A PRIORI KNOWLEDGE IN PERSPECTIVE: NAMING, NECESSITY AND THE ANALYTIC A POSTERIORI

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I

IN THE FIRST PAPER IN THIS SERIES[1] I attempted to do away with some common misconceptions concerning Kant's doctrine of a priori knowledge. By using the approach of Philip Kitcher as a sounding-board, I was able not only to provide a Kantian response to Kitcher's criticisms of the apriorist understanding of mathematical knowledge, but also to sketch in the process some of the essential tenets of Kant's epistemological framework. In this sequel, I will begin by reviewing and expanding my discussion of Kant's terminology. In Section II, I will then discuss the epistemological framework developed by Saul Kripke.[2] In Section III, I will demonstrate that the two frameworks overlap to a large extent, but that because they define the same terms in different ways, one must be adjusted to correspond to the other before any intelligible debate between the two can be carried out. After adjusting Kripke's terminology to fit Kant's framework, I will suggest a perspectival reconstruction of Kant's framework which takes into consideration Kripke's most important insights. Finally, in Section IV, I will provide a response to Kripke's unfair treatment of Kant, after which I will return to the subject of mathematical knowledge and

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[1] See "A Priori Knowledge in Perspective: Mathematics, Method and Pure Intuition," *The Review of Metaphysics* 41 (September 1987): 3–22.

[2] As developed in *Naming and Necessity* (Oxford: Basil Blackwell, 1980), references to which will be abbreviated '*NN*' and included in the text. The following works of Philip Kitcher, used extensively in the first part of this series, will also occasionally be referred to in the text using abbreviations: "Kant and the Foundations of Mathematics" (*KFM*), *The Philosophical Review* 84 (1975): 23–50; "A Priori Knowledge" (*APK*), *The Philosophical Review* 89 (1980): 3–23; "Apriority and Necessity" (*AN*), *The Australasian Journal of Philosophy* 58 (1980): 89–101; and *The Nature of Mathematical Knowledge* (*NMK*) (Oxford: Oxford University Press, 1984).

Review of Metaphysics 41 (December 1987): 255–282.

argue that, although mathematical knowledge is synthetic a priori from one perspective, it is also analytic or a posteriori from other (equally valid) perspectives, and that in some cases it is even *analytic a posteriori*.

For Kant, the two distinctions between a priori and a posteriori and between analytic and synthetic are fundamental, irreducible and not coextensive.[3] "A priori knowledge" refers to the necessary and universal conditions which the subject imposes upon the object in the very act of experiencing it. A posteriori knowledge, by contrast, is the conscious knowledge which only arises contingently, as a result of particular experiences. Analytic knowledge is that knowledge whose propositional expression is true (given an accepted meaning for the terms involved) solely by virtue of logical laws. And synthetic knowledge "goes beyond" such logical clarification and is positively informative; its source of new information (being non-logical) is intuition. (As we saw in the first part of this series, these terms can also refer to methods of argumentation, in which case the analytic a priori method indicates movement from the universal to the particular, and the synthetic a posteriori method indicates the opposite pattern. In the former method the terms used are not up for revision, while in the latter they are. We also saw that Kant introduced a new, synthetic a priori method.) Combining these two distinctions gives rise to the four possible types of knowledge in Kant's epistemological framework: (1) synthetic a priori knowledge is *intuitive and necessary*, (2) synthetic a posteriori knowledge is *intuitive and contingent*, (3) analytic a priori knowledge is *conceptual and necessary*, and (4) analytic a posteriori knowledge would be *conceptual and contingent*, if it were possible. Nearly all philosophers (Kant included) have assumed with little or no argument that the fourth combination is impossible; but I have argued elsewhere (see note 3) that analytic a posteriori knowledge is not self-contradictory, and seems actually to play an important role in Kant's system. The further elucidation of what this rather enigmatic combination of terms might mean is one of the main purposes of this paper.

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[3] I discuss these two distinctions and how they interrelate in my article "Knowledge and Experience: An Examination of the Four Reflective 'Perspectives' in Kant's Critical Philosophy," *Kant-Studien* 78 (1987): 170–200. The present paragraph is a summary of the detailed interpretation presented in this article.

This description of the four kinds of knowledge delineated by Kant's epistemological framework is potentially misleading in one respect: its use of the words 'contingent' and 'necessary' needs to be carefully explained. We could say that 'contingent' means "could have been otherwise" and 'necessary' means "cannot be otherwise,"[4] but this still leaves open the question of what kind of possibility counts as an exception. In *The Critique of Pure Reason*, Kant says a priori knowledge provides "strict necessity, such as mere empirical knowledge cannot supply" (*CPR*, A2). This implies that there are other, "looser" types of necessity which Kant would allow some other application. He then contrasts "empirical universality" with "strict universality", and explains that the latter is "inseparable" from (strict) necessity, these being the two "sure criteria of *a priori* knowledge" (*CPR*, B4). And he explains that "a proposition which in being thought is thought as *necessary* . . . is an *a priori* judgment" (*CPR*, B3). Strict (epistemological) necessity, then, includes logical necessity, but not empirical necessity (i.e., what might nowadays be called "physical necessity"). When Kant associates the empirical with the contingent—as when he says "empirical, and therefore contingent" (*CPR*, B5)—he is obviously already assuming the strict version of this distinction. Clearly, therefore, in the framework outlined in the previous paragraph, the terms 'necessary' and 'contingent' must be taken solely in Kant's "strict" sense.

Kant's explanation of the necessary character of a priori knowledge indicates that not just *any* sense of the word 'necessary' will suffice to define a judgment as a priori. What he says is that everything a priori is necessary (and universal) in the *strict* sense. Thus his claim that mathematical judgments "carry with them necessity, which cannot be derived from experience" (*CPR*, B14) need not be taken to imply "that knowledge of necessity cannot be derived from experience" (*AN*, 100n); for Kant means that such judgments carry with them a *type* of necessity which cannot be derived from experience, so it is only *a priori* necessity which cannot be so derived. It is wrong, therefore, to assume that Kant

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[4] Kant uses this definition of necessity in *Critique of Pure Reason*, trans. Norman Kemp Smith (London: The Macmillan Press Ltd., 1933), p. 3. References to the *Critique* will be indicated by the abbreviation '*CPR*', followed by a page number.

himself "uses 'necessary' and 'a priori' interchangeably," even though this way of reading him has become so widespread that most commentators and critics "have followed suit, taking the equivalence of the notions of necessity and a priority for granted" (*AN*, 89; *NMK*, 32). As we shall see, the tendency to forego any distinction between types of necessity in favour of some supposedly all-encompassing one (such as "true in all possible worlds" [*KFM*, 24; see also *AN*, 92 and *NN*, *passim*]) can lead not only to a misinterpretation of Kant, but to an improper analysis of certain types of truth.

