God and Infinity: Directions for Future Research

Philosophical investigation—and, in particular, metaphysical investigation—is rarely advanced through the consultation of dictionaries. However, in the present case, it will repay us to begin by considering the entries for "infinite", "infinity" and "the infinite" that are found in the OED.

Infinite adj. (Omitting obsolete and archaic uses)

1. Having no limit or end (real or assignable); boundless, unlimited, endless; immeasurably great in extent, duration, or other respect. Chiefly of God or His attributes; also of space, time, etc., in which it passes into the mathematical use.
2. In loose or hyperbolical sense: Indefinitely or exceedingly great; exceeding measurement or calculation; immense, vast.
3. *Math.* Of a quantity or magnitude: Having no limit; greater than any assignable quantity or magnitude (opp. to finite). Of a line or surface: Extending indefinitely without limit, and not returning to itself in any finite distance (opp. to closed).
4. *Infinite series*: a series of quantities or expressions which may be indefinitely continued without ever coming to an end (but may or may not have a finite value or 'limit' to which it approaches as more and more terms are taken).
5. *Gram.* Applied to those parts of the verb which are not limited by person or number.

Infinite absol. or as sb.

1. That which is infinite or has no limit; an infinite being, thing, quantity, extent, etc. Now almost always in the sing. with *the*; esp. as a designation of the Deity or the absolute Being.
2. *Math.* An infinite quantity.

Infinity n.

1. The quality or attribute of being infinite or having no limit; boundlessness, illimitableness (esp. as an attribute of Deity)
2. Something that is infinite; infinite extent, amount, duration, etc.; a boundless space or expanse; an endless or unlimited time.
3. *Math.* Infinite quantity: denoted by the symbol ∞. Also, an infinite number (of something).
4. *Geom.* Infinite distance, or that portion or region of space which is infinitely distant: usually in phr. *at infinity*.

The Oxford English Dictionary Vol V. H-K.

As these entries make clear, the words "infinite" and "infinity" have a number of overlapping uses and meanings. This overlapping of uses and meanings can—it seems—

be explained, at least in part, by appeal to historical considerations. However, it is a matter for investigation whether this overlapping of uses and meanings constitutes an impediment to certain kinds of inquiries and theoretical activities. We might think of this investigation as one kind of prolegomenon to serious discussion of the proper uses of the words "infinite" and "infinity"—and the concepts that these words express—in theological contexts.

1. A Question for Investigation

According to reliable authority[1], the origins of our words "infinite" and "infinity" can be traced back to the Greek word *peras* (πέρας), which can be translated by "limit", or "bound", or "frontier", or "border", and which has connotations of being "clear" or "definite". The Greek word *to apeiron* (άπειρον)—the "negation" or "opposite" of *peras*—thus can be understood to refer to that which is unlimited, or boundless, or—in some cases—unclear and indefinite.

When the word *to apeiron* makes its first significant recorded appearance—in the work of Anaximander of Miletus—it is used to refer to "the boundless, imperishable, ultimate source of everything that is" (Moore (1998:772). Thus, in this early usage, the word *to apeiron* has connotations—"imperishable", "ultimate source of everything"—that are quite separate—or, at any rate, separable—from considerations about the absence of "limits", or "bounds", or "frontiers", or "borders", or "clarity" or "definiteness".

As Moore (1998:773) points out, most of the Greeks associated much more negative connotations with *to apeiron* than are evident in the early usage of Anaximander: for the Pythagoreans, and—at least to some extent, for Plato—*to apeiron* "subsumed … all that was bad …; it was the imposition of limits on the unlimited that accounted for all the numerically definite phenomena that surround us". Again, on this kind of usage of the term, *to apeiron* has connotations—"chaotic", "irrational", "disorderly"—that are quite separate—or, at any rate, separable—from considerations about the absence of "limits", or "bounds", or "frontiers", or "borders", or "clarity" or "definiteness".

In current English, we have the adjective "infinite", the noun "infinity", and the substantive "the Infinite". The standard use of the substantive form is "as a designation of the Deity or the absolute Being"; and so, of course, there is one standard use of the adjectival and noun forms that rides piggyback upon this standard use of the substantive form. It seems to me that it is plausible to see the current use of the substantive "the Infinite" as a direct descendent of Anaximander use of the word *to apeiron* with more or less the same connotations—"imperishable", "ultimate source of everything"—except, of course, that *to apeiron* is personalised, i.e., taken to have personal attributes and attitudes, in Christian theology.

However, in current English, we also have uses of the adjective and noun forms that are not obviously related to the standard use of the substantive form. In particular, there are uses of these terms in mathematics, including geometry, and applications of these terms to space and time, in which most of the connotations associated with the substantive form

seem to play no role at all. While these uses of the term do have more or less clear connections to the absence of "limits", or "bounds", or "frontiers", or "borders", they have very little to do with considerations about the absence of "clarity" or "distinctness", and nothing at all to do with considerations about "the ultimate, imperishable, source of everything".

It is not clear to me whether this separation of considerations was achieved by the Pythagoreans. Given their metaphysical belief that the positive integers are the ultimate constituents of the world, it is a plausible conjecture that they did not recognise the discussion of "limits", or "bounds", or "frontiers", or "borders"—and the application of these terms to, say, space and time—as a separate topic for investigation in its own right. But, whatever the truth about this matter may be, it seems that some of the contemporaries and immediate successors of the Pythagoreans *did* come to see the discussion of these topics as an independent subject matter. It is, I think, plausible to view Zeno's paradoxes as a contribution to such a discussion; and, even if that is not so, it is surely right to see Aristotle's treatment of infinity as an investigation of "limits" and "bounds"—in the context of space, time and matter—in their own right. (In *Physics*, Book III, Aristotle makes mention of Anaximander's views about "the ultimate source of everything". But those views are entirely incidental to the theory of "limits" and "bounds" that Aristotle proceeds to elaborate and defend.)

However, once it is recognised that the investigation of "limits" and "bounds"—in the context of space, time and matter—is a legitimate subject matter in its own right, then various questions arise about the application of the results of *that* investigation to the subject matter with which Anaximander was primarily concerned: "the ultimate source of everything". Even if it is true—as I think it is—that the historical entanglement of talk about "limits" and "bounds" with talk about "the ultimate source of everything" persists into the present, it is important to ask whether this entanglement has any *essential* significance for either the investigation of "limits" and "bounds" as a subject matter in its own right, or for the investigation of "the ultimate source of everything" (as a subject matter in its own right).

Prima facie—at least!—there seems to be good reason to think that the investigation of "the ultimate source of everything" has no *essential* or *ineliminable* significance for the investigation of "limits" and "bounds" as a subject matter in its own right. Modern logical, mathematical, and physical theories depend upon no substantive theological assumptions. No serious, standard text in logic, or mathematics, or the physical sciences begins with a chapter on "theological preliminaries" or "theological assumptions".[2] Moreover, the same point holds true for serious textbook discussions of infinities *in* logic, and mathematics, and the physical sciences: there is no theological prolegomenon that is *required* for examinations of Conway numbers, or renormalisation in quantum field theory, or Kripke models for intuitionistic logic, or any other particular topic in this domain.

