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FREGE, SENSE AND LIMITED RATIONALITY

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In this paper, I will discuss a well-known oscillation in Frege's conception of sense. My point is only partially concerned with his two different criteria of sense identity, and touches upon a more specific point: what happens if we apply Frege's intuitive criterion for the difference of thoughts to logically equivalent sentences? I will try to make a schematic argument here that will preempt any endeavor to make Frege more coherent than he really is. In sections A and B, I will present two alternative Fregean ways to treat the sense of logically equivalent sentences. Frege really oscillated between two alternative conceptions of sense, and his inability to detect the contrast between the two alternative conceptions is partly due to his strong conception of rationality. To apply the criterion of difference of thoughts to logical matters, we may also use a weak notion of rationality, or at least a notion of rationality of human agents, with limited computational resources. The distinctions towards which Frege was striving are better understood nowadays from the point of view of the treatment of limited rationality, which imposes itself even in logical matters.

A. **Traditional setting of the definition of sense: 1892**

In "Über Sinn und Bedeutung" [1892, p.32], Frege defines the sense of a sentence as the thought expressed by it. He then elaborates what has been called the principle of intuitive difference of thoughts. The definition and argument are as follows:

(1) DEFINITION**:** The *sense of a sentence* is the thought expressed by the sentence.

(2) ARGUMENT**:** *The principle of intuitive difference of thought*: If it is possible to understand two sentences and coherently believe what one expresses while not believing what the other expresses, then those sentences express different senses, or different thoughts (see Evans 1982, pp.18-21). Sainsbury 1999 works upon this idea, speaking of '*rational co-tenability*'.

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In belief contexts, co-substitutability requires identity of sense — not of reference. We need to know exactly what the sense of a sentence is; however the definition of the sense of a sentence as the thought expressed by that sentence is only programmatic. We have an intuitive negative criterion of *difference* of thoughts: sentences that are not substitutable *salva veritate* in belief contexts are supposed to have different senses. To have a positive criterion of *identity*, we need to go the other way around: which kinds of sentences are substitutable in any context without loss of truth value? One widely discussed criterion for sense-identity is general substitutability in indirect contexts. Still, a long debate has resulted in doubts that a clear definition of sense-identity can be attained in this way.[1] The two main attempts developed by Frege to define a criterion of sense identity are logical equivalence and immediate recognizability.[2] However I will not discuss the general problem of criteria of identity in Frege, but will just elaborate on a more limited point: I will check which problems are posed by the negative criterion of difference of thoughts if applied in the context of beliefs about logically equivalent sentences. After that I will see how much these problems help to clarify some tensions in Frege's conception of sense.

The negative criterion of difference of thought is exemplified by Frege's famous example of "Über Sinn und Bedeutung": "...the thought in the sentence 'The Morning Star is a body illuminated by the Sun'

[1]We may think of the traditional attempts by Carnap and model theoretic semantics, and of the difficulties found in using intentional isomorphism in a definition of meaning (Mates's puzzle). Beaney in [1] has given some interesting suggestions to relativize substitutability to different kinds of contexts, and not only to intentional and hyper-intentional contexts. The point however is only programmatic, and there is not yet a clear application of this program.

[2]The two main lines of research of criteria of sense identity are, on the one hand, the criterion of intensional equivalence, and on the other, a criterion of immediate recognizability. The first can be derived by a classical definition in a letter to Husserl of 1906 ([14, p.105]), which attributes sense identity to sentences that lead to a contradiction when assigned different truth values (and that happens with logically equivalent sentences). The second is stated in a remark of 1906 (see [13, 213], but see also [13, 227]) and is normally interpreted as saying that sense identity or equipollence is immediate recognizability. On the contrast between the two lines of research of criteria of sense identity, see [6, p.323], [21], [1, pp.228-229]. We might also speak of a third criterion, the identity of deductions derivable from the expression, as stated at the beginning of *Begriffsschrift*. This notion of sense may be made compatible with the notion of sense as truth condition, but not with the criterion of *immediate* recognizability (given that we cannot immediately recognize all possible consequences).

