ПРОБЛЕМИ НА ТЕОРИЯТА НА ПОЗНАНИЕТО

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THE KOCHEN – SPECKER THEOREM IN QUANTUM MECHANICS: A PHILOSOPHICAL COMMENT (PART 2)[2]

Abstract: The text is a continuation of the article of the same name published in the previous issue of *Philosophical Alternatives*. The philosophical interpretations of the Kochen – Specker theorem (1967) are considered. Einstein's principle regarding the „consubstantiality of inertia and gravity" (1918) allows of a parallel between descriptions of a physical micro-entity in relation to the macro-apparatus on the one hand, and of physical macro-entities in relation to the astronomical mega-entities on the other. The Bohmian interpretation (1952) of quantum mechanics proposes that all quantum systems be interpreted as dissipative ones and that the theorem be thus understood. The conclusion is that the continual representation, by force or (gravitational) field between parts interacting by means of it, of a system is equivalent to their mutual entanglement if representation is discrete. Gravity (force field) and entanglement are two different, correspondingly continual and discrete, images of a single common essence. General relativity can be interpreted as a superluminal generalization of special relativity. The postulate exists of an alleged obligatory difference between a model and reality in science and philosophy. It can also be deduced by interpreting a corollary of the theorem. On the other hand, quantum mechanics, on the basis of this theorem and of Von Neumann's (1932), introduces the option that a model be entirely identified as the modeled reality and, therefore, that absolutely reality be recognized: this is a non-standard hypothesis in the epistemology of science. Thus, the true reality begins to be understood mathematically, i.e. in a Pythagorean manner, for its identification with its mathematical model. A few linked problems are highlighted: the role of the axiom of choice for correctly interpreting the theorem; whether the theorem can be considered an axiom; whether the theorem can be considered equivalent to the negation of the axiom.

Key words: Kochen – Specker theorem, relativity, entanglement, model and reality, Bohmian interpretation of quantum mechanics, axiom of choice.

Резюме: Разглеждат се философски интерпретации на теоремата на Коушън и Шпекър (1967). Айнщайновият принцип за „единосъщието на инерция и гравитация" (1918) позволява паралелизъм на описанието на физически микро-обект спрямо макро-уреда, от една страна, и на физически макро-обекти по отношение на астрономически мега-обекти. Бомовата интерпретация (1952) на квантовата механика позволява квантовите системи да се тълкуват като дисипативни и на тази основа да се осмисля теоремата. Заключението е: гладкото представяне чрез сила или поле между частите на система, взаимодействащи си чрез тях, е еквивалентно на сдвояване между тях при дискретно представяне. Гравитацията (силовото поле) и сдвояването са две различни, съответно гладък и дискретен образ на

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една-единствена обща същност. Общата теория на относителността може да се интерпретира като свръхсветлинното обобщение на специалната. В науката и философията съществува постулатът за принципно неотстранимата разлика между модел и реалност. Той може да се изведе чрез тълкуване на едно от следствията на теоремата. От друга страна, квантовата механика – на основание на теоремата, както и на предхождащата я на фон Нойман (1932) – въвежда възможността за отъждествяване на модел и реалност и оттук: за абсолютно познание на реалността от човека: една нестандартна хипотеза в епистемологията на науката. Заедно с това самата реалност необходимо започва да се разбира като математическа, т.е. по питагорейски, поради отъждествяването ѝ с нейния математически модел. Извеждат се няколко свързани проблема: фундаменталната важност на аксиомата за избора при интерпретиране на теоремата; статусът на теоремата: възможно ли е да се интерпретира като аксиома; дали теоремата не може да са разглежда като еквивалентна на отрицание на аксиомата.

II. A few philosophical comments about the Kochen – Specker theorem

If you return to a quantum reading of GR and take into account „the consubstantiality of inertia and gravity" (Einstein 1918: 241), you should mean that not only descriptions by gravitational and any potential field are equivalent to each other mathematically (or if we allow ourselves to express so, consubstantial), but from our viewpoint the quantum description of the same object to a mega-object is equivalent to them, too. Clearly, the theory of representations, offering a mathematical language to identify and explain the isomorphism of potential (gravitational) and quantum randomness, along with that, guides us to their eventual consubstantiality. If quantum mechanics by Ψ-function shares an equivalent discrete description of any potential field, and general relativity has already enacted the indistinguishability of arbitrary potential and gravitational field, we have no more choice except concluding that Ψ-function should be an equivalent, but discrete description of gravitational field: For example, then the descriptions of the universe by its Ψ-function or by metrical tensor in any point of space-time are at least isomorphic, and following an Einsteinian kind of Pythagorean ontology, one in essence, besides.