But what about the other direction? Does the investigation of "limits" and "bounds" as a subject matter in their own right have some *essential* or *ineliminable* significance for the

investigation of "the ultimate source of everything"? It is, of course, well-known that some of those who have investigated "limits" and "bounds" as a subject matter in their own right have supposed that this investigation does have *important* consequences for the investigation of "the ultimate source of everything". (This is true, for example, of Cantor.) But the question that I wish to take up, in the remainder of this paper, is whether it is true—and, if so, in what ways it is true—that those who wish to investigate "the ultimate source of everything" need to equip themselves with the fruits of an investigation of "limits" and "bounds" as a subject matter in their own right.

2. Predicates and Properties

There is a range of different views that those who believe that there is a unique "ultimate source of everything" take concerning the language that they use when they talk about "the ultimate source of everything". We can illustrate some of the range of views by considering simple subject-predicate sentences of the form "God is F", where "F" is a relatively simple and unstructured predicate.

There are, of course, questions about the interpretation of the word "God". Since I don't wish to focus upon *those* questions here, I shall simply assume that we can take it for granted that "God" is a proper name, and that the reference of this name is fixed by the description "the ultimate source of everything". (Others who accept the general account that is suggested here will prefer different reference-fixing descriptions, e.g. "the omnipotent, omniscient, perfectly good creator of the world *ex nihilo*" (Swinburne (1979:8)) or "the thing that is, in an objectively normative manner, the proper object for religious attitudes" (Sobel (2004:10)). For the purposes of the present discussion, nothing turns upon the exact phrasing of the reference-fixing description.) Of course, this account leaves it open that there is nothing that satisfies the reference-fixing description; if there is nothing that satisfies the reference-fixing description, then the name is empty.

Among those who suppose that "God" is not an empty name, there is a wide range of views about the *understanding* that it is possible for people to have of the properties that are possessed by the being who bears the name. Some suppose that we cannot grasp (apprehend, understand) *any* of the properties of God. Others suppose that we cannot *fully* grasp (apprehend, understand) any of the properties of God, but that we can have a *partial* or *incomplete* grasp (apprehension, understanding) of *some* of the properties of God. Yet others suppose that, while we can fully grasp (apprehend, understand) *some* of the properties of God, there are other properties of God of which we can—as a matter of logical or metaphysical necessity—have no more than a partial or incomplete grasp. And—perhaps—there are some who suppose that, while there are properties of God of which we remain—and will always remain—ignorant, there is no *logical* or *metaphysical* barrier to our grasping (apprehending, understanding) any of those properties. (There is, of course, a related range of views about the *knowledge* that it is possible for people to have concerning which properties are, in fact, possessed by the being who bears the name. Naturally, it should be borne in mind that it is one question whether we can (fully) understand (grasp, apprehend) a property, and quite another question whether we have what it takes to be able to determine whether or not God in fact *possesses* that property.)

The range of views concerning what it is possible for us to *say* about the properties that God possesses depends, in part, upon the views that we take about the understanding of God's properties, and, in part, upon the theory of predication—and, in particular, upon the theory of the relationship between predicates and properties—that we adopt. Amongst theories of properties and predication, the most important distinction for us—for present purposes—is between *luxuriant* theories that suppose that every well-functioning predicate expresses a property (or universal) and *sparse* theories that suppose that there are many perfectly well-functioning predicates that fail to express properties (or universals) even though these predicates can be truly predicated of at least some objects. If we adopt a luxuriant theory of properties and predication, then we shall suppose that whenever we make a true claim of the form "God is F", the predicate "F" expresses a property that is possessed by God. However, if we adopt a sparse theory of properties and predication, then we can suppose that, at least sometimes, when we make a true claim of the form "God is F", there is no property that is expressed by the predicate "F" that is possessed by God.

If we suppose that we cannot grasp (apprehend, understand) *any* of the properties of God, and if we adopt a luxuriant theory of properties and predication, then it surely follows that we cannot say anything at all about God. Indeed, this combination of views seems incoherent; for, in order to fix the reference of the name "God", we need to make use of some predicates that we take to be true of that which bears the name. If we claim that those predicates express properties, and yet also claim that we cannot grasp (apprehend, understand) any of the properties of God, then we have lapsed into self-contradiction.[3] Here, I assume that one does not understand a predicate unless one grasps (apprehends, understands) the property that is expressed by that predicate; if I don't know which property is expressed by a predicate, then I cannot make meaningful use of that predicate to express my own thoughts. (Note, by the way, that I am not here assuming that the property that is expressed by a predicate is required to be the *literal* content of the predicate. It could be that, in the case in question, the use of the predicate is *metaphorical* or *analogical*. However, I am assuming that one does not grasp (apprehend, understand) a metaphorical or analogical use of a predicate unless one understands *which* property is being attributed to the subject of the predication by the metaphor or analogy in question. This is not quite rejection of the view that there can be irreducible (essential) metaphors or analogies; however, it is the view that, where there are irreducible (essential) metaphors or analogies, these arise because of limitations upon our powers of *representation* and *expression*, and not because of limitations upon our powers to grasp (apprehend, understand) the properties that are possessed by things.)

If we suppose that we cannot grasp (apprehend, understand) any of the properties of God, and if we adopt a sparse theory of properties and predication, then there will be things that we can say truly of God. Perhaps, for example, we can truly say that God is self-identical, while denying that there is any such thing as the property (universal) of "being self-identical". While this view does not collapse quite so immediately into self-contradiction, it is not clear that this view can be seriously maintained. In particular, it seems doubtful that one can plausibly allow that the predicates that are used in the kinds

of reference-fixing descriptions mentioned earlier—and the predicates that are entailed by those predicates that are used in the kinds of reference-fixing descriptions mentioned earlier—fail to express properties. Consider Swinburne's definition. Any being that is omnipotent, omniscient, perfectly good, and creator of the world *ex nihilo* will be good, powerful, possessed of knowledge, creative, and so forth (i.e. will be such that the predicates 'good', 'powerful', 'possessed of knowledge', 'creative', and so forth, can be truly predicated of it). But can it be plausibly maintained that *none* of these are properties (universals)? Sparse theories of properties (universals) must satisfy the constraint that, among the properties (universals) over which they quantify, there are those properties (universals) that constitute the basic building blocks for *our* world. It is not, I think, plausible to suppose that not one of the predicates that can be truly applied to God expresses a property (universal).

The view that, while we cannot *fully* grasp (apprehend, understand) any of the properties of God, we can have a *partial* or *incomplete* grasp (apprehension, understanding) of *some* of the properties of God, seems to me to be subject to much the same kinds of difficulties as the view that we cannot grasp (apprehend, understand) any of the properties of God. On the one hand, if we adopt a luxuriant theory of properties and predication, then this view will have us saying that we have no more than a partial or incomplete grasp (apprehension, understanding) of such properties as self-identity, existence, uniqueness, and the like. And, on the other hand, even if we adopt a sparse theory of properties and predication, then this view will have us saying that we have no more than a partial or incomplete grasp of the properties that are expressed by predicates such as "is good", "knows", "is powerful", "is creative", and the like. Neither of these views seems to me to be at all attractive.

