A Remark on Probabilistic Measures of Coherence

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Abstract

In recent years, some authors have proposed quantitative measures of the coherence of sets of propositions. Such *Probabilistic Measures of Coherence (PMCs)* are, in general terms, functions that take as their argument a set of propositions (along with some probability distribution) and yield as their value a number that is supposed to represent the degree of coherence of the set. In this paper, I introduce a minimal constraint on PMC theories, called 'the weak stability principle' (WWSP) and show that any correct, coherent and complete PMC cannot satisfy WWSP. As a matter of fact, the argument offered in this paper can be applied to any coherence theory that uses a priori procedures. I briefly explore some consequences of this fact.

1 Introduction

Recently, following an insight by C.I. Lewis [16], a number of formal, quantitative explications of the notion of coherence have been proposed under the name of *Probabilistic Measures of Coherence (PMCs)*. Such PMCs take as their arguments the probabilities of the propositions in the set whose coherence is to be established (and the probabilities of their Boolean combinations, if necessary) and yield as value some number that represents the degree of coherence of the aforementioned set. If an all-or-nothing answer is desired, a threshold can be adopted to decide whether the set is coherent or not.

These theories of set coherence can be found, for instance, in Shogenji [24], Olsson [19], Fitelson [9] and Douven and Meijs [7].[1] Thus, for example, Shogenji [24] proposes to measure the coherence of a given set of propositions $\{B_1, B_2, \ldots, B_n\}$ as follows:

$$C_S(\{B_1, B_2, \ldots, B_n\}) = \frac{Pr(B_1 \wedge B_2 \wedge \ldots \wedge B_n)}{Pr(B_1) \times Pr(B_2) \times \ldots \times Pr(B_n)}$$

Shogenji's measure equals 1 when all the propositions considered are jointly independent. Thus, 1 is a neutral point with respect to the degree of coherence. A value of the function which is greater than 1 indicates that the set of propositions is coherent,

[1] See also Glass [10], Meijs [17], Schupbach [23], Roche [21], Schippers [22] and Koscholke [13].

and a value of the function between 0 (included) and 1 (not included) indicates that the set is incoherent.

Olsson [19] presents another example of a PMC. According to Olsson, the degree of coherence of a given set of propositions $\{B_1, B_2, \ldots, B_n\}$ must be measured in the following way:

$$C_O(\{B_1, B_2, \ldots, B_n\}) = \frac{Pr(B_1 \wedge B_2 \wedge \ldots \wedge B_n)}{Pr(B_1 \vee B_2 \vee \ldots \vee B_n)}$$

In this case, $C_O(\{B_1, B_2, \ldots, B_n\}) = 0$ when $Pr(B_1 \wedge B_2 \wedge \ldots \wedge B_n) = 0$ and $C_O(\{B_1, B_2, \ldots, B_n\}) = 1$, which is the maximal degree of coherence, when $Pr(B_1 \wedge B_2 \wedge \ldots \wedge B_n) = Pr(B_1 \vee B_2 \vee \ldots \vee B_n)$.

PMC theories can be seen as ways of making the notion of coherence more precise than it customarily is in more traditional discussions, according to which sets of propositions are coherent when, for instance, the propositions in the set "hang together" or they "mutually support each other". These more precise characterizations of coherence are interesting in themselves, and, if successful, they could be fruitfully used in other areas, especially in coherentist theories of epistemic justification (see, for instance, BonJour [4] and Shogenji [25]).

PMC theories have been criticized on several grounds (see, for example, Siebel [26, 27], Akiba [2], Bovens and Hartmann [5], Olsson [20] and Koscholke and Schippers [14]). In this paper I introduce a minimal constraint on PMC theories and use it to show that there are no coherent, correct and complete PMC theories.

The minimal constraint I am going to use is the following one, which I call 'the weak stability principle':[2]

> (WSP) If a given set of propositions Γ logically imply a proposition ϕ, then adding ϕ to Γ does not decrease the degree of coherence of Γ.

WSP is generally presupposed as a sound principle in the literature on PMC theories. On the one hand, some authors defend the claim that adding or substracting logical implications from a set of propositions should not affect its degree of coherence (see, for example, Moretti and Akiba [18, p. 76] and Douven and Meijs [7, pp. 417-418]) and, hence, *a fortiori*, accept WSP.

