense. The mathematical problem whose emergence Wittgenstein presents as analogous has a curious history which is worth mentioning in order to help clarify his point. During most of Wittgenstein's life, and certainly at the time of making this remark, the development of pi has not been extended beyond the first few hundred digits, and these do not include the sequence "777". Yet shortly before his death the occurrence of the sequence "7777" has been discovered at the 1589th place (Reitwiesner 1950). I do not think it would have mattered much to Wittgenstein had he known this, for what is essential has not changed: all we have for pi is a rule of expansion, and though we (or rather, our computers) can make calculations that extend this expansion indefinitely, there is no mathematical method of answering, in advance of the calculations (which are always finite in number) whether a certain sequence of numbers will or will not appear in the infinite expansion of pi (RFM V §41; cf. PI §516). I myself discovered the answer to the question by checking a record of the first computer-generated calculation of pi; what Wittgenstein rejects is surely not the possibility of doing that, but the possibility of answering the question in a reasoned, mathematical manner, namely by means of proof. His point is that where such a proof cannot be given, what we are facing is not a genuine mathematical question, but the mere appearance of one. Like riddles, mathematical problems may deceive us into treating them as solvable, but like Carroll's riddle, some mathematical problems may be unsolvable – and we might not be able to tell the difference.

That we may be caught up in riddles that have no solution, and that it may be that only after a very long time, and only indirectly, that we may realize this, is evidenced by the two Euclidean problems that Wittgenstein often appeals to as examples – the trisection of the angle and the construction of the heptagon (*PR* §151, §156; *PI* §334, §463, §517). What is puzzling about these problems is that the mathematicians who have worked on them, eminently rational men and women, have taken themselves to be in pursuit of something real, and yet the referents of the seemingly meaningful subject terms of their problems (e.g. "the trisection of the angle") has eventually been proven not to exist. But if there really never was such thing as trisection, what were these mathematicians looking for, this whole time – what was the object of their thoughts? There is a sense in which these mathematicians, just like those readers of Carroll who attempted to explain why a raven is like a writing desk, pursued nothing but a mirage. They were, as it were, caught up in a riddle, and unbeknownst to them, it was an unsolvable riddle, a riddle that – as 6.5 puts it – does not exist.

That proofs of impossibility have eventually been discovered which show that these problems are

unsolvable demonstrates clearly that earnest searching does not imply meaningfulness, and that we may indeed be "mistaken in thinking that we understand a question" (PI §517). More generally, it clarifies the sense in which our language is not transparent to us; as language users, we are vulnerable to the threat of illusions of sense.

That this threat might devolve into full-blown skepticism is a danger Wittgenstein is keenly aware of, and understanding his response to this form of skepticism is crucial for understanding how, despite his rejection of realism, Wittgenstein does not end up with an unrealistic account of our intellectual engagements in mathematics and in philosophy. Wittgenstein's response to skepticism does not consist in denying the truth of the skeptic's claim, that we are not always able to tell whether our earnest intellectual pursuits are meaningful or not. Indeed, this is the shape of the real difficulty to which Wittgenstein wishes to open our eyes. But this does not mean that Wittgenstein accepts the conclusion the skeptic draws from this – that our intellectual pursuits are hopelessly meaningless. His point, rather, is that we can learn to live with the difficulty without completely giving up on the notions of meaning and of truth.²⁴ What is needed is a manner of rejecting realism which stops short of succumbing to skepticism.

7. The Recoil from Realism

There are important differences between Wittgenstein's treatments of mathematical meaning in the middle and in the later period of his career, where a greater emphasis is laid on the role played by the application of a mathematical sentence in fixing its content. Importantly, the claim that Wittgenstein seems to assert unequivocally in the early 30s, "if you want to know what is proved, look at the proof' (*PG* p. 301 and 369; cf. *PR* §163), seems to be framed much more cautiously later on: "I once said: 'If you want to know what a mathematical proposition says, look at what its proof proves'. Now is there not both truth and falsehood in this?" (*RFM* VII §10). It is debatable whether this and similar remarks indicate a decisive turnabout in Wittgenstein's thought.²⁵ As I see it, there is no radical break between the middle and late period, and the later remark can be taken to make explicit something which was already implicit in Wittgenstein's earlier discussions. For the whole point of comparing nonsensical expressions with riddles (which is central in both the middle and the late period) is to show that what would be revealed to be meaningless from the external perspective (i.e. from the strictly logical perspective) may form, for the one occupying the internal perspective, the object of earnest work.

