

UNIVERSITY OF OTAGO

# Logic and Normativity

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## Abstract

What is the relationship between logic and thought? One view is that logic merely describes how people think. But this view – called ‘psychologism’ – cannot be quite right. Logic cannot describe how people reason, because although people can reason well, they can also reason badly. The obvious response is to say that logic does not describe how people do think, but rather prescribes how they ought to think. If logic describes how people ought to reason, then if the premises of a logical argument imply the conclusion of that argument and you believe the premises, then you ought to believe the conclusion. According to classical logic the premise, ‘grass is green’ implies the conclusion, ‘the sky is blue or the sky is not blue’, but it seems absurd to say that because I believe that grass is green, I ought to believe that the sky is blue or the sky is not blue. What has gone wrong here? Should the principle, ‘if the premises imply the conclusion and you believe the premises, then you ought to believe the conclusion’ be changed? If so, perhaps it would be more correct to say, ‘if the premises imply the conclusion and you believe the premises, then you have reason to believe the conclusion’. Or is classical logic to blame? Are we mistaken in thinking that ‘grass is green’ implies ‘the sky is blue or the sky is not blue’. I examine a number of arguments that relate to this question. I argue that classical logic deserves a philosophy of logic that does not imply that classical logic is not proper logic.

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## Introduction

In its broadest use in everyday language, 'logic' refers to careful reasoning. One might say, "You're not being logical" in everyday language to express the meaning, 'You haven't thought about that carefully and as a result you have made an error'. 'Logic' usually refers to the study of deductive logic, but it can be broadened to capture inductive and abductive logic. The word logic is also used to refer to formal systems. Each of these uses is legitimate, but it can lead to confusion. This thesis deals with the concepts 'logic' and 'obligation', so before I begin, I will discuss how I use the two words. When I use the word 'logic' in this thesis, I am usually referring to deductive logic, and formalisation is never far from my mind. I try to avoid speaking in a way that will associate logic with good reasoning – since that is the very association I wish to discuss. I also speak of 'a logic', meaning a particular system of formal deduction: the syllogistic is a logic; the propositional calculus is a logic; the predicate calculus is a logic.

With regard to 'obligation', it is not clear that epistemic obligations are distinct from moral obligations, but it is worth considering that they might be. Susan Haack says, "In the celebrated debate between Clifford and James, it is their shared failure to distinguish epistemological from ethical justification which creates the false impression that one must choose either the morally over-demanding account proposed in 'The Ethics of Belief' or the epistemologically over-permissive account proposed in 'The Will to Believe'" (Haack 2001, p. 22). The famous quote from William Clifford's "The Ethics of Belief" is, "It is wrong always, everywhere, and for anyone, to believe anything upon insufficient evidence"(Clifford 1877, para. 17). In "The Will to Believe" William James attacks Clifford's position, but there is not an equally famous summary of his position. As a summary of James' position Haack suggests, "it is not always wrong *either* epistemologically *or* morally to believe on insufficient evidence"(Haack 2001, p. 28). In "The Ethics of Belief Reconsidered", Haack wonders what the relation of epistemic to ethical appraisal might be, and argues, "The relation of epistemic to ethical justification is ...less intimate than partial identity, more intimate than analogy"(Haack 2001, p. 22). I will interpret the 'ought' in 'If  $q$  is a consequence of  $p$ , if you believe  $p$  you ought to believe  $q$ ' as an epistemic obligation and not a moral one. If you fail this duty, you are a less than ideal epistemic agent, but not a less than ideal moral agent. Since there is no clear evidence to the contrary, we must consider it possible that we can have obligations from different sources and those obligations might conflict; you might be epistemically obliged to believe  $q$  but morally obliged to believe  $\sim q$ . It is also an open question as to whether obligations from the same source might conflict, and for my purposes, I shall assume that they can.

The central theme of this thesis is, “What is the relation between logic and thinking?” I begin with a discussion of Psychologism and Normativism, two philosophies of logic that regard the relationship between logic and thinking as the distinguishing characteristic of logic. Psychologism is the view that logic describes the reasoning process, while Normativism is the view that logic describes the ideal reasoning process. Normativism is a natural consequence of rejecting psychologism, and most modern explanations of what logic is about contain a normative assumption. In spite of this normativism has escaped serious philosophical treatment, and normativism may have greater impact on logic than the initial explanations of what logic is about assume. First, though relevant logic has been a topic under discussion for some years, one argument for relevant logic rests heavily on normativism. If normativism is discarded, then this argument collapses, but if normativism is assumed this argument presents a serious challenge to classical logic. Next, in his article “Hume’s Master Argument” Adrian Heathcote presents an argument that depends heavily on normativism and pushes for the conclusion that there can be *valid* inductive arguments, and contra Hume, there is no Is/Ought gap.

That logic describes the ideal reasoning process implies that there is some true statement roughly of the form ‘if p implies q, and you believe p, then you ought to believe q’. This and similar principles come under debate in three places: in Heathcote’s article, and Charles Pigden and Alan Musgrave’s criticisms of that article, all found in the book, *Hume on Is and Ought*; in Broome’s article “Normative Requirements”, to which Heathcote refers; and in an unpublished draft of a conference paper given by John MacFarlane entitled “In What Sense (if any) is Logic Normative for Thought?” available on his website. When discussing these principles, there is a stable pattern of argument that goes, “Well if you object to ‘...then you ought to q’ surely you won’t object to ‘...then you have a reason to q’”. MacFarlane presents an original method for dealing with these principles that circumvents this argument form. I expand upon his method, and discuss objections to various principles.

Normativism might be true, and if it is and there is no principle that produces true norms when combined with the rules of classical logic, then it rules out classical logic. However, if one wishes to retain classical logic, and discard normativism instead, then one must explain what classical logic is about in a way that does not imply normativism. To do so, one must explain what logic is about without referring to reasoning. It does not mean that logic has no relationship with reasoning, but the relationship with reasoning does not explain what logic is about.

# Logic says you ought to believe...

## *Psychologism and Normativism*

What is the relationship between logic and thinking? “Logic is descriptive of the reasoning process” is a straightforward philosophy of logic. J.S. Mill is often cited as a proponent of this view, though it has been pointed out that, “Critics and interpreters of Mill’s philosophy of logic have been unable to reach a verdict on the question whether Mill was a psychologistic thinker” (Kusch 2009). Mill says that logic is “the Science, as well as the Art, of reasoning; meaning by the former term, the analysis of the mental process which takes place whenever we reason, and by the latter, the rules, grounded on that analysis, for conducting the process correctly” (Mill 1882, pp. 17–18). This statement seems to capture the idea that logic has both descriptive and normative components. Mill makes the point that Archbishop Whately, an eminent logician of the time, considers syllogising to be “the philosophical analysis of *the* mode in which all men reason, and must do so if they reason at all” (Mill 1882, p. 142). Mill argues that syllogistic logic does not constitute all or even most of the reasoning process. He says:

And is the syllogism, to which the word reasoning has so often been represented to be exclusively appropriate, not really entitled to be called reasoning at all? This seems an inevitable consequence of the doctrine, admitted by all writers on the subject, that a syllogism can prove no more than is involved in the premises. Yet the acknowledgment so explicitly made, has not prevented one set of writers from continuing to represent the syllogism as the correct analysis of what the mind actually performs in discovering and proving the larger half of the truths.

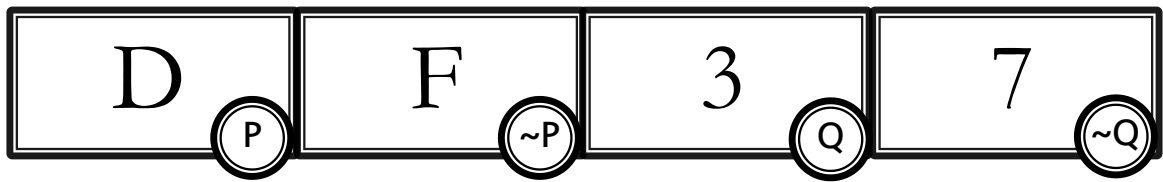
(Mill 1882, p. 139)

This becomes a criticism of the syllogistic if one adopts a psychologistic philosophy of logic. It is not clear how far Mill meant to carry this idea as a criticism of syllogising, but if one were to construe it as a criticism of syllogistic logic, the argument would be:

- (1) Laws of logic describe the reasoning process
- (2) People do not reason according to the laws of syllogistic logic
- (3) Therefore, the laws of syllogistic logic are false.

Classical logic offers a more refined analysis of argument than syllogistic logic, and it is tempting to think of classical logic as descriptive. However, it is reasonable to view the Wason selection task as proof that it is not. The standard Wason selection task involves a descriptive conditional and four cards representing each of  $P$ ,  $\sim P$ ,  $Q$ , and  $\sim Q$ , and an instruction to select the cards that must be turned to determine whether the conditional is violated.

If A person has a 'D' rating then his documents must be marked code '3'.



- One side of a card tells a person's letter rating and the other side tells a person's number code.
- Which of the cards must be turned to determine if a person violates the rule?

Classical logic entails the selection of the P, and  $\sim$ Q cards. Yet, most people will select only the P card, or the P and Q cards. This is evidence that classical logic is not descriptive, and the same psychologicistic argument can be made:

- (1) Laws of logic describe the reasoning process
- (2) People do not reason according to the laws of classical logic
- (3) Therefore, the laws of classical logic are false.

So, if classical logic is not descriptive is that a criticism? The standard reply is 'No'. Classical logic is not descriptive, but that does not mean that it is not correct, because logic is not about the way that people actually think. If we reject the conclusion that the laws of classical logic are false, then we must reject premise (1) – psychologism. The argument for the rejection of psychologism is:

- (1) If laws of logic were descriptive of the reasoning process then people would reason this way
- (2) But people do not reason this way
- (3) Therefore, the laws of logic are not laws of thought.

It is important to note that both arguments have a particular kind of strength – neither argument can be genuinely defeated. They represent a divergence of opinion on the nature or purpose of logic. If the nature or purpose of logic is to describe how people think then the psychologicistic argument holds. The anti-psychologicistic argument amounts to the opinion that the nature or purpose of logic is not to describe how people think because if that were the case then the laws of logic would be incorrect. A normative argument reinforces the anti-psychologicistic position: people reason incorrectly as well as correctly, so the purpose of logic is not to tell us how we reason but how we ought to reason. In other words, logic is normative and not descriptive – the cards indicated by classical logic are in fact the cards that would need to be checked to determine rule violation. Q does not need to be checked because the rule would not be violated even if a document marked with code '3' did not have a 'D' rating. To say that logic is normative is to say that logic is descriptive of the ideal reasoning process. If logic is descriptive of the ideal reasoning process it seems correct to say that if a person A believes premise p and if premise p implies conclusion z, then there is some sense in which A ought to believe z.



## ***Classical Logic is not Logic***

The initial association of logic with reasoning results in psychologism, but the rejection of psychologism while maintaining the association with reasoning leads naturally to normativism. Gilbert Harman expresses doubts about the association between logic and reasoning. He says, “There is a tendency to identify reasoning with proof or argument in accordance with rules of logic. Given that identification, logic obviously has a special role to play in reasoning. But the identification is mistaken. Reasoning is not argument or proof. It is a procedure for revising one’s beliefs, for changing one’s view” (Harman 1984, p. 107). If reasoning is a procedure for revising one’s beliefs and logic is identified with reasoning, normativism is a natural result. If one believes such-and-such, then, according to the rules of logic, one ought to believe thus-and-such.

Frege phrases this new normativism in a way that echoes the old psychologism. He says, “[Laws of logic] have a special title to the name ‘laws of thought’ only if we mean to assert that they... prescribe universally the way in which one ought to think if one is to think at all” (Frege 1982, p. 12). Most introductory logic courses come with a dose of naive normativism. For instance, Copi and Cohen open their *Introduction to Logic* with, “Logic is the study of the methods and principles used to distinguish correct from incorrect reasoning” (Copi & Cohen 2009, p. 4) Where an introductory logic text does not speak of reasoning, it will speak of argument. For instance, P.D. Magnus opens with, “Logic is the business of evaluating arguments, sorting good ones from bad ones. ...A logical argument is structured to give someone a reason to believe some conclusion” (Magnus 2008, p. 1). Likewise, Rod Girle opens his *Introduction to Logic* with, “Logic is chiefly concerned with argument” (Girle 2002, p. 1). Harman claims that reasoning is a procedure for revising one’s beliefs, and the same might be said of argument - that argument is a procedure for persuading another to revise their beliefs.

Inference is another concept commonly associated with logic and which leads naturally to normativism. This becomes clear as Stephen Read tries to give an account of what logic is. He says, “The central topic of the philosophy of logic is inference, that is, logical consequence, or what follows correctly from what. What conclusions may legitimately be inferred from what set of premises?” (Read 1995, p. 1). The concept of inference is inseparable from the practice of inferring, and it seems quite reasonable to suppose that the practice of inferring is normative: that there are ways one ought to infer and ways one ought not to infer.

