The Spanish Mathematician Ventura Reyes Prósper and his connections with Charles S. Peirce and Christine Ladd-Franklin

Jaime Nubiola and Jesús Cobo

jnubiola@unav.es
Department of Philosophy
University of Navarra
E-31080 Pamplona, Spain

This is a previously unpublished paper describing the relations between the almost unknown Spanish mathematician Ventura Reyes Prósper (1863-1922) with Charles S. Peirce and Christine Ladd-Franklin. Critical feedback and new details about those connections would be greatly appreciated and duly acknowledged in subsequent versions.

Two brief papers from Reyes Prósper published in *El Progreso Matemático* 12 (20 December 1891), pp. 297-300, and 18 (15 June 1892) pp. 170-173 on Ladd-Franklin, and on Peirce and Mitchell, respectively, are translated for first time into English and included at the end of the paper.

Introduction[1]

The Spanish mathematician Ventura Reyes y Prósper (1863-1922) is one of the most important figures in the spread of information about mathematical logic in the late 19th century[2]. Despite being mentioned by C. I. Lewis in the final bibliography of a *A Survey of Symbolic Logic*[3], and by Alonzo Church in his bibliography of *Journal of Symbolic Logic*[4], almost nothing was known in the United States about this surprising mathematician of Toledo, Spain, who corresponded with some of the best mathematicians of his times and silently faded away. His work has also been totally neglected in Spain. This situation started to change with the excellent study on Reyes Prósper of Juan Antonio del Val in *Teorema*, which was reviewed by Jorge J. E. Gracia in the United States[5], and especially with the recent monograph of Jesús Cobo[6].

The aim of this paper is two-fold: On the one hand, to provide a general presentation of Ventura Reyes Prósper and of some remaining evidence of his connections with Charles S. Peirce and Christine Ladd-Franklin; on the other hand, to present the first English translation of the papers that Reyes dedicated to Ladd-Franklin in 1891 and to Peirce and his student Oscar H. Mitchell in 1892 in the pioneering Spanish mathematics journal *El Progreso Matemático*.

The first of these two papers, "*Cristina Ladd Franklin. Matemática americana y su influencia en la lógica simbólica*" (Christine Ladd Franklin, American Mathematician

and her Influence in Symbolic Logic), is the only non-American reference in the scarce bibliography about her work in logic[7]. Having studied with Charles Peirce in Johns Hopkins, Christine Ladd-Franklin was the first American woman to become involved in symbolic logic, and when she turned her attention to psychology, also the first American woman to become an outstanding experimental psychologist[8]. The second paper highlights Oscar Howard Mitchell (1851-89), Peirce's gifted student, whose life and logical contributions were presented by Randall Dipert in 1994[9]. Dipert started to prepare, with the help of H.-N. Castañeda, an English translation of that paper, but it was never published. The publication in the *Arisbe Website* of this new English translation can be seen as an eulogy from Spain for this American logician who died in the prime time of his youth. Reyes laments: "It is a great pity that a scholar like Mitchell, of whom so much could be expected, passed away at 38 years of age".

An introduction to Ventura Reyes Prósper

Ventura Reyes Prósper has been increasing in importance over time. A view of Reyes Prósper as one of the outstanding Spanish scholars of his time has supplanted the early image of him, contradictory and picturesque, that highlighted some mysterious aspects of his life. Reyes Prósper was born on May 31, 1863, in Castuera (Badajoz, Spain), to a family of middle class. He was remarkable in his secondary studies and brilliant in the University of Madrid where he studied Natural Sciences between 1879 and 1883. On 1885 Reyes received his Ph. D. with honors. His dissertation, published in 1886, was the first systematic catalogue of the birds of the Iberian Peninsula and the Balearic Islands[10].

Reyes had a vast intellectual curiosity; he was eager to know and to learn. His fondness for mathematics was developed during his university years autodidactically, reading foreign mathematical books and journals especially those from Germany. He always exhibited a flair for languages, and this enabled him to start during those same years copious correspondence with scientists of different countries. He asked them questions, asked for news and communicated to them his discoveries. He corresponded at least with Blasius, Georg Cantor, Marcel Dieulafoy, Henri Douvillé, Von Hayck, Aemilius Hübner, W. E. Johnson, Bray Kempe, Dairoku Kikuchi, Felix Klein, Christine Ladd-Franklin, Hugh MacColl, Joseph Murphy, Albino Nagy, Moritz Pasch, Giuseppe Peano, Charles S. Peirce, Ernst Schröder, Adolf Schulten, Joseph Marie de Tilly, John Venn, Andreas Voigt and the brothers Hermann and Otto Wiener[11]. In the poor, but pretentious, mathematical Spanish atmosphere of his time, still subdued to the most retrograde derivations from French rationalism, Reyes was a rare specimen, a science freelancer and a lonely man.

Soon he became excited by the study of non euclidean geometry, totally unknown at this time in Spain. On December, 1886 he wrote a short note "Sur la géométrie non-euclidienne" that was published in the prestigious *Mathematische Annalen* of Leipzig[12]. In spite of the title, the note —as A. Bernalte has pointed[13]— was a contribution to the projective geometry, related to the properties of the harmonic quatern.

During the summer of 1887 Reyes Prósper made a trip to Germany with his brother Eduardo, a learned botanist who later held a chair of Botany at the University of Madrid and was also the director of the Jardín Botánico of that city.

