

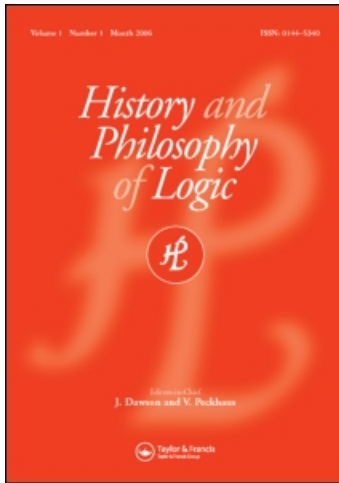
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Anti-psychologism about Necessity: Friedrich Albert Lange on Objective Inference

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In the nineteenth century, the separation of naturalist or psychological accounts of validity from normative validity came into question. In his 1877 *Logical Studies* (*Logische Studien*), Friedrich Albert Lange argues that the basis for necessary inference is demonstration, which takes place by spatially delimiting the extension of concepts using imagined or physical diagrams. These diagrams are signs or indications of concepts' extension, but do not represent their content. Only the inference as a whole captures the objective content of the proof. Thus, Lange argues, the necessity of an inference is independent of psychological accounts of how we grasp the content of a proposition.

1. Introduction

Friedrich Albert Lange (1828–1875) studied a wide range of subjects over his lifetime: pedagogy and its history, the history of the teaching of gymnastics, psychology, and moral statistics. Lange attended the university in Bonn from 1848 to 1851 to study theology, philosophy, and philology. Afterwards, he taught in a gymnasium in Köln (until 1855) and then taught as Privatdozent for 2 years at the Bonner Universität. There, in the summer of 1857, he held a seminar on the history of materialism that attracted 19 participants.¹ That was the beginning of a book project, the *History of Materialism and Critique of Its Present Significance* (*Geschichte des Materialismus und Kritik seiner Bedeutung in der Gegenwart*). The *Geschichte* appeared in 1865 and went through multiple editions, though only two in Lange's lifetime.²

Lange's final work, *Logical Studies: A Contribution to the New Foundation of Formal Logic and Epistemology*, appeared posthumously in 1877.³ As the subtitle suggests, Lange's work is not intended solely as a logical text, but also as an analysis of the contributions of logic to epistemology more generally. Lange's aim is to give an account of the material facts that allows for the contribution of logic to history and to science, without positing any theoretical entity beyond the target phenomena. Lange is an opponent of conceptual or categorical realism, of Platonism, and of the Kantian appeal to pure intuitions of space and time.⁴ Lange has an intriguing position in the history of philosophy, as a critic of the neo-Aristotelian tendency in mid-nineteenth-century philosophy and logic as represented by

¹ See von Kloeden 1992 and Köhnke 1991.

² These were edited and introduced by Hermann Cohen. While there is a clear historical influence, Lange should not be considered a Marburg neo-Kantian (see Ollig 1979, 2.1.2).

³ *Logische Studien: Ein Beitrag zur Neubegründung der Formalen Logik und der Erkenntnistheorie*.

⁴ Lange also had a profound influence on the birth of neo-Kantianism through his protégé Hermann Cohen, the founder of the Marburg School – Lange's chosen successor in Marburg after his death and editor of the *Logical Studies*. Cohen provides a few tantalizing facts in his introduction to the original 1877 edition. The manuscript was completed 3 weeks before Lange's death in 1875, but represents only half of Lange's original project. Furthermore, Lange was working on the revised second edition of the *History of Materialism* at the same time that he was writing the *Logical Studies*. As Cohen points out, there are compelling points of contact between the *History of Materialism* and the *Logical Studies*. Lange's argument in the *History of Materialism* was that we have access only to those facts available in empirical experience, that is, only observed phenomena. Nonetheless,

Adolf Trendelenburg and Friedrich Überweg and also as a critic of the epistemology of the founder of empirical psychology, Johann Friedrich Herbart. This essay will seek not only to examine Lange's argument that logical inference can serve as a grounding for epistemology, but also to evaluate his significant place in the history of the changing relationship between logic, epistemology, and science in nineteenth-century Germany.

The outline of Lange's argument in the *Logical Studies* is as follows. He gives a general definition of apodictic, or formally certain, knowledge. He concludes that all apodictic reasoning is the same in kind and, thus, that the formal truths of logic and mathematics rest on the same kind of reasoning. This conclusion leads Lange to abandon Kant's distinction between the analytic judgments of logic and arithmetic and the synthetic judgments of geometry and natural science. After a discussion of the Aristotelian logic and how it can be revised to apply to a science of extension rather than to content, Lange concludes his first chapter with the claim that even the law of non-contradiction rests on inner spatial observation. In the chapters that follow, Lange derives the major forms of the syllogism using Euler–Gergonne diagrams.⁵ As Venn remarks later, Lange achieved this result before Venn himself did so using Venn diagrams, which are more restricted than Euler diagrams.⁶

Lange concludes his book with an intriguing argument. The justification for the derivation of necessary relations between facts does not rest on *a priori* conceptual resources or universal laws, but rather, the security of the logical method rests on the norms governing spatial reasoning and spatial observation.⁷