Normativism can lead to an argument for the rejection of classical logic. This argument is made in the following passage from *Entailment: the logic of relevance and necessity*:

Imagine, if you can, a situation as follows. A mathematician writes a paper on Banach spaces, and after proving a couple of theorems he concludes with a conjecture. As a footnote to the conjecture, he writes: "In addition to its intrinsic interest, this conjecture has connections with other parts of mathematics which might not immediately occur to the reader. For example, if the conjecture is true, then the first order functional calculus is complete; whereas if it is false, then it implies that Fermat's last conjecture is correct." The editor replies that the paper is obviously acceptable, but he finds the final footnote perplexing; he can see no connection whatever between the conjecture and the "other parts of mathematics," and none is indicated in the footnote. So the mathematician replies, "Well, I was using 'if ... then -' and 'implies' in the way that logicians have claimed I was: the first order functional calculus is complete, and necessarily so, so anything implies that fact - and if the conjecture is false it is presumably impossible, and hence implies anything. And if you object to this usage, it is simply because you have not understood the technical sense of 'if ... then -' worked out so nicely for us by logicians." And to this the editor counters: "I understand the technical bit all right, but it is simply not correct. In spite of what most logicians say about us, the standards maintained by this journal require that the antecedent of an 'if ... then -' statement must be relevant to the conclusion drawn. And you have given no evidence that your conjecture about Banach spaces is relevant either to the completeness theorem or to Fermat's conjecture.

(Anderson & Belnap 1975, pp. 17–18)

Both premises intuitively have something going for them. The association of logic with the 'laws of thought' or the principles of good reasoning is a long one, and it does provide a simple and intuitive philosophy of logic. In classical logic, the definition of validity is "There is no interpretation such that the premise(s) is true and the conclusion false". If there is no interpretation of 'the first order functional calculus is complete' that yields a falsehood then, the argument from ' $2 + 2 = 4$ ' to 'the first order functional calculus is complete' is sound - valid with a true premise. Nevertheless, it is an unconvincing argument – the premise and the conclusion do not seem to have anything to do with one another. We also know that one ought not reason from ' $2 + 2 = 4$ ' to 'the first order functional calculus is complete'. This argument parallels the psychologistic critique of logic:

- (1) Logic describes how we ought to reason
- (2) But we ought not reason according to the rules of classical logic
- (3) So, classical logic is not proper logic

Ermanno Bencivenga tells a story about logic that begins, "In the beginning, logic was about arguments. ... There was a stable feeling that some of these arguments were good and others bad, that in some cases the premisses [sic] supported the conclusion and in others they didn't; logic was supposed to codify, and possibly explain, this feeling" (Bencivenga 1999, p. 6). Certainly, the codification and explanation of what makes an argument good or bad is a valuable and interesting exercise, although, it seems that classical logic does not do this, at least, not in any intuitive way.

## *Two Other Consequences of Normativism*

Three conclusions may follow from normativism: classical logic ought to be rejected; deduction and induction are on a par; and there is no Is/Ought gap. I have already discussed the argument that classical logic ought to be rejected. The second conclusion that might follow from normativism is that deduction and induction are on a par. If validity is a normative notion in the sense that there is some identity between an argument's being valid and believing the premises obliging one to believe the conclusion, then if that same belief-obligation relation holds between the premises and conclusion of an inductive argument then an inductive argument can be valid. Another way this argument might be put is: the obligation to believe the conclusion of an inductive argument if one believes the premises is identical to the obligation to believe the conclusion of a deductive argument if one believes the premises, therefore deduction and induction are on a par. This is exactly the way Heathcote argues. He gives the argument:

A believes premises  $p_1 \dots p_n$ .  
These (deductively or inductively) imply conclusion  $q$ .  
 $\therefore$  A ought to believe  $q$

and then says, "So we have a puzzle: on the one hand we have an argument form ... that Hume would likely accept, and on the other we have Hume's is-ought stricture - which has been widely accepted as true - to show that we can't deduce an ought statement from a set of ought-free statements. Yet we seem to have done it... Which is true?" (Heathcote 2010, p. 107). If the same bridge principle holds for both deductive and inductive arguments it does strongly imply that induction is valid. It is of course possible that there are different bridge principles for deduction and induction, in which case this argument will not hold. It is more likely that if there is such a principle then it is the same for deduction and induction. Say, for instance, that I live in the village where there is one albino crow, but every crow I have ever seen was black, and every crow that everyone else has ever seen, has been black. There would be something seriously epistemically wrong with me if I did not behave as if, and expect that 'all crows are black' is true because 'every crow I have ever seen was black' is true, and 'every crow that everyone else has ever seen has been black' is true. It would not be more of an epistemic failure if I refused to accept that 'every crow I have ever seen was black and every crow that everyone else has ever seen has been black' is true because 'every crow I have ever seen was black' is true, and 'every crow that everyone else has ever seen has been black' is true. Denying either consequence is epistemically irresponsible in precisely the same way. If there is an importantly normative component to validity, then it seems reasonable to say that a good account of validity must make room for inductive validity as well as deductive.

The final consequence of normativism is that it might be possible to get an 'Ought' from an 'Is'. For normativism to be correct there must be some true bridge principle. If there is a bridge principle, and the principle is analytic, then it will likely allow for a bridging of the Is/Ought gap.

Then Heathcote would be right when he claims, “The general answer to Hume’s problem is that the normativity of sound reasoning yields arguments that do *validly* derive *ought*-statements from *is*-statements”(Heathcote 2010, p. 110). The key question is whether the bridge principle is analytic. Analicity is difficult to define; roughly, if something is analytic it is true by virtue of the meanings of the words involved. A better way to break down the significant question for the bridging of the Is/Ought gap is to ask whether the bridge principle contains substantive moral content in the way that a genuine moral claim does; or whether the bridge principle carries no genuine moral content. If the principle has genuine moral content, then no bridging of the Is/Ought gap occurs, because there is at least one premise with moral content - the bridge principle. However, if the bridge principle were a statement of fact, but not a statement of a moral fact, then, it would allow us to move from statements of fact to statements of obligation. Pigden makes this point saying, “If the inference is to be formally valid there must be an epistemic ‘ought’ *somewhere* in the premises, presumably in the bridging principle, but if the bridging principle is analytic in something like the old-fashioned sense (that is true in virtue of the meanings of words) we may still have an inference to a substantive epistemic ‘ought’ from *substantively* ought-free premises. Not so if the bridging principle is synthetic, for in that case the conclusion would depend on a substantive evaluative principle” (Pigden 2010, p. 140).

Similarly, in “On Heathcote Against Hume’s Law”, Norva Lo makes the point that the key question is the analicity of the principle when she says, “Valid counter-examples to Hume’s Law can be readily supplied by using Hume’s own non-rationalist ... bridging principle...: A (morally) ought to do such-and-such iff A’s doing such-and-such will elicit approval in spectators occupying, what Hume (T, 3.3.1. 30/590) calls, ‘the common point of view’. Suppose this principle is granted as necessarily true. Then it will be valid to infer from an instance of the analysans (an ‘is’-statement) to the corresponding instance of the analysandum (an ‘ought’-statement)” (Lo 2010, p. 127).

If there is an analytic bridge principle, then Hume is wrong about the Is/Ought gap, at least with regard to epistemic obligation. It is also worth recalling how similar the epistemic bridge principles are to the principles of practical rationality: it is only a very small step from, ‘one ought to believe such-and-such’ to ‘one ought to do such-and-such’. This gap is even smaller if one thinks that belief is a kind of action. It would seem that if one adopts some bridge principle or other the only way to maintain the Is/Ought gap (if one so desires) is to claim that the bridge principle in question carries genuine moral content.

# Heathcote, Broome, and MacFarlane

## *Heathcote and the Normativity of logic*

In “Hume’s Master Argument”, Heathcote’s primary thesis is that Hume uses similarly structured arguments in favour of both inductive scepticism and the Is/Ought gap. Heathcote gives an Ockhamist reformulation of that common argument structure. Though Pigden challenges the choice of Ockhamist principles for his reformulation of the argument, Heathcote’s point that the two arguments are importantly similar goes unchallenged. What is challenged, is Heathcote’s contention that Hume was wrong about both induction and the Is/Ought gap. Heathcote’s Ockhamist reconstruction of Hume’s argument for inductive scepticism is:

Let  $F$  be some statement about the future; let  $\Phi$  be a set of necessary truths and  $\Phi'$  be that same set augmented by a number of statements that report (past) sensory experience. Then  $\Phi$  does not entail  $F$  by 8) and 9) [of Ockham’s inferential relations], because  $F$  is not necessary. The lemma that proves that  $F$  is not necessary and is therefore either contingent or impossible is as follows:  $\neg F$  implies no contradiction, which it would if  $F$  were a necessity... We may assume that  $\Phi'$  is consistent, since it is just  $\Phi$  plus sentences reporting sensory experience. If  $F$  were impossible then it could not follow from  $\Phi'$  by 1) [of Ockham’s inferential relations]. But (and call this (\*)) if  $F$  is contingent then it still does not follow from  $\Phi'$  because its negation is consistent with  $\Phi'$  (This last part from 6 [of Ockham’s inferential relations]). Therefore  $F$  cannot follow from either  $\Phi$  or  $\Phi'$   
(Heathcote 2010, p. 97)

Therefore, one cannot deductively derive statements about the future either from necessary truths or from necessary truths supplemented by truths about the past. However, one can deductively derive statements about the future from necessary truths supplemented by truths about the past and *causal truths*. Heathcote points out this sub-dilemma. He says, “Either the premise  $\Phi'$  consists of a summation of my past experience – in which case the premise is not sufficient to deductively entail the proposition that concerns unobserved events; or,  $\Phi'$  is supplemented by a number of statements concerning the relation of cause and effect (in effect, generalizations) but these statements, though they now will deductively entail propositions that concern unobserved events, will necessarily beg the question” (Heathcote 2010, p. 106). An inductive argument can be rephrased as a deductively valid argument by adding an extra premise, in which case the argument against induction turns on where the extra premise comes from. As Musgrave remarks, “Hume’s problem with this is where the extra premise comes from – it is not knowable *a priori*, and it is not knowable from experience because arguments from experience all presuppose it” (Musgrave 2010, p. 121).

There is a parallel with the Is/Ought case. In the case of the Is/Ought gap, Hume’s charge is that in order to derive statements about obligation, one must include at least one statement about obligation in the premise set. So, where Heathcote’s critics attack his attempt to bridge the

Is/Ought gap, they validate inferences from statements of fact to statements of obligation by adding an extra premise. The argument then turns to whether the extra premise is analytic. If this extra premise is analytic, it is simply a claim about what it means to believe, or to be obliged, which introduces the possibility of bridging the Is/Ought gap because we may "...have an inference to a substantive epistemic 'ought' from substantively ought-free premises" (Pigden 2010, p. 140). If, however, this bridging principle is synthetic, an argument containing it "...does not bridge the Is/Ought Gap, because it has a general normative principle among its premises" (Musgrave 2010, p. 124). If the bridge principle is synthetic and true, there could be valid deductive arguments about the ethics of belief that have true conclusions. If the bridge principle is analytic and true, there could be valid deductive arguments about the ethics of belief that violate the Is/Ought stricture. This would be true for any bridge principle that is granted analytic status. Nevertheless, Heathcote's critics argue, the Is/Ought gap is not bridged because not only is the principle not analytic, it is false.

Musgrave's critique of Heathcote begins with, "He [Heathcote] tells us that if the premises of a valid argument are true, then so is the conclusion. He then says that we can 'express it differently' by saying, 'if someone believes the premises then they ought to believe the conclusion'. But this normative principle of the ethics of belief is not another way of expressing the logical principle. The two are quite different. The logical principle is true, the normative principle false" (Musgrave 2010, p. 122). Musgrave offers an initial objection (which he attributes to Greg Restall) to the argument that can be assigned the scheme:  $Bp \wedge (p \vDash q) \rightarrow OBq$ . This scheme accurately describes the argument that Heathcote first proposes. The objection points to a self-licensing problem. Since any  $p$  is a deductive consequence of itself,  $Bp \wedge (p \vDash q) \rightarrow OBq$  implies that I ought to believe anything I do in fact believe. So  $Bp \wedge (p \vDash q) \rightarrow OBq$  must be false, since it is not true that it is a consequence of holding a belief that I am obliged to hold that belief (Musgrave 2010, p. 122). Heathcote starts out with  $Bp \wedge (p \vDash q) \rightarrow OBq$ , but he abandons it in favour of  $(p \vDash q) \rightarrow (OBp \rightarrow OBq)$  so this objection, while forceful, is not attacking the heart of Heathcote's argument.

Musgrave also challenges an intuition that both Heathcote and Broome share regarding believing contradictions. Musgrave raises this challenge after proposing an alternative principle to that used by Heathcote. He says, "We might try 'If you believe the premises of a valid argument, then you ought not *at the same time* to believe the negation of the conclusion'" (Musgrave 2010, p. 123), but says that this principle is not true because it is just a special case of the false principle 'you ought not to believe contradictions'. To support the claim that the principle that you ought not to believe contradictions is false, Musgrave gives the example that one generally ought to believe statements on public notices. Therefore, Musgrave contends, if a statement on a particular public notice contains an implicit contradiction that one fails to notice, one ought to believe the statement.

Thus, Musgrave gives a plausible circumstance where one ought to believe a contradiction. Of course, once one notices that contradiction then one ought not to believe that statement (Musgrave 2010, p. 123).