Once we proceed to views that allow that we can fully grasp (apprehend, understand) *some* of the properties of God, the kinds of difficulties that we have been exploring thus far lapse. So long as we allow that the properties that we appeal to—or that are entailed by those properties that we appeal to—in fixing the reference of the name "God" are among the properties that we can fully grasp (apprehend, understand), then we have no (immediate) reason to fear that our theory of the fixing of the referent of the name "God" is self-contradictory, or incoherent, or evidently inadequate. Certainly, this is clear if we allow that *all* of the properties that we appeal to—*and* that are entailed by those properties that we appeal to—in fixing the reference of the name "God" are among the properties that we can fully grasp (apprehend, understand). But, plausibly, the consequence remains clear even on weaker readings of the condition: if all the properties that we appeal to—and all of the properties that we *in fact* infer from those properties that we appeal to—in fixing the reference of the name "God" are among the properties that we can fully grasp (apprehend, understand), then we have no reason to fear that our theory of the fixing of the referent of the name "God" is self-contradictory, or incoherent, or evidently inadequate. Indeed, it *may* even be plausible that the consequence remains on *much* weaker readings of the condition: for the most important constraint here is just the our theory of the fixing of the reference of the name "God" should not impute partial or incomplete grasping (understanding, apprehension) of predicates in cases where we have good independent reason to insist that there is full grasp (understanding,

apprehension) of those predicates. And, of course, *this* constraint can be satisfied even if some of the predicates that are used in the fixing of the reference of the name "God" are only partially or incompletely grasped, and even if many of the predicates that are entailed by the predicates that are used in the fixing of the reference of the name "God" are only partially or incompletely grasped, so long as there are some *other* predicates that can be truly applied to the object picked out by the reference-fixing description that are fully grasped.[4]

Of course, the discussion to this point does not exhaust the questions that arise concerning the views that those who believe that there is a unique "ultimate source of everything" take concerning the language that they use when they talk about "the ultimate source of everything". In particular, I've said nothing thus far about the view that there are properties of God of which we can—as a matter of logical or metaphysical necessity—have no more than a partial or incomplete grasp. This view is the subject of the next section of my paper.

3. Understanding Properties

There are various foundational debates about properties and predicates that have so far gone without mention in our discussion. Amongst these hitherto undiscussed debates, the most important for present purposes are (i) the various debates about the tenability of quantification over predicate position; and (ii) the debates about whether there is a non-pleonastic sense in which predicates have properties as semantic values. In the previous section of this paper, the discussion takes it for granted that there is a non-pleonastic sense in which predicates have properties as semantic values—the idea that there is a distinction between luxuriant and sparse theories of properties lapses if this assumption is rejected—and the discussion also takes it for granted that we can make intelligible quantification over predicates, talking freely about the existence of properties of various kinds, and so forth. If either, or both, of these suggestions is rejected, then we shall need to seriously reconsider the terms of that previous discussion.

If we reject the claim that there is a non-pleonastic sense in which predicates have properties as semantic values—and if we adopt, instead, the proposal that properties are no more than the ontological shadows of meaningful predicates—then it is not clear that we can even make sense of the idea that there are properties of God of which we can—as a matter of logical or metaphysical necessity—have no more than a partial or incomplete grasp. If to be a property is just to be the ontological shadow of a meaningful predicate in a human language, then there are no properties that elude our understanding. While it is perhaps consistent with the suggestion, that there are no properties that elude our understanding, that the expressive power of our language is susceptible of indefinite improvements, it is not clear that the idea that the expressive power of our language is susceptible of indefinite improvement of itself is sufficient to support the claim that, as a matter of logical or metaphysical necessity, we have only a partial or incomplete understanding of God. At the very least, it seems to me that some investigation is needed of the consequences of deflationary semantics for the claim that there are properties of

God of which we can—as a matter of logical or metaphysical necessity—have no more than a partial or incomplete grasp.

If we reject the claim that there can be intelligible quantification over predicate position—or even if we insist on the claim that the best choices for canonical notation and logic are based on languages in which there is no quantification over predicate position—then, again, it is not clear that we can even make sense of the idea that there are properties of God of which we can, as a matter of logical or metaphysical necessity, have no more than a partial or incomplete grasp. If we cannot intelligibly quantify over predicate position, then we cannot make sense of any claim of the form "there are properties of God which …," and hence, in particular, cannot make sense of the claim that there are properties of God of which we can—as a matter of logical or metaphysical necessity—have no more than a partial or incomplete grasp. While there are reasons to think that we should allow that quantification over predicate position is not merely intelligible but actually acceptable—and, indeed, required in order to allow us to say some of the things that we want to be able to say—a fully carried out project into the foundations of claims about God and infinity would need to include some investigation of these matters.

Suppose, however, that we allow that there is a non-pleonastic sense in which predicates have properties as semantic values, and that there can be intelligible quantification over predicate position—and, perhaps, that the best choices for canonical notation and logic are based on languages in which there is quantification over predicate position. What should we then say about the claim that there are properties of God of which we can—as a matter of logical or metaphysical necessity—have no more than a partial or incomplete grasp? While we cannot hope to adequately address this problem here, perhaps we can make a few useful preliminary observations.

From the outset, it is important to distinguish between the claim that it is logically or metaphysically necessary that there are properties of God of which we can have no more than a partial or incomplete grasp, and the claim that there are properties of God of which it is logically or metaphysically necessary that we can have no more than a partial or incomplete grasp. The former claim could, for example, be true of things other than God if, for example, those things have infinitely many logically independent properties and we are only capable of fully and completely grasping a finite range of properties; and the former claim could be true of things other than God if there is no upper bound to the number of logically independent properties that are possessed by different things, but there is an upper bound to the number of properties that we can fully and completely grasp; and so forth. On the other hand, the latter claim can only be true if there is something about the nature of a particular property that causes it to be the case that that property is resistant to our full and complete understanding. I take it that it is this latter claim that is primarily of interest to us in the present context.

It is often said—or suggested, or implied—that there are properties of God that are resistant to our understanding in the sense that we cannot understand what it would be like to possess those properties. So, for example, it is sometimes said that we cannot

understand what it would be like to be omniscient. (We might think that Dennett uses this observation in order to undermine the knowledge argument against physicalism: because we don't know what it is like to be omniscient, we are in no good position to judge what Mary would know if it were true that she knew all the physical truths about the world.) However, I take it that this kind of ignorance—ignorance about what it would be like to possess a certain kind of property—is perfectly compatible with full and complete knowledge about which property it is that is in question. Supposing that, for example, it is true that to be omniscient is to know every proposition that it is logically possible for one to know, *given that* there are the weakest possible constraints on what it is logically possible for one to know, then one can have full and complete knowledge about what omniscience is even if one cannot even begin to imagine (picture, "understand from the inside") what it would be like to be omniscient.

