Infinite Opinion Sets and Relative Accuracy

Ilho Park and Jeamin Jung*

Abstract

We can have credences in an infinite number of propositions—that is, our opinion set can be infinite. Accuracy-first epistemologists have devoted themselves to evaluating credal states with the help of the concept of 'accuracy'. Unfortunately, the infinite opinion sets, under several innocuous assumptions, yield several undesirable results, some of which are even fatal, to accuracy-first epistemologists. Moreover, accuracy-first epistemologists cannot circumvent these difficulties in any standard way. In this regard, we will suggest a non-standard approach, called a relativistic approach, to accuracy-first epistemology and show that such an approach can successfully circumvent undesirable results while having some advantages over the standard approach.

Keywords: Infinite Opinion Sets; Local and Global Accuracy Measure; Accuracy; Relative Accuracy; Accuracy-first Epistemology; Probabilism

1   Introduction

Sometimes one can have opinions on a finite number of propositions. For instance, suppose that an agent, called John, is told that his friend, called Paul, has a favorite French movie that was made in 2021. Then it is possible for him to have credences in each of the propositions Paul's favorite French movie of 2021 is x, for all French movies x which was made in 2021. In this case, John's opinion set can be finite. Let us call a set of propositions in which an agent has a credence *her opinion set*. Can an agent's opinion set be infinite? Considering our cognitive limitations, one may think that opinion sets should be only finite. Sometimes, however, we can have credences in an infinite number of propositions. To illustrate, suppose that John is told that Paul has a favorite natural number. Then it is possible for him to have credences in each of the propositions, Paul's favorite number is

n, for all finite natural numbers n. In this case, John's opinion set can be infinite.[1]

Then how do we epistemically evaluate an agent's credal state whose opinion set can be either finite or infinite? Recently, many philosophers have devoted themselves to evaluating credal states with the help of the concept of accuracy. Such philosophers are often called *accuracy-first epistemologists* (*accuracy-firsters* for short). They think that accuracy plays a central role in epistemically evaluating the credal states. According to them, the higher your credences in truths and the lower your credences in falsehoods, the better off you are all epistemic things considered. It is noteworthy that many accuracy-firsters have restricted their attention to credence functions whose opinion sets are *finite*.[2] Under such a restriction, they have provided several interesting accuracy-based arguments that, many think, vindicate various epistemic principles quite successfully.[3] In particular, accuracy-firsters seem to succeed in vindicating Probabilism, which says that all and only probabilistically coherent credence functions are epistemically rational.

However, as pointed out above, an agent's opinion set can be *infinite*. Then, do such arguments also work for credence functions that are defined on an infinite opinion set? In particular, can we vindicate Probabilism for credence functions on an infinite opinion set? Unfortunately, it seems not. As will be evident, the infinite opinion sets yield several undesirable results, some of which are even fatal, to accuracy-first Probabilists. For example, we will argue in what follows that, contrary to what accuracy-firsters expect, they cannot help failing to expel all and only probabilistically incoherent functions from the class of rational credence functions when the credence functions are defined on an infinite opinion set.[4]

These difficulties, we think, have to do crucially with ways of epistemically comparing one credence function with another. Accuracy-firsters have so far explored various ways of measuring accuracy and comparatively evaluating credence functions. All of them, as far as we know, measure *separately* the accuracies of two different credence functions and then determine the epistemic betterness between them on the basis of the accuracies so measured. We will call this the *standard approach to accuracy-first epistemology*. As will be shown, the standard approach fails to circumvent the aforementioned difficulties unless some innocuous assumptions are rejected.

We will proceed as follows. In Section 2, we introduce a few constraints on accuracy measures and two principles to rank various credence functions. In Section 3, we show

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[1]This example is from Pettigrew (2016).

[2]For example, see Carr (2015), Greaves and Wallace (2006), Joyce (1998), Joyce (2009), Leitgeb and Pettigrew (2010*a,b*), Pettigrew (2016), Pettigrew (2018), Predd et al. (2009), and Talbot (2019). Admittedly, there are some accuracy-firsters' works dealing with credence functions whose opinion sets are infinite. See Easwaran (2013), Huttegger (2013), and Kelley (forthcoming), for example. Among these works, we will mainly consider Kelley (forthcoming) in more detail, below.

[3]For example, on the assumption of a finite opinion set, Pettigrew (2016) attempts to justify Probabilism, Plan Conditionalization, the Principal Principle, and the Principle of Indifference.

[4]It is noteworthy that Kelley (forthcoming) provides a generalized argument for Probabilism. However, Kelley's argument is based on different assumptions from ours. We will consider this point in Section 5.

that, given an infinite opinion set, the constraints and principles (described in Section 2) entail several undesirable results for accuracy-firsters. In Section 4, we consider some possible standard ways out and conclude that none of them are plausible. In Section 5, we suggest our non-standard approach to accuracy-first epistemology and show how our approach circumvents the undesirable results in a plausible way.

Before we proceed further, some preliminary remarks about credence functions are in order. Throughout, credence functions, which are regarded as representing credal states, will be denoted by $\mathbf{c}$, $\mathbf{c}^*$, $\mathbf{c}^i$, etc. Credence functions are assumed to assign a real number in [0,1] to each proposition in a given opinion set $\mathcal{F}$. In this paper, a proposition is identified with a finite or infinite subset of a set of all possible worlds $\mathcal{W}$. We assume $\mathcal{W}$ to be infinite, and so propositions are also infinitely many. Our discussion mainly focuses on a credence function whose opinion set is *countably* infinite. Such a credence function can be identified with a sequence. Thus, a credence function $\mathbf{c}$ on a countably infinite opinion set $\mathcal{F} = \{A_1, A_2, \cdots\}$ can be represented as an infinite sequence $(c_1, c_2, \cdots)$. When there is no danger of confusion, 'c_i' will be used to refer to a credence that a credence function $\mathbf{c}$ assigns to a proposition A_i—that is, $\mathbf{c}(A_i) = c_i$.

Some credence functions are probabilistically coherent, while some are not.[5] The coherence is usually regarded as a characteristic of credence functions defined on a particular kind of opinion set. That is to say, when we say that a credence function is coherent, its opinion set is often assumed to be a σ-algebra—i.e., the set is closed under a (countable) truth-functional combination. Admittedly, this definition can be extended to credence functions whose opinion set is not a σ-algebra, as follows.

Coherence. A credence function $\mathbf{c}$ on $\mathcal{F}$, which is a subset of the power set of $\mathcal{W}$ is coherent if and only if there is a credence function $\mathbf{c}^+$ on $\mathcal{F}^+$ such that:

1. $\mathcal{F} \subseteq \mathcal{F}^+$, and $\mathcal{F}^+$ is a σ-algebra;

2. $\mathbf{c}^+(A) \in [0, 1]$ for any $A \in \mathcal{F}^+$, and $\mathbf{c}^+(\mathcal{W}) = 1$;

3. $\mathbf{c}^+(A) = \mathbf{c}(A)$ for any $A \in \mathcal{F}$; and

4. (Countable Sum) $\mathbf{c}^+(\bigvee_{A_i \in \mathcal{A}} A_i) = \sum_{A_i \in \mathcal{A}} \mathbf{c}^+(A_i)$ for any countable set $\mathcal{A} \subseteq \mathcal{F}^+$ whose members are (pairwise) disjoint.

This definition can be applied to a credence function whose opinion set is countably infinite, but is not a σ-algebra.

2   Accuracy Measures and Epistemic Betterness

Let us begin with making some notes about accuracy-first epistemology. Following accuracy firsters, we will accept what they call Alethic Vindication (or Alethic Principle).

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[5]In what follows, we will omit the modifier 'probabilistical(ly)' if there is no danger of confusion.

According to the principle, the ideal credence function, to which a credence function at a given world should strive to be as close as possible, is the truth function at that world. In this paper, an ideal credence function on $\mathcal{F}$, i.e., a truth function, at a world w will be denoted by '$\mathbf{v}^w$', which assigns 1 to $A_i \in \mathcal{F}$ when A_i is true at w, and assigns 0 to A_i otherwise. They can also be represented by a countable sequence $(v_1^w, v_2^w, \cdots)$ such that $\mathbf{v}^w(A_i) = v_i^w$ for any $A_i \in \mathcal{F}$. It is noteworthy that the truth-functions are all coherent.

On the other hand, accuracy-firsters often introduce two kinds of accuracy measures: *local* and *global* measures. A local accuracy measure, which will be denoted by '$\mathfrak{s}$', gives us the accuracy of a single credence at a given world. In particular, '$\mathfrak{s}(1, c_i)$' and '$\mathfrak{s}(0, c_i)$', respectively, refer to the accuracy of a credence c_i at the world where A_i is true, and at the world where A_i is false. A global accuracy measure is devised in order to formulate the accuracy of an overall credal state, i.e., a credence function. In what follows, such a global accuracy measure, which will be denoted by '$\mathfrak{G}$' or '$\mathfrak{R}$', is assumed to be generated from a local accuracy measure in some particular ways, which will be explained in detail later.[6]

Accuracy-firsters have suggested several constraints on accuracy measures, and then narrowed the class of legitimate measures down. Moreover, they formulate some principles governing our comparative evaluations among credence functions using the legitimate measures. In what follows, we will put forward such constraints and principles.

2.1 Constraints on Accuracy Measures

Let us start with some innocuous constraints on a legitimate local accuracy measure $\mathfrak{s}$. Here are two constraints:

Continuity. If $\mathfrak{s}$ is a legitimate local accuracy measure of a credence x, then $\mathfrak{s}(1, x)$ and $\mathfrak{s}(0, x)$ are all continuous of $x \in [0, 1]$.

Monotonicity. If $\mathfrak{s}$ is a legitimate local accuracy measure of a credence x, then $\mathfrak{s}(1, x)$ is a strictly increasing function of $x \in [0, 1]$, and $\mathfrak{s}(0, x)$ is a strictly decreasing function of $x \in [0, 1]$.

Keeping it in mind that accuracy-firsters want to measure the accuracy on the basis of closeness to the truth, we may easily find these two constraints are uncontroversial.

The above constraints say nothing about the maximum and minimum values of the local accuracy. We can strengthen Monotonicity as follows:

Monotonicity$^+$. If $\mathfrak{s}$ is a legitimate local accuracy measure of a credence of x, then it holds, in addition to Monotonicity, that

$$\mathfrak{s}(0, 1) < 0 < \mathfrak{s}(0, 0) \text{ and } \mathfrak{s}(0, 1) < 0 < \mathfrak{s}(1, 1).$$

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[6]In this paper, we will introduce two kinds of global accuracy measure: the non-relativistic global measure and the relativistic global measure. The former will be denoted by '$\mathfrak{G}$', and the latter by '$\mathfrak{R}$'. The difference between them will be explained in Section 5.

This requires, unlike Monotonicity, that the maximum value (and the minimum value) should be positive (and negative). Someone may think that these requirements are a matter of stipulation. In the next sections, however, we will see that Monotonicity$^+$ leads to a serious problem for accuracy-firsters, considering credence functions on an infinite opinion set.

Monotonicity$^+$ just says that the maximum and minimum values should be positive and negative, respectively. They say nothing about the upper and lower bounds of the local accuracy. For the sake of our discussions, we will assume that:

Finiteness. If $\mathfrak{s}$ is a legitimate local accuracy measure of a credence x, then $\mathfrak{s}(1, x) \in (-\infty, \infty)$ and $\mathfrak{s}(0, x) \in (-\infty, \infty)$ for any $x \in [0, 1]$.