Heathcote gives two consequences of his original principle:  $(p \rightarrow q) \rightarrow (OBp \rightarrow OBq)$ . They are:  $(p \rightarrow q) \rightarrow (OBq \vee \sim OBp)$ , and  $(p \rightarrow q) \wedge OBp \rightarrow Obq$  (Heathcote, 2010, p. 113). Pigden's critique is aimed at:  $(p \rightarrow q) \rightarrow (OBq \vee \sim OBp)$ ; he thinks the arrows are intended in the strong sense that even if the antecedent doesn't happen to be true, the consequent still follows, such as 'if I were to stand in the rain I would get wet.' So, Pigden writes:  $(p \rightarrow q) \rightarrow (OBq \vee \sim OBp)$ , which he believes captures Heathcote's intended meaning more accurately. Pigden argues that this principle cannot be correct, saying, "Suppose  $p$  represents a collection of contingent propositions relevant to our concerns for which we have excellent evidence. And suppose that  $q$  represents some trivial logical consequence of  $p$  (for example:  $(p \vee m) \& (p \vee p) \& (p \vee r) \& (p \wedge p)$ ). Then it may well be the case that we are obliged to believe  $p$  but not obliged to believe  $q$ , since it would surely be silly to fill our heads with such useless junk" (Pigden 2010, p. 141). This objection is the objection from triviality. Musgrave gives the same objection saying, "Suppose I believe  $P$ . The argument from  $P$  to ' $P$  or  $Q$ ', for any  $Q$ , is valid. Ought I to waste time formulating and coming to believe all the idiotic disjunctions of the form ' $P$  or  $Q$ '?" (Musgrave 2010, p. 122). Applying a bit of paraphrasing to the arguments above, the argument is:

$$\begin{aligned} & (P \equiv Q) \rightarrow (OBP \rightarrow OBQ) \\ & p \equiv (p \vee q) \\ & OBp \rightarrow OB(p \vee q) \\ & OBp \\ & \therefore OB(p \vee q) \quad \text{for any arbitrary } q? \end{aligned}$$

This does not seem like a plausible conclusion because it does not seem plausible that one could be obliged to believe trivial disjunctions, even if they are true.

### ***Broome and practical rationality***

Before putting forward his own principle, Heathcote attempts to undermine an argument of Broome's which Heathcote summarises as: "The normative force of giving an argument can never be such that, given the premises we ought to believe the conclusion. He [Broome] thinks it is a variety of the familiar modal fallacy: confusing  $p \rightarrow \Box q$  with  $\Box (p \rightarrow q)$ " (Heathcote, 2010, p. 110). Broome's paper "Normative Requirements", to which Heathcote refers, deals with much of the same material found in the debate between Heathcote and his critics, though Broome's interest has rather different motivations. Broome is primarily interested in practical reasoning, and the point of this article is to argue for the introduction of a new relation called 'normative requirement' into discussions of normativity with the relations 'ought' and 'a reason'.

Broome represents ‘p normatively requires q’ as ‘p requires q’, he contrasts this with ‘ought’ and ‘a reason’, for which he writes ‘p oughts q’ and ‘p reasons q’. We could re-read these as ‘p requires you to q’, ‘p obliges you to q’, and, ‘p gives you a reason to q’. Broome explains that if p gives you a reason to q, then if p is true, then you have a reason to q. Similarly, if p obliges you to q, then if p is true, then you are obliged to q (Broome 1999, pp. 400–401). Broome represents ‘if p is true, then you have a reason to q’ as ‘ $p \rightarrow Rq$ ’ and ‘if p is true, then you are obliged to q’ as ‘ $p \rightarrow Oq$ ’. ‘p oughts q’ and ‘p reasons q’ are detachable: the combination of ‘ $(p \text{ oughts } q) \rightarrow (p \rightarrow Oq)$ ’ and ‘p’ will allow us to arrive at ‘Oq’, likewise, the combination of ‘ $(p \text{ reasons } q) \rightarrow (p \rightarrow Rq)$ ’ and ‘p’ will allow us to arrive at ‘Rq’. Broome also explains that ‘p’ and ‘Oq’ could both be true rendering ‘ $p \rightarrow Oq$ ’ true, but ‘p oughts q’ might still be false.

Broome then explains that if p requires you q then you ought to see to it that if p is true, so is q. Broome represents ‘you ought to see to it that if p is true, so is q’ as ‘ $O(p \rightarrow q)$ ’ and the overall relation is:  $(p \text{ requires } q) \rightarrow O(p \rightarrow q)$ . The upshot of this is that ‘p requires q’ is non-detachable, you cannot get from ‘p requires q’ and ‘p’ to ‘Oq’. Broome says, “The reasons relation [p reasons q] and the requires relation [p requires q] are both, in a sense, weakenings of the oughts relation. [p oughts q]” (Broome 1999, p. 411). Broome uses the ‘oughts’, ‘reasons’, and ‘requires’ notation because he thinks there is a significant difference between these and the typical logical notation. He says that ‘p requires q’ implies ‘ $O(p \rightarrow q)$ ’, but it is not equivalent to ‘ $O(p \rightarrow q)$ ’ because ‘p requires q’ contains a determining relation as well as a material conditional. Broome says that the determining relation is somewhat analogous to causation, saying that you are required to q *because of* p (Broome 1999, p. 400). As I have already mentioned, Broome says that, ‘p requires q’ does not imply ‘ $p \rightarrow Rq$ ’ or ‘ $p \rightarrow Oq$ ’, (Broome 1999, p. 402). Broome also says, “It must be a feature of the logic of normative requirement that  $O(\sim(p \wedge r))$  is derivable from ‘p requires q’ and ‘r requires  $\sim q$ ’ (Broome 1999, p. 412).

Broome discusses the English idiom that misrepresents the logic involved in conditional propositions that contain modality, for instance, ‘If it is raining, it must be thawing’ and relates that to statements like, ‘If you believe p, you should believe q’. He argues that reinterpreting ‘If you believe p, you should believe q’ as ‘If you believe p you ought to believe q’ or ‘If you believe p you have a reason to believe q’ misrepresents the underlying logic (Broome 1999, p. 411). Broome argues that the relationship between believing something and believing its consequence should be represented as ‘Bp requires Bq’ which he says does not mean ‘ $Bp \rightarrow OBq$ ’ or ‘ $Bp \rightarrow RBq$ ’, (Broome 1999, p. 402). Broome says, “Suppose you notice there must be a normative connection between believing something and believing one of its consequences. But suppose you also notice that believing something does not make it the case that you ought to believe its consequence”



(Broome 1999, p. 411). So, Broome's intended audience agrees that there is a logic–belief bridge principle of some sort but rejects  $Bp \wedge (p \rightarrow q) \rightarrow OBq$ .

Heathcote interprets Broome's 'p requires q' as ' $O(Bp \rightarrow Bq)$ ', which is not strictly accurate as Broome states that the two are not equivalent, because there is a causal relation between belief in p and belief in q. Heathcote says:

The meaning of  $Bp$  requires  $Bq$ , Broome tells us, is that *you ought to see to it that if you believe p you believe q*. The ought operator governs the proposition that follows: if you believe p, you believe q. But this doesn't seem right. Take *reductio ad absurdum* again, and suppose that you believe the premises (you can't see the contradiction): it is not the case that you ought to see to it that since you believe p you believe q. You ought not to believe p, nor q. There is no obligation to believe an explicit contradiction merely because you mistakenly believed an implicit one. And you can't say well, consistency requires you to believe the explicit contradiction in this case - because consistency requires you not to believe a contradiction at all!

(Heathcote 2010, p. 112)

What Heathcote's objection points out is that  $O(Bp \rightarrow Bq)$  implies that, if p is inconsistent, one ought to bring it about that if one believes p (which is inconsistent) one believes everything. Broome would agree with Heathcote's assessment that mistakenly believing a contradiction does not oblige one to believe the consequences of that contradiction. Heathcote's criticism misses the spirit of Broome's paper, which is to introduce a new relation *designed* to capture our intuitions on this matter. Broome would answer that the 'requires' relation simply could not hold between an inconsistent premise and everything else, and so one could never derive  $O(Bp \rightarrow Bq)$  from that relation. However, Heathcote's criticism does give us ample reason not to accept  $O(Bp \rightarrow Bq)$  on its own.

Heathcote also objects that, "Broome's non-detachable  $Bp$  requires  $Bq$  has the absurd consequence that no one should ever believe a theorem of mathematics. ... At best, according to Broome, one should say that one ought to see to it that if one believes the axioms of mathematics (whatever they are now!) then one believes Fermat's Last Theorem." But, Heathcote complains, the bridge principle should oblige one to believe Fermat's Last Theorem *simpliciter* (Heathcote 2010, p. 112). So, Heathcote's complaint is that this principle doesn't oblige you to believe a conclusion that you should be obliged to believe. Of course, this may not be a repugnant conclusion if you believe that you ought not to believe anything without adequate justification, because the obligation to believe Fermat's Last Theorem *simpliciter* requires you to believe Fermat's Last Theorem – even though you have forgotten all the axioms of mathematics. The problem with  $O(Bp \rightarrow Bq)$  is not what it does not require of you, but rather what it *does* require of you. MacFarlane's objection to  $O(Bp \rightarrow Bq)$  makes this plain:

[It] implies that you ought either to cease believing the axioms of Peano Arithmetic or come to believe all the theorems as well. We humans are presumably incapable of the latter. Even if a genie could grant us the capacity for arithmetical omniscience, it's not clear we'd have reason to accept it. Only a small number of the theorems are likely to be of any practical or theoretical use to us; why must we clutter up our minds with all the rest? So it seems that the only feasible way for a human to comply with the norm would be by ceasing to believe the Peano axioms. The same argument could be made with regard to any inferentially fertile set of axioms. But it's crazy to think that logic (together with these premises about human capacities) forbids us from believing such axioms, even if they are consistent and true!

(MacFarlane 2004, p. 11)

Heathcote uses his critique of Broome to present his alternative:  $(p \rightarrow q) \rightarrow (OBp \rightarrow OBq)$ .

Heathcote's critics focus on this principle and the critiques that they give of Heathcote's principle look remarkably like the critique Heathcote gives of Broome's principle, that this principle cannot be true because it has unintuitive consequences.

### ***MacFarlane and bridge principles***

In Heathcote's discussion of the Is/Ought gap and Broome's discussion of practical rationality, principles of a similar form come under scrutiny. Suggesting a new principle that varies the deontic operator or the polarity is a common tactic in both debates. For instance, given an argument to the effect that though I desire the lettuce, and I believe that crossing the road is the only means to getting the lettuce, it doesn't seem to follow that I am obliged to cross the road, someone might respond, "Well you might not be obliged to cross the road, but surely you have a reason to". Likewise, given an argument to the effect that even if I believe the premises and I believe the conclusion it is not the case that I ought to believe the conclusion, someone might respond, "Perhaps you're not obliged to believe the conclusion but surely you ought not to disbelieve it".

In 2004, Macfarlane gave a conference paper entitled "In What Sense (If Any) Is Logic Normative for Thought?" which is available on his website. Given that Macfarlane's paper is only an unpublished draft, MacFarlane cannot be held to any of the particulars, nevertheless the paper is notable for the systematic approach to the formulation of bridge principles. He gives three variable terms: the scope of the deontic operator, the type of operator, and the polarity; and constructs a table of potential bridge principles by varying these terms. The types of deontic operators that MacFarlane considers are 'obligation', 'permission' and 'a reason'. The scope of the deontic operator refers to whether the deontic operator occurs only in the antecedent, in both the antecedent and consequent, or scopes over an entire conditional. The polarity refers to whether the principle instructs you to believe, or not to disbelieve. MacFarlane gives a table of the bridge principles generated by varying the terms:

If  $A, B \models C$ , then...

*C* Deontic operator embedded in consequent.

*o* Deontic operator is strict obligation (ought).

$Co^+$  If you believe *A* and you believe *B*, you ought to believe *C*.

$Co^-$  If you believe *A* and you believe *B*, you ought not to disbelieve *C*.

*p* Deontic operator is permission (may).

$Cp^+$  If you believe *A* and you believe *B*, you may believe *C*.

$Cp^-$  If you believe *A* and you believe *B*, you are permitted not to disbelieve *C*.

*r* Deontic operator is has (defeasible) reason for.

$Cr^+$  If you believe *A* and you believe *B*, you have reason to believe *C*.

$Cr^-$  If you believe *A* and you believe *B*, you have reason not to disbelieve *C*.

*B* Deontic operator embedded in both antecedent and consequent.

*o* Deontic operator is strict obligation (ought).

$Bo^+$  If you ought to believe *A* and believe *B*, you ought to believe *C*.

$Bo^-$  If you ought to believe *A* and believe *B*, you ought not to disbelieve *C*.

*p* Deontic operator is permission (may).

$Bp^+$  If you may believe *A* and believe *B*, you may believe *C*.

$Bp^-$  If you may believe *A* and believe *B*, you are permitted not to disbelieve *C*.

*r* Deontic operator is has (defeasible) reason for.

$Br^+$  If you have reason to believe *A* and believe *B*, you have reason to believe *C*.

$Br^-$  If you have reason to believe *A* and believe *B*, you have reason not to disbelieve *C*.

*W* Deontic operator scopes over whole conditional.

*o* Deontic operator is strict obligation (ought).

$Wo^+$  You ought to see to it that if you believe *A* and you believe *B*, you believe *C*.

$Wo^-$  You ought to see to it that if you believe *A* and you believe *B*, you do not disbelieve *C*.