A good example of the one-sided view of necessity which is to be avoided is defended by Quinton in "The *A Priori* and the Analytic."[5] He defines "the necessary as that which is true in itself, no matter what" (*APA*, 112; see also p. 109) and the contingent as that which "is true dependently on or because of something else, something outside itself" (*APA*, 109). These two terms "make an exclusive and exhaustive division of the realm of truths" (ibid.). At one point in his discussion of "the analytic thesis" (i.e., "the doctrine that all *a priori* statements are analytic") (*APA*, 107), Quinton describes his position in terms of "the principle that all necessity is logical" (*APA*, 124), which he upholds so long as this "is interpreted broadly, as saying that all necessary truths implicitly define their terms or are reducible to those that do" (*APA*, 125). The reasoning behind this view, apparently, is as follows: necessity is a type of truth; truth is a property of propositions; propositions are true or false only in virtue of their logical form; therefore, all necessity is logical, for the truth it describes is assumed by the meanings of the terms. On this basis he argues that the analytic-synthetic and a priori-a posteriori distinctions both "coincide with the distinction of the necessary from the contingent" (*APA*, 109).

Quinton's epistemological framework, though representative of a position often defended by analytic philosophers, is incoherent inasmuch as it rests on a vacuous definition of necessity. For if the necessary is really that which is true "no matter what" (i.e., from any perspective), then there would be no necessary truths whatso-

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[5] Anthony Quinton, "The *A Priori* and the Analytic," in P. F. Strawson, ed., *Philosophical Logic* (Oxford: Oxford University Press, 1967), 107–28; reprinted from *Proceedings of the Aristotelian Society* 64 (1963–4): 31–54; abbreviated hereafter by '*APA*'.

ever. Any type of necessary truth can be regarded as falsifiable (and thus contingent) if the foundation (or perspective) on which it rests is removed or altered, even if the terms defined by the proposition itself retain the same meaning. For example, if we are willing to deny that scientific laws describe physical necessities for our particular world, then it would not be physically necessary that "Unprotected human beings cannot survive prolonged exposure to a 100°C environment." As it is, most of us would agree that the word 'cannot' in this proposition describes a physical necessity, even though it is obviously conceivable (i.e., *logically* possible) that human beings might have been unaffected by such high temperatures. Or again, if we deny that the law of causality describes a transcendental law which limits how we can interpret our experience, then the transcendental necessity referred to in the proposition "This phenomenon *must* have been caused by *something*" is rendered transcendentally contingent. Even propositions whose necessity clearly is logical, such as "All bachelors are unmarried,"[6] would be made logically contingent if we were to deny that "$A = A$" expresses a fundamental logical law. To conflate all these types of necessity, just because we express such truths in words, ignores the fact that it is ultimately not the words alone, but the *relation* between a proposition and its context (i.e., the *perspective* which is assumed in uttering it) that determines its necessity or contingency.

Truths which are true "no matter what," or which "implicitly define their terms or are reducible to those that do" [*APA*, 124–25] should not be classified as "necessary," but as "analytic." Similarly, Quinton's definition of 'contingent' (*APA*, 109; quoted above) is almost identical to one of Kant's definitions of 'synthetic'. His definition of *a priori* as that which "rests on intuition" (*APA*, 108) also seems closer to Kant's understanding of 'synthetic', until Quinton explains that by "intuitive" he means that the truth of a

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[6] This proposition, incidentally, is not necessary in the loose, empirical, sense of "true in all possible worlds" (see note 7). For we can conceive of a world in which some or all of those whom we now call "bachelors" do marry (in which case, of course, the persons to whom we refer would no longer be described as bachelors). In this empirical sense, the truth expressed by the proposition in question is contingent, even though from the logical perspective (i.e., attending only to the meanings of the words) it is necessary.

proposition "is a condition of understanding the terms it contains" (*APA*, 108)—and this, of course, is not the meaning of 'a priori', but of 'analytic'. Quinton adds that " '*a priori*' means either, widely, 'non-empirical' or, narrowly, following Kant, 'necessary'. To assimilate these is to make the questionable assumption that all non-empirical truths are necessary" (*APA*, 108). But this just isn't true. Kant defines 'a priori' in neither of these two ways; instead, he requires that anything a priori is *both* non-empirical *and* necessary. And this leaves open the possibility of establishing an *empirically* necessary class of truths as well as a non-empirical, non-necessary class. All that is required by Kant's definition is that if such classes are established, they must not be called "a priori."

Mentioning Quinton's framework and exposing its weakness in this first section has helped to refine our understanding of Kant's epistemological framework; but it also serves as a helpful preparation for Section II, where we shall examine a similar framework adopted by Kripke in *Naming and Necessity.* There are important differences; but this brief treatment of one of Kripke's analytic precursors will make it easier to see where he is breaking with his own tradition—though, interestingly enough, some of the most important points which come to fruition in Kripke's work actually have their seed in "The *A Priori* and the Analytic" (see e.g., *APA*, 110–11, 119–20). Let us now turn to the arguments presented by Kripke, paying special attention to the framework upon which they rest.

II

Kripke's main purpose in *Naming and Necessity* is to open our minds to the possibility of the "necessary *a posteriori*" and "contingent *a priori*" (*NN*, 38) and to provide concrete evidence for the existence of truths of these sorts. He does this by discussing various ways of interpreting the function of names. The "classical" view Kripke ascribes to Frege and Russell is that "a proper name" is simply "a definite description abbreviated or disguised" (*NN*, 27). The problem he finds with this view is that it purports to provide a theory of naming yet its account is not always consistent with the way names are ordinarily learned and used. "If 'Moses' *means* 'the man who did such and such', then, if no one did such and

such, Moses didn't exist" (*NN*, 58). This clearly won't do, because we would want to admit that (for instance) Nixon would have been Nixon whether or not he ever became president, even though many people would associate his name with the description "the U.S. President in 1970" (*NN*, 40–49).

To explain why it makes sense to think of Nixon as having decided at some point not to go into politics, but not to think of him as having not been Nixon, Kripke introduces the term 'rigid designator'. He defines 'designator' as "a common term to cover names and descriptions" (*NN*, 24); it is "a *rigid designator* if in every possible world it designates the same object" (*NN*, 48). The object designated—that is, "the thing named by a name (or uniquely satisfying a description)"—is called the "referent" (*NN*, 86n). The crux of Kripke's theory is that "*names* are rigid designators" (*NN*, 48), so that it is not the meaning of a particular description, but the existence of a particular referent which is of primary importance. By distinguishing in this way "between using a description to give a meaning and using it to fix a reference" (*NN*, 5; see also *AN*, 95–96), Kripke preserves some use for the description theory. Although it is inadequate as a theory of meaning, he accepts that it is often adequate as an explanation of what happens in "cases of initial baptism" (*NN*, 96n; see also 135).

Kripke describes this perspectival shift as a change from viewing necessity as "a notion of epistemology" to viewing it in the context "of metaphysics" (*NN*, 35; see also 122–23, 135). Unfortunately, he never explains what he means by "metaphysical," nor how he can coherently combine metaphysical terms with terms he admits to be epistemological to produce the locutions "necessary a posteriori" and "contingent a priori" (as in *NN*, 56). What he does say is that his version of the necessary-contingent distinction has a certain "intuitive content" (*NN*, 41–42). This seems to mean that he intends to use 'necessity' as a description of the status of empirical objects or acts rather than of items of knowledge. (If so, the word 'ontological' would have been more suitable, especially since, as it turns out, his supposedly *meta*physical necessity is often applied to empirical objects in such a way as to render it indistinguishable from what is ordinarily called *physical* necessity.) This interpretation is supported by his general usage, as, for example, in the following:

> If we can't imagine a possible world in which Nixon doesn't have a certain property, then it's a necessary condition of someone being Nixon. . . .
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> . . . The question of whether Nixon might have not been a human being, given that he is one, is not a question about knowledge, *a posteriori* or *a priori*. It's a question about . . . what might have been the case [in some possible world]. (*NN*, 46–47)

The *empirical* nature of such necessity (and contingency) is highlighted by Kripke's frequent reference to "possible worlds." For, "A possible world is *given by the particular descriptive conditions we associate with it*" (*NN*, 44). To associate necessity with the implications of such *givenness* ties it inexorably to the empirical world.[7] Hence, instead of "metaphysical" (or "ontological") necessity, I shall refer to Kripke's special version as "empirical necessity."