In our physical world a model can be represented by dissipative system from chaos theory. By it we could read as the alleged hidden parameter of quantum mechanics any „gently swinging the wings of a butterfly" in the appliance that leads to the measured result about the investigated quantum object in a dissipative, but causal way, anyway. Such an approach very reminisces Bohm (Bohm 1952: 171)'s interpretation. However according to Kochen – Specker's theorem and its corollaries a coherent system is impossible to be reduced to any de-coherent state. Consequently, dissipation can be accomplished only by the system environment or in other words, by the system non-standard, entangled, external parts, i.e. by the device as which all the rest can be considered. If we think of the universe as a whole without parts or environment, then all other possible states of its force it to leave the coherent state in favor of the single real state. Who accomplishes a choice? According to the axiom of choice a system can make the choice by itself. According to the Kochen – Specker theorem it is impossible to do that by itself, and the choice is only forced by its environment or by its possible states. Therefore we should again speak of a Skolemian type of the relativity of freedom and necessity, this time.

Our central interest is the pending identification of the alleged superluminal area of SR (the tachyons theory) with GR on the base of the already proposed identification of any potential field (GR), but also with one-to-one discrete morphism, and hence with a certain superluminal speed transferring us into the real domain of Minkowski space (SR).

The idea of interpreting GR as the „theory of tachyons", i.e. as the superluminal generalization of SR, is not only quite new and unexpected, but suggests even having to be particularly highlighted. The reasons are several:

Even the very name and the intention of *general* relativity suggest that it to be interpreted as a generalization of SR as to non-inertial (or arbitrary) reference frame. Along with that the quoted already many times principle of Einstein's relativity (1918: 241) constrains all physical movement to diffeomorphisms, so that non-inertial systems which are obtained from one another by a discrete (quantum) leap are not considered. Just because of that the idea of a possible generalization of the principle or theory of relativity as well regarding such a kind of reference frame is able to be suggested. Furthermore, it seems that the notion of relative speed cannot be defined in a nontrivial way since it is always infinite, and besides, violates the principle of no exceeding the light sped in free space, correspondingly, Lorentz invariance.

It is useful to emphasize a relationship with the representation theorem (Riesz 1907) to try to clarify how the already sketched identification of GR with the superluminal extension of SR (the theory of tachyons), first, is easy to be transferred as isomorphism between pseudo-Riemannian and Minkowski space, and secondly, gains some redundancy of the thought of „curved" Hilbert space, since that isomorphism can be conducted, namely by the Riesz theorem, also between usual standard and alleged „curved" Hilbert space.

Suffice it to introduce the concept of covariant world line matching the usual contravariant one if and only if the curvature of pseudo-Riemannian space in any point of the world line is zero. Obviously, the condition is met for each world line in Minkowski space.

If you introduce a more natural and reasonable, and one can say, and traditional principle of invariance of physical laws in the transition from quantum micro-object to measuring instrument (in particular, it implies the mega-interpretation of quantum mechanics), which establishes the equivalence of any usual continual and a new, discrete description of the surrounding macro-physical reality in relation to mega, i.e. universe objects. Its relativity affects the equivalence as of the relation of micro and macro and of macro and mega as of the relation of discrete and continuous models.

This approach is similar to that of Schrödinger: Ψ-function to be interpreted as a „list of expectation" (Schrödinger 1935: 827-828), i.e. as a „description". He also discussed it as „reality", therefore in an epistemological sense, which is opposite. There is more information in the world line than in the Ψ - function and it should regard as reality from this perspective. From the other hand, we cannot recover the real world line from all the set of experiments, as we have not any empirical or experimental access the alleged hidden parameter, which we could call the moment of projection and which appears to be due to an uncontrollably random superposition of a large number of contributing factors associated with macro-measuring: if we did so, we would be near to Bohm (Bohm 1952, I: 171)'s position.

There is always a morphism, whose physical interpretation is the time reversion, swapping places of calculation and meta-calculation, or respectively, of the co- and contravariant world line as well as, of the set of all subset of a given one and it itself, which is rather unexpectedly. That will be allowed only if any set is always a set of subsets of another.

The conclusion is: *The continual representation by force or (gravitational) field between parts, interacting by means of it, of a system is equivalent to entanglement between them if representation is discrete. Gravity (force field) and entanglement are two different, correspondingly continual and discrete, images of a single common essence.*

Here we encounter a Skolemian type of relativity between discrete and continual models, between a system as an indivisible whole and as an ensemble or even sum of its components, between entanglement and (incl. gravitational) forces. On the pole of philosophical reflection, continual and discrete, countable and countless, „curved" and „flat", physical, logical and mathematical, material and algorithmic, holistic and calculate models proved to be really united, but splitting up into these polar images, which should be different only and seemingly rather only by tradition. It is Hilbert space that manages to display that unity.