Moreover, it is important to see that the verificationist assertion that meaning is constituted by the manner of establishing the mathematical claim, from which it follows that unproven conjectures lack meaning, is put forth in the context of a dialectical engagement. This dialectical movement starts with the recoil from realism, proceeds to articulate its antirealist antithesis, goes on to observe the difficulties this gives rise to, and then shifts from the concern with the content of statements to a concern with the role these statements play in our engagement with them, where this engagement is seen from a dynamic, historical perspective.²⁶

This becomes clear when we consider the specific contexts in which Wittgenstein makes the provocative claim: if you want to know what is proved, look at the proof. This claim forms part of his effort to bring out the grounds for his rejection of the realist view according to which what mathematical work consists in is discovery, not construction, and according to which what proof does is establish truth, rather than establish sense:

Why do I say that we don't discover a proposition like the fundamental theorem of algebra, and that we merely construct it? – Because in proving it we give it a new sense that it didn't have before. Before the so-called proof there was only a rough pattern of that sense in the word-language. (*PG* p. 374)

One would like to say: the proof changes the grammar of our language, changes our concepts. It makes new connexions, and it creates the concept of these connexions. (It does not establish that they are there; they do not exist until it makes them.) (*RFM* III, §31)

By contrast to the realist, Wittgenstein points out that the meaning of the mathematical expression depends on whether it has been given a determinate role within a mathematical language. This is not a matter of mere convention, but a logical matter: before the proof is given, the language does not enable us to make certain connections, and hence prevents us from assigning the expression with the sense that it will have once the proof is given. A proof, like a solution to a riddle, transforms our language.

In the context of this recoil from realism Wittgenstein seems to fully embrace verificationism, and

thus seems to succumb to the "paradox" "that there are no difficult problems in mathematics, since if anything is difficult it isn't a problem" (*PR* §151) – which would seem to render inexplicable the mathematicians' earnest engagement. Indeed, as part of his negative characterization of mathematical problem-solving, Wittgenstein even casts doubt on the assumption that unproven conjectures count as proper objects of belief:

Ask yourself: What does it mean to *believe* Goldbach's conjecture? What does this belief consist in? In a feeling of certainty as we state, hear or think the conjecture? (That would not interest us.) And what are the characteristics of this feeling? Why, I don't even know how far the feeling may be caused by the conjecture itself. Am I to say that belief is a colour tone of our thoughts? Where does this idea come from? Well, there is a tone of believing, as of doubting. I should like to ask: how does the belief engage with this conjecture? Let us look and see what are the consequences of this belief, where it takes us. "It makes me search for a proof of the conjecture." – Very well; and now let us look and see what your searching really consists in! Then we shall know what believing the conjecture amounts to. (*PI* §578)

Belief, ordinarily understood as the affirmation of a determinate content, cannot be the attitude we have towards an unproven conjecture, since it has no determinate content that we can be said to affirm or deny. The avowal of one's belief in such conjectures must therefore be treated differently; what the avowal should be taken to indicate is that one is caught up in the riddle that the conjectures pose. Like the words that make up a riddle, the expressions that make up the conjecture may indeed act as "stimuli" and as "signposts" for mathematical investigation (*PG* p. 371), but they do not function as words do in ordinary descriptions and in ordinary questions. Just like the expressions of riddles, we cannot say what it is that the expressions of the mathematical problems really are about, before our investigation is concluded.

It is crucial, however, that the paradox which seems inevitable for verificationism – that difficult problems lack sense, and hence are not problems at all – does not have traction against a view which is sensitive to the variety of kinds of problems that there are, and to the variety of kinds of searching that they may give rise to. And it is precisely such differences that Wittgenstein is concerned to bring out, as can be seen in the following paragraph:

My explanation mustn't wipe out the existence of mathematical problems. That is to say, it isn't as if it were only certain that a mathematical proposition made sense when it (or its opposite) had been proved. (This would mean that its opposite would never have a sense (Weyl).) On the other hand, it could be that certain apparent problems lose their character as problems – the question as to Yes or No. (PR §148)²⁷

When he denies that mathematicians are solving problems when they engage with a conjecture for whose proof they do not possess a method, Wittgenstein primarily has in mind the cases in which mathematical problems resemble philosophical problems – where they are merely "apparent problems", which might eventually "lose their character as problems", namely if it would be revealed that they do not have a solution. But unlike his treatment of philosophical problems in the *Tractatus*, Wittgenstein is well aware that in the mathematical case, the opposite might occur, even though from our present, internal perspective, we are not able to tell.