According to Del Val, Reyes met the mathematicians Ferdinand Lindemann and Felix Klein in that journey[14]. It is certain that he met Hermann A. Schwarz[15], and it seems very probable that he visited Moritz Pasch in Giessen. As is well known, in these decades German universities were the main destination of academic pilgrimage for scholars of other European countries and of the United States. In fact the two American correspondents with Reyes Prósper visited Germany. Peirce visited Berlin several times in his European sojourns, and Ladd-Franklin spent her husband's sabbatical year researching the theory of color at Göttingen and Berlin (1891-92), and returned in 1894 and in 1901-02[16].

Reyes' second paper on mathematics was extracted by *Mathematische Annalen* from a letter of him to Moritz Pasch, dated in Madrid in 1888. The title given to the paper was "Sur les propriétés graphiques des figures centriques"[17]. In fact it is Reyes' most important mathematical paper and his greatest contribution to geometry. In that paper Reyes gave a proof of Desargues' theorem[18], also referred to as the theorem on homologous triangles, in such a ingenious way that it seemed to Pasch "the simplest imaginable"[19]. Pasch included Reyes' proof of Desargues' theorem in the Spanish edition of his *Vorlesungen über neuere Geometrie*, trying, in spite of the previously quoted words, to simplify it even more, but, in fact, obscuring it and lessening its original elegance[20]. Pasch always acknowledged his debt to the beautiful proof of Reyes Prósper. In 1926, forty years after Reyes' letter on Desargues theorem and three years after the death of Reyes, Pasch wrote in the foreword to the second German edition of his *Vorlesungen:* "One of the main goals that I tried to achieve in its time with this book was this: the introduction of points, straight lines and planes were inappropriate to lay the foundations of pure geometry. The ideas I put to work to get this aim were very complicated, but later I was able to simplify them considerably employing an ingenious idea from Reyes y Prósper (1888)"[21].

On 1891 the first Spanish mathematical journal was founded in Zaragoza with the name *El Progreso Matemático.* It was published by professor Zoel García de Galdeano, an enthusiastic popularizer of mathematics in Spain. In the first series of this journal, which lasted until 1895, Reyes Prósper contributed a collection of papers, usually brief, that were largely fruits of the knowledge acquired through his vast correspondence. In the stale atmosphere of the Spanish science of his time, typically characterized by the rarity and inconsistency of creative attitudes and by an almost exclusively focus on outdated problems, the fourteen papers from Reyes Prósper in *El Progreso Matemático* are surprisingly fresh. They exhibit noteworthy scientific qualities: wit, simplicity and elegance[22], based on first hand information of the forefront of mathematics. However, these papers are scarcely creative, and the author's purpose seems to be, above all, to popularize and teach. The papers can be classified in three groups according to subject: I) Geometry, with seven papers; II) Logic, six papers; and III) Arithmetic-Logic, only one paper[23]. As Gracia noted, these paper "furnish evidence as well of Reyes y Prósper's thorough acquaintance with the logical literature of the time and of the personal contacts he maintained with eminent logicians such as Peirce, Peano, Schröder and others"[24]. With these papers Reyes Prósper become the first promoter and popularizer in Spain of the modern studies of formal logic, but his effective influence seems to have been very limited in scope[25].

Besides the two journals already mentioned, Reyes Prósper, along his career as professor in schools of secondary studies (Teruel, 1891-92; Cuenca 1892-98; Toledo, 1898-1922), published also in *Archivo de Matemáticas, Revista de la Real Academia de Ciencias de Madrid, Revista de la Sociedad Matemática Española, Revista Matemática*

Hispano-Americana, Bulletin de la Société Physico-Mathématique de Kasan, Bulletin de Mathématiques Élémentaires, Mathesis, and *The Educational Times.* His rate of publications gradually decreased and the character of his writings leaned more and more towards the history and bibliography of mathematics. The receptive and understanding side of his scientific personality was totally open, but the creative side resulted in laziness. It is surprising that a person so well prepared in mathematics, with fondness and scientific qualities, having published before being 25 years-old in prestigious foreign journals, able to maintain a copious reading of mathematical novelties and an active correspondence with well known scientists of his time, suffered so big apathy that prevented him to launch projects of greater scope. His papers, most of them interesting, give only a hint of his ability and his talent. In this laziness seems to be hidden the key of the deepest contradictions of his complex character. Reyes was a good-natured man, extremely cultured, capable of pleasant conversation, but pusillanimous and untidy. He dragged along a lonely and eccentric life until his death in Madrid on 27 November, 1922.

Letter to Peirce

The reading of Ernst Schröder's *Der Operationskreis des Logikkalkuls* (Leipzig, 1877) produced a "very big impression"[26] upon Reyes Prósper and it was the source of his interest in the studies of formal logic. It is not clear whether Reyes visited Schröder during his journey to Germany in 1887, but in any case the works of the German logician and the correspondence with him seem to have been the primary source of his information about the logic scene of his time, in particular, about the American logicians and what he used to call "the circle of Baltimore".

Reyes, eager of news, started probably in 1889, a correspondence with Christine Ladd-Franklin, whose first letters are not known to us. It was probably Ladd-Franklin who provided wide and detailed information about the logicians from John Hopkins, and specially about Charles S. Peirce, who had been her professor of logic there. On the 5th of March, 1890 Reyes Prósper wrote a concise and very respectful letter to Peirce, which is the only one remaining from a probably larger correspondence[27]. The letter, whose original is in French, says the following[28]:

Mr. Charles S. Peirce

Sir:

Having started the study of the Algebra of Logic and having already in my power your two Memoirs included in the *American Journal of Mathematics*, vol. III and vol. VII[29], and willing to get your other writings on Mathematical Logic, I am informed by the references that Mad. Christine Ladd Franklin has communicated to me that probably it has been published a volume under the title *Contributions to Logic* with a reprinting of your works.