Once we have the distinction between (i) the possession of full and complete knowledge of what a property F *is* and (ii) the possession of full and complete knowledge of what it would be *like* to possess property F, we can apply this distinction to the question whether we should want to assent to the claim that there are properties of God of which it is logically or metaphysically necessary that we can have no more than a partial or incomplete grasp. As we noted in the previous paragraph, it seems quite reasonable to allow that there are properties of God of which it is logically or metaphysically necessary that we have no more than a partial or incomplete grasp of what it would be *like* to possess those properties. However, it is much less obvious that it is reasonable to allow that there are properties of God of which it is logically or metaphysically necessary that we have no more than a partial or incomplete grasp of what those properties *are*. At the very least, I think that it is clear that there is room for much further fruitful investigation of this issue.

4. Infinite Domains and Infinite Degrees

At the end of the first section of this paper, I said that the primary question to be investigated herein is whether it is true—and, if so, in what ways it is true—that those who wish to investigate "the ultimate source of everything" need to equip themselves with the fruits of an investigation of "limits" and "bounds" as a subject matter in their own right. *Prima facie*, at least, there are various syntactically simple claims that many believers have been inclined to make that suggest that those who wish to investigate "the ultimate source of everything" *do* need to equip themselves with the fruits of an investigation of "limits" and "bounds" as a subject matter in their own right. On the one hand, believers often claim that God is infinite. On the other hand, believers often claim that God is omnipotent, and omniscient, and omnipresent, and eternal, and perfectly good, and sole creator of the universe, and so forth. All of these claims, when interpreted in a straightforward and literal way, strongly suggest that believers must actually be relying upon the results of investigations of "limits" and "bounds" as subject matters in their own right.

Now, of course, it might be said that these various claims should not be interpreted in a straightforward and literal way. However, I take it that the discussion in the previous two

sections of this paper strongly supports the view that believers ought not to take such a line. While believers can perfectly well maintain that a *complete* characterisation of God is beyond our imaginative and conceptual capacities, such believers are obliged to allow that we have the capacity to provide an intelligible—literal, straightforward—description that fixes the referent of the name "God". Of course, some will not be persuaded that this is so. No matter; those not persuaded should think of this inquiry as conditional in form: what should those who suppose that it is straightforwardly and literally true that God is omnipotent, and omniscient, and omnipresent, and eternal, and perfectly good, and sole creator of the universe, and so forth, allow that investigations of "limits" and "bounds" as subject matters in their own right contribute to their understanding of these claims?

There are straightforward ways in which literal interpretations of the claims that God is omnipotent, God is omniscient, God is omnipresent, God is eternal, God is perfectly good, God is the sole creator of the universe, and so forth, *suggest* involvement with investigations of "limits" and "bounds" as subject matters in their own right. To say that God is omnipotent is, at least roughly, to say that it is within God's power do anything that it is logically possible for God to do. To say that God is omniscient is, at least roughly, to say that God knows the truth status of every proposition for which it is logically possible that God know the truth status of that proposition. To say that God is omnipresent is to say that every spatiotemporal location is present ("available") to God. To say that God is eternal is to say that every time is present ("available") to God. (Some say, rather, that God is sempiternal, i.e., that God exists at every time. My formulation is neutral on the question whether God is *in* time.) To say that God is perfectly good is to say, *inter alia*, that there is no moral obligation, or moral duty, or moral good to which God fails to pay due accord. To say that God is the sole creator of the universe is to say that God is the sole original creator of all contingently existing things. And so forth. In every case, the attribution of one of these properties to God brings with it quantification over a domain of objects—and, in each case, there is then a serious question to address concerning the measure or cardinality of that domain.

Consider the case of omniscience. If we suppose that God knows the truth status of every proposition for which it is logically possible that God know the truth status of that proposition[5], then a natural question to ask is: how many propositions are there concerning which God knows the truth status? Before we try to investigate the question, we need to tighten it up a little. In the case of human beings, it is a reasonable conjecture that there is a quite small bound on the number of propositions that are *explicitly* represented by a human agent over the course of a typical human life. Of course, it *might* be that the finite number of propositions that are explicitly represented over the course of a typical human life *entail* an infinite number of propositions that *might* then be said to be implicitly represented over the course of a human life. However, at least on standard accounts of divine knowledge, there is no corresponding distinction in the case of God's knowledge: every proposition that God knows is a proposition of which God has explicit representation (or, perhaps better, direct acquaintance). But, if every proposition that God knows is a proposition of which God has explicit representation (or with which God has direct acquaintance), and if God knows the truth status of every proposition for which it is logically possible that God know the truth status of that proposition, then one might

think that there is good reason to suppose that God has explicit representation of (or direct acquaintance with) infinitely many distinct propositions. For, at the very least, it seems *implausible* to suppose that there are only finitely many distinct propositions concerning which God can have knowledge of truth value.

There are, of course, many subtleties here. While those of us of a Platonist bent may be inclined to suppose that even a natural language such as English has the capacity to represent infinitely many distinct propositions—consider, for example, the propositions expressed by the sentences *1+1=2*, *1+2=3*, *1+3=4*, etc.—there will be at least some radical finitists who deny that this is so. (Perhaps, they might say, there is no good reason to suppose that the operations that are invoked in the "specification" of infinite lists of well-formed sentences of English are total!) Moreover, while those of us of a Platonist bent may also be inclined to suppose that there *are* infinitely many distinct propositions that could be expressed by sentences of English, there will be at least some intuitionists and constructivists who deny—at least in the case of the example given above—that there are propositions that exist independently of the actual construction or tokening of the relevant sentences in some language. (Perhaps, that is, they might say, there is merely a potential infinity of propositions that can be expressed by sentences of English.) However, regardless of the correct position to take concerning the expressive capacities of natural human languages such as English, there are also questions about the nature of the representational properties that are attributed to God that also need to be taken into account. If we suppose—as standard Christian theology would have it—that there is nothing *potential* in God, then it seems that there is good reason to deny that it is possible to apply a constructivist or intuitionistic—or even radically finitist or formalist—account of mathematical truth and mathematical ontology to God's knowledge or to the propositions that are known by God. Of course, we might wonder whether it is appropriate to suppose that God has a language of thought—or, indeed, whether it is appropriate to suppose that God has beliefs or other representational states of that kind—but, no matter how these matters are resolved, it seems at least *prima facie* plausible to suppose that the attribution of omniscience to God will lead us to claim that there *are* infinitely many distinct propositions that are known to God. (We shall return to the consideration of some of the relevant subtleties that are raised by the discussion of omniscience in the next section of the paper.)