2. Epistemology and science

In the first chapter of the *Logical Studies*, Lange presents himself as responding to the main proponents of *Erkenntnisstheorie* in Germany, especially Johann Friedrich Herbart and Adolf Trendelenburg. Herbart, Kant's successor in Königsberg, was one of the founders of modern experimental psychology, along with Gustav Theodor Fechner and Wilhelm Wundt.⁸ Herbart set himself the problem of constructing a mathematical rational psychology that would serve as a foundation for experiment, but also for epistemology.⁹ His rational and empirical psychology developed in tandem, as is in evidence in his *Psychology Textbook* (*Lehrbuch der Psychologie*), the first part of which is on rational psychology and the second part on empirical psychology.¹⁰

Trendelenburg, a professor at the then-Friedrich-Wilhelms-Universität in Berlin, published his *Logical Investigations* (*Logische Untersuchungen*) in 1840.¹¹ The *Logical Investigations* went through two more revised editions, in 1862 and 1870, and was for decades Trendelenburg's only systematic text, the work that was consulted by philosophers interested in his philosophy (Dilthey, Dühring, Riehl, Überweg, Vaihinger, and Cohen, among them).

The *Erkenntnisstheorie* movement took itself to be responding to Kant, but also, and in Kant's spirit, to respond to the best of the then-contemporary science and logic. In fact, as is

any materialist history or theory must account for the fact that the observed facts must fit into some larger explanation that goes beyond physical description.

⁵ Ernst Schröder traces the first use of such spatial diagrams to Euler's *Letters to a German Princess* (Euler 1847 [1768–1772]), but both Lange and Venn point out that they had been in use even earlier, by Juan Luis Vives, for instance (Lange 1877, §2, p. 30; Schröder 1890, §3, p. 155). Hamilton 1860, Lecture XIV, p. 180 traces them to a German source, *Weise 1691*, which Hamilton cites as being published posthumously in 1712.

⁶ Venn 1881, 17n.

⁷ Cf. Lange 1887, p. 149. I am grateful to an anonymous reviewer for reference to this passage.

⁸ See Boring 1950, pp. 253–254. Also, see Ribot 1886 [1879].

⁹ For a detailed discussion of Herbart's empirical psychology, see Hatfield 1991, 117ff.

¹⁰ Herbart 1850.

¹¹ For an appreciation of Trendelenburg's logic and epistemology, see Peckhaus 2006, 100ff.

becoming better known, this movement was involved in significant developments in logic in the nineteenth century. As Volker Peckhaus points out,

Application of the diagrammatic methods of the syllogism proposed, e.g. by the 18th century mathematicians and philosophers Leonard Euler, Gottfried Ploucquet, and Johann Heinrich Lambert, presupposed quantification of the predicate. The German psychologistic logician Friedrich Eduard Beneke (1798–1854) suggested quantifying the predicate in his books on logic,¹² the latter of which he sent to Hamilton.¹³

Lange was well aware of Beneke's work and cites him in the *History of Materialism*, and, of course, Lange's use of diagrammatic methods to derive the major forms of the syllogism is in the same tradition.

However, Lange engages critically with *Erkenntnisstheorie* on fundamental and significant logical and epistemological questions. First, Lange rejects Herbart's claim that the empirical laws of synthesis, or combination, of representations can be the matter of epistemology. Second, however, Lange rejects Trendelenburg's argument that logic is linked with the analysis of being. Lange thus places himself between Herbart's direct realism about representation and Trendelenburg's neo-Aristotelianism, aiming to provide a third option.

In the introductory discussion on *Erkenntnisstheorie*, Lange responds to Herbart and to Trendelenburg. Lange argues against Herbart that an empirical account of mental synthesis can never capture the normative aspect of the laws of thought. Our representations do not have absolute *objective* position. We experience bare appearances or phenomena, which are constituted by their place in the space of our consciousness. Lange allows that synthesis of representations can be a part of the process of our coming to be aware of them, but he argues that we must abstract away from that synthesis in formal reasoning.

Here, Lange's response to Herbart follows Trendelenburg's. But Lange responds critically to Trendelenburg as well. While Trendelenburg suggests that logic and the sciences have a common normative aspect, Lange points out that this is in tension with an Aristotelian analysis of the categories. If categorial analysis can reach out to being, then the methods of logic and of the empirical sciences in determining necessity cannot be the same – empirical science does not analyze being through pure methods of thought. But Lange objects to the notion that a scientist must study metaphysics, or know the laws of logic, to be able to construct necessary inferences:

According to the metaphysicians' account, Faraday has no knowledge of magnetism, Meynert no knowledge of the construction of the brain, Helmholtz's doctrine of the sensations of tone or his physiological optics are not science, because they are not deduced from *φύσει προτερον*, but are built on experiments; maybe, at most, those elements that can be deduced solely and entirely from mathematical principles can be called 'science' (Lange 1877, 6).

A major project of the *Logical Studies* is to show that all the Aristotelian syllogisms go through even if we abandon the *a priori* specification of content for concepts or the *a priori* postulation of logical laws. Lange develops a response to Herbartian psychologism that is intended as an alternative to Trendelenburg's.

