Wittgenstein's *Reductio*

Gilad Nir

Forthcoming in *Journal for the History of Analytic Philosophy*

Abstract

By means of a *reductio* argument, Wittgenstein's *Tractatus* calls into question the very idea that we can represent logical form. My paper addresses three interrelated questions: first, what conception of logical form is at issue in this argument? Second, whose conception of logic is this argument intended to undermine? And third, what could count as an adequate response to it? I show that the argument construes logical form as the universal, underlying correlation of any representation and the reality it represents. I further show that the argument seeks to undermine core commitments of Frege's and Russell's. But the *reductio*, as I read it, is not intended to establish the falsity of any of their specific assumptions. Rather, it aim is to make manifest the indeterminacies that underly the language in which these assumptions are framed, and establish the need for a transformation of that language. So understood, Wittgenstein's argument exemplifies his idea that philosophy is not a theory, but an activity of elucidation. The interpretation I propose bears on one of the central debates in the literature, namely how we should understand Wittgenstein's contention that his elucidations succeed despite being nonsensical.

1. Introduction

A central claim of Wittgenstein's *Tractatus Logico-Philosophicus* is that while logical form "shows itself" whenever we use propositions, "What *can* be shown *cannot* be said" (TLP 4.121-4.1212).[1] Indeed, Wittgenstein glosses his so-called "fundamental thought" with the claim that "the *logic* of the facts cannot be represented" (4.0312). These ideas are notoriously difficult to make sense of, for it is unclear whether in saying of anything that it cannot be said we do not end up saying something about it after all. In other words, it is unclear whether with these claims Wittgenstein attempts to say something which by his own lights can only be shown but cannot be said, or whether his claims are simply nonsensical, and do not convey any content at all, in which case it would not be clear what role they might have. My aim in this paper is to investigate the line of argument in the context of which Wittgenstein frames these notorious claims. I argue that the argument forms part of the dialectical strategy by means of which Wittgenstein seeks to subvert Frege's and Russell's approach to logic.

Wittgenstein's argument presents itself as a *reductio ad absurdum* of the assumption that

---

[1] References to the *Tractatus Logico-Philosophicus* (Wittgenstein 1960) are abbreviated as 'TLP' followed by paragraph numbers, except for the "Author's Preface" which is cited by page number (26-27).

logical form can be represented. Starting from the distinction between the content and form of representation, Wittgenstein argues that no representation can represent its own form (TLP 2.16-2.174). But given the further assumption that logical form underlies any possible representation (2.18, 3.032) it seems to follow that there can be no representation of logical form (4.12).

I propose to distinguish two ways of understanding the upshot of Wittgenstein's argument. On what I will call the standard reading of the argument, its goal is to show that the assumption that logic can be represented is incompatible with principles that make up Wittgenstein's own account of logic and representation; the argument is taken to record Wittgenstein's reasons for rejecting that assumption.[2] But this gives rise to a peculiar predicament: the conclusion that logical form cannot be the topic of assertions seems to follow from premises that involve assertions about logical form. As a remedy to this predicament, the standard reading appeals to Wittgenstein's distinction between what can be said and what can be shown (4.1212), since this distinction seems to be meant to allow Wittgenstein to indirectly convey substantive insights concerning the logical form of language and reality, while avoiding the obstacle that logic cannot be directly represented. Thus although according to the standard reading the grounds of Wittgenstein's theory are ineffable, it is a theory that he is taken to thereby provide.

And yet in spelling out his conception of philosophy Wittgenstein expressly says that philosophy is "not a theory but an activity" which essentially consists in elucidations (TLP 4.112). Philosophical elucidations, Wittgenstein explains, aim to effect a clarification of our language, and thereby allow us to overcome our philosophical confusions. Moreover, far from holding that his elucidations convey a commitment to substantive philosophical theses, in the penultimate paragraph of the book Wittgenstein urges us to throw away the ladder of elucidations on which we have climbed up, for once we have reached clarity, we should no longer have any use for them (6.54). Taking these remarks as their starting point, proponents of the so-called "resolute" reading of the *Tractatus* reject the standard reading's construal of Wittgenstein's aims.[3] The alternative construal of Wittgenstein's *reductio* that I propose in this paper is meant to lend support to the resolute approach.

The alternative reading of the argument, which I will defend, is guided both by Wittgenstein's methodological reflections in the *Tractatus* as well as by his later reflections on the different ways in which one might react to the discovery of a contradiction in mathematical as well as in philosophical contexts. The inconsistencies that a *reductio* argument exposes, on this reading, reflect the presence of pervasive confusions that underlie our use of language, rather than the falsity of this or that premise of the argument. Not only the expressions of the inconsistent theory, but the very questions which gave rise to them are thereby recognized to have lacked a determinate sense. This is a general point Wittgenstein makes in the Preface to the *Tractatus*: the manner in which philosophical problems tend to be posed, Wittgenstein there says, reflects a misunderstanding of the logic of our language (TLP 27 and 3.323-4). The proper response to the discovery

---

[2] Proponents of the standard reading include Anscombe 1959 162, Geach 1976, Hacker 2001b 146, Williams 2004 21, Nordmann 2005 65, White 2006 122, and Zalabardo 2015 84, as well as Russell 1960 21-23, whose reading I discuss in Section 4.

[3] Diamond 1991b; Kremer 2002; Conant and Diamond 2004; Narboux 2014. For an overview of the debate between standard and resolute readers of the *Tractatus*, see Bronzo 2012.

of such underlying confusion, he goes on to suggests, is the transformation of the use of language within which those problems are couched (4.112). As a result, the problems would not be solved, but would rather be made to disappear (6.52-6.521 and 6.54). Importantly, in order to achieve this effect, no philosophical theses need to be relied on. Clarity will transpire by removing confusions, not by metaphysical discovery (cf. 6.53).

The Tractarian *reductio*, as I understand it, specifically aims to expose the indeterminacies that underly the way in which Frege and Russell frame their conception of logic, and thereby to point out the need for a transformation of their philosophical language. His goal is not, therefore, to show that some of their theses about logic and representation are false whereas others are true, but rather to show that there is no genuine, meaningful question to which the theses advanced by his predecessors provide an answer. Furthermore, since the elucidatory expressions in which the *reductio* consists themselves draw on the indeterminate use of language that they help us overcome, their success as elucidations requires that we come to recognize that they, too, are nonsensical. Wittgenstein therefore recommends that the ladder provided by the *reductio* ultimately be thrown away.

The structure of the paper is as follows. In Section 2 I reconstruct the ostensible form of Wittgenstein's *reductio*, which the standard reading takes at face value. In Section 3 I assess the extent to which Frege and Russell can be said to be committed to the premises on which the argument draws. In Section 4 I consider Russell's attempt to circumvent the *reductio* by rejecting one of the central premises of the argument while keeping the rest intact. In Section 5 I motivate the alternative construal of the role of *reductio* arguments, drawing on the later Wittgenstein's reflections on the philosophy of mathematics, and I show that the germ of this construal is already to be found in Frege's and Russell's reflections on the admissibility of indirect proofs in laying the foundations of logic. In Sections 6 and 7 I argue that this alternative construal fits the Tractarian *reductio*, and I then clarify the way in which my suggestion lends support to the resolute reading of the *Tractatus*.

2. The Ostensible Structure of Wittgenstein's *Reductio*

My initial aim is to spell out the ostensible form of the Tractarian *reductio*. I call it the *ostensible* form, for upon further reflection it will become difficult to see how one could accept the implications of the argument while continuing to treat the premises from which it takes its start as intelligible. Nonetheless, the dialectical strategy of the *Tractatus* seems to target the kind of reader who, like Frege and Russell, does not suspect that there is anything amiss with the premises of the argument, apart from their possibly being false.

The argument can be broken down into two main steps. In the first step, Wittgenstein distinguishes between the *content* of a representation and the *form* of representation that it employs, and argues that in order for a representation to represent its own form of representation, it would have to employ a different form of representation than the one that it purports to represent. He concludes that no picture can represent its own form of representation. Nonetheless, it still seems possible, at this stage of the argument, that a form of representation of one picture could be represented by means of a different picture, which employs a different form. In the second step of the argument this possibility is blocked. It is argued that all forms of representation presuppose

logical form, and from this the conclusion is drawn that logical form cannot be represented.

The paradigmatic, basic case of representation that Wittgenstein addresses in the first step of the argument is that of a picture. According to Wittgenstein, if a picture is to count as a representation at all, at least two conditions must be met. I call the first the condition of *Common Form*:

> 2.16 In order to be a picture a fact must have something in common with what it pictures.
> 2.17 What the picture must have in common with reality in order to be able to represent it after its manner—rightly or falsely—is its form of representation.

