

# The Earth and the Aleph\*

Josh Parsons - 11 Nov 2013

## 1. The story of the Aleph

In his (1993), David Sanford describes a situation that he believes to be coherent, but inconsistent with “standard” formal mereologies. In Jorge Luis Borges' story “The Aleph”<sup>1</sup>, an object, the Aleph, is described that is inside Beatriz Viterbo's house, and has in turn, inside it, the whole universe. The Aleph is on Earth – is part of the Earth – but the Earth is also part of it. This is inconsistent, Sanford says, with the principle of anti-symmetry: that if  $x$  is part of  $y$ , and  $y$  is part of  $x$ , then  $x=y$ .

I would like to refine Sanford's example a little. Sanford simply quotes Borges, and Borges' story focuses on the fantastic nature of the Aleph. But the more we think of the Aleph as fantastic and weird, and unlike any other object in the universe, the easier it is to think of the scenario as one in which the Aleph “contains” the universe in some non-mereological sense; or in which the Aleph only *appears* to have the Earth as a proper part. The best and most difficult version of a Sanford-style counter-example would go like this: Suppose that, prior to the events of Borges' story, Beatriz Viterbo discovered the Aleph for herself. Looking at her cellar staircase through a powerful electron microscope, she finds a tiny apparent replica of the Earth among the microscopic constituents of the 19th step. Looking closer, she sees an even tinier apparent replica of herself in her house, looking into a tiny electron microscope. Zooming out, she sees that the tiny Earth is surrounded by a Solar System, and indeed by a tiny apparent replica of the entire known universe, all forming a microscopic part of the 19th step. Looking into the sky with a powerful radio telescope, she sees that what she thought was the universe is in fact a part of a gigantic intergalactic replica of her cellar, as seen from the 19th step. Being a parsimonious reasoner, she concludes that the apparent replicas are one and the same thing – that the Earth she found in her staircase is the Earth she lives on; that the gigantic step in the sky is the one in her house; and she is right (says the story).<sup>2</sup>

Let us call the tiny apparent replica of the known universe the Aleph. On this version of the story, the Aleph *is* the known universe, and contains the Earth in just the sense that the known universe contains the Earth; the Earth is a proper part of it. The Aleph is itself a proper part of the 19th step, and is so in just the same sense that other microscopic constituents of the step are proper parts of it too. By “proper part” here, I mean part in a sense that excludes both numerical identity and material coincidence. The leg of a statue is a proper part of it, but the statue is not a proper part of itself, and nor is the clay it is made of. Proper parthood of an object goes with being, as I put it above, “strictly within” an object, and with occupying a proper subregion of the space that object occupies.<sup>3</sup> It is apparently true in the story that the Aleph is both a proper part of the Earth, and that the Earth is a proper part of it.

The principle of anti-symmetry that Sanford mentions is in fact frequently denied in formal mereologies that wish to accommodate pairs of distinct objects that materially coincide, or “are made of the same stuff” – a statue and the clay it is made of, persons and their bodies. So there is no difficulty about devising a formal mereology that does that. But the Earth and the Aleph are not like

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1 Originally published in Spanish in 1949, but widely translated and republished, for example in (Borges 2000).

2 The alternative, less parsimonious, hypothesis open to Viterbo is that she is part of an unbounded infinite series of universes, nested inside each other like Russian dolls. Another reason, besides parsimony, to reject this hypothesis is that it is inconsistent with unrestricted mereological composition. (Bohn 2009)

3 None of this is intended in any way as a definition of “proper part”, only as a gloss to help you get the idea.

coincident but distinct objects: they occupy different places in the mereological hierarchy; the Earth is strictly within the Aleph, and the Aleph strictly within the Earth; they are not “right on top of each other” in the way that coincident entities are supposed to be. The story puts pressure not just on the anti-symmetry of parthood, but on a more indubitable principle, the anti-symmetry of proper part: that if  $x$  is proper part of  $y$ , then  $y$  is not a proper part of  $x$ . Sanford is right to think that this is a radical departure from the “official view” of mereology.