Getting once again on a properly philosophical position, we have to raise the issue of mapping between description and reality, or on the poles of inevitable speculation, between subject and object, in our case: a quantum physical object described by Ψ-function. For classical philosophy, the description is always different from reality, in most cases it is not more than an extremely rude and imperfect copy.

Is there a mathematical structure to model both an object and its description and which can display the relationship of identity between them? Hilbert space matches such a type. We could look at world line as an object by itself, and on Ψ-function as its description. On the one hand, we could add to Ψ-function the corresponding hypermaximal operator or a set of such ones so as to get the object just as the movement of a material point or as the change of the quantities characteristic of it. On the other hand, the very Ψ-function choses exactly a single world line as the movement of a material point and in a sense it is that description which coincides with the object, it is a „phenomenon". So, in this second case, we deal with the phenomenon of the thing in relation to the latter. The phenomenon can play the role of logical subject in spite of being variable in time and having many physical quantities, which are not phenomena as the Ψ-function has to be complemented by the corresponding self-adjoint operator characterizing a concrete quantity. However, any quantity can be considered as the substance of the thing, and any else as its predication.This allows us to make identification, by which to move ourselves to the position of classical ontology and logic, by taking the entire class of specific quantities as the very thing, since each one of them can be considered as that substance – a phenomenon, to which the rest are predications. The prospect of an also Skolemian relativity of phenomenon and object, of individual and class is now outlined in front of us.

The made just discussion should be generalized. Not only the mechanical movement can be seen in the logical and discursive perspective of description, but conversely, any logic of something or a concrete discourse is isomorphic of some mechanical movement and consequently, we can direct to them also from this position, again by means of Ψ-function, but already discussed as an iterating and iterative set of projective operators in the spirit of von Neumann's classical foundation of quantum logic.

The notion of time as in physical as in philosophical sense allows us also to correlate infinite and finite choice: infinite choice mathematically guaranteed by the axiom of choice and embodied in the totality of time usually named eternity, and finite choice which is empirically given to any human being in present.

One could formulate the following principle of a rather philosophical nature: description is not more complex than described (reality), the cardinal of the former is less than or equal to that of the latter. Description can be treated as encoding the described, and the physical quantity of information as the product of those two factors.

Along with the above, 'forces in reality' can adequately and equivalently be represented as the entanglement of descriptions. Jumps, „miracles" from the discrete description have a parallel *continual* physical description only within which they are able to be thought as physical forces or fields according to the setting or prejudices of modern science. A-causal description by means of feedback from the future is systematically overlapped by causal one, which has as if the crucial advantage of successful scientific predictions, which do not influence, do not „shift" the future by their very nature.

The notions of subjective or objective probability allow for us to distinguish correspondingly subjective or objective time being reconstructed on the base of them (e.g. as the expectation of a subject or the frequency a given event to occurs). In both cases, time is assumed discrete as it jumps from the present to the fixed future moment when the event is going to occur.

In the case of objective probability interpreted e.g. as frequency, a statistics is available as to the realization of the event in many cases: the numbering of the latter may be regarded as a parameter or as a name of many parameters.

The case of subjective probability means a unique event, which rejects the possibility of any hidden parameter because of its indivisibility. An iterative procedure can be imagined such that it converges on a hypothetical absolute subject gradually covering more and more real ones who form a common (to call it „expert") opinion on the probability for the target event to take place in reality.

We can express the hypothesis that the two limits each tending to infinity correspondingly of individual events or actors will coincide, besides suggesting its fundamental, axiomatic character.

Another option is its negation, namely that there is in general a mismatch between the value of the objective and subjective probability, possibly varying from one to another event: We will call it ontological difference. Let us emphasize that we define a strictly quantitative expression for 'ontological difference': the difference or the ratio between the limits (in infinity) of subjective and objective probability, to which a difference or a ratio of the restored to their base discrete (subjective and objective) times will correspond.

However, if you stand on the position of a match, therefore continuity between subjective and objective time, you could weaken the hypothesis of smooth transition between them: It is only the infinite values that have to coincide, but not both first derivatives correspondingly „on the right" and „on the left", which both must exist. The limits of converging by subjective and by objective probability do not coincide.

Such an option split the concept of ontological difference by the hypothesis of the identity/ non-identity of infinity: we will denote it as „logical difference", and being different from „ontological difference", it suggests a topic about „onto-logical difference", of course, in a partly quantitative and emphasizing its limited aspect.