A form of representation is the medium within which certain features of the picturing fact become significant. For by being common to the structure of the picture and the structure of the depicted fact, the form enables their systematic correlation (TLP 2.15-2.151). A relatively simple example for this is the case of spatial pictures that represent spatial states of affairs, where what is correlated are two structures that have spatial form; a more complicated example is the case of the gramophone record whose physical shape pictures a musical piece (4.0141), where the depicting fact and the depicted fact do not belong to the domain of a single form. The principle of Common Form thus states that what is common to all pictures, including those in which the picture and the depicted belong to radically heterogenous domains, is the availability of systematic correlation—a projection or mapping of the elements and structure of one fact onto the elements and structure of another (3.11-3.1432).

The second condition that must be met by any pictorial representation is what I shall call *Outsideness*:

> 2.173 The picture represents its object from outside (its standpoint is its form of representation), therefore the picture represents its object rightly or wrongly.

The basic idea behind Outsideness is that representation as such involves a distinction between two coordinated but distinct facts — the picture and the pictured. One thing this implies is that nothing can count as its own representation. This sounds so trivial that one might think it is not even worth mentioning, but as we shall see it is far from being inconsequential.[4]

Given Common Form and Outsideness, it results that no picture can represent its own form of representation. For whatever a picture represents, it must be "outside" of, but at the same time, any picture must be "within" the common form that it shares with its object. Wittgenstein concludes from this that the form of representation that enables a picture to depict some range of facts is not representable by that picture, or any picture of that same form:

---

[4] See also TLP 4.041 and 5.61, as well as the the illuminating discussions of this feature of Wittgenstein's account of picturing in Diamond 1991b 186-193, Sullivan 2001 106 and and Zalabardo 2015 74. In the next section I continue to discuss Outsideness and its role in Wittgenstein's *Tractatus*, as well as in the work of his predecessors.

> 2.174 The picture cannot place itself outside of its form of representation.

However, even though the form of a representation cannot be part of its content, it must somehow be operative in our use of the picture if we are to interpret it as the picture it is. This drives Wittgenstein to qualify his conclusion:

> 2.172 The picture, however, cannot represent its form of representation; it shows it forth.

Let us leave aside, for the moment, the task of coming to grips with Wittgenstein's distinction between what is said and what is shown forth in a picture. What is crucial for present purposes is that Wittgenstein does not say (at least not yet) that the form of representation which shows forth in a picture could not be represented *at all*. Indeed, what he says seems to leave it open that this could be done by means of a different picture, which employs a different form of representation. Thus a regress of forms of representation seems to loom; and the second step of the argument can be taken to block this regress.

The second step of the argument shifts the focus from pictorial representation to representation as such, including representation by means of propositions, in which, according to Wittgenstein, thought essentially consists (TLP 3, 4). This shift involves a complete abstraction from the natural correlations that may be found between the elements and structure of ordinary pictures and the elements and structure of the facts they depict: whereas in pictorial representation a red element in the picture might depict a red object by virtue of being red, the correlation of a proposition and the fact it represents does not depend on any material similarities. What remains, despite the abstraction, is the least common factor which any representation must have with what it depicts. This is what Wittgenstein calls logical form:

> 2.18 What every picture, of whatever form, must have in common with reality in order to be able to represent it at all—rightly or falsely—is the logical form, that is, the form of reality.
> 2.181 If the form of representation is the logical form, then the picture is called a logical picture.
> 2.182 Every picture is also a logical picture.

Wittgenstein here introduces the principle of the universality of logical form (or Universality, for short). Like spatiality, which informs all spatial representations, logical form is thought of as that which informs any representation of reality, and in particular, all language and thought. In articulating what this bare medium of correlation involves, Wittgenstein explains that the structures of any picture and any depicted fact must have, at a minimum, the same "logical (mathematical) multiplicity" (4.04, 5.475). Of this multiplicity, too, Wittgenstein says that it cannot be represented at all, since "one cannot get outside it in the representation" (4.041).

There are two interrelated features of Universality on which the *reductio* argument turns: the idea that logical form underlies all representation, without exception, and the idea that in speaking of logical form we are speaking of a single, unified form, or at any rate, a single network of interconnected forms (TLP 2.18, 2.181, 3.032, 4.12, 4.121, 5.47, 5.511 and 6.124). This

is not to deny that there are places in the *Tractatus* in which the term 'logical form' admits of the plural, that is, places in which Wittgenstein speaks of the different logical forms of different types of sentences, e.g. sentences involving predicates ranging over different number of arguments (3.315, 3.327, 4.0031, 4.128, 6.23). This plural notion of logical form is a central topic of Russell's 1913 manuscript (Russell 2013), and Wittgenstein's early *Notebooks* directly responds to Russell with an argument directed against the idea that logic can say anything about such forms (Wittgenstein 1984 2; cf. TLP 5.55-5.555). But the passages in which the Tractarian *reductio* is presented abstract from the particularity of logical forms and directly target the representability of logical form as such (this is particularly evident in TLP 2.18-2.182, cited above, and 4.12, which is cited below).[5]

I will have more to say about Universality in the next section, where I consider the vexed question of the extent to which anything like it may be attributed to Frege and Russell; I would here like to focus on the role it plays in the *reductio*. The regress of forms of representation which initially seemed possible in the case of pictorial forms of representation is blocked as soon as one introduces the principle of Universality. For it follows from Universality that there is no such thing as an *ill*ogical form of representation—there can be no *common* factor which does not involve that which is minimally and universally common (cf. TLP 3.03-3.0321, 5.4731, and Wittgenstein 1984 108). And it follows from Outsideness that if there is no such thing as standing outside logical form, then there is no representing it, at all. The entire second step of the argument appears, in highly condensed form, at TLP 4.12:

> 4.12 Propositions can represent the whole reality, but they cannot represent what they must have in common with reality in order to be able to represent it—the logical form.
> To be able to represent logical form, we should have to be able to put ourselves with the proposition outside logic, that is outside the world.

The argument of 4.12 may be reconstructed as follows. The three principles of representation which I spelled out above seem to serve as uncontested premises in the argument. For convenience, I repeat them here:

> *Common Form*: a representation and what it represents are correlated with one another by virtue of having a common form.
> *Outsideness*: a representation is distinct from what it represents.
> *Universality*: logical form is the least common denominator of all correlation.

Let us call the assumption which is to be rejected by means of the *reductio*, on this construal, *Representability*. The first step of the reconstructed argument, (A), consists in the assertion of

---

[5] If the Tractarian *reductio* were concerned with the representability of each particular logical form, for the argument to produce a contradiction further assumptions would have to be added, e.g. that a representation that has one logical form could not have enough in common with other logical forms to count as their representation.

this assumption:

> (A) *Representability:* We can make a representation of logical form.

Together with *Outsideness*, Representability yields (B):

> (B) Such a representation of logical form would be *outside* its putative object, i.e. outside logical form.

I note again that the notion of logical form which is relevant for the argument, as I reconstruct it, is the singular, not plural; this explain why Wittgenstein says that "To be able to represent logical form, we should have to be able to put ourselves with the proposition outside logic." Furthermore, note that although Wittgenstein does not fully spell out what he means by "standing outside," it seems minimally to imply the distinction between two separate facts, such that no fact could be "outside" itself. When applied to the relation between a representation and its form, it implies that a representation which is outside of a certain form does not have that form. This construal of Outsideness would explain why Wittgenstein moves, in TLP 4.12 from (B) to

> (C) Such a representation of logical form would not itself have logical form.

Or in the words of 4.12, such a representation would be "outside logic." Now given Universality, according to which logical form is the least common denominator of all forms of representation, it follows that

> (D) Such a representation of logical form would not have anything in common with logical form.

But since this contradicts the principle of Common Form, it seems that we must reject Representability and conclude

> (E) We cannot make a representation of logical form.

In the words of 4.12, "Propositions… cannot represent… the logical form". For in trying to force a proposition to take the universal form of representation as its object, satisfying the condition of Outsideness requires that we violate the condition of Common Form, and vice versa. But nothing can be both *within* and *outside* its form. Hence nothing could count as a representation of logical form. Representability, it seems, is thereby shown to be false.

3. Whose Views are Targeted by the *reductio*?

Standard readings of the Tractarian *reductio* often assume that what is at stake in it is the status of a theory that Wittgenstein himself advocates. For example, Peter Geach says Wittgenstein deals himself a "self-mate" (Geach 1976 54); Peter Hacker takes Wittgenstein to be "sawing off the branch upon which he is sitting" (Hacker 2001a 102); and Alfred Nordmann explicitly states

that Wittgenstein subjects his own views to a *reductio* (Nordmann 2005 65). To avoid inconsistency, Wittgenstein is taken by these standard readers to conclude that some of his theoretical commitments, though true, cannot be expressed; the distinction between saying and showing is supposedly meant to "ameliorate" the difficulty exposed by the *reductio* (Williams 2004 21; cf. Hacker 2001b 146).[6] By contrast, I will here argue that Wittgenstein targets not his own but Frege's and Russell's views.[7] I will then proceed to clarify in what way Wittgenstein addresses these views; what he aims for, I will argue, is not to expose these views as false but as nonsensical, and this calls for an alternative construal of the nature of the *reductio* argument.