What goes for omniscience goes for the other properties that I listed above. On plausible interpretations of the simple subject-predicate sentences that I listed, it is highly natural to suppose that the truth of any one of those sentences brings with it a commitment to infinite domains of objects and/or infinite magnitudes of degreed properties. At the very least, it is *prima facie* plausible to suppose that there are infinitely many different possible actions that an omnipotent being can perform, and that there are infinitely many different tasks that have been carried out by a sole creator of all contingently existing things; and it is *prima facie* plausible to suppose that a four-dimensionally omnipresent being is present to an infinite volume of space-time (and this because it is plausible to suppose that the universe is open in the future); and it is *prima facie* plausible to suppose that an eternal being is present to an infinite extent of times (again because it is plausible to suppose that the universe is open in the future); and it is *prima facie* plausible to

suppose that a perfectly good creator has created a world of infinite value (because it would be unworthy of such a being to create a world of lesser value than some other world that it might have created); and so forth. Moreover, of course, the commitment to infinite magnitudes of degreed properties seems evident on its face in the case of the claim that God is infinite (though see the following section for discussion of some of the difficulties that are raised by this claim).

There has been considerable recent philosophical activity that has sought to apply recent mathematical discussions of infinity to the divine attributes that are currently under discussion. In particular, there is a considerable literature on omniscience that draws upon Cantorian theories of the infinite (mostly drawing upon or discussing ideas that were first canvassed by Patrick Grim (1991)). However, even in the case of omniscience, there has been no *systematic* study of the kind that would be needed to address the kinds of questions that I have been raising in the present section of this paper. There is a large program of research here waiting to be carried out.

5. God and Infinity

Amongst the various claim listed for consideration in the previous section, the claim that *God is infinite* raises special difficulties. As we noted initially, some might suppose that this claim is only to be interpreted in a loose or metaphorical sense: what it really means to say is that God is imperishable, or unchanging, or the source of everything, or the like. Of course, if this is all that the claim that God is infinite is taken to really mean, then understanding of the claim that God is infinite will not be enhanced by considerations drawn from an investigation of "limits" and "bounds" as subject matters in their own right. But I do not think that it is plausible to suppose that this is all that those who now claim that God is infinite mean to assert; certainly, it is not all that *many* of those who now claim that God is infinite mean to assert. From this point, I shall proceed under the assumption that those who claim that God is infinite mean to assert something that is susceptible of explanation in terms of considerations drawn from an investigation of "limits" and "bounds" as subject matters in their own right.

Perhaps the most plausible way to interpret the claim that God is infinite is to take it to be the claim that God is infinite in certain respects. Some might think that it should be taken to be the claim that God is infinite in *every* respect; but—unless we have some very subtle way of determining what counts as a respect—it seems likely that this further claim will have untoward consequences. For example, there are few who would wish to claim that God has infinitely many parts; or that God consists of infinitely many distinct persons; or that God has created infinitely many distinct universes; or the like. And surely there are *none* who would wish to make contradictory claims—e.g. that God is infinitely small and infinitely large; or infinitely heavy and infinitely light; or infinitely knowledgeable and infinitely ignorant; and so forth. But, if we take the claim that we are interested in to be the claim that God is infinite in *certain* respects, then, of course, we shall naturally wish to inquire about the nature of those relevant respects.

One natural thought is that, for any degreed property that it is appropriate to attribute to God, God possesses that property to an infinite degree. However, there are reasons for thinking that this thought is not obviously correct. Suppose that God is three-dimensionally omnipresent, and that three-dimensional omnipresence is taken to be understood in terms of presence to every volume of space. It should not be a consequence of the claim that God is infinite that God is present to an infinite volume of space—for it may be that we want to deny that it is even possible for the volume of space to be infinite; and, even if we do not wish to deny that it is possible for the volume of space to be infinite, we should surely allow that we do not currently have overwhelming reason to think that the spatial volume of our universe is infinite. Yet the property of being present to a volume of n m^3 *is* a degreed property: something could be present to a volume of 1 m^3; something could be present to a volume of 2 m^3; something could be present to a volume of 3 m^3; and so forth.

The fix here is not hard to see. Rather than suppose that, for any degreed property that it is appropriate to attribute to God, God possesses that property to an infinite degree, we should say rather that, for any degreed property that it is appropriate to attribute to God, God possesses that property to the maximal or minimal possible extent that is consistent with the obtaining of other relevant facts, or else God possesses that property to an infinite degree. In the case of presence to volumes of space, God is present to the most inclusive volume of space; if that volume of space happens to be finite, then God is present to a finite volume of space—N m^3—but there is nothing objectionable about the fact that God does not possesses this degreed property to an infinite extent.

Some philosophers might have thought it preferable to try for a different fix. Suppose that we have some acceptable way of distinguishing between the intrinsic—or perhaps *non-relational*—properties of God and the extrinsic—or perhaps *relational*—properties of God. Then, amongst the degreed properties, it may seem right, at least initially, to say that whether extrinsic properties of God are infinite in degree can depend upon what it is that God is accidentally (contingently) related to under those properties. However, on this line of thought, it will then seem that whether the intrinsic properties of God are infinite in degree *cannot* depend upon what it is that God is accidentally (contingently) related to under those properties—since, by definition, those properties are non-relational—and so it will seem reasonable to insist that, in these cases, God must possess the properties to an infinite degree.

But consider, again, the example of God's knowledge. We've already seen that it is at least *prima facie* plausible to claim that there are infinitely many distinct propositions whose truth value is known to God. But it does not *immediately* follow from this *prima facie* plausible claim that it is *prima facie* plausible to attribute some kind of infinite faculty to God. For whether we should say that we are here required to attribute some kind of infinite faculty to God *might* be thought to turn up whether we are required to attribute knowledge of the truth value of infinitely many logically independent propositions to God. And—given that the attribution of omniscience requires that God knows (more or less) every logically independent proposition—that in turn would invite assessment of exactly how many logically independent propositions there are. Various

subtleties now arise. What, exactly, do we mean by logically independent propositions?[6] Are the propositions that it is possible that p and that it is possible that q logically dependent propositions for any propositions that p and that q? If we assume that the correct logic for modality is S5, then, for any proposition that p, if it is possible that p, then it is a necessary truth that it is possible that p. If we suppose that all necessary truths are logically dependent, then we shall arrive at the view that only some collections of contingent propositions are mutually logically independent. Yet, even if we accept the—controversial—assumptions required to arrive at this view, it is not clear whether we should then go on to draw the further conclusion that there are only finitely many logically independent propositions that are known by God. (Moreover, even if we do conclude that there are only finitely many logically independent propositions that are known by God, we might still think that, if there can be nothing potential in God, God is required to have explicit representations of infinitely many distinct propositions.)

If we suppose that God's omniscience requires that God is related to infinitely many contingently true propositions, and if we also suppose that this entails that God has infinitely many distinct explicit representations, then we might suppose that there are intrinsic properties of God that are infinite in degree, even though there is also a sense in which these intrinsic properties are dependent upon the world in which God is located. It seems plausible to think that the counting of representational states is an intrinsic matter—how many distinct representational states one has at a given time supervenes merely upon how one is at that time, and not at all upon how the rest of the world is—even though there are causal relations that hold between representational states and the world that contribute to the determination of how the number of distinct representational states that one is in varies over time. Thus, whether God is related to infinitely many contingently true propositions can be both a question about an intrinsic property of God—how many distinct explicit representations does God have—and yet also a question about how the world in which God is located (since the number of God's distinct explicit representations of contingently true propositions simply reflects the complexity of the world in which God is located).