The realist objects that in veering towards verificationism Wittgenstein ends up with an unconvincing picture of the mathematician's earnest work.²⁸ This objection receives at least part of its force from the fact that, as we have seen, in speaking of unsolved problems Wittgenstein sometimes ignores the difference between the third and fourth kinds of problems distinguished above, the solvable and the unsolvable ones. By contrast, the realist might point to historical cases that fall under the third kind, such as Fermat's last theorem, where a method of solution has eventually been found.²⁹ Surely, the realist suggests, we would not want to say that such problems lacked sense altogether, before they were solved. So in general, the fact that we do not yet have a method for solving a particular problem should not be taken to indicate its meaninglessness.

It is precisely this that Wittgenstein finds unrealistic in the realist's approach to mathematical meaning. For the realist, what matters is how things would look like in hindsight, or from God's point of view; the realist pretends to be able to observe mathematical language from the external perspective of the divine riddler, who already possesses solutions to all our problems. Wittgenstein, by contrast, asserts: "We know as much as God does in mathematics" (*LFM* p. 104; cf. *PR* §174; *RFM* V §41; *PI* §352, §426). A genuine realist*ic* approach, of the kind Wittgenstein seeks to instill in us, requires that we attend to how things look from *within* the situation in which the riddle is experienced as a riddle, that is, experienced as something which possesses the

"character" of a problem. Consider the following remark:

The schoolboy that lacked the equipment for answering the second question [a problem of higher trigonometry], couldn't merely not answer it, he couldn't even understand it. (It would be like the task the prince set the smith in the folk tale: Fetch me a 'Fiddle-de-dee' [ein Klamank].) (PR §150; cf. LFM p. 206, PG p. 379)

In the folktale Wittgenstein alludes to, the prince sets the smith an impossible task in order to take advantage of him (his riddle is deceitful in all three senses distinguished above). For the way in which the prince spells out the task includes a word which lacks meaning, thus rendering the entire expression nonsensical.³⁰ It is this, Wittgenstein suggests, that resembles our situation when faced with an unsolved problem of mathematics. With respect to unproven conjectures, we are all in the same situation as the student who couldn't merely not answer the question that was posed to him, but couldn't even understand it. Like the schoolboy, and like the smith in the folktale, it is not impossible that we would eventually find a way to give a new sense to a string of signs which at the moment does not yet amount to a statement of a problem. But from our present perspective, that is, viewed realistically, it has no sense. Indeed, even God could not be counted on to endow our senseless words with sense, behind our backs as it were; if they are to have sense at all, a transformation of our language would be required, as a result of which they could no longer count as being the same words. And it is up to us, not up to a benevolent God, to effect such transformations.

8. The Shifting Limits of the Imaginable

As we have seen, Wittgenstein rejects realism because in his eyes it is not realistic enough. His own realistic picture does not consist in denying that there are difficult problems in mathematics, however, nor does he recommend that we stop trying to solve such problems. Rather, his point is that the difficulties are of a completely different kind when we do not have a method of solution, and that both the searching and the finding of a solution, in such cases, have a different form from the searching and finding that empirical questions give rise to. Seeking a proof in mathematics, like solving riddles, is a struggle to transform the language in which the task is spelled out. Wittgenstein's account of mathematical riddles thus brings into view the temporal, historical dimension of our life with language and the creative nature of the activity that brings about its

transformation.

Consider again Wittgenstein's discussion of the fairytale about the peasant's wise daughter, to which the king poses a riddle: come neither naked nor dressed. In comparing this riddle to mathematical problems, Wittgenstein notes that in both cases, the form of the challenge is "Do something which I shall be inclined to accept as a solution, though I do not know now what it will be like" (*AWL* 185–6). For in both cases, reaching a solution requires that we transform our language and thereby alter the range of possibilities of making sense which are available in it:

The question arises: Can't we be mistaken in thinking that we understand a question?