On the other hand, most authors think that, if two propositions are logically equivalent (to wit, they logically imply each other), then they form a paradigmatic case of information hanging together in the appropriate way in order to count as coherent. This idea usually appears in the literature under the form of maximality constraints on coherence theories that require information sets to be maximally coherent when the

[2]This is a weakening of—and is named after—a principle introduced in Moretti and Akiba [18, p. 76]. Moretti and Akiba defend the notion that any PMC theory must meet their Stability Principle (SP): "No [set of propositions] changes its degree of coherence unless the believer adds any essentially new information to the set or drops any essentially old information from it." (Moretti and Akiba [18, p. 76]). Furthermore, they claim (and show, case by case) that the main PMC proposals available in the literature fail to meet SP and, therefore, should be rejected. Moretti and Akiba consider that adding logical consequences is a paradigmatic example of not adding essentially new information. Thanks to Manolo Martínez for very helpful comments on this point.

propositions contained therein are all logically equivalent (see, for example, the *Equivalence Desiderata* in Meijs [17, p. 235] and Fitelson [9]). Be that as it may, in this case WSP is also vindicated.[3]

In this paper, I offer an informal argument which shows, with the use of a slightly weaker version of WSP, that in general any PMC theory, Γ, as described above, such that it is a coherent, correct (that is, it yields the desired results with respect to the coherence of any given set) and complete (in the sense that it can decide, for any set of propositions, B, its degree of coherence) cannot satisfy WSP on pain of contradiction. The argument uses some of the ideas in classical diagonalization arguments such as those in Gödel [11] and Tarski [29].[4]

The paper is organized as follows. The next section introduces the main argument; in Section 3 I consider two objections to the argument; and finally, in the last section, I briefly discuss some consequences of the puzzle posed by the argument.

2 The Argument

One of the main reasons to maintain WSP is that given a set of propositions Γ and a proposition ϕ logically implied by Γ, adding ϕ to Γ should not decrease its degree of coherence because, in an epistemologically significant sense, ϕ was already present in Γ. The sense in which ϕ was already present can be made more clear if we think of an ideal believer S (who clearly sees the implicit logical consequences of Γ) who, given Γ and ϕ as above, just needs to apply some *a priori* procedures or algorithms (maybe some logical calculus) to obtain ϕ from Γ.

One of the insights I am going to use in the argument that follows is that the very same reason given concerning logical consequences can be given for the claims made *within PMCs themselves* regarding the coherence of sets of propositions.This is because such claims depend on the application of certain *a priori* mathematical algorithms, which can be supposed to be carried out when necessary by an ideal believer. As a matter of fact, I also claim that the same reasons apply to any propositions whose truth can be established with *a priori* procedures (cf. my fn. 8 below). Accordingly, I propose to weaken the notion of *logical implication* in WSP to a broader notion of *implication* which can encompass the claims made within the frameworks of PMCs about the coherence of sets of propositions and claims whose truth can be established with *a priori* procedures. In this way, I obtain then the following principle:

> (WWSP) If a given set of propositions Γ imply (in the aforementioned sense) a proposition ϕ, then adding ϕ to Γ does not decrease the degree of coherence of Γ.

Thus, suppose Γ is the set of propositions that constitute a certain PMC theory, as

[3] Thanks to an anonymous reviewer for suggesting this motivation for WSP.

[4] The result in this paper is more general than that achieved by Moretti and Akiba [18]; while they show that the PMC theories available in the literature cannot satisfy SP one by one (see my fn. 2 above), the argument I present herein depends only on a general characterization of PMC proposals and the weaker WSP. Furthermore, although I focus on PMC theories, the argument in this paper is applicable to any theory that establishes the coherence of sets of propositions through the use of *a priori* procedures.

described in section 1 above.[5] I do not need to dwell on the particular characteristics of PMC theories beyond the fact that they characterize such functions.[6] Now, Γ will typically characterize some function (in Shogenji's measure, for instance, it would be the function C_S) which will take as its arguments the probabilities of the propositions to be evaluated and the probabilities of their Boolean combinations and yield a number as a result. In the case of Shogenji's theory described above, Γ would contain, among many others, some proposition along the lines of <if a set of propositions A is coherent, then $C_S(A) > 1$>.[7]

Suppose now that Γ yields the result that a given set A is coherent; that is, after applying the function given by Γ to the propositions in A the result is above the threshold for coherence established by the theory.[8] Then, by WWSP, adding the proposition that A is coherent to Γ should not decrease the degree of coherence of Γ. Furthermore, for this particular application of WWSP, if we concede that Γ includes all the mathematics necessary to work with the values in the probabilistic measure at hand, then we can claim that $< A$ is coherent $>$ is just a logical consequence of Γ (so that in this case WSP is sufficient).