I hope you will condescend to clarify this point, or to order to send me that book or other memoirs if copies are remaining; I address to you asking my apologies.

Accept you the expression of my deepest respect.

Dr. Ventura Reyes Prósper

My address is Calle de San Bernardo, nº 56 2º derecha Madrid

Madrid 5 of March of 1890.

Although Peirce's answer is not known –and it, as well as the rest of the correspondence received by Reyes, should be considered lost, perhaps definitively– it is certain that some letter in response existed. Reyes mentioned Peirce regarding logicians who he thanked for their information and materials sent[30]. For his part, Peirce, in a review of Schröder, writes that he is informed about Reyes's project of translating into Spanish the *Vorlesungen über die Algebra der Logik (Exakte Logik)*[31]. There is also the evidence of the two copies of the small volume *Studies in Logic by Members of the Johns Hopkins University*, edited by Charles Peirce in 1883[32], a copy of the Circular of John Hopkins University, number 15, of May, 1882, and several offprints from Peirce that are kept in the private library of Reyes located at present in the Biblioteca de Matemáticas del Consejo Superior de Investigaciones Científicas (Madrid, Spain).

In the papers on logic published by Reyes Prósper in *El Progreso Matemático*, his references to Peirce are constant. Since the first paper, "Machine Ratiocination", where Peirce is called "venerable master"[33], until the "Project of Classification of the Logic-Symbolic Writings, Specially the Postboolean", in which Reyes establishes a Peircean group, made by "the publications from Grassmann, Hugh McColl, part of the recent work of Peirce and his students of the school of Baltimore (with the exception of Mitchell), and the ones from the wise professor of Karslruhe Mr. Schröder, later to his *Operationskreis des Logikkalkuls*, and the writings of Giuseppe Peano"[34]. In that same article Reyes included the early writings of Peirce in the "Boolean group", and besides, he considered other two groups related with the American logician: that of the relatives, in which are to be included the "most beautiful memoirs" of Peirce, and "the group of link between the logic of the absolute and the logic of the relatives", field in which Peirce had reached the logic culmination, following, and improving, the methodology devised by his student Oscar H. Mitchell[35].

For Reyes Prósper, modern logic started with George Boole. Coming from Boole's "gigantic work", two vigorous schools had been developed at the end of the century: the American school, "of so very useful and witty enterprise", centered in Peirce and connected partly with British logicians as Hugh MacColl, and the German one, whose capital figure was Schröder. The distinction between both "schools" was for Reyes mainly methodological: "There is an essential difference between the treatment of logic between MacColl and Peirce, for one side, and Schröder in the other side. The first grounds the calculi of identities on the propositional calculi. The method of Schröder is more general"[36].

Correspondence with Ladd-Franklin

When in August of 1891 Reyes wrote his first paper for *El Progreso Matemático*, it is certain that he already corresponded with "the charming figure of Christine Ladd-Franklin"[37]. Probably most of Reyes' news and information about the school of Baltimore that he mentioned very often in his papers has its source in Ladd-Franklin letters. In fact, the opening remark of Reyes' paper on Ladd-Franklin in December 1891 shows that he is well informed that "Christine Ladd [...] by chance is at present

in Europe" and throughout the text acknowledges his debt to Ladd as his source of information.

Christine Ladd had studied at Johns Hopkins from 1878 to 1882, in spite of the fact that the University was closed to women, thanks to the influence of the eminent mathematician James J. Sylvester. Her stay at Johns Hopkins turned her attention first to logic and later to visual processes: "It was the lectures of that splendid though erratic genius, Charles S. Peirce, that aroused her interest in logic"[38]. She wrote a dissertation on an original method for reducing all syllogisms to a single formula, which she called the "antilogism" or the "inconsistent triad" of propositions, that appeared in *Studies in Logic by Members of the Johns Hopkins University*. Although Ladd had completed all requirements for a Ph. D., as Johns Hopkins would not award degrees to women, she did not receive the degree until 1926 at the Johns Hopkins semicentennial. She married on August, 1882 with her instructor Fabian Franklin, who stayed in Johns Hopkins as professor of mathematics until 1895 when he became editor of the Baltimore *News*. They had two children: the first died in infancy, and the second was a daughter named Margaret. After Ladd-Franklin left Johns Hopkins in 1882, she continued to publish in logic and theory of color vision. Her interest in this area had began with her investigation in 1886 on the nature of the horopter, a mathematical question concerning binocular vision, and spread throughout her life.

Regrettably, most of the correspondence between Reyes and Ladd —probably initiated in 1889 and continued at least until May 1896— seems lost. Nevertheless, in the papers of Christine Ladd and her husband Fabian Franklin, given in 1962 to the Columbia University Libraries, there are kept three letters from Reyes addressed to Ladd-Franklin in 1895 and 1896[39]. These letters are written in French, in a plain and correct style. Their texts are brief and laconic, but they suggest cordiality manifested in charming details such as the interchange of some small gifts (drawings, postcards) and in personal confidences of some intimacy (health, family events).

