Newton on Active and Passive Quantities of Matter

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Abstract

Newton published his deduction of universal gravity in *Principia* (first ed., 1687). To establish the universality (the particle-to-particle nature) of gravity, Newton must establish the additivity of mass. I call 'additivity' the property a body's quantity of matter has just in case, if gravitational force is proportional to that quantity, the force can be taken to be the sum of forces proportional to each particle's quantity of matter. Newton's argument for additivity is obscure. I analyze and assess manuscript versions of Newton's initial argument within his initial deduction, dating from early 1685. Newton's strategy depends on distinguishing two quantities of matter, which I call 'active' and 'passive', by how they are measured. These measurement procedures frame conditions on the additivity of each quantity so measured. While Newton has direct evidence for the additivity of passive quantity of matter, he does not for that of the active quantity. Instead, he tries to infer the latter from the former via conceptual analyses of the third law of motion grounded largely on analogies to magnetic attractions. The conditions needed to establish passive additivity frustrate Newton's attempted inference to active additivity.

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1 Introduction

Newton's heralded 'deduction' of the law of universal gravity occurs in the first seven propositions of Book III of *Principia* (1687, 1713, 1726). Roughly, the first five establish that the centrally directed force ('gravity') upon an orbiting body toward a central body is inversely proportional to the square of its distance from the central body; the sixth that gravity is directly proportional to the quantity of matter of the orbiting body; and the seventh that gravity is directly proportional to the quantity of matter of the central body. Newton's law of gravity thus involves three components: the two mass terms in the numerator and the distance-squared term in the denominator.

There is different empirical evidence for the claim that gravity is proportional to the quantity of matter of the orbiting body than there is for the claim that it is proportional to the quantity of matter of the central body.[1] Newton establishes the former claim in part by substantiating a principle underlying the widespread observation that all bodies fall at the same rate. Bodies of different quantities of matter would fall at the same rate, Newton reasons, if gravity were to proportion itself to their quantities of matter.[2] To substantiate this for terrestrial bodies, III.6 describes the two-pendulum experiment which in part rules out that gravity is proportional to bodies' qualitative properties, such as their forms and textures.[3] For celestial bodies, Newton reasons in III.6 that the non-

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[1] Newton's evidence for the inverse-square claim is discussed in Harper 2011, esp. 114-126 and 139-142, *et passim*; Wilson 1970; and Smith 2002.

[2] Borrowing apt language from Émilie du Châtelet 1759:

> Since attraction proportions itself [*se proportionne*] to the mass of the attracting body, & to that of the attracted body, one should conclude from this that it belongs to each part of matter, & that all the parts of which a body is composed mutually attract one other; for if attraction did not belong to each part of matter, it would not follow the ratio of the masses. (51-2, my trans.)

[3] References to *Principia* [P] will be to Newton 1999 and 'III.6.1', e.g., denotes Corollary 1, Proposition 6 of Book III. After summarizing the two-pendulum experiment in the body of the proposition, III.6.1

eccentric and 'extremely regular motion' of, e.g., Jupiter's satellites shows that the action of the sun on unequal bodies of that system produces equal 'accelerative gravities,' i.e. gravity proportions itself to the bodies at a given distance, and so makes unequal bodies at equal distances describe equal spaces in equal times.[4] How Newton establishes the claim concerning the central body, by contrast, has been a matter of great controversy, since he relies upon an application of the third law of motion which itself seems to presuppose the law of gravity.[5]

Newton understands these arguments to have established more than just the proportionality of gravity to the quantities of matter of the orbiting and central bodies. In particular, he takes them to have established *universal* gravity: that every two *particles* interact directly as their masses and inversely as the square of their distance. Hence in III.6, he extends his reasoning to apply to all the parts of the attracted body: 'But further, the weights of the individual parts of each planet toward any other planet are to one another as the matter in the individual parts.' And after concluding III.7 with a significant '*Q.E.D.*' (the first such

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begins: 'Hence, the weights of bodies do not depend on their forms and textures.' For the seventeenth-century doctrines of explanation in terms of forms and textures, see Nadler 2000. The experiment runs as follows: Newton sets in motion two pendulums 'entirely alike' ('*omnino paria*') in weight, shape, and air resistance containing at their centers of oscillation equal weights ('*pondus*') of formally and/or texturally different materials (e.g., gold and wood). The air resistance will act equally upon what we now call the inertial quantity of matter ('*copia materiae*' in Newton's language in the earliest extant manuscript description of this experiment, MS Add. 3965.5a, f. 25v). Meanwhile, gravity will act equally upon equal weights ('*pondus*'). Were air resistance to cause one pendulum to swing slower, '*copia*' and '*pondus*' would be empirically distinguishable. Newton's experiment shows they are empirically indistinguishable: see Fox 2016. For more on this experiment and Newton's developing notions of weight and quantity of matter: Stein 1990b; Wilson 1999; Biener and Smeenk 2012, 32-33.

[4] The reasoning here uses I.65, a proposition discussed later (note 11). See P, Definition 7: 'The accelerative quantity of centripetal force is the measure of this force that is proportional to the velocity which it generates in a given time.'

[5] See Cotes to Newton, Letter LXXX, Feb. 18, 1713 in Edelston 1850; Kant 1786, 4:515; Koyré 1965, 273ff.; Stein 1990a; Stein 1991; Densmore 1999, 104-111; Harper 2002; Harper 2011, 346-55; Harper 2013; Biener and Smeenk 2012; and Smith 2013. Remarkably, III.7 and its two corollaries undergo absolutely no revisions in any edition (or in Newton's own interleaved copy). In the sequence III.1-7 and their corollaries leading to the statement of universal gravity, it is the only one without revisions. See Koyré and Cohen 1972, 576-77.

declaration of Book III), Newton begins the first corollary with the claim: 'Therefore the gravity toward the whole planet arises from and is compounded of the gravity toward the individual parts.'

This paper focuses on Newton's evidence for universality.[6] In particular, I investigate the following problem. In order to secure universal, particle-to-particle interaction, the evidence establishing the proportionality of gravitational force to each mass term must do extra work. It must also establish the additivity of mass. I call 'additivity' the property a body's quantity of matter has just in case, if gravitational force is proportional to that quantity, the force can be taken to be the sum of forces proportional to each particle's quantity of matter. Where little m is the mass of the orbiting or attracted body, big M is the mass of the central or attracting body, S is the collection of all particles composing the relevant body, and d is the distance to the body or the particle, Newton must establish

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[6] Newton recognizes that the commitment to universality sets him at variance with accepted modes of mechanical explanation. For example, Newton's most important contemporaneous reader, Christiaan Huygens, in the course of advancing his own rival mechanical account of gravity, unambiguously rejects universality:

> ...I am not in agreement with a Principle which [Mr Newton] supposes in this calculation and elsewhere, namely, that all the small parts that one could imagine in two or more different bodies attract one another or tend to approach one another mutually. This I cannot admit, because I believe that I see clearly that the cause of such an attraction is not explicable at all by any principle of Mechanics or of the rules of motion... (Huygens 1690, 159, my trans.)

In the General Scholium added in the second edition, Newton acknowledges that universal gravity does not lend itself to mechanical explanation:

> Indeed, this force [of gravity] arises from some cause that penetrates as far as the centers of the sun and planets without any diminution of its power to act, and that acts not in proportion to the quantity of the *surfaces* of the particles on which it acts (as mechanical causes are wont to do) but in proportion to the quantity of *solid* matter...Gravity toward the sun is compounded of the gravities toward the individual particles of the sun... (P, 943)

both entailments:[7]

$$F^g \propto \frac{m}{d^2} \qquad \Rightarrow \qquad F^g = \sum_{i \in S} F_i \qquad \text{where each} \quad F_i \propto \frac{m_i}{d_i^2}$$

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What follows addresses the following two questions. First, in what sense are these entailments distinct if gravitating bodies are simultaneously attracted and attracting? Second, to what extent does Newton succeed in establishing these entailments?