differs from that in the sentence 'The Evening Star is a body illuminated by the Sun'. Anybody who did not know that the Evening Star is the Morning Star might hold the one thought to be true, the other false." [1892 p.32] The argument, stated as it is, can be applied to different examples, even to logical ones. Take the following[3]:

(3) EXAMPLE**:**
John believes that $A \to B$
$A \to B$ is equivalent to $\neg(A \wedge \neg B)$
* John believes that $\neg(A \wedge \neg B)$

Let us assume that John, having studied some logic and already knowing the (classical) sense of logical constants, does not acknowledge immediately that $A \to B$ is equivalent to $\neg(A \wedge \neg B)$. Therefore he may hold that $A \to B$ and disbelieve that $\neg(A \wedge \neg B)$. Therefore, we should conclude that $A \to B$ and $\neg(A \wedge \neg B)$ have different senses, and express different thoughts.

Against this interpretation, it is arguable to say that these cases are cases of self evident equivalences. These cases should then fall under the criterion of sense identity as immediate recognizability. However, what is immediately recognizable for you is not immediately recognizable for me. We touch here on the problem of our limitations in understanding senses. If you say that John does not grasp the sense of the sentences completely, we might answer that this is the typical relation between a speaker and the sense of sentence: even the most expert mathematicians don't fully grasp the sense of mathematical formulas. The grasp of a sense is always partial (see [17]). An interpretation of this aspect of Frege's ideas brings about the need to take into account our computational limitation while discussing sense and sense identity. We might then accept in principle the idea that the two sentences $A \to B$ and $\neg(A \wedge \neg B)$ express different senses.

This would imply that we may consider $A \to B$ and the negation of $\neg(A \wedge \neg B)$ rationally co-tenable. How could we accept that? In the case of empirical inquiry we have the lack of knowledge of an astronomical truth, the identity of the Morning Star and the Evening Star. In the case of logical inquiry we have the lack of knowledge of a logical equivalence, notwithstanding the assumption that John understands both sides of the equivalence. Somebody might say that if he does not realize that the formulas are logically **equivalent**, he does not understand their sense. This criticism can be challenged. In fact,

[3]Where * means that the conclusion does not necessarily follow from the premises. On the contrary, apparently, $A \to B$, $(A \to B) \longleftrightarrow \neg(A \wedge \neg B) \vdash \neg(A \wedge \neg B)$.

if he understands the sense of the logical constants, he has a grasp of the sense of the formulas: he knows that $A \rightarrow B$ is true only if either A is false or B is true, and $\neg(A \wedge \neg B)$ is true if it is false that A is true and B is false.[4] However, grasping the sense of each sentence, he may not realize the logical equivalence because he misses the identity of the resulting truth tables. It may easily happen to beginners in a logic course, and the case may be more plausible if we think of more complex logically equivalent formulas, which require a certain amount of calculation to detect their equivalence.

To ask just how "rational" John's co-tenability of alternative beliefs might be implies a discussion of rationality. I do not want to enter into a general discussion here on the topic of rationality[5]; I am content to claim that John is rational if he accepts a method of decision (truth table) and he is ready to change his idea when the teacher shows him that the truth tables of the two sentences are the same. Therefore, it is reasonable to accept that he may make a mistake of computation, and still remain rational.

Summarizing: even if John knows the meaning of the logical constants "$\neg$", "$\rightarrow$" and "$\wedge$", he may not know that $A \rightarrow B$ and $\neg(A \wedge \neg B)$ have the same truth condition and that they are translatable into each other. The difference from the original example by Frege in "Über Sinn und Bedeutung" is that the limited knowledge of the (ignorant) astronomer regards an empirical matter, while the limited knowledge of the (slow-in-calculation) John regards a logical matter. But once we get involved in the problem of belief, we have to take into account the attitudes of speakers and their limited knowledge even in logical matters. I assume, therefore, a weak requirement of rational co-tenability, which admits failures in a rational person regarding logical matters. Let me elaborate briefly on that, with the help of Frege.