On my version of the story, the Aleph is just one member of a cycle of proper parthood that also includes the Milky Way, the Solar System, the Earth, Viterbo's house, and so on. The Aleph, in my version of the story, was the “tiny apparent replica” of the known universe found by Viterbo herself. But what counts as the “known universe” depends on who does the knowing. If someone who only knows of the Solar System discovered the Aleph, they would be amazed to find a tiny apparent replica of the Solar System among the microscopic constituents of the 19th step. If an intelligent ant that never left the cellar discovered the Aleph, it would be amazed to find a tiny apparent replica of the cellar. If we take seriously the idea that everything there is a part of the 19th cellar step of Viterbo's house, then there is nothing special about the aggregate of galaxies that we think of as the “known universe” – no sense in which it and it alone is the “biggest thing there is”. Anything that has the Aleph as a part can be consistently regarded as “the universe”.

A consequence of this is that there cannot be two Alephs; that is, it would not be coherent for the story to continue “and Viterbo found another tiny replica of the universe in the 18th step, and had just the same sort of evidence that it was the whole universe too”. When Viterbo looked into the sky with the radio telescope and receives evidence that she is part of the Aleph, she saw an intergalactic apparent replica of her cellar as if from the point of view of the 19th cellar step. That tells her that the Aleph is on the 19th step and not (e.g.) the 18th. To find two Alephs in the same way, she would have to see something inconsistent through the radio telescope.<sup>4</sup>

## 2. Sanford's challenge

What should mereologists make of the story of the Aleph? I think that there are four general approaches that could be taken.

First, there is what is perhaps Sanford's own view – that all formal mereology rests on a mistake; that there are no general conceptual truths of the mereological relation that can be stated purely in the language of “part-whole” together with first-order logic. If we are not to accept that conclusion, then what can we say about the story? – I call this question “Sanford's challenge”.

Second, there is the *dismissive reply* to Sanford's conference given by van Inwagen – that the story of the Aleph contains conceptual falsehoods, but gives the impression of coherence because “the author of a fantasy has the power to in confer 'truth the story' on known conceptual falsehoods” (van Inwagen 1993, 229). Other mereologists have been surprisingly quick to accept this view.<sup>5</sup> Without wanting to get into the philosophy of literature, it's not obvious to me that van Inwagen is right in his diagnosis. It does not seem possible for an author to make it true in a fiction that there are married bachelors. (On the other hand, Borges was a literary genius, and if anyone could do this, he could).

More importantly, my reconstruction of the story presents a more powerful argument than Sanford did, and one that cannot so easily be rebutted. I am not just asking you to consult your intuitions to decide that a work of literature is internally consistent. The story also describes a way in which a clearly consistent set of evidence could favour a scenario in which there is a mereological

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4 Though, interestingly, what Borges describes seeing in the Aleph is inconsistent in the same way that Viterbo's radio telescope readout would have to be – he sees the universe from every point of view at once. (Note that this is different from what Viterbo sees through the microscope in my version of the story).

5 See for example Casati and Varzi (1999, 35–36).

circularity.<sup>6</sup> But scenarios cannot be favoured by evidence unless they are consistent. In any case, even if both these objections to the dismissive reply fail, it would be nice to explore other possible replies to Sanford's argument.

Third, there are *deflationary replies* that explain away the apparent cycle of proper part relations in the story. For example, perhaps the right way to think of the Aleph is as a wormhole in space that leads from Viterbo's house to a point on the edge of the known universe. Looking through the Aleph, you see the known universe as if from a great distance; but the known universe is not part of the Aleph. This way of thinking about what's going on fits Borges' original story better than the dismissive reply above, I think.<sup>7</sup> But it does not fit my version of the story very well. In my version of the story, it would be arbitrary to single out the Aleph for this kind of explaining away. For the reasons I described above, anything that has the Aleph as a part can be regarded as “the known universe” and discovered to have a tiny apparent replica of itself inside it. It's no less plausible that the Aleph has the Earth as a part, than it is that the Solar System does.

Fourth, there are *straight replies* that attempt to concoct a formal mereology among the models of which can be found something that seems to match the story of the Aleph (as well as something that seems to match the actual world). It is to devising a straight reply that I now turn.

### 3. Straight replies: Cotnoir and Bacon

I want to discuss two straight replies to Sanford's challenge: the first is given by Cotnoir and Bacon in their (2012); the second is original to this paper. Both Cotnoir and Bacon's and my approach can be seen as starting from the idea that the story of the Aleph is a counter-example to the anti-symmetry of proper part. There is, however, an immediate problem with this move that the two replies deal with in very different ways.