In the next section, I distinguish the entailments both by how the mass terms are empirically measured and by what evidence Newton has to establish the additivity of each mass term so measured. The two-pendulum experiment, accepted principles of statics, and I.65 help establish the first entailment, but no empirical experiments are available to Newton directly to establish the second. Newton nevertheless recognizes that these entailments occasion distinct evidence problems and successfully frames experimental programs which historically come to provide support for the second entailment.

In the subsequent section, I provide an account of how Newton confronts the evidence problem for the second entailment in the manuscript version of his initial argument for universal gravity, dating from spring 1685. In place of empirical evidence, Newton offers conceptual

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[7] Expressing the additivity property of this physical quantity as an 'entailment' is meant to represent that the antecedent is relevant to the consequent. In particular, a measurement procedure satisfying the antecedent conditions the interpretation of the sum in the consequent. For example, the procedure of weighing on a double-pan balance has a natural concatenation operation: putting a atop b on the same pan yields a composed object, c, which can be assigned a quantity (a weight) the same way that a and b independently can. The concatenation operation and other features of this measurement procedure condition how a quantity assignable by the procedure (the weight of c) can be taken to be a sum of other quantities assignable by the same procedure (the weight of a and that of b, where a and b compose c).

analyses of the third law of motion and reasoning by analogy to magnetic attractions. These efforts are not entirely satisfactory to establish the second entailment.

I focus on the initial argument for a number of reasons. The final argument in III.7 is compressed to such an extent that it partly masks its own logic; the initial argument may help unpack it. The central claims of III.6-7 use warrants from the same propositions of Book I as are used in counterpart places of the initial argument; how the initial argument relies on these propositions may help deepen our understanding of how the final argument does. A significant portion of the initial argument consists in conceptual analyses of the third law of motion which are entirely eliminated in the final argument, even though III.7.1 recaps - in reverse order - much of the deductive sequence of the initial argument and retains an appeal to magnets which is germane to the eliminated parts. Close scrutiny of these parts of the initial argument may thus help illuminate how the final argument goes beyond it and qualifies as a 'deduction' of universal gravity.

2 Active and passive quantities of matter

Why should the evidence for the first entailment be different than that for the second? To answer this question, I would like to distinguish two quantities of matter. By 'passive quantity of matter' I mean that quantity of it in proportion to which a body is acted upon by another. By 'active quantity of matter' I mean that quantity of it in proportion to which a body acts upon another.[8] In the case of gravitational force, we can measure the

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[8] By 'act' I understand impressed force; see Definition 4: 'Impressed force is the action exerted on a body to change its state either of resting or of moving uniformly straight forward,' (P, 405). These expressions are not the same as what we now call 'active' and 'passive gravitational mass'. It is an open question how they relate. For reasons of space I also make no serious claims about how they might relate to Newton's concept of the 'inertia of the mass'. On an interpretation I adopt above, Newton conceptually separates weight [*pondus*] and inertial quantity of matter [*copia materiae*] in the two-pendulum experiment. But as

passive quantity through our parochial concept of weight. How much a body weighs on a double-pan balance, for instance, indicates how much matter there is which is acted upon by gravitational force. What Newton calls the body's 'absolute measure' of centripetal force is proportional to its active quantity of matter: how strongly a body draws others to it indicates how much matter there is which attracts. This measure is given for instance by the constant Keplerian harmonic ratio exhibited by each body's satellites. The ratio, $\frac{A^3}{P^2}$, where A is the mean distance of an orbiting body and P its period, measures the strength of a central body's acceleration field. The higher the value, the shorter the period of a satellite at a given distance. The value exhibited by the sun's satellites can be compared, for instance, to the value exhibited by Jupiter's satellites to give the ratio of the central bodies' active quantities of matter. My contention in this section is that there is different evidence to attribute additivity to each of the quantities so measured.[9]

It is a relatively unproblematic principle from the science of weights that the total passive quantity of matter of some body is a sum of the passive quantities of matter of its parts.[10] This is related to the fact, mentioned above, that gravity has an accelerative quantity: at

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I understand the separation, it concerns experiment-building and the conditions on measurement: which phenomena can be taken as measuring each quantity. Similarly, I argue Newton distinguishes active and passive quantities as those which correspond to different measurement procedures: we measure passive quantity on a balance (for ordinary objects), and measure active quantity by orbital periods.

[9] The distinction I draw is between two quantities and the grounds for their additivity, not two roles played by matter. The literature has distinguished, in the case of gravity, the passive role of matter in the change of its inertial state of motion by an externally impressed force (characterized by Law II) from the active role of matter involved in considering gravitational attractions as true interactions (characterized by Law III). See, for instance, the discussion of 'active dispositional' gravity in McMullin 1978, 59-61. McMullin draws his distinctions (gravity as active or passive, actual or dispositional), however, within the question of whether gravity is essential to bodies. There is then the further question of Newton's quest for 'agencies' or 'powers' responsible for the active role of matter. Newton famously expresses this in Query 31 (initially 23) of *Opticks* (375ff.). On this see also Stein 2002, 289 *et passim*; and Janiak 2007.

[10] Archimedes's proof for the law of the lever requires 'distributing the two unequal weights analyzed into rational component parts over the extended beam uniformly so that we have a case of equal weights at equal distances,' (Clagett 1959, 11). (For criticism of these redistribution assumptions, see Mach 1883, 13-20; and Dijksterhuis 1956.) Newton seeks nevertheless to substantiate this principle for the passive quantity. His two-pendulum experiment specifically rules out *texture* as a component of the passive quantity of matter (III.6.1). See note 16 on 'texture'.

equal distances unequal bodies are equally accelerated. Gravity proportions itself to their passive quantities of matter, and parts of bodies fall independently of one another. Hence in a system of weights sufficiently far from a gravitational source, presupposing uniform gravity along parallel lines, the total passive quantity of matter is the sum of the passive quantities of matter of all its parts, where the sum comprises its weight toward that source. In such contexts, passive quantity of matter is additive.[11]

The active quantity is not so unproblematically additive. If A attracts B in proportion to A's quantity of matter, can the total force toward A be taken to be the sum of lesser forces each proportional to and directed toward a different particle of A, for all the particles of which A is composed?[12] An affirmative answer to this question would go beyond the science of weights, which traditionally avoids any hypotheses analyzing the physical source of gravity.[13] One way of answering the question would be to add matter to an attracting

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[11] Newton provides a general, mathematically precise characterization of such contexts. In I.65 and its corollaries, Newton spells out necessary and sufficient conditions for a system of orbiting bodies to be treated as if it were a quasi-isolated system relative to some external gravitational source ('...no change whatsoever of the motion of the bodies attracted among themselves will result from their attractions toward the greater body...'): that it be sufficiently far for the differences among the lengths and among the inclinations of the lines drawn from its parts to the source to be less than any assigned value. Supposing gravity acts uniformly on each part of the system in proportion to its quantity of matter, then if its action is along lines as nearly equal and as nearly parallel as needed, the passive quantity of matter of the system can be taken to be unproblematically additive.