The first of these letters, dated in Cuenca the 10th of April of 1895, seems to have been written after a long break in their correspondence due to family reasons and health problems. Reyes' interest in formal logic had probably diminished since 1893, but had not extinguished totally. In fact, he did not finish his translation into Spanish of Schröder's *Vorlesungen* —or perhaps he was not able to find a way to publish it—, and he never again wrote an article popularizing logic after his "*La lógica simbólica en Italia*"[40] published in February of 1893. Curiously, it ended by expressing Reyes' wish that the "Baltimorean logic grow in the native land of the Caesars and the martyrs". Resuming the correspondence with Christine Ladd-Franklin could perhaps mean a revival of Reyes Prósper's curiosity for symbolic logic: the first letter suggests this and the third one seems to confirm it.

The first of the three letters, translated into English, says:

Madame Christine Ladd Franklin

Madame!

I pray you to accept my excuses, because I have not answered your letter nor I have returned you the issues of <u>Mind</u>[41]. But I have suffered a lot from digestive atony, and specially I have been grieving over the death of my beloved mother. I pray you to send me

your exact address. I have sent you some stereoscopic views of Cuenca (Spain) and its outskirts for your children. Are you still involved in Logic and Mathematics? Have you published something new?

Your affectionate and sorrowful servant

Prof. Dr. Ventura Reyes Prósper
My address is: Calle Calderón de la Barca 107, Cuenca (España)
10, IV, 1895.

When this letter reached the address of the envelope (Johns Hopkins University, Baltimore, Maryland), Christine Ladd was in her second sojourn in Europe. According to the envelope the letter was sent to Paris, and from Paris to Switzerland (Bains de Montbarry, près Bulle), where Ladd-Franklin probably received it.

The second remaining letter is also dated in Cuenca on the 26th of June of 1895, two months and a half later than the previous one. Reyes Prósper begins his letter giving thanks to Christine for her answer. Probably Ladd-Franklin had answered Reyes' letter from Switzerland as soon as she had received the letter of April, 10. Reyes' letter says:

Madame Christine Ladd

Madame.

I am grateful for your letter and for sending me your very interesting work on Psychology which I have read with pleasure. I am grateful also to Miss Margaret Franklin for her drawing, and I am sending for her a bunch of painted flowers.

I wish to know your memoir about the mystical hexagram of Pascal[42].

Prof. Dr. Ventura Reyes Prósper
Cuenca (España)
26 June 95.

The "very interesting work on Psychology" to which Reyes refers is probably "*Eine neue Theorie der Lichtempfindungen*", whose offprint is kept in Reyes' library[43]. It is a translation into German of the paper that Ladd-Franklin read at the Second International Congress of Experimental Psychology in London in 1892 in which she presented a new theory on color vision giving a better account of the available set of experimental facts than the two prevailing alternative theories of her time[44]. "For some years" —wrote the expert psychologist Dorothea Hurvich in 1971— "this theory received considerable attention and support, and though later research has gone beyond the photochemistry of the retina to stress the importance of the complex neural network associated with the visual system, Dr. Ladd-Franklin's emphasis on the evolutionary development of increased differentiation in color vision still has validity"[45].

This letter, in spite of its brevity, is especially expressive for the tenderness shown by the interchange of drawings: Ladd-Franklin sends a drawing from her

daughter Margaret[46], who was then eleven years old, and Reyes corresponded with another drawing, probably painted by himself[47].

The last letter, dated also in Cuenca on the 2nd of May of 1896, confirms the permanence of Reyes' interest in logic. Translated into English, it goes:

Madame Christine Ladd Franklin

Madame!

It is a long time since I received your last letter. I have not received news from you over the past many months. I am pleased to write and to request you to inform me about new publications on Symbolic Logic, specially those of the logicians of Baltimore. Has Mr. Charles Santiago Peirce published anything new on Logic or Mathematics? Where could I find a biography of his father Benjamin Peirce?[48] Have you perhaps published your handbook of deductive and inductive logic?[49] It will be really pleasant to me to receive news of the progress in Logic.

Did you receive my letter with a postcard answering your last letter?

I thank you again for the drawing sent and I pray you to greet the little painter. Be always assured of the high esteem and consideration of your very humble servant.

Dr. Ventura Reyes Prósper
Cuenca (España) 2, V, 1896.

The content of this letter suggests that Christine Ladd had not written again to Reyes since the spring of 1895. If she did not answer to this letter of Reyes, that letter should be considered as the end of the correspondence between them. With this Reyes would have lost his main source of information about American logic, and at the same time a powerful stimulus to his lonely adventure through the field of symbolic logic.

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Below are the translation into English of the papers that Ventura Reyes Prósper published in *El Progreso Matemático* 12, pp. 297-300 (20 December 1891), and 18, pp. 170-173 (15 June 1892) on Ladd-Franklin, and on Peirce and Mitchell, respectively. Minor misspellings and errors are corrected silently. Footnotes are from Reyes Prósper.

CHRISTINE LADD FRANKLIN

AMERICAN MATHEMATICIAN AND HER INFLUENCE ON SYMBOLIC LOGIC

Christine Ladd (now the wife of Mr. Franklin, professor of Mathematics in Johns Hopkins University, Baltimore) who by chance is at present in Europe, is one of the most distinguished disciples of the renowned logician Charles Santiago Peirce, a former lecturer of the aforementioned University.

Since some years ago this lady has been concerned a lot and brilliantly with Geometry, Algebra, Logic, Physiology and Instruction. We are going to review very quickly the meritorious works of such a distinguished author.