¹² *Syllogismorum analyticorum origines et ordinem naturalem*, Berlin: Mittler, 1839 and *System der Logik als Kunstlehre des Denkens*, 2 volumes, Berlin: F. Dümmler, 1842.

¹³ Peckhaus 1999, p. 437.

2.1. Herbart

In the *Psychology Textbook*, Herbart argues that observable variations in ‘intensity or force [*Kraft*]’ of representations can be analyzed mathematically.¹⁴ For instance, a sensation of a finger being pressed into the eyelid can vary in intensity, depending on how hard the finger is pressed. This sensation varies regularly with respect to the intensity of its stimulus. Since these attributes can be represented as a variation, Herbart argued, the qualitative properties of our representations can be evaluated mathematically. For instance, the relationship between the force of pressure on the eyelid and the intensity of sensation could be graphed as a set of values that have a determinate mathematical relationship to each other. Herbart thinks that there are principles, or laws, of the variation of response with respect to stimulus.

For Herbart, then, psychologists can construct the equivalent of a mechanical system for representation that treats psychological phenomena as objective events governed by laws. The mathematical foundations of this system will be a ‘mental statics (the mathematics of qualitatively separate ideas varying in intensity) and a mental dynamics (the mathematics of ideas varying in time and intensity)’.¹⁵ Herbart treats representations as objects in the world equivalent to physical forces and thought that they could be given a similar mathematical analysis. In fact, Herbart argues that the laws of the mind’s activity can be given, just as Newton gave the laws of external phenomena. Herbart takes this quite literally – according to him, it is, in principle, possible to give an account of how thoughts or representations attract or repel each other.

Like Herbart, Lange begins his formal analysis with the material facts present in consciousness, the facts revealed by ordinary perception but also by scientific observation and experiment.¹⁶ These facts occur in the psychological space of consciousness, the ‘sensorium’, which Lange glosses as the table of a camera obscura. These facts are the *analysanda*. They are the product of synthesis – the process of combining aspects of representations according to a rule.

However, while Lange thinks that synthesis takes place necessarily in experience, he does not think that there is an adequate empirical explanation for it, and, therefore, he argues, an empirical account of synthesis of representations cannot be the basis of an epistemology.¹⁷ Lange rejects Herbart’s direct realism, in favor of an argument that the problem of epistemology is precisely to explain the possibility of proofs of necessary or objective synthesis.

2.2. Trendelenburg

The neo-Aristotelian Trendelenburg was among Herbart’s most prominent opponents. In his *Logical Investigations* of 1840, Trendelenburg argues that Herbart’s mistake is to concentrate only on thought, not on being, and to confine his investigations to the side of thought. Significantly for Lange’s account, Trendelenburg links his analysis to Herbart’s argument about positions in space. First, Trendelenburg cites Herbart:

In sensation, absolute position is at hand, even if one is not aware of it. It must be generated in thought first, through the removal of its contradictory [i.e. relative

¹⁴ See, for example, *Boring 1950*, p. 255.

¹⁵ Cf. *Wozniak 1999*.

¹⁶ Lange’s first chapter, *Formale Logik und Erkenntnistheorie*, contains much intriguing material about Aristotle’s logic, his method of reasoning, and his metaphysics. I will not evaluate Lange’s particular claims here – for such an evaluation, see, for example, *Stewart 1878*.

¹⁷ Cf. also Lange’s response to Heinrich Czolbe in the fourth chapter of the *History of Materialism*: ‘If we call the act of transition from physical multiplicity to psychological unity a synthesis, then this synthesis remains equally inexplicable, whether it refers to the union of the numerous discrete points of a complete image, or to the mere spatially distributed conditions of the image’ (*Lange 1881* [1866], p. 213).

position]. For thought itself, torn from sensation, posits this withdrawal only tentatively and with reservations. To remove these reservations means to explain something as being [*als seiend*].¹⁸

Trendelenburg continues:

These words [...] describe nothing other than the immediate and almost forced *necessity* of the object in sensation, and that is mediated and freely generated in thought. Thus if being is described as absolute position, that expression certainly describes it as a necessity independent of thought, but acknowledged nonetheless, and contains no element whatsoever that has to do with the specific nature and characteristics of being. Here, the independence of being always is considered only with respect to human sensation and thought. This determination cannot be valid as a concept of a thing [*Sache*] that represents the law of being [*Dasein*] or the development of becoming [*des Werdens*]; for despite its absolute position it is relative through and through, and indeed it arose from the relationship to representation.¹⁹

Trendelenburg argues that Herbart's logic of sensation must be replaced with a logic of content. However, he argues that conceptual analysis cannot be carried out on its own as an independent science, and thus he rejects attempts by the logicians Twisten and Drobisch to put such a science on a firm footing.²⁰

Instead, Trendelenburg sees logic as the guiding set of norms for the production of science.