Newton also characterizes the relatively unproblematic nature of passive additivity in II.20.6, where he contrasts absolute gravity with gravity combined with buoyancy:

> By absolute gravity the parts of all fluids and bodies gravitate in their places, and thus the sum of the individual weights is the weight of the whole. For every whole is heavy, as can be tested in vessels full of liquids, and the weight of the whole is equal to the sum of the weights of all the parts, and thus is composed of them.

[12] Active additivity is a more general representation of the second entailment above. It is more general in the sense that the so-called orbiting body and the so-called central body can each be assigned both active and passive quantities of matter. Letting X^+ be a schematic variable for some body's active quantity of matter, active additivity for gravity says:

$$F^g \propto \frac{X^+}{d^2} \qquad \Rightarrow \qquad F^g = \sum_{i \in S} F_i \text{ where each } F_i \propto \frac{X_i^+}{d_i^2}$$

[13] See, e.g., Roberval: '...I will do my best to imitate Archimedes, who on this issue of weight posits as a principle or postulate the invariable fact, upheld through all past centuries up to present, that there are heavy bodies...without taking the trouble to know thoroughly the principles and causes of heaviness...'

body and check whether the magnitude and direction of its attraction are proportionally adjusted. But Newton recognizes the difficulty of measuring attractions among terrestrial objects.[14] Needless to say, it is impossible in 1685 to add matter to a planet and observe any change in the harmonic ratio exhibited by its satellites.[15] This creates an evidence problem for a premise central to Newton's argument for the universality of gravity.

The additivity of the active quantity must be established to rule out the physical possibility that the force proportional to this quantity may depend on the relative configuration of the body's particles ('texture').[16] It is conceivable that different textural configurations of

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(Mémoire of August 7, 1669, in Huygens 1937, 629-30, my trans.).

[14] See *Liber secundus*, §22: 'It will perhaps be said that by this law all bodies must attract each other, contrary to experience in terrestrial bodies. But I respond that there is no experience at all in terrestrial bodies,' (trans. Smith forthcoming, with slight modifications). Newton nevertheless describes two experiments by which gravitation between terrestrial bodies may be measured: the time for two homogeneous one-foot globes to approach one another in free space from a quarter-inch away, and the deviation of a plumb-line at the base of a large mountain. He calculates that these measurements would remain insensible: the first would require at least a month, and the second would deviate less than two arc minutes. Newton uses the dispiriting calculations of §22 (implying the futility of empirical measurement of terrestrial attractions) to motivate 'perhaps...disput[ing] about lesser bodies in the following manner [*in hunc modem*],' that is, in the manner of §23 (discussed at length below): conceptual analyses of Law III and reasoning by analogy to magnetic attractions. For corrected calculations for the first experiment, see Poynting 1894, 10. Bouguer 1749 and Maskelyne 1775 actually performed versions of the plumb-line experiment.

[15] Though Newton acknowledges the relevance of this kind of test in III.7.1:

> For every attraction toward a whole arises from the attractions toward the individual parts. This will be understood in the case of gravity by thinking of several smaller planets coming together into one globe and composing a larger planet.

In a parallel passage in *Liber secundus* §25, he writes:

> If Jupiter and its Satellites were to come together and be formed into a single globe, each of them would doubtless continue to draw [*trahere*] each of the others as before; and conversely, if the body of Jupiter were resolved into several globes, it must be believed [*credendum est*] that these would then draw one another no less than they now draw the Satellites.

[16] I have in mind what Boyle calls 'texture' (see also Antsey 2000, 48):

> That when *diverse* of them are consider'd together, there will necessarily follow here Below both a certain *Position* or *Posture* in reference to the Horizon (as Erected, Inclining, or Level) of each of them, and a certain *Order*, or placing before, or behind, or besides one another; (as when in a company of Souldiers, one stands *upright*, the other *stoops*, the other *lyes along* upon the Ground, they have various *Postures*; and their being plac'd *besides* one another in Ranks, and *behind* one another in Files, are Varieties of their *Order*:) and when many of these small parts are brought to Convene into one Body from their *primary Affections*, and their Disposition, or *Contrivance* as to *Posture* and *Order*, there results That, which by one

attractive particles may produce different mechanisms which independently and varyingly (and therefore non-additively) contribute to the body's total attraction, much like Newton represents forces of resisting media in Book II.[17] And Newton does find evidence that there are forces proportional to mass without additivity in cases of real-world impact. The total force between two impacting bodies involves not just the two masses but also the coefficient of restitution proper to the kind of material.[18] Equal quantities of different materials will not in general produce equal forces upon impact, and the total force of impact cannot be taken to be the sum of independent, lesser impacts from each particle. In the Scholium to the Laws of Motion, Newton frames the experiment used to determine this coefficient in his proof for what we would now call the conservation of momentum (what he calls 'quantity of motion') in inelastic collisions (P, 425ff.).

Newton recognizes that attributing additivity to active quantity of matter is a non-trivial commitment raising mathematical and experimental questions. One question, raised and partly answered in the proposition following the additivity claim in III.7.1, is whether the inverse-square component holds accurately ('*accurate*') or only very nearly so ('*quam proxime*') at smaller distances, where the varying situations and strengths of independently attracting particles raise the possibility that the sum of the inverse-square forces toward

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> Comprehensive Name we call the Texture of that Body. (Boyle 1666, 99-100; see also 116-17).

[17] In Book II Newton uses pendulum-decay and vertical fall experiments to disaggregate how two or three mechanisms contribute to total resistance and whether they may be mathematically represented as terms which vary with different powers of velocity: what Newton sometimes calls 'tenacity' or 'lack of lubricity' with v^0, 'friction' with v, and 'inertia' with v^2 (see Smith 2001 and 2005). This mathematical framework represents a superposition of different force laws which interacts experimentally in different ways with the three textural models Newton uses (in all three editions) for fluids: rarefied, elastic, and continuous fluids. The analogy I make is not to draw attention to Newton's investigation of force laws for centripetal motions other than inverse-square or their superposition, but rather to the fact that textural differences may affect the accuracy of the mathematical term-wise expansion of total centripetal force (additivity).

[18] 'This coefficient purports to describe the degree of plasticity of the collision, and is usually defined as the ratio of the final to initial relative velocity components of the striking objects in the direction normal to the contact surfaces. Values of $e = 1$ and $e = 0$ denote the idealized concepts of perfectly elastic and plastic impact, respectively.' Goldsmith 1960, 4.

those particles does not constitute an inverse-square force toward any fixed center of action.[19] (Let 'center of action' denote the point where the extended body's entire mass may be thought to be situated so as to generate the same inverse-square acceleration field.) Relying on I.75-76, which prove that a body consisting of inverse-square attracting particles and with spherically symmetric density variation has a center of action at its geometric center, Newton concludes in III.8 that two homogeneous globes with active additivity attract with a force inversely proportional to the square of the distance between their centers.[20] A related question, which he suggests will require experimental investigation, is the more general case of non-spherical bodies. In Book I, Section 13 (I.85-93), he broaches the question of where to situate the center of action of an arbitrary body. Among his conclusions is that at least some non-spherical bodies consisting of inverse-square attracting particles cannot be replaced by point particles with a quantity of matter parameter at their centers of action, because no such fixed centers exist. Call this conclusion the 'center of action problem'. In the case of a regular cylinder consisting of inverse-square attracting particles, for example, the total attractive force is not an inverse-square force toward any single fixed point; rather, the force upon an exterior particle lying on a line extending the cylinder's axis varies directly as the cylinder's length and inversely as the product of the given attracted particle's distances from the nearest and furthest edges (I.91.1).