*p* Deontic operator is permission (may).

$Wp^+$  You may see to it that if you believe *A* and you believe *B*, you believe *C*.

$Wp^-$  You may see to it that if you believe *A* and you believe *B*, you do not disbelieve *C*.

*r* Deontic operator is has (defeasible) reason for.

$Wr^+$  You have reason to see to it that if you believe *A* and you believe *B*, you believe *C*.

$Wr^-$  You have reason to see to it that if you believe *A* and you believe *B*, you do not disbelieve *C*.

–*k* (As suffix to one of the above) antecedent of bridge principle is “If you know that...”

MacFarlane examines these principles as a way of exploring what it means to say that an inference is *valid* or that a conclusion *follows logically* from some premise (MacFarlane 2004, p. 1). Macfarlane’s interest seems primarily driven by a desire to understand more about the nature of the disagreement between the relevantists and the classicists<sup>1</sup> about what validity is. MacFarlane says, “Relevantists will concede that *ex falso quodlibet* is necessarily truth-preserving, so that it would be valid if the classicists were right that logical consequence is necessary truth-preservation. It’s just that that’s not what validity is” (MacFarlane 2004, p. 3). He quotes Graham Priest saying that anyone who reasoned from an arbitrary premise to the infinity of prime numbers would not last long in an undergraduate mathematics course (Priest 1979, p. 279) and says, “Priest assumes here that if an argument is valid, it is always correct to reason from its premises to its conclusion” (MacFarlane 2004, p. 4). This suggests that the relevantist believes that true validity produces plausible norms according to a bridge principle, and since classical logic does not produce plausible norms according to a bridge principle, it cannot represent true validity.

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<sup>1</sup> Relevantists and Classicists are proponents of relevant logic and classical logic respectively.

## Formulating a Bridge Principle

### *My Bridge Principles*

Initially I constructed a table of schemas based on MacFarlane's table bridge principles, particularly so that I would have something that could be compared to the principles discussed by Heathcote, Musgrave, and Pigden, but I found that table unsatisfactory. I began with the antecedent conditions: the premise implies the conclusion; you believe that the premise implies the conclusion; and, you believe the premise. I included the condition 'you believe that the premise implies the conclusion' to pre-empt the objection that it does not seem likely that I should be obliged to believe the conclusion if I fail to notice that it follows from what I believe. I also included a polarity that MacFarlane explicitly excludes, 'ought not believe', he excluded it because, he says, "The Fregean identification of 'disbelieving p' with 'believing not-p' is controversial. Dialetheists reject it, because they think one should sometimes believe both p and not-p (when p is both true and false), though one should never both believe and disbelieve p" (MacFarlane 2004, p. 8). I included 'ought not believe' because excluding it from the beginning seems to be conceding too much. My notation also differs from that used before. Heathcote used ' $(p \rightarrow q) \rightarrow (OBp \rightarrow OBq)$ ', ' $(p \rightarrow q) \wedge OBp \rightarrow OBq$ ' and ' $(p \rightarrow q) \rightarrow (OBq \vee \sim OBp)$ ' which Pigden reinterpreted as ' $(p \rightarrow q) \rightarrow (OBq \vee \sim OBp)$ '; but the bridge principle is really about inferences so ' $(p \rightarrow q)$ ' should be ' $(p \vDash q)$ '. ' $(p \vDash q) \rightarrow (OBq \vee \sim OBp)$ ' would be too strong, for if p does not imply q, why should I be obliged to believe q or not obliged to believe p? Yet it is not certain that the conditional that the bridge principle contains is the material conditional, so I shall use ' $\rightarrow$ ' and not ' $\supset$ '. My initial table of bridge principles was:

#### **Deontic operator embedded in consequent.**

##### **Deontic operator is strict obligation (ought).**

- [1]  $\varphi \vDash \psi \ \& \ B(\varphi \vDash \psi) \ \& \ B\varphi \rightarrow OB\psi$
- [2]  $\varphi \vDash \psi \ \& \ B(\varphi \vDash \psi) \ \& \ B\varphi \rightarrow O \sim \sim B\psi$
- [3]  $\varphi \vDash \psi \ \& \ B(\varphi \vDash \psi) \ \& \ B\varphi \rightarrow O \sim B \sim \psi$

##### **Deontic operator is permission (may).**

- [4]  $\varphi \vDash \psi \ \& \ B(\varphi \vDash \psi) \ \& \ B\varphi \rightarrow PB\psi$
- [5]  $\varphi \vDash \psi \ \& \ B(\varphi \vDash \psi) \ \& \ B\varphi \rightarrow P \sim \sim B\psi$
- [6]  $\varphi \vDash \psi \ \& \ B(\varphi \vDash \psi) \ \& \ B\varphi \rightarrow P \sim B \sim \psi$

##### **Deontic operator is has (defeasible) reason for.**

- [7]  $\varphi \vDash \psi \ \& \ B(\varphi \vDash \psi) \ \& \ B\varphi \rightarrow RB\psi$
- [8]  $\varphi \vDash \psi \ \& \ B(\varphi \vDash \psi) \ \& \ B\varphi \rightarrow R \sim \sim B\psi$
- [9]  $\varphi \vDash \psi \ \& \ B(\varphi \vDash \psi) \ \& \ B\varphi \rightarrow R \sim B \sim \psi$

**Deontic operator embedded in both antecedent and consequent.**

**Deontic operator is strict obligation (ought).**

- [10]  $\varphi \models \psi \ \& \ B(\varphi \models \psi) \ \& \ B\varphi \ \& \ O B\varphi \rightarrow O B\psi$
- [11]  $\varphi \models \psi \ \& \ B(\varphi \models \psi) \ \& \ B\varphi \ \& \ O B\varphi \rightarrow O \sim \sim B\psi$
- [12]  $\varphi \models \psi \ \& \ B(\varphi \models \psi) \ \& \ B\varphi \ \& \ O B\varphi \rightarrow O \sim B \sim \psi$

**Deontic operator is permission (may).**

- [13]  $\varphi \models \psi \ \& \ B(\varphi \models \psi) \ \& \ B\varphi \ \& \ O B\varphi \rightarrow P B\psi$
- [14]  $\varphi \models \psi \ \& \ B(\varphi \models \psi) \ \& \ B\varphi \ \& \ O B\varphi \rightarrow P \sim \sim B\psi$
- [15]  $\varphi \models \psi \ \& \ B(\varphi \models \psi) \ \& \ B\varphi \ \& \ O B\varphi \rightarrow P \sim B \sim \psi$

**Deontic operator is has (defeasible) reason for.**

- [16]  $\varphi \models \psi \ \& \ B(\varphi \models \psi) \ \& \ B\varphi \ \& \ O B\varphi \rightarrow R B\psi$
- [17]  $\varphi \models \psi \ \& \ B(\varphi \models \psi) \ \& \ B\varphi \ \& \ O B\varphi \rightarrow R \sim \sim B\psi$
- [18]  $\varphi \models \psi \ \& \ B(\varphi \models \psi) \ \& \ B\varphi \ \& \ O B\varphi \rightarrow R \sim B \sim \psi$

**Deontic operator scopes over whole conditional**

**Deontic operator is strict obligation (ought).**

- [19]  $\varphi \models \psi \ \& \ B(\varphi \models \psi) \rightarrow O (B\varphi \rightarrow B\psi)$
- [20]  $\varphi \models \psi \ \& \ B(\varphi \models \psi) \rightarrow O (B\varphi \rightarrow \sim \sim B\psi)$
- [21]  $\varphi \models \psi \ \& \ B(\varphi \models \psi) \rightarrow O (B\varphi \rightarrow \sim B \sim \psi)$

**Deontic operator is permission (may).**

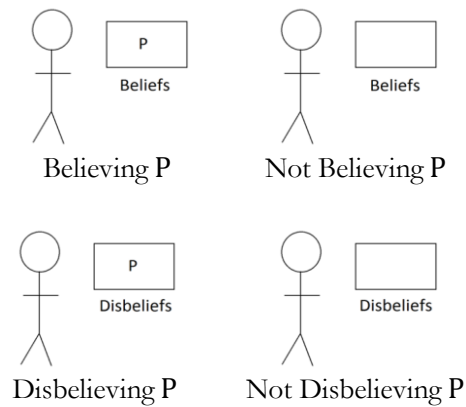
- [22]  $\varphi \models \psi \ \& \ B(\varphi \models \psi) \rightarrow P (B\varphi \rightarrow B\psi)$
- [23]  $\varphi \models \psi \ \& \ B(\varphi \models \psi) \rightarrow P (B\varphi \rightarrow \sim \sim B\psi)$
- [24]  $\varphi \models \psi \ \& \ B(\varphi \models \psi) \rightarrow P (B\varphi \rightarrow \sim B \sim \psi)$

**Deontic operator is has (defeasible) reason for.**

- [25]  $\varphi \models \psi \ \& \ B(\varphi \models \psi) \rightarrow R (B\varphi \rightarrow B\psi)$
- [26]  $\varphi \models \psi \ \& \ B(\varphi \models \psi) \rightarrow R (B\varphi \rightarrow \sim \sim B\psi)$
- [27]  $\varphi \models \psi \ \& \ B(\varphi \models \psi) \rightarrow R (B\varphi \rightarrow \sim B \sim \psi)$

I found this table unsatisfactory because in representing the bridge principles in this way I introduced translation artefacts. I intended these to be read as schematic representations of English sentences – each term to be directly replaceable by an English word with all of the vagueness of English sentences of this type. Expressing these sentences in schemata and then returning them to English introduces an ambiguity.  $O \sim \sim B P$  could be translated as ‘you ought not disbelieve P’ or ‘you ought to make it the case that you bring about the negation of not believing P’. While the second is inelegant, it allows for a consistent translation across different schemas.

Yet, the phrasing ‘...ought not disbelieve...’ is used to overcome the objection that you could not possibly be obliged to believe every logical consequence (including the trivial consequences) of the things you currently believe. If ‘P implies Q’ and ‘you ought to believe P’, then even though Q is some consequence so trivial that it seems over demanding to say that you are obliged to believe Q, still, it could be the case that you are obliged not to disbelieve Q. This intuition rests on the idea that belief is a contentful mental state.



In order to fulfil ‘not believing’  $P$  must be absent from the ‘beliefs’ box. Therefore, the negation of ‘not believing’ seems to amount to having  $P$  in the belief box – i.e. believing  $P$ . This is not the same as ‘not disbelieving’. Like belief, disbelief is a contentful mental state, so, ‘not disbelieving’ is fulfilled when  $P$  is absent from the ‘disbelief’ box. Therefore, ‘...you ought to make it the case that you bring about the negation of not believing ...’ is equivalent to ‘...you ought to make it the case that you believe...’ in a way that ‘...you ought to make it the case that you do not disbelieve...’ is not. Not disbelieving  $P$  involves not having  $P$  occur among your disbeliefs, but the negation of not believing would involve having  $P$  occur among your beliefs. The point of the ‘ought not disbelieve’ construction was to evade the objection from triviality by not requiring a positive belief. So perhaps the same objective could be achieved by ‘...you ought to make it the case that you do not believe the negation of...’.

Macfarlane excluded the ‘ought not believe’ principles on the grounds that, “The Fregean identification of ‘disbelieving  $p$ ’ with ‘believing not- $p$ ’ is controversial” (MacFarlane 2004, p. 8). One challenge for those who adopt the Fregean identification is that believing something might not commit one to disbelieving its negation. The reasoning behind this is that disbelief is a positive stance, so demanding that for each belief there is a disbelief in its negation effectively doubles the work necessary to believe any given thing. If disbelieving  $\sim P$  is equivalent to believing  $\sim\sim P$  (by the Fregean identification), then if for each belief  $P$  one is not obliged to disbelieve  $\sim P$ , then one is likewise not obliged to believe  $\sim\sim P$  (by the Fregean identification). So, to allow for both the Fregean identification and the idea that believing something might not commit one to disbelieving its negation, believing  $P$  must be compatible with not believing  $\sim\sim P$ . If believing  $P$  is compatible with not believing  $\sim\sim P$ , then believing  $\sim\sim P$  is not reducible to believing  $P$ . This is so because if we permit that believing  $\sim\sim P$  reduces to believing  $P$ , but allow that believing  $P$  does not commit one to believing  $\sim\sim P$ , means that the initial belief,  $\sim\sim P$ , is lost. Presumably, those who adopt the Fregean identification have reductionist motivations for doing so, finding that one reduction prevents another it is a disappointing result.

To permit the Fregean identification, for each belief  $P$  one must also disbelieve  $\sim P$ , which is to say that for each belief  $P$  one also believes  $\sim\sim P$ . The objection to this was that this would double the amount of work required to have a belief, but this objection fails if to believe  $\sim\sim P$  is just to believe  $P$ . Then, to ensure that the ‘...you ought to make it the case that you do not believe the negation of...’ structure serves its intended purpose – avoiding the objection from triviality – not believing  $\sim P$ , must be incompatible with disbelieving  $P$ . This would mean that not disbelieving  $P$  is equivalent to not believing  $\sim P$ . So if the Fregean identification is true, ‘ought disbelieve  $\sim P$ ’ is equivalent to ‘ought to believe  $P$ ’, and ‘ought not disbelieve  $P$ ’ is equivalent to ‘ought not believe  $P$ ’.