For some mathematical proofs do lead us to say that we *cannot* imagine something which we believed we could imagine. (For example, the construction of a heptagon.) They lead us to revise what counts as the domain of the imaginable. (*PI* §517)

I can't ask whether an angle can be trisected with ruler and compasses, until I can see the system 'Ruler and Compasses' as embedded in a larger one, where the problem is soluble; or better, where the problem is a problem, where this question has a sense.

This is also shown by the fact that you *must* step outside the Euclidean system for a proof of the impossibility.

A system is, so to speak, a world. (PR §152)

Up to the introduction of the proof Wittgenstein describes, each the systems that make up the entirety of our mathematical language formed an insulated language game, providing a limited range of possibilities of making sense, and a limited range of methods of searching. The proof forged new connections between these mathematical systems, and thereby altered the limited "domain of the imaginable" that our language opened up.

The unique role of open mathematical problems, seen from this perspective, is that they invite us to reach beyond the limited, present domain of sense and transform it. This is what distinguishes the difficulty of mathematical problem from that of routine questions:

The discovery of the connection between two systems wasn't in the *same* space as those two systems, and if it had been in the same space, it wouldn't have been a discovery (but just a piece of homework). (*PR* §158)

By contrast to the routine homework assignment of the schoolboy who merely repeats patterns with which he is acquainted, facing real mathematical difficulty requires a break with our past practices. In solving mathematical problems, we modify the space of sense, and thus "step outside" it. The progress we thereby make is non-linear and revolutionary.

The creativity of mathematical inquiry cannot be accounted for in terms of retracing the logical connections which are implicit in what we already understand. To bring out this point Wittgenstein contrasts the narrow sense of "searching" for a solution, which he construes as an "analytic" process, with the "synthetic" activity through which new connections between systems and between concepts may be established:

And 'search' must always mean: search systematically. Meandering about in infinite space on the look-out for a gold ring is no kind of search.

You can only *search* within a system: And so there is necessarily something you *can't* search for. (*PR* §150)

But if [the mathematician] has no kind of system, either in written or unwritten symbols, then he can't search for a solution either, but at best can only grope around. – Now, of course you may find something even by random groping. But in that case you haven't searched for it, and, from a logical point of view, the process was synthetic; whereas searching is a process of analysis. (*PR* §151)

The radicality of the difference Wittgenstein wishes to draw leads him to describe the "synthetic" activity through which mathematicians engage with their problems as "random groping" (cf. *PR* §156) and as "messing about" (*AWL* p. 221). He might thereby appear to *ridicule* the work of mathematicians. Indeed this is precisely how the realist reacts to Wittgenstein's claim that in engaging with unproved conjectures, the mathematician engages with expressions that lack sense.

And admittedly, Wittgenstein does not in this context add any remark resembling the one he makes at the conclusion of "A Lecture on Ethics", that an earnest engagement with expressions which we cannot understand may nonetheless deserve our utmost respect, and should not be ridiculed. But the mere absence of such a remark in the present context need not be taken to mean that his aim is to ridicule the manner in which mathematical discovery is made. ³¹ Wittgenstein's startling characterizations of "synthetic" mathematical search are meant to free us from an unrealistic view of what mathematicians are up to, and thereby to allow us to acknowledge what is truly worthy of respect in their achievements.

9. Conclusion

There are important affinities as well as important differences between the role of the analogy between riddles and mathematical problems and the role of the analogy between riddles and philosophical problems. Although in both contexts we may be misled, and take meaningless expressions to spell out a task worthy of our earnest engagement, it is according to Wittgenstein only in the mathematical domain that such engagement might bring about genuine progress. While philosophical nonsense is nothing but a riddle without solution, only some of the riddles of mathematics are unsolvable and not all mathematical searching is without hope. The challenge is to acknowledge this point without denying the true nature of the difficulty mathematicians face, and without denying the nature of the difficulties we all face, in various other domains of thought. Our real difficulty is that we cannot tell in advance whether we will ever find a solution to the riddles with which we are confronted. But this does not relieve us of the duty to try.