Newton thinks that the center of action problem, rooted in a contrast between attractions to spherical and to non-spherical bodies, alongside the related question of the figure of the earth, opens the way to an experimental program that may come to provide empirical evi-

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[19] My emphasis on the relation between additivity and the existence of centers of action diverges from the reading of III.8 proposed by Densmore 1995, who emphasizes that in this proposition Newton rules out the possibility that the force toward the closest particles near the surface of an attracting body approaches infinity.

[20] See Newton to Halley, 20 June 1686: '...I never extended ye duplicate proportion lower then [sic] to ye superficies of ye earth & before a certain demonstration I found ye last year have suspected it did not reach accurately enough down so low...' Turnbull 1960, 2:435.

dence for the additivity of active quantity of matter. This, in turn, would provide evidence for universal, particle-to-particle gravity. I will explain Newton's thinking concerning such prospective evidence by way of an analogy to the case of constrained motion by weight.

Huygens seeks to determine the length of a simple pendulum isochronous to a given compound pendulum, which is called the center of oscillation of the latter. Its length – i.e., where 'the gravity of its weight is to be thought of as gathered together at one point' (Huygens 1673, part IV, def. III) – can be determined in the case of the circular compound pendulum but not generally for the cycloidal compound pendulum. This is a case of determining the range of situations in which a passive quantity of matter distribution may be reduced to a point so the motions generated in a given time *of* the distribution and *of* the point are equal. Analogously, Newton seeks to determine whether compound bodies have unique centers of action – i.e., where attractive matter is to be thought of as gathered together at one point to generate the same inverse-square acceleration field. This is a case of determining the range of situations in which an active quantity of matter distribution may be reduced to a point so the velocities generated in a given time *by* the distribution and *by* the point are equal. Huygens's center of oscillation can be found when an isochronous simple pendulum is constructed, and isochrony is measurable. Newton's center of action has no immediate empirical test. But it is plausible to see I.92 and the question of the figure of the earth as framing experiments to reveal the constraints on a range of centers of action.[21] Deriving the law of attraction for particles set outside irregular bodies is also an

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[21] Smith (ms) emphasizes that Proposition 92 ('Given an attracting body, it is required to find the ratio by which the centripetal forces tending toward each of its individual points decrease') is the statement of a research program, which Newton puts forward even before learning of the variation in surface gravity from Halley in summer 1686. The true figure of the earth and the resulting variation in surface gravity could possibly provide evidence that a body's weight is the sum of inverse-square attractions to the earth's individual particles (Smith 2011, 275-8). Newton's recognition of this possibility may explain his fastidious amplifications across editions of the data in III.19-20. For a historical overview see Todhunter 1873. See Maupertuis 1738, esp. 206-7, for how the figure of the earth may adjudicate between Newton's theory of universal gravity and the Huygens 1690 account of gravity as an effect of bodies' differential velocities.

analytical project connected to the development of potential theory: equipotential surfaces about a body provide a generalization of the distance parameter in inverse-square acceleration fields about a point. Finally, Lagrange's and Laplace's works late in the 18th century show serious and pervasive acceptance of the additivity of active quantity of matter, which they both treat as important evidence for universal, particle-to-particle gravity.[22]

In this section, I have distinguished active and passive quantities of matter both by how they are empirically measured and by what evidence Newton has to establish their additivity. To conclude the section, I would like to explain why the evidence available to Newton for passive additivity (two-pendulum experiment, principles from statics, I.65) cannot straightforwardly count as evidence for active additivity. A natural thought suggests otherwise. For a gravitating body, how much matter there is which weighs should, in some sense of 'equal', also be equal to how much matter there is which attracts. Now, Newton has evidence that a part's weight is to the body's weight as the part's passive quantity of matter is to the body's passive quantity of matter. Why does he not thereby have license to infer that a part's attraction is to the body's attraction as the part's active quantity of matter is to the body's active quantity of matter? Newton's grounds for passive additivity cannot themselves straightforwardly count as grounds for active additivity because of what I call

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[22] Lagrange 1773 expresses a clear link between additivity and the figure of the earth:

> One must now add to these forces deriving from the Sun's attraction those that come from the Earth's attraction; and since we want to have consideration of the non-sphericity of its figure, it is necessary to consider in particular the attraction of each particle of the earth on the moon and to look for the forces which result from them. (348, my trans.)

And Laplace 1798 also connects additivity to the evidence for universal gravity:

> The attractive property of celestial bodies does not belong to them only as aggregates [*en masse*]; rather, it is proper to each of their molecules [*molécules*]. If the sun only acted upon the center of the earth without particularly attracting each of its parts, there would result within the ocean very different and incomparably larger oscillations than those which we observe. The weight of the earth toward the sun is therefore the result of the weights of all of its molecules which consequently attract the sun in the ratio of their respective masses. (Part I, Bk II, 122, my trans.)

the ‘accelerative problem’, which I now describe.

Consider the empirical measures of passive quantity available to Newton. The double-pan balance used to compare weights and the pendulums of the two-pendulum experiment are taken to be immersed in a field acting uniformly along parallel lines. This condition allows one to infer that if the beam is at rest in a direction normal to the field then the weights are equal, and that if the pendulums are isochronous then weight measures passive quantity of matter. Relatedly, for a system of bodies to be taken to be a quasi-isolated system with passive additivity in the sense of I.65 (see note 11), the source must be at a ‘great distance’ from the system, producing ‘equal accelerative forces...along parallel lines’ (P, 568).[23] For example, Newton treats the Jovian system as quasi-isolated relative to the sun because the orbits of its moons are very nearly concentric. This indicates that the sun is sufficiently far to produce effectively equal and effectively parallel accelerative forces (or ‘accelerative gravities’ (III.6)) on all parts of that system. Newton’s empirical measures of passive quantity of matter, and therefore also his grounds to attribute additivity to this quantity so measured, take the acceleration field to be effectively uniform within the experimental region of bodies assigned this quantity.

One way to establish active additivity in this context would consist in showing that the field is produced by every part of the source acting independently and in proportion to the part’s quantity of matter, and that the field strength is a linear combination of the accelerations toward each part. Within the experimental region of bodies immersed in a field taken to be effectively uniform, however, the actions of the source’s parts cannot be

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[23] This is connected to Corollary 6 to the Laws of Motion, expressing so-called non-inertial relativity:

> If bodies are moving in any way whatsoever with respect to one another and are urged by equal accelerative forces along parallel lines, they will all continue to move with respect to one another in the same way as they would if they were not acted on by those forces. (P, 423)

discerned as independent. The source is taken to act as if it were a single center of action at a great distance, effectively without parts. In this context, it is not obvious how to justify treating the source's active quantity of matter as the sum of the active quantities of its parts without assuming active additivity from the outset.

Therefore, Newton cannot infer that the source's active quantity of matter is additive (second entailment) merely on grounds that the passive quantity of a body immersed in its field is additive (first entailment). Instead, given his empirical access to the passive quantity, the conditions Newton must assume for passive additivity – equal, accelerative forces – frustrate a straightforward inference to the source's active additivity. Call this the 'accelerative problem'. Newton nevertheless attempts an inference from passive to active additivity in his initial deduction of universal gravity. In the next section, I will reconstruct and assess this attempt. As we will see, one of Newton's possible means of inference – from the passive additivity of some body to the active additivity of the gravitational source – runs into the accelerative problem, while the other possible means of inference – from the passive additivity of some body to the active additivity of that very body – runs into the center of action problem.