May John rationally believe that the two sentences above express different thoughts? He may, if we admit the possibility of limitations or even mistakes in his ability of computing. If the above-mentioned

[4]He understands both sides because, by assumption, he understands the meaning of the logical constants. The problem I am going to discuss is not directly connected with the problem of the meaning we attribute to logical constants. The problem does not change if we use natural deduction, or we interpret logical constants in an intuitionistic way. The point pertains to our beliefs and our limited ability to compute.

[5]I think of recent discussions about the relation between rationality and computational limitations, starting with [4]. See also [15], for a discussion on the relevance of computational complexity with respect to the definition of tractable competence.

sentences have different senses, and what is different in them are the computations to reach the truth condition, we should conclude that the sense of a logical sentence is given by the computational procedure needed to reach the truth condition expressed by the sentence. This sounds coherent with the Fregean claim that "2+2=4" and "$2^2 = 4$" express different senses [1893, §2]. They are equations, which are supposed to be reduced to equivalent logical formulas; however they express two different procedures to reach the same reference (the truth value). Somebody might understand and know how two mathematical formulas should be computed, while being unable to understand whether they will give the same result. Even clearer, and wholly analogous to our above mentioned claims, is Frege's claim in a letter to Russell written in 1904 for which 7=7 and $(5^2 \times 211 - 4)/753 = 7$ "do not have the same value for our knowledge" ([14, p.247]). Given that he has assumed that sense has to do with cognitive value (1892, p.25), he clearly implies here that the two formulas have different senses.

Certainly it is not rational to believe p and $\neg p$ at the same time[6]. However, when the computation required to understand that a certain sentence is logically equivalent to another is too complex for our limited ability, we may rationally — and provisionally — hold that the two sentences express different thoughts, unless a better computation proves the opposite is true. In other words, when we see two sentences that *appear* to be very different in logical form, and whose immediate composition we recognize as correct, it is rational to suspend judgment until we can check whether or not they are logically equivalent. A rational agent should keep a reasonable attitude: if he already believes one sentence, he may disbelieve the other, ready to change his opinion with further inquiry and evidence (we can consider this step as a typical case of default reasoning).

In conclusion, if we interpret principle (2) in a *weak* sense, and accept dealing with John's limited access to computational resources and his limited rationality, we may accept that John can rationally believe and disbelieve two logically equivalent sentences. Since he may not be able to completely perform the relevant operations, we therefore have to conclude that:

[6]Certainly, when our student, John, believes that $A \to B$ and disbelieves that $\neg(A \wedge \neg B)$, we may rationally hold that "John believes that $A \to B$" and that "John does not believe that $\neg(A \wedge \neg B)$". Does this peculiar version of Kripke's puzzle compel us to say that John believes $p \wedge \neg p$? I will not go into that here; however we may show John that, if he wanted to keep such beliefs, then he would be compelled to believe that $p \wedge \neg p$. And this is normally the step that makes John update his beliefs.

$$A \to B \quad \text{has a different sense than} \quad \neg(A \wedge \neg B)$$

B. **Setting of the definition of thought in 1906**

In a letter to Husserl in October 1906, Frege explicitly says that $A \to B$ and $\neg(A \wedge \neg B)$ express the same thought. In this letter he gives a definition of thought as the content shared by equipollent sentences [14, p.102]. Equipollent sentences may differ psychologically (difference in tone); a set of equipollent sentences may be given in normal form. However, different forms may have different uses (for pragmatic reasons to make deduction more perspicuous). The following are some relevant quotations (all from [14, p.102]):

(1) Definition of Sense: "equivalent sentences have something in common in their content, and this is what I call the thought they express... The rest I call the coloring and the illumination of the thought."

(2) Differences among Equipollent Sentences: "Judged psychologically, the analyzing proposition is of course always different from the analyzed one, and all logical analysis can be brought to a halt by the objection that the two sentences are merely equipollent (...) For it will not be possible to draw a clear recognizable limit between merely equipollent and congruent sentences."

(3) Normal Form and Use of Differences: "[Given logical analysis] all that would be needed would be a single standard sentence for each system of equipollent sentences, and any thought could be communicated by such a standard sentence. For given a standard sentence everyone would have the whole system of equipollent sentences, and he could make the transition to any one of them whose illumination was particularly to his taste."