According to Kripke, 'analytic', 'a priori' and 'necessary', together with their counterparts, are all "categories of truth" (*NN*, 34); yet he never makes it clear how his non-epistemological use of 'necessity' can be used to draw conclusions about "necessary *truth*" —a notion generally not associated with empirical objects as such,

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[7] The fallacy which is often committed by those who avail themselves of "possible worlds" logic is that they mistakenly assume that a truth which is determined by this means to be *necessarily* true is therefore *logically* necessary, when in fact, its truth is only *empirically* necessary. The reason for this is that the law of identity only carries necessity in this context when the terms used are interpreted as rigid designators (see *NN*, 3); and to impose this condition is to introduce empirical criteria into the judgment. For it is only true that $(x)(y)(x = y \supset \Box x = y)$ if 'x' and 'y' (e.g. Hesperus and Phosphorus) are both *assumed* to refer rigidly to "some fixed individual" (*NN*, 9)—that is, to the same *empirical* object (e.g., Venus) (*NN*, 20). And the factual ground of this assumption means that the necessity of the identity could never be established by means of logic alone. I am not assuming that such identities are "contingently identical" (*NN*, 4–5), but rather that the type of necessity they do in fact possess is *empirical*—which means, of course, that from the logical perspective alone such identities *are* properly regarded as contingent. Indeed, one of the contributions of *Naming and Necessity* is to have shown that, in order to produce any sort of *necessity*, possible worlds semantics must be *tied* to the empirical world in some explicit, extra-logical way. All talk of "possible worlds," therefore, is talk about "(abstract) *states*" of *empirical* objects (see *NN*, 16–7). (Unfortunately, Kripke occasionally confuses matters by claiming that the necessity he is dealing with is not "mere physical necessity," but refers to that which "is actually necessary *tout court*" [*NN*, 164]. This indicates that he himself was not fully aware of the limited applicability of his conclusions [see Section III].)

but with the logical structure of propositions. Presumably, the phrase 'necessary a posteriori truth' describes the status of a proposition in which the referent is empirically necessary (i.e., the proposition is true in all possible worlds), even though the subject's knowledge of its truth is gained only through particular experience(s). Likewise, the phrase 'contingent a priori truth' apparently describes the status of a proposition in which the referent is empirically contingent (i.e., the proposition is not true in all possible worlds), even though the subject knows it is true in this world without having to experience the object itself.

The legitimacy of this way of combining ontology and epistemology, or the real and the logical, in assigning categories of truth, is the subject of Kant's criticism of Leibniz (*CPR* 319–36), and will be called into question in Section III. For now we can ignore this problem and simply note that Kripke completes his framework by stipulating "that an analytic statement is . . . true by virtue of its meaning," in such a way that "something which is analytically true will be both necessary and *a priori*" (*NN*, 39). In light of his emphasis on the "distinction between a priority and [empirical] necessity" (*NN*, 110), we can therefore summarize Kripke's quasi-epistemological framework as follows:

1. *Necessary a posteriori* truth is true in all possible worlds, but must be learned through experience
2. *Contingent a posteriori* truth is true in only some possible worlds, and must be learned through experience (hence it can be called *synthetic*[8])
3. *Necessary a priori* truth is true in all possible worlds, and can be known without appealing to experience (hence it can be called *analytic*)
4. *Contingent a priori* truth is true in only some possible worlds, but can be known without appealing to experience

Categories (2) and (3) are comparatively unproblematic. Categories (1) and (4), by contrast, are plagued with a host of difficulties.

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[8] Kripke never to my knowledge defines 'synthetic' in this (or any other) way, no doubt because he has little interest in discussing the nature of this "contingent a posteriori" category of truth. Nevertheless, the parallelism between this use of 'synthetic' and the meaning he stipulates for 'analytic' (see *NN*, 39, quoted above) should be obvious.

The remainder of this section will be devoted to a sketch of Kripke's arguments for the legitimate application of the latter pair to various sorts of propositions.

Kripke's exploration of the realm of necessary a posteriori truths concentrates on propositions describing discoveries of "theoretical identities"—that is, "identities involving two rigid designators" (*NN*, 140), such as 'Hesperus is Phosphorus'. (These names were used to refer rigidly to the evening star and the morning star, respectively, before astronomers discovered that both are appearances of Venus [*NN*, 28].) Now, once the identity of Hesperus and Phosphorus is accepted as a "given" fact, the truth of the proposition "Hesperus is Phosphorus" must be regarded as empirically necessary, for "there couldn't have been" a world "in which Hesperus wasn't Phosphorus" (*NN*, 103). This is the case so long as we regard identity not epistemologically as "the relation which holds between two names when they designate the same object," but metaphysically (i.e., empirically) as "the relation between an object and itself" (*NN*, 107–08). Kripke admits that "two distinct bodies might have occupied, in the morning and the evening, respectively, the very positions actually occupied by Hesperus-Phosphorus-Venus" (*NN*, 143; see also 104), but he stresses that this "would not have been a situation in which Hesperus was not Phosphorus," because as long as these names are regarded as rigid designators, "at least one of them, and maybe both, would not have been Hesperus" (*NN*, 109). The conclusion, then, is "that identity is not a relation which can hold contingently between objects" (*NN*, 154), because if two objects *are* identical (i.e., are one and the same empirical object, viewed from two different perspectives), they *must be* identical (*NN*, 3).

Most philosophers before Kripke would have assumed that 'Hesperus is Phosphorus', if necessary, must also be analytic, and therefore a priori as well. But Kripke denies this on the grounds that our discovery "that we have tagged the same planet twice . . . is empirical" (*NN*, 100). All we can know "*a priori*, by philosophical analysis" is "that *if* such an identity statement is true it is necessarily true" (*NN*, 109). Once we recognize that such names function as rigid designators, he argues, we must admit that "we do not necessarily know *a priori* that an identity statement between [them] is true" (*NN*, 101). On the contrary, such knowledge usually arises a posteriori, so the propositions describing the discovery of

such theoretical identities must be regarded as necessary a posteriori truths.