This constraint says that any local accuracy of a credence has a finite lower and upper bound. Some local measures in the market satisfy it, some do not. For example, Finiteness holds for the local version of the Brier score, but not for the local version of the Logarithmic measure. Is there any reason to deny this constraint? As of now, we will not address this question. Rather, we just assume it here, and will reconsider it in Section 4.[7]

Heretofore, we have taken a look at some constraints imposed on the local accuracy measures. As stated, accuracy can be assigned to a credence function. In order to measure the global accuracy, some accuracy-firsters formulate a way of generating the global accuracy from the local counterpart. The following is one of such ways.[8]

Simple Additivity. If $\mathfrak{s}$ is a legitimate local accuracy measure, then a legitimate global accuracy measure of a credence function $\mathbf{c}$ on a countable opinion set $\mathcal{F}$ at a world w, i.e., $\mathfrak{G}_{\mathcal{F}}(\mathbf{c}, w)$, is generated from $\mathfrak{s}$, as follows:

$$\mathfrak{G}_{\mathcal{F}}(\mathbf{c}, w) = \sum_i \mathfrak{s}(v_i^w, c_i).$$

Here, the modifier 'simple' is attached to distinguish it from what we will call Relativistic Additivity in Section 5. Simple Additivity says that the global accuracy of a credence function $\mathbf{c}$ on $\mathcal{F}$ should be given by the sum of the local accuracy of each credence in the opinion set $\mathcal{F}$.[9] Are there any epistemic reasons that lead us to endorse this constraint? There are several arguments for and against it.[10] We leave this issue aside and just assume, for the

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[7] Finiteness, which is formulated by Pettigrew (2016, 37), is about the global accuracy measure, not the local measure, while the above Finiteness is just about the local one. In this paper, we do not assume that the global accuracy should be finite. Note that, given Simple Additivity formulated below, the global version of Finiteness entails the local version, but the converse does not hold. For this reason, it may be said that our Finiteness is weaker than what is formulated by the aforementioned author.

[8] Many accuracy-based arguments hinge on Simple Additivity. For examples, see Kelley (forthcoming), Leitgeb and Pettigrew (2010a,b), Predd et al. (2009) and Pettigrew (2016).

[9] Note that this constraint does not assume that the opinion sets are finite.

[10] Some critical discussions may be found in Carr (2015), Fallis and Lewis (2019), Talbot (2019), etc. Pettigrew (2022) provides new arguments for accuracy-first epistemology without appealing Simple Additivity.

sake of argument, that the constraint is one of the adequate constraints on the relationship between the local and global accuracy measures. We will reconsider this point in section 4.

Let us now turn to the last constraint that is called (Strict) Propriety.

Simple Global Propriety. If $\mathfrak{s}$ is a legitimate local accuracy measure, $\mathfrak{G}$ is the legitimate global accuracy measure generated from $\mathfrak{s}$, and $\mathbf{c}$ is a coherent credence function on a countable opinion partition $\mathcal{F}$, then it holds that, for any credence function $\mathbf{c}^*(\neq \mathbf{c})$ on $\mathcal{F}$,
$$\sum_{A \in \mathcal{F}} \mathbf{c}(A)\mathfrak{G}_{\mathcal{F}}(\mathbf{c}, w_A) > \sum_{A \in \mathcal{F}} \mathbf{c}(A)\mathfrak{G}_{\mathcal{F}}(\mathbf{c}^*, w_A)$$
which is equivalent to
$$\sum_{A \in \mathcal{F}} \mathbf{c}(A) \sum_i \mathfrak{s}(v_i^{w_A}, c_i) > \sum_{A \in \mathcal{F}} \mathbf{c}(A) \sum_i \mathfrak{s}(v_i^{w_A}, c_i^*),$$
when Simple Additivity holds for $\mathfrak{s}$ and $\mathfrak{G}$.

Here, 'w_A' refers to a world where A is true and $\mathbf{c}^*$ might be an incoherent credence function.[11] The modifier 'simple' is attached to distinguish it from what we will call Relativistic Global Propriety in Section 5. Simple Global Propriety says that each credence function should expect itself to have the uniquely maximal accuracy. As far as we know, every accuracy-firster takes it to be an epistemically fundamental constraint on accuracy measures. This constraint is about the global accuracy. Its local counterpart can be easily formulated as follows:

Local Propriety. If $\mathfrak{s}$ is a legitimate local accuracy measure of a credence x, Then, it holds that, for any $y \, (\neq x)$,
$$x\mathfrak{s}(1, x) + (1 - x)\mathfrak{s}(0, x) > x\mathfrak{s}(1, y) + (1 - x)\mathfrak{s}(0, y).$$

Local Propriety entails Simple Global Propriety when the opinion set of $\mathbf{c}$ is finite. Interestingly, such a logical relation does not hold when the credence function has an infinite opinion set. We will revisit this point in Section 3.

It is noteworthy that Pettigrew's new arguments are restricted in credence functions on a finite opinion set. Simple Additivity without such a restriction is given in Kelley (forthcoming). A critical discussion about such a version can be found in Easwaran (2013, 125).

[11]Note that the above formulation is restricted in the credence functions on a countable opinion *partition*. Accuracy-firsters, of course, do not make such a restriction. However, this restriction does not undermine our main points.

2.2 Epistemic Betterness

The accuracy-firsters, with the above nuts and bolts in hand, attempt to rank various credal states, i.e., credence functions. That is, they provide a few epistemic principles that govern the epistemic betterness between two credence functions. The following is one of such principles.

Truth-directedness. Suppose that $\mathcal{F}$ is the opinion set of $\mathbf{c}$ and of $\mathbf{c}^*$, and $\mathfrak{s}$ is a legitimate local measure. Then, $\mathbf{c} \prec_w \mathbf{c}^*$ if

    a. $\mathfrak{s}(v_i^w, c_i) \leq \mathfrak{s}(v_i^w, c_i^*)$ for all $A_i \in \mathcal{F}$, and

    b. $\mathfrak{s}(v_i^w, c_i) < \mathfrak{s}(v_i^w, c_i^*)$ for some $A_i \in \mathcal{F}$.

Here, '$\mathbf{c} \prec_w \mathbf{c}^*$' means that $\mathbf{c}^*$ is epistemically better than $\mathbf{c}$ at a world w.[12] This principle seems to be an almost-constitutive part of accuracy-first epistemology. Nevertheless, it is easy to find a case where Truth-directedness is silent on the epistemic betterness between two credence functions.

Of course, using the global accuracy measures, we can easily formulate another principle that renders the relation of epistemic betterness to be more complete. Here is such a principle.

Simple Global Betterness. Suppose that $\mathcal{F}$ is the opinion set of $\mathbf{c}$ and $\mathbf{c}^*$. Suppose also that $\mathfrak{s}$ is a legitimate local accuracy measure and that $\mathfrak{G}$ is the legitimate global accuracy measure generated from $\mathfrak{s}$. Then, $\mathbf{c} \preceq_w \mathbf{c}^*$ if

$$\mathfrak{G}_{\mathcal{F}}(\mathbf{c}, w) \leq \mathfrak{G}_{\mathcal{F}}(\mathbf{c}^*, w)$$

which is equivalent to

$$\sum_i \mathfrak{s}(v_i^w, c_i) \leq \sum_i \mathfrak{s}(v_i^w, c_i^*),$$

when Simple Additivity holds for $\mathfrak{s}$ and $\mathfrak{G}$.

Here, '$\mathbf{c} \preceq_w \mathbf{c}^*$' means that $\mathbf{c}^*$ is at least as epistemically good as $\mathbf{c}$ at a world w.[13] When the opinion set $\mathcal{F}$ is finite, Simple Global Betterness, with Monotonicity (or Monotonicity$^+$), entails Truth-directedness. However, such an entailment does not hold when the opinion set $\mathcal{F}$ is infinite. In the next section, we will discuss this point in detail.

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[12] In this paper, the credal state is individuated by a credence function and an opinion set. Thus, it is more exact to say that $\mathbf{c}^*$ on $\mathcal{F}$ is epistemically better than $\mathbf{c}$ on $\mathcal{F}$ at a world w. Note also that the epistemic betterness is relative to what a local accuracy measure is given. Thus, it is even more exact to say that $\mathbf{c}^*$ on $\mathcal{F}$ is epistemically better than $\mathbf{c}$ on $\mathcal{F}$ at a world w relative to a local accuracy measure $\mathfrak{s}$, which may be symbolized as '$\mathbf{c}_{\mathcal{F}} \prec_w^{\mathfrak{s}} \mathbf{c}_{\mathcal{F}}^*$'. For the notational simplicity, however, we will omit the subscript '$\mathfrak{s}$' and the subscript '$\mathcal{F}$' if there is no danger of confusion.

[13] According to Simple Global Betterness, when $\mathfrak{G}_{\mathcal{F}}(\mathbf{c}, w) = \mathfrak{G}_{\mathcal{F}}(\mathbf{c}^*, w)$, it holds that $\mathbf{c} \preceq_w \mathbf{c}^*$ and $\mathbf{c}^* \preceq_w \mathbf{c}$—or, simply $\mathbf{c} \sim_w \mathbf{c}^*$. In words, this means that $\mathbf{c}^*$ is as epistemically good as $\mathbf{c}$ at a world w. When $\mathfrak{G}_{\mathcal{F}}(\mathbf{c}, w) < \mathfrak{G}_{\mathcal{F}}(\mathbf{c}^*, w)$, Simple Global Betterness entails that $\mathbf{c} \preceq_w \mathbf{c}^*$ and $\mathbf{c} \nsim_w \mathbf{c}^*$—or, simply $\mathbf{c} \prec_w \mathbf{c}^*$.

3   Countably Infinite Opinion Sets

Given countably infinite opinion sets, the above-considered constraints and principles entail many undesirable results, some of which are even fatal, to accuracy-firsters.

Let us start with considering the following proposition.[14]

Proposition 3.1. Let $\mathcal{F} = \{A_1, A_2, A_3, \cdots\}$ be a countably infinite opinion partition. Suppose that $\mathfrak{s}$ is a legitimate local accuracy measure that satisfies Continuity, Monotonicity$^+$, and Finiteness. Suppose also that $\mathfrak{G}$ is the legitimate global accuracy measure generated from $\mathfrak{s}$ in accordance with Simple Additivity. Then, *all coherent credence functions on $\mathcal{F}$ are infinitely accurate at every w.*

This proposition itself is bad news for accuracy-firsters. According to Proposition 3.1, any coherent credence functions on a countably infinite opinion partition have the same global accuracy, i.e., infinite accuracy, to each other, regardless of how the world turns out. Then, it should be said that the accuracy measures, which satisfy the relevant constraints, are *useless* in comparatively evaluating credence functions on an infinite opinion partition.