One certainly speaks of the methods peculiar to particular sciences, in the sense of different methods, [for instance] of the logic of mathematics, of natural science, or of jurisprudence. These distinct paths are forged by the one thought, which in various forms always nestles up to the object to grasp it. In the sciences the one thought is only given a push in various ways to find ever new skills, which must give up the object as if captured. But only one skill reveals itself through all these [methods], and in all of them thought is revealed to be only one thing, an entity that can be powerful with few tools. *In all these thought seeks the necessary; never does it bring contradiction with it, but, on the way, thought uses contradiction to determine the necessary.* If we want to develop a path to necessity or to come closer to the path of knowledge of the necessary and to name the degree of resemblance to necessity using rigorous methods, that is how to make the method of science into a science. And if the methods appear in the object of science, but are not given in it, but rather have their general basis in the thought that works through the objects: this works toward the task of finding its origin in the being of thought. In this way the metaphysics of each science leads to logic, to the investigation of thought which produces science.²¹

Lange is impressed by Trendelenburg's statement, emphasized above, that non-contradiction is a tool used in determining the necessary truths of science. Giving a foundation for logic no longer consists in listing a set of necessary *a priori* laws. Instead, logic is taken to be a methodology for the sciences, and the necessity of logic is not independent of its use as a tool for scientific inference.

However, Lange objects to Trendelenburg's argument that logic is a 'logic of content' and argues that the new 'logic of extension' is superior. Lange's goal in the *Logical Studies*

¹⁸ Herbart 1950, §206, cited Trendelenburg 1862, pp. 174–175.

¹⁹ Trendelenburg 1862, p. 175.

²⁰ Trendelenburg 1862, Introduction; cf. also Haaparanta 2009a, 2009b, p. 212.

²¹ Trendelenburg 1862, pp. 9–10, emphasis added.

is to construct a theory of the role of logic in knowledge that takes our access to necessity to be through the norms governing inference. Unlike Trendelenburg, Lange argues that it is possible to give such an account without appealing to the *a priori* or apodictic status of the laws governing inference. Instead, as I will show in the following section, for Lange, the norms of inference emerge from our ability to construct a given proof.

3. The logical studies

In what follows, I analyze Lange's account in the *Logical Studies*, paying particular attention to his argument that the law of non-contradiction is synthetic, and will examine responses to his view by Ernst Schröder, John Venn, and J. Stewart. I will examine Lange's justification for his argument that the apodictic should not be ranked above the assertoric. Lange argues that an apodictic major premise is not needed to construct a valid deductive inference. More generally, he rejects theories that require grasping of necessary *a priori* logical laws to grasp a necessary inference.

Lange's account in the *Logical Studies* is based on his reading of the 'new' English logic of extension, as opposed to the 'traditional' Aristotelian logic of content.

Above all we see that Aristotle's logic indeed can be described as a logic of content, in such eminent contrast to the modern-nominalist logic of extension especially at home in England (*Lange 1877*, p. 12).

In Lange's view, all formal definitions are given in terms of extension. Extension can be described either as the literal extension of a figure in space or as a class of objects contained in a general concept (*Lange 1877*, p. 145). Lange argues that the 'new logic' is a logic of extension rather than of content. In particular, the new practice of determining the extension of a concept rather than its essential characteristics allows Lange to argue that a single activity, that of determining and evaluating the extension of a concept, is fundamental to apodictic reasoning. Lange does not mean to refer only to the literal *drawing* of a diagram (a demonstration in psychological space) when he speaks of determining the extension of a concept by means of demonstrations in intuition.

Through mathematics (for Lange, both geometry and algebra) and logic, we can manipulate our descriptions of the material facts in formally regular ways. These manipulations are what Lange refers to as 'apodictic' reasoning. Lange defines apodictic reasoning as follows:

[W]e cannot allow anything to count as truly apodictic in a science that is supported by whichever 'proofs' there may be, but only those [proofs], that any person, who has understood the sense of the relevant assertions, must also truly accept and will accept; that, therefore, *over which there can be no further argument* (*Lange 1877*, p. 3, my trans.).

In a review of *Logical Studies* for *Mind*, J. A. Stewart points out the ramifications of this view:

The fact that metaphysicians are not agreed, proves that we must not look for the apodeictic in their various systems, for the apodeictic is self-evident and beyond dispute. The metaphysicians have had it so much their own way since Aristotle's time that the mere *form* of deduction has come to be identified with the apodeictic, however disputed in each system the principles may be and the conclusions derived from them. The professor of a systematic metaphysic thus elevates himself above the man of science to whom he denies the apodeictic. It is the object of

Lange in the present work to vindicate against this professorial apodeictic that of μαθηματική ακριβολογία.²²

For Lange, apodictic reasoning is reasoning that follows independently of any other substantive commitments to principles, axioms, or propositions. Lange uses his definition of apodicticity as a weapon against dogmatism: he argues that no *metaphysical* system can lay claim to an apodictic system of reasoning, whereas most mathematical theories can. The goal of Lange's formal logic is to make logic more like mathematics in the following sense: rather than appealing to a deductive and closed system of axioms, the point is to provide a grounding for a particular method of thought.