Whether Frege and Russell were committed to the views that Wittgenstein attributes to them is a controversial matter. In particular, there is an ongoing debate over whether a "universalist conception of logic" should be attributed to Frege and Russell, that is, a conception of logic as the science concerned with the most general and universally applicable truths—it is worth noting that several distinct points are run together in speaking of such a "universalist conception", including in particular the two principles which I distinguished between above, Universality and Representability.[8] Since the question to what extent Frege and Russell espoused such a conception of logic cannot be fully settled in the present context, I propose to pursue a more modest aim, namely to clarify Wittgenstein's intent, rather than the success of his critique. What I hope to show is that, given many things Frege and Russell do say, it was not implausible of Wittgenstein to take his predecessors to have espoused such views.[9]

---

[6] For the resolute readers' critique of this construal of the distinction see Conant and Diamond 2004 47ff.; I return to this issue in Section 7, below.

[7] A notable precursor of my proposal is Ricketts 1985, who argues that the *Tractatus* exposes a tension in Frege, namely the tension between Frege's idea that logic plays a constitutive role in guiding all acts of judgment (which is roughly equivalent to what I called Universality) and Frege's construal of the logical axioms as substantively contentful (which is roughly equivalent to the assumption of Representability).

[8] A further claim often made by those who take Frege and Russell to be universalists, on which I do not wish to take a stance here, is that it follows from it that Frege and Russell are prevented from engaging in logical metatheory. Interpreters who defend the ascription of some such version of universalism to Frege and Russell include Van Heijenoort 1967, Goldfarb 1982, Ricketts 1985, Hylton 1990 and 2005, and Kuusela 2019. The claim that Frege is prevented from engaging in logical metatheory is contested by Stanley 1996, Tappenden 1997 and Heck 2007; on this see the response by Weiner 2005. Scholars who argue that not all aspects of the universalist conception of logic are attributable to Russell include Proops 2007 (who nonetheless makes clear that some of these aspects are present in Russell's earlier texts, especially the *Principles of Mathematics*) as well as Korhonen 2012 and Blanchette 2013. I discuss some further points made by Proops in fn. 13, below.

[9] Indeed the very fact that there is such a debate as the one described in the previous footnote goes to show that it is not unnatural, and in fact quite tempting to read Frege's and Russell's work in ways that involve the attribution to them of something like the principle of Universality—which supports my claim that Wittgenstein himself had reasons to take them to endorse it.

Let us begin with *Common Form*. A version of this principle is appealed to in Frege's argument that his formal language is preferable to Boole's. According to Frege, his Begriffsschrift makes it easy to recognize the correspondence between the way its signs are concatenated and "the structure of concepts" which they express (Frege 1979b 12-13).[10] Russell appeals to a similar notion of common form in his multiple-relation account of judgment. One of the roles he gives to the element of judgment which he calls "logical form" is to guarantee that all its other elements are combined "in the right order", i.e. in a way that corresponds to the structure of what is judged (Russell 2013 116). Furthermore, in his "Introduction" to the *Tractatus*, Russell says that Wittgenstein's claim that logical form is common to the proposition and the fact it depicts is "In certain elementary ways… obvious." (Russell 1960 xi). And, if I am right to suggest, as I will below, that Frege and Russell can plausibly be taken to endorse Universality, it is trivial to derive from this a commitment to the weaker principle of Common Form.

What about *Outsideness*? As the word "therefore" in 2.173 (cited above) makes clear, with this principle Wittgenstein attempts to single out the feature of representations which enables them to be truth-evaluable: it is because of the independence of the picture from what it depicts that it is capable of depicting states of affairs which do not actually exist, and in those cases to be evaluable as false. In this connection, Wittgenstein goes on to argue that the very meaningfulness of propositions, their having a sense, depends on their not presupposing their own truth (TLP 4.061; cf. 2.0211 and 3.24).[11] Now even though the early Russell propounded an account of judgment which fails this test, both Frege and (slightly later) Russell advance theories that aim to meet the condition of Outsideness, so understood. Thus by means of his distinction between sense and reference, Frege guarantees that the meaningfulness of sentences is independent of their truth-value (Frege 1997). And once Russell abandons his earlier account of propositions, he turns to develop a theory of judgment that aims to incorporate *Outsideness* without introducing Fregean senses.[12]

To what extent can the intended targets of Wittgenstein's argument—Frege's and Russell's conceptions of logic—be taken to assume the Universality of logical form, understood as the claim that all thought and representation are governed by a single form or an interconnected network of forms, such that one cannot step outside it and yet still count as representing? Frege

---

[10] And see the discussion of the affinity between Wittgenstein's and Frege's theories of representation in Johnston 2017.

[11] Zalabardo 2015 (74) offers a different construal of the principle I call Outsideness. According to him for a picture to be truth-evaluable, the very existence of the picture must not imply the existence of that which it depicts. This condition is not met in the case of attempts to depict logical form, since such attempted depictions would have the very form which they depict, and this would mean that the existence of the depictions would guarantee the existence of what they depict. I find this interpretation illuminating, but I believe there is not enough textual evidence to support it.

[12] For Russell's early view see Russell 1973, and the discussion in Hylton 1990 243-275. For the mature view, see Russell 1905 and especially 2013, with which Wittgenstein was intimately familiar.

says that logic is concerned not with any truth in particular, but with the "laws of truth" which are at the same time the "laws of thought." These laws are not merely normative for all thinking (Frege 1984 351) they are constitutive of all judgment and inference (Frege 1979a 3). Since logical laws constitute the activity of justification, there is no external standpoint from which one can justify any of these laws; all one can do is show how each of them interconnects with the others (Frege 2013 xvii). Logic is for Frege, in this specific sense, the study of the single, unified, and absolutely general form of all thinking. This becomes particularly explicit when Frege addresses the question whether there might be such a thing as thought which is governed by an alternative logic, in the context of Frege's polemic against the psychologistic logicians of his time, whose construal of logical laws in terms of natural laws governing the operation of brains renders these laws contingent (Frege 2013 xvi). The psychologistic logicians, Frege points out, thereby make room for the possibility of logical aliens, i.e. beings whose form of thinking corresponds to laws that contradict ours. But in the absence of agreement on common logical criteria, we could not evaluate what these aliens do in light of their laws as either correct or incorrect; and given that correctness is the constitutive goal of thinking, the very idea that what they do deserves to be called thinking, at all, is thereby shown to be incoherent. Frege further holds that to deny even a single basic principle of logic is no longer to engage in thinking; what it amounts to, Frege holds, is mere confusion (Frege 1960 21).

In his *Principles of Mathematics*, Russell similarly holds that logic is concerned with the principles that govern "inference in general" (Russell 1903 11). Like Frege, Russell is lead from this commitment to Universality to the claim that it is not merely wrong, but *impossible* to coherently judge what is logically false. One place in which this becomes evident, both in Frege and in Russell, is their arguments against the employment of indirect proofs (the genus to which proofs by *reductio* belong) in laying the foundations of logic. Frege maintains that we cannot meaningfully assume the falsity of a logical axiom, even for the purpose of deriving a contradiction, as this would require suspending the universal principles which constitutively govern our acts of thinking and judging (Frege 1980 79 and 182, Frege 1984 335, Frege 1960 21; and see Ricketts 1985 11). Russell, for his part, argues that assuming the falsity of a logical principle, e.g. in the context of an attempted indirect proof of their independence, would introduce an incoherence into our body of thought, which could not be insulated and contained. He reasons that in allowing the negation of a logical principle into the proof, we would also legitimate the invalid forms of reasoning that are derivable from it. Russell's worry is that nothing we would go on to

say in such a context would count as properly established (Russell 1903 15; cf. 454).[13]

Russell's views on logic shift considerably over the years, and there is an ongoing debate over whether and to what extent he can be taken to be committed to something like Universality.[14] The outcome of this debate should not matter for my present purposes, however, since the question I am primarily concerned with is not what Russell actually thought at each and every point in his career, but whether Wittgenstein could plausibly take him to advocate Universality. The evidence I just cited, while perhaps not sufficient for conclusively settling the first question, is sufficient for answering the latter. Wittgenstein had reasons (at the very least, apparent reasons) to take Russell to be committed to Universality. Moreover, as I will show in more detail in the next section, in his confrontation with Wittgenstein's *reductio* Russell proposes that one could save the assumption that logical form is representable if one denied Universality. What he says there implies that Universality is a commitment that he previously made and that he now finds it very difficult to disown (Russell 1960 23). So Russell himself seems to confirm my line of reading.