Moreover, we can show, with the help of Proposition 3.1, that some logical relations between the constraints on accuracy measures and between the principles about epistemic betterness may fail to hold, depending on the cardinality of the opinion sets. Local Propriety, as noted, entails Simple Global Propriety when the opinion set is finite. However, this logical relation does not hold any longer when the opinion set is countably infinite. Suppose that $\mathbf{c}$ and $\mathbf{c}^*$ are coherent credence functions defined on a countably infinite opinion partition. Then, according to Proposition 3.1, $\sum_w \mathbf{c}(w)\mathfrak{G}_{\mathcal{F}}(\mathbf{c}^*, w) = \infty$, irrespective of whether or not a local measure from which $\mathfrak{G}$ is generated satisfies Local Propriety. Thus, every coherent credence function on such a partition has the same expected accuracy to each other, and so Simple Global Propriety cannot help failing for such functions. To put it another way, we can say that Continuity, Monotonicity$^+$, Simple Additivity, Finiteness, and Simple Global Propriety are collectively inconsistent when the opinion set is a countably infinite partition.

A similar thing goes with the principles about epistemic betterness. Consider two credence functions $\mathbf{c}$ and $\mathbf{c}^*$ that have a finite opinion set in common. As stated, when Truth-directedness evaluates $\mathbf{c}$ to be epistemically better than $\mathbf{c}^*$, Simple Global Betterness also reaches the same verdict. In regard to infinite opinion sets, however, this logical relation does not hold any more. See the following example.

Example 3.2. The assumptions about the local and global accuracy measures are the same as ones in Proposition 3.1. Let A_i be the proposition that Paul's favorite natural number is i. Here it is assumed that his favorite natural number uniquely exists. Let $\mathcal{F}$ be the set of A_is. Note that this set is a countably infinite opinion partition. Let w_i

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[14]The proofs of this proposition and others are given in Appendix.

be a world where $A_i \in \mathcal{F}$ is true. Now, consider the following two coherent credence functions $\mathbf{c}^1$ and $\mathbf{c}^2$, and a truth-function $\mathbf{v}^{w_1}$ such that:

$$\mathbf{c}^1 = (1/2^1, 1/2^2, 1/2^3, 1/2^4, \cdots);$$
$$\mathbf{c}^2 = (1/2^2, 1/2^1, 1/2^3, 1/2^4, \cdots);$$
$$\mathbf{v}^{w_1} = (1, 0, 0, 0, \cdots).$$

These functions are assumed to be defined on $\mathcal{F}$. According to Truth-directedness, then, it holds that $\mathbf{c}^2 \prec_{w_1} \mathbf{c}^1$ since

$$\mathfrak{s}(1, c_1^2) < \mathfrak{s}(1, c_1^1),$$
$$\mathfrak{s}(0, c_2^2) < \mathfrak{s}(0, c_2^1), \text{ and}$$
$$\mathfrak{s}(v_i^{w_1}, c_i^2) = \mathfrak{s}(v_i^{w_1}, c_i^1) \text{ for any } i \geq 3.$$

However, Simple Global Betterness leads us to a different verdict since $\mathfrak{G}_{\mathcal{F}}(\mathbf{c}^1, w) = \mathfrak{G}_{\mathcal{F}}(\mathbf{c}^2, w) = \infty$ according to Proposition 3.1.

This example clearly shows that Truth-directedness and Simple Global Betterness conflict with each other under the assumptions in Proposition 3.1.

The above results, we think, seem sufficient to jeopardize accuracy-first epistemologists. However, an even more fatal result can be drawn from Proposition 3.1. To illustrate, let us first consider the following example:

Example 3.3. The same assumptions as Example 3.2 are made. Let z be a credence such that $0 < z < 1$ and $\mathfrak{s}(0, z) = 0$. Note that Continuity and Monotonicity$^+$ jointly ensure the existence of such a credence. Now, consider an *incoherent* credence functions $\mathbf{c}^z = (z, z, z, \cdots)$ on $\mathcal{F}$. It is obvious that $\mathbf{c}^z$ is incoherent. (Note that $\sum_i^\infty c_i^z = \sum_i z = \infty \neq 1$.) Then, it follows from Simple Additivity that, for any i, $\mathfrak{G}_{\mathcal{F}}(\mathbf{c}^z, w_i) = \mathfrak{s}(1, z) + \mathfrak{s}(0, z) + \mathfrak{s}(0, z) + \cdots = \mathfrak{s}(1, z)$, which is finite according to Finiteness. So, it can be said that the credence function $\mathbf{c}^z$ is finitely accurate at every world.

This example shows that, when credence functions are defined on a countably infinite opinion partition, some incoherent credence functions are *finitely* accurate at every world.

As stated, accuracy-firsters are to vindicate Probabilism, appealing to the following decision-theoretic principle:[15]

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[15]The following formulation of the dominance principle omits a condition related to dominating credence functions. As some authors like Pettigrew (2016) have noted, the dominance principle in question should be regarded as wrong if the dominating functions turn out to be epistemically irrational. So, such authors impose a constraint on the dominating functions. For instance, Pettigrew (2016, 24) says that the dominating credence functions should not be *extremely modest* in the sense that the functions should not expect itself to be less accurate than any other functions. However, this omission does not yield any serious problem to

Dominance. A credence function $\mathbf{c}$ is epistemically irrational if there is another credence function $\mathbf{c}^*$ such that $\mathbf{c}^*$ strongly accuracy-dominates $\mathbf{c}$ relative to a legitimate global accuracy measure—that is, for any w, $\mathbf{c} \prec_w \mathbf{c}^*$ relative to a legitimate global accuracy measure.

Fortunately, we can say with the help of Dominance and Proposition 3.1 that the credence function $\mathbf{c}^z$ in Example 3.3 is epistemically irrational, as accuracy-firsters expect.

However, not all incoherent credence functions on a countably infinite opinion partition are finitely accurate at every world. Consider the following example.

Example 3.4. The same assumptions as the above examples are made. Consider an *incoherent* credence functions $\mathbf{c}^0 = (0, 0, 0, \cdots)$ on $\mathcal{F}$. Then, it follows from Simple Additivity that $\mathfrak{G}_{\mathcal{F}}(\mathbf{c}^0, w_i) = \mathfrak{s}(1, 0) + \mathfrak{s}(0, 0) + \mathfrak{s}(0, 0) + \cdots$, for any w_i. Note that Monotonicity$^+$ and Finiteness entail that $-\infty < \mathfrak{s}(1, 0) < 0$ and $\mathfrak{s}(0, 0) > 0$. Thus, we have that $\mathfrak{G}_{\mathcal{F}}(\mathbf{c}^0, w_i) = \infty$. So, it can be said that the credence function $\mathbf{c}^0$ is infinitely accurate at every world.

Thus, Dominance and Proposition 3.1 do not entail that the incoherent credence function $\mathbf{c}^0$ is epistemically irrational. As noted, one of the main projects of accuracy-firsters is to expel all incoherent credence functions from the class of the epistemically rational credence functions. As shown in the above considerations, however, this project cannot help facing with a fatal problem when the relevant opinion sets are countably infinite.

Some accuracy-firsters may cope with the above conclusion by saying that at least the coherent credence functions on a countably infinite opinion partition remain epistemically rational because they are infinitely accurate, and so that Probabilism may be still vindicated in a weak sense. However, a little consideration reveals that this response is unsatisfactory. In particular, when we turn our attention to a countably infinite opinion set that is not a partition, we can see that some coherent credence functions are strongly dominated by another function. See the following example.

Example 3.5. The same assumptions as the above examples are made, except that A_i is the proposition that Paul's favorite natural number is not less than i. Let $\mathcal{F}^*$ be the set consisting of A_is. Note that $\mathcal{F}^*$ is countably infinite but is not a partition. Then, at least one of A_is is true, and $A_{i+1} \vdash A_i$ for any i. It should be emphasized here that, for any world w, there is a k such that A_i is true when $i \leq k$, and A_i is false otherwise. So, the truth-function at a world w is identified with a sequence that consists of finitely many 1s and infinitely many 0s. For instance, when and only when the value of k at a world w is 2, the truth-function on $\mathcal{F}^*$ at w, i.e., $\mathbf{v}^w$, is identified with $(1, 1, 0, 0, 0, \cdots)$. Consider the credence function $\mathbf{c}^{z+} = (1, z, z, \cdots)$, where z is a credence such that

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the discussions that follow, we think. Note that the dominating credence functions appearing in this section are all infinitely accurate at every world, and so they are not extremely modest.

$\mathfrak{s}(0, z) = 0$. (Revisit Example 3.3 for this kind of credence.) It is not hard to see that $\mathbf{c}^{z+}$ on $\mathcal{F}^*$, unlike $\mathbf{c}^z$ on $\mathcal{F}$, is coherent. Let w_k be a world where A_is are true when $i \leq k$, and A_is are false otherwise. Then, we obtain from Simple Additivity and the definition of the credence z that: for any w_k,

$$
\begin{aligned}
\mathfrak{G}_{\mathcal{F}^*}(\mathbf{c}^{z+}, w_k) &= \sum_i \mathfrak{s}(v_i^{w_k}, z) \\
&= \mathfrak{s}(v_1^{w_k}, 1) + \sum_{i=2}^{k} \mathfrak{s}(v_i^{w_k}, z) + \sum_{i=k+1}^{\infty} \mathfrak{s}(v_i^{w_k}, z) \\
&= \mathfrak{s}(1, 1) + \sum_{i=2}^{k} \mathfrak{s}(1, z) + \sum_{i=k+1}^{\infty} \mathfrak{s}(0, z) \\
&= \mathfrak{s}(1, 1) + (k - 1)\mathfrak{s}(1, z).
\end{aligned}
$$

According to Finiteness, $\mathfrak{s}(1, 1)$ and $\mathfrak{s}(1, z)$ are finite, and thus $\mathfrak{G}_{\mathcal{F}^*}(\mathbf{c}^{z+}, w_k) < \infty$ for any w_k. Now, consider another coherent credence function $\mathbf{c}' = (1, 0, 0, 0, \cdots)$. Note that: for any w_k,

$$
\mathfrak{G}_{\mathcal{F}^*}(\mathbf{c}', w_k) = \mathfrak{s}(1, 1) + \sum_{i=2}^{k} \mathfrak{s}(1, 0) + \sum_{i=k+1}^{\infty} \mathfrak{s}(0, 0).
$$

Finiteness says that $\mathfrak{s}(1, 1)$ and $\mathfrak{s}(1, 0)$ are in $(-\infty, \infty)$, and Monotonicity$^+$ entails that $\mathfrak{s}(0, 0) > 0$. Thus, we have that $\mathfrak{G}_{\mathcal{F}^*}(\mathbf{c}', w_k) = \infty$ for any w_k. As a result, it holds that $\mathfrak{G}_{\mathcal{F}^*}(\mathbf{c}^z, w_k) < \mathfrak{G}_{\mathcal{F}^*}(\mathbf{c}', w_k)$ for any w_k.

This example clearly shows, under the relevant assumptions, that there are some coherent credence functions on an infinite opinion set that are strongly accuracy-dominated relative to a legitimate global accuracy measure by another function on that opinion set.[16] Hence, we should conclude that Probabilism cannot be vindicated even in the weak sense that is stated above, when credence functions on infinite opinion sets are considered.