The Fregean identification might be true. To avoid taking a stand one way or another I include both versions in the table. Presumably, those who disagree with the Fregean identification would not accept the ‘ought not believe’ variants as this would probably lead to a contradiction with the principle that it is permissible to believe both  $p$  and  $\sim p$ . To those who agree with the Fregean identification the ‘ought not disbelieve’ duplicates the meaning of ‘ought not believe’ and ‘ought disbelieve the negation’ duplicates the meaning of ‘ought to believe’, and so they would only need to consider one of each of those.

Next, a natural reading for  $O\sim B\sim\psi$  would be ‘ought not believe the negation of  $\psi$ ’, but it is not clear whether ‘not’ in this sentence is attached to ‘ought’ or ‘believe’. If ‘not’ is attached to ‘believe’ then  $O\sim B\sim\psi$  expresses an obligation to be in the state of not believing. If you are forbidden to  $\psi$  then it ought to be the case that you don’t  $\psi$ , ordinarily you are forbidden to  $\psi$ , you may not  $\psi$ , and you have an obligation to not  $\psi$  are equivalent. If one thinks that the obligation to be in the state of not believing commits one to a thing that is ‘the state of not believing’ but one thinks there is no such thing, one might prefer to use ‘forbidden’ as an operator. Then one might say that there is only one ‘belief-state’ – believing – which you might be forbidden to be in.

For the second version of my table of schemas, I also re-considered the antecedent conditions for the bridge principle. I dropped  $\phi\equiv\psi$ , as an antecedent condition, because it limits the cases to which the bridge principle would apply, yet this might not be the correct approach. In the following section, I discuss considerations that would be relevant to the selection of a bridge principle’s antecedent conditions.

Finally, I also replaced bridge principles with the deontic operator in both the antecedent and consequent with bridge principles with an exclusive disjunction in the consequent. I think bridge principles with the deontic operator in both the antecedent and consequent are more properly considered with the most appropriate antecedent conditions for a bridge principle. Rather than bridge principles with the deontic operator in both the antecedent and consequent, I consider

bridge principles with a disjunctive consequent, because they may allow for direction of belief revision, because it is not always the case that I ought to believe the conclusion, sometimes I ought to disbelieve the premises instead.

For all this, the table I produce is only a rough sketch, a starting point. For this reason, I have made what would be odd choices if my aim were to announce which principles are true. I have followed a principle of proliferation. I've included 'disbelief' alongside 'not believing', but the correct choice would be one or the other – depending on whether one accepts or rejects the Fregean identification of 'disbelieving P' with 'not believing P'; and I include 'forbidden' as an operator even though 'forbidden' is more usually represented as 'O~' or '~P'. In general, one need only consider one negative polarity variant per deontic operator, so some judicious crossing out might leave one considering only [1], [5], [9], [10], and [13] (where [13] stands as the negative polarity variant of both [1] and [5]) of the [C] principles. The second version of my table of schemas is

**Deontic operator embedded in consequent**

**Deontic operator is strict obligation**

- [1]  $B(\varphi \vDash \psi) \& B\varphi \rightarrow OB\psi$
- [2]  $B(\varphi \vDash \psi) \& B\varphi \rightarrow O \sim B \sim \psi$
- [3]  $B(\varphi \vDash \psi) \& B\varphi \rightarrow OD \sim \psi$
- [4]  $B(\varphi \vDash \psi) \& B\varphi \rightarrow O \sim D \psi$

**Deontic operator is permission**

- [5]  $B(\varphi \vDash \psi) \& B\varphi \rightarrow PB\psi$
- [6]  $B(\varphi \vDash \psi) \& B\varphi \rightarrow P \sim B \sim \psi$
- [7]  $B(\varphi \vDash \psi) \& B\varphi \rightarrow PD \sim \psi$
- [8]  $B(\varphi \vDash \psi) \& B\varphi \rightarrow P \sim D \psi$

**Deontic operator is 'a reason'**

- [9]  $B(\varphi \vDash \psi) \& B\varphi \rightarrow RB\psi$
- [10]  $B(\varphi \vDash \psi) \& B\varphi \rightarrow R \sim B \sim \psi$
- [11]  $B(\varphi \vDash \psi) \& B\varphi \rightarrow RD \sim \psi$
- [12]  $B(\varphi \vDash \psi) \& B\varphi \rightarrow R \sim D \psi$

**Deontic operator is forbidden**

- [13]  $B(\varphi \vDash \psi) \& B\varphi \rightarrow FB \sim \psi$
- [14]  $B(\varphi \vDash \psi) \& B\varphi \rightarrow F \sim B \psi$
- [15]  $B(\varphi \vDash \psi) \& B\varphi \rightarrow FD \psi$
- [16]  $B(\varphi \vDash \psi) \& B\varphi \rightarrow F \sim D \sim \psi$

**Deontic operator embedded in consequent**

**Deontic operator is strict obligation**

- [17]  $B(\varphi \vDash \psi) \& B\varphi \rightarrow OB\psi \vee O \sim B \varphi$
- [18]  $B(\varphi \vDash \psi) \& B\varphi \rightarrow O \sim B \sim \psi \vee O \sim B \varphi$
- [19]  $B(\varphi \vDash \psi) \& B\varphi \rightarrow OD \sim \psi \vee O \sim B \varphi$
- [20]  $B(\varphi \vDash \psi) \& B\varphi \rightarrow O \sim D \psi \vee O \sim B \varphi$

**Deontic operator is permission**

- [21]  $B(\varphi \vDash \psi) \& B\varphi \rightarrow PB\psi \vee P \sim B \varphi$
- [22]  $B(\varphi \vDash \psi) \& B\varphi \rightarrow P \sim B \sim \psi \vee P \sim B \varphi$
- [23]  $B(\varphi \vDash \psi) \& B\varphi \rightarrow PD \sim \psi \vee P \sim B \varphi$
- [24]  $B(\varphi \vDash \psi) \& B\varphi \rightarrow P \sim D \psi \vee P \sim B \varphi$



**Deontic operator is ‘a reason’**

- [25]  $B(\varphi \vDash \psi) \& B\varphi \rightarrow RB\psi \vee R\sim B\varphi$
- [26]  $B(\varphi \vDash \psi) \& B\varphi \rightarrow R\sim B\sim\psi \vee R\sim B\varphi$
- [27]  $B(\varphi \vDash \psi) \& B\varphi \rightarrow RD\sim\psi \vee R\sim B\varphi$
- [28]  $B(\varphi \vDash \psi) \& B\varphi \rightarrow R\sim D\psi \vee R\sim B\varphi$

**Deontic operator is forbidden**

- [29]  $B(\varphi \vDash \psi) \& B\varphi \rightarrow FB\sim\psi \vee FB\varphi$
- [30]  $B(\varphi \vDash \psi) \& B\varphi \rightarrow F\sim B\psi \vee FB\varphi$
- [31]  $B(\varphi \vDash \psi) \& B\varphi \rightarrow FD\psi \vee FB\varphi$
- [32]  $B(\varphi \vDash \psi) \& B\varphi \rightarrow F\sim D\sim\psi \vee FB\varphi$

**Deontic operator scopes the conditional****Deontic operator is strict obligation**

- [33]  $B(\varphi \vDash \psi) \rightarrow O(B\varphi \supset B\psi)$
- [34]  $B(\varphi \vDash \psi) \rightarrow O(B\varphi \supset \sim B\sim\psi)$
- [35]  $B(\varphi \vDash \psi) \rightarrow O(B\varphi \supset D\sim\psi)$
- [36]  $B(\varphi \vDash \psi) \rightarrow O(B\varphi \supset \sim D\psi)$

**Deontic operator is permission**

- [37]  $B(\varphi \vDash \psi) \rightarrow P(B\varphi \supset B\psi)$
- [38]  $B(\varphi \vDash \psi) \rightarrow P(B\varphi \supset \sim B\sim\psi)$
- [39]  $B(\varphi \vDash \psi) \rightarrow P(B\varphi \supset D\sim\psi)$
- [40]  $B(\varphi \vDash \psi) \rightarrow P(B\varphi \supset \sim D\psi)$

**Deontic operator is ‘a reason’**

- [41]  $B(\varphi \vDash \psi) \rightarrow R(B\varphi \supset B\psi)$
- [42]  $B(\varphi \vDash \psi) \rightarrow R(B\varphi \supset \sim B\sim\psi)$
- [43]  $B(\varphi \vDash \psi) \rightarrow R(B\varphi \supset D\sim\psi)$
- [44]  $B(\varphi \vDash \psi) \rightarrow R(B\varphi \supset \sim D\psi)$

**Deontic operator is forbidden**

- [45]  $B(\varphi \vDash \psi) \rightarrow F(B\varphi \supset B\sim\psi)$
- [46]  $B(\varphi \vDash \psi) \rightarrow F(B\varphi \supset \sim B\psi)$
- [47]  $B(\varphi \vDash \psi) \rightarrow F(B\varphi \supset D\psi)$
- [48]  $B(\varphi \vDash \psi) \rightarrow F(B\varphi \supset \sim D\sim\psi)$

Because looking at the principles with a strict search and replace method in mind revealed flaws in the earlier schemas, I have translated these principles to English in a way that sacrifices elegance for consistency. Particularly, instead of the more elegant ‘ought to’ or ‘ought not’ I use ‘obliged to bring it about that’ and ‘obliged to bring it about that you do not’. This works well for the principles that have a conditional in the consequent, ‘ought to if p then q’ clearly makes no sense. Further, it ensures that ‘may’ and ‘not’ are not joined together to produce a bridge principle that means ‘you are forbidden to...’ instead of ‘it is permissible for you to not...’.

**Deontic operator in the consequent.****Deontic operator is strict obligation**

- [1] If you believe that the premises imply the conclusion and you believe the premises, then you are obliged to bring it about that you believe the conclusion.
- [2] If you believe that the premises imply the conclusion and you believe the premises, then you are obliged to bring it about that you do not believe the negation of the conclusion.
- [3] If you believe that the premises imply the conclusion and you believe the premises, then you are obliged to bring it about that you disbelieve the negation of the conclusion.
- [4] If you believe that the premises imply the conclusion and you believe the premises, then you are obliged to bring it about that you do not disbelieve the conclusion.

**Deontic operator is permission.**

- [5] If you believe that the premises imply the conclusion and you believe the premises, then you may bring it about that you believe the conclusion.
- [6] If you believe that the premises imply the conclusion and you believe the premises, then you may bring it about that you do not believe the negation of the conclusion.
- [7] If you believe that the premises imply the conclusion and you believe the premises, then you may bring it about that you disbelieve the negation of the conclusion.
- [8] If you believe that the premises imply the conclusion and you believe the premises, then you may bring it about that you do not disbelieve the conclusion.

**Deontic operator is 'a reason'.**

- [9] If you believe that the premises imply the conclusion and you believe the premises, then you have reason to bring it about that you believe the conclusion.
- [10] If you believe that the premises imply the conclusion and you believe the premises, then you have reason to bring it about that you do not believe the negation of the conclusion.
- [11] If you believe that the premises imply the conclusion and you believe the premises, then you have reason to bring it about that you disbelieve the negation of the conclusion.
- [12] If you believe that the premises imply the conclusion and you believe the premises, then you have reason to bring it about that you do not disbelieve the conclusion.

**Deontic operator is forbidden.**

- [13] If you believe that the premises imply the conclusion and you believe the premises, then you are forbidden to bring it about that you believe the negation of the conclusion.
- [14] If you believe that the premises imply the conclusion and you believe the premises, then you are forbidden to bring it about that you do not believe the conclusion.
- [15] If you believe that the premises imply the conclusion and you believe the premises, then you are forbidden to bring it about that you disbelieve the conclusion.
- [16] If you believe that the premises imply the conclusion and you believe the premises, then you are forbidden to bring it about that you do not disbelieve the negation of the conclusion.

**Exclusive disjunction in the consequent.****Deontic operator is strict obligation.**

- [17] If you believe that the premises imply the conclusion and you believe the premises, then you are either obliged to bring it about that you believe the conclusion or you are obliged to bring it about that you do not believe the premises.
- [18] If you believe that the premises imply the conclusion and you believe the premises, then you are either obliged to bring it about that you do not believe the negation of the conclusion or you are obliged to bring it about that you do not believe the premises.
- [19] If you believe that the premises imply the conclusion and you believe the premises, then you are either obliged to bring it about that you disbelieve the negation of the conclusion or you are obliged to bring it about that you do not believe the premises.
- [20] If you believe that the premises imply the conclusion and you believe the premises, then you are either obliged to bring it about that you do not disbelieve the conclusion or you are obliged to bring it about that you do not believe the premises.

**Deontic operator is permission.**

- [21] If you believe that the premises imply the conclusion and you believe the premises, then you may either bring it about that you believe the conclusion or you may bring it about that you do not believe the premises.
- [22] If you believe that the premises imply the conclusion and you believe the premises, then you may either bring it about that you do not believe the negation of the conclusion or you may bring it about that you do not believe the premises.
- [23] If you believe that the premises imply the conclusion and you believe the premises, then you may either bring it about that you disbelieve the negation of the conclusion or you may bring it about that you do not believe the premises.
- [24] If you believe that the premises imply the conclusion and you believe the premises, then you may either bring it about that you do not disbelieve the conclusion or you may bring it about that you do not believe the premises.