The problem is this: the proper part relation is commonly supposed to be transitive as well as anti-symmetric – if  $x$  is a proper part of  $y$  and  $y$  is a proper part of  $z$ , then  $x$  is a proper part of  $z$ . If there are counter-examples to anti-symmetry – cases of  $x$  and  $y$  such that  $x$  is a proper part of  $y$  and  $y$  a proper part of  $x$  – then it follows that  $x$  (and  $y$  for that matter) is a proper part of itself. This however runs contrary to what I said earlier about the concept of “proper part”. Proper part, I said, means “part in a sense that excludes... numerical identity”; by that I meant that it should be a conceptual truth that nothing is a proper part of itself. To put this point more abstractly, the proper part relation is commonly supposed to be a strict ordering: transitive, anti-symmetric, and irreflexive. We are trying to relax the requirement of anti-symmetry while keeping as much as else as we can. But every transitive irreflexive relation is, perforce, anti-symmetric; so to deny that the proper part relation is anti-symmetric, we must either deny that it is transitive or that it is irreflexive.

Cotnoir and Bacon make the latter move. On their view, proper part is transitive, but neither anti-symmetric nor irreflexive. Their paper presents an ingenious generalisation of classical mereology called Non-wellfounded Mereology (NWM) which allows for this, and which they apply to a number of philosophical problems other than Sanford's challenge. I won't present the details of their system here – the following objections to it depend only on what I've said about it so far. It seems to me this reply to Sanford's challenge suffers from three problems, which I discuss in order of increasing severity.

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6 My argument here is analogous to Shoemaker (1969), which argues that since a consistent set of evidence could favour the existence of a global freeze – a period of time during which nothing changed – the hypothesis that a global freeze occurred must be metaphysically possible (and a fortiori consistent).

7 Actually, neither fit it that well. But then, some aspects of Borges' story don't really square that well with the idea that the Aleph even appears to have everything as a part. As mentioned above, when Borges (the character) looks into the Aleph he sees possible point of view. But that's not either like seeing the universe as if it were a part of the Aleph, nor like seeing the universe as if from a great distance.

*First problem: losing our grip on the concept of proper part.* It's all very well to say that the proper part relation is not necessarily irreflexive, but do we have a clear grasp of what that means? What is the difference between those things that are proper parts of themselves and those that are not? Is there any way of explaining what "proper part" means, in natural language terms, without assuming (as I did in my explanation) that nothing is ever a proper part of itself? These are rhetorical questions, but I'm not sure that I know their answers. At least more work needs to be done here.

The second and third problems are more serious.

*Second problem: distinguishing circularity from coincidence.* Cotnoir and Bacon's theory is intended to allow for two individuals to materially coincide (e.g. a statue and the clay it is made of). They do this by saying that for two individuals to coincide is for those individuals to have at least one proper part and have all and only the same proper parts (and indeed this is one standard definition of coincidence). However, it also follows on their theory that two individuals that coincide are proper parts of each other. There is thus no mereological difference between a set of things that are all part of a cycle of proper parthood (e.g. in the story of the Aleph: the known universe, the Earth, and Beatriz Viterbo's house) and a set of things that materially coincide (e.g. the statue, the lump of clay). In both cases, the things in the set are all mereologically indistinguishable from each other. But intuitively (insofar as one can have intuitions here) the mereological relationship between the Earth and the Aleph is very different from the relationship between the statue and the clay.

*Third problem: the "betweenness" problem.* The mereological relation has a "betweenness" structure that, in the normal case, is implicitly represented in the part-whole relation. The Solar System is "mereologically between" the Milky Way galaxy and the Earth – moving from whole to part down the mereological hierarchy, you pass through the Solar System on the way between the Milky Way and the Earth. If we can assume that the proper part relation is anti-symmetric, then mereological betweenness can be analysed in terms of proper part:  $x$  is between  $y$  and  $z$  iff  $x$  is a proper part of  $y$  and  $z$  is a proper part of  $x$ . (You may wish to check that this works for the Solar system, the Milky Way and the Earth.) Insofar as I can make sense of the story, I imagine that these betweenness facts obtain between some but not other triples of objects of which the Aleph is a part. For example, Viterbo's house is between the Earth and the Aleph; and, because the Aleph is the known universe, the Milky Way is between the Aleph and the Earth. But Viterbo's house is *not* between the Aleph and the Earth; and the Milky Way is *not* between the Earth and the Aleph. If you move from whole to part, starting from the Aleph, you get to the Earth *before* you get to Viterbo's house, and not vice versa. If you move from whole to part, starting from the Earth, you get to the Aleph *before* the Milky Way, and not vice versa. These facts of mereological betweenness seem to me to be essential to understanding the story. Without them I cannot imagine the Aleph being any different from the Earth; I am forced to imagine them coinciding. So there are differences of mereological "betweenness" between the Earth and the Aleph and Viterbo's house, and the Milky Way; but these differences are not represented by any facts about what is a proper part of what, on the assumption that proper part is transitive, and the Earth and the Aleph are proper parts of each other.