3 *Liber secundus* and Three Versions of Law III

Before Newton settles on a three-book form for *Principia* in summer 1686, he composes a two-book treatise with 'an earlier version of book 3 in popular form, so that it might be more widely read' (P, 793). Drafted in the early part of 1685, these are titlted *De motu corporum, Liber primus* and *Liber secundus*. After learning of Hooke's priority claim on the inverse-square proportion, Newton then 'translate[s] the substance of the earlier version [of Book 3, i.e. *Liber secundus*] into propositions in a mathematical style, so that they may be read only by those who have first mastered the principles,' (*ibid.*). Some versions of *Liber secundus* are published posthumously under the titles *De Mundi Systemate liber Isaaci Newtoni* and *Treatise of the System of the World*.[24]

Sections 18-26 of *Liber secundus* correspond to the *Principia*'s deduction of universal gravity. §§18-19 establish that force is proportional to the attracted body ($F^g \propto m$), §§23-24 establish that force is proportional to the attracting body ($F^g \propto M$), §25 concludes that 'the force of a whole globe is composed of the forces of all the particles' ($F^g = \sum_{i \in S} F_i$ where each $F_i \propto \frac{M_i}{d_i^2}$), and §26 states the full law of gravity.[25] The conclusion of §25 expresses the consequent of what I have called the additivity of active quantity of matter, an important premise for universality. In this section, I will show that Newton does not have any direct evidence for active additivity in *Liber secundus.* Instead, in §§20, 21, and 23, he provides conceptual analyses of the third law of motion grounded largely on reasoning by analogy to magnetic attractions.[26] These analyses in the course of this initial argument for universality are striking because they are entirely absent from the final argument in III.1-7. The purpose of these analyses appears to be to license inferring the additivity of active quantity of matter on the basis of the independently established additivity of passive quantity of matter. After summarizing these analyses, I will assess whether they succeed in their purpose.

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[24] Newton 1728. The editors of both Latin and English published editions made the text appear much closer to *Principia* Book III than the manuscript of *Liber secundus* would on its own suggest.

[25] '*…et propterea ex particularum viribus componi vim globi totius*'. For a study of this deductive sequence, see Smith (forthcoming b). The manuscript shows that the contents of §26 were initially a part of §25.

[26] In all editions of *Principia*, Law III reads: 'To any action there is always an opposite and equal reaction; in other words, the actions of two bodies upon each other are always equal and always opposite in direction,' (417). It is identically worded in the fullest extant version of *Liber primus* ('LL_β', MS Dd. 9.46), whose composition, however, postdates *Liber secundus.* The first eight leaves - including the definitions and laws of motion - of the version of *Liber primus* ('LL_α') which predates *Liber secundus* are missing. For labeling and dating see Cohen 1971, 310ff.

It will be helpful to begin by sketching the deductive structure of §§18-26. First, Newton establishes that gravity is proportional to passive quantity of matter for both celestial (§18) and terrestrial (§19) bodies. Since the planets are attracted to the sun by an inverse-square force producing 'regular' orbits (following the Keplerian harmonic ratio), Newton reasons that the circumsolar force upon the planets must proportion itself to their quantities of matter. He then observes that the moons within planetary systems also orbit their central bodies regularly, and so the circumsolar force must proportion itself to the weights (he uses '*ponderum*') of, e.g., Jupiter and all of its satellites as one system. To this observation Newton applies what becomes I.65 in *Principia*. This licenses treating the bodies composing such internally regular planetary systems as parts of a quasi-isolated system weighing toward the external source (see end of section 2 above and note 11). This, in turn, licenses attributing additivity to celestial passive quantity of matter: there is proportional action upon the planets' quantity of matter 'and thus also upon the particles of that ~~heaviness [*ponderis*]~~ $_\wedge$quantity$_\wedge$ of which the Planets are composed,' (§18, insertion and deletion in original manuscript). In the following section, Newton summarizes the two-pendulum experiment, according to which weight is a measure of terrestrial passive quantity of matter (see note 3). Combined with accepted principles from the science of weights (see note 10), Newton's experiment licenses attributing additivity to terrestrial passive quantity of matter. Thus, §§18-19 not only establish that the force of gravity is proportional to passive quantity of matter, but also that that quantity is additive. This substantiates the first entailment required for universality.

Before arguing for parallel claims regarding active quantity of matter (§§23-25), Newton provides two conceptual and seemingly metaphysical analyses of Law III (§§20-21), discussed below.[27] These raise the question of §22 (see note 14): what evidence is there that Law

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[27] For a metaphysical reading of Newtonian force reconciling the published English edition of *Liber se-*

III holds for attractions between terrestrial bodies? Newton concedes that measurements of terrestrial interactions would be exceedingly difficult, but concludes §22 by suggesting nevertheless to 'dispute about lesser bodies in the following manner' (i.e., in the manner of §23). But §23 provides yet another conceptual analysis of Law III. So Newton's preoccupation between §§18-19 on passive quantity of matter and §§23-25 on active quantity of matter is to develop three analyses of Law III. I will now describe these three analyses and determine how they may figure into Newton's inference to active additivity.

3.1 Analyses of Law III

The manuscript of *Liber secundus* includes a heavily reworked version of §23. It is written out in Newton's hand (the rest is in his amanuensis') partly on the verso side of the preceding folio, and then entirely crossed out and replaced. The deleted §23 (titled 'That forces tend toward all terrestrial bodies') is informative. It contains at least four successive attempts at the argument eventually retained in the undeleted §23 (titled 'That forces proportional to the quantity of matter nevertheless tend toward all terrestrial bodies'). A version of this argument is added to the second edition *Principia*'s Scholium to the Laws of Motion as further proof that attractions are governed by Law III (P, 428). The first three attempts, [A]-[C], offer an analysis of how Law III applies to the parts of one body. The fourth, [D], generalizes these and is retained almost verbatim in the undeleted §23. A new fifth argument, [E], only appears in the undeleted §23.

A. Let a globe representing the earth be cut by two parallel planes equidistant from the center. If an outer part were slowly and slightly withdrawn, the middle would yield to the 'urging weight [*cederet urgenti ponderis partis*]' of the other outer part and would tend

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cundus (i.e., *A Treatise of the System of the World*) with *Principia*, see Schliesser 2011.

toward the withdrawn part. And if an outer part and the rest 'were forcibly [*violenter*] held back a certain distance from one another,' when let go, 'both bulks would rush headlong [*ruerent*] toward one other and thus have the power of mutual gravitation [*in se invicem adeoque gravitatione mutua pollent*].'[28]

B. Newton argues for the conclusion that the attractive forces of the two outer parts from [A] accompany them when moved apart from a more general principle that 'the affections and operations of bodies depend on bodies [*pendent affectiones & ope<ra>tiones corporum a corporibus*]' and 'will accompany moved bodies [*comitabuntur corpora translata*].' Here, Newton explicitly mentions magnets: 'the magnetic force follows [*sequitur*] a loadstone.'[29]

C. It is possible for one of the parts to be moved far enough to create a new equilibrium. Specifically, if an outer part is moved so that 'its action...would cease [*cessaret*] on account of too great a distance,' the once-middle would not yield to the weight of the other outer

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[28] Gilbert uses '*ruit*', from *ruo*, technically to describe an effect of magnetic conjoint action ('*conactus*'): that iron 'rushes headlong [*ruit*] toward the loadstone, without curves and turnings,' expresses the conjoint 'action of each towards unity' (1600, 68). '*Ruo*' has a few meanings: to fall with violence, to rush down, to fall down, to tumble down, and to go to ruin (Lewis & Short 1879). Gilbert's description of magnetic *conactus*, one sentence before one of his uses of '*ruit*', is quoted by Charleton 1654 (387). McGuire & Tamny 1983 (6, 20) and Westfall 1962 emphasize Charleton's influence on Newton's knowledge of magnetism in his Trinity notebooks from 1663-6. Thus there is a possible indirect connection between Newton's use of '*ruerent*' in the deleted §23 and Gilbert's technical use in his chapter on conjoint action: it signifies a direct falling, without curves and turnings. The word '*ruo*' does not appear in any form in *Principia* (according to the concordance – covering all editions and all of Newton's notations in his copies – which I. B. Cohen had produced with the variorum edition of the *Principia*, as per George E. Smith).