4 Some Possible Ways Out

In order to avoid these difficulties, accuracy-firsters should deny that epistemically rational agents can have an infinite opinion set, and/or deny at least one of the aforementioned

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[16]Interestingly, there is also an *incoherent* credence function that strongly accuracy-dominates the credence function $\mathbf{c}^{z+}$. Consider a credence function $\mathbf{c}^{in} = (0, 1, 0, 0, \cdots)$. This is obviously incoherent since $\mathbf{c}^{in}(A_2) > \mathbf{c}^{in}(A_1)$ while $A_2 \vdash A_1$. On the other hand, it follows from Finiteness and Monotonicity$^+$ that, for any w_k, $\mathfrak{G}_{\mathcal{F}^*}(c^{in}, w_k) = \mathfrak{s}(1, 1) - \mathfrak{s}(1, 0) + \sum_{i=1}^{k} \mathfrak{s}(1, 0) + \sum_{i=k+1}^{\infty} \mathfrak{s}(0, 0) = \infty$. As a result, we have that $\mathfrak{G}_{\mathcal{F}^*}(\mathbf{c}^{z+}, w_k) < \mathfrak{G}_{\mathcal{F}^*}(\mathbf{c}^{in}, w_k)$ for any w_k. However, someone may think that it does not follow from this result that $\mathbf{c}^{z+}$ is epistemically irrational. This is because $\mathbf{c}^{in}$ should be ensured to be epistemically rational in order for us to derive such a conclusion, but it is not clear that the incoherent function $\mathbf{c}^{in}$ can be regarded as rational. For a comment related to this, see footnote 15.

constraints. In this section, we will consider four ways out of the undesirable results and show that none of them are promising.[17]

4.1 Way out 1: Rejecting Infinite Opinion Sets

The first way out for accuracy-first epistemology is to deny that a rational agent's opinion set can be countably infinite. But why? If you exclude an infinite opinion set without any explanation of why an opinion set should be limited like this, you would do something like what Lakatos (1976) dubs 'monster-barring'.

4.2 Way out 2: Rejecting Finiteness

The second way out is to deny Finiteness. In this view, the minimum or maximum value of local accuracy should be negatively or positively infinite—that is, $\mathfrak{s}(0,1) = \mathfrak{s}(1,0) = -\infty$, or $\mathfrak{s}(0,0) = \mathfrak{s}(1,1) = \infty$. In this regard, it is noteworthy that, although the other constraints are all accepted, Proposition 3.1 cannot hold if there is no lower bound of the local accuracy. To see this, consider a coherent credence function $\mathbf{c} = (1,0,0,0,\cdots)$ that is defined on a countably infinite opinion partition $\mathcal{F}$. Suppose that $\mathfrak{s}$ and $\mathfrak{G}$ endorse the assumptions in Proposition 3.1 except Finiteness. In particular, let's assume that $\mathfrak{s}(0,1) = -\infty$. Under such assumptions, we cannot say that $\mathfrak{G}_{\mathcal{F}}(\mathbf{c}, w) = \infty$ for any w. This is because there are worlds where the global accuracy of $\mathbf{c}$ is *undefined*.[18]

However, this way out is of no help to accuracy-firsters for at least two reasons. First, there are some difficulties to which we cannot properly respond even if rejecting Finiteness. Reconsider Example 3.2. When the other constraints are given, we can derive, without appealing to Finiteness, that $\mathfrak{G}_{\mathcal{F}}(\mathbf{c}^1, w_1) = \mathfrak{G}_{\mathcal{F}}(\mathbf{c}^2, w_1) = \infty$.[19] On the other hand, Truth-directedness says, regardless of Finiteness, that $\mathbf{c}^1$ is epistemically better than $\mathbf{c}^2$. Thus, even though we reject Finiteness, we can still say that Simple Global Betterness and Truth-directedness conflict with each other under the relevant assumptions.

Moreover, this way out has epistemically unacceptable implications even about finite opinion sets when the other constraints remain intact. To illustrate, suppose that John and Paul share a finite opinion set $\mathcal{F} = \{A_1, \cdots, A_n\}$ of $n(\geq 2)$ propositions, which are all true at w. John has a credence of 0 in A_1 and a credence of 0.99999 in all the other propositions. Paul has a credence of 0.00001 in all propositions. Suppose that Finiteness does not hold—in particular, $\mathfrak{s}(1,0) = -\infty$. (Note that this assumption and Monotonicity (or

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[17]In what follows, we will not consider giving up on Continuity. The constraint is innocuously structural.

[18]For example, when $\mathbf{v}^{w_2} = (0,1,0,0,\cdots)$, $\mathfrak{G}_{\mathcal{F}}(\mathbf{c}, w_2) = \mathfrak{s}(0,1) + \mathfrak{s}(1,0) + \sum_i^\infty \mathfrak{s}(0,0) = -\infty - \infty + \infty$, which is mathematically undefined.

[19]Suppose that Continuity, Monotonicity$^+$, and Simple Additivity. Then, we have that $\mathfrak{G}_{\mathcal{F}}(\mathbf{c}^1, w_1) = \sum_j \mathfrak{s}(v^{w_1}, c^1) = \mathfrak{s}(1, 1/2^1) - \mathfrak{s}(0, 1/2^1) + \sum_{i=1}^\infty \mathfrak{s}(0, 1/2^i)$. On the other hand, Continuity and Monotonicity$^+$ entail that $\lim_{i \to \infty} \mathfrak{s}(0, 1/2^i) = \mathfrak{s}(0,0) > 0$. Note that if $\lim_{i \to \infty} a_i > 0$, then $\sum_i^\infty a_i = \infty$. Thus, we have that $\mathfrak{G}_{\mathcal{F}}(\mathbf{c}^1, w_1) = \infty$. Similarly, we can also derive that $\mathfrak{G}_{\mathcal{F}}(\mathbf{c}^2, w_1) = \infty$.

Monotonicity$^+$) entail that $\mathfrak{s}(1, x) > -\infty$ for any $x \in (0, 1]$.) Then, it follows from Simple Additivity and Simple Global Betterness that Paul's credal state is epistemically better than John's, because the global accuracy of John's credal state is negatively infinite but the global accuracy of Paul's credal state is not. However, this result seems to run counter to our epistemic intuition. It seems quite counter-intuitive that the loss of epistemic goodness due to just one minimally accurate credence cannot be recovered no matter how many very accurate credences are obtained.

4.3   Way out 3: Rejecting Monotonicity$^+$

The third way out is to reject Monotonicity$^+$. Monotonicity$^+$, unlike Finiteness, plays an indispensable role in deriving each result in Section 3. So, rejecting Monotonicity$^+$ may be said to be a better maneuver than rejecting Finiteness.[20] However, this way out is also unsatisfactory for a few reasons.

First, we would like to emphasize that many authors do not cast any doubt on Monotonicity$^+$.[21] In particular, Leitgeb and Pettigrew (2010a) and Pettigrew (2016) shows that the local version of the Brier score and its positive linear transformations are all legitimate. (Note that the positive linear transformations of the Brier score can endorse Monotonicity$^+$.) In the context related to finite opinion sets, Monotonicity$^+$ does not lead accuracy-firsters to be in a predicament. How about in the context related to infinite opinion sets? Is there any epistemic rationale to think that the minimum or maximum of local accuracy should be different depending on the cardinality of the relevant opinion set? It is quite strange that there is such a thing.

Moreover, as rejecting Finiteness does, this way out also has epistemically unacceptable implications. To illustrate, suppose that John's opinion set consists of only 100 true propositions while Paul's set consists of 1,000 true propositions including the 100 propositions in John's set. Moreover, suppose that John and Paul have the same credences in the 100 propositions, and that Paul has very high credences (say 0.999) in the remaining 900 propositions. Now, let's assume that Monotonicity$^+$ does not hold—in particular, assume that the maximum value of local accuracy, i.e., $\mathfrak{s}(1, 1)$, is 0. Then, it follows from Simple Additivity and Simple Global Betterness that John's credal state cannot help being epistemically better than Paul's, because $\mathfrak{s}(1, x) < 0$ for any $x(< 1)$, and so any credence other than 1 in truths makes one's overall credal state epistemically worse. However, it seems intuitive that it is at least sometimes epistemically rational to have very accurate

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[20]However, we can give another example showing that, even if Monotonicity$^+$ is rejected, Truth-directedness and Simple Global Betterness still conflict with each other under the relevant assumption. For such an example, see Example 5.4.

[21]For example, see Carr (2015), Easwaran (2013), Leitgeb and Pettigrew (2010a), Pettigrew (2016, 2018), and Talbot (2019). In particular, Easwaran (2013), who deals with credence functions on an infinite opinion set in a different context from us, does not require that the minimum or maximum value of local accuracy is 0.

credences in some contingent propositions. Thus, we should say that this implication is hard to accept. A similar reasoning, *mutatis mutandis*, goes with the assumption that the minimum value of local accuracy is 0.

4.4 Way out 4: Rejecting Simple Additivity

The last way out that deserves our attention is to reject Simple Additivity (and so Simple Global Betterness). Is there any alternative way of generating global accuracy from its local counterpart?[22]

Of course, yes. What is often called Averaging is regarded as an arch rival of Simple Additivity. Suppose that a credence function $\mathbf{c}$ is defined on a finite opinion set $\mathcal{F}$. According to Averaging, it should hold that $\mathfrak{G}_{\mathcal{F}}(\mathbf{c}, w) = (1/|\mathcal{F}|) \sum_i \mathfrak{s}(v_i^w, c_i)$. That is, it interprets the global accuracy as an average of local accuracies rather than a sum of them. Note that this version of Averaging, as it stands, cannot apply to credence functions on an infinite opinion set. Instead, it should be reformulated, for the sake of our discussions, as follows:

Infinite Averaging. If $\mathfrak{s}$ is a legitimate local accuracy measure, then a legitimate global accuracy measure of a credence function $\mathbf{c}$ on a countably infinite opinion set $\mathcal{F}$ at a world w, i.e., $\mathfrak{G}_{\mathcal{F}}(\mathbf{c}, w)$, is generated from $\mathfrak{s}$, as follows:

$$\mathfrak{G}_{\mathcal{F}}(\mathbf{c}, w) = \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \mathfrak{s}(v_i^w, c_i).$$

Then, can this way of generating the global accuracy from its local counterpart rescue accuracy-firsters?

Unfortunately, it cannot. Reconsider Example 3.2. Even though Infinite Averaging is given, it still holds that $\mathfrak{G}_{\mathcal{F}}(\mathbf{c}^1, w_1) = \mathfrak{G}_{\mathcal{F}}(\mathbf{c}^2, w_1)$, and thus the difficulty in Example 3.2 remains intact.[23] So, replacing Simple Additivity with Averaging cannot be taken as a good maneuver to avoid the aforementioned difficulties.