Against this background Frege asks whether $A \to B$ and $\neg(A \wedge \neg B)$ are equipollent, that is, given the definitions above, whether they express the same thought:

> "With regard to the question whether the sentence 'if A then B' is equipollent with the sentence 'it is not the case the A without B' [...] we have four combinations:
> True True / True False / False True / False False
> Of these, the first, third and fourth are compatible with the sentence 'If A then B', but not the second. We therefore obtain by negation: A is true and B is false, or: A holds, without B holding, just as on the right hand side [...] If we consult my *Begriffsschrift*, which is

> now 28 years old, we find the answer to such a question without further ado."[14, pp.103-104]

The conclusion is that the two sentences are equipollent and therefore express the same thought. Frege recalls here his original *Begriffsschrift*; however, he writes this passage just a few years after his definition of thought as the sense of a sentence. We may therefore conclude that Frege considers the sense of a sentence, at least in this instance, as the truth condition given by the peculiar conventions in his logical system[7]. The criterion seems to be in fact logical equivalence.

In a letter to Husserl of December 1906, Frege speaks of the need to look for an "objective criterion for recognizing a thought as the same", for "without such a criterion a logical analysis is not possible" [14, p.105]. In this letter he states the claim that two sentences express the same thought if they lead to a logical contradiction if assigned different truth values [14, p.105]. These ideas follow some interesting remarks of 1897, where he claims that in the transformation of a conditional sentence into its contraposed sentence the sense remains unchanged, because "after the transformation the sentence gives no more and no less information than before"[13, p.166].

Transformations of logically equivalent sentences (for example, a conditional and its contraposition) preserve sense identity; but logically equivalent sentences are those sentences that lead to a contradiction when assigned different truth values in accordance with laws of logical equivalence. Therefore, the criterion of sameness of logical equivalence

[7]Somebody might react by saying that the idea of sense as truth condition was already clear in *Grundgesetze*, one year after the definition of the intuitive criterion of the difference of sense. However his explicit definition of sense as dealing with truth conditions does not help in clarifying the question. In the first volume of *Grundgesetze* he speaks of the sense of a sentence or formula as the sense of the name of a truth value, and he says that with our stipulations we determine under which conditions a proper name refers to the truth. He concludes by saying that the sense of a sentence is "the thought that those conditions are satisfied" (1893, §32, p.50). What does it mean? Either it means that giving the sense we give a representation of the truth conditions, or it means that giving the sense we give a representation of the steps which compute the truth conditions, given our stipulations. The first is more compatible with the definition of sense in Wittgenstein's *Tractatus*, the second with a more procedural notion of sense. The history of logic after Frege developed the first aspect, and used a *strong* notion of truth condition, given in an ideal setting from the point of view of an ideal omniscient mind. Recent discussions on computational tractability tend to recover aspects of the other alternative, on the basis of the difference given by Marr in [19] between the computational level and the algorithmic level. The first deals with the input and output of functions; the second with the procedures associated with the function.

(same truth conditions) is another way to express the criterion given in terms of logical contradiction[8].

Keeping this in mind, together with the Wittgensteinian development of sense as truth condition, we may easily be convinced to accept that

$$A \to B \quad \text{has the same sense as} \quad \neg(A \wedge \neg B)$$

C. **Comparing the two views**

From section A we have that "$A \to B$" and "$\neg(A \wedge \neg B)$" express different thoughts; from section B we have that "$A \to B$" and "$\neg(A \wedge \neg B)$" express the same thought.

We have here two notions of thought or sense of a sentence, one cognitive or epistemic, the other semantic or ontological.[9] We might say that the intuitive criterion of sameness of sense as immediate recognizability (see footnote 1) might help us in choosing between the two options, but it doesn't. In fact, depending on what we intend with sense, the criterion of immediate recognizability gives two different answers:

a: if sense is defined as the computing procedure which gives us the truth table, understanding the sense amounts to realizing that the two procedures are different;

b: if sense is defined as the truth condition, understanding the sense amounts to realizing that the two sentences have the same truth conditions.