What should we make of these problems? I can see two likely morals. First, we should deny that the proper part relation is transitive, or else pay more attention to a closely related relation that is not transitive; that way we can continue to affirm that it is irreflexive (solving the first problem) and hopefully solve the second and third problems as well. Second, we would do well to pay attention to the ternary relation I have called "betweenness", which has hitherto been neglected in treatments of formal mereology.

## 4. Straight replies: resolutionism

My reply is as follows. Notice each of the things which have the Aleph as a part is equally a candidate to be regarded as “the universe”. We can think of this as a kind of metaphysical indeterminacy – the world “has not decided” which of those things is the totality of all there is. A good way of representing an indeterminate state of affairs is by representing all the determinate states of affairs that that it is indeterminate between, and we can apply this technique to the case of the Aleph.

Think about all the models of classical mereology that resemble the mereological structure of the story of the Aleph, but in which different “candidates” really are the universe – the fusion of all that exists – in each model. In one model, for example, the Aleph is the universe, and this thus not a part of Viterbo's cellar stairs (that model perhaps resembles the mereological structure of the actual world). In another model, the Milky Way is the universe, and is thus not part of the Aleph (which is part of Viterbo's cellar stairs). In yet another model, Viterbo's house is the universe, and the Aleph (with the Milky Way as a part of it) is part of the house. None of these models of classical mereology individually represents the mereological structure presented in the story, but collectively they do. I will call each of these models a *resolution* – each resolution resolves any mereological circularity by representing some member of each mereological cycle as maximal within its cycle (i.e. having all other members of its cycle as proper parts).

So the mereological structure of the story of the Aleph consists in a set of resolutions – one resolution for each individual that has the Aleph as a part, and with each such individual having all other individuals as parts *in some resolution*. The mereological structure of the actual world – or any other world that does not have any mereological cycles in it – an acyclical world, for short – consists in a set of resolutions with exactly one member.

When we say that  $x$  is a proper part of  $y$  (without reference to any resolution) we may take that to mean  $x$  is proper part of  $y$  in some resolution. (And similarly,  $x$  is part of  $y$  can be taken to mean  $x$  is part of  $y$  in some resolution). Since each resolution is a model of classical mereology, and since an acyclical world has exactly one resolution, then if the world is acyclical, then the proper part relation satisfies the axioms of classical mereology (in particular, it is irreflexive, transitive, and anti-symmetric).

Now consider what we can say about proper part in the story of the Aleph (and thus in general). The proper part relation must still be irreflexive: because every resolution is a model of classical mereology, on no resolution is anything a proper part of itself. However, the proper part relation is not antisymmetric, since there can be some  $x$  and  $y$  such that  $x$  is a proper part of  $y$  on one resolution, and  $y$  a proper part of  $x$  on another. In particular, there will be some resolution in which everything is a part of the Earth (and in particular, in which the Aleph is a proper part of the Earth) and another resolution on which everything is part of the Aleph (and, in particular, in which the Earth is a proper part of the Aleph). So the Earth and the Aleph are mutual proper parts – which is a straightforward way of saying that the Earth and the Aleph are involved in a mereological cycle of the kind described in the story. Finally, the proper part relation is not transitive: if  $x$  is a proper part of  $y$  on one resolution, and  $y$  a proper part of  $z$  on another, there is no guarantee that  $x$  is a proper part of  $z$  on either or any resolution. In particular, though the Aleph is a proper part of the Earth on some resolution, and the Earth a proper part of the Aleph on another, it does not follow that there is any resolution on which the Aleph is a proper part of itself (indeed, for the reasons given above, there cannot be any such resolution).