We think, though, that rejecting Simple Additivity is broadly on the right track. Then, how do we reject it? To seek a satisfactory way of rejecting Simple Additivity, we should pay attention to the standard approach to accuracy-first epistemology. Accuracy-firsters' comparative evaluation between two credence functions typically takes two steps. First, they measure *separately* the accuracies of two different credence functions using Simple

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[22]There are some ways of measuring the accuracy of a credence function without appealing to Simple Additivity. What is often called the *spherical rule* is a good example. Suppose that $\mathcal{F}$ is an opinion partition, and that $\mathbf{c}$ is defined on $\mathcal{F}$. Then, the rule says that $\mathfrak{G}_{\mathcal{F}}(\mathbf{c}, w) = c_w / \sqrt{\sum_i c_i^2}$, where c_w is a credence in the proposition that is true at w. However, we do not seriously consider this rule in this paper. This is because the spherical rule applies to only credence functions on a partition, and so it is of little use to the accuracy-firster's project to vindicate Probabilism for credence functions defined on various opinion sets. See Fallis and Lewis (2019)

[23]Recall that $\mathbf{c}^1 = (1/2^1, 1/2^2, 1/2^3, 1/2^4, \cdots)$, $\mathbf{c}^2 = (1/2^2, 1/2^1, 1/2^3, 1/2^4, \cdots)$ and $\mathbf{v}^{w_1} = (1, 0, 0, 0, \cdots)$. Suppose that the global accuracy of these credence functions are generated from its local

Additivity (or Averaging). Second, they determine the epistemic betterness between them on the basis of the accuracies so measured and Simple Global Betterness. We will call this a *standard approach to accuracy-first epistemology*. As many accuracy-firsters have shown, this standard approach works well in the context related to finite opinion sets. However, this approach faces serious difficulties when considering epistemic betterness between two credence functions on infinite opinion sets. According to Proposition 3.1, the accuracies of the relevant credence functions, which are *separately measured*, are equal to each other, and so we cannot help reaching some undesirable verdicts like the one in Example 3.2. Therefore, if we are to comparatively evaluate two credence functions on an infinite opinion set by means of the accuracy, then we should refuse to take the first step in question. How can we do this?

5   The Relativistic Approach

5.1   A Clue from a Practical Decision Theory

We get a strong clue from Colyvan (2008), which suggested an ingenious response to a practical decision-theoretical difficulty. The decision problem devised and considered by Colyvan is related to coin tossing in the so-called St. Petersburg Game and its nearby neighbor. A fair coin is tossed repeatedly until the first head appears. In this regard, the following two games are offered to players.

- St. Petersburg Game: Players get $\$2^n$ when the first head appears on the nth toss.

- Petrograd Game: Players get $\$(2^n + 1)$ when the first head appears on the nth toss.

It is very intuitive that the Petrograd game is practically better than the St. Petersburg game. Indeed, a practical version of the Dominance principle fits well with this intuition. On the other hand, the Maximization of Expected Utility, which is one of the core decision principles, runs counter to our intuition. Since the expected utilities of the two are all *infinite*, the principle cannot say that the Petrograd Game is practically better than the St. Petersburg Game. To wrap up, the difficulty of this decision problem is due to two

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counterpart in accordance with Infinite Averaging. Then we have that:

$$\mathfrak{G}_{\mathcal{F}}(\mathbf{c}^1, w_1) = \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \mathfrak{s}(v_i^{w_1}, c_i^1) = \lim_{n \to \infty} \frac{1}{n} \left[\mathfrak{s}(1, 1/2^1) - \mathfrak{s}(0, 1/2^1) + \sum_{i=1}^{n} \mathfrak{s}(0, 1/2^i) \right]$$

$$= \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \mathfrak{s}(0, 1/2^i)$$

$$= \lim_{n \to \infty} \frac{1}{n} \left[\mathfrak{s}(1, 1/2^2) - \mathfrak{s}(0, 1/2^2) + \sum_{i=1}^{n} \mathfrak{s}(0, 1/2^i) \right] = \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \mathfrak{s}(v_i^{w_1}, c_i^2) = \mathfrak{G}_{\mathcal{F}}(\mathbf{c}^2, w_1).$$

features. The first one is that the two core decision principles conflict with each other, and the second one is that the conflict at hand is mainly due to the infinite expected value.

After presenting this decision problem and its difficulty, Colyvan suggests an ingenious way out, called Relative Expectation Theory. This theory relies on the concept of *the relative expected utility*. Roughly speaking, the relative expected utility of one option over another is the average of the differences in utilities between the two options. With this kind of expected utility in hand, the theory says that, if the relative expected utility of one option over another is greater than 0, then the former is practically better than the latter. Unlike the Maximization of Expected Utility, this approach does not run counter to our intuition—that is, the Petrograd game can be said to be practically better than the St. Petersburg game, since the relative expected utility of the former over the latter is greater than 0. Moreover, it can be shown that the relative expectation theory is equivalent to the traditional expectation theory, when the (non-relativistic) expected utilities of the relevant options are *finite*. In this sense, Colyvan's theory can be said to be *conservative*.

Let us return to our problems. Interestingly, some difficulties presented in Section 3 have striking structural similarity to the one which Colyvan presented. For instance, Example 3.2 shows, similar to Colyvan's decision problem, that two core principles about epistemic betterness, i.e., Truth-directedness and Simple Global Betterness, conflict with each other, and that the conflict is mainly due to the infinitely accurate credence functions. This structural similarity may lead accuracy-firsters to a similar way out to Colyvan's suggestion.

Our suggestion depends on an entirely new way of evaluating epistemic betterness, which is somewhat similar to Colyvan's relative expectation theory. Recall what we have called the standard approach to accuracy-first epistemology. To comparatively evaluate the epistemic betterness between two credence functions, this standard approach appeals to the global accuracy of each credence function measured *separately* from any other credence functions. However, our new approach to epistemic betterness hinges on what we will call the *relative global accuracy*, which refers to the accuracy of a given credence function that is measured *relative to another credence function*. In this sense, our suggestion can be called the *relativistic approach* to accuracy-first epistemology. [24]

Before proceeding further, we should emphasize that our approach is entirely new to accuracy-first epistemologists, but it is also *conservative* in at least two senses. First, as will be shown below, our approach leads us to the same results as those obtained from the non-relativistic approach when our attentions are restricted to *finite* opinion sets. Second, it will also be shown that our approach can derive almost all results that are drawn from the existing works about *infinite* opinion sets. In this regard, it should be noted that Kelley (forthcoming) has recently developed a generalized accuracy-based approach related to

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[24]In order to distinguish the standard approach from ours, we will, if necessary, add the modifier 'non-relativistic' to the name of the relevant norms and principles.

infinite opinion sets. Our approach differs from hers in that the first, not the second, can embrace Monotonicity$^+$—in particular, the first does not rule out that the maximum value of local accuracy can be greater than 0 while the second heavily depends on the assumption that the maximum value is equal to 0. That said, our approach can derive almost all of what Kelley's approach tells us, as will be explained in Section 5.3.

5.2   Relative Global Accuracy

Let us begin with defining what we call the Relative Global Accuracy. This kind of accuracy is intended to measure the degree to which a credence function $\mathbf{c}$ on $\mathcal{F}$ is globally more accurate than another function $\mathbf{c}^*$ on $\mathcal{F}$ at a world w. Such a relative global accuracy will be denoted by '$\mathfrak{R}_{\mathcal{F}}(\mathbf{c}, \mathbf{c}^*, w)$'. The legitimate relative measure $\mathfrak{R}$ could be generated from a legitimate local measure $\mathfrak{s}$ in a similar way to what (non-relativistic) Simple Additivity requires of us. Here is such a way:

Relative Additivity. If $\mathfrak{s}$ is a legitimate local accuracy measure, then a legitimate relative global accuracy measure of a credence function $\mathbf{c}$ over a credence function $\mathbf{c}^*$ on a countable opinion set $\mathcal{F}$ at a world w, i.e., $\mathfrak{R}_{\mathcal{F}}(\mathbf{c}, \mathbf{c}^*, w)$, is generated from $\mathfrak{s}$, as follows:
$$\mathfrak{R}_{\mathcal{F}}(\mathbf{c}, \mathbf{c}^*, w) = \sum_i \left(\mathfrak{s}(v_i^w, c_i) - \mathfrak{s}(v_i^w, c_i^*) \right).$$

Here, $\mathcal{F}$ may be finite or infinite. This constraint says that the relative global accuracy of $\mathbf{c}$ over $\mathbf{c}^*$ at w should be given by the sum of the differences between the local accuracy of c_i and of c_i^* at w.[25] At first blush, the relative global accuracy so defined appears to be nothing but the difference between two non-relativistic global accuracies, and thus it seems to make little contribution to avoiding the difficulties provided in the previous section. However, this first impression will turn out to be incorrect.

Let us begin with formulating some relativistic versions of the constraints and principles in Section 2, as follows:

Relative Global Propriety. If $\mathfrak{s}$ is a legitimate local accuracy measure, $\mathfrak{R}$ is the legitimate relative global accuracy generated from $\mathfrak{s}$, and $\mathbf{c}$ is a coherent credence function on a countable opinion partition $\mathcal{F}$, then it holds that, for any credence function $\mathbf{c}^*(\neq \mathbf{c})$ on $\mathcal{F}$,
$$\sum_{A \in \mathcal{F}} \mathbf{c}(A) \mathfrak{R}_{\mathcal{F}}(\mathbf{c}, \mathbf{c}^*, w_A) > 0$$
which is equivalent to
$$\sum_{A \in \mathcal{F}} \mathbf{c}(A) \sum_i \left(\mathfrak{s}(v_i^{w_A}, c_i) - \mathfrak{s}(v_i^{w_A}, c_i^*) \right) > 0,$$

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[25]We assume Finiteness. Thus, the relative accuracy in question can be said to be mathematically well-defined.

when Relative Additivity holds for $\mathfrak{s}$ and $\mathfrak{R}$.

Relative Global Betterness. Suppose that $\mathcal{F}$ is the opinion set of $\mathbf{c}$ and of $\mathbf{c}^*$. Suppose also that $\mathfrak{s}$ is a legitimate local measure and that $\mathfrak{R}$ is the legitimate relative global accuracy measure generated from $\mathfrak{s}$. Then, $\mathbf{c} \preceq_w \mathbf{c}^*$ if

$$\mathfrak{R}_{\mathcal{F}}(\mathbf{c}, \mathbf{c}^*, w) \leq 0$$

which is equivalent to

$$\sum_i \left(\mathfrak{s}(v_i^w, c_i) - \mathfrak{s}(v_i^w, c_i^*) \right) \leq 0,$$

when Relative Additivity holds for $\mathfrak{s}$ and $\mathfrak{R}$.

In what follows, we will show that these relativistic versions can successfully avoid the aforementioned difficulties.

Before seeing this, note first that, when the opinion set is finite, the two global measures $\mathfrak{G}$ and $\mathfrak{R}$ are related to each other, as follows:

Proposition 5.1. Suppose that $\mathcal{F}$ is a *finite* opinion set of $\mathbf{c}$ and $\mathbf{c}^*$. Suppose also that $\mathfrak{s}$ is a legitimate local measure, and that $\mathfrak{G}$ and $\mathfrak{R}$, respectively, are generated from $\mathfrak{s}$ in accordance with Simple Additivity and with Relative Additivity. Then, $\mathfrak{G}_{\mathcal{F}}(\mathbf{c}, w) \lesseqgtr \mathfrak{G}_{\mathcal{F}}(\mathbf{c}^*, w)$ *if and only if* $\mathfrak{R}_{\mathcal{F}}(\mathbf{c}, \mathbf{c}^*, w) \lesseqgtr 0$.

The proof of this proposition is very straightforward. It is also easy to see, with Proposition 5.1 in hand, that Simple Global Propriety and Simple Global Betterness are, respectively, equivalent to Relative Global Propriety and Relative Global Betterness when the opinion set is finite.

Moreover, we can prove the following proposition:

Proposition 5.2. Suppose that $\mathfrak{s}$ is a legitimate local accuracy measure that satisfies Continuity, Monotonicity (or Monotonicity[+]), and Finiteness. Suppose also that $\mathfrak{R}$ is the legitimate relative global accuracy measure generated from $\mathfrak{s}$ in accordance with Relative Additivity. Then, *Local Propriety entails Relative Global Propriety*.