**Deontic operator is 'a reason'.**

- [25] If you believe that the premises imply the conclusion and you believe the premises, then you either have reason to bring it about that you believe the conclusion or you have reason to bring it about that you do not believe the premises.
- [26] If you believe that the premises imply the conclusion and you believe the premises, then you either have reason to bring it about that you do not believe the negation of the conclusion or you have reason to bring it about that you do not believe the premises.
- [27] If you believe that the premises imply the conclusion and you believe the premises, then you either have reason to bring it about that you disbelieve the negation of the conclusion or you have reason to bring it about that you do not believe the premises.
- [28] If you believe that the premises imply the conclusion and you believe the premises, then you either have reason to bring it about that you do not disbelieve the conclusion or you have reason to bring it about that you do not believe the premises.

**Deontic operator is forbidden.**

- [29] If you believe that the premises imply the conclusion and you believe the premises, then you are either forbidden to bring it about that you believe the negation of the conclusion or you are forbidden to bring it about that you believe the premises.
- [30] If you believe that the premises imply the conclusion and you believe the premises, then you are either forbidden to bring it about that you do not believe the conclusion or you are forbidden to bring it about that you believe the premises.
- [31] If you believe that the premises imply the conclusion and you believe the premises, then you are either forbidden to bring it about that you disbelieve the conclusion or you are forbidden to bring it about that you believe the premises.
- [32] If you believe that the premises imply the conclusion and you believe the premises, then you are either forbidden to bring it about that you do not disbelieve the negation of the conclusion or you are forbidden to bring it about that you believe the premises.

**Deontic operator scopes over whole conditional.****Deontic operator is strict obligation.**

- [33] If you believe that the premises imply the conclusion then you are obliged to bring it about that if you believe the premises then you believe the conclusion.
- [34] If you believe that the premises imply the conclusion then you are obliged to bring it about that if you believe the premises then you do not believe the negation of the conclusion.
- [35] If you believe that the premises imply the conclusion then you are obliged to bring it about that if you believe the premises then you disbelieve the negation of the conclusion.
- [36] If you believe that the premises imply the conclusion then you are obliged to bring it about that if you believe the premises then you do not disbelieve the conclusion.

**Deontic operator is permission.**

- [37] If you believe that the premises imply the conclusion then you may bring it about that if you believe the premises then you believe the conclusion.
- [38] If you believe that the premises imply the conclusion then you may bring it about that if you believe the premises then you do not believe the negation of the conclusion.
- [39] If you believe that the premises imply the conclusion then you may bring it about that if you believe the premises then you disbelieve the negation of the conclusion.
- [40] If you believe that the premises imply the conclusion then you may bring it about that if you believe the premises then you do not disbelieve the conclusion.

**Deontic operator is 'a reason'.**

- [41] If you believe that the premises imply the conclusion then you have reason to bring it about that if you believe the premises then you believe the conclusion.
- [42] If you believe that the premises imply the conclusion then you have reason to bring it about that if you believe the premises then you do not believe the negation of the conclusion.
- [43] If you believe that the premises imply the conclusion then you have reason to bring it about that if you believe the premises then you disbelieve the negation of the conclusion.
- [44] If you believe that the premises imply the conclusion then you have reason to bring it about that if you believe the premises then you do not disbelieve the conclusion.

**Deontic operator is forbidden.**

- [45] If you believe that the premises imply the conclusion then you are forbidden to bring it about that if you believe the premises then you believe the negation of the conclusion.
- [46] If you believe that the premises imply the conclusion then you are forbidden to bring it about that if you believe the premises then you do not believe the conclusion.
- [47] If you believe that the premises imply the conclusion then you are forbidden to bring it about that if you believe the premises then you disbelieve the conclusion.
- [48] If you believe that the premises imply the conclusion then you are forbidden to bring it about that if you believe the premises then you do not disbelieve the negation of the conclusion.

While this is a long list, it is not exhaustive. I use only the antecedent conditions ‘you believe that the premises imply the conclusion’ and ‘you believe the premises’, however, there are other candidate conditions: that the premises really do imply the conclusion, that you ought to believe the premises, and that the premises are true. In addition, in the ‘E’ principles, the second disjunct contains the same deontic operator as the first and the polarity ‘do not believe’; both might be varied to create more principles.

There is also a general problem about the way that the deontic operators are used in philosophical discussion: obligation is very strict, and ‘a reason’ is very weak, but the middle ground seems open. If I ought to brush my teeth every night, does it follow that I ought to brush my teeth tonight? It seems that the obligation for me to brush my teeth tonight is weaker than the obligation for me to brush my teeth every night – certainly the cost of not brushing my teeth tonight is much less than the cost of not brushing my teeth every night. While it seems clear that I have a reason to brush my teeth tonight, saying that I have a reason to brush my teeth every night seems implausibly weak. Perhaps a deontic operator that is weaker than ‘ought’ but stronger than ‘a reason’ should be implemented in a bridge principle?

Another point to consider is the epistemic operators. In the bridge principles, I have referred to belief only, and omitted knowledge, but the two are not the same. In the bridge principle it is possible to specify the two conditions for knowledge that are widely agreed upon – truth and belief, but knowledge is generally thought to have a third condition. What that condition is, is under dispute. Introducing knowledge or grades of belief might allow one to be sensitive to the impact that the initial conditions might have on the direction of belief revision. To satisfy, ‘If you believe that the premises imply the conclusion and you believe the premises then either you are obliged to bring it about that you believe the conclusion or you are obliged to bring it about that you do not believe the premises’, you must do one or the other but it is not clear which. Having moderate belief that the premises imply the conclusion, weak belief in the premises, and strong disbelief in the conclusion, suggests that you should stop even weakly believing the premises.

Finally, there is causation and time indexing to consider. It seems that the antecedent conditions must occur first, before the obligation arises. This is probably driven by the intuition

that having the beliefs you do is the cause of the obligation(/permission/reason) to believe the consequences. So perhaps  $B(\varphi \vDash \psi) \& B\varphi \rightarrow OB\psi$  should be translated as: Because you believe that the premises imply the conclusion and you believe the premises, you are obliged to bring it about that you believe the conclusion.

### *Testing Bridge Principles*

A particular advantage afforded by categorising bridge principles this way is that it allows one to predict what the weaknesses of a newly suggested bridge principle are likely to be. For example: Heathcote's  $(p \rightarrow q) \wedge OBp \rightarrow OBq$  has the same characteristics as Macfarlane's  $Bo^+$  and is therefore likely to be subject to the same criticisms, and Broome's  $(p \rightarrow q) \rightarrow O(Bp \rightarrow Bq)$  has the same relation to  $Wo^+$  and likewise one can expect it to fall to the same criticisms.

There are some general comments about the consequences of the various possible antecedent conditions. In the first place, if the antecedent conditions contain both  $Bp$  and  $OBp$ , then, "...logic is only normative for those whose beliefs are already in order—that is, for those who believe what they ought to believe (or may believe, or have reason to believe). To the unfortunate others, logical norms simply do not apply" (MacFarlane 2004, p. 10). Next, if the antecedent conditions contain  $B(p \rightarrow q)$  or 'if you know that  $(p \rightarrow q)$ ', then it seems that one is only governed by the norms of logic to the extent that one has beliefs about logic. Which means that "The more ignorant we are of what follows logically from what, the freer we are to believe whatever we please—however logically incoherent it is" (MacFarlane 2004, p. 12). If there is the antecedent condition that  $p$  really does imply  $q$ , then, if the premises imply the conclusion, but you don't believe that they do, would you still be obliged to believe the conclusion? What if the premises do not imply the conclusion, but you believe that they do? Surely, you are being a more responsible epistemic agent if you believe the conclusion than if you do not — even though the conclusion does not follow. So, a difficult question about the antecedent conditions is whether one's logical obligations follow from one's beliefs about logic or the logical facts.

The first test for a bridge principle is that it must not entail that belief is self-licensing. Broome says:

Suppose a proposition  $q$  follows from a proposition  $p$  by a valid inference. That is to say:  $p \vDash q$ . Now suppose you believe  $p$ . Then a process of correct reasoning will bring you to believe  $q$ . ... However, it is not necessarily the case that you ought to believe  $q$ , nor that you have a reason to believe  $q$ . For example, suppose you ought not to believe  $p$ , though you do. Then it plainly may not be the case that you ought to believe  $q$  or that you have a reason to believe  $q$ . So we cannot say either that:  $Bp$  oughts  $Bq$  or that  $Bp$  reasons  $Bq$ . To reinforce this point, remember that  $p$  itself is a consequence of  $p$ . A belief in  $p$  is plainly not self-justifying, so it cannot be that either:  $Bp$  oughts  $Bp$  or  $Bp$  reasons  $Bp$ .  
(Broome 1999, p. 405)

This is the point that Musgrave makes against  $(p \models q) \wedge Bp \rightarrow OBq$ : this principle entails that having a belief is sufficient for an obligation to have that belief. The same can be said of  $B(p \models q) \wedge Bp \rightarrow OBq$ ; believing a thing and believing that it is a logical consequence of itself obliges one to believe it.

The second test for a bridge principle is that it must not be excessively demanding. A principle will be excessively demanding when a competent reasoner could not reasonably be expected to comply with the norm. This test finds voice in the triviality objection to  $(p \models q) \wedge OBp \rightarrow OBq$ : could ‘q’ be some statement so trivial that an obligation to believe ‘q’ would be excessively demanding? This same test is present in the charge that for  $(p \models q) \rightarrow O(Bp \rightarrow Bq)$  the best way to comply with this norm is to cease having beliefs. Because the logical consequences of any given premise are infinite, this objection is relevant wherever the principle requires that you adopt some positive belief or disbelief stance. So,  $(p \models q) \wedge OBp \rightarrow OBq$ ,  $(p \models q) \rightarrow O(Bp \rightarrow Bq)$ ,  $Co^+$ ,  $Bo^+$ ,  $Wo^+$ , [1], [3], [14], [16], [17], [19], [30], [32], [33], [35], [46], and, [48] must all be examined with this objection in mind.

The next thing to consider for a potential bridge principle is whether it is strict enough. As Broome says, “The relation between believing p and believing q is strict. If you believe p but not q, you are definitely not entirely as you ought to be” (Broome 1999, p. 406). There is a real sense in which principles where the deontic operator is permission or ‘a reason’ are not strict enough – you have reason, or permission to draw the conclusion but you are not required to do so. This runs counter to intuition. While there might be cases where conclusion drawing is not required, there would also be cases where you would be failing in your epistemic duty if you did not believe the conclusion. The problem with permission and ‘a reason’ is that they do not sanction against failure to comply with the principle.

Some principles are designed to allow for believing that the premises imply the conclusion, and believing the premises, but not taking a stand on the conclusion one way or the other. This is the intuitive response to the objection that a bridge principle is excessively demanding. But it has the unintended consequence of making it permissible to be logically obtuse, to acknowledge an argument as valid, but refuse to take any kind of action regarding premises or conclusion. Being logically obtuse should not be an acceptable general tactic, because if it were, how would we achieve persuasion? Surely, persuasion occurs when someone agrees that the premises imply the conclusion and agrees with the premises, because then, in some sense, they must believe the conclusion.

Yet, the intuition that it is not permissible to be logically obtuse stands in direct opposition to the intuition that in some situations of paradox we are not required to find a resolution. As Harman says:

Even the rule “Avoid inconsistency!” has exceptions. ...On discovering one has inconsistent beliefs, one might not see any easy way to modify one’s beliefs so as to avoid the inconsistency, and one may not have the time or ability to figure out the best response. In that case, one should (at least sometimes) simply acquiesce in the contradiction while trying to keep it fairly isolated. I would think this is the proper attitude for most ordinary people to take toward many paradoxical arguments. Furthermore, a rational fallible person ought to believe that at least one of his or her beliefs is false. But then not all of his or her beliefs can be true, since, if all of the other beliefs are true, this last one will be false. So in this sense a rational person’s beliefs are inconsistent. It can be proved they cannot all be true together.

(Harman 1984, pp. 108–109)

In this situation, a person might believe that contradictory premises imply every conclusion, but also have rationally chosen to believe contradictory things (he does not have the time or ability to sort it out), but it would be irrational for him to come to believe everything because of believing a contradiction. Here the rational course of action is to refuse to draw the conclusion that he believes follows from his beliefs, but this is the exception, not the rule.

The objections from strictness, excessive demands, logical obtuseness, and inconsistency avoidance point to a tension among our intuitions. In some cases, failing to believe the conclusion means that you fail in your epistemic duty, but in others, it is permissible for you to not believe the conclusion; but the bridge principle requires that you adopt a consistent practice one way or the other. This is not the only tension among our intuitions.

The other major tension involves an obligation to believe the truth. If one believes things that are false, or believes things for frivolous reasons, does one have an obligation to believe their logical consequences? If one believes a false thing for good reasons then it seems reasonable to say that one does have an obligation to believe the logical consequences of that belief. It also seems that if one has only frivolous reasons to believe a true thing then one does not have an obligation to believe it. Further, if one does believe something for frivolous reasons it seems reasonable to say that one might have no obligation, reason, or permission to believe its logical consequences. The bridge principles I have given do not distinguish between things believed for good reasons and things believed for frivolous reasons. Even if one had a principle that required belief in only the significant consequences of things believed for good reasons, the question, “To what extent do we have an obligation to believe the truth?” remains. Imagine that everything that a person believes, he believes for good reason, and that he has no contradictory beliefs, but everything he believes is false. According to his bridge principle he should believe every significant consequence of his beliefs, if he does this, has he fulfilled his epistemic obligations, or is there some further epistemic obligation to believe true things which would mean that he has failed to fulfil his epistemic obligations?