If, then, we take the claim that God is infinite to be the claim that God is infinite in certain respects, then *perhaps* we can say something like the following: For any degreed property that it is appropriate to attribute to God, God possesses that property to the maximal or minimal possible extent that is consistent with the obtaining of other relevant facts, or else God possesses that property to an infinite degree.[7] If there is some sense in which a degreed property is relational, then it may be that whether that property is infinite in degree depends upon what it is that God is accidentally (contingently) related to under that property; but where there is no sense in which a degreed property is relational, it cannot be that whether that property is infinite in degree depends upon what it is that God is accidentally (contingently) related to under that property. Since it is possible for an intrinsic property to nonetheless be, in some senses, relational, it should not be thought that, merely because a degreed property is intrinsic, it cannot be that whether that property is infinite in degree depends upon what it is that God is accidentally (contingently) related to under that property.

Perhaps, though, we shouldn't take the claim that God is infinite to be the claim that God is infinite with respect to *all* of an appropriately restricted class of degreed properties; perhaps, rather, we should take the claim that God is infinite to be the claim that God is infinite in *certain very particular* respects. I don't have any clear suggestion to make about what these very particular respects in which God is infinite might be; perhaps, though, further investigation of this line of thought might turn up some interesting results.

Anselm refers to God by the formula "that than which no greater can be conceived". It is not impossible that one might think that the claim that God is infinite should be tied to the sense of greatness that is implicated in St. Anselm's formulation. Surely St. Anselm would have agreed with the claim that God is infinitely great; and surely it is not utterly implausible to think that modern theories of mathematical infinity might be pressed into service in the understanding of this claim. Alas, however, it is not clear what sense should be interpreted to "greatness" in Anselm's formulation. (Indeed, this is a much debated question in the recent literature on this topic.) Thus, while we might make progress on what is meant by "*infinitely* great", it is less clear that we will make progress on what is mean by "infinitely *great*". (I continue with this theme in the next section.)

Of course, it should not be thought that the above discussion exhausts the kinds of considerations—never mind the details of the considerations—that should be raised in the course of an examination of the claim that God is infinite, when that claim is given a straightforward, literal interpretation that draws upon the investigation of "limits" and "bounds" as subject matters in their own right. As in previous sections of this paper, I claim to have done little more than to indicate where there is further work that needs to be done.

6. God and the Transfinite

Throughout the paper to this point, I have made free use of the expressions "infinite" and "infinity" in talking about domains of objects and magnitudes of degreed properties. But, of course, even if one grants that we can make sense of talk of "infinite" domains of objects and "infinite" magnitudes of degreed properties, one might insist that—in the light of the development of Cantor's theory of the transfinite—one needs to bring far more precision to this kind of talk than our initial discussion has recognised. In particular, given that there is a hierarchy of infinite cardinals, it seems that we need to ask about the particular infinite cardinals that might be thought to be appropriate to the characterisation of God's properties.

One way—among many—into this topic is by way of some reflection upon the theory of numbers developed in Conway (1976)[8]. In Conway's system, there is a "gap"—"**On**"— which lies at the end of the number line. Intuitively, **On**—i.e. {**No**│ }, where "**No**" is shorthand for the entire number line—is "greater" than all of the numbers, including, in particular, all of Cantor's infinite cardinals. Anything that has a magnitude that is property characterised by **On** will have a magnitude that is not properly characterised by any number, however large.[9]

In Conway's theory of numbers—as in Cantor's theory of ordinal numbers—we have a sequence of numbers ordered by the "greater than" relation, including a series of (special) limit ordinals that can be identified with the distinct infinite cardinals of Cantor's theory of cardinal numbers. Thus, if we are talking about "infinite" quantities in the context of Conway's theory of numbers—or Cantor's theory of infinite cardinals—the question will always arise about the size of the infinity under consideration. Moreover, if one wishes to "exceed" any limitations that might be placed upon size, then one will be driven to talk about things that are not properly considered to be numbers at all. (I suspect that this point is linked to the idea—to be found in many versions of set theory—that there are collections that form only proper classes and not sets, because they are "too big" to be collected into sets.)

If, then, one is to say that God is "infinite" in such and such respects—or that God is "infinite" (*sans phrase*)—the question will always arise about the size of the infinity under consideration. Given the discussion above—in section 5—one might think that the appropriate thing to say is something like this: for any degreed property that it is appropriate to attribute to God, God possesses that property to the maximal or minimal possible extent that is consistent with the obtaining of other relevant facts, or else God possesses that property to an unlimited (unquantifiable, proper-class-sized) extent. Under this reformulation, we allow that it is possible that God's possessing a given property to a certain maximal extent forces us to say that God possesses that property to a given infinite cardinal degree. (If, for example, there are $\aleph_{15}$ true propositions, then God knows $\aleph_{15}$ true propositions; in that case, God's knowledge is infinite, but it is not unquantifiably infinite.) However, we also allow that, at least until further considerations are brought to bear, it remains an open question whether God possesses some properties to an unquantifiable extent. (If, for example, there are proper class many—**On**—true propositions, then God's knowledge is unquantifiably infinite.)

There may be some pressures that nudge theologians in the direction of saying that God does possess at least some properties to an unquantifiable extent. Suppose, for example, that we accept Anselm's formulation: "God is that than which no greater can be conceived". Since it seems that we can conceive of creatures who possess some great-making properties that are of unquantifiable extent, there is at least some reason to suspect that we will be driven to the conclusion that God possesses those properties to an unquantifiable extent. However, once again, this is a matter for more careful investigation.

7. Checking for Consistency

In the introduction to Oppy (2006), I hinted at the existence of an argument for the conclusion that "there is no conception of the infinite that can be successfully integrated into relatively orthodox monotheistic conceptions of the world" (xi). About this hint, I wrote: "Since all that this brief introduction aims to do is to make it seem plausible that there is a *prima facie* interesting question to address, I shall leave further discussion of this argument to the future." (xiii) In this section of the present paper, I reproduce the earlier discussion, and provide some further comments upon it (though certainly not on the scale envisaged in the just quoted remark).

Here's what I wrote:

> If we are strict finitists—and thus reject all actual and potential infinities—then we are obliged to say that God is finite, and that the magnitudes of the divine attributes are finite. But what reason could there be for God to possess a given magnitude to degree N rather than to degree N+1? More generally, how could a finite God be the kind of endpoint for explanation that cosmological arguments typically take God to be?