As noted in Section 3, Local Propriety entails Simple Global Propriety when the opinion set is finite, but this logical relation does not hold for the infinite opinion sets. However, Proposition 5.2 says that the logical relation between Local Propriety and its global version still holds even for infinite sets, when the corresponding global version is formulated by means of Relative Global Propriety.

A similar point goes with the logical relation between Relative/Simple Global Betterness and Truth-directedness. Revisit $\mathbf{c}^1$, $\mathbf{c}^2$, and $\mathbf{v}^{w_1}$ in Example 3.2. As explained, $\mathbf{c}^2 \prec \mathbf{c}^1$ according to Truth-directedness, but $\mathbf{c}^2 \nprec \mathbf{c}^1$ according to Simple Global Betterness.

That is, in this example, Simple Global Betterness conflicts with Truth-directedness. What about Relative Global Betterness? Note that:

$$\mathfrak{R}_{\mathcal{F}}(\mathbf{c}^1, \mathbf{c}^2, w) = \sum_{i}^{\infty}(\mathfrak{s}(v_i^w, c_i^1) - \mathfrak{s}(v_i^w, c_i^2))$$
$$= (\mathfrak{s}(1/2, 1) - \mathfrak{s}(1/2^2, 1)) + (\mathfrak{s}(1/2^2, 0) - \mathfrak{s}(1/2, 0)),$$

which is positively finite, given Monotonicity (or Monotonicity[+]). Thus, Relative Global Betterness leads us to the same verdict as one led by Truth-directedness. More generally, we can prove the following proposition:

> **Proposition 5.3.** Suppose that $\mathfrak{s}$ is a legitimate local accuracy measure that satisfies Continuity, Monotonicity (or Monotonicity[+]), and Finiteness. Suppose also that $\mathfrak{R}$ is the legitimate relative global accuracy measure generated from $\mathfrak{s}$ in accordance with Relative Additivity. Then, *Relative Global Betterness entails Truth-directedness*.

As noted, Simple Global Betterness entails Truth-directedness when the opinion set is finite, but this logical relation does not hold for the infinite opinion sets. However, such a logical relation holds for any opinion set when Relative Additivity and Relative Global Betterness are accepted.

5.3 Probabilism for Credences on Infinite Opinion Sets

As stated, one of the main projects of accuracy-firsters is to expel all incoherent credence functions from the class of rational credence functions. However, this project cannot help failing under the constraints and principles given in Section 2, since some incoherent credence functions are infinitely accurate at every world, and so cannot be strongly accuracy-dominated by any other coherent functions.

In this regard, we should pay attention to a recent accuracy-based vindication, given by Kelley (forthcoming), of Probabilism for credence functions on infinite opinion sets. She proves, for example, that, when $\mathcal{F}$ is countably infinite and $\mathbf{c}$ is an incoherent credence function on $\mathcal{F}$, $\mathbf{c}$ is strongly accuracy-dominated relative to a legitimate global accuracy measure by a coherent function. Here, the legitimate global accuracy measure is generated from a legitimate local one in accordance with Simple Additivity—in other words, the global accuracy measure is defined as being non-relativistic.

At first blush, her results seem to conflict with our claims in Section 3 that there is an incoherent credence function that is *not* strongly accuracy-dominated relative to a legitimate non-relativistic global measure by any credence function. Moreover, it could be said, if Kelley's arguments are sound, that our relativistic approach is of little theoretical use since the difficulties in Section 3 can be properly avoided without introducing the concept of relative accuracy. Furthermore, someone may criticize our approach in that, contrary

to Kelley's approach, the relativistic approach does not vindicate Probabilism for credence functions on infinite opinion sets. In what follows, we will explain that such worries about our approach are somewhat immature.

Note first that our *local accuracy measure* differs, in an important way, from what Kelley suggests.[26] Following Predd et al. (2009), Pettigrew (2016), and so forth, Kelley formulates her local accuracy measures using the so-called bounded one-dimensional Bregman divergence.[27] We will call such a measure a *zero-based local accuracy measure*.

Zero-based Local Accuracy. $\mathfrak{s}^0$ is a zero-based local accuracy measure if

- $\mathfrak{s}^0(1, x) = -\mathfrak{d}(1, x)$; and
- $\mathfrak{s}^0(0, x) = -\mathfrak{d}(0, x)$,

where $\mathfrak{d}$ is a bounded one-dimensional Bregman divergence.

Here, $\mathfrak{d}$ is generated from a function $\varphi : [0, 1] \to \mathbb{R}$ that is continuous, bounded, strictly convex on [0,1], and continuously differentiable on (0,1). Moreover, when $\mathfrak{d}$ is generated from φ, it holds that: for any $x, y \in [0, 1]$,

$$\mathfrak{d}(x, y) = \varphi(x) - \varphi(y) - \varphi'(y)(x - y).$$

It should be emphasized here that Kelley's definition of local accuracy measures cannot endorse Monotonicity[+]—that is, the above definition entails that $\mathfrak{s}^0(1, 1) = -\mathfrak{d}(1, 1) = 0$ and $\mathfrak{s}^0(0, 0) = -\mathfrak{d}(0, 0) = 0$, regardless of the generating function φ.[28] This is one reason why we dub Kelley's local measure a *zero-based* one.[29]

As mentioned, Monotonicity[+] plays an indispensable role in deriving each result in Section 3. For this reason, Kelley's approach may be said to rescue accuracy-based probablism at the cost of Monotonicity[+]. However, keeping the arguments for Monotonicity[+] in Section 4 in mind, we could not say conclusively that Kelley's approach is theoretically better than our relativistic approach. Moreover, we have some other arguments against Kelley's approach, and for our relativistic approach.

First, we would like to point out that Kelley's approach suffers from a similar problem to the one explained in Example 3.2.

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[26]In this paper, our discussions are given on the basis of accuracy. However, many accuracy-firsters including Kelley provide their works using the concept of *inaccuracy*. This difference, we think, is not substantial to our discussions. Below, we reformulate Kelley's works using the concept of accuracy, not inaccuracy.

[27]More exactly, she defines the global accuracy using the bounded one-dimensional Bregman $\mathfrak{d}_\varphi$ that is generated from a function φ. Kelley assumes that φ is bounded and so is $\mathfrak{d}_\varphi$. Thus, it can be said that her work, like ours, assumes Finiteness. Kelley (forthcoming, footnote 9) said that the assumption of boundedness is made just for a technical reason.

[28]Suppose that $\mathfrak{d}$ is generated from φ. Then, it holds that $\mathfrak{d}(1, 1) = \varphi(1) - \varphi(1) - \varphi'(1)(1 - 1) = 0$. A similar thing goes with $\mathfrak{d}(0, 0) = 0$.

[29]Note that, except Monotonicity[+], the zero-based local accuracy measures satisfy the other aforementioned constraints on local accuracy measures, i.e., Continuity and Finiteness.

Example 5.4. The same assumptions as Example 3.2 are made, except that Monotonicity, not Monotonicity$^+$, holds, and that A_i is the proposition that Paul's favorite natural number is not greater than i. In particular, assume that the global accuracy measure $\mathfrak{G}$ is generated from a zero-based local accuracy measure $\mathfrak{s}^0$ in accordance with Simple Additivity. Let $\mathcal{F}^+$ be the set consisting of A_is. Note that $\mathcal{F}^+$ is countably infinite but is not a partition. Then, at least one of A_is is true, and $A_i \vdash A_{i+1}$ for any i. Now, consider the following two (incoherent) credence functions $\mathbf{c}^{1+}$ and $\mathbf{c}^{2+}$, and a truth-function $\mathbf{v}^{w+}$ that are defined on $\mathcal{F}^+$:

$$\mathbf{c}^{1+} = (1, 0, 0, 0, \cdots);$$
$$\mathbf{c}^{2+} = (1, 1, 0, 0, \cdots); \text{ and}$$
$$\mathbf{v}^{w+} = (1, 1, 1, 1, \cdots).$$

Then, Truth-directedness entails that $\mathbf{c}^{1+} \prec_{w+} \mathbf{c}^{2+}$ since $\mathfrak{s}^0(v_i^{w+}, c_i^{1+}) = \mathfrak{s}^0(1, 0) < \mathfrak{s}^0(1, 1) = \mathfrak{s}^0(v_i^{w+}, c_i^{2+})$ when $i = 2$, and $\mathfrak{s}^0(v_i^{w+}, c_i^1) = \mathfrak{s}^0(v_i^{w+}, c_i^2)$ otherwise. However, Simple Global Betterness entails that $\mathbf{c}^{1+} \not\prec_{w+} \mathbf{c}^{2+}$. This is because $\mathfrak{s}^0(1, 1) = 0$ and so,

$$\mathfrak{G}_{\mathcal{F}^+}(\mathbf{c}^{1+}, w^+) = \mathfrak{s}^0(1, 1) + \sum^{\infty} \mathfrak{s}^0(1, 0)$$
$$= \sum^{\infty} \mathfrak{s}^0(1, 0)$$
$$= 2\mathfrak{s}^0(1, 1) + \sum^{\infty} \mathfrak{s}^0(1, 0) = \mathfrak{G}_{\mathcal{F}^+}(\mathbf{c}^{2+}, w^+).$$

This example clearly shows that Truth-directedness and Simple Global Betterness still conflict with each other, even if we accept the zero-based measure to be a legitimate local accuracy measure and so reject Monotonicity$^+$.

What about our relativistic approach? Interestingly, our relativistic approach is free from the above conflict, irrespective of whether Monotonicity$^+$ is assumed or not. To see this, suppose that $\mathfrak{R}$ is a relativistic global accuracy measure generated from a local accuracy measure $\mathfrak{s}$ in accordance with Relative Additivity. In particular, let's assume that $\mathfrak{s}$ endorses Continuity, Finiteness, and Monotonicity (or Monotonicity$^+$). Then, we have that:

$$\mathfrak{R}_{\mathcal{F}^*}(\mathbf{c}^1, \mathbf{c}^2, w^+) = \sum^{\infty} \left(\mathfrak{s}(v_i^w, c_i^1) - \mathfrak{s}(v_i^w, c_i^2) \right)$$
$$= \mathfrak{s}(1, 0) - \mathfrak{s}(1, 1) < 0.$$

Hence, it follows from Relative Global Betterness that $\mathbf{c}^{1+} \prec_{w+} \mathbf{c}^{2+}$. Unlike Simple Global

Betterness, Relativistic Global Betterness does not conflict with Truth-directedness.[30] This feature of the relativistic approach must be regarded as a theoretical advantage that Kelley's approach lacks.

Some readers may argue, though, that our approach lacks another important theoretical virtues that Kelley's approach has. Note again that Kelley presents a few successful arguments for Probabilism while our relativistic approach seems to have no such positive arguments yet. Thus, if we cannot make any argument for Probabilism, then it might be said to be theoretically better to abandon Monotonicity$^+$ and adopt Kelley's approach.