Now consider the betweenness problem. We can adequately define “betweenness” in the following way:  $y$  is mereologically “between”  $x$  and  $z$  iff there is some resolution  $r$  such that  $y$  is a proper part of  $x$  in  $r$ , and  $z$  is a proper part of  $y$  in  $r$ . This avoids the betweenness problem, because, in the story of the Aleph, we may suppose, there is no single resolution in which the Milky Way is a proper part

of the Earth and the Aleph a proper part of the Milky Way. So the Milky Way is not “between” the Earth and the Aleph. On the other hand, there is a resolution on which the Milky Way is a proper part of the Aleph and the Earth a proper part of the Milky Way; and another resolution on which the Earth is a proper part of the Milky Way, and the Aleph a proper part of the Earth – so, the Milky Way is “between” the Aleph and the Earth, and the Earth is “between” the Milky Way and the Aleph.

## 5. Conclusion

Back to the big picture: now that we've seen a straight reply to Sanford's challenge, does it seem good enough? Should we say that the coherence of the story of the Aleph teaches us that the part-whole relation is strictly speaking, not transitive? Or should we take the modus tollens, and say that the ugliness of the best straight reply available is reason to deny that the story is coherent?

I find it hard to decide; but I am not disturbed by this. The purpose of formal mereology (as I understand it – and I think that this purpose is implicit in Sanford's critique of formal mereology as well) is to separate certain of the conceptual truths concerning the mereological relations (namely those that can be formalised in a purely mereological language) from the substantive truths. This project is of utility to philosophers because it enables philosophers who disagree about substantive matters to at least agree about what the consequences of each others theories are. It would be nice if we could devise a universal mereology whose theorems would all be universally agreed to be conceptual truths. Unfortunately, that is impossible – not because there are no such truths but because of the requirement for universal agreement. Philosophers may disagree about what is conceptually true, just as they disagree about what is substantively so. We may also find ourselves uncertain about what is conceptually true. In fact, what's fun and philosophically rich about the story of the Aleph and many of Borges' other works is precisely that they create that kind of uncertainty.

So, I remain undecided on whether the story of the Aleph is coherent. But at least, I am convinced, it poses no threat to the project of doing formal mereology in general.

## Appendix: a formal treatment

In this section I present a first-order theory that matches the informal discussion in the text. It will be helpful to begin by sketching an axiomatisation of classical mereology in order to make the relationship between the theory I describe, and classical mereology clearer.

The primitive of our axiomatisation of classical mereology shall be  $<$  “is part of”, and there shall be the following defined predicates and operators:

$x \ll y \equiv x < y \wedge \neg y < x$	“x is a proper part of y”
$x \circ y \equiv (\exists z)(z < x \wedge z < y)$	“x overlaps y”
$(\text{Fu } y)(\phi[y]) \ x \equiv (\forall z)(x \circ z \leftrightarrow (\exists y)(\phi[y] \wedge y \circ z))$	“x is a fusion of the $\phi$ s”

The axioms of classical mereology shall be as follows:

(Refl)	$x < x$
(Asym)	$x < y \wedge y < x \rightarrow x = y$
(Trans)	$x < y \wedge y < z \rightarrow x < z$
(SSP)	$\neg x < y \rightarrow (\exists z)(z < x \wedge \neg y \circ z)$
(GSP)	$(\exists x)(\phi[x]) \rightarrow (\exists y)((\text{Fu } x)(\phi[x])y)$

SSP here stands for “strong supplementation principle” and GSP for “general sum principle”. The significance of these principles is outside the scope of this paper; for a discussion see (Simons 1987,

25–37) or (Casati and Varzi 1999, 38–47). Note that the definition of the operator Fu and the axiom GSP slightly stretch the usual syntax of first-order logic: the expression “(Fu y)(φ[y])” is to be understood as a complex predicate (with y standing in for an otherwise unbound variable, and φ[y] being an open sentence containing a free occurrence of y). This is normal in first-order presentations of classical mereology: see the citation to Simons above.

Though I've made < (“part”) primitive, and ≪ (“proper part”) defined (this is formally cleaner, as ≪ does not play any role in the axioms) it would be equally possible to make ≪ primitive and < defined, as indeed Simons does; that might better fit the text of this paper, in which most attention is paid to the concept of “proper part”.