In a beautiful memoir aimed to elucidate certain points of view of the great English logician and mathematician Augustus De Morgan, she already initiated her preferences for logic studies, so much that, when indicating that the meaning of the word function in her text is much broader than the commonly accepted, she introduces the relatives as an example of this. So beautiful work was included in the *American Journal of Pure and Applied Mathematics*, vol III, pp. 209 to 226, and some analysis of this work are to be found in several mathematics journals and magazines, like for example in the *Bulletin des Sciences Mathématiques et Astronomiques*, published by Darboux in Paris. The original notations which Christine proposes in her memoir have been adopted afterwards in the memoir published later by Peirce with the title *Algebra of Relatives*.

During the year 1883, Santiago Peirce edited his students' work and his own in a small but beautiful book with the title *Studies in Logic by Members of the Johns Hopkins University*. Since then Christine Ladd has been devoting herself, uninterruptedly it seems, to the calculus ratiocinator. One of the articles included in the small book with the title *On the Algebra of Logic* is written by her. The works of George Boole had run out of print and were now somewhat out-dated, thus there was a need for the reader not expert in the subject a very brief exposition of modified Boolean theories[1] following the example of what in Germany did the famous professor Schröder, a learned scientist whom we think also to deal with. However, Mrs. Ladd's work is not only an exposition of facts already known, but also she introduces very interesting novelties thanks to her inventive ability. Boole treated Logic dealing with its problems by means of equations, what, besides the complexity of the copula "equals", was also presenting several other inconveniences. All these led Santiago Peirce to decide to use implications, that is subsumptions. For her part, Christine treated the problems by means of «exclusions», adopting a copula different from which was already used, including the ones which Morgan had indicated. This has some advantages in some determined cases with respect to the other used methods.

In this regard, I have to say that whatever mathematical treatment of a logical given problem will just be a repetition in a different symbolic language, of the facts that for a same problem are always the same. Varying the method the symbolic language is changed. What has been written in a different way is simply translated into the new one, just like a polyglot book, whose content is always the same. Boole's Logic, Ladd's, Mitchell's, Peirce's or Schröder's are absolutely one and the same; only the signs used and the grammar of the signs change. This is because the origin of all these laws, as was pointed out by Benjamin Peirce in one of his works, is the «divine source of all geometry», which is immutable, inflexible and eternal.

To the copula of exclusion also corresponds the negative copula, and it is very interesting to see how through these two combinations the old syllogisms, which were considered as particular cases of more complicated problems, are solved with amazing facility .

Christine Ladd does not pay attention in her short work aforementioned to certain more delicate and annoying questions which are now being debated. To the contrary, the context suggests that she considers self-evident the foundations in which she bases her development. Thus, for instance, she does not examine the fundamental part of the

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[1] That is, leaving aside the numbers and understanding logical addition as Stanley Jevons and Peirce do, as it is nowadays generally done, with the only exception of the distinguished English scholar, Mr. John Venn.

distributive laws of logical multiplication, one question which later on has triggered a debate between Santiago Peirce and Schröder. Indeed, the published book was intended more to find methods to replace the less practical methods of Boole than to clarify certain critical points of formal Logic.

It is also interesting the part of the work, which we are now reviewing, relative to the universe of discourse. An inadvertent erratum made Schröder not to pay attention on the important results there obtained, as it can be seen in a note published in *Mathematische Annalen*.

Our author proposed or solved several problems in the mathematics section of the English newspaper *The Educational Times*. However, we will not deal with these questions.

Christine Ladd's most recent works on logic which I am aware of, were published in the *American Journal of Psychology* and in *Mind*, the English journal of Psychology.

In the first of them, entitled "On Some Characteristics of Symbolic Logic" (*Am. J. of Psychology*, 1889), and I am indebted to her kindness for allowing me to read it as well as the second, she outlines the general ideas regarding the copulas and she claims for her ill-fated colleague Oscar Howard Mitchell[2] an honor which can never be fully appreciated by logicians who are not at the same time mathematicians.

The work published in the *Mind* (vol. XV, n. 57) was indeed noteworthy. The author proposes in it new notations ingeniously designed and which include all the copulas of De Morgan. If some day the use of the notations of this memoir and the methods which Santiago Peirce propounds in his paper "On the Algebra of Logic: A Contribution to the Philosophy of Notation" are generalized, this could lead to the disappearance of all analogy between logical symbols and those that are being used in other mathematical disciplines. Thus, the signs + and x could possibly be substituted by those which Mrs. Ladd uses as signs of the *complement* and *partient* copulas.

I know that perhaps one day the Franklin couple will publish a treatise on logic which may include simultaneously the deductive and the inductive logic, and I could only wish that it would be done soon, because from the clear talents of both authors, we could learn many good things.

Christine Ladd also wrote about other subjects like her research on the mystical hexagram, her *A Method for Experimental Determination of the Horopter,* and *The Education of Woman in the Southern States* included in a book edited by Annie Nathan Meyer.

Before I end, I wish to extend now my greetings to the lady and ask from her an apology for whatever errors dealing with her work I might have committed.

Prof. Dr. Ventura Reyes y Prósper

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[2] The words of Newton about professor Cotes can also be said of him: «If he had lived, we might have known something».

CHARLES SANTIAGO[1] PEIRCE
and
OSCAR HOWARD MITCHELL

I am going to present a quick review of the mathematical logic works of these such distinguished North American scholars, concerning myself with both at the same time given that each has had a reciprocal effect upon the other. The great logician Santiago Peirce was Mitchell's master, and the works of Mitchell have had, in their turn, an influence over the latest works of his notable teacher.

Charles Santiago Peirce is the son of one of the most distinguished American mathematicians, professor Benjamin Peirce, of the University of Harvard.