> If we are potential infinitists—i.e. if we reject all actual infinities, but allow that some entities and magnitudes are potentially infinite—then it seems that we will be obliged to say that God is potentially infinite, and that the magnitudes of the divine attributes are potentially infinite. But what kind of conception of God can sustain the claim that God is susceptible of improvement in various respects? If God possesses a magnitude to degree N even though God could possess that magnitude to degree N+1, surely God just isn't the kind of endpoint for explanation that cosmological arguments typically take God to be.

> If we are neither strict finitists nor potential infinitists, then it seems that we must be actual infinitists, i.e. we must suppose that God is actually infinite, and that the magnitudes of the divine attributes are actually infinite. But is there a conception of the infinite that can sustain the claim that God is actually infinite, and the claim that the magnitudes of the divine attributes are actually infinite without undermining the kinds of considerations to which orthodox cosmological arguments appeal in attempting to establish that God exists? Indeed, more generally, are there conceptions of the infinite that can sustain the claim that God is actually infinite, and the claim that the magnitudes of the divine attributes are actually infinite *tout court*? Moreover, if there is a conception of the infinite that can sustain the claim that God is actually infinite, can this conception of the infinite also sustain the idea of an incarnate God, and the idea that there is an afterlife in which people share the same abode as God?

As I noted at the end of the previous section, it seems to me that there are pressures that drive theologians in the direction of claiming that God possesses some properties to an unquantifiable—"more than proper class many", "**On**"—extent. But, if that's right, then it seems that theologians should not look with any fondness on those philosophical views that deny that we can form a coherent conception of actually infinite domains and actually infinite magnitudes. Rather than side with formalists, or radical finitists, or constructivists, or intuitionists, or those who insist that there are none but merely potential infinities, believers in God should say instead that there can be domain and properties that are "unquantifiably infinite", i.e. not measurable by any of the cardinalities that are to be found in Cantor's paradise. So, I take it, the direction of thought that is expressed in the first two paragraphs of the above quotation is acceptable without qualification (though there is much more to be said in defence of the main theses outlined therein). Furthermore—and for the same reasons—I take it that the line of thought that is expressed in the first part of the third paragraph is also acceptable.

However, when we turn to the question of traditional arguments for the existence of God, matters are rather more interesting than the above compressed presentation allows. What is true is that there are *some* traditional arguments for the existence of God—e.g. one *a priori* version of the kalām cosmological argument—in which it is explicitly assumed that there can be no actual infinities. *Those* arguments cannot be defended consistently with the adoption of the conception of divine infinity articulated in the previous section of this paper. But, of course, there are many other arguments—including many other cosmological arguments—that make no such (implicit or explicit) assumption about the impossibility of actual infinities. *These* arguments are not impugned by the considerations about infinity to which I have been here adverting. (They may be impugned by *other* considerations about infinity; but that's another story.) It would be an interesting project to run an inventory of arguments about the existence of God, to determine where considerations about infinity come up, to check to see how those arguments fare under the kinds of considerations that were adduced earlier.

When we turn to matters such as the idea of an incarnate God, and the idea that there is an afterlife in which people share the same abode as God, there is yet a further raft of considerations that comes into view. It is not easy to reconcile the suggestion that God is actually infinite with the idea that God took on a finite physical form. It is not easy to reconcile the idea that God's abode is infinite with the idea that that abode is inhabited by finite physical creatures (such as ourselves). At the very least, there is clearly an interesting possible project that investigates the ways in which particular Christian doctrines—concerning, e.g., incarnation, trinity, atonement, and so forth—are affected by particular theories about the ways and respects in which God is actually infinite. As I suggested at the outset, a full investigation of the implications for theology, of an investigation of "limits" and "bounds" as a subject matter in its own right, is likely to be very prolonged indeed.

8. A Concluding Stocktake

As I said initially, this paper is intended to be a kind of prolegomenon to the discussion of infinity in theological contexts. What I have tried to do is to raise various kinds of issues in a preliminary way, without in any way supposing that my comments upon these issues constitute decisive verdicts. Perhaps it will be useful, in closing, to provide a summary of the range of issues that has been canvassed (and of the opinions that I have expressed).

First, there are issues that cluster around the question of the *normative significance* that mathematical and theological investigations of the infinite have for one another. On the one hand, it seems to me to be highly plausible to think that mathematical investigations of infinity do have significant consequences for theology that should be recognised on all sides. (Of course, this claim relies upon some contentious assertions about the properly realistic interpretation of theological talk. More about this anon.) On the other hand, it seems to me to be equally plausible to suppose that *only* those who actually accept relevant theological presuppositions will suppose that theological investigations of infinity have significant implications for mathematics and the physical sciences. As I

noted towards the end of section 1 above, there are no substantive results in contemporary logic or mathematics or physics that depend essentially upon theological assumptions: one is not *required* to make theological assumptions in order to earn entitlement to the axiom of choice, or the fundamental theory of calculus, or the theory of general relativity, or whatever.[10]

Second, there are issues that cluster around the *interpretation* of theological talk. There is a long tradition of claiming that much—or even all—theological talk is metaphorical or analogical, or, at any rate, not susceptible of a straightforward realist construal. I've suggested that it is highly plausible to think that mathematical investigations of infinity will have significant consequences for theology only if theological talk is given a straightforwardly realist construal. At the very least, it very hard to see how one could think that mathematical analyses of the infinite bear at all on theological talk about the infinite if this latter talk is all taken to be merely metaphorical, or analogical, or the like.

Third, there are issues that cluster around the *limits* that one might wish to impose upon straightforwardly realistic theological talk because of alleged limitations in our capacities to fully and completely understand the central objects of theological talk. I've suggested that it is not at all obvious that it is reasonable to allow that there are properties of God of which it is logically or metaphysically necessary that we have no more than a partial or incomplete grasp of what those properties *are*. However, if there are properties of God of which it is logically or metaphysically necessary that we have no more than a partial or incomplete grasp of what those properties *are*, then that suggests one kind of limitation upon the application of mathematical investigations of infinity in theological contexts that will need to be respected.

Fourth, there are issues that cluster around the *identification* of those parts of theology where it is plausible to suppose that mathematical investigations of infinity will have significant consequences. I take it that the obvious place to look is the discussion of divine attributes. There are many divine attributes that seem to involve some kind of imputation of infinite magnitude to properties or infinite domains of entities. While I mentioned a few plausible candidates—omniscience, omnipotence, omnipresence, eternity, perfect goodness, sole creation of the world *ex nihilo*—I don't pretend that this list is either systematically generated or exhaustive. However, it should not be supposed without further investigation that there are no other parts of theology—i.e. apart from discussion of the divine attributes (and, of course, the arguments for and against the existence God)—where mathematical investigations of infinity will have significant consequences.

Fifth, there are issues that cluster around the *application* of the results of mathematical investigations of infinity to those parts of theology where it is plausible that mathematical investigations of infinity will have significant consequences. How exactly, can or do mathematical accounts of infinity contribute to the analysis, or understanding, or explanation of particular divine attributes (or of particular arguments concerning the existence of God)? What commitments to infinite magnitudes of properties or infinite domains of entities are plausibly incurred by way of the attribution of particular divine

attributes (or the adoption of particular arguments concerning the existence of God)? What kinds of infinities are involved in those cases where there are commitments to infinite magnitudes of properties or infinite domains of objects incurred by way of the attribution of particular divine attributes (or the adoption of particular arguments about the existence of God)?