I now turn to the assumption which the *reductio* purports to refute, namely that logical form can be represented (A). To what extent and in what sense can Frege and Russell be taken to aim at representing logical form? The core of Frege's and Russell's logicist projects can be roughly described as the attempt to reduce the entirety of mathematical knowledge to the axioms which capture the most fundamental and self-evident laws of thought. The logical axioms are thus treated as sources of substantive knowledge. Drawing analogies between logical laws and scientific laws, Frege and Russell both treat logical propositions as substantive, contentful truths, i.e. as propositions that represent something, however general or abstract their content might be. In Frege's case, the axioms are taken to capture the descriptive laws of the realm of truth (Frege 1984b 351); Russell, for his part, says that "logic is concerned with the real world just as truly as zoology, though with its more abstract and general features" (Russell 1919 169).

Wittgenstein's *reductio* undercuts the idea that logical propositions amount to substantive

---

[13] Proops 2007 (21-24), who contests the attribution to Russell of a universalist approach to logic, argues for a different interpretation of this passage. One useful point he makes is that Russell is here concerned only with the axioms of the calculus of propositions, not with all of the principles that Russell calls logical axioms. Proops then further argues that the passage only aims to rule out a weaker "non-demonstrative independence proofs" and that it should not be taken to bear on the possibility of demonstrative independence proofs of those axioms. Whether this is a correct interpretation of the passage or not, Proops does not deny that Russell holds it to be impossible to coherently assume the falsity of the axioms of the propositional calculus. The argument on which Russell relies in making this point is not entirely clear, as Proops himself points out, but what Proops fails to emphasize is that whether Russell's argument is successful or not, it is very clear that Russell accepted its conclusion, namely that (some) axioms of logic cannot be coherently denied. And it is also clear that the reasons Russell gives for this have to do with the role that these axioms play as principles of reasoning. So even Proops should not deny that the passage reflects Russell's adherence to some form of the principle of Universality.

[14] See the literature cited in Footnote 7 above. Proops 2007 admits that among Russell's texts, the early *Principles* is the one to which some version of Universality must be ascribed.

representations of the logical form of all thought. To better see what kind of view Wittgenstein rejects it would be helpful to briefly look at the alternative that he himself proposes. In the *Tractatus* Wittgenstein construes the nature of the propositions of logic in a manner radically different from that in which he construes the contentful propositions that make up our scientific knowledge (4.111, 6.111). He insists that logical propositions are empty of all content: they are senseless signs, mere "tautologies" (TLP 4.46). For unlike ordinary, contingent propositions, in logical propositions the representational relations to reality "cancel one another" (4.462). This does not mean that in Wittgenstein's view logical propositions represent something *other than* the facts that can be ordinarily represented (as proponents of the standard reading might have it). Logical propositions fail to amount to any sort of picture at all, and it is only in a very attenuated sense that they deserve to be called propositions, or even to count as true or false. Thus in pre-Tractarian manuscripts Wittgenstein simply denies that logical propositions have a truth value, saying that they are "neither true nor false" (Wittgenstein 1984 109), and although by the time of writing the *Tractatus* he proposes to count tautologies as true and contradictions as false, he often uses scare-quotes in ascribing truth-values to them (TLP 6.125 and 6.1223).

Though they are not *representations* of logical form, there is, for Wittgenstein, a different sense in which logical propositions can be said to make logical form manifest. A logical proposition such as "p v ~p" shows that this particular way of combining its elements renders the result empty of all content, no matter what content each of the elements might have when it appears in other contexts (TLP 6.121). But as Wittgenstein is quick to point out, it is not only through logical propositions that logical form shows itself — any proposition must be able to show its own logical form, for otherwise the proposition would not be a determinate, truth-evaluable representation at all. And since there is nothing that only logical propositions can express, they are *in principle* dispensable (6.122, 6.126), even if in practice, as Wittgenstein grants, they may be useful (6.1262). For example, by means of logical equivalences such as De Morgan's law, ~(p & q) ≡ (~p v ~q), we can simplify what would otherwise be much more complicated chains of inferences, e.g. the derivation of ~q from p and ~(p & q). The role of the logical proposition in this context is not to represent anything, but to facilitate substitutions and thereby to make our genuine representations and the inferences that involve them more perspicuous.[15]

Despite the rejection of Representability, however, Wittgenstein goes on in later parts of the *Tractatus* to introduce what he calls the general propositional form (4.5), in which the "essence of all description" consists (5.4711); he proposes to spell out this form in terms of a variable which any truth-functional combination of elementary propositions falls under (6). This is not the place to attempt to provide a full account of this idea; nonetheless, I wish to allay the worry that the introduction of such a variable violates the rejection of Representability. When Wittgenstein explains how variables can bring out what is characteristic of formal concepts (e.g. object, concept, fact, function, number; cf. 4.1272), he distinguishes the relation between a variable and its instances from the relation between a concept and the objects that fall under it; the latter relation is precisely what descriptive propositions express, whereas the former is something

[15] A parallel point applies to mathematical equations: they do not convey any facts, but instead serve to facilitate substitutions and thereby to make our reasoning more perspicuous. See 6.211 and Kremer 2002 298, as well as Diamond 2019 134.

which is not said by propositions, but is shown by the fact that we accept one proposition as an appropriate substitution for another (4.126). Accordingly, whatever it is that the introduction of the variable which captures the general propositional form is meant to do—however we wish to construe the way this variable gives us an overview of propositional expressions, and thereby helps draw the limits of the significant use of language—it is clear that Wittgenstein does not mean, by its introduction, to provide a *picture* of the logical form of the propositions, or to provide a substantive characterization of the realm to which logic applies. For in order to do anything like the latter, Wittgenstein reminds us, we would have to step "outside" logic (5.61).[16]

4. Russell's Response

The way Russell responds to the *reductio* in the introduction he wrote for the *Tractatus* provides further support for the claim that Wittgenstein's argument targets views endorsed by his predecessors. Russell construes Wittgenstein's argument as a standard *reductio* of the assumption of Representability. And he proposes to save this assumption by rejecting one of the other premises of the argument, namely Universality.[17] In consequence, Russell unleashes the regress which the introduction of Universality blocked:

> These difficulties suggest to my mind some such possibility as this: that every language has, as Mr. Wittgenstein says, a structure concerning which *in the language*, nothing can be said, but that there may be another language dealing with the structure of the first language, and having itself a new structure, and that to this hierarchy of languages there may be no limit. … Such an hypothesis is very difficult, and I can see objections to it which at the moment I do not know how to answer. I do not see how any easier hypothesis can escape from Mr. Wittgenstein's conclusions. (Russell 1960 23)

Russell here suggests that one could represent the logical form of one language by employing another language, whose logical form is different from that of the first. He finds the consequences of the rejection of universality difficult to swallow (which might plausibly be taken to indicate his own prior commitment to Universality), but he does not think that the regress is hopelessly absurd. Perhaps this is because he takes the regress to be merely "potential", rather than an "actual" regress, such that it does not threaten the intelligibility of the very idea of lan-

---

[16] See Diamond 2011, who argues that the sole purpose that the specification of the variable (the general form of the proposition) serves is to aid in the activity of the clarification of our use of language, and reaching such clarity does not depend on making any substantive claims about logical form or about language.

[17] One might also try to save the assumption in other ways, e.g. by rejecting Outsideness. But Russell does not consider such alternatives, and for reasons of space I shall ignore them here. In the following sections I consider a different understanding of the nature of *reductio* arguments which disavows all moves of this kind.

guage.[18]

But in fact it is quite unclear what Russell could mean in saying that each language in the hierarchy has a different structure (or a different logical form) than the one that precedes it (which is to be distinguished from the idea that each language is restricted to a single type of objects). If this is supposed to allow Russell to save Representability, each such linguistic structure would have to be incommensurable with the structure of any of the other languages in the hierarchy, since if there were anything in common to them all, then placing them in a hierarchy would not be of much help—there would still be this common infrastructural element that all languages presuppose, and which, by Outsideness, none of these languages are able to represent. But if there is no such common structure, by what right might Russell call any of them "language"? The intuition I would like to invoke here is the same one that Davidson (1974) famously draws on: by treating a putative language as incommensurable, and hence as untranslatable to one's own, one loses the right to assume that what one is treating of is a language—a vehicle of representation and thought—at all. Indeed, Russell seems, inconsistently, to imply that there *is* something common to all the languages in the hierarchy, since he says that each of them "deals" with some things which the others cannot deal with. But it is precisely this "dealing with"—that is, the common notion of representing—that Russell's languages must not share, if the hierarchy is truly to provide an escape from the *reductio*.