Both father and son have enriched science with numerous notes and works that have appeared in various scholarly publications, especially in the memoirs and proceedings of the learned societies and academies of America, having both contributed to the task of creating and of giving character to American scholarship, and freeing it from being a European tributary.

Leaving aside the works of Peirce's father, only one of which, that I am aware of, could be related to the logic of relatives (I refer to his admirable book *The Linear Associative Algebra*), I am going to indicate rapidly the merits of his son in the area of Logic, his preferred object of studies, although certainly not being unfamiliar to him many other sciences.

Santiago Peirce has obtained his instruction in Logic principally from, it appears to me, the writings of the eminent mathematician and logician Augustus De Morgan, for whom he professes such admiration that in one of his works Peirce declares that De Morgan seems to be for him (as indeed he is) one of the principal logicians ever to have existed. Peirce is the successor in scholarship of De Morgan, and his main claims to be credited are without doubt his work on the logic of relatives, logic of which the English logician is the father.

Peirce, in his earliest works titled, *On an Improvement in Boole Calculus of Logic, On the Natural Classification of Arguments, On a New List of Categories* limits himself to introduce some notable modifications in equational logic or Boolean calculus, some of which had been pointed out by Stanley Jevons, and to illustrating Aristotelian logic.

Where his eminent, scientific personality truly appears is, as I see it, in his beautiful memoir *The Logic of Relatives*, full of exceedingly interesting logical doctrine, and which has not perhaps received the attention of scholars as much as it deserves. In this memoir, published in the proceedings of the National Academy of Sciences of Washington in the year 1870, the implicative copula is introduced, after which it will be almost exclusively adopted by our author in preference to the copula *equals* which is of a more complex nature.

As the title indicates, the author is primarily concerned with the Logic of relatives, dealing very little with the Logic of the absolute. It is difficult to convey an idea of the rich content of this work and of the original and deep concepts that are therein propounded. Peirce knows how to find exceedingly curious connections between things that at first glance seem not to exist.

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[1] Although it may seem strange, his first name is in English and his second is in Spanish; I do not know why.

Peirce's most important publication is his memoir *On the Algebra of Logic* published in the *American Journal of Mathematics* vol. III, 1880. In this paper he encodes in symbols all of the ancient Logic, and afterwards establishes Boolean calculus, availing himself with great astuteness of the implicative formulas, instead of following the reverse course, that is, instead of founding calculus by means of postulates and deducing from there the syllogisms as individual cases.

The eminent German logician Schröder is of the opinion that the procedures of Peirce are perhaps similar to that of writing a grammar in the same language in which one wishes to teach, since the propositional calculus is precisely the base of his scientific building.

Peirce's work gave rise to a correspondence with Schröder in regard to the second distributive law of logical multiplication, as has been already related in my paper on the latter (*Progreso Matemático* n. 14).

It is unanimously recognized by all that the memoir that now concerns us has had the privilege of blazing a new trail in the area of symbolic Logic and marks an era in the history of this science.

Meanwhile Peirce, lecturer in Logic at Johns Hopkins University in Baltimore (for many titles renowned), had gathered around him some notable disciples, Gilman, Allan Marquand, Christine Ladd y Oscar Howard Mitchell, publishing in the year 1883, in Boston by Little, Brown and Co., the splendid little book *Studies in Logic* by members of the Johns Hopkins University, in which the joint work of master and disciples is brought together. Peirce contributed to this book with the following notes: *A Theory of Probable Inference* (splendid exposition of inductive logic). *On a Limited Universe of Marks. The Logic of Relatives* (where in part he expounds what was already stated in his other memoir of the same title, and in addition, discusses the findings of Mitchell).

Mitchell, for his part, contributed with his memoir: *On a New Algebra of Logic*, an extract of which had already been published in the *Johns Hopkins University Circulars*. The ideas of Mitchell, who upon creating the theory of the multiple dimensions of Logic, arrived at what had been until him an impossibility, that is, to link in a common body of doctrine the logic of the absolute and the logic of the relative, were adopted immediately by his master. It is a great pity that a scholar like Mitchell, of whom so much could be expected, passed away at 38 years of age in Marietta (Ohio) where he was a teacher of mathematics. I don't know if after 1883 he published anything more regarding Logic; I am only aware of some works regarding the theory of numbers.

In the *American Journal of Mathematics* vol. 7, 1884, Peirce published with the title of *On the Algebra of Logic, A Contribution to the Philosophy of Notation* a complete summary of his doctrines, employing the discovery of Mitchell, but with a different notation to that used by the inventor. In this summary the differences that exist between the exposition of questions in Logic adopted by the logicians of Baltimore and the exposition that is adopted by the German scholars, are pointed out more and more. The former prefer to reduce everything to the calculus of propositions, the latter to the calculus of classes, considered from a different point of view to that of the Americans.

Posterior to the aforementioned papers, I am not aware of anything written about Logic, except for a small note included in the number I of the *American Journal of Psychology* titled *On the Logical Machines*. There are other brief works prior to 1884 by Mr. Peirce, but I think that the most important ones are those I have already indicated.

May Mr. Peirce receive, with apologies for the mistakes which I may have committed, a testimony of sincere admiration sent by a foreigner from the other side of the oceans.

God willing, may *the splendid day soon dawn in which someone illuminates with a beam of light the many questions of the logic of relatives.*

Professor Dr. Ventura Reyes Prósper.

Madrid, the 3rd of June, 1892.