In what follows, I will set out Lange's schema of that method in four steps. The first step is his argument that all truly apodictic reasoning is the same in kind. The second step is his argument that since the truths of mathematics (always meaning both arithmetic and geometry, for Lange) and logic are the same in kind, Kant's distinction between analytic and synthetic reasoning fails. The third step is his claim that even logical results alleged to be analytic, such as the law of non-contradiction, are synthetic. That is, such results must be demonstrated or constructed.²³ The last step is the argument that rests on a key distinction between the descriptive psychological law of non-contradiction, which does not require demonstration, and the normative logical law, which does.

3.1. *The apodictic*

Lange's definition of apodicticity, at one and the same time, widens and restricts the scope of what counts as apodictic. It restricts it in the following sense: metaphysical systems of first principles are ruled out, since they require substantive *a priori* presuppositions. Lange's definition widens the extension of apodictic reasoning as well, though, because he does not restrict apodictic logical truth to analytic propositions, as Kant does. That will mean a profound revision to Kant's doctrine on the distinction between the truths of logic and mathematics and a blow to the claim that all logical relations rest on the law of non-contradiction. Lange makes this point by arguing that apodictic reasoning is all the same in kind and, thus, that there can be no principled distinction between analytic logical principles and synthetic mathematical reasoning:

It is known that Kant took *mathematical* judgments in general to be synthetic, and that to him, mathematics is the primary proof that there are judgments that are synthetic, but at the same time are connected with the consciousness of *necessity* and thus, as Kant concluded, are not empirical. This theory was new; but since then the view has been prevalent that even mathematical judgments are analytic and rest on the principle of contradiction. This view finds energetic adherents even to this day, to whom Kant himself essentially handed a weapon in the form of his view of the purely analytic nature of formal logic. In fact, it can be proven that *the nature of all necessarily valid judgments is essentially one and the same* and that it follows from this that they are either all synthetic, or all analytic, or that this distinction in the Kantian, exclusive sense is not correct at all. In fact, the latter is the case. (*Lange 1877*, p. 9)

As can be seen from the remarks on Kant, Lange is arguing here that Kant's demonstrations that mathematics can be apodictic and synthetic *a priori* did not go far enough. Lange contends that Kant should have argued further that *all* apodictic reasoning *a priori* is synthetic,

²² *Stewart 1878*, p. 112.

²³ Cf. the account given in *Thiel 1994*, pp. 110–113.

including the fundamental principles of formal logic. Lange's definition of apodicticity depends on the notion of necessary demonstration: that, as in a mathematical proof, the steps of reasoning must be such that the inference is obvious and true to anyone who has understood the presuppositions. For Lange, formal logic must follow this method, but he argues that logical demonstrations can be carried out *a priori*.

Kant's doctrine in the area of mathematics is along the right lines, Lange says. The problem comes in when Kant makes too sharp a distinction between sensibility and the understanding in his doctrine of synthesis,

for it can be shown, that 'intuition' of any object in general cannot arise without the cooperation of spontaneity. Conversely, even in the most abstract objects there is no thought without intuition. (*Lange 1877*, p. 9)

Lange puts the distinction between his own account resting on empirical intuition and Kant's resting on pure intuition as between abstract and formal thought. For Kant, pure intuition is necessary for abstract thought: for thought of an object abstracting away from everything but extension and form [*Ausdehnung und Gestalt*] (*Lange 1877*, p. 132). There are several reasons why Lange cannot accept such an account. The first is that he finds the concept of pure intuition unimaginable (*Lange 1877*, pp. 132–133). The second is his rejection of the distinction between matter and form in the *History of Materialism* (*Lange 1881* [1866], 193ff. and 224ff.). Without that difference, the distinction between empirical and pure intuition becomes cloudy as well, because we cannot separate the formal aspects of perception that remain in pure intuition from the material aspects. Any formal logic, or any formal reasoning at all, must then, on Lange's account, rest on constructions in empirical intuition with the help of the spontaneity of the understanding. Lange argues that while there can be purely formal reasoning, there can be no reasoning whatsoever that abstracts away from intuition or observation completely.

All apodictic reasoning, then, is reasoning that follows from the immediate grasp of the sense of the premises, governed by the norms that shape reasoning using the extension of concepts.

3.2. *The analytic–synthetic distinction and non-contradiction*

Lange draws the following conclusions from his abandonment of the analytic–synthetic distinction. We could argue (from the standpoint of what Lange calls the 'confirmed Aristotelian') that because Aristotle's doctrines contain apodictic reasoning, they are, therefore, the basis of formal logic (*Lange 1877*, p. 7).²⁴ The argument would extend even to Aristotle's metaphysical-teleological doctrines, his cosmology, for instance. We would be able to give such an Aristotelian theory of formal logic a foundation by appealing to Kant's twin doctrines that the principles (*Lehrsätze*) of logic are analytic and rest on the principle of non-contradiction.