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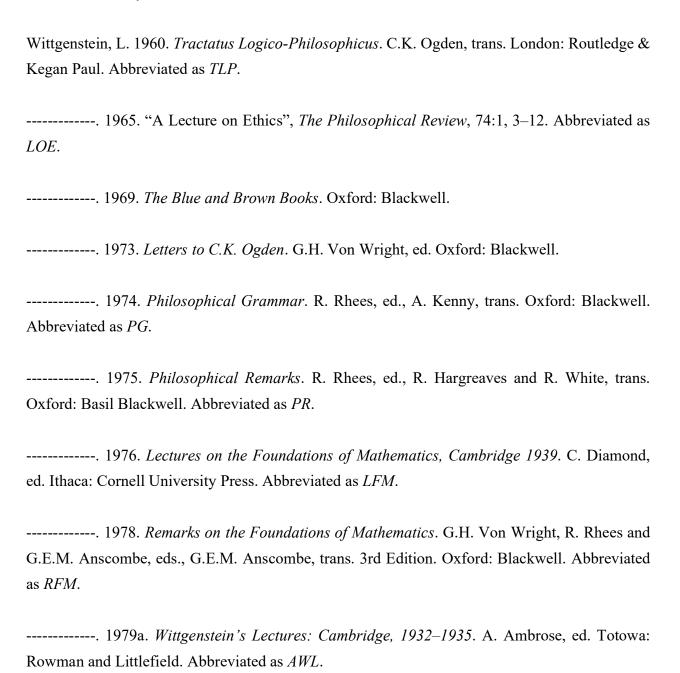
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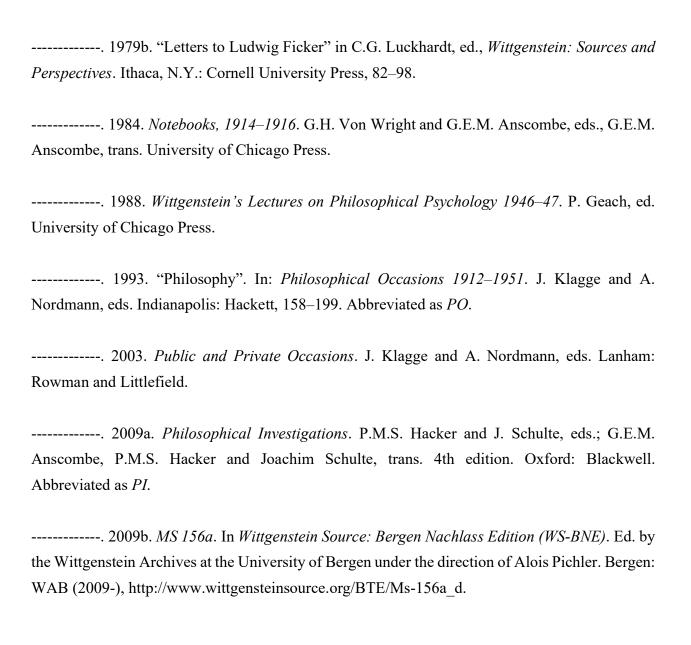
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¹ Wittgenstein's *Tractatus Logico-Philosophicus* (Wittgenstein 1960) is hereafter referred to as *TLP*, and cited by paragraph number, except when citing the Preface, pp. 26-27. Other bibliographical abbreviations used in this essay are as follows:

AWL Wittgenstein's Lectures: Cambridge, 1932-1935 (Wittgenstein 1979a)

LOE "A Lecture on Ethics" (Wittgenstein 1965)

LFM Lectures on the Foundations of Mathematics (Wittgenstein 1976)

PI Philosophical Investigations (Wittgenstein 2009)