NOTES

[1] The authors are indebted with Sara F. Barrena, Vincent Larson, Alice Ramos and Juan Carlos Torchia Estrada for their help, and especially with Erlito Maraya and Kevin O'Hagan for the initial English translation of Reyes' papers. We are grateful also to an anonymous referee of the *Transactions of the Charles S. Peirce Society*, and to the Houghton Library and to Columbia University (Christine Ladd-Franklin Papers, Rare Book and Manuscript Library) for their permission to translate Reyes Prosper's letters to Peirce and Ladd-Franklin, respectively.

[2] He was the only Spaniard mentioned in the first issue of the *Transactions* as someone who had corresponded with Peirce and whose correspondence, amongst others, was sought by the then recently created Peirce Society ("Some Correspondents of Charles Sanders Peirce", *Transactions of the Charles S. Peirce Society* I (1965), p. 30). Reyes Prósper was included in that list because one letter from him to Peirce of 5th of March of 1890 was kept in the Houghton Archives and listed in the Robin Catalogue with number L 372: *Annotated Catalogue of the Papers of Charles S. Peirce*, by Richard S. Robin (Amherts, MA: University of Massachusetts Press, 1967).

[3] Clarence Irving Lewis, *A Survey of Symbolic Logic* (Berkeley: University of California Press, 1918), p. 402

[4] Alonzo Church, "A Bibliography of Symbolic Logic", *Journal of Symbolic Logic* 1 (1936), p. 143.

[5] Juan Antonio del Val, "Los escritos lógicos de Ventura Reyes y Prósper (1863-1922)", *Teorema* 3 (1973), pp. 315-354; Jorge J. E. Gracia, "Review of del Val: 'The Logical Writings of Ventura Reyes y Prósper (1863-1922)'", *Mathematical Reviews* 50 (1975), #9519. That paper was preceded by Juan Antonio del Val, "Un lógico y matemático español del siglo XIX: Ventura Reyes y Prósper", *Revista de Occidente* 12 (35), (1966), pp. 252-261.

[6] Jesús Cobo, *Ventura Reyes Prósper* (Badajoz: Departamento de Publicaciones Diputación Provincial, 1991). See also Jesús Cobo, "Ventura Reyes Prósper", in *Biografías y semblanzas de profesores. Instituto 'El Greco' de Toledo (1845-1995)*, edited by I. E. S. "El Greco", Toledo, 1999, pp. 213-219.

[7] Judy Green, "Christine Ladd-Franklin (1847-1930)", in *Women of Mathematics: A Biographic Source Book*, Louise. S. Grinstein and Paul J. Campbell, eds. (Westport, CN: Greenwood Press, 1987), p. 128.

[8] Thomas C. Cadwallader and Joyce V. Cadwallader, "Christine Ladd-Franklin (1847-1930)", in *Women in Psychology: A Bio-bibliographic Sourcebook*, Agnes N. O'Connell and Nancy F. Russo, eds. (New York: Greenwood Press, 1990), p. 222.

[9] Randall R. Dipert, "The Life and Logical Contributions of O. H. Mitchell, Peirce's Gifted Student", *Transactions of the Charles S. Peirce Society* XXX/3 (1994), pp. 515-542.

[10] Ventura de los Reyes y Prósper, "Catálogo de las aves de España, Portugal e Islas Baleares",

Anales de la Sociedad Española de Historia Natural XV, 1 (1886), pp. 5-109.

[11] Jesús Cobo and Jaime Nubiola, "Cuatro cartas americanas. Correspondencia de Ventura Reyes Prósper con Charles S. Peirce y Christine Ladd-Franklin", *Llull* 20 (1997), pp. 765-766, n. 3.

[12] Ventura Reyes y Prósper, "Sur la géométrie non-euclidienne", *Mathematische Annalen* 29 (1887), pp. 154-156.

[13] Antonio Bernalte et al, "Introducción de las geometrías no-euclídeas en España", in *Estudios sobre Historia de la Ciencia y de la Técnica*, M. Esteban Piñeiro et al, eds. (Valladolid: Junta de Castilla y León, 1988), vol. II, p. 973.

[14] Juan Antonio del Val, "Un lógico y matemático español del siglo XIX", p. 255.

[15] Antonio Bernalte et al, "Introducción de las geometrías no-euclídeas en España", p. 974.

[16] Laurel Furumoto, "Joining Separates Spheres: Christine Ladd-Franklin, Woman-Scientist", *American Psychologist* 47 (1992), p. 179; "Ladd-Franklin, Christine, in *The Psychological Register*, Carl Murchison, ed. (Worcester, MA: Clark University Press, 1929), p. 134.

[17] Ventura Reyes Prósper, "Sur les propriétés graphiques des figures centriques. (Extrait d'une lettre adressée à Mr. Pasch.)", *Mathematische Annalen* 32 (1888), pp. 157-158.

[18] For a presentation of Desargues' theorem, see Boris A. Rosenfeld, *A History of Non-Euclidean Geometry* (New York: Springer-Verlag, 1988), pp. 138-140.

[19] "auf denkbar einfachste Art.". From Pasch's answer to Reyes' letter published in the same issue of *Mathematische Annalen*, p. 159.

[20] Moritz Pasch, *Lecciones de Geometría Moderna*, (Madrid: Junta para Ampliación de Estudios e Investigaciones Científicas, 1913, p. 58). Pasch's book was originally published in Leipzig, Teubner, 1882.

[21] Moritz Pasch, *Vorlesungen über neuere Geometrie*, 2nd edition (Berlin: Julius Springer, 1926), p. vi.