One might be tempted to help Russell to a less problematic construal of the hierarchy of languages, namely the one that modern semantic theory, going back to Tarski, draws on (e.g. Tarski 1944). But this would be anachronistic; let me briefly point out what I take to be the crucial difference. Whereas Russell still assumes that the languages that make up the hierarchy are meaningful, interpreted languages, modern logicians sharply distinguish the syntax and semantics of a language. The construction of the syntactic structures that make up the hierarchy does not presuppose any semantic notions; rather, the latter are subsequently defined in a way that makes them relative to each such system of signs. This is sufficient for overcoming the objection I raised with respect to Russell's hierarchy: identifying each of the languages that make up the modern semantic hierarchy does not presuppose or imply that these languages involve any single, semantic notion of representation, and so it does not imply that they share a common logical form.[19]

To the extent that the modern semantic approach makes itself immune to Wittgenstein's *reductio*, it achieves this by completely changing the subject. For unlike Russell's hierarchy, the modern semantic approach severs the connections between the idea of language and the idea of representation, of reasoning, of thought and of inference, to which Russell still seems to hold on. Rather than vindicating Russell's proposal, this reveals how radically one must break with Russell's framework—how radically one must change the meanings of the terms underlying the as-

---

[18] For the distinction between potential and actual regress see Russell 1903 51.

[19] Tarski's reasons for introducing the hierarchy were different, of course: he was hoping to thereby provide an adequate solution to the semantic paradoxes, particularly the liar paradox. Russell's hierarchy would clearly fail to meet this goal, as well. By contrast, Carnap 2000 282 is explicitly concerned to show that the modern semantic approach is not vulnerable to the difficulties raised by Wittgenstein's *reductio*.

sumption that logical form can be represented—in order to bypass the difficulties exposed by the *reductio*.[20] What begins to emerge from these considerations is the shape of an alternative construal of the role that *reductio* arguments may play. Namely, they may serve to manifest the need for a complete transformation of the terms in which a certain philosophical problem is couched; fully overcoming the contradiction exposed by the *reductio* requires the introduction of a radically new philosophical approach. I now turn to develop this construal of the *reductio*.

5. The Alternative Construal of the Role of *Reductio* Arguments

Russell's way of responding to Wittgenstein's *reductio* assumes that the sense of each of its premises and assumptions can be held fixed independently of that of the others. On Russell's approach, despite the rejection of Universality, what the other premises say about logic and representation and the very meaning of terms such as 'represent' (or 'deal with') and 'logical form' are supposed to remain unchanged. Similarly, on the standard construal of Wittgenstein's argument, which can be found in Hacker, Geach and Nordmann, despite the rejection of Representability, what the premises indicate is taken to be true, even though such truth cannot be properly represented.By contrast, I would now like to suggest a radically different way of construing the role of the Tractarian *reductio*, according to which what it shows is not that logical form cannot be represented, but that no determinate sense has been given to the terms in which we frame the very question whether we can represent logical form.[21] A *reductio* argument, in this sense, may signal the incoherence of the use of language in the context of which a certain philosophical problem (as well as the proposed solution to it) comes to *seem* meaningful. The proper response to this discovery is not the rejection of any single assumption, but the transformation of the language, and therewith the overcoming of the illusion of meaningfulness to which that language gave rise. This is not to say that there are no contexts in which *reductio* arguments do function in the manner in which they are standardly construed, that is, contexts in which they serve as proofs that establish the falsity of a specific assumption without ushering any radical change of meaning. Rather, the point is merely that there are certain contexts in which the effect of *reductio* arguments is precisely such a transformation of language, and that Wittgenstein's critique of Frege and Russell is one such context.

The later Wittgenstein's observations on the effects that proofs of impossibility may have on our understanding of the nature of a mathematical problem shed light on the idea, which as I will argue below is already present in the *Tractatus*, that demonstrating the unsolvability of a problem may prompt us to transform our use of language and thereby to realize that there was no

---

[20] I return to reflect on the continuities and discontinuities in the history of modern logic in the concluding section of this paper.

[21] Both Sullivan 2004 34 and Conant 2007 56 point out (as a consideration against standard readers) that whereas a *reductio* proof, standardly construed, establishes the falsity of a proposition that we understand, the *Tractatus* does not purport to establish the falsity of philosophical propositions, but to expose their nonsensicality, i.e. their not being propositions. This is correct, but as I will argue, there is an alternative way of construing the role of *reductio* arguments which perfectly fits the purpose of the *Tractatus*, so understood.

substantive problem there to be solved, in the first place.[22] One example that Wittgenstein repeatedly invokes in this connection is the trisection of the angle—a Euclidean problem that exercised mathematical minds for millennia, until it was proven, in the 19th century, to be unsolvable. Wittgenstein draws our attention to the effect that the discovery of the proof that it is impossible to trisect the angle using Euclidean methods has on our description of what the mathematicians who were bothered by this problem were up to: from our vantage point, to say that they were looking for the trisection of the angle would be quite misleading, since as we know, there really is not such a thing for which one might look (Wittgenstein 2009 §463-4). Rather, it seems more apt to say that they were guided by a defective grasp of what can be looked for; to make sense of their behavior, we must take them to have been under the *illusion* that certain combinations of words meant something determinate, and that by means of them they managed to pose an intelligible problem. Notably, prominent 19th century mathematicians themselves observed, in considering such impossibility proofs, that they do not provide a solution to the original problem, but rather transform our understanding of it. For rather than answering a question of the form "what is the solution…", the impossibility proofs answer the question "is there a solution for…". Thus David Hilbert observed that by means of such proofs certain problems "have finally found fully satisfactory and rigorous solutions, although in another sense than that originally intended".[23]

To avoid the contradiction that the *reductio* argument exposes we may choose to modify our language in different ways. For one, we can reject the problematic phrase from our language, by becoming alert to the fact that no determinate meaning has been assigned to phrases such as "the euclidean method for trisecting an angle".[24] Alternatively, we can go on to alter our use of words in such a way that the problematic expression remains in use but no longer leads us to the contradiction. Thus the pythagorean proof of the impossibility of finding the rational number whose square equals 2 (which may indeed be construed as a *reductio*) ultimately led mathematicians to reject the idea that any number must be expressible as a ratio of two whole numbers, but it did not lead them to stop assuming that the phrase "the square root of 2" is meaningful (though we can imagine different languages and different mathematical systems which would result had they chosen the latter response). The effect of such impossibility proofs is thus not to provide an answer to the original problem, but to make that problem disappear.

It is worth recalling that Frege and Russell themselves see the presence of a contradiction in a system of thought as an indication for the indeterminacy of the use of language that underlies such a system, and they thereby anticipate Wittgenstein's construal of the effect that *reductio*

---

[22] See Wittgenstein 1974 387-392, Wittgenstein 2009 §334 and §§463-4; the effects of *reductio* in the mathematical context is further discussed in Wittgenstein 1975 184–185, Wittgenstein 1976 86–9, and Wittgenstein 1978 185 and 371. For insightful discussions, see Diamond 1991c and Floyd 1995.

[23] Hilbert 1901, cited in Lützen 2009 390.

[24] Cf. Wittgenstein 2009 §500: "When a sentence is called senseless, it is not, as it were, its sense that is senseless. Rather, a combination of words is being excluded from the language, withdrawn from circulation."

arguments may have. As I noted above, both Frege and Russell object to the use of indirect proofs in logic, for the reason that the assumption of the falsity of a logical axiom threatens the soundness of our reasoning.[25] Frege, in particular, sees the presence of a contradiction, even when it is artificially produced in the context of a proof, as an indication that one has not made a fully meaningful use of language. Thus in the context of his polemic against Hilbert, Frege complains that in Hilbert's proofs of the independence of the geometrical axioms, "the word 'axiom', as [Hilbert] uses it, fluctuates from one sense to another without his noticing it" (Frege 1979c 247). In other words, Hilbert seems to Frege to suffer from what the *Tractatus* would later diagnose as the marks of philosophical nonsense: an indeterminate use of words which sustains an illusion of meaning (TLP 5.4733).[26]

Wittgenstein suggests that it is not merely difficult, but impossible to retrospectively specify what one took one's words to mean, after one has realized that their words imply a contradiction. When this is discovered, Wittgenstein observes, what one is prone to say is "That's not the way I meant it" (Wittgenstein 2009 §125; cf. §334). In other words, one is driven to completely retract what one originally meant so as to render one's past behavior coherent in light of one's present understanding. Only after such a revision has taken place can one be said to truly mean anything determinate by one's words; whereas before it, and before the contradiction was made explicit, one cannot be said to have meant anything determinate by it, at all.

Note, moreover, that *reductio* argument themselves draws on the terms whose indeterminacy they lead their target audience to acknowledge and overcome. That is, the expressions that make up the argument are janus-faced.[27] For those whose language the argument aims to transform, each of these expressions may not seem problematic at first, at least not until the moment at which the contradiction is revealed; so they would take the argument to bring out compelling inferential relations between these putatively meaningful expressions. But for us, who are no longer caught up in that defective use of language, the steps made in the argument may no longer appear to be motivated at all. Since we have undergone the relevant transformation, and reached greater clarity, we may no longer take the argument to expose a contradiction in our language. Indeed, since our language has already transformed in ways that prevent the contradiction from arising, we can simply throw away the argument, along with the indeterminate uses of language that it led us to overcome. Nothing substantive would thereby be lost.