Therefore, the criterion of immediate recognizability does not help us to choose (even if Frege used it to support the sense identity of logically equivalent sentences). How does one evaluate this situation in Frege's work? We have three basic options:

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[8]I take this idea from a manuscript paper by Massimo Grassia (Columbia University).

[9]Apparently this is not a novel discovery, and it is standard since at least [1, p.227] with the terminology of epistemic and semantic sense (see also [20]). It is apparent that this ambiguity stands behind many different contemporary attempts to distinguish two aspects of meaning, like the pairs intention-intentional structure (Carnap), content-character (Kaplan), thought-sense (Perry), truth conditional-ingredient sense (Dummett), and in general semantic content *vs.* modes of presentation. Somebody might say that we have a third basic idea of sense in Frege, besides cognitive sense and truth conditional sense: the idea of inferential potential so clearly stated at the beginning of *Begriffsschrift* (§3). However it might also be argued that this notion is neutral between the two basic alternatives, depending on what you take as acceptable inferences.

i. **Frege has made a mistake.** Either the first definition is wrong and Frege was mistaken in attempting an impossible, more finely grained definition of identity (congruence) of thoughts, or the second definition is wrong and it is a mistake in Frege's point of view.

But:

To reject the conclusion of section (B) implies the rejection of the basic ideas of the difference between sense and tone (sense pertains to identity of consequences) which is basic from the *Begriffshrift*. Equipollent sentences are such that their difference pertains to tone; in the above-discussed example, the difference between the two logically equivalent sentences can be considered similar to the difference between "the Greeks defeated the Persians" *vs.* "the Persians were defeated by the Greeks": different tone, same sense or same consequences.

To reject the conclusion of section (A) implies the rejection of the basic idea of cognitive value as it developed inside the explanation of the distinction between sense and reference, which is basic to the entire logicist project. Cognitive value is relevant in belief contexts. The information given by two logically equivalent sentences, even if identical as far as consequences are concerned, can be conceived as different if we are concerned with our epistemic access or with our ability to grasp them more or less completely.

ii. **Frege changed his mind.** We might think that Frege tampered with the idea of sense identity based on cognitive value, dealing with co-substitutability in indirect contexts, which is therefore more finely grained than logical equivalence. He decided later that the only way to have a definition of sense was to have the definition be something similar to the truth condition — as defined by Wittgenstein in his *Tractatus*, read by Frege in 1918. In his later works, like "Gedankengefüge", possibly influenced by the *Tractatus*, Frege often speaks of equivalence of thoughts regarding pairs of logically equivalent expressions, elaborating on the theme discussed in his letter to Husserl. He shows there how transformation of logically equivalent sentences such as "$A \rightarrow B$" and "$\neg(A \wedge \neg B)$" happens "without altering the sense" [12, p.48].

But:

Even in the latest papers we may find a trace of the cognitive notion of sense, with a criterion of identity more refined than logical equivalence. It is enough to look at the work "Negation." Here, in arguing that the double negation of a sentence gives the same truth value as the original sentence, Frege distinguishes between "the two thoughts A

and the negation of the negation of A" [11, p.157]. It is easy to conclude that if A and $\neg\neg A$ are two different thoughts and at the same time they are logically equivalent, there is a conception of identity of thoughts which cannot be given by logical equivalence.

iii. **Frege did not realize the clash between these two different conceptions.** Most critics have attempted to preserve Frege's coherence by distinguishing different levels of analysis[10]. Even so, it is difficult to avoid a feeling of dissatisfaction about the contrast given above. If different preoccupations were at work in Frege's mind[11], he was unable to detect the conflict among them. We should therefore look for the psychological and theoretical reasons why Frege never became aware of the apparent contrast.

One of the main contrasts is given by the kind of worries on which he was actually working at different times: on the one hand, the problem of distinguishing the same thought in different disguises, and on the other hand, the problem of distinguishing different thoughts in expressions with the same reference. His distinction between sense and tone lead him toward a conception of sense as truth condition, while his discussion of the distinction between sense and reference led him to elaborate the notion of sense as cognitive value. As soon as we enter the problem of belief and limited knowledge, there is no way to avoid the clash between the two conceptions. In fact, there is an ambiguous realm between tone and sense, where the cognitive power of the speaker is taken into account: the *grasping* of sense is a mysterious act, "the most mysterious process" which connects psychology to logic, the subjective with the objective ([13, p.157]. Here Frege oscillates.