[22] Jesús Cobo, "La obra científica de Ventura Reyes Prósper", *Toletvm* 30 (1994), p. 190.

[23] Jesús Cobo, "La obra científica de Ventura Reyes Prósper", pp. 191-192.

[24] Jorge J. E. Gracia, "Review of del Val: 'The Logical Writings of Ventura Reyes y Prósper'", #9519.

[25] Juan Antonio del Val, "Los escritos lógicos de Ventura Reyes y Prósper (1863-1922)", p. 326.

[26] Ventura Reyes Prósper, "Ernesto Schröder. Sus merecimientos ante la lógica, su propaganda lógico-matemática, sus obras", *El Progreso Matemático* 2 (14), (1892), p. 33.

[27] That letter, kept in the Houghton Library under the number L 372, is erroneously dated in Robin's catalogue as of 5 March 1891.

[28] Jesús Cobo and Jaime Nubiola have published the original text and an Spanish translation in "Cuatro cartas americanas", pp. 759 and 764.

[29] These are Peirce's papers "On the Algebra of Logic" (1880) and "On the Algebra of Logic: A Contribution to the Philosophy of Notation" (1885): Offprints of both papers are kept in

Reyes Prósper's library in the Unidad de Matemáticas de la Biblioteca del Consejo Superior de Investigaciones Científicas, in Madrid (Spain) bounded in a single volume numbered L-1628.

[30] Ventura Reyes Prósper, "Proyecto de clasificación de los escritos lógico-simbólicos especialmente de los post-boolianos", *El Progreso Matemático* 2 (20), (1892), p. 230.

[31] Charles S. Peirce, *Contributions to 'The Nation'*, K. L. Ketner and J. E. Cook, eds. (Lubbock: Texas Tech Press), vol. I, p. 111: "We learn that a Spanish logician has undertaken a translation of it. For an English-speaking public, a somewhat different presentation of the subject would be preferable". This project of Reyes did not go further.

[32] (Boston: Little & Brown).

[33] Ventura Reyes Prósper, "El raciocinio a máquina", *El Progreso Matemático* 1 (2), (1891), p. 217.

[34] Ventura Reyes Prósper, "Proyecto de clasificación de los escritos lógico-simbólicos, especialmente los postbolianos", p. 231.

[35] Ventura Reyes Prósper, "Proyecto de clasificación de los escritos lógico-simbólicos, especialmente los postbolianos", p. 231.

[36] Ventura Reyes Prósper, "Ernesto Schröder. Sus merecimientos ante la lógica, su propaganda lógico-matemática, sus obras", p. 33.

[37] Ventura Reyes Prósper, "El raciocinio a máquina", p. 217. Reyes' letter to Peirce provides evidence of that connection.

[38] "Ladd-Franklin, Christine", in *The Biographical Cyclopaedia of American Women*, E. C. Lee and H. C. Wiley, comp. (New York: Williams-Wiley, 1928), vol. 3, p. 137. This entry was prepared from typescripts submitted by Ladd-Franklin in 1926.

[39] Jesús Cobo and Jaime Nubiola, "Cuatro cartas americanas", pp. 761-765.

[40] Ventura Reyes Prósper, "La lógica simbólica en Italia", *El Progreso Matemático* III (26), (1893), pp. 41-43.

[41] Probably these were some issues of *Mind* that had been sent him by Christine Ladd and Reyes had to give them back. Reyes mentioned in his paper of 1891 on Ladd Franklin (see below) that he was "indebted to her kindness for allowing me to read" her work "Some Proposed Reforms in Common Logic", published in *Mind* 15 (1890), pp. 75-87. In Reyes' Library there are kept offprints of that paper (dedicated to him by Ladd), and of Ladd's "Review of Schröder's *Vorlesungen über die Algebra der Logik (Exakte Logic)*", *Mind* n. s. 1 (1892), pp. 126-132.

[42] This is Ladd's paper "The Pascal Hexagram", *American Journal of Mathematics* 2 (1879), pp. 1-12.

[43] C. Ladd-Franklin, "Eine neue Theorie der Lichtempfindungen", *Zeitschrift für Psychologie* IV (1892), pp. 211-221. Published in English in the *Proceedings of the International Congress of Experimental Psychology*, (London: Williams & Norgate, 1892), pp. 103-108.

[44] "Ladd-Franklin, Christine", *The Biographical Cyclopaedia of American Women*, p. 139.

[45] Dorothea J. Hurvich, "Ladd-Franklin, Christine", in *Notable American Women 1607-1950*, (Cambridge, MA: Harvard University Press, 1971) vol. 2, p. 355.

[46] Margaret L. Franklin (1884-c.1961) became active in the women's suffrage movement and published in 1913 *The Case for Woman Suffrage: A Bibliography* (New York: National College Equal Suffrage League).

[47] In the Reyes family there were good painters, his brother Eduardo and his uncle Buenaventura Reyes Corradi. He was very fond to painting and by the testimony of some of his pupils it is known that Reyes painted well.

[48] Reyes mentioned Benjamin Peirce in the two papers translated below. On passing, in the first about Christine Ladd, and very favorably in the second about his son Charles. Reyes qualifies Benjamin Peirce as "one of the most distinguished American mathematicians" and mentions "his admirable book *The Linear Associative Algebra*". A copy of that book, edited by Charles Peirce in New York in 1882, is kept in Reyes' Library.

[49] As Reyes refers in his paper of 1891, he knew from Ladd-Franklin that she had the project to prepare with her husband "a treatise on logic which may include simultaneously the deductive and the inductive logic".