- PO Philosophical Occasions (Wittgenstein 1993)
- PG Philosophical Grammar (Wittgenstein 1974)
- PR Philosophical Remarks (Wittgenstein 1975)
- RFM Remarks on the Foundations of Mathematics (Wittgenstein 1978)
- ² The terms "substantial" and "austere" were proposed by James Conant (2002), who draws on Diamond 1991a. Conant and Diamond are the most prominent adherents of the austere approach. A leading defender of the substantial approach is Hacker (2001).
- ³ One important qualification to the claim that Wittgenstein inherits Frege's conception of nonsense must be added. Wittgenstein does not accept Frege's identification of the subjective with the *merely* psychological. The subjective realm, according to Wittgenstein, is the totality of the speaker's language (cf. 5.6-5.61); the possibility of nonsense indicates that a language may be, unbeknownst to its speaker, a defective one. I am indebted to Jonathan Soen for pushing me to clarify my position on this issue; I develop it further in Sections 4 and 5.
- ⁴ On the austere side, see Dain (2008) and Bronzo (2011); on the substantial side, see Hacker (2001) as well as Glock (2004).
- ⁵ There are two examples Wittgenstein gives in *PI* §352: the philosophical question whether our sensations of pain correspond to private mental objects, and the mathematical question whether a certain sequence of numerals, e.g. 7777, occurs anywhere in the infinite development of pi. I discuss Wittgenstein's treatment of this mathematical problem in Section 6, below.
- ⁶ Wittgenstein refers to Hertz's *Principles* in the *Tractatus*, though he does not mention this specific paragraph. For an illuminating discussion of the manner in which the method of the *Tractatus* may nonetheless be taken to reflect the Hertzian insights contained in this paragraph, see Kremer 2012. In his middle and later period, Wittgenstein explicitly discusses this paragraph multiple times, for instance in the methodological, "Philosophy" chapter of the *Big Typescript* (*PO* p. 181); in *The Blue and Brown Books* (Wittgenstein 1969, p. 26 and especially p. 169), and in his discussions in the Cambridge Moral Science Club Meeting on 23.2.1939 and 14.11.1946 (Wittgenstein 2003, p. 379 and p. 399). And see the next footnote.
- ⁷ Wittgenstein included the last sentence of Hertz's paragraph as the motto for the *Philosophical Investigations* in earlier manuscripts, but he eventually replaced it with a quotation from Nestroy, which, like Hertz's, addresses a misconception about the nature of progress: "The trouble about progress is that it always looks much greater than it really is". See McGuinness (2008, p. 303).
- ⁸ I discuss this issue in Nir (forthcoming).
- ⁹ This is a slightly modified formulation of the riddle discussed by Pagis (1996, p. 97).
- ¹⁰ On the vanishing of the problem upon its solution, see PR §157, as well as PG p. 193, cited below.
- ¹¹ Translation emended.
- ¹² This point and much of the inspiration behind my discussion here is owed to Diamond (1991b).
- ¹³ In Wittgenstein's comment on the translation of 6.5 the phrase "the riddle" is pulled out of its original context, and this creates the misleading impression that Wittgenstein takes the problems of philosophy, too, to be concerned with "higher meaning". But note that in the context of 6.5, the existence of the riddle is flatly denied: "The riddle does not exist".
- ¹⁴ See the sources cited by Gardner in Carroll (2000, fn. 5, pp. 71–73).
- ¹⁵ Three relatively well-known examples of illegitimate riddles of this sort are Samson's riddle in *Judges* 14:5–9, the riddle Odin poses to King Heidrik in the *Hervarar saga*, and, in modern literature, Bilbo's riddle in Tolkien's *The Hobbit*. For an illuminating discussion of Samson's riddle see Bal (1988); on Odin's riddle see Heller-Roazen (2013, p. 77); and on *The Hobbit* and the place of riddles in the Anglo-Saxon culture from which it is inspired, see Roberts (2013).
- 16 I discuss this riddle in more detail in Section 7. Wittgenstein refers to it in PR §150, in PG p. 379 and in LFM p. 206. In PG Wittgenstein gives as the source a folktale collected by Busch (1910). See also footnote 36, below.
- ¹⁷ "Oh, how good it is that nobody knows, that I am called Rumpelstiltskin".

¹⁸ Whether Rumpelstiltskin's challenge should count as a riddle at all may be doubted. What speaks in favor of treating it as one is, firstly, that the role it occupies in the tale is the one that is ordinarily reserved for unlawful riddles – apparent riddles which in fact consist in empirical questions whose solution is known to the one who poses them alone. Secondly, in the fairytale the queen first goes through all the names she and her subjects can come up with, and yet is unable to find the answer; since from the queen's perspective all the possible answers to Rumpelstiltskin's question have been exhausted, his challenge assumes the form of a riddle. For a different interpretation of why Wittgenstein took Rumpelstiltskin's words to convey profound evil, see Diamond 2002, p. 171.

¹⁹ This is a point made by Stanley Cavell with respect to Wittgenstein's later philosophy; see Cavell (1999, p. 175). A convincing reading of the *Tractatus* as mounting a critique of modern, science-oriented culture is found in Cahill (2018).