D. **Conclusions**

Frege did not have a clear grasp of the contrast, probably because of his fear of letting psychology intrude into logical matters. However, he hints at the problem in the letter to Husserl of October 1906. Here, as we have seen, he presented an example of an application of the idea that

[10]The different attempts follow often very different claims, distinguishing analysis and decomposition ([6])), epistemological and ontological claims ([5]), ontological and logical claims ([16] and [3]).

[11]The general idea is that many different worries are behind the attempts of giving a criterion of identity of thoughts. Following the strategy hinted at in the paper we might distinguish different categories of synonymous pairs of sentences. Think for instance of synonymous sentences which (i) differ in tone, (ii) differ in grammatical construction, (iii) are paraphrases or elucidations (iv) are logical transformations. See [21].

two logically equivalent sentences have the same sense. After having declared that $A \to B$ and $\neg(A \wedge \neg B)$ are "equipollent" he goes on to ask a new question:

"Now are these sentences also congruent?"

The answer says:

"This could well be debated for a hundred years or more. At least I do not see what criterion would allow us to decide this question objectively." [14, pp.104-105]

The answer seems to imply that Frege recognizes the *need* for a kind of equivalence between sentences that is stronger than his definition of equipollence as "having the same thought". What does Frege mean by "congruent"? We have seen in section B that he thought it difficult to draw a sharp line between equipollence and congruence. On the other hand, a canonical representation of a class of equipollent sentences does not impede the possibility of choosing one particular sentence of the class which is more suitable to the taste or the needs of the user. We may study the peculiar aspect of that particular sentence which makes the sentence different from most of the others belonging to the same class, and define this peculiar aspect inside a psychological theory. We can then study the possibility of a kind of equivalence which pertains to psychology or to the study of mental processes. When Frege claims that there is no *objective* criterion, he alludes to (or at least his assertion implies) the possibility of what he might have called a *subjective* criterion of identity, that is, a criterion linked to psychology or to pragmatic affairs.

One hundred years have passed. Do we have better or clearer ideas? Since Frege, psychological and computational studies have changed considerably, posing the problem of identity criteria for computational procedures, or for computational complexity. In short, today, instead of speaking of a subjective criterion, we may speak of a criterion laid down in terms of cognitive studies (let us say, for the sake of simplicity, a criterion about the same formulas in a language of thought, given in a computational representation of the mind). This kind of "objective" analysis of psychological processes was beyond Frege's horizon. However, his intellectual acumen was strong enough to individuate the place where the discussion should have been developed, that is, the belief contexts, or more generally the contexts in which we face the problem of the speaker's limited access to information.

My paper aims to make a historical point about Frege. To give an answer to a contemporary problem is apparently beyond the possibility of this paper and of Frege's conceptual machinery, especially the

relation between computational complexity and realistic psychological models of rationality[12]. Frege wanted to deal with the ideal rational mind of a perfect omniscient knower. Many efforts today are devoted to devise the rationality of the limited knower: "If we look at the kinds of mistakes people make, the kinds of problems people run into, and the corners that are cut to get around them, we will find modifications to classical logic that ensure the computational tractability of the associated thinking"[13].

Dealing with the problems of treating limited knowledge and limited rationality, we are more aware today of the problems Frege found in his attempt to devise a formal language, mainly thought for mathematical reasoning. However, since the *Begriffsschrift* ([8, pp.V-VI]) he always thought of his attempt as a first step of a much wider application of his symbolism, on the track given by Leibniz's grandiose program. Developments in computational models of the mind or in a formalized model of commonsense reasoning are not contradictory to the wider project hinted at in the "Introduction" of the *Begriffsschrift*. In his later work, he detected some of the problems we continue to debate today, keeping his peculiar stance to separate the psychological from the logical. Our problem is how far and how much we may keep this distinction in developing an analysis of the cognitive aspects of language use and still pursue his attempt to give a non-naturalized framework for the analysis of thought.

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