The examples Kripke gives of contingent a priori truths are all concerned with those aspects of a name whose truth is "given by the way its reference is fixed" (*NN*, 135). One such example concerns the designator 'one meter', which is stipulated at a certain point in time (t_0) "to be the length of S where S is a certain stick or bar in Paris" (*NN*, 54). Kripke asks: "Is it then a necessary truth that stick S is one meter long at time t_0?" (*NN*, 54). He answers that it is not, because (for instance) "if heat had been applied to this stick S at t_0, then at t_0 stick S would not have been one meter long" (*NN*, 55). His point is correct, as long as we remember once again that "the phrase 'one meter' . . . is meant to designate rigidly a certain length in all possible worlds, which in the actual world happens to be the length of stick S at t_0" (*NN*, 55; see also 75). In the same way the description 'the greatest man who studied under Plato' can be used to fix the referent of the rigid designator 'Aristotle', without making it an empirically necessary fact that the man Aristotle decided to study philosophy (he could have become a fisherman).

The a priori status of such reference-fixing propositions is put forward in a rather tentative way by Kripke (see, e.g., *NN*, 38). "It would seem", he says, that a person who "used stick S to fix the reference of the term 'one meter' . . . knows automatically, without further investigation, that S is one meter long" (*NN*, 56); that is, "he knows it *a priori*." Likewise, if a person first learns of Aristotle by learning that he was Plato's greatest pupil, the latter will serve as a kind of a priori "definition" of the name 'Aristotle' (*NN*, 57). Kripke's point here is that nonrigid designators can serve to fix the reference of rigid designators, and that fixing the reference in this way gives rise to knowledge which is "in some sense" a priori (*NN*, 63n). To guard against the claim that some "reformulation" of the meaning of a priori might be able to preserve the traditional thesis "that everything *a priori* is necessary," he merely stipulates that he will "use the term '*a priori*' " solely in connection with "statements whose truth follows from a reference-fixing 'definition' " (*NN*, 63–4n).

Of primary concern to Kripke throughout his discussion is the analysis of the activity of naming. As we have seen, both the necessary a posteriori and the contingent a priori turn out, on the basis of his framework, to be of vital importance to a proper under-

standing of this fundamental human activity. His interest is not so much with the *meaning* of names such as 'quarks', but with determining the status of a person's knowledge of the truth of the proposition that "quarks are called 'quarks' " (*NN*, 69–70). Although "it's trifling to be told that Socrates is called 'Socrates'," this does not mean that "it is necessary or analytic," even though it may be known a priori (*NN*, 72). For "whatever this relation of *calling* is is really what determines the reference and not any description" (*NN*, 70). When naming or learning to use a name, the only way to avoid getting stuck in a meaningless circle is to appeal to "some independent determination of the referent" (*NN*, 73). Kripke proposes a theory according to which a person learns how to use a name "by virtue of his membership in a community which passed the name on from link to link" in "a chain of communication" (*NN*, 91). But there is no need to examine the details of this theory here, since our interest is in the framework he uses to define a priori (and other sorts of) knowledge. Having completed our sketch of his main arguments, we can now proceed to discuss their implications with respect to Kant's framework.

III

The well-known arguments of Kripke, as summarized in the previous section, offer quite a challenge to any traditional account of the nature of a priori knowledge—not the least to the epistemological framework developed by Kant (see Section I). They purport to illuminate types of a priori and a posteriori knowledge which Kant never took into consideration. Although Kripke claims that his conclusions contradict Kant's (see Section IV), we should not make such a judgment too hastily since the framework upon which they are based is quite different from that which Kant proposed. Indeed, this difference threatens to preclude any intelligible communication between proponents of these two approaches. The only way to avoid this danger will be to translate one set of conclusions into the other framework, for in this way alone can the relationship between the two approaches be accurately assessed. Kant's framework has the advantage not only of being traditional and therefore more familiar, but also of avoiding Kripke's ~~pres~~ Critical mixture of "metaphysical" and epistemological perspectives, the legitimacy of

which he never defends. Accordingly, I will begin this section by translating Kripke's arguments into their equivalents in the Kantian framework, and assessing the extent to which both can maintain their legitimacy. As a result, I will propose a new epistemological framework which takes into consideration the conclusions of both of these approaches. This will serve as a preparation for examining, in Section IV, the legitimacy of Kripke's criticism of Kant, and for developing a more balanced view of the status of mathematical knowledge.

At first sight, the main difference between the frameworks proposed by Kant (see Section I) and Kripke (see Section II) is that the latter goes beyond the former in taking into consideration the implications of *empirical* necessity and contingency. This is reflected by the significant fact that the roles played by the "analytic-synthetic" and "necessary-contingent" distinctions are interchanged in the two frameworks. Whereas Kant's framework employs 'analytic' and 'synthetic' as terms which have an equal status with 'a priori' and 'a posteriori', they appear in Kripke's framework as secondary labels for two of his four categories of truth. And whereas Kant employs 'necessary' and 'contingent' in a strict sense to help describe the difference between 'a priori' and 'a posteriori', Kripke uses them in a broad sense as terms of primary importance. In order to make an accurate translation of Kripke's classifications into their Kantian counterparts we must carefully scrutinize the way he uses his terms in each case, and then search Kant's framework for the classification which corresponds most closely to this usage.

The second and third categories in both frameworks are quite similar. To say that "synthetic a posteriori knowledge is . . . contingent" is hard to distinguish from saying that "contingent a posteriori truth is . . . synthetic." Likewise, Kant's "analytic a priori knowledge is . . . necessary" is quite similar to Kripke's "necessary a priori truth is . . . analytic." There are some differences between these pairs, but these will be easier to discern by concentrating on the differences between the two types of first and fourth categories.

We saw in Section II that Kripke defines "necessary a posteriori" as the category of truth which holds between two rigid designators when they refer to the same empirical object. Transferring Kripke's empirical interpretation of propositions such as "Hes-

perus is Phosphorus" into Kant's purely epistemological categories involves explicating the meaning of each term by replacing it with a description of that which it designates rigidly. Doing this yields either a false proposition or one which is analytic a priori, depending on how the designated object is described: "The appearance of Venus in the evening is the appearance of Venus in the morning" is false (because the terms are taken to designate rigidly the appearance rather than the object which appears), whereas "Venus is Venus" and "Venus is Hesperus (i.e., the appearance of Venus in the evening) and Phosphorus (i.e., the appearance of Venus in the morning)" are analytic a priori. (Given that $A = B$ and $A = C$, the equation $B = C$ can be reduced to the logical identity, $A = A$.) Thus when Kripke says, "One does know *a priori*, by philosophical analysis, that *if* such an identity statement is true it is necessarily true" (*NN*, 109), we can translate his point into Kant's framework by saying, "One does know analytically that *if* such an identity statement is true it is true a priori." This indicates that the "necessity" of such theoretical identities is, in fact, analytic necessity (in *Kant's* sense) after all. Nevertheless, our translated version of the necessary a posteriori seems rather trivial compared to the rich implications Kripke draws out of this class of truths. For Kripke is able to account for the element of *discovery* which is implied in such statements of theoretical identity.