Lange argues that such a strategy would not work in the nineteenth century, because it has been demonstrated that the principles of logic are not analytically true. So, we cannot base formal logic on the Aristotelian logic by appealing to what we might call the face value of logical propositions, that is, whether formally speaking they conform to the principle of non-contradiction or not. This is, broadly speaking, what Lange calls giving a logical grammar. He does not argue that such a practice is unfruitful, only that it does not capture a feature of (then-)modern formal logic that he wants to isolate from 'überlieferten', or traditional, logic. The feature is that 'the new logic' is a 'Logik des Umfangs' – a logic of the *extension* of sets or classes, rather than a logic of the *content* of concepts or categories.

²⁴ I would like to thank Cecilia Grayson for assistance with translation here.

Lange then makes his most significant argument, from this perspective: that even the principle of non-contradiction is synthetic (rests on ‘spatial intuition’) and not analytic (*Lange 1877*, p. 27). Lange argues that there is a *psychological* law of non-contradiction relevant to only what can be revealed to us in first-person experience – this principle determines the limits of a possible empirical experience. Such a principle itself does not require demonstration in spatial observation (*Lange 1877*, p. 28). The psychological law is what Lange calls a ‘Naturgesetz’ – a descriptive or natural law. It determines the limits of possible experience, but does not give prescriptions about consistency, for example. So the psychological law of non-contradiction tells us that an object cannot have contradictory properties at the same time, that a man cannot be old and young at the same time, for example. However, at the level of theory, the psychological law is not prescriptive, for Lange. He points out that it is no barrier to a possible experience that we have contradictory *explanations* of things (*Lange 1877*, p. 27). While Lange does not give this example, one could appeal to the competing theories of light, one of which explains light as emissions of streams of particles, while another explains light as transverse waves in a medium. The fact that there are competing explanations does not affect whether or not we can experience light.

The relevant contrast here is relatively intuitive. The psychological law of non-contradiction says that we cannot draw a figure that is black all over and white all over at the same time. But it is not impossible to construct inconsistent theories or explanations of the phenomena.

Here, Lange’s epistemological reasoning takes an innovative turn. He argues that the goal of a diagrammatic proof is to resolve normative conflicts about explanation using the psychological, naturalist law of non-contradiction as a tool of inference. In other words, Lange argues that it is possible to construct a diagram that resolves inconsistencies at the level of explanation, by showing that some assumption of the explanation leads to a conflict with the naturalist law of non-contradiction, that is, with the conditions for a possible experience.

Here, Lange invokes the ‘normative law’ (*Normalgesetz*) of non-contradiction. The *Normalgesetz* of non-contradiction goes beyond what we can experience to posit a formal property of the relations between our observations and our explanation:

Now if we should consider the same law as foundation of *logic*, we should take it as a *normative law* of all thought, since, as *descriptive law*, it is effective even without our awareness; for in this case, as in any other, we require all the other axioms of typical observation to persuade us. That the whole is greater than the part, that equals added to the same yield equals, we *see* and therefore we believe. Any example you like contains generality in it, because we see it in motion immediately [*sofort als beweglich sehen*], and *acquire the conviction that any thinkable alteration to the form and quantity of the observed must take place in the same way*. Similarly, in a spatial image of some sort, be it in a concrete case or in a mere linear schema, we see that I cannot affirm and deny the same of the same objects. The same image becomes typical right away, only without any image at all the formula remains empty to me, and I acquire neither the conviction of its unconditional validity, nor even merely actual insight into its sense (*Lange 1877*, pp. 28–29, emphasis added).

The source of the relevant normativity is the passage emphasized. A well-constructed logical demonstration can yield proof that the given can be manipulated formally only in a particular way and, thus, that the demonstration yields results about the limits of *cognition*, not just of experience.

In his *Lectures on the Algebra of Logic*, Ernst Schröder identifies one of the main goals of the logic of the time: '[w]e must distinguish physical-physio-psychic, "psychological" or subjective formal necessity [*Denknotwendigkeit*] from "logical" or objective formal necessity' and remarks,

In striving for our goals, we can thus take comfort in the belief that under certain knowable conditions the objective necessity that we are searching for will always also become subjective. Namely, if we are talking about the unification of unmediated contradictions, then both formal necessities will always occur together. In this context, F.A. Lange remarks very aptly, 'The law of non-contradiction is the point at which the *natural law* of thought comes into contact with the *normative law*'.²⁵

Most of the logical work that Lange does is to support the method of showing that conflicts with the natural law can be the basis for demonstrations that have normative force. Lange is concerned to show that logical relations are similar to geometrical constructions in the following respect: that they preserve what we know already and yet give us rules to construct new proofs that go beyond the given. To that end, Lange deduces all the modes of the syllogistic from what are now called Euler–Gergonne diagrams (*Thiel 1994*, p. 105). John Venn achieved the same result slightly later by means of his own diagrams.²⁶ The apodicticity of proofs on that basis will rest on Lange's support for his claim that deductions by means of determining the extension of a concept can be constructed so that mappings of conversions and manipulations of their extension depend on no further substantive assumptions than the bare activity of constructing the proof.

3.3. *The problematic and the apodictic*

Thus, the plausibility of Lange's account rests on his ability to show that proofs can be constructed which (1) support substantive conclusions but (2) are, in fact, formally equivalent to the given premises. Lange argues that two requirements for diagrammatic proof allow him to demonstrate this. The first is that logic and mathematics (again, algebra and geometry) determine only *extensions* (both of concepts and of figures) rather than content. The second is that all conversions between extensions should be reversible or, more precisely, neutral.