[29] On most mechanical accounts of the motion of heavy bodies, forces do not accompany bodies in this way. The Huygens 1690 quote above (note 6) continues as follows: 'Nor am I at all persuaded of the necessity of the mutual attraction of whole bodies, having shown that, were there no Earth, bodies would not cease to tend toward a center by what we call their gravity,' (Huygens 1690, 159). In contrast, Newton's argument here echoes attractionist theories like Gilbert 1651, 144: 'Position is nothing. It does not exist; all power resides in bodies themselves,' (quoted in Gaukroger 2002, 100n). See also the Introduction to *Astronomia nova* in Kepler 1609:

> Heavy bodies (let us place [*collocemus*] most particularly the Earth at the center of the world) are not carried [*feruntur*] toward the center of the world as the center of the world, but as the center of a kindred [*cognati*] round body, namely the Earth. Therefore, wherever the Earth were placed, or wherever it were transported by its animal faculty, it is always toward it that heavy bodies are carried. (25, my trans.)

part (as in [A]); instead, they would exhibit a new equilibrium. Newton cites the first law of motion to assert that 'perpetually accelerated motion in a straight line must not be granted to forces of nature.' When the outer part is removed at some small distance, the remainder's fall 'will demonstrate attraction [*attractionem*]'; and 'by not falling' when far 'will demonstrate that attractive force [*vi<m> attractivam*] is diminished by distance.' Though magnets are not mentioned here, it is plausible to associate the notion of a magnetic 'sphere of activity' with the possibility of moving an attractive part far enough that its action *cease*.[30]

[A]-[C] argue for the interactivity of attractions, the inherence of force within bodies, and the importance of the first law of motion to equilibrium conditions on isolated systems. They also reveal, as with [D] and §21 discussed below, that magnets play some fundamental evidentiary role in Newton's developing analyses of Law III and, by extension, in the broader argument for additivity which these analyses support.

D. In a more general presentation of [A], Newton argues that the parts of a globe with one planar cut will gravitate toward one another equally. For since one part leans on and presses the other with all its weight, the other must sustain that weight and remain unmoved—and so must have an equal endeavor in the opposite direction. Newton writes, 'The parts therefore urge each other equally by their weights, that is, are drawn toward each other equally (as the third Law requires) and thus if separated from one another and

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[30] In §21, Newton argument for the mutuality of magnetic attraction is that when a magnet is removed, 'nearly the whole force of the iron ceases [*cessat*]'. In a draft conclusion of the *Opticks* (cited in Cohen 1966, 179-80), Newton writes, 'Hypoth. 1. The particles of bodies have certain spheres of activity, within which they attract or shun one another. For ye attractive vertue of the whole magnet is composed of ye attractive vertues of all its particles & the like is to be understood of the attractive vertues of electrical & gravitating bodies.' This notion can be traced back to Hooke's 'sphere of activity', Wilkins's 'sphere of vigor', Charleton's 'sphere of Diffusion', Ridley's 'orbe ... of vigour and power', and through them back eventually to Gilbert's '*orbis virtutis*'. See Hooke 1678, 228-9; Charleton 1654, 389; Wilkins 1648; Ridley 1613, 47; Gilbert 1600. For an overview see Bennett 1981.

let go would fall toward each other with velocities that would be reciprocally as the bodies.' There is again an appeal to magnets: 'It is legitimate [*licet*] to test and observe all this in a loadstone.' The final sentence of [D] is not included in the undeleted §23, though the rest of [D] is repeated verbatim: 'Its [the loadstone's] attracted [*attracta*] part does not propel the attracting [*trahentem*] part but resists and is sustained.' Significantly, alongside this sentence in the margin there is a postil, written in Newton's hand and subsequently crossed out, designating the title of what would have been §24: 'Illustration by means of magnetic attraction [*attractionem*]'.[31] This indicates that Newton was intending to adduce yet further grounds from magnetic attractions in the heart of his initial argument for universal gravity.

E. This argument from the undeleted §23, and not present in the deleted §23, is an application of [D] to a situation when the planar cut picks out 'some small body on the earth's surface' or a 'particle'. Newton makes a puzzling argument that two conceptually distinct quantities (the attraction of the particle toward the earth and that of the earth toward the particle) are both proportional to one and the same quantity (the matter of the particle):

> Now let ACB represent some small body on the earth's surface, and since ~~the velocity~~ the mutual attractions of this particle and the rest of the earth ACD are equal, and yet the attraction of the particle toward the earth (that is, its weight [*pondus*]) is as the matter of the particle [*ut materia particulae*] (as has been proved by the pendulum experiment), the attraction of the earth toward the particle will also be as the matter of the particle [*ut materia particulae*], and thus the attractive force of every terrestrial body is as the quantity of matter [*ut quantitas materiae*] in it.

Both the 'attraction of the particle toward the earth' and the 'attraction of the earth toward the particle' are said to be 'as the matter of the particle [*ut materia particulae*]'. Newton cites the two-pendulum experiment of §19 to justify the first proportionality. He does not re-apply that experiment to the 'attraction of the earth toward the particle', which would

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[31] Newton's numbering was off, so the postil in the margins of the manuscript reads 'XXV'.

yield its proportionality to the matter of the earth. Instead, he applies some version of Law III – evoked in the second clause of the passage quoted (‘*attractiones in se mutuò sunt aequales*’) – to infer that the ‘attraction of the earth toward the particle’ is also ‘as the matter of the particle [*ut materia particulae*]’.

I suggest the expression ‘as the matter of the particle’ is used in two senses. The first usage is passive. It denotes a quantity in proportion to which the particle is acted upon. Hence the first ‘attraction’ is specified by the two parentheticals, which name it ‘weight’ and connect it to its empirical measure.[32] The second usage is active. It denotes a quantity in proportion to which the particle acts upon the earth. The second ‘attraction’ is not named ‘weight’, nor is it connected to an empirical measure (understandably, given the calculations of §22 - see note 14) apart from its inferred association with the passive quantity.

The succession of arguments [A]-[E] from §23 therefore culminates in a version of Law III which licenses inferring one proportionality from another. In particular, the analysis proposed licenses inferring that if force is proportional to the passive quantity of some body, then it is also proportional to the active quantity of that very body. I will now turn to the distinct analyses of Law III offered in §§20 and 21.