Can Kant's framework be used to account for the element of discovery? I think it can, provided we keep in mind the perspectival nature of his epistemological classifications. For the common source of the two appearances of the planet Venus, which Kripke rightly regards as empirically "necessary," and the scientist's discovery of this fact, which Kripke rightly regards as "a posteriori," are both examples of what Kant would classify as "synthetic a posteriori knowledge." In the first place Kripke's use of the word 'necessary' has to do entirely with what Kant would call "synthetic" matters—that is, matters which go beyond logic and the meaning of words and are concerned with the objects of intuition, with "what might have been the case" (*NN*, 47). And secondly, *any proposition* containing a term which is assumed to be a rigid designator is a posteriori in Kant's framework, because interpreting a word as a rigid designator means regarding it as inextricably tied to the object *as experienced*—that is, to the empirical object itself rather than to some description of it. Given Kripke's definition of 'ana-

lytic' as that which is both empirically "necessary" and "a priori", this would entail that there are few, *if any*, analytic truths.[9]

Kripke avoids this undesirable implication of his framework by stipulating (in *NN*, 13–14n) that *a priori*, which for Kant indicates that a proposition's truth can be justified independently of particular experiences, will now refer to "statements whose truth follows from a reference-fixing 'definition'." But this is unnecessary. The undesirable conclusion can just as well be avoided by giving up the enigmatic requirement that analytic truths be both empirically necessary and a priori. The traditional view of analyticity, to which Kant subscribed, is that analytic truth is that which is true in virtue of its logical form, given the *meaning* of the terms used (but not their *reference*, as is required by Kripke's inclusion of empirical necessity). As we have seen, theoretical identities are analytic only when the meanings of the words are assumed to be given. Thus Kant's framework can do justice to the discovery of theoretical entities by describing them as synthetic a posteriori when they are viewed from the empirical perspective of their *discovery* (cf. *NN*, 100), even though they are analytic a priori when viewed from the logical perspective of their *meaning*. (Before Hesperus was discovered to be Phosphorus in disguise, the statement "Hesperus is Phosphorus" could have been regarded as a contingent hypothesis, but not a necessary truth. It's empirical necessity is apparent only when the identity of the two rigid designators is taken as given, at which point the analytic a priori status of the same sentence, interpreted in terms of its meaning, also becomes recognizable.)

Kripke's account of the contingent a priori concerns reference-fixing propositions, such as "One meter is the length of stick S at t_0," which also use a rigid designator, so it is difficult to see how they could be a priori when assessed in terms of the Kantian framework. As we have seen, by "a priori" Kripke means that the truth of such propositions "follows from a reference-fixing 'definition'." Yet for Kant, anything which follows from a definition is not (as such) a priori, but *analytic*.[10] What sense can we make, then, out of

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[9] Along these lines, Quinton argues in *APA*, 111, that *all* a priori truths are contingent.

[10] Indeed, Kant regards definitions as the epitome of analytic truth. See L. W. Beck, "Kant's Theory of Definition," *The Philosophical Review* 65 (1956): 179–91; reprinted in R. P. Wolff, *Kant* (London: Macmillan, 1968), 23–36.

the supposedly *contingent* nature of such propositions? Kripke regards them as contingent because the description used to fix the reference (in the present case, "the length of stick S at t_0") is a *non*rigid designator, so the state of the empirical object to which it refers can be different in other possible worlds. For Kant, however, it would not be the contingent nature of the empirical object (which he would readily admit), but its relation to the knowing subject which would determine the status of the proposition. In order to fix the reference of a name a person must have (at least the possibility of) a concrete *experience* of the object in question, so a proposition expressing the results of a reference-fixing judgment must be a posteriori. (What sense would it make to fix the reference of 'one meter' by referring to a stick in Paris, but stipulating that no one is able to experience the stick in question? It certainly would not suffice to fix the reference of a rigid designator.) Hence Kripke's analysis of the contingent a priori awakens the Kantian to the possibility of *analytic a posteriori knowledge.*

This seems to pose a problem for the traditional Kantian framework, on the basis of which analytic a posteriori knowledge has almost always been denied as a contradiction in terms.[11] Quinton is one of the few who bothers to provide an explicit argument against the analytic a posteriori: he reasons that "if [a statement] is true in virtue of its meaning there is no room left for experience to have any effect on it and if it is true in virtue of experience then its meaning cannot have been sufficient to determine its truth" (*APA*, 110). But such arguments depend on the assumption that our analysis is limited to a consideration of the status of propositions in which the meanings of the terms is given, whereas Kripke encourages us to extend our view to the status of propositions used in the act of naming or of learning to use a name —i.e., in the act of *fixing a reference.* This extension of the scope of epistemological frameworks represents Kripke's lasting contribution to this branch of philosophy, even though the framework he

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[11] Kripke's framework disallows the analytic a posteriori by definition, since "analytic" is stipulated to mean that which is "both necessary and *a priori*" (*NN*, 39). He admits at one point, however, that his definition of analyticity may be too strict, in which case something very much like the analytic a posteriori is suggested: "If statements whose *a priori* truth is known via the fixing of a reference are counted as analytic, then *some analytic truths are contingent*" (*NN*, 122n, emphasis added).

employs in doing so is less than adequate. We need not transcend epistemology in favour of metaphysics or ontology in order to analyse the status of reference-fixing propositions. For they can be explained most adequately in terms of the analytic a posteriori category of knowledge.

Accepting the analytic a posteriori as a legitimate epistemological category enables us to distinguish, in a way which neither Kant nor Kripke succeeded in doing, between the status of "naming" and "defining." To name requires that we adopt a *practical* perspective, according to which we act "as if" (or stipulate that) a certain object is to be rigidly designated by a certain word. That is, we subsume an object as experienced (a posteriori) under a given concept (analytically). To define, by contrast, requires that we adopt a *logical* perspective, according to which we devote all our attention to accumulating a set of properties which describe a concept uniquely. That is, we subsume a set of general characteristics (a priori) under a given concept (analytically). Naming is in certain key respects *prior* to the synthetic a posteriori knowledge which we gather from the empirical world, whereas defining generally comes *after* and on the basis of a good deal of empirical information.[12] (We therefore honor babies by naming them, but honor the elderly or the dead by telling stories, or even writing biographies, about their lives.)

Once the possibility of analytic a posteriori knowledge is admitted on the basis of an analysis of naming, various other types of propositions can also be included in this category. Propositions which use a word or words in some "counter-conventional" way, such as is often found in poetry and other creative activities, "may be striking enough to recommend a new convention" (*APA*, 118) and so earn the status of analytic a posteriori. Another example, taken from philosophy, is Descartes' famous *cogito* argument, which is cogent only if it is regarded from this *practical* perspective. To say

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[12] The primacy of names over definitions bears a close resemblance to Kant's doctrine of the primacy of "practical reason" over theoretical (or "speculative") reason (see, e.g., *Critique of Practical Reason*, 119–21 [Berlin Academy numbering]). I discuss the possibility of regarding Kant's "practical perspective," especially as it is expounded in the second *Critique*, as giving rise to analytic a posteriori belief in "Knowledge and Experience," section 3, where I also discuss Kant's reasons for failing to recognize its analytic a posteriori status (see note 3, above).

"I think, therefore I am" only expresses a necessary truth if it is taken as an analytic a posteriori principle: "I know from experience (a posteriori) that I can and do think, and this implies (analytically) that I exist." The mistake of Descartes and others who regarded such arguments as establishing the *a priori* certainty of the immateriality of the soul, or even the a priori certainty of one's own existence,[13] is, as Kant himself argues in the Paralogisms (*CPR*, A341–405, B399–432), to confuse logical implication (i.e., analyticity) with real implication (i.e., syntheticity). Of course, interpreting the *cogito* as a practical proposition divests it of much of its philosophical interest, inasmuch as it no longer confers *absolute* (a priori) certainty on its conclusion.