This construal of the role of *reductio* arguments thus helps clarify what the Tractarian *reductio* is meant to achieve, and how well it serves the declared aims of the *Tractatus* as the resolute reading understands them. Such arguments serve to draw our attention to the indeterminacies that underly our use of language, and indeed it is these indeterminacies which, according to the *Tractatus,* are the source of our apparent philosophical problems (TLP 3.323-3.324, 4.003 and

---

[25] See section 3, above.

[26] It is a separate question whether or not Frege is correct to think that Hilbert's proof procedures involve assuming the falsity of an axiom, and whether his use of the word "axiom" is indeed indeterminate. For a discussion of the complex issues involved here, see Tappenden 2000.

[27] For the related idea that in the context of Tractarian elucidations the use of signs is merely "transitional" see Diamond 2000 157.

6.53); the task of philosophical elucidation is to render our expressions clearer (cf. 4.112), that is, to alter our use of language, and thereby to make these apparent problems disappear (cf. 6.521). As a result of such a transformation of our language, Wittgenstein reasons, the elucidatory expressions themselves would come to be recognized as nonsensical, and thus be thrown away (6.54). The effect these elucidations would have on us—the clarity in the use of language that we would thereby gain—would be revealed by our no longer feeling the desire to speak in those problematic ways (7).

The Tractarian *reductio* with which this paper is concerned is but one of several lines of arguments in the *Tractatus* in which Wittgenstein pursues this program. To very briefly mention one other clear example, in the 5.53s Wittgenstein argues that it is only because of confusions surrounding the interpretation of the identity sign that Russell was pushed to introduce the axiom of infinity, and that this axiom itself gives rise to further irresolvable problems. Wittgenstein then proposes to address the difficulty by transforming our language in a manner which would allow us to dispense with the identity sign; as a result, he argues, all of the problems surrounding the axiom of infinity would disappear (5.535). However, in order to spell out this suggestion, e.g. in oder to clarify what it is that the identity sign is *not* required to expressed, Wittgenstein himself must engage in ways of speaking that are bound to mislead (cf. 4.1272). So his own elucidations of identity must be thrown away, once they have achieved their goal, namely once we have adopted a new use of language in which the old problems of identity no longer arise, and in which there is no longer any need for such elucidations.

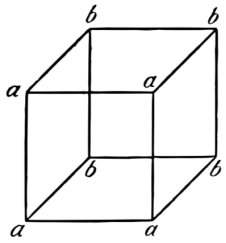
6. The Alternative Construal of the Tractarian *Reductio*: The First Step

Our situation in philosophy, according to the *Tractatus*, is just like the one in which mathematicians find themselves before they have discovered that their assumptions lead to a contradiction, and hence before they have realized the need to revise the terms they draw on in their assumptions. As we have just seen, it is the official aim of the *Tractatus* to expose the unnoticed ambiguities and indeterminacies that underlie the nonsensical propositions of philosophy, and to lead us, by means of elucidations, to overcome the appearance that the problems which these propositions purport to answer are genuine problems. The Tractarian *reductio* is designed to serve this goal insofar as it shows that the idea of representing logical form is incoherent, and that its philosophical attraction is merely illusory. Representability only appears to make sense because of the indeterminacy in the use of words with which this idea has been framed. The kind of transformation of language with which Wittgenstein proposes to remedy this defect does not consist in our coming to see Representability as false; rather, it consists in our no longer taking this assumption to make any sense, and hence in our throwing it away, removing it from our language.

With this in mind, let us reconsider the first step of the Tractarian *reductio.* Wittgenstein here seems to argue, on the basis of Common Form and Outsideness, that it is impossible for a picture to represent its own form of representation. But upon further reflection, I will now show, what he brings us to see is that the thought of making such a representation is neither true nor false, but incoherent.

On Wittgenstein's view, pictures are individuated not only by reference to the elements and structure that make up the depicting fact, but also by reference to the form of representation

that is employed in "projecting" such a fact onto the depicted fact. Consider, in this connection, the case of the Necker cube, discussed at TLP 5.5423:



On one way of seeing it, this diagram represents one cube (the one in which the points marked with 'a' make up the face closest to the viewer), while on another way of seeing it, it represents another cube (the one in which the points marked with 'b' form the face closest to the viewer).[28] Representing each of the two distinct cubes requires a distinct form of representation, by means of which the same spatial structure — the same arrangement of points and lines — combine into a representation of a different content. These two distinct forms of representation cannot themselves be elements of a single pictorial fact, for if they were, we would not be able to see it as representing once this cube, once that cube, where in each case we only have in view one completely determinate cube. Indeed, if you try to situate both represented cubes in a single three-dimensional coordinate system, it will become evident that not all of their vertices coincide. This shows that in the uninterpreted diagram, each point is merely an ambiguous sign, which is used in different ways in each of the two pictures, to depict different spatial locations. It is only in the context of each of the working pictures of each of the two cubes that each point in the diagram truly counts as a symbol for a determinate spatial location. Conversely, it is only by equivocation that we call the diagram "the Necker cube", as though it were a single, determinate picture of any cube. The diagram, considered apart from its interpretations, i.e. considered apart from the two forms of representation that may be employed in order to project it unto a three dimensional space, is not yet a picture.

This point extends to representation in general. When it comes to language, Wittgenstein distinguishes the mere sign from the significant symbol, and argues that the meaning of signs can only be identified when one considers their role within the context of the significant use of a propositional picture (TLP 3.3, 3.321). According to Wittgenstein, it is precisely by failing to pay heed to such shifts in the context of use and by ignoring the ambiguities that result from them that the attraction of philosophical nonsense is sustained (3.323-3.324, 5.4733).[29]

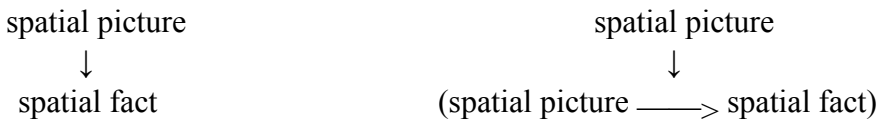
In view of these distinctions, suppose that there could be a single *spatial* picture which depicted its own form of representation. To do that, it would have to serve two radically distinct roles. Qua spatial picture, it would have to employ a form of representation that correlates spatial

---

[28] With a little more difficulty, we can even see it as a two-dimensional hexagon with lines running through it. For the sake of brevity, I leave this possibility out of account here.

[29] On Wittgenstein's diagnosis of philosophical nonsense see Diamond 1991b 197 as well as Conant and Diamond 2004 62.

aspects of the depicting fact with spatial aspects of the fact that it depicts. But qua representation of a form of representation, it would have to employ a form of representation that correlates a spatial fact with a fact of a different order of complexity, i.e. the fact which consists in the *correlation* between two spatial facts. The difference can be illustrated as follows:



Inasmuch as a spatial picture performs the first role, it employs a form of representation that is simply not cut out for performing the second. And even if a single pictorial fact might *seem* to perform two such radically distinct roles, keeping in mind the lesson learned from the discussion of the Necker diagram, we should understand such a fact as a merely ambiguous and indeterminate sign, which takes part in two distinct pictures, rather than counting it as a single picture that represents its own form of representation.

In the specific case under consideration in the first step of the Tractarian *reductio*, a philosophically tempting idea seems to suggest itself in the phrase "a picture which represents its own form of representation." Its attractiveness depends, however, on our failing to notice the ambiguity of the expressions that appear in this phrase (particularly, "a picture" and "its own form"). The standard reading of Wittgenstein's argument takes the phrase to refer to a coherently specified possibility, which the first step of the *reductio* shows not to be realizable. But if the phrase cannot be unabmiguously read as referring to any single picture, then what we are ultimately meant to realize is not the truth or falsity, but the nonsensicality of the claim that a picture cannot represent its own form of representation.

7. The Alternative Construal of the Tractarian *Reductio*: The Second Step

One of indeterminacies of meaning which the second step of the Tractarian *reductio* exposes comes to the surface in step (D) of the argument, where we explicitly entertain the possibility of a representation (of logical form) that has nothing in common with what it purports to represent. But the very idea that we can call anything a representation while depriving it of even the most minimal correlation with that which it represents disintegrates upon reflection. In other words, in proposing that we could represent logic illogically, we lose our grip on the very idea of representation. Indeed, the indeterminacy that becomes explicit at this point runs through the entire argument, all the way back to its starting point, Representability. For to say that (and similarly to ask whether) we can represent logical form involves a merely equivocal use of the terms "logical form" and "represent". In saying it, we purport to distinguish the form and content of a single, determinate representation of something which is distinct from it, but we then take that form to be the content that that picture represents. It is worth noting that the ambiguous manner in which we treat logical form in this context is reminiscent of the kind of confusion Frege criticizes when he argues that in attempting to speak about concepts and distinguish them from objects (e.g. by saying "no concept is an object") we willy-nilly talk of concepts as though they were objects

which fall under first-order predicates, and in doing so we distort their essentially predicative nature; as a result, we fail to say anything determinate about concepts.[30] Analogously, in speaking of logical form as that which is represented by a proposition whose form it is, we ignore the formal role it is supposed to play in that representation, namely that of enabling the correlation of the picture and what it depicts.