## *Which is true, logic or the bridge principle?*

Heathcote's critics aim to prove that Heathcote's bridge principle is false. The principle and one or more logical theorems take the place of premises in a valid argument, and then the critic argues that the conclusion of this argument is false, therefore, the principle must be too. The first strategy that should be employed against this argument is to search for a bridge principle that can be combined with the principles of classical logic without producing apparently false normative statements. However, a thorough investigation of the potential principles reveals just how unlikely it is that a suitable candidate will be found. There is another weakness to the argument that the bridge principle is false. This weakness is due to the *reductio ad absurdum* form of the argument, the falsity of the conclusion assures us that at least one of the premises must be false, but it does not say which. Pigden and Musgrave claim that it is the bridge principle that is false, but one could also claim that it is ' $p \models (p \vee q)$ ' that is false. This is essentially the relevantist's argument: normativism is true, so classical logic must be false. It does follow, if normativism is true (and there is no true bridge principle) then classical logic is false. Yet, through the falsity of the conclusion, we can also establish that if classical logic is true, then normativism is false. Given the semantic structure of classical logic ' $p \models (p \vee q)$ ' is an analytic truth, but that does not suffice to establish classical logic as true.

Establishing the truth of any particular set of logical rules is a perplexing exercise. An early argument that touches on this problem is in "What the Tortoise said to Achilles" (1895) by Lewis Carroll. The claim is, using a certain tactic, the Tortoise can indefinitely, and legitimately, refuse to accept the conclusion of an argument. Achilles forwards the argument:

- A Things that are equal to the same are equal to each other.
- B The two sides of this Triangle are things that are equal to the same.
- Z The two sides of this Triangle are equal to each other.

But, the Tortoise answers, the conclusion depends on an assumed premise:

- C If A and B are true, Z must be true.

So the Argument is reformed with this premise added:

- A Things that are equal to the same are equal to each other.
- B The two sides of this Triangle are things that are equal to the same.
- C If A and B are true, Z must be true.
- Z The two sides of this Triangle are equal to each other.

But, again the tortoise answers, Z depends on an assumed premise:

- D If A and B and C are true, Z must be true.

We can give the argument a modern spin:

- A'  $A \supset B$
- B' A
- Z' B



This rests on the truth of:

$$C' \quad A \supset B, A \vDash B$$

In her article “The Justification of Deduction” Susan Haack argues that one cannot justify ‘ $A \supset B, A \vDash B$ ’ without circularity. Haack aims to demonstrate this circularity. First she gives the justification of *modus ponens*, then she gives a parallel justification for a fictional rule, *modus morons* (otherwise known as the fallacy of affirming the consequent), and then challenges the reader to defend the validity of *modus ponens* while maintaining the invalidity of *modus morons*. The justification of *modus ponens* that Haack gives is: “Suppose C (that ‘A’ is true and that ‘ $A \supset B$ ’ is true). If C then D (if ‘A’ is true and ‘ $A \supset B$ ’ is true, ‘B’ is true). So, D (‘B’ is true too.)” (Haack 1976, p. 114). And the justification for *modus morons* is: “Suppose D (if ‘ $A \supset B$ ’ is true, ‘B’ is true). If C, then D (if ‘A’ is true, then, if ‘ $A \supset B$ ’ is true, ‘B’ is true). So, C (‘A’ is true)” (Haack 1976, p. 115). Haack says that arguments that show the validity of *modus ponens* by the truth table will fail because, “...arguments from the truth-table to the justification of a rule of inference are liable to employ the rule in question” (Haack 1976, p. 117). Stephen Read refers to the same objection when he says, “The truth-condition... for a conjunctive statement of the form ‘A and B’ is itself a conjunction - ‘A and B’ is true if ‘A’ is true and ‘B’ is true. Is there a vicious regress, or even a circularity here? Are we already presupposing what we are trying to explain[?]” (Read 1995, p. 21).

So when we consider the argument:

$$\begin{aligned} & (\varphi \vDash \psi) \rightarrow (OB\varphi \supset OB\psi) \\ & p \vDash (p \vee q) \\ & OBp \supset OB(p \vee q) \\ & OBp \\ & \therefore OB(p \vee q) \quad \text{for any arbitrary } q? \end{aligned}$$

Given that ‘ $OB(p \vee q)$ ’ could be false, the argument is invalid, but, ‘ $(\varphi \vDash \psi) \rightarrow (OB\varphi \supset OB\psi)$ ’ is not the only candidate for rejection, ‘ $p \vDash (p \vee q)$ ’ is up for grabs too. If we assert that ‘ $(\varphi \vDash \psi) \rightarrow (OB\varphi \supset OB\psi)$ ’ is true, then we would have to give up ‘ $p \vDash (p \vee q)$ ’. If we assert that ‘ $p \vDash (p \vee q)$ ’ is true, then we must give up ‘ $(\varphi \vDash \psi) \rightarrow (OB\varphi \supset OB\psi)$ ’. Since it seems unlikely that any bridge principle can be true at the same time as all the rules of classical logic, either we must abandon classical logic, or we must not explain what classical logic is about in a way that implies that some bridge principle is true.

Psychologism can be rejected because people reason illogically as well as logically, but it is also plausible to reject psychologism because rather than being a philosophy of logic that explains classical logic and the syllogistic, it rules them out for failing to describe the way people think. One can reject normativism on the same grounds – normativism is legitimate enough as a philosophy of logic, but it rules out classical logic for failing to describe the way people ought to think. So then, the problem is to give a philosophy of logic that explains what classical logic is about.

# Rejecting Normativism

## *Logical Connectives and Meaning*

One way to tackle the question of what classical logic is about, is to explain what makes  $(A \supset B), A \vDash B$  true, and  $(A \supset B), A \not\vDash B$  false. Arthur Prior raises a similar question by way of his article, “The Roundabout Inference-Ticket”. In this article Prior introduces an operator ‘tonk’ about which he says, “Its meaning is completely given by the rules that (i) from any statement  $P$  we can infer any statement formed by joining  $P$  to any statement  $Q$  by ‘tonk’ (which compound statement we hereafter describe as ‘the statement  $P$ -tonk- $Q$ ’), and that (ii) from any ‘contonktive’ statement  $P$ -tonk- $Q$  we can infer the contained statement  $Q$ ” (Prior 1960, p. 39). A logic containing this operator would be trivial in the sense that anything implies everything. Prior introduces ‘tonk’ to refute the idea that the meaning of a connective is given entirely by its introduction and elimination rules. One could hold that it is so, but Prior’s challenge is to demonstrate a principled way to avoid the trivial logic. In “Roundabout the Roundabout Inference-ticket”, J.T. Stevenson responds with the point that ‘tonk’ is in violation of the rules for connectives with a truth tabular semantics; and Nuel Belnap’s response, in “Tonk, Plonk and Plink”, is to say that introduction and elimination rules must be in harmony with one another. Both responses could be seen as a straight concession of Prior’s point that the meaning of a connective is not *entirely* given by its introduction and elimination rules. Stevenson’s response abandons any attempt at locating the meaning of the connective in the introduction and elimination rules, while Belnap tries to maintain the spirit of inferentialism – Roughly, the view that the meaning of a connective is given by its introduction and elimination rules. And in “Rule-circularity and the Justification of Deduction”,

Tennant remarks that, “Tonk has the grammatical features of a binary connective, to be sure; but it has no sense, no semantic value at all” (Tennant 2005, p. 638). In a logic composed of binary connectives with a two-valued semantics ‘tonk’ is not a legitimate connective because it has no semantic value. The semantics of a logic place constraints on what counts as a legitimate connective as well as fixing the meaning of those connectives.

Tennant uses a proof theoretic method to show why ‘tonk’ is not a legitimate connective and he claims that the same thing will go for other binary connectives in a two-valued logic (Tennant 2005, p. 626). Stevenson also argues that ‘tonk’ has no semantic value, but he uses a truth tabular semantics to do it. He says, “[Prior] gives the meanings of connectives in terms of permissive rules, whereas they should be stated in terms of truth-function statements in a meta-language” (Stevenson 1961, p. 127). This method can be adapted to show the truth of  $(A \supset B), A \vDash B$ , and the falsity of  $(A \supset B), A \not\vDash B$ . The truth table for  $\supset$  is:

A	B	$A \supset B$
T	T	T
T	F	F
F	T	T
F	F	T

We can say there is no interpretation under which B is false where A and  $A \supset B$  are true, thus ' $(A \supset B), A \vDash B$ ' is true. However, given the truth table of ' $\supset$ ' ' $(A \supset B), A \vDash B$ ' must be false because there is an interpretation under which A is false where B and  $A \supset B$  are true.

A	B	$A \supset B$
T	T	T
T	F	F
F	T	T
F	F	T

The truth of ' $(p \supset q) \wedge p \vDash q$ ' and falsity of ' $(p \supset q) \wedge q \vDash p$ ' is necessitated by the truth table that gives the meaning of ' $\supset$ '. Given the semantic meaning of an operator and the definition of validity, we can sort inferences containing that operator into the valid and the invalid. Given the logic to which the sentence  $(p \supset q) \wedge q \vDash p$  belongs, we can say that it is false; what justifies the semantics being this way?

### ***Logic is a Science***

This question could be answered a number of ways, but there is one that is often overlooked. In the article "Logic and Reasoning" Gilbert Harman discusses the same subject that has been the focus of my thesis: the relationship between logic and reasoning. Harman suggest that it might be possible to view logic as having no special relationship to reasoning, He says:

In this view logic is merely a body of truths, a science like physics or chemistry, but with a more abstract subject matter and therefore a more general application. This is an extreme view that no one seems to hold in an unqualified way, which is surprising, since the view seems to be quite viable. Frege may seem to take the extreme view when he says the laws of logic are laws of truth and since he attacks "psychologism"; but he also says the laws of logic "prescribe universally the way in which one ought to think if one is to think at all", which is to reject the extreme view. Similarly, Quine may seem to advocate the extreme view when he says logic is a science of truths. But he also sees a special connection between logic and inference when he says one needs logic to get to certain conclusions from certain premises. As far as I have been able to determine, other philosophers who may seem at one place to put forward the extreme view that logic is a science, a body of truths, go on someplace else to say that logic has a special role to play in reasoning.

(Harman 1984, pp. 109–110)

Musgrave would endorse the "extreme view", as Harman calls it, that logic is a science. He says, "It has long been well-known that logical facts are one thing, psychological facts about what we believe another. ...It is less well-known that logical facts are one thing, and normative principles

about what we ought to believe another. Just as one cannot ‘read off’ psychology from logic, so also we cannot ‘read off’ the ethics of belief from logic” (Musgrave 2010, p. 123). While arguing the same point in an earlier paper Musgrave says, “...the logician’s task is not, as the Wittgensteinians seem to suppose, merely to describe the ways people think or talk. The logician’s prime concern is with the validity or invalidity of arguments. The distinction between valid and invalid arguments cannot be drawn from an empirical inquiry into the way people in fact argue, for people argue invalidly as well as validly” (Musgrave 1999, p. 87). I have argued that connecting logic with arguments inclines one to identify validity with goodness-of-argument and invalidity with badness-of-argument, and that this identity does not hold. If we took this quote at face value, Musgrave would seem to be conforming to Harman’s point that many may seem at one place to put forward the extreme view, but at others seem to say that logic has a special role to play in reasoning. Perhaps the explanation to Harman’s puzzle that no one seems to endorse the view that logic is a science in an unqualified way is that there is a path between the two. Giving an explanation of what logic is about as a body of truths, and then explaining how this body of truths relates to reasoning. So what can be made of the idea that logic is a science? We can look to Frege for hints on how to spell out logic as a science. Harman seems to me to be mistaken to take the quote from Frege that “[The laws of logic] prescribe universally the way in which one ought to think if one is to think at all” to mean that Frege does not endorse the view that logic is a science. The passage to which Harman refers is a very dense passage. Frege appears to be considering what sense could be made of the phrase ‘laws of thought’. He first considers that logic might be descriptive of reasoning and quickly dismisses that interpretation. He then considers that logic might be descriptive of how we ought to reason, and dismisses that because it begins with a confusion between something’s being taken to be true with it being true. The passage expresses a very particular idea of truth, and a hint at how this idea might relate to thinking. He says:

Being true is different from being taken to be true, whether by one or many or everybody, and in no case is to be reduced to it. There is no contradiction in something’s being true which everybody takes to be false. I understand by ‘Laws of logic’ not psychological laws of takings-to-be-true, but laws of truth. If it is true that I am writing this in my chamber on the 13th of July, 1893, while the wind howls out-of-doors, then it remains true even if all men should subsequently take it to be false. If being true is thus independent of being acknowledged by somebody or other, then the laws of truth are not psychological laws: they are boundary stones set in an eternal foundation, which our thought can overflow, but never displace. It is because of this that they have authority for our thought if it would attain to truth. They do not bear the relation to thought that the laws of grammar bear to language; they do not make explicit the nature of our human thinking and change as it changes.