I will now proceed with the argument. First, consider the following set A:

$$A = \Gamma \cup \{< A \text{ is not coherent} >\}$$

where Γ is, as before, the set of propositions that constitute a PMC theory. As I have said, I will suppose that Γ is a coherent and correct theory that yields the desired results with respect to the coherence of any given set. I will also suppose that Γ is complete, in the sense that it can decide, for any set B, whether B is coherent or not.

Now, I can reason as follows. Suppose, first, that A is coherent. Then, Γ, if it is to be a correct and complete theory concerning coherence, must yield <A is coherent>. That means, by WWSP, that the degree of coherence of the following set, A', cannot be less than the degree of coherence of A, for I am simply adding to A one of its consequences:

$$A' = \Gamma \cup \{< A \text{ is not coherent} >\} \cup \{< A \text{ is coherent} >\}$$

But A' is clearly inconsistent and, consequently, it can hardly be coherent. Since, on the one hand, the degree of coherence of A' cannot be less than the degree of coherence

[5] I am assuming, for the moment, that theories can be expressed as sets of propositions in the sense defended in, for instance, Dummett [8, p. 405]. I will return to this issue in Section 4. Thanks to Manolo Martínez and an anonymous reviewer for helpful discussion on these points.

[6] Of course, given that the collection of all propositions is so large that it does not constitute a set, I am supposing some suitable restriction such that it does constitute a set, so that the procedures of PMC theories are functions. Thanks to Peter Pagin for advancing this worry to me.

[7] The symbols '$<$' and '$>$' surrounding a given expression, e, are used to indicate an expression referring to the propositional constituent expressed by e. Thus, when e is a sentence, '$< e >$' means *the proposition that e*.

[8] Strictly speaking, in order to evaluate the coherence of A, I also need the probabilities of the members of A: information that is not given in Γ. However, in this part of the paper I will idealize the situation and I will suppose that the notion of probability at work is such that we can have *a priori* access to it (the classical Laplacian or the logical Carnapian interpretation of probability). This means that, given (WWSP), the fact that I can access *a priori* to the relevant probabilities, and given the discussion on page 3, I can always add the propositions that contain the needed probabilistic information to a set without decreasing its degree of coherence, so that, in order to keep the argument simpler, such information can, for the moment, be ignored. At the end of Section 3 and the Appendix, I will return to this question.

of A and, on the other hand, the latter is coherent (by supposition) while the former is not, we have reached a contradiction. So, the supposition that A is coherent must be dropped: A is not coherent.

I can therefore conclude that A is not coherent. That means that Γ, which we are supposing is a sound and complete theory, must yield <*A* is not coherent>.

Hence, by WWSP, the degree of coherence of A cannot be less than the degree of coherence of the set

$$A'' = \Gamma,$$

for $A - A'' = \{< A \text{ is not coherent} >\}$ follows from A'' by virtue of the *a priori* procedures of the theory represented by Γ. Since I have just concluded that A is not coherent and that the degree of coherence of A must not be less than the degree of coherence of A'', I cannot but conclude that A'' is also not coherent. That is, I have just proved that Γ is not coherent. Since Γ is coherent by supposition, we reach a contradiction.

3 Two objections

At this point, it can be immediately replied that the set A is not well defined. The reason for this is that A contains a proposition that mentions A itself. However, this worry is not justified; there are very natural and harmless cases in which a set of propositions contains a proposition mentioning the set itself. For instance, I can claim that the set of propositions expressed by the sentences that constitute this paper is coherent or, at least, that I hope it is. It seems hard to deny that the set of propositions expressed by the sentences in this paper is well defined just because I mentioned that very same set. It is easy to come up with other ordinary harmless situations where a given set of propositions mentions that very same set. For example, it may be written in a book that the book itself is interesting, so that the proposition expressed by this claim involves the book itself. It would not be reasonable to claim that, in this case, the book becomes unintelligible or that the sentence claiming that the book is interesting does not express any proposition.