Given the legitimacy of "analytic a posteriori" as an epistemological category, how is it related to Kant's special "synthetic a priori" category? We saw in Section I that Kant defines a priori knowledge in terms of "strict" necessity and universality. We then saw in Section II that Kant's strict necessity is quite different from Kripke's empirical necessity, inasmuch as the main criterion for the latter is "true in all possible worlds," whereas the main criterion for the former would be something like, "that without which experience *in our world* would be impossible." One way of expressing this difference is to say that for Kant 'experience' is a *rigid* designator, since "in our world" refers not to any of its empirical details, but to the general fact that concepts and intuitions together are needed to produce empirical knowledge, and so to define an experience. For Kripke, on the other hand, 'experience' is nonrigid, since it refers to any *conceivable* kind of experience. As a result, Kripke applies the word 'necessary' in many instances which do not apply in Kant's strict sense, because Kant reserves this label for the *general condi-*

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[13] In *AN*, 92–93 Kitcher argues for the apriority of "I exist": "so long as I think about the issue and my belief is a product of my reflection upon it, . . . I know a priori that I exist." Following Kripke, he regards this as an example of the contingent a priori. But the exercise of "thinking about the issue" and forming a "belief" makes it a posteriori, while the fact that the belief is already implied as "a product of my reflection" makes it analytic. Elsewhere Kitcher mentions "I exist" along with "I have some beliefs" and "There are thoughts" as examples of contingent a priori propositions (*APK*, 18; *NMK*, 30). Yet each of these would be more accurately described, at least in terms of my expanded version of Kant's framework, as analytic a posteriori propositions: their self-evident truth depends on an analysis of what is implied by certain given experiences.

tions which must be imposed upon the world (synthetic a priori) by a human subject in the process of constructing experience. The synthetic a priori and analytic a posteriori are therefore similar classes of knowledge insofar as both are concerned with conditions imposed on the world by the subject (in contrast to the analytic a priori and the synthetic a posteriori classes, which are concerned with information which can be drawn out of, or deduced from, what we find in experience), but they differ by virtue of the fact that one imposes *general* conditions (a priori) with intuitive (synthetic) content, whereas the other imposes *particular* conditions (a posteriori) with conceptual (analytic) content.[14]

Once we take into consideration the perspectival character of epistemological frameworks, we can see that any given proposition can be analysed in terms of several different classes, depending on the systematic context in which it is used. The difficulty (or total inability) many modern philosophers have in assigning a status to a specific proposition, and the resulting tendency to regard such epistemological distinctions as outlining blurred degrees of knowledge, rather than clear-cut classifications, is due largely to the neglect of the crucial requirement of specifying the context, or perspective, assumed. In analysing the status of theoretical identities, such as "Hesperus is Phosphorus," we have already seen that a single proposition can be classified in different ways. An extended example should clarify the perspectival relationship between the four classes in Kant's epistemological framework, while at the same time providing a summary of the reconstruction of that framework made in this section with Kripke's help.

Imagine some primitive ancestor of ours putting a finger into a fire and, noticing the extreme difference between the resulting sensation and that gained by immersing that finger in a nearby stream, exclaiming, "That in the fire which caused me this pain I

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[14] An interesting practical example of the analytic a posteriori and synthetic a priori can be found in the religious rites of initiation and sacrificial offering, respectively. Interpreted as an example of the former, the Christian ritual of infant baptism represents the official *naming* of a baby: a conceptual (analytic) content, the name, is imposed upon the baby through a specific transforming experience (a posteriori). The celebration of the Eucharist, on the other hand, represents God's (a priori) imposition of a universal (synthetic) "yes" upon mankind—an acceptance which is repeatedly celebrated because it does not depend on any *particular* experience of the accepted individual.

shall call 'heat' " (or some primitive equivalent). Or recall the parallel experience which is repeatedly witnessed today, when a parent fixes the reference of the word 'hot' by using the stove as an object lesson for a young inquisitive child: the child might learn 'stove hot' without realizing at first that this name can be applied in other situations as well. These experiences are examples of what I have called analytic a posteriori knowledge. Once the naming of 'heat' is understood sufficiently well so that we no longer need to refer to the fire or the stove, but know instead that 'heat' designates rigidly a property of *anything* which is hot, our knowledge becomes synthetic a posteriori. If our knowledge ever becomes so refined that we feel confident in proposing a *definition* of 'heat', this would be expressed in an analytic a priori proposition. And if we were somehow able to prove that heat is a necessary condition for the combination of concepts and intuitions to produce the sort of empirical knowledge which characterizes our experience, then the resulting knowledge would be synthetic a priori.

Suppose now that, because you were so brilliant as a young child, the authorities snatched you from your parents' influence just as you were about to touch the stove for the first time, and forced you to live a very sheltered life, concentrating all your energies on learning the intricacies of theoretical physics. In particular, suppose you had never learned the word 'heat' and had never been in contact with extreme temperatures, having lived the rest of your life up until now in the protective environment of a laboratory. One day, as you are flipping through an old textbook to find a formula, your eye happens to spot the sentence, "Heat is molecular motion." You know all about the motion of molecules, having studied their properties, but never before had you thought of calling this phenomenon by any one name. For you, in this situation, this proposition would express an analytic a posteriori truth: all in a flash of insight it would dawn on you that your experience of the motion of molecules (a posteriori) can be subsumed under a single name (analytically). When you share this discovery with a less sheltered colleague, he informs you that "heat" is a phenomenon which everyone experiences, even those who know nothing about theoretical physics. You respond to this startling suggestion by spending the next few days observing what is common to all your experiences of molecular motion, and soon you learn that a certain type of sensation, which you have been experiencing all along in

various (subtle) degrees, is actually caused by the motion of molecules. 'Heat' then becomes for you not just an arbitrary name, but a rigid designator referring to anything which causes this type of sensation which you have just learned to recognize (see *NN*, 136). The proposition "Heat is molecular motion" is now synthetic in virtue of the need to go beyond the concept of "heat" to reach the description "molecular motion," and a posteriori in virtue of the role of particular experiences in your newfound knowledge (cf. *NN*, 153). Your discovery that two apparently distinct phenomena are in fact two perspectives on one and the same phenomenon is here expressed as a theoretical identity between two rigid designators. But you now wish to formalize your knowledge in the fixed form of a definition. To do so, you abstract from the reference of these terms and make a claim about the *meanings* of the words themselves. This time when you state, "Heat *is* molecular motion," you are proposing a definition which can be accepted without further experience (a priori) and on the basis of the concepts alone (analytic).

Similar stories can be told which include a final excursion into the synthetic a priori as well. For example, young philosophy students who read Kant's first *Critique* and realize for the first time that "every alteration must have a cause" (*CPR*, B5) hold this idea in their minds in a way which can be described as analytic a posteriori, especially if they are not too familiar with this use of the term 'alteration'. Running through their various experiences of "alterations," they might then reaffirm that, "every alteration (in my experience, a posteriori) does indeed have a (synthetic) cause." If they set the book down when the going gets tough, they might assent to the proposition, but regard it as necessary in virtue of its logical form: 'every alteration' might seem to contain within it the concept of "cause," in which case their assent to the proposition would be on an analytic a priori basis. But, those who are able to make it through to Kant's detailed transcendental arguments for the necessity and universality of this proposition (*CPR*, B232–56) may find themselves agreeing that it deserves to be assigned the status of an item of synthetic a priori knowledge, a condition for the possibility of experience.