(Frege 1982, p. 13)

Frege has a realist conception of truth. Truth is independent of our ability to speak or think. “I don’t exist” might be inherently contradictory because “I exist” is implied each time I speak it or think it – nevertheless that does not mean that “I don’t exist” cannot be true. For me to go about

actually thinking that it is possible that I don't exist would be an offence against good reason, but belief in a proposition matters not at all to its ultimate truth or falsity. How does the realist conception of truth relate to logic as a science? Stephen Read explains that for the realist, "Truth is objective. The world is a world of facts which make our judgements objectively true or false" (Read 1995, p. 7). Read explains that this understanding of truth has two important features: it is epistemically unconstrained, and is committed to ontological realism. He says, "Truth, on the realist picture, is epistemically unconstrained. Propositions have truth-values without regard to the possibility of our finding them out. ...The link with ontological realism is an attempt to explain that independence. What is it, independent of us, that makes true propositions true?" (Read 1995, p. 203). The ontological realist is committed to truth-makers – that which makes true propositions true. Read explains that objective truth-values lead to bivalence, because, "It is natural to think that either an object exists or it doesn't... [and] by linking the condition for truth of a proposition to a corresponding object... we are naturally led to Bivalence ...every proposition is true or false – and is so, regardless of our ability to discover it" (Read 1995, p. 11). From the idea that every proposition is either true or false, it is a straight shot to the binary operators in a two valued semantics, and after defining a valid inference as one in which it is impossible for the premises to be true and the conclusion false, propositional logic follows neatly.

### ***Classical Logic is one thing, and the ethics of belief quite another***

There is another way in which logic might be related to reasoning, which I have so far not considered, it will seem obviously true, but it does nothing to support the normative hypothesis. Remember that the normative hypothesis is usually brought in to explain what logic is about: "Logic", it says, "tells us how we ought to reason"; or, "Logic is about inference: what may be legitimately inferred from what"; or, "Logic is about the validity or invalidity of arguments" – which conjures up discussion and right-reasoning not schemas and proofs that may bear only family resemblance to real-world arguments. Frege says laws of logic, "...have authority for our thought if it would attain to truth" (Frege 1982, p. 13). Priest makes this same suggestion when he says, "We reason about all kinds of situations. We want to know what sorts of things hold in them, given that we know other things; or what sorts of things don't hold, given that we know other things that don't. If we reason validly then, by the definition of validity, we can be assured that reasoning forward preserves the first property, and that reasoning backwards preserves the second. Validly is how one ought to reason if one wants to achieve these goals. The obligation is, then, hypothetical rather than categorical" (Priest 1999, p. 202). Because the way that the criterion of validity is structured, it ensures that if one reasons validly, then one can never reason from truth to falsehood. This might explain the relationship between logic and reasoning, but it cannot be used as an explanation of what logic is about. It does not explain what logic is about because it comes out as

the rather watery statement, “If you want to achieve the ends that this logic is designed to achieve, then you ought to use this logic”. It is true enough, but one wonders what it is supposed to reveal; perhaps that you should view different logics as you would different kinds of hammers – all of a similar kind, but some are more fit for particular purposes than others?

Accepting the hypothetical imperative might also incline one to the view that because the rules of classical logic are the way they are, we ought to reason in accordance with them. This is another way to approach the combining of the rules of logic with a bridge principle to produce a norm. If it contradicts a standing social norm, that does not indicate that the norm produced was false, rather that indicates that we have a norm that we ought not to have. Logic is not normative in the sense that it is intended to produce sentences that describe ideal reasoning. Logic is normative in the sense that it prescribes ideal reasoning. Again, there seems to be nothing immediately wrong with this view, but there are many logics, and all of them, even the trivial logic would be prescriptive in this way. This makes it a silly way to explain what logic is about. If you produced a trivial logic and then carefully explain that your conclusion is true because you reached by following the rules of you logic, whomever you explain this to is going to be very annoyed with you. In order so some logic to be prescriptive this way we must all agree that the logic we use is the right logic. Naturally, I cannot say that this is the right logic because according to this logic all its rules are true, as there are other logics that say just the same about their own rules.

For developing the view that logic is a science Harman makes the encouraging comment that, “So, as far as I can see, no serious objections have ever been raised to the view that logic is a science, like physics or chemistry, a body of truths, with no special relevance to inference except for what follows from its abstractness and generality of subject matter” (Harman 1984, p. 111). Logic as a science might be spelt out with a combination of a realist conception of truth and a hypothetical imperative. A hypothetical imperative is very flexible. It allows for the view that logic is one thing and the ethics of belief another. The hypothetical imperative may occur in an Ethics of Belief along with a collection of other principles only coming into force when some other principle requires it. Once you have explained a logic, based on a theory of truth it is also no longer silly to suggest that norms of reasoning should be revised to conform to that logic. Of course, it might be a little extreme to suggest that all norms for reasoning strictly follow the rules of that logic, but it seems reasonable to allow for exceptions. Generally, you should not believe contradictions, but there are circumstances where it is permissible. If logic is decoupled from the ethics of belief, then ethics of belief might contain general rules along the lines of, ‘you ought to reason validly’, but these rules could have exceptions. While doing this we must be careful that we do not once again associate logic with reasoning. In arguing against the standard account of validity Priest says, “The alternative account turns this procedure on its head. The notion of truth is taken as primary.

Validity is defined in terms of truth-preservation and, so defined, is taken to provide the norms for a social practice, inferring” (Priest 1979, p. 297). To develop the notion of logic as a science, one must take the first two steps - take the notion of truth as primary, and define validity in terms of truth preservation, but one must resist the third step - taking these to provide the social norms for the practice of inferring. The social norms for the practice of inferring might be informed by logic but they are not necessarily bound to it.

## Conclusion

What is the relation between logic and thinking? The initial view was psychologism: the view that logic is descriptive of the reasoning process. It is common now to say, “Logic does not describe how we reason (because we can reason badly), rather, logic describes how we ought to reason”, but no one has really examined whether this really provides a better philosophy of logic than psychologism. I refer to the view that logic is descriptive of the ideal reasoning process as normativism, because it implies that there is some true normative proposition roughly of the form ‘if  $p$  implies  $q$  and you believe  $p$ , then you ought to believe  $q$ ’. The psychologistic critique of syllogistic or classical logic is that it fails to describe how we reason. Of course, in principle, formal methods are just as capable of describing how we do reason, as they are capable of describing how we ought to reason. It is not wrong to take psychologism seriously, and if you do, then logics that do not describe reasoning are failing in a significant way. Normativism uses the same argument structure as psychologism. The normative critique of logic is that it does not describe how we ought to reason. To respond to this argument, one must either, admit that the logic in question suffers from a significant failure, or one must reject normativism. There are a number of consequences to normativism. The first and most significant is that classical logic is failing in a significant way – it seems impossible to reconcile classical logic and a bridge principle with the norms of reasoning, but if normativism is true, then one would expect that one could arrive at the norms of reasoning by combining logic and a bridge principle. The second consequence is that there is a real possibility of bridging the Is/ought gap. Finally, one could argue that inductive arguments are valid.

In “Hume’s Master Argument”, two of Heathcote’s arguments turn on the truth of normativism. His first argument is that inductive arguments can be valid. This is a consequence of normativism if the same normative proposition that is true for deductive arguments is true for inductive arguments. Inductive arguments are valid because under normativism, validity is about the truth of a normative proposition like ‘if  $p$  implies  $q$ , and you ought to believe  $p$ , then you ought to believe  $q$ ’, and not about necessary truth preservation. The second argument is that Hume’s Is/Ought gap is bridged. This is a consequence of normativism if one grants that the normative proposition is analytic, or that is true but purely descriptive, because then it seems that we would be able to move from statements of fact to statements of obligation. While Heathcote’s argument is relevant for determining the consequences of normativism, it is also relevant because it opens debate on principles of theoretical rationality (the normative proposition that would have to be true for normativism to be true), which is also referred to as a ‘bridge principle’ because of the potential for a bridging of the Is/Ought gap. Musgrave and Pigden both criticise Heathcote’s bridge principle, saying that the principle is false because when combined with the laws of logic, the norms produced are false.



Because there is no established literature for Heathcote to turn to when examining principles of theoretical rationality, Heathcote refers to Broome's discussion of principles of practical rationality in "Normative Requirements". Heathcote criticises Broome's principle (although at this point it is less Broome's principle and more Broome's principle adapted to fit within Heathcote's discussion of theoretical rationality) and puts forward his own. This is a reasonable turn because principles of theoretical rationality and practical rationality have similar structural features. For example, 'if you believe  $p$  is a means to  $q$ , and you desire  $q$ , then you ought to  $p$ ', or 'if you believe  $p$  is a means to  $q$ , and you ought to  $q$ , then you ought to  $p$ ', are two potential principles of practical rationality. These are clearly very similar to 'if  $p$  implies  $q$  and you believe  $p$ , you ought to believe  $q$ '. The similar structural features provide an intuitive reason to believe that objections to the one kind of principle can be adapted to be objections to the other kind of principle. I examine Broome's discussion of practical rationality.

MacFarlane too refers to principles of theoretical rationality as bridge principles, possibly because such a principle would be a bridge between logic and the ethics of belief (or the norms of thought). In "In What Sense (If Any) Is Logic Normative for Thought?" MacFarlane uses a systematic method to produce bridge principles for examination. MacFarlane uses three variables to create a table of eighteen candidate bridge principles, which can be doubled by changing the antecedent condition 'the premises imply the conclusion' to 'you know that the premises imply the conclusion'. The method for developing bridge principles that MacFarlane uses is helpful for systematically examining bridge principles. Ultimately, there are too many variables to create a neat table of all the potential bridge principles, but the method does allow a more systematic consideration of each of the variables. What is revealed through such a consideration is that the most plausible bridge principles are very complex sentences. This complexity makes it very hard to determine the truth of the bridge principles in question, but there are intuitive reasons to suspect that many of the bridge principles are not true. There is also a real possibility that a bridge principle will either, state that there is an obligation where there isn't one, or fail to capture some obligation that is there – a bridge principle will either be too strong or too weak, leaving none that are just right.

Nevertheless, bridge principles cannot be dismissed entirely, because the argument used to criticise bridge principles has a *reductio* structure. Pigden and Musgrave hold the bridge principle responsible for the inconsistency between the predicted norm and the actual norm, but there are other legitimate strategies. One strategy would be to claim that the problem lies with the social norms, which should be adjusted to be in line with logic and a bridge principle. While this would render logic normative, it is unclear why this should be a significant, even defining, characteristic of logic. Particularly since this strategy can be used to defend any logic/bridge principle combination

– even the trivial logic, and if there is anything that ought not to inform our norms for inferring it is the trivial logic.

Another much more powerful strategy for responding to the logic/bridge principle *reductio*, is to say that the logic is just as open to revision as the bridge principle. This strategy assumes that the purpose of logic is to tell people how they ought to think, and the existence of a bridge principle is a natural consequence of that. It is not specific about the bridge principle that needs to be chosen, nor does it specify a logic, but it is clear that an important adequacy criterion for logic is that in all reasoning circumstances people genuinely feel the normative requirements placed on them by their logic. This approach takes normativism seriously, and produces a normative critique of a logic. This approach has strong backing, because unless we can produce a justification for the rules of logic, it seems that preferring the simple elegant philosophy of logic that normativism provides would be best. However, justifying the rules of logic is no simple matter. One area of debate that may shed some light on the rules of logic is the discussion of the connective ‘tonk’. These discussions generally conclude that ‘tonk’ – an intuitively repugnant rule because it allows one to move from any premise to any conclusion – is not a legitimate connective, because the semantics of the logic rule it out. From this, we can infer that the rules of logic are justified by the logic’s semantics. Then the question is, “what is the reason for this semantic structure?”

Psychologism can be rejected because people reason illogically as well as logically, but it is also plausible to reject psychologism because it rules out classical logic and the syllogistic for failing to describe the way people think. One can reject normativism on the same grounds – normativism is legitimate enough as a philosophy of logic, but it rules out classical logic for failing to describe the way people ought to think. So then the glaring question is, “What is classical logic about?”

One possible avenue for justifying a semantic structure is to appeal to a theory of truth. Further explaining a theory of truth and how that relates to a logic may serve as an explanation of what that logic is about. Bivalence is an important characteristic of classical logic, and an appeal to a theory of truth that is committed to truth-makers and the idea that something is true quite independently of any person’s ability to know it, might justify having a bivalent semantics. Of course, this avenue is perused after rejecting the idea that logic describes how people ought to reason, but this does not mean that it cannot inform the norms for reasoning. If logic does not describe how people ought to reason, this does not mean that people have no obligation to reason logically. But the obligation to reason logically comes from the ethics of belief and we could expect that they are general rules that may have exceptions; the ethics of belief does not play any role in explaining what logic is about.

It is possible to explain what logic is about by appealing to the idea that logic tells us how we ought to reason; this implies that some bridge principle is true. It seems likely that there is no

combination of classical logic and a bridge principle that produces true normative statements, and this challenges classical logic's claim to being a proper logic. If one maintains that classical logic is a proper logic then one must explain what logic is about without appealing to the idea that logic tells us how we ought to reason. It is possible to explain what logic is about without appealing to the idea that logic tells us how we ought to reason. This does not mean that there is no relationship between logic and reasoning – but this relationship cannot explain what classical logic is about.

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