Fortunately, however, the relativistic approach can derive (almost) all of the results shown in Kelley (forthcoming).[31] To see this, let us define a legitimate local accuracy measure that can embrace Monotonicity$^+$, as follows:

Local Accuracy. $\mathfrak{s}^+$ is a legitimate local accuracy measure if

- $\mathfrak{s}^+(1, x) = \alpha - \lambda\mathfrak{d}(1, x)$; and

- $\mathfrak{s}^+(0, x) = \alpha - \lambda\mathfrak{d}(0, x)$,

  where α is a non-negative real number, λ is a positive real number, and $\mathfrak{d}$ is a bounded one-dimensional Bregman divergence.

It is obvious that $\mathfrak{s}^+$ satisfies Continuity and Finiteness, and does not rule out Monotonicity$^+$. It is worthwhile noting that $\mathfrak{s}^+$ is a positive linear transformation of the corresponding zero-based local measure $\mathfrak{s}^0$—in particular, it holds that $\mathfrak{s}^+ = \alpha + \lambda\mathfrak{s}^0$.

With this kind of legitimate local measure in hand, we can derive a proposition about the relationship between simple and relative global accuracy measures, as follows:

Proposition 5.5. Suppose that $\mathcal{F}$ is an opinion set of $\mathbf{c}$ and $\mathbf{c}^*$. Suppose further that $\mathfrak{s}^0$ is a zero-based local accuracy measure, and that $\mathfrak{s}^+$ is a legitimate local accuracy measure that is a positive linear transformation of $\mathfrak{s}^0$. Suppose still further that $\mathfrak{G}^0$ and $\mathfrak{R}^+$, respectively, are generated from $\mathfrak{s}^0$ in accordance with Simple Additivity, and from $\mathfrak{s}^+$ in accordance with Relative Additivity. Then, *for any credence function* $\mathbf{c}$ *and* $\mathbf{c}^*$, $\mathfrak{G}^0_{\mathcal{F}}(\mathbf{c}, w) \lesseqgtr \mathfrak{G}^0_{\mathcal{F}}(\mathbf{c}^*, w)$ *if and only if* $\mathfrak{R}^+_{\mathcal{F}}(\mathbf{c}, \mathbf{c}^*, w) \lesseqgtr 0$, *unless* $\mathfrak{G}^0_{\mathcal{F}}(\mathbf{c}, w)$ *and* $\mathfrak{G}^0_{\mathcal{F}}(\mathbf{c}^*, w)$ *are all negatively infinite.*

Here, $\mathcal{F}$ may be finite or infinite. This proposition clearly says that, unless the global accuracies of two credence functions are all negatively infinite, the epistemic betterness between credence functions on the basis of $\mathfrak{G}^0_{\mathcal{F}}$ and Simple Global Betterness is *ordinally equivalent* to the epistemic betterness on the basis of $\mathfrak{R}^+_{\mathcal{F}}$ and Relative Global Betterness.

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[30]Indeed, we have already proved this point. See Proposition 5.3.

[31]Why 'almost'? When the non-relativistic global accuracies of two credence functions are all negatively infinite, our relativistic approach may lead us to a verdict different from one led by Kelley's approach. In such a case, the verdict led by the former is more epistemically plausible than the one by the letter. See Example 5.4.

Thus, Kelley's non-relativistic accuracy-based arguments with $\mathfrak{G}^0$ can be paraphrased to our relativistic arguments with $\mathfrak{R}^+$, and so the relativistic approach can derive, without abandoning Monotonicity$^+$, (almost) all of the results from Kelley's approach.

6 Concluding Remarks

We have shown why accuracy-firsters need to epistemically evaluate our credal states in a relativistic way rather than a (standard) non-relativistic way. Our opinion set is either finite or infinite. Given a finite opinion set, our relativistic approach is able to do everything the non-relativistic approach can do. Moreover, given an (countably) infinite opinion set, our relativistic approach is able to do more things than the non-relativistic approach can do. In particular, it ensures the compatibility of two main principles, i.e., Truth-directedness and Global Betterness, and provides, without abandoning Monotonicity$^+$, a plausible way to vindicate Probabilism for credence functions on an infinite opinion set.

Appendix

A Proof of Proposition 3.1

Let $\mathbf{c} = (c_1, c_2, \cdots)$ be a coherent credence function on a countably infinite opinion partition $\mathcal{F}$. Here, c_is are all non-negative. So, we can rearrange, without any loss of generality, the infinite sequence $(c_1, c_2, c_3, \cdots)$ into another infinite sequence $\mathbf{p} = (p_1, p_2, p_3, \cdots)$ such that, for any $i \in \mathbb{N}$, $p_i \geq p_{i+1}$. Let A_i be a proposition in $\mathcal{F}$, to which $\mathbf{p}$ assigns p_i. On the other hand, the world w at which A_k is true can also be represented by an infinite sequence $(v_1^w, v_2^w, v_3^w, \cdots)$ such that $v_i^w = 1$ when $i = k$, and $v_i^w = 0$ otherwise. As defined, the sequence $(p_1, p_2, p_3, \cdots)$ is non-increasing. Then, only two cases are possible.

- Case 1: There is an N such that $p_i = 0$ for any $i > N$.

- Case 2: $p_i > 0$ for any p_i, and the sequence converges to 0.

If neither of these cases holds, then the sequence cannot be coherent. Consider a sequence in which p_is are all positive, but $\lim_{n \to \infty} p_n > 0$. Here we need to note a simple mathematical fact that $\sum_{n=1}^{\infty} a_n = \infty$ if $\lim_{n \to \infty} a_n = \epsilon > 0$. With this mathematical fact in hand, then, we obtain that $\sum_{n=1}^{\infty} p_n = \infty$, and so that such a sequence violates the probability axiom—in particular, Countable Sum.

Now, let us see how accurate the coherent credence functions are in each case. First, suppose that Case 1 holds. Then, it follows from Simple Additivity that, for any w,

$$\mathfrak{G}_{\mathcal{F}}(\mathbf{c}, w) = \mathfrak{G}_{\mathcal{F}}(\mathbf{p}, w) = \sum_{i=1}^{\infty} \mathfrak{s}(v_i^w, p_i) = \sum_{i=1}^{N} \mathfrak{s}(v_i^w, p_i) + \sum_{i=N+1}^{\infty} \mathfrak{s}(v_i^w, 0).$$

Note that, according to Finiteness, the local accuracy of each credence is finite. Meanwhile, only one proposition of A_is is true at each world. Thus, $\sum_{i=N+1}^{\infty} \mathfrak{s}(v_i^w, 0)$ must be infinite for any w since $\mathfrak{s}(0,0) > 0$ according to Monotonicity$^+$. Hence, it follows $\mathfrak{G}_{\mathcal{F}}(\mathbf{c}, w) = \sum_i^{\infty} \mathfrak{s}(v_i^w, c_i) = \infty$ for any w.

Second, let us suppose that Case 2 holds, which says that $\lim_{n \to \infty} p_n = 0$. Then, Continuity and Monotonicity$^+$ entail that $\lim_{n \to \infty} \mathfrak{s}(p_n, 0) = \mathfrak{s}(0,0) > 0$. On the other hand, it follows from Simple Additivity that, for any w,

$$\mathfrak{G}_{\mathcal{F}}(\mathbf{c}, w) = \mathfrak{G}_{\mathcal{F}}(\mathbf{p}, w) = \sum_{i=1}^{\infty} \mathfrak{s}(v_i^w, p_i) = \mathfrak{s}(1, p_w) - \mathfrak{s}(0, p_w) + \sum_{i=1}^{\infty} \mathfrak{s}(0, p_i),$$

where p_w is a credence that $\mathbf{p}$ assigns to a proposition that is true at w. Thus, $\mathfrak{G}_{\mathcal{F}}(\mathbf{c}, w)$ must be infinite for any w, since $\lim_{n \to \infty} \mathfrak{s}(0, p_n) > 0$ and so $\sum_i^{\infty} \mathfrak{s}(0, p_i) = \infty$. $\qquad \square$

A Proof of Proposition 5.1

As stated in the main text, the proof is straightforward. Only the following is required: when $\mathcal{F}$ is finite, $\mathfrak{R}_{\mathcal{F}}(\mathbf{c}, \mathbf{c}^*, w) = \mathfrak{G}_{\mathcal{F}}(\mathbf{c}, w) - \mathfrak{G}_{\mathcal{F}}(\mathbf{c}^*, w)$ for any w. $\qquad \square$

A Proof of Proposition 5.2 and Some Relevant Remarks

Here, we will prove that, when the opinion sets are countable, Relative Global Propriety follows from Local Propriety, but Simple Global Propriety cannot.

Suppose that $\mathbf{c}$ is a coherent credence function on a countable opinion partition $\mathcal{F}$, and $\mathbf{c}^*$ is a different credence function on $\mathcal{F}$ from $\mathbf{c}$. Suppose also that a legitimate relative global accuracy measure $\mathfrak{R}$ is generated from a legitimate local accuracy measure $\mathfrak{s}$, which satisfies Continuity, Monotonicity (or Monotonicity$^+$), and Finiteness, in accordance with

Relative Additivity.

$$\sum_{A\in\mathcal{F}}\mathbf{c}(A)\mathfrak{R}_{\mathcal{F}}(\mathbf{c},\mathbf{c}^*,w_A)$$

$$=\sum_{A\in\mathcal{F}}\mathbf{c}(A)\sum_{A'\in\mathcal{F}}\left(\mathfrak{s}(\mathbf{v}^{w_A}(A'),\mathbf{c}(A'))-\mathfrak{s}(\mathbf{v}^{w_A}(A'),\mathbf{c}^*(A'))\right)$$

$$=\sum_{A'\in\mathcal{F}}\sum_{A\in\mathcal{F}}\mathbf{c}(A)\left(\mathfrak{s}(\mathbf{v}^{w_A}(A'),\mathbf{c}(A'))-\mathfrak{s}(\mathbf{v}^{w_A}(A'),\mathbf{c}^*(A'))\right)$$

$$=\sum_{A'\in\mathcal{F}}\Big[\sum_{A=A'}\mathbf{c}(A)\left(\mathfrak{s}(\mathbf{v}^{w_A}(A'),\mathbf{c}(A'))-\mathfrak{s}(\mathbf{v}^{w_A}(A'),\mathbf{c}^*(A'))\right)$$

$$+\sum_{A\neq A'}\mathbf{c}(A)\left(\mathfrak{s}(\mathbf{v}^{w_A}(A'),\mathbf{c}(A'))-\mathfrak{s}(\mathbf{v}^{w_A}(A'),\mathbf{c}^*(A'))\right)\Big]$$

$$=\sum_{A'\in\mathcal{F}}\Big[\mathbf{c}(A')\left(\mathfrak{s}(1,\mathbf{c}(A'))-\mathfrak{s}(1,\mathbf{c}^*(A'))\right)+\mathbf{c}(\neg A')\left(\mathfrak{s}(0,\mathbf{c}(A'))-\mathfrak{s}(0,\mathbf{c}^*(A'))\right)\Big]$$

$$=\sum_{A'\in\mathcal{F}}\Big[\left(\mathbf{c}(A')\mathfrak{s}(1,\mathbf{c}(A'))+\mathbf{c}(\neg A')\mathfrak{s}(0,\mathbf{c}(A'))\right)-\left(\mathbf{c}(A')\mathfrak{s}(1,\mathbf{c}^*(A'))+\mathbf{c}(\neg A')\mathfrak{s}(0,\mathbf{c}^*(A'))\right)\Big]$$