Immediately following the summary of the two-pendulum experiment, which establishes the proportionality or ‘agreement [*analogiam*]’ of weight and passive quantity (see note 32), Newton gives a short argument in §20 (titled ‘Unanimity of agreements [*Analogiarum consensus*]’) which I quote in full:

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[32] At the end of §19, after summarizing the two-pendulum experiment, Newton writes:

> In these experiments, in bodies of the same weight [*pondus*], a difference of matter that would be even less than a thousandth of the whole could have been clearly noticed. ~~Because of this agreement [*analogiam*], I have throughout designated the quantity of matter in each individual body by the word *pondus*, using the name of the measure for the thing measured, as is common custom.~~

> And since the action of centripetal force upon the attracted [*attractum*] body, at equal distances, is proportional to the matter in this body, it is reasonable also to grant [*rationi etiam consentaneum est*] that it is proportional as well to the matter in the drawing [*trahente*] body.[33] For the action is mutual, and makes the bodies by a mutual endeavor [*conatu mutuo*] (by Law 3) approach one another [*ad invicem*], and accordingly it must be in conformity with itself on the body in both places [*sibi ipsi conformis esse debet in corpore utroque*]. One body can be considered as attracting [*attrahens*], and the other as attracted [*attractum*], but this distinction is more mathematical than natural. The attraction [*attractio*] is really that of either of the two bodies towards the other$_\wedge$, and thus of the same kind in both$_\wedge$.

The first sentence sets out the main inference Newton motivates: if the action on an attracted body is proportional to its matter (§§18-19), then the action is also proportional to the matter of the other (attracting) body. This inference concerning quantities is motivated by the seemingly qualitative principle which follows: because the mutual action causes a mutual endeavor in both bodies to approach, the action must conform to itself at both places. If the action is proportional to one body, conforming to itself on the body in the other place is presumably taken to imply being proportional to that body as well. Notice this is different from §23. Here the §20 analysis of Law III licenses inferring that if the force is proportional to the passive quantity in one body, then it is proportional to the active quantity in the other body.

Picking up on the topic of the last lines of §20, Newton offers a different, more longwinded analysis of Law III in §21. He substantiates his analysis with an extended discussion of magnets:

> And although, in a pair of Planets, the action of each on the other can be distinguished and can be considered as two [*binae*] actions by which each draws [*trahi*] the other: ~~they are not two but a simple operation between two termini.~~

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[33] Here and elsewhere, Newton seems to be systematically alternating between '*trahere*' (to pull or draw) and '*attrahere*' (to pull or draw to something), though the sense of this alternation is not clear.

> ~~By the contraction of one rope insofar as between~~ yet inasmuch as these are intermediate [*intermediae*], they are not two but a simple operation between two termini. Two bodies can be drawn to each other by the contraction of a single rope between them. The cause of the action is paired [*gemina*], namely the disposition [*dispositio*] of each of the two bodies; the action is likewise paired, insofar as it is upon two bodies [*in bina corpora*]: but ~~the operation by which the Sun~~ insofar as it is between two bodies it is simple and single ... By the action by which the Sun draws Jupiter, Jupiter and the Sun endeavor to approach each other (by Law 3), and by the action by which Jupiter draws the Sun, Jupiter and the Sun also endeavor to approach each other. Moreover, the Sun is not attracted [*attrahitur*] by a twofold action towards Jupiter, nor Jupiter by a twofold action towards the Sun, but there is one action between them by which both approach each other. Iron draws a Loadstone [*magnetum*] just as much as a Loadstone draws iron. For any iron in the vicinity of a Loadstone draws other iron also. But the action between the Loadstone and the iron is simple, and Philosophers consider it as simple; the operation of the iron upon the Loadstone is the very operation of the Loadstone between itself and the iron, by which both endeavor to approach each other. This is manifest from the fact that if the loadstone is removed, nearly the whole force of the iron ceases. In this way conceive that a simple operation, arising from their conspiring [*conspirante*] nature, is exerted between two Planets, and this will hold in the same way for both [*eodem modo se habebit*] and thus, being proportional to the matter existing in one of them, will be proportional to the matter in the other.

The goal is to concede that the actions of two bodies on one another do have a dual aspect, but to insist nevertheless that they are fundamentally single and simple. The action is paired because it is ‘upon’ two bodies: both are caused to move, and the cause, being a disposition in both, is itself also paired. The action, however, is said to be single because it is ‘intermediate’ or ‘between’ two bodies: a single mutual endeavor to approach. Moreover, just as the bodies attract by a single, simple operation, so too are they attracted by a single, simple operation. There is one operation by which they draw and are drawn together. Newton then substantiates his analysis by discussing magnets. He appeals to the authority of ‘Philosophers’ to justify treating the actions of the loadstone and iron as simple and single, and describes an easy experiment to make ‘manifest’ this point. He finally enjoins the reader to conceive of gravity in this way, and thereby to conclude that the action between two gravitating bodies is proportional to both of their quantities of matter.

We have three distinct analyses of Law III in §§20, 21, and 23. §20 analyzes Law III to license inferring that if the force is proportional to the passive quantity of the 'attracted' body, it is also proportional to the active quantity of the other ('attracting') body. §21 analyzes the law less to infer than to describe the nature of gravitational interaction as a simple action. §23 analyzes Law III to license inferring that if the force is proportional to the passive quantity of one body, it is also proportional to the active quantity of the very same body.

To what is Law III, for attractions, meant to apply? Does it license a point-of-view switch on the quantity of the action, as proportional to both 'attracted' and 'attracting' bodies? Or a point-of-view switch on the nature of the action, as dual but also single? Or a point-of-view switch on the quantity of a single body, as itself both passive and active? We are now in a position to appreciate the complexity of the reasoning involved in these analyses of Law III, and to discern how they figure into the argument that Newton is attempting to develop.

3.2 Assessment

What I wish to assess is whether Newton has established the additivity of active quantity of matter. He expresses the consequent of active additivity in §25: 'It has already been established that these forces arise from the universal nature of matter, and therefore that the force of a whole globe is composed of the forces of all the particles.' Before determining whether the analyses of Law III discussed above may justify this conclusion, it will be instructive to situate the conclusion in relation to the prior sections.

Recall that §22 raises the problem of measuring terrestrial interactions (see note 14) and suggests to reason about them in the manner of §23, which uses the heavily reworked analysis of Law III described above to establish that force is proportional to terrestrial active quantity of matter. Following this, §24 makes two arguments that force is proportional to celestial active quantity of matter. The two arguments appear relevant to the active additivity conclusion in §25, so I begin with them.

The first argument is an assertion which attempts to extend the results of §23: since force is proportional to active quantity of matter for terrestrial bodies of all forms, it must be proportional to active quantity of matter universally. As the two-pendulum experiment rules out forms and textures as components of passive quantity of matter, so this assertion functions to rule out forms and their modifications as components of active quantity of matter. (Newton's reference in the §25 conclusion to the 'universal nature of matter' may be connected to this assertion.) He writes that the assertion 'is proved also for celestial bodies' by the second argument.

The second argument of §24 uses what becomes I.69, an important proposition also cited in III.7. The proposition says that for any system of bodies any pair of which is subject to Law III and whose attractive forces have accelerative measure, the force toward any one body is as its active quantity of matter. The condition that a body's attractive forces are accelerative indicates here that the body is the source of a field: by proportioning themselves to the passive quantities in the field, the attractive forces produce equal accelerations at equal distances from the source. So §24 uses what becomes I.69 to infer that the field strength is proportional to the source's active quantity of matter.[34]

It does not follow, however, that the field results from independently attracting parts of the source. In fact, I.69 itself makes no reference to parts. As the 'accelerative problem' suggests above, the actions of the parts of the source cannot be distinguished merely on grounds that the accelerative forces proportion themselves to the passive quantities of matter within the field. So neither argument from §24 directly establishes the additivity conclusion of §25.[35] Do Newton's analyses of Law III enable him to establish this conclusion?