Given the ambiguities that underly it, Representability cannot truly be taken to specify any determinate possibility that we should affirm or deny, and its negation in (E) is just as indeterminate as it. Both are nonsense, in the specific sense that we have failed to assign the signs they involve a determinate meaning (TLP 6.53), "even if we believe that we have done so" (5.4733). Indeed, the indeterminacy in the use of the expressions "logical form" and "representation" casts doubt on the intelligibility of the other principles on which the argument depends—Common Form, Outsideness and Universality. In framing them, too, our words did not succeed in saying anything, either rightly or falsely. The *reductio* fulfills its role as soon as it dispels the illusory appearance that they did.

This construal of the Tractarian *reductio* neatly exemplifies the kind of elucidatory activity that 4.112 and 6.54 describe as the method of the book, and it is in this respect that my interpretation speaks in favor of the resolute reading of the *Tractatus*, and against its standard reading. The standard readers of the Tractarian *reductio* construe its upshot as the denial of an intelligible but false assumption, namely that logical form can be represented. The *reductio* proves, for such readers, that attempts to represent logical form would fail, and that the expression of such failure would consist in nonsense; but it is not nonsensical, according to standard readers, to say of these failed expressions what it is they fail in, namely the representation of logical form.[31] By contrast, the Tractarian *reductio*, on my construal, and in line with the resolute reading, might initially seem to its reader to rely on substantive premises from which one then derives a conclusion, but it is ultimately meant to lead the reader to realize that the premises which they took to be meaningful lacked a determinate sense. They are nonsense, on this construal, not because they manage to determinately refer to logical form and fail to say something which cannot be said of it, but rather because there is no determinate "it" that they say anything about.[32] In other words, the goal that philosophers such as Frege and Russell have set themselves, to represent logical form, was no goal at all—even if their language allowed them to frame phrases that gave the appearance that it were. But since nothing was determinately meant by such expressions, nothing substantive is being denied when these expressions are rejected as nonsensical; just as we do not feel deprived of anything when we are told that we cannot "put an event into a hole".[33]

Indeed, standard readers take Wittgenstein to hold that his own elucidatory propositions, despite their nonsensicality, are not "mere nonsense", but nonsense which somehow conveys a

---

[30] See Frege 1984a 186 ff.; and cf. TLP 4.126-4.1272.

[31] E.g. Geach 1976; Hacker 2001a; Williams 2004 and Nordmann 2005.

[32] Resolute readers similarly reject the idea that we can determinately say what it is of which something cannot be said, and that anything determinate is left of the idea of logical form once we have overcome our initial confusion; see Diamond 1991b 181 and 198, Kremer 2001 42, and Conant and Diamond 2004 65.

[33] This example of nonsense is discussed in Wittgenstein 1984 108.

determinate content, namely that which it tries to say, but cannot.[34] Without this, it seems to the standard reader, it would not be possible to explain how the author of the *Tractatus* could hope to achieve *any* philosophical aim whatsoever by means of his nonsensical elucidations. This assumption underlies the way Peter Hacker, the main proponent of the standard reading, responds to the resolute reading. Hacker seems to think that the resolute readers themselves take Wittgenstein to advance an argument whose conclusion is that elucidatory expressions are mere nonsense and must be thrown away; he then objects that such an argument would depend for its success on premises that are conveyed by Wittgenstein's elucidatory propositions, i.e. by means of expressions which the resolute readers take Wittgenstein to ultimately repudiate, but which cannot be thrown away if such an argument is to succeed (Hacker 2001a 113). Hacker here assumes that there is nothing else that elucidations might be taken to do, apart from supplying premises for an argument of this sort. But this mischaracterizes the way the resolute reading conceives of the work of elucidation.

Cora Diamond argues, in response to Hacker's objection, that the *effect* which the *Tractatus* strives to achieve—the effect which I here described in terms of the transformation of the language of the philosopher—does not relate to the elucidations that bring this effect about in the same way that a conclusion of an argument relates to the premises from which it follows; clarity, once achieved, is not undermined by throwing away the ladder of unclear propositions which led to it (Diamond 2014 17). The alternative construal of Wittgenstein's *reductio* that I propose helps clarify how elucidations do that. The steps in the *reductio* draw on the language which they ultimately lead us to transform, and to that extent they themselves involve indeterminate uses of language. The apparent contradiction the *reductio* exposes manifests the defects of our current use language; and once we transform that language and overcome those defects we may come to see the argument itself as defective. But in realizing this and in throwing the argument away, along with the philosophical expressions whose nonsensicality the argument made manifest, we would not be repudiating the clarity that we achieved—rather, we would thereby reaffirm it.

8. Conclusion

The true force of Wittgenstein's *reductio*, on the reading proposed in this paper, does not consist in refuting the assumption that logic can be represented, on the basis of other assumptions which are grounded in ineffable insights. Rather, its force consists in making manifest the need for a thoroughgoing transformation of the philosophical language within which all these assumptions seem to make sense. Once this transformation is complete, the question concerning the representability of logical form would no longer seem relevant, and the appearance that in order to answer it we must rely on ineffable insights into the nature of logic and representation would equally lose its force. Of course the very expression "representation of logical form" might then go on to acquire a different, benign meaning. But if it does, that meaning would no longer be the one that Frege, Russell and Wittgenstein's standard readers expect this phrase to have.

We can see the modern semantic approach to logic as one way in which such a transformation of Frege's and Russell's philosophical language can be achieved; a distinctive feature of this modern approach is that it severs the internal connection between logic and thought which

---

[34] See e.g. Hacker 2001a 111 and esp. 140: "What one *means* when one tries to state these insights is perfectly correct, but the endeavour must unavoidably fail."

informs, in various ways, Frege's, Russell's and Wittgenstein's work. We must be wary, however, of treating the modern semantic approach as the only, necessary consequence of the history which preceded it. Indeed, the historical development of modern logic can be fruitfully compared with the kind of scientific revolutions with which Kuhn (1962) is concerned; this is what van Heijenoort seems to point to, in saying that modern logical theory evolved by making a "sharp break with the Frege-Russell approach to the foundations of logic" (1967 328). Whether it might still be possible for us nowadays to conceive of logic in terms of its role for thought, and yet neither to equivocate in our use of these words nor to appeal to mysterious intuitions of the ineffable, remains an open question. My hope in this paper has been to make clear that this was the issue with which Wittgenstein was concerned.

Acknowledgements

I presented previous versions of this paper at the University of Chicago, the University of Leipzig and the University of Jena. I thank the participants of these events for helpful comments. I am especially indebted to Michael Kremer, Jim Conant, Cora Diamond, Tyke Nunez, David Löwenstein, Steven Methven, Wim Vanrie, Gino Margani, Martijn Wallage, Nethanel Lipshitz, Amichai Amit, and Abigail Akavia. Work on this paper was supported by a Minerva Fellowship of the Minerva Stiftung Gesellschaft für die Forschung mbH.

References

Anscombe, G.E.M. 1959. *An Introduction to Wittgenstein's Tractatus*. London: Hutchinson.

Black, Max. 1964. *A Companion to Wittgenstein's Tractatus*. Cornell University Press.

Blanchette, Patricia A. 2013. "From Logicism to Metatheory." In *The Palgrave Centenary Companion to Principia Mathematica*, edited by Bernard Linsky and Nicholas Griffin. Basingstoke: Palgrave Macmillan Press, 59-78.

Bronzo, Silver, 2012. "The Resolute Reading and Its Critics: An Introduction to the Literature." *Wittgenstein-Studien* 3 (1): 45-80.

Carnap, Rudolph, 2000. *Logical Syntax of Language*, translated by Ametha Smeaton. London and New York: Routledge.

Conant, James, 1992. "The Search for Logically Alien Thought: Descartes, Kant, Frege, and the Tractatus." *Philosophical Topics* 20 (1): 115-80.

———, 2007. "Mild-Mono Wittgensteinianism" In *Wittgenstein and the Moral Life: Essays in Honor of Cora Diamond*, edited by Alice Crary. Cambridge, Mass: MIT Press, 174-217.

Conant, James and Cora Diamond, 2004. "On Reading the *Tractatus* Resolutely." In *Wittgenstein's Lasting Significance*, edited by Max Kölbel and Bernhard Weiss. London: Routledge, 42-97.

Davidson, Donald, 1974. "On the Very Idea of a Conceptual Scheme." *Proceedings and Addresses of the American Philosophical Association*, 47: 5-20.

Diamond, Cora, 1991a. "What Nonsense Might Be." In *The Realistic Spirit: Wittgenstein, Philosophy, and the Mind.* Cambridge, MA: MIT Press, 95-114.