Furthermore, the needed effect can be achieved with propositions which mention themselves, with which some people feel more comfortable.[9] Having a proposition which mentions itself does not seem, at least immediately and without further philosophical development, harmful. Think, for example, of cases such as <this proposition is an abstract object>, or even <all propositions are structured entities>. To develop this idea further, we can follow Barwise and Etchemendy [3], who claim that there is no reason to suppose that there are things to which we cannot refer to, at least if those things can be made salient in some way or other; then, we can always use the demonstrative 'this' to refer to whatever has been made salient. This implies that "you can refer to any proposition whatsoever" (Barwise and Etchemendy [3, p. 15]) with the phrase 'this proposition'. In particular, nothing prevents us from using the phrase 'this proposition' embedded in a sentence to refer to "the very same proposition the

[9] As examples of authors who see self-referential propositions as unproblematic, see Horwich [12] and Barwise and Etchemendy [3].

embedding sentence is used to express" (Barwise and Etchemendy [3, p. 15]). What this means is that the only reason we may have to reject circular sentences would be a general prohibition against circularity, which (as examples like <this proposition is an abstract object> show) is not reasonable without further philosophical development.[10]

Let us therefore suppose that circular propositions are well defined and perfectly meaningful. Next, consider the following circular proposition, λ:

$$< \text{the set whose only elements are } \lambda \text{ and the members of } \Gamma \text{ is not coherent} >$$

which can now be used to define the set B as follows:

$$B = \Gamma \cup \{< \text{the set whose only elements are } \Gamma \text{ and } \lambda \text{ is not coherent} >\}$$

Note that the set whose only elements are Γ and λ is B and hence B contains a proposition that ascribes non-coherence to B. Hence, if circular propositions are well defined and meaningful, and sets like B above can be described using circular propositions, such sets are also well defined and meaningful. The argument in this paper can be executed with B and so, having a set which contains a proposition that mentions the very same set is not (at least not obviously) problematic.

The second objection I want to consider is the following one. Throughout the paper I have idealized the situation and I have supposed that the information on the probability of the elements of A is available, so that the necessary calculations using Γ can always be carried out.[11] At this point, somebody could complain that such an idealization cannot be made. In order to address this worry, let me represent the required information about the probabilities of the elements of A with a set Δ of propositions of the form <Pr(y)=x>, where y is a proposition in A. Then, the argument above can be reproduced with the following superset of Δ:

$$C = \Gamma \cup \Delta \cup \{< C \text{ is not coherent} >\}$$

in order to prove that the following set cannot be coherent:

$$C'' = \Gamma \cup \Delta$$

I think this result is already disturbing, for it shows that whenever we take a correct, coherent and complete PMC theory, we can turn the set formed by its propositions into

[10] Barwise and Etchemendy [3] also offer a model for circular propositions. They call 'Russellian propositions' (with no substantive historical intention) the claims about the world we make when we use sentences. Such claims have as constituents, in the simplest of the cases, just an object and a property; in which case, they constitute the claim that the object in question has the property indicated. Barwise and Etchemendy represent such propositions with set-theoretic objects. Of course, if we model circular propositions with set-theoretic objects, we need a consistent and coherent set theory without the axiom of foundation that allows for a given set to have itself as a member. Such an axiomatization of set theory, presented in Aczel [1], exists and is well known.

[11] This is a general problem any proposal using PMC theories has to meet (see Siebel [26, p. 336]). See also the discussion of *Case 3* in the Appendix and footnote 19 below for worries related to the cardinality of Δ. Thanks to an anonymous reviewer for pressing this point.

an incoherent one just by adding to it the members of Δ and there does not seem to be any reason why this should be the case.

To proceed, we must now ask ourselves what notion of probability we are using. Notice that if we are working with a classical or logical notion of probability, then we can have *a priori* access to Δ. That means that, by WWSP, adding Δ to any set should not decrease its degree of coherence and hence, since C'' is not coherent, neither is Γ.

This leaves as open the question concerning whether the same result would follow if we were working with a notion of probability according to which we could not have *a priori* access to the information provided by Δ —say, frequency or propensity interpretations of probability.[12] In this case the argument must be considerably weakened in order to achieve the conclusion that Γ is incoherent. However, I still think it retains its interest. I discuss it in the Appendix.

4 Discussion

The argument presented in this paper can be understood as a *reductio* argument. I took an arbitrary PMC theory, Γ, and I supposed that it was coherent, correct and complete. Since I concluded that it cannot be coherent, a contradiction appeared. Hence, some of the hypotheses used in the argument are not true. There are at least four points that I think can be considered in order to avoid this negative result.