Sixth, there are issues that cluster around the *consistency* or stability of uses of the results of mathematical investigations of infinity in theology. Once we have in view a map of the ways in which the results of mathematical investigations of infinity have been—or could be—applied across a range of theological domains, we are then in a position to ask whether those results have been—or would be—applied in a consistent manner across those domains. I've suggested, for example, that there are serious questions to be asked about the consistency of the treatment of infinity in some of the standard arguments for the existence of God with the treatment of infinity in some of the standard analyses of the divine attributes.

Seventh, there are issues that cluster around the question of the *normative significance* that philosophical and theological investigations of the infinite have for one another. I take it that theological hypotheses can have significant consequences for philosophical debates about the ways (if any) in which infinity is present in the world. I sketched an argument which suggests that standard theological hypotheses bring with them a range of commitments to actual infinities and to Platonist interpretations of contested philosophical domains. If this is right, then, for example, the adoption of standard theological hypotheses has important consequences for the debates about formalism, finitism, intuitionism and Platonism in the philosophy of mathematics for those who take these theological hypotheses seriously.

Eighth, there are issues that cluster around the *application* of the results of mathematical investigations of infinity to specific parts of Christian theology and doctrine—e.g. to discussions of trinity, incarnation, immortality, and so forth. To the extent that there has been prior discussion of the *application* of the results of mathematical investigations of infinity to theology, this discussion has tended to focus on questions about generic divine attributes, i.e. divine attributes as these are conceived on most monotheistic conceptions of God. However, it seems to me that there are bound to be questions that are quite specific to Christian theology and doctrine for which investigation of "limits" and "bounds" as a subject matter in its own right has important consequences.[11]

While the examination of infinity in theological contexts is doubtless not itself an *infinite* task, it is abundantly clear—even from this relatively superficial and incomplete overview—that there is *plenty* of work to be done.[12]

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Endnotes

[1] See, for example, Barrow (2005), Benardete (1964), Moore (1990), Owen (1967), and Rucker (1982).

[2] Meyer (1987) offers a "proof" that the claim that God exists is logically equivalent to the Axiom of Choice. However—even setting aside the evidence of tongue placed securely in cheek—it is clear that Meyer offers no more than a patch for *one* of the holes in the argument of Aquinas' second way. Lewis (1991) is typical of much technical literature in logic, mathematics, and the physical sciences: it contains a range of references to God, but none that is *essential* to the theory of parts of classes that Lewis elaborates and defends.

[3] There are many complex issues—concerning, in particular, the doctrine of divine simplicity—that arise here. If we suppose that there must be unity of truth-makers for claims involving simple predicates, then it seems to me that the doctrine of divine simplicity stands refuted unless we allow that we can grasp *the* divine property. However, if we allow that there can be diversity of truth-makers for claims involving simple predicates, then perhaps we can allow that there is a sense in which we understand simple predications that are made of God—since we understand the words that are used in making these predications—even if there is another sense in which we don't understand these predications—because we cannot fully understand that in virtue of which these predications are true. In the remainder of this paper, I shall ignore considerations about divine simplicity.

[4] There are a number of interesting questions to be raised here about the propriety of using reference-fixing descriptions that contain predicates that one does not fully understand. There are also interesting questions here about the relationship between entailment and (full) understanding, and the relationship between devising analyses and possessing (full) understanding. And, perhaps most importantly of all, there are fundamental questions to ask about what is involved in the full—and in the partial or incomplete—grasping (understanding, apprehension) of properties. Much work remains to be done to achieve clarity on all of the relevant issues that arise in connection with these questions.

[5] Note that, if we suppose that God knows the truth status of every proposition for which it is logically possible that God knows the truth status of that proposition, we do not suppose—but rather take no stance on the claim—that there are propositions for which it is not logically possible that God knows their truth status. My formulation here is meant to be neutral on, for example, the question whether Gödel's limitative theorems would have application in the case of God.

[6] There are many important and interesting questions that arise here. In particular, there are questions about whether we can understand talk of "logical independence" that is not tied to the specification of particular linguistic resources. If we specify a language and a proof theory (or model theory), then we can give an account of logical independence for the logical system thus specified. But what are we to make of talk of "logical independence" that is not thus tied to specification of a particular logical system? There are other

places in these notes where some will suppose that what is said makes no sense because these kinds of foundational questions about languages, syntax and interpretation have not been addressed. I take it that this points to yet another area of inquiry that cannot be avoided in a full examination of the implications for theology of investigations of "limits" and "bounds" as subject matters in their own right. (Some might think that there are also questions about the logic that is proper to discussion of God, just as some have supposed that there are questions about the logic that is proper to discussion of quantum mechanics. My own view is that classical logic is the proper logic for discussions of both quantum mechanics and God.)

[7] Even this formulation is at best provisional. I have already noted that "consisting of N persons" is a degreed predicate. If we allow that it is possible that a being consist of three persons, how can we deny that it is possible that a being consist of four persons? Yet, somehow, the Christian theologian needs to be able to defend the claim that God consists of exactly three persons. I shall not speculate here about further refinements to the principle that I have begun to formulate.

[8] For a reasonably brief exposition of Conway's theory, see Oppy (2006: 42-4).

[9] There is at least a loose sense in which **On** can be identified with Cantor's "absolute infinity". Like Cantor's absolute infinity, **On** "lies beyond" all of the transfinite numbers. However, there are claims that are sometimes made about Cantor's absolute infinity that are clearly not true of **On**. In particular, it should be noted that there is nothing inconceivable about **On**; and nor is it the case that **On** cannot be either uniquely characterised or completely distinguished from the transfinite numbers. (Cf. Rucker (1982:53)) On the contrary, **On** is a gap rather than a number, and it is distinguished, in particular, by the fact that it "exceeds" all of the transfinite numbers. (I think that the claims in question are no more plausible in the case of Cantor's absolute infinity; but it is, I think, even more clear that they are not plausible in the case of **On**.)

[10] There have been some recent expressions of interest in, and support for, the notion of "theistic science". I take it that the key idea here is that there might be significant scientific—or logical, or mathematical—results that depend essentially upon theological assumptions. If, for example, the fine-tuning of the cosmological constants is best explained—and only explainable—by the hypothesis of intelligent design, then that explanation *might* count as an example of "theistic science". Thus, the claim that I have made in the main text is not entirely uncontroversial.

[11] Perhaps because of the nature of my own interests, I have focused here particularly upon considerations from logic, philosophy of language, and metaphysics. But there are also interesting *epistemological* issues that are raised by questions about the infinite—for some introduction to these considerations, see, for example, Thomson (1967) and Lavine (1993)—and, in particular, by questions about the infinite in the context of theology.