———, 1991b. "Throwing Away the Ladder." In *The Realistic Spirit*, 179-204.

———, 1991c. “Riddles and Anselm’s Riddle.” In *The Realistic Spirit*, 267-289.
———, 2000. “Ethics, Imagination and the Method of the Tractatus.” In *The New Wittgenstein*, edited by Alice Crary and Rupert J. Read. London: Routledge, 149-173.
———, 2011. “The *Tractatus* and the Limits of Sense”, in *The Oxford Handbook of Wittgenstein*, edited by Oskari Kuusela and Marie McGinn, eds., Oxford University Press, 238-273.
———, 2014. “Addressing Russell Resolutely” in *Philosophical Topics* 42:2, 13-43.
———, 2019. *Reading Wittgenstein with Anscombe, Going On to Ethics*. Harvard University Press.
Floyd, Juliet, 1995. “On Saying What You Want to Say: Wittgenstein, Gödel and the Trisection of the Angle.” *From Dedekind to Gödel: The Foundations of Mathematics in the Early Twentieth Century,* edited by Jaakko Hintikka. Dordrecht: Kluwer Academic Publishers, 373-426.
Frege, Gottlob, 1960. *The Foundations of Arithmetic*, translated by John L. Austin. Second revised edition. New York: Harper Torchbooks.
———, 1979a. “Logic” in *Posthumous Writings*, edited by Hans Hermes et al., translated by Peter Long and Roger White. Chicago: University of Chicago Press, 1-8.
———, 1979b. “Boole’s Logical Calculus and the Concept Script.” In *Posthumous Writings*, 9-46.
———, 1979c. “Logic in mathematics.” In *Posthumous Writings*, 203-50.
———, 1980. *Philosophical and Mathematical Correspondence,* edited by Gottfried Gabriel et al., translated by Hans Kaal. Chicago: University of Chicago Press.
———, 1984a. “On Concept and Object.” In *Collected Papers on Mathematics, Logic, and Philosophy*, edited by Brian McGuinness, translated by Peter Geach. Oxford: Blackwell, 182-194.
———, 1984b. “Thoughts.” In *Collected Papers on Mathematics, Logic, and Philosophy*, translated by Peter Geach and R.H. Stoothoff, 351-372.
———, 1997. “On *Sinn* and *Bedeutung.*” In *The Frege Reader*, edited and translated by Michael Beaney. London: Blackwell, 151-171.
———, 2013. *Basic Laws of Arithmetic,* translated by Philip. A. Ebert and Marcus Rossberg. Oxford: Oxford University Press.
Geach, Peter T. 1976. “Saying and Showing in Frege and Wittgenstein.” *Acta Philosophica Fennica* 28: 54-70.
Goldfarb, Warren, 1982. “Logicism and Logical Truth.” *Journal of Philosophy* 79 (1): 692-695.
Hacker, Peter M. S., 2001a. “Was He Trying to Whistle It?” In *Wittgenstein: Connections and Controversies*. Oxford: Clarendon Press, 98-140.
———, 2001b. “When the Whistling Had to Stop.” In *Wittgenstein: Connections and Controversies*. 141-69.
Heck, Richard G., 2007. “Frege and Semantics.” *Grazer Philosophische Studien* 75 (1): 27-63.
Hylton, Peter, 1990. *Russell, Idealism, and the Emergence of Analytic Philosophy.* New York: Oxford University Press.
———, 2005. “Logic in Russell’s Logicism.” In *Propositions, Functions and Analysis: Selected Essays on Russell’s Philosophy.* Oxford: Oxford University Press, 49-82.

Johnston, Colin, 2017. “The Picture Theory.” In *A Companion to Wittgenstein*, edited by Hans Johann Glock and John Hyman. Chichester: John Wiley & Sons, 141-158.
Korhonen, Anssi, 2012. “Logic as a Science and Logic as a Theory: Remarks on Frege, Russell and the Logocentric Predicament.” *Logica Universalis* 6 (3–4): 597-613.
Kremer, Michael, 2001. “The Purpose of Tractarian Nonsense.” *Noûs* 35 (1): 39-73.
———, 2002. “Mathematics and Meaning in the *Tractatus.*” *Philosophical Investigations* 25 (3): 272-303.
Kuhn, Thomas, 1962. *The Structure of Scientific Revolutions.* Chicago: University of Chicago Press.
Kuusela, Oskari, 2019. *Wittgenstein on Logic as the Method of Philosophy: Re-examining the Roots and Development of Analytic Philosophy*. Oxford: Oxford University Press.
Lützen, Jesper 2009. “Why was Wantzel overlooked for a century? The changing importance of an impossibility result.” *Historia Mathematica* 36: 374–394.
Narboux, Jean-Philippe, 2014. “Showing, the Medium Voice, and the Unity of the *Tractatus.*” *Philosophical Topics* 42 (2): 201-62.
Nordmann, Alfred, 2005. *Wittgenstein's* Tractatus*: An Introduction*. Cambridge: Cambridge University Press.
Proops, Ian, 2007. “Russell and the Universalist Conception of Logic.” *Noûs* 41 (1): 1-32.
Ricketts, Thomas, 1985. “Frege, The Tractatus, and the Logocentric Predicament.” *Noûs* 19 (1): 3-15.
Russell, Bertrand, 1903. *The Principles of Mathematics.* Cambridge: Cambridge University Press.
———, 1905. “On Denoting.” *Mind* 14 (56): 479-93.
———, 1960. “Introduction.” In Wittgenstein 1960 7-23.
———, 1973. “Meinong’s Theory of Complexes and Assumptions.” In *Essays in Analysis*, edited by Douglas Lackey. London: Allen and Unwin, 21–76.
———, 2013. *Theory of Knowledge: The 1913 Manuscript.* Edited by Kenneth Blackwell and Elizabeth Ramsden Eames. London and New York: Routledge.
Stanley, Jason, 1996. “Truth and Metatheory in Frege,” *Pacific Philosophical Quarterly* 77: 45-50.
Sullivan, Peter, 2001. “A Version of the Picture Theory.” In *Ludwig Wittgenstein. Tractatus logico-philosophicus*, edited by Wilhelm Vossenkuhl. Berlin: Akademie Verlag.
———, 2004. “What is the *Tractatus* about?” In *Wittgenstein's Lasting Significance*, edited by Max Kölbel and Bernhard Weiss. London: Routledge, 28-41.
Tappenden, Jamie, 1997. “Metatheory and Mathematical Practice in Frege.” Philosophical Topics, 25 (2): 213-163.
———, 2000. “Frege on Axioms, Indirect Proofs, and Independence Arguments in Geometry: Did Frege Reject Independence Arguments?” *Notre Dame Journal of Formal Logic* 41 (3): 271-315.
Tarski, Alfred, 1944. “The Semantic Conception of Truth: And the Foundations of Semantics.” *Philosophy and Phenomenological Research* 4 (3): 341-376.
van Heijenoort, Jean, 1967. “Logic as Calculus and Logic as Language.” *Synthese* 17 (1): 324-330.

Weiner, Joan, 2005. “Semantic Descent.” *Mind* 114: 324-54.
White, Roger, 2006. *Wittgenstein’s Tractatus Logico-Philosophicus: A Reader’s Guide*. London: Continuum.
Williams, Meredith, 2004. “Nonsense and Cosmic Exile.” In *Wittgenstein’s Lasting Significance*, edited by Max Kölbel and Bernhard Weiss. London: Routledge, 1-27.
Wittgenstein, Ludwig, 1960. *Tractatus Logico-Philosophicus*, translated by C. K. Ogden and F.P. Ramsey. London: Routledge & Kegan Paul.
———, 1974. *Philosophical Grammar*, edited by Rush Rhees, translated by Anthony Kenny. Oxford: Blackwell.
———, 1975. *Philosophical Remarks*, edited by Rush Rhees, translated by Roy Hargreaves and Roger White. Oxford: Blackwell.
———, 1976. *Lectures on the Foundations of Mathematics, Cambridge 1939*, edited by Cora Diamond. Ithaca: Cornell University Press.
———, 1978. *Remarks on the Foundations of Mathematics,* Third Edition, edited by Georg H. Von Wright, Rush Rhees and G. E. M. Anscombe, translated by G.E.M. Anscombe. Oxford: Blackwell.
———, 1984 *Notebooks, 1914-1916*, edited by Georg H. Von Wright and G. E. M. Anscombe, translated by G.E.M. Anscombe. Chicago: University of Chicago Press.
———, 2009. *Philosophical Investigations*, Fourth Edition, edited by Peter M.S. Hacker and Joachim Schulte, translated by G.E.M. Anscombe, P.M.S. Hacker and Joachim Schulte. Oxford: Blackwell.
Zalabardo, Jose, 2015. *Representation and Reality in Wittgenstein’s* Tractatus. Oxford: Oxford University Press.