To answer this, consider the following table, where A and B designate two bodies in a situation of gravitational interaction. Let the plus and minus signs designate the bodies' active and passive quantities of matter.

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[34] In the Scholium to I.69, Newton again appeals to magnets to motivate active additivity:

> By these propositions we are directed to the analogy between centripetal forces and the central bodies toward which those forces tend. For it is reasonable that the forces directed toward bodies depend on the nature and the quantity of matter of such bodies, as happens in the case of magnetic bodies. And whenever cases of this sort occur, the attractions of the bodies must be reckoned by assigning proper forces to their individual particles and then taking the sums of these forces. (P, 588-9)

A version of this scholium from a manuscript ('MS_x') among the closest to the composition of *Liber secundus* §§18-26 can be found in Whiteside 1974, 179ff. It is not identical but it does include a reference to magnetic bodies.

[35] The restriction in §25 to 'a whole globe' is repeated in §26, where Newton states for the first time the full law of gravity: 'the motive force ... is as the product of the quantities of matter in the two globes divided by the square of the distance between their centers, by Prop. 46, Corol. 4.' That proposition is numbered I.76 in *Principia*, which, as discussed above, only proves a restricted kind of additivity: if inverse-square forces are already taken to tend toward each particle of a spherically symmetric body, their linear combination results in an inverse-square force to the center. Thus of the two propositions cited in the vicinity of §25 (I.69 and I.76), the first is not in the position to prove active additivity while the second proves a special case. It is significant in this connection that, as Smith (ms) suggests, Newton added I.79-84 (non-inverse-square attractions toward particles composing spheres), I.91 (attraction of particle on any axis of revolution toward the body of revolution), and I.92 (see note 21) to *Liber Primus* after composing §§18-26 of *Liber secundus*.

A	B
$+$	$-$
$-$	$+$

All four quantities are present in a single interaction: this seems to be one upshot of §21. According to the interpretations I proposed above, here are the inferences licensed by the other sections, understanding, e.g., ‘A^-’ as A’s passive quantity of matter:

§20

1. $F^g \propto A^- \qquad \rightarrow \qquad F^g \propto B^+$

2. $F^g \propto B^- \qquad \rightarrow \qquad F^g \propto A^+$

§23

3. $F^g \propto A^- \qquad \rightarrow \qquad F^g \propto A^+$

4. $F^g \propto B^- \qquad \rightarrow \qquad F^g \propto B^+$

§24

1*. $F^g \propto \frac{A^-}{d^2} \qquad \rightarrow \qquad F^g \propto \frac{B^+}{d^2}$

2*. $F^g \propto \frac{B^-}{d^2} \qquad \rightarrow \qquad F^g \propto \frac{A^+}{d^2}$

The last two inferences are labeled as they are because they are formally refined versions of (1) and (2). Recall, moreover, that §§18-19 not only establish that the force of gravity is proportional to passive quantity of matter, but also that that quantity is additive. Thus it establishes the following entailments and their antecedents:

§§18-19

5. $F^g \propto \frac{A^-}{d^2} \quad \Rightarrow \quad F^g = \sum_{i \in S} F_i$ where each $F_i \propto \frac{A_i^-}{d_i^2}$

6. $F^g \propto \frac{B^-}{d^2} \quad \Rightarrow \quad F^g = \sum_{i \in S} F_i$ where each $F_i \propto \frac{B_i^-}{d_i^2}$

Having enumerated the inferences licensed by the analyses of Law III, we can now determine whether Newton has the logical resources to infer the consequent of the additivity of active quantity of matter (the target thesis of §25) from the independently established additivity of passive quantity of matter.

One possible strategy is to try to infer the active additivity of B on the basis of the passive additivity of A. But this runs into the accelerative problem, as discussed just above in relation to the inferences licensed by §24. The most these inferences would seem to yield is the proportionality of force to active quantity of matter; that is, they yield the antecedent of active additivity, but neither the entailment itself nor, therefore, its consequent.

Another possible strategy is to try to infer the active additivity of A on the basis of the passive additivity of A. It may proceed as follows. By §§18-19, we know that the force on A is proportional to the passive quantity of matter and inversely to the distance squared ($F^g \propto \frac{A^-}{d^2}$). By (5) we can infer that this inverse-square force can be taken to be the sum of forces on A's particles, where for any particle the force on it is proportional to the passive quantity of matter of that particle and inversely to the distance-squared ($F^g = \sum_{i \in S} F_i$

where each $F_i \propto \frac{A_i^-}{d_i^2}$). We can apply (3) to each F_i to infer that each such force is also proportional to the active quantity of matter of that particle and inversely to the distance-squared. Recall that the analysis of §23 underlying inference (3) represents the body as of negligible size (a 'particle'). So each of the denominators (d_i^2) in this inferred aggregate of attractions would be well-defined and computed from B to the situated particle i. The aggregate takes this form: $\frac{A_1^+}{d_1^2}, \frac{A_2^+}{d_2^2}, ..., \frac{A_n^+}{d_n^2}$. Can Newton then conclude that this inferred aggregate of attractions would constitute a sum equal to an inverse-square total attraction to A?

This line of reasoning does circumvent the accelerative problem because it does not use the passive additivity of A to infer the active additivity of B. Instead, it uses the passive additivity of A to infer the active additivity of A. But this reasoning runs into the center of action problem. As Book I, Section 13 demonstrates, some non-spherical bodies consisting of inverse-square attracting particles have no centers of action. Each individual inverse-square attraction in the inferred aggregate is well-defined because the distance is computed between B and that particle. But this does not yet entitle us to infer a linear combination of these attractions yields a total inverse-square force to a center of action associated with A.

Thus, at the time of Newton's initial argument for universal gravity, he does not seem to have non-prospective evidence for the additivity of active quantity of matter. This is in contrast to passive additivity, which is established in §§18-19 with mathematical and empirical arguments. Reflection on the conditions required to attribute passive additivity, however, shows that an inference from passive to active additivity is not straightforward.

4 Conclusion

Newton is committed to establishing the additivity of mass as a premise in his deduction of universal, particle-to-particle gravity. There are, however, two kinds of mass which enter into the expression for the force of gravity. I have distinguished these kinds of mass by the measurement procedures available to Newton. A double-pan balance can measure what I call 'passive quantity of matter', while acceleration field strength (indicated by Keplerian orbital periods) can measure what I call 'active quantity of matter'.

These procedures also constrain the available evidence for the additivity of each quantity so measured. Measurement on a balance assumes a uniform field. So Newton's evidence for passive additivity (two-pendulum experiment, principles from statics, I.65) assumes this, too. The assumption amounts to a condition that the gravitational source is effectively a point at a great distance. But active additivity in full generality entails that the source has parts which attract independently. Hence establishing active additivity on the basis of passive additivity is not straightforward.

I have suggested these conceptual and evidential issues help clarify Newton's strategy in his initial deduction of universal gravity in §§18-26 of *Liber secundus* (early 1685). Newton here provides at least three analyses of Law III grounded largely on analogies to magnetic attractions. I have shown these analyses attempt to underwrite inferences from passive to active additivity, and have given reasons to think that the attempt is not entirely successful. The insufficiency of these analyses might partly explain why they are eliminated from the final deduction of universal gravity in III.1-7 of *Principia* (1687, first ed.).

Newton nevertheless frames experimental and mathematical programs which historically do come to provide support for active additivity. Investigations into the figure of the earth and resulting variations in surface gravity, in particular, come to provide empirical support for active additivity. And in broaching the question of whether irregular bodies have centers of action (I.85-93), Newton also anticipates questions later addressed in potential theory.

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