First, it is interesting to note that basically the same argument as in Section 2 would show that A cannot be consistent, by taking Γ as a consistency theory and WWSP as a principle concerning consistency theories.[13] This suggests that the notion of coherence could have a structure similar to that of the notion of consistency and hence it would share some of the limitative results in Gödel [11] that affect the latter. Thus, mirroring the first incompleteness theorem, which, very roughly, says that if a theory is consistent, then it is incomplete, we can claim that PMC theories are incomplete—in the sense that they cannot decide on the coherence of certain sets—unless they are incoherent. In this case, the argument offered in this paper would show that, for any PMC account, a set can be found such that it cannot be established that the set is coherent and it cannot be established that the set is not coherent. This means that the functions at the core of all PMC proposals will necessarily be partial. It is also worth noticing that, as I said at the beginning, the argument I defend in this paper can be applied to any coherence theory that uses *a priori* procedures. This way of reading the argument is, to my mind, the most reasonable one; so the argument defended in this paper should be read as an incompleteness result involving the notion of coherence. This means that, if the argument is sound, it is *impossible* to have a complete PMC theory (regardless of the idealization involved).

Second, we can adopt a strategy that is analogous to that followed by Alfred Tarski to solve the Liar Paradox and save the truth predicate from inconsistencies (Tarski

[12] With respect to subjective probability, I will suppose that the subjective probability functions of agents are guided by expert functions (in the sense of van Frassen [31]) which match some of the objective interpretations. In any case, for my purposes, it is sufficient for subjective probabilities to follow the probability axioms and suppositions 1-4 in the Appendix. Thanks to an anonymous reviewer for pressing this question.

[13] Thanks to Elia Zardini for suggesting this line of thought.

[29]). To wit, we could restrict coherence to sets of propositions which do not involve coherence themselves. As in the case of Tarski, this would, however, represent a serious impairment of the notion of coherence. We could try to overcome this difficulty by giving a characterization of the collection of propositions (which now might themselves involve coherence) to which coherence can be applied *safely*, so to speak. Although, that might prove to be a difficult task.[14]

Third, we can reject the assumption that PMC theories can be properly expressed as sets of propositions, in which case they are not the sort of entities to which PMC theories apply and, consequently, the argument does not even get off the ground. The main alternative conception to the syntactic approach to theories used in this paper is the semantic approach, according to which a theory, rather than being a collection of truth-bearers, is a class of models (see, for example, Suppe [28]). Although I will not delve into the details here, let us suppose that somebody defends such a semantic approach to theories. Even if PMC theories can be coherently seen as classes of models, it seems to be the case that, given a PMC theory $\mathbb{P}$, the argument offered in this paper will let us prove the incompleteness of any collection of propositions satisfied by the models that characterize $\mathbb{P}$ (which, in a sense, suggests the incompleteness of $\mathbb{P}$ itself). Be that as it may, more work is needed in order to see which notion of theory best applies to PMC theories and whether the argument introduced in this paper can be successfully applied to them.[15]

Finally, since a *modus ponens* for a philosopher can be a *modus tollens* for another, we can take the argument to prove that WWSP is not correct. I do not think this is a very promising way out, for WWSP seems a very reasonable principle to adopt.[16]

Appendix

I need to show that, given that $C'' = \Gamma \cup \Delta$ is not coherent, neither is Γ. Furthermore, recall that I am now supposing that we do not have *a priori* access to Δ, since, as I said, Δ contains the probabilistic information required to assess the coherence of A and we are now assuming an interpretation of probability according to which assessments of probabilities are not logically necessary. Now, since by supposition Γ is coherent, there must be some proposition in Δ which, once added to Γ, leads to incoherence; for Γ is coherent while C'' is not. In this Appendix I will show that no proposition in Δ can be the responsible for the incoherence of C'', which means that the source of incoherence of C'' must be Γ itself.

In the argument below I will use the following two suppositions:

[14]See Kripke [15] for a discussion of the problems of Tarski's proposal and for the theory of truth that Kripke worked out that is immune to the Liar Paradox.

[15]Thanks to an anonymous reviewer for suggesting this line of thought.

[16]I am grateful to José Martínez Fernández, Manolo Martínez, Elia Zardini and two anonymous reviewers for all their extremely helpful comments to previous versions of this paper. I also want to express my gratitude to Peter Pagin, Gonçalo Santos and Michael Schippers. Please note that inclusion in the acknowledgements does not imply endorsement by those named of any claim defended in this paper. During the writing of the paper, I have benefitted from the project FFI2015-70707P of the Spanish Ministry of Economy, Industry and Competitiveness on *Localism and Globalism in Logic and Semantics*.

> *Supposition 1*: Adding a claim about the high probability of some of the propositions in a coherent theory, T, to T does not turn it into an incoherent theory.