We might be tempted to respond to these stories with the traditional claim that such propositions have the status they have only in virtue of their internal structure, regardless of the perspective we adopt (perhaps mistakenly) towards them. Many propositions

do indeed fit more naturally into one perspective, so that determining the status of propositions might be pictured in terms of a progressive penetration of the boundaries of a set of concentric circles, until we find the next boundary insuperable.[15] At this point we will have reached what might be called the "ultimate" status of the proposition. But it is still crucial to emphasize that we cannot determine the status of *any* proposition (not even "$A = A$") unless we know what perspectival context is assumed. Kant's epistemological framework actually defines not merely four types of knowledge in abstraction, but also four perspectives, or *ways of knowing*, out of which each type of knowledge arises: the transcendental, empirical, logical and practical perspectives, respectively. In determining the status of any proposition, therefore, the philosopher's first task is to determine which of these perspectives is assumed, for only this will provide an adequate preparation for assessing the status of a given proposition.

IV

By now it should come as no surprise to find that nearly all of Kripke's explicit criticisms of Kant are rendered invalid because of

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[15] Pressing this analogy a little further provides a helpful way of distinguishing between Kant's own use of his framework in *The Critique of Pure Reason* and my use of a perspectival reconstruction of it in this paper. Kant's main concern is with examining the nature and extent of our synthetic a priori knowledge, so this category represents for him the largest circle of judgments (the "bounds of sense") and defines his overall method. The analytic a priori and synthetic a posteriori categories represent progressively smaller circles of judgments which are relevant to Kant's task in the first *Critique*. And the smallest circle is occupied by the "ideas" introduced in the "Dialectic," where theoretical reason ultimately dies, in the sense that it gives way to practical reason (i.e., to the analytic a posteriori). By contrast, my goal in this paper has not been the Critical and synthetic a priori one of outlining the transcendental conditions for the possibility of experience in general, but the analytic a posteriori one (via Kripke) of naming the classes in a modern reconstruction of Kant's epistemological framework. Thus the largest circle in my approach has been the analytic a posteriori category which governs the process of naming as such. When following this approach the synthetic a posteriori and analytic a priori categories represent progressively smaller circles of refinement in our understanding and use of names. And the smallest circle is now occupied by the comparitively few synthetic a priori judgments which can be established through the efforts of the Critical Philosopher.

his failure to take into account the differences in their respective epistemological frameworks. By assuming that Kant's arguments can be criticized in terms of a foreign framework, Kripke consistently misrepresents Kant's position. I will therefore begin this final section by examining what Kripke's criticisms actually come to, when Kant's own framework is given a fair hearing. I will then return briefly to the subject of *mathematical* knowledge, discussed in the first part of this series (see note 1), and attempt to provide a coherent account of its perspectival status.

Kripke's explicit criticism of Kant focuses on Kant's claim that the proposition "Gold is a yellow metal" is analytic,[16] which Kripke regards as an "extraordinary" example, "because it's something I think that can turn out to be false" (*NN*, 39). He suggests that if it were discovered some day that, for instance, an optical illusion has been preventing us from realizing that the substance we have been calling "gold" is actually *blue*, we would not be justified in saying, "Gold does not exist" (*NN*, 118), as would seem to be the logical conclusion if the original sentence is analytic. Moreover, although 'yellow metal' might be the description we originally use to fix the reference of 'gold', the discovery of other yellow metals, such as fool's gold, requires us to search for other properties in order to describe it uniquely (*NN*, 119). Kripke emphasizes that it is an "empirical matter whether the characteristics originally associated with the [natural] kind [i.e., with gold] apply to its members universally . . . , and whether they are in fact jointly sufficient for membership in the kind. (The joint sufficiency is extremely unlikely to be *necessary*, but it may be *true*. . . .)" (*NN*, 137). Thus he concludes that Kant's proposition "is not even *a priori*, and whatever necessity it has is established by scientific investigation; it is thus far from analytic in any sense" (*NN*, 123n).

For Kant, however, such facts are irrelevant to the task of determining the epistemological status of the proposition. He would have admitted that gold might turn out not to be yellow; but because it is generally accepted as a given fact that gold *is* (i.e., appears to everyone to be) yellow, he would say that "All gold is yellow, though we might some day discover that it is blue" is just as analytic as "All gold is yellow." For the yellowness of gold is un-

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[16] *Prolegomena to Any Future Metaphysics* (New York: Bobbs-Merrill, 1950), 267 (Berlin Academy numbering).

derstood to be *given* as a part of its definition—an assumption which can be justified even today by noting the presence of the terms 'yellow' and 'metal' in the first definition of 'gold' given in any good dictionary. (Kripke does speculate that Kant might have been led to regard this proposition as analytic by believing "that 'gold' could be *defined* as 'yellow metal' " [*NN*, 134]. But his neglect of the perspectival character of knowledge causes him to downplay the significance of this possibility.) Both Kant and Kripke would agree that the yellowness of gold is an empirical and contingent fact, a fact which could in principle be refuted by some unexpected empirical discovery. The difference between them comes in how they interpret this fact. According to Kripke's framework, any proposition associating gold with yellowness is contingent and therefore cannot be analytic. Kant's framework, on the other hand, allows such an association to be empirically contingent and yet to merit an analytic a priori status, in virtue of its (assumed) role in defining 'gold'.

Kripke's criticism turns on the claim that 'gold', like any name, "is a rigid designator, whose reference is fixed by its 'definition' " (*NN*, 136). As a result, the only way for a proposition to be analytic, according to his framework, is for the subject to be coextensive with (i.e., uniquely defined by) the predicate. Thus, "Yellow is a color" could not be analytic, because blue is also a color, so "a color" would not ordinarily suffice to fix the reference of 'yellow'. For Kant, by contrast, a proposition is analytic if the *meaning* of the subject is entirely contained within the *meaning* of the predicate. This is why Kant says, "Gold is *a* yellow metal" and not, "Gold *is* yellow metal." The analyticity of Kant's sentence does not preclude, but *assumes* the possibility of other yellow metals, for it assumes that the meaning of the word 'gold' includes "member of the class of yellow metals." By saying that 'Gold is a yellow metal' is analytic, Kant is merely saying that it is true (*not* empirically necessary, as Kripke assumes [*NN*, 137]) that the conventional meaning of 'gold' includes "yellow metal." If this is proved wrong by some unexpected empirical discovery concerning the nature of the kind of empirical object to which 'gold' *refers* when it is regarded as a rigid designator, then the definition would simply need to be changed to fit the newfound facts.