The reason for these two requirements is to demonstrate that reasoning using diagrams, which is constrained by the natural law of non-contradiction, can yield normative results. For that to be the case, we may make no substantive presuppositions in diagrammatic inferences, either about the nature of the subject or of the object, or about the first principles that describe their relations. Instead, in our explanation of the facts, we are able to take only our ability to construct certain relations, independently of substantive assumptions, as our starting point. These relations, then, must be demonstrable *sui generis*: they must themselves be the proof of their validity. If you can construct a proof of x, then x has validity over the domain (extension) of which it has been demonstrated.

The ability to reverse and to convert one's judgments is a basic element of Lange's logic, and, accordingly, examples of conversion are prominent in the *Logical Studies* (chapter 1

²⁵ Schröder 1890, pp. 11–13, citation from Lange 1877, pp. 27–28.

²⁶ Venn gave his first presentation of the Venn diagrams in an article 'On the Forms of Logical Proposition' for *Mind* in 1880 (see bibliography). In 1881, he published *Symbolic Logic*, which contained a revised version of that article as its first chapter. There we find a footnote: 'When the substance of this chapter was first written out for *Mind* I was unable to ascertain that any attempt had been made to reconstruct the syllogistic figures upon this propositional scheme. I have since found that almost exactly the same results as are given here had been already obtained by F.A. Lange, in his admirable *Logische Studien*, though from a somewhat different point of view' (*Venn 1881*, 17n).

pp. 8–10; all of chapter 3, ‘Das particulare Urtheil und die Lehre von der Umkehrung der Urtheile’).

In chapter 3, Lange considers the conversion of a particular affirmative judgment. Where B is a categorical judgment (some As are B) and c is a conversion of that judgment (some Bs are A), Lange argues that drawing a spherical diagram demonstrates the possibility of converting the judgment:

The *particular judgment* contains the possibility of all conceptual relationships, with the single exception of separation. Basically, this is to say nothing more than the negation of negation and it is left entirely undecided, which of the positive conceptual relationships obtain. The rule of conversion follows from this easily. For if [a] S either is identical to P, or [b] a part of P, or, conversely, [c] P crosses part of S or, finally, [d] S crosses the sphere of P, then it follows, that also P either is identical to S, or S is part of P, or P part of S, or, finally, P crosses the sphere of S.²⁷ The conceptual relationships of the form a and d are directly convertible, b and c take their places, and so already the four named conceptual relationships are given as possible and only negation is ruled out. Thus, it follows that the particular affirmative judgment is convertible also. That result follows, without analysis of particular cases, immediately from observation; for if some S are P, then one can only envision it, to be persuaded, that at least these among P are S; the remaining forms of the spheres can be as you will. (*Lange 1877*, p. 68)

This passage reveals two essential facts about Lange’s view: first, that all particular affirmative judgments are fully reversible, and second, that this reversal is meant to follow ‘without analysis of particular cases, immediately from observation’.²⁸

According to Lange’s theory, we are able to convert one particular judgment to another apodictically, that is, without reference to the content (or even the particular extension) of the concept. However, we are able to do so only with reference to a particular problem or task. Here an important part of Lange’s theory is revealed: the apodictic only reveals itself as a possible avenue of reasoning, given a certain problem. Thus, the logical, as well as the geometrical, relations that Lange employs are often relative to a given intellectual problem.

Lange allows for various kinds of ‘media’ as ‘Veranschaulichungsmittel’ – that is, as symbolic illustrations or conventional signs for the extension of a concept or for the inclusion or exclusion of a term. The key is that the extension of the sign be demonstrated in the proof. For example, when Lange gives Aristotle’s reversal of categorical negation, he explains it as follows. Aristotle explains the reversal as, ‘[w]e know that no As are B. Now if we take any G²⁹ that belongs to A, it cannot be true that G belongs to B. Therefore no Bs are A’ (*Aristotle 2009* [ca. 350 B.C.E.], chapter 2, cited in Greek in *Lange 1877*, p. 9). This is a paradigm case of the kind of conversion that Lange wants to say is apodictic. However, its demonstration rests on a sensible instantiation (*Veranschaulichung*) which Lange says clearly is the *letter G* (*Lange 1877*, p. 9).

The final concern of Lange’s project in the *Logical Studies* is to clarify the relation between the ‘problematic’ and the ‘apodictic’ in giving a proof. We are led to think of geometrical demonstrations here, but we can consider the question logically as well. In the ‘problematic’ and the ‘apodictic’, Lange is separating the aspects of an inference or

²⁷ Lange defines S as subject and P as predicate.

²⁸ As *Venn 1881* remarks, it is also a minor advance that Lange is able to distinguish (b) and (c) from each other and then to give their conversion validity (p. 6n).