> *Supposition 2*: Adding the claim that one of its consequences has probability 1 to a coherent theory, T, does not turn T into an incoherent theory.[17]

Let us now see which are the propositions that constitute Δ and whether they can be responsible for turning Γ into an incoherent set. They are propositions of the form $X_\Delta =< Pr(Y) = x >$ where $0 \leq x \leq 1$ and Y is a conjunction of propositions belonging to C.[18] This means that Y is a conjunction of propositions that can belong to $\Gamma \cup \Delta \cup \{< C \text{ is not coherent} >\}$. Let us see all the possible cases.

Case 1. Suppose, first, that $Y \in \Gamma$. Then we are just adding a claim about the probability of one of the propositions of the theory, which is supposed to be correct. Hence, if Γ is a good theory, we just add a claim that says that one of the propositions in Γ has a high probability. Now, by Supposition 1, this should not transform C'' into an incoherent set. Thus, Y cannot be an atomic sentence of Γ.

Case 2. Suppose secondly that $Y =< C \text{ is not coherent} >$. Notice that we just proved (without free premises) this proposition and hence it has a probability of 1. Now, by Supposition 2, this should not transform C'' into an incoherent set.

Case 3. Suppose now that $Y \in \Delta$. The propositions in Δ are all of the form $<Pr(Z) = x>$ and thus, if $Y \in \Delta$, X_Δ will be of the form $<Pr(Pr(Z) = x) = y>$. I am supposing that the information in Δ is true; that is, if $<Pr(Z) = 0.5> \in \Delta$, then $Pr(Z) = 0.5$. That means that the probability of $Pr(Z) = 0.5$ can reasonably be taken to be 1[19]; which means that, in this case, X_Δ has a probability of 1.

To finish *Case 3* I will now need two more suppositions:[20]

> *Supposition 3*: Adding a claim that is independent of a coherent theory, T, to T should not transform the theory into an incoherent one.

> *Supposition 4*: Γ is neutral with respect to how probabilities are distributed by any given measure and hence it should be independent with respect to claims concerning Δ.[21]

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[17] I do not want to claim that these suppositions do not have any counterexamples. Take, for instance, some kind of coherent nominalist theory that denies the existence of propositions. It seems that, adding to such a theory a claim about the high probability of some of its propositions, would turn the theory into an incoherent one; thus, such a theory would be a counterexample to Supposition 1. Nevertheless, this counterexample is not relevant to Γ and I think we can confidently hope that other counterexamples will be of a similar nature and, hence, irrelevant for the purposes of this paper.

[18] I am using the fact that all the probabilities of the relevant Boolean combinations used in PMC theories can be defined using the probabilities of the conjunctions of the atomic propositions.

[19] I am using the following principle for higher-order probability as defended, for example, in Uchii [30]:

$$Pr(p) = x \text{ if, and only if } Pr(Pr(p) = x) = 1$$

Carnap himself seems to think the same when he claims that "any statement on probability or estimation is, if true, analytic" (Carnap [6, p. 181]).

[20] Thanks to Elia Zardini for very useful comments on this point.

[21] I think this makes sense, for I am now supposing that Δ is not given *a priori*, and I do not see why a theory of coherence should be committed to some non *a priori* probabilistic measure above another.

Now, Supposition 4 implies that X_Δ, which in this case is just a claim about the accuracy of Δ, is independent of Γ and, hence, by Supposition 3, it should not be responsible of the incoherence of C''.

Case 4. Finally, suppose that $Y = < \alpha_1 \wedge \cdots \wedge \alpha_n >$ where each $\alpha_i \in \Gamma \cup \{< C \text{ is not coherent} >\} \cup \Delta$. As we have seen, each proposition in Δ has probability 1 and the same happens with <*A* is not coherent>. Thus, by probabilistic rules, the probability of Y above is the same as the probability of the conjunction of those α_i such that $\alpha_i \in \Gamma$ which, since the theory is supposed to be correct, must be high. Now, by Supposition 1, just adding to Γ a claim about the high probability of the conjunction of some of its propositions should not transform C'' into an incoherent set.

Hence, if no information in Δ can be responsible for the incoherence of C'' in spite of the fact that $C'' = \Gamma \cup \Delta$ is an incoherent set, then the only option available is that Γ itself is an incoherent set. Thus, no correct and coherent PMC theory using algorithms with Boolean combinations of probabilities as described at the beginning of this remark can follow WWSP.

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