Inasmuch as "Gold is a yellow metal" has a similar form to "Heat is molecular motion," it fits into my perspectival reconstruc-

tion of Kant's framework in much the same way. In the rare instances in which this sentence is actually used to fix the reference of 'gold', its status would be analytic a posteriori. The scientist's affirmation that, in spite of the latest attempts to find a species of blue gold, "Gold is [i.e., still appears to be] a yellow metal" is a synthetic a posteriori judgment. Kripke's arguments for the contingent status of this sentence demonstrate only that it *may* be viewed from this synthetic a posteriori perspective. If we now treat 'gold' as a rigid designator defining that which is "yellow metal," then (paraphrasing Kripke in *NN*, 142–43) the inaccurate statement that gold might turn out to be blue would have to be replaced by the statement that it is logically possible that there should have been something blue with all the other properties ordinarily ascribed to gold. Finally, if the sentence is taken as an explanation of part of the conventionally accepted meaning of 'gold', then, as Kant rightly judged, its ultimate status is analytic a priori.

The special criticism of Kant's view of mathematical knowledge by Kripke (*NN*, 158–60) also suffers from his lack of consideration for the perspectival character of all knowledge. First, Kripke wrongly states that Kant's use of 'necessary' in *Critique of Pure Reason*, B3–4 implies "that if a proposition is known to be *necessary*, the mode of knowledge not only *can* be *a priori* but *must* be" (*NN*, 159), without explaining that this is true *only* for "strict" (non-empirical) necessity. He then cites as a supposed counter-example the fact that "one can learn a mathematical truth *a posteriori* by consulting a computing machine, or even by asking a mathematician." But as we have seen, the synthetic a posteriori situation in which one learns a truth of any sort has nothing (necessarily) to do with its ultimate status. (Indeed, how would it be possible to *learn* a mathematical truth in any way other than an a posteriori way?) Finally, Kripke claims that mathematical statements are necessary a posteriori (cf. Kitcher), inasmuch as "any empirical knowledge of their truth is automatically empirical knowledge that they are necessary." What he fails to understand is that, according to Kant's framework, this would be classified as a *synthetic* a posteriori fact, a fact about empirical necessity.

Is Kant then justified in regarding all mathematical knowledge as ultimately synthetic a priori? I think not. For he too has failed at this point to distinguish clearly between different epistemologi-

cal perspectives. He argues that 7 + 5 = 12 must be synthetic in virtue of the fact that we cannot discover '12' merely by analysing '7 + 5 =', but must make use of some intuition as well (*CPR*, B15–16). The traditional alternative to this view is that mathematical propositions are all eventually reducible to logical ones, and are therefore analytic a priori. Another recently popular alternative, backed by Kitcher and Kripke and discussed in the first part of this series, is that mathematical necessity is empirical, so that mathematical propositions are synthetic a posteriori. My perspectival reconstruction of Kant's epistemological framework provides a way of acknowledging an element of truth in all these approaches. Kant's worry about the number 12 is illegitimate for several reasons. First, in all a priori judgments the meanings of all the terms are *assumed* to be given, so 7 + 5 = 12 would be *analytic* a priori: if one knows already what '7', '+', '5', '=' *and* '12' mean, then one will have no need for intuition of any sort to know that 7 + 5 = 12. Second, if one does not know what '12' means, but knows all about the meanings of '7', '+', '5' and '=', then the act of assigning the name '12' to the solution will be analytic a posteriori. The reference of '12' will in this case be initially fixed by identifying it with 7 + 5. Knowledge concerning the empirical details surrounding this situation, as well as that concerning the mathematician's (or for that matter, the number mystic's) laborious search for other uses of '12', will be classified as synthetic a posteriori. Once 7 + 5 = 12 is believed to be empirically necessary, the mathematician can attempt to prove it, thus establishing it as an analytic a priori truth.[17]

But there can be no justification, so far as I can see, for proceeding to the inner circle and assigning a synthetic a priori status

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[17] Kitcher does show some awareness of the chronological development of knowledge from the practical to the empirical, logical and transcendental perspectives when he says (in *NMK*, 9) that "brief reflection on the natural sciences [the empirical] will remind us that enterprises which begin with practical problems may end in theories which have little practical utility." However, instead of using this idea to support the perspectival nature of knowledge, he assumes that anything with a practical starting point cannot be a priori. Approaches to mathematical knowledge such as the one Kitcher ends up developing are inspired, no doubt, by an admirable desire to give due recognition to the complexities which typically characterize such historical situations. But to argue that because something is historical it is nothing other than historical is the crudest sort of positivism.

to *particular* mathematical propositions. The philosophical examples of the synthetic a priori which Kant gives are all general principles rather than specific facts. Shouldn't the same hold true for mathematical knowledge? Kant contradicts himself in this respect in *The Critique of Pure Reason*, where he heads a section with the title "In All Theoretical Sciences of Reason Synthetic *A Priori* Judgments Are Contained *as Principles*" [emphasis added], yet heads the first subsection "*All mathematical judgments, without exception, are synthetic*" (*CPR*, B14). He never explains why natural science (*CPR*, B17–18) and metaphysics (*CPR*, B18) contain synthetic a priori judgments only as general principles, yet in mathematics all judgments whatsoever deserve this status. What he should have argued is that certain fundamental mathematical laws, as synthetic a priori principles, must be imposed upon experience by the knowing subject in order for mathematics itself to be possible. Within Kant's system the first such law might have been something like "The pure intuition of space and time has a mathematical form." A few other alogical propositions could be added to this in order to provide a basis on which the laws of logic could be used to derive less fundamental mathematical laws, and (potentially) the whole corpus of mathematical truth.

Kant was wrong, therefore, to have based his arguments for the synthetic a priori nature of space and time *in general* on the need for an explanation of the supposed synthetic a priori nature of *particular* mathematical (especially geometrical) judgments. Had he noticed this incongruity between his use of the general and the particular, he could have strengthened his overall argument considerably by arguing instead from the need to *ground* particular mathematical judgments on a synthetic a priori foundation to the parallel need to ground our particular experiences of bits of space and time on the synthetic a priori foundation of the pure intuition of space and time in general. Not only would he have circumvented those who deny the legitimacy of his overall theory in the "Aesthetic" simply on the grounds that his argument from geometry is mistaken, but he may have also been able to propose a theory of mathematical knowledge which was more adequate in and of itself —one which could be used to support his transcendental idealism without denying the validity of the work of philosophers such as Frege and Russell, who demonstrated the analytic a priori status of particular mathematical propositions. This in turn would have

forced him to be more clear on just what he meant by his rather obscure theory of pure intuition (see Section IV of the previous paper), and to keep it more obviously distinct from his discussions of the particular manifestations of empirical intuition which form a part of our everyday experience.

In spite of his over-emphasis on synthetic a priori knowledge, Kant did provide us with an invaluable epistemological framework. In these articles I have both used this framework to criticize the works of Kitcher and Kripke and used the modern developments they represent to expand and reconstruct Kant's framework on an explicitly perspectival basis. On the one hand, I have affirmed that the synthetic a posteriori, and even more so, the analytic a posteriori, are categories of knowledge which are relevant to any discussion of the nature of a priori knowledge, so they deserve far greater attention than they have been given by philosophers in the past. And on the other hand, I have suggested that those who *are* devoting their attention to these neglected areas should not go to the other extreme and claim they are giving the concept of apriority "the burial it deserves" (*AN*, 101), but, recognizing the perspectival character of knowledge, should join forces with the apriorist in a concerted effort to explicate the manifold nature of human knowledge.

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