²⁰ Wittgenstein 1979b, p. 94. A similar point is made by Kremer 2013.

²¹ We have already encountered the idea that philosophical problems reflect the incoherent demands that we ourselves place, in the remark from *PO* (p. 161) and in the passage by Hertz cited above. For an illuminating discussion of this theme in Wittgenstein's work, see Diamond (1991c).

It is worth noting that Wittgenstein's critique of philosophical nonsense in the *Tractatus* and his insistence that all metaphysical propositions lack sense can be taken to reflect requirements that he himself poses in advance, namely that there is only one kind of necessity – logical necessity (6.37) – and that logical claims are themselves senseless, and hence lack the kind of substance that the metaphysician yearns for. These requirements are controversial not only since the modern metaphysician proposes that there are other sources of necessity, but also since the early Wittgenstein seems to conceive of logic rather narrowly. These are issues I cannot get into here.

²² In a much later remark Wittgenstein similarly construes his task in terms of showing that philosophical riddles are unsolvable, and hence that they are only apparent riddles: "The only way to deal with a puzzle is to get someone to see it's *not* a puzzle. . . . Unless you can show that a puzzle is not a puzzle you are left with what really are puzzles: a puzzle is something with no solution" (Wittgenstein 1988, pp. 347–348).

²³ I distinguish the shape of an ordinary argument by *reductio* from the shape of Wittgenstein's arguments in the *Tractatus*, along these lines, in Nir (2022).

What learning to live with it might consist in cannot be fully fleshed out here, but I believe that the disjunctivist response to skepticism that John McDowell develops in his discussions of Wittgenstein's later philosophy is the place to start. See McDowell (1998a and 1998b), as well as Conant (2013).

²⁵ E.g. Potter (2011) suggests that the later Wittgenstein rejected the views expressed in his middle period. For another discussion which tends in this direction, see Floyd (2021). With respect to the middle period, Floyd writes: "But his *Satzsystem* conception [in the middle period] imbibes from the TLP the idea of sharpness of sense, putting it together with an idea of mathematical sentences as embedded in calculi that determine their senses. The result *does* wipe out the meaningfulness of unproved conjectures" (Floyd 2021, p. 47). With respect to his later period, she writes that "Later Wittgenstein advocates a more fluid approach to sense-individuation" (p. 67). Two views which emphasize the continuities in Wittgenstein's thought are Diamond (1999) and Säätelä (2011). For a relevant quote from Diamond's essay, see the next footnote.

²⁶ I here draw on a suggestion made by Diamond (1999): "What I have called the verificationist strand in Wittgenstein's thought is properly seen as a matter of a philosophical technique, a technique for redirecting attention; the usefulness of the technique depends on the particular kinds of problem with which we may be confronted" (p. 128).

²⁷ For a different reading of this paragraph, see the quote from Floyd (2021) in footnote 29, above.

²⁸ For instance Potter writes that Wittgenstein "has nothing very convincing to say about what mathematicians are doing when they try to prove Goldbach's conjecture" (Potter 2011, p. 129).

²⁹ Notoriously, Wittgenstein himself mentions Fermat's last theorem as an example for a meaningless conjecture in PR (§150). For discussion, see Säätelä (2011, p. 177).

³⁰ As Wittgenstein presents the riddle here, the word the prince uses is "Klamank", which has no meaning in German. But in some earlier manuscripts in which Wittgenstein mentions it, and also in Busch's version on which Wittgenstein draws, the word the prince uses is "Klamauk", which is in fact a colloquial German

word stemming from the Berliner dialect (which Wittgenstein might have been unfamiliar with) whose meaning is clamor. Nonetheless it is noteworthy that in the fairytale, the prince himself does not seem to know the solution to the riddle that he poses, and he in fact hopes that it could not be solved. On the transmission of this riddle in Wittgenstein's manuscripts see Rothhaupt (2013, p. 22, fn 57).

³¹ Recall that Wittgenstein says that unproven conjectures serve as "signposts" for "investigations" and as "stimuli" for "constructions" (PG 370-1). So his point cannot be to deny that mathematicians should to continue to engage with unproven conjectures. See the helpful discussion of this topic in Säätelä (2011).