Suppose now that Local Propriety holds—that is, for any $A'\in\mathcal{F}$,

$$\mathbf{c}(A')\mathfrak{s}(1,\mathbf{c}(A'))+\mathbf{c}(\neg A')\mathfrak{s}(0,\mathbf{c}(A'))>\mathbf{c}(A')\mathfrak{s}(1,\mathbf{c}^*(A'))+\mathbf{c}(\neg A')\mathfrak{s}(0,\mathbf{c}^*(A')).$$

Thus, we have that $\sum_{A\in\mathcal{F}}\mathbf{c}(A)\mathfrak{R}_{\mathcal{F}}(\mathbf{c},\mathbf{c}^*,w)>0$. So, we can conclude that Relative Global Propriety follows from Local Propriety irrespective of whether the opinion sets are finite or countably infinite. $\square$

Then, what about Simple Global Propriety? As explained in the main text, it is obvious that Simple Global Propriety does not follow from Local Propriety since, when $\mathcal{F}$ is a countably infinite opinion partition, $\mathfrak{G}_{\mathcal{F}}(\mathbf{c},w)=\infty$ for any coherent credence function $\mathbf{c}$. Moreover, we can explain this in another way. Note that we can derive Simple Global Propriety from Local Propriety using the above equation if it holds that

$$\sum_{A\in\mathcal{F}}\mathbf{c}(A)\mathfrak{G}_{\mathcal{F}}(\mathbf{c},w_A)-\sum_{A\in\mathcal{F}}\mathbf{c}(A)\mathfrak{G}_{\mathcal{F}}(\mathbf{c}^*,w_A)=\sum_{A\in\mathcal{F}}\mathbf{c}(A)\mathfrak{R}_{\mathcal{F}}(\mathbf{c},\mathbf{c}^*,w_A)$$

However, this cannot hold when $\mathfrak{G}_{\mathcal{F}}(\mathbf{c},w_A)$ and $\mathfrak{G}_{\mathcal{F}}(\mathbf{c}^*,w_A)$ are infinite.

A Proof of Proposition 5.3 and Some Relevant Remarks

Let us first prove Proposition 5.3. Suppose that $\mathcal{F}$ is the opinion set of $\mathbf{c}$ and $\mathbf{c}^*$, $\mathfrak{s}$ is a legitimate local measure satisfying Continuity, Monotonicity (or Monotonicity$^+$), and Finiteness, and $\mathfrak{R}$ is a legitimate relative global measure generated from $\mathfrak{s}$ in accordance with Relative Additivity. Suppose further that Relative Global Betterness holds. Suppose still further that

(a) $\mathfrak{s}(v_i^w, c_i) \leq \mathfrak{s}(v_i^w, c_i^*)$ for all $A_i \in \mathcal{F}$, and

(b) $\mathfrak{s}(v_i^w, c_i) < \mathfrak{s}(v_i^w, c_i^*)$ for some $A_i \in \mathcal{F}$.

In order to prove Proposition 5.3, it is sufficient to show that it follows from (a) and (b) that:

$$\sum_i \left(\mathfrak{s}(v_i^w, c_i) - \mathfrak{s}(v_i^w, c_i^*) \right) < 0. \tag{$\dagger$}$$

This is because ($\dagger$), with Relative Additivity and Relative Global Betterness, entails that $\mathbf{c} \prec_w \mathbf{c}^*$. It is very trivial that (a) and (b) jointly entail ($\dagger$) irrespective of whether $\mathcal{F}$ is finite or countably infinite. As a result, we conclude that Relative Global Betterness entails Truth-directedness. $\square$

When $\mathcal{F}$ is finite and $\mathfrak{G}$ is generated from the above $\mathfrak{s}$ with accordance with Simple Additivity, ($\dagger$) entails that $\mathfrak{G}(\mathbf{c}, w) < \mathfrak{G}(\mathbf{c}^*, w)$. Thus, we can say that Simple Global Betterness entails Truth-directedness when the opinion sets are finite. However, when $\mathcal{F}$ is infinite, ($\dagger$) cannot entail that $\mathfrak{G}(\mathbf{c}, w) < \mathfrak{G}(\mathbf{c}^*, w)$ and so Simple Global Betterness does not entail Truth-directedness, as shown in Example 3.2.

A Proof of Proposition 5.5

Suppose that $\mathcal{F}$ is an opinion set of $\mathbf{c}$ and $\mathbf{c}^*$. Suppose further that $\mathfrak{s}^0$ is a zero-based local accuracy measure, and that $\mathfrak{s}^+$ is a positive linear transformation of $\mathfrak{s}^0$ such that $\mathfrak{s}^+ = \alpha + \lambda \mathfrak{s}^0$. Here, α is a non-negative real number and λ is a positive real number. Suppose still further that $\mathfrak{G}^0$ and $\mathfrak{R}^+$, respectively, are generated from $\mathfrak{s}^0$ in accordance with Simple Additivity, and from $\mathfrak{s}^+$ in accordance with Relative Additivity. Lastly, let us assume that at least one of $\mathfrak{G}_\mathcal{F}^0(\mathbf{c}, w)$ and $\mathfrak{G}_\mathcal{F}^0(\mathbf{c}^*, w)$ is greater than $-\infty$.

It should be emphasized here that $\mathfrak{G}_\mathcal{F}^0(\mathbf{c}, w)$ and $\mathfrak{G}_\mathcal{F}^0(\mathbf{c}^*, w)$ cannot be greater than 0. This is because the maximum values of $\mathfrak{s}^0(1, x)$ and $\mathfrak{s}^0(0, x)$ are all zero. Then, only two cases are possible: (i) $\mathfrak{G}_\mathcal{F}^0(\mathbf{c}, w)$ and $\mathfrak{G}_\mathcal{F}^0(\mathbf{c}^*, w)$ are all non-positively finite; (ii) one of the two is non-positively finite and the other is negatively infinite. Let us consider each case.

Case (i). Suppose that $\mathfrak{G}_\mathcal{F}^0(\mathbf{c}, w)$ and $\mathfrak{G}_\mathcal{F}^0(\mathbf{c}^*, w)$ are all non-positively finite. Then, it ob-

tains that:

$$\lambda \left[\mathfrak{G}_{\mathcal{F}}^0(\mathbf{c}, w) - \mathfrak{G}_{\mathcal{F}}^0(\mathbf{c}^*, w) \right] = \lambda \left[\sum_{\mathcal{F}} \mathfrak{s}^0(v_i^w, c_i) - \sum_{\mathcal{F}} \mathfrak{s}^0(v_i^w, c_i^*) \right]$$

$$= \sum_{\mathcal{F}} \left[\lambda \mathfrak{s}^0(v_i^w, c_i) - \lambda \mathfrak{s}^0(v_i^w, c_i^*) \right]$$

$$= \sum_{\mathcal{F}} \left[\left(\alpha + \lambda \mathfrak{s}^0(v_i^w, c_i) \right) - \left(\alpha + \lambda \mathfrak{s}^0(v_i^w, c_i^*) \right) \right]$$

$$= \sum_{\mathcal{F}} \left[\mathfrak{s}^+(v_i^w, c_i) - \mathfrak{s}^+(v_i^w, c_i^*) \right]$$

$$= \mathfrak{R}_{\mathcal{F}}^+(\mathbf{c}, \mathbf{c}^*, w)$$

Note that λ has been assumed to be positive, and that $\mathfrak{s}$ and $\mathfrak{s}^+$ satisfy Finiteness. Then, this equation entails that $\mathfrak{G}_{\mathcal{F}}^0(\mathbf{c}, w) \gtreqless \mathfrak{G}_{\mathcal{F}}^0(\mathbf{c}^*, w)$ if and only if $\mathfrak{R}_{\mathcal{F}}^+(\mathbf{c}, \mathbf{c}^*, w) \gtreqless 0$, as required.

Case (ii). Suppose that one of $\mathfrak{G}_{\mathcal{F}}^0(\mathbf{c}, w)$ and $\mathfrak{G}_{\mathcal{F}}^0(\mathbf{c}^*, w)$ is negatively infinite. In particular, suppose that $\mathfrak{G}_{\mathcal{F}}^0(\mathbf{c}, w)$ is non-positively finite, but $\mathfrak{G}_{\mathcal{F}}^0(\mathbf{c}^*, w)$ is negatively infinite. Thus, $\mathfrak{G}_{\mathcal{F}}^0(\mathbf{c}, w) > \mathfrak{G}_{\mathcal{F}}^0(\mathbf{c}^*, w)$. So, it is sufficient to show, for our purpose, that $\mathfrak{R}_{\mathcal{F}}^+(\mathbf{c}, \mathbf{c}^*, w)$ is positive. It is mathematically trivial that, if $\sum_i^\infty a_i = k \in \mathbb{R}$ and $\sum_i^\infty b_i = \infty$, then $\sum_i^\infty (a_i + b_i) = \infty$. So, we easily have that $\mathfrak{R}_{\mathcal{F}}^+(\mathbf{c}, \mathbf{c}^*, w) > 0$.

From the above considerations, therefore, we obtain Proposition 5.4. $\square$

References

Carr, J. R. (2015), 'Epistemic expansions', *Res Philosophica* **92**(2), 217–236.

Easwaran, K. (2013), 'Expected accuracy supports conditionalization–and conglomerability and reflection', *Philosophy of Science* **80**(1), 119–142.

Fallis, D. and Lewis, P. J. (2019), 'Accuracy, conditionalization, and probabilism', *Synthese* **198**(5), 4017–4033.

Greaves, H. and Wallace, D. (2006), 'Justifying conditionalization: Conditionalization maximizes expected epistemic utility', *Mind* **115**(459), 607–632.

Huttegger, S. M. (2013), 'In defense of reflection', *Philosophy of Science* **80**(3), 413–433.

Joyce, J. M. (1998), 'A nonpragmatic vindication of probabilism', *Philosophy of science* **65**(4), 575–603.

Joyce, J. M. (2009), Accuracy and coherence: Prospects for an alethic epistemology of partial belief, *in* 'Degrees of belief', Springer, pp. 263–297.

Kelley, M. (forthcoming), 'On accuracy and coherence with infinite opinion sets', *Philosophy of Science* .

Lakatos, I. (1976), *Proofs and Refutations: The Logic of Mathematical Discovery*, Cambridge and London: Cambridge University Press.

Leitgeb, H. and Pettigrew, R. (2010*a*), 'An objective justification of bayesianism I: Measuring inaccuracy', *Philosophy of Science* **77**(2), 201–235.

Leitgeb, H. and Pettigrew, R. (2010*b*), 'An objective justification of bayesianism II: The consequences of minimizing inaccuracy', *Philosophy of Science* **77**(2), 236–272.

Pettigrew, R. (2016), *Accuracy and the Laws of Credence*, Oxford University Press UK.

Pettigrew, R. (2018), 'The population ethics of belief: In search of an epistemic theory X', *Noûs* **52**(2), 336–372.

Pettigrew, R. (2022), 'Accuracy-first epistemology without additivity', *Philosophy of Science* **89**(1), 128–151.

Predd, J., Seiringer, R., Lieb, E., Osherson, D., Poor, H. V. and Kulkarni, S. (2009), 'Probabilistic coherence and proper scoring rules', *IEEE Transactions on Information Theory* **55**(10), 4786–4792.

Talbot, B. (2019), 'Repugnant accuracy', *Noûs* **53**(3), 540–563.