Above, I said that the mereological structure of the world consists in a non-empty set of resolutions, where each resolution is a model of classical mereology. I will now show how to represent that structure using a first-order theory. The primitive of this theory shall be the ternary relation of “- is part of - in the resolution -”. We write this as follows: “x<<sub>r</sub>y” is to be read “x is part of y in r”. The theory will therefore be a form of *ternary mereology*; compare for example (Thomson 1983).

It will be handy to speak of individuals overlapping in a resolution, or some individual being a fusions of some individuals in a resolution, so let us define those in a parallel way to the way we did above:

$$\begin{aligned} x \ll_r y &\equiv x <_r y \wedge \neg y <_r x && \text{“x is a proper part of y in r”} \\ x \circ_r y &\equiv (\exists z)(z <_r x \wedge z <_r y) && \text{“x overlaps y in r”} \\ (Fu_r y)(\phi[y]) x &\equiv (\forall z)(x \circ_r z \leftrightarrow (\exists y)(\phi[y] \wedge y \circ_r z)) && \text{“x is a fusion of the } \phi\text{s in r”} \end{aligned}$$

The ternary relation of “betweenness” discussed in the text can also be defined. Since we are now considering a ternary mereology, this becomes a 4-ary relation:

$$y \text{ is between } x \text{ and } z \text{ in the resolution } r \equiv x \ll_r y \wedge y \ll_r z$$

It will also be useful to have the monadic predicate “is a resolution”, which I will write R. Since each thing is part of itself (in every resolution), and there cannot be an empty domain in first-order logic, it's reasonable to define R as follows: r is a resolution iff there is some x such that x is part of x in r:

$$Rr \equiv (\exists x)(x <_r x) \quad \text{“r is a resolution”}$$

Each resolution is to be a model of classical mereology; that is for each resolution r, the binary relation <<sub>r</sub> should satisfy the axioms of classical mereology given above; that is, each of the following should be theorems of our ternary mereology. Let us posit them as axioms:

$$\begin{aligned} (\text{Refl}) \quad & Rr \rightarrow x <_r x \\ (\text{Asym}) \quad & Rr \rightarrow (x <_r y \wedge y <_r x \rightarrow x=y) \\ (\text{Trans}) \quad & Rr \rightarrow (x <_r y \wedge y <_r z \rightarrow x <_r z) \\ (\text{SSP}) \quad & Rr \rightarrow (\neg x <_r y \rightarrow (\exists z)(z <_r x \wedge \neg y \circ_r z)) \\ (\text{GSP}) \quad & Rr \rightarrow ((\exists x)(\phi[x]) \rightarrow (\exists y)((Fu_r x)(\phi[x])y)) \end{aligned}$$

What exactly is a resolution? Since, for our intended application of this theory, each resolution represents a different individual as the universe, we can identify resolutions with the individuals that are the fusion of everything *in that resolution*. Since the axioms above do not guarantee this, let's have an additional axiom that does:

$$(U) \quad Rr \rightarrow (Fu_r x)(x=x)r$$

Axiom U is not really necessary in order to obtain desirable consequences; but without it there are ways in which models of the theory may differ which do not correspond to any difference that is significant in the philosophical interpretation of those models. Take, for example, a model that represents the mereological structure of the story of the Aleph, on which U is satisfied. There are a very large number of models resulting from permuting the extension of the ternary relation in that

model with regard to the resolution argument only (e.g. one on which the Earth plays the role of the resolution in which the Aleph is the universe, and vice versa). These models really do differ as regards what is part of what in what resolution, but do not differ in any way that matters. It would be best if only one permutation of that kind were allowed, and U is one way to do that.

Also, we need to guarantee that there is at least one resolution (“the mereological structure of the world consists in a *non-empty* set of resolutions”):

(N)  $(\exists x)(Rx)$

We can now define *binary* part and proper part in terms of their *ternary* namesakes, and “betweenness” in terms of ternary parthood in the way suggested in the text:

$x < y \equiv (\exists r)(x <_r y)$

$x \ll y \equiv (\exists r)(x \ll_r y)$

$y$  is between  $x$  and  $z \equiv (\exists r)(y$  is between  $x$  and  $z$  in  $r) \equiv (\exists r)(x \ll_r y \wedge y \ll_r z)$

Binary part and proper part are reflexive and irreflexive respectively, neither are anti-symmetric or transitive. The need for N is seen in ensuring that binary part is reflexive.

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