²⁹ We can take G as a singular term or as a subset of A, though there is disagreement among commentators. I am indebted to Stephen Menn for pointing out the disagreement, but also for making clear that it is immaterial in this context.

construction that are constructed or given (the premises, or the original drawing) from the aspects that follow apodictically.

In Lange's presentation, the 'given', in a formal proof, is that which we have assumed to be true – the premises of an inference, for example. The 'given' is what is problematic. It determines the limits within which the problem may be solved. In chapter 4, 'Syllogistic', Lange argues that the distinction between problematic and apodictic allows him to make substantive revisions to the Aristotelian methodology. Lange claims, first, that it is a mistake to rank the apodictic above the assertoric ($\delta\iota\sigma\tau\iota/\sigma\tau\iota$). In particular, he argues that it is wrong to assume that only syllogisms with an apodictic major premise can follow apodictically (Lange 1877, p. 95).

Why is this relevant to the problem of inference? Lange's assertion that formal reasoning is always relative to material facts applies equally to a case in which the material facts are substantive hypothetical assumptions, such as premises or a geometrical figure. This application yields the principle that as long as our construction proceeds only according to the rules that Lange has set out for apodictic reasoning – that is, that we make no substantive assumptions in the reasoning process itself but only make conversions, manipulate proportions, and so on – then we have the right to construct apodictic proofs that have assertoric propositions as their major premise. When we see how Lange explains it, this conclusion will sound actually quite familiar:

Now if the major premise is assertoric, while the minor premise is apodictic, then the demand of this apodicticity is not related at all to the particular content of the inference [*Schlufssatz*], but only on the *right to construct it*. (Lange 1877, p. 95, emphasis in original)

On the one hand, the question is of validity: in which cases, assuming the premises true, can we deduce the conclusion? A further point is that inferences must be truth preserving – if the problematic elements of our inference turn out to be correct, then an apodictic inference must preserve that truth (cf. Lange 1877, p. 97). The fact that an inference follows apodictically must be established by the fact that given the problematic elements (premises), we have the right, incorporating no further assumptions, to construct the conclusion.

4. Conclusion

Lange's intent in writing the *Logical Studies* was to give a foundation for *Erkenntnistheorie* that avoids psychologism and Aristotelianism. But his more specific goal was to show that the explanation of necessary inference does not require tracing proofs back to the first principles or to *a priori*, analytic laws of logic. The trick, for Lange, was to give an explanation of necessary inference that did not appeal to such a logicist or Aristotelian account, which nonetheless gives an account of necessity not reducible to materialism or psychologism.

Lange can give an account of why the inferences he allows are objective, in the sense of not subjective. There are two reasons why Lange can argue that his account of inference is objective: he separates the problematic, or 'given', elements of a proof from the inference itself, and he argues that the necessity of an inference rests only on our right to construct it and not on the content of the premises. Our grasp of the content of the propositions is necessary only to take hold of the given or problematic elements of the proof: the conversion or manipulation of the formal aspects of the problematic elements needs no independent psychological or naturalist explanation.

Lange appeals heavily to a diagrammatic proof structure in giving his account. Here, he appeals to 'immediately evident' properties that can be read from a diagram, but arguably he does not give a solid foundation for this procedure. After all, if the axioms or laws

of logic are not true or analytic *a priori*, then what guarantees that a given conversion or manipulation of an image will be valid?

Lange's answer, as given above, is his assertion that a valid manipulation or conversion is done under the condition that it is possible to prove that any such manipulation will not alter the 'form and quantity of the observed' (Lange 1877, pp. 28–29). This strategy appears to sit uneasily with Lange's rejection of the matter–form distinction overall, as noted above. If Lange were to respond that the form of the observed is imposed conventionally, then we are back at the start of the objection: does Lange have an explanation for why the rules governing the manipulation of conventional forms are necessary?

Two historical points are worth making in closing. The first is that Lange's account of the relationship between the problematic and the apodictic and his account of the postulation of a 'Veranschaulichung' to complete a proof have more in common with the methods of Greek mathematics than his rejection of the Aristotelian methodology in chapter 1 appears to allow. The relationship between features of a geometrical diagram and the manipulation of those features in analysis, for instance, is treated as a metaphor for the philosophical method in Plato and Aristotle.³⁰ Lange was responding, not to the historical Aristotle, but to Aristotle's nineteenth-century interpreters, most prominently Friedrich Überweg, who was Lange's colleague in Bonn from 1852 to 1862, and Adolf Trendelenburg. Lange's rejection of the Aristotelian methods does not imply a rejection of the Euclidean methods of postulation and diagrammatic analysis, in particular.

Secondly, as should be evident from the above, Lange's account anticipates Quine's objections to Carnap's logicist and empiricist program. Lange's objections to the Kantian distinction between analytic and synthetic methods of reasoning, his argument that scientific inference is on a level with logical inference, and his insistence that all necessary inference is the same in kind are all anticipations of Quine. It is intriguing, then, why Lange did not abandon the notion that a certain set of logical inferences, the syllogistic, could be given an intuitively evident demonstration. As a result, Lange's account of inference is an interesting and historically significant blend of naturalism and objectivism.

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³⁰ See Menn 2002 and Netz 2003, 253ff.

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