But which is the case realized in quantum mechanics if we look at it as a check on that: which of the above suggestions as theoretical possibility has taken place in reality?

There is a statistics (frequency) as in the case of objective probability, but it precludes any hidden parameter, as in that of subjective probability. One (and perhaps the only possible) solution is matching the subjective and objective probability, i.e. zero logical and therefore onto-logical difference. We are going to denote that case as „quantum probability" and understand exactly matching the subjective and objective probability.

We can add that the concept of probability seen as the ratio of infinites in a finite limit allows to clarify how and why its involvement in quantum mechanics gains a definite quantitative expression for any discretization as regards the infinite speeds obtained as a result because of the zero time to perform any quantum leap. While infinity is not empirically and therefore experimentally attainable, it is already subjective probability (of an absolute subject), postulated as coinciding with the objective one and by means of it, that is.

While and if we have assumed that present is before the infinity of past, and that of future is after it, we could use the term of „non-finite" for a similar status after arithmetization. Non-finite sets are built as follows: that set whose set of subsets is countable is non-finite. *Present is located in non-finiteness*: after any finiteness, but before any infinity. It is the relation between non-finiteness and infinity that is represented as probability and can describe discreteness.

When we generalize the concept of „trajectory" from three-dimensional to Hilbert space, we replace 'point' (an element of a function) with 'function'. The concept of „continuous trajectory" must therefore be reviewed. In our examination by means of qubits, continuous trajectory refers to all of them and thus is limited to the continuous path of points. Furthermore, *the change of probability per time unit is directly interpreted by the quantum of action, i.e. for the values smaller than it*. We have to pay special attention to the text in italics, because, by the assumed and interpreted relationship between energy and probability, we can transform sub-quantum into super-quantum reality including the macro-world. Roughly speaking, the entire universe can look like the inside of one quantum; inversely, the super-quantum area can be considered as the outside of the universe: It is very interesting that the vicinity of any its internal point is able to be discussed as its outside. Indeed, Hilbert space interpreted as a space of $\boldsymbol{\Psi}$-functions equivalent to world lines has precisely this property, which can be called fractal probably not only metaphorically. Each line describes an eternity of the universe as the continuity of past, present and future: note that the description originates from one reference frame. The impossible and self-contradictory external movement of the universe is described as the internal one: It is enough to „change the Gestalt" and look at each Ψ-function like a description of the state of the universe and only secondarily, and as a result of the former like a description of a physical object inside. *The universe consists (internally) of its (external) states*. The transition from stochastically determined chaos to probabilistic quantum states converts its parts into its states.

Therefore, the Skolemian relativity of continuity and discreteness is easily to be transformed into the relativity of externality and internality permitting a single common description. Then energy corresponds to the values of change within a quant or a discrete jump. On the other hand, we have used the probabilities for infinity: the probability is a bridge, through which we are able to think uniformly as infinitely great as infinitely small quantities as infinitesimal, but in terms of discreteness embedded by quantum.

Since in our approach, action corresponds to the change of probability, the principle of least action is converted into a principle of least probability change, which is intuitively perfectly acceptable: the transition from one to another Ψ-function must take place through the least change of probability. In particular, in the case of the transformation corresponding to a physical quantity (i.e. by a hypermaximal operator), the amendment of probability is zero, and we have already noted that all transformation of that class may be related to the present of the object possessing that Ψ-function. Time can be described as the density of probability for transiting from one to another Ψ-function, in which the former is interpreted as the present of the quantum object in question, while the latter as future or past states. The probability density function will be maximal in the present, to the maximum of which the past values will grow monotonically by different slope, and those of the future in turn will decrease, forming a typical bell shaped probability density curve. Negative probability will be then naturally interpreted as a probability for converting in past states, i.e. back in time, while the complex one will be referred to the time axis distortion.

The fundamental constants – the Plank one and that of the light speed in vacuum – are a natural units set allowing the parts of them to be interpreted as probabilities, and continuity to be transformed into the internality of a discrete (quantum) jump.

If we mean to type a summary of the principle of least action, it appears that may cover a wider area, namely Hilbert space, and as long as the amendment of the probability in time is interpreted as the frequency of de Broglie wave. Such an interpretation is natural because probability, thought even elementarily, is defined as the ratio of the part of the alternatives accepted as favorable (whether in anticipation or as a statistical frequency) of a choice towards all of them. Then de Broglie wave corresponds to the amendment of the positive alternatives if the set of all the alternatives remains the same (in other words, the change of the favorable ones is for the account of the negative ones). Such an approach, besides suggesting an unusual perspective on energy, respectively matter, assumes that the law of energy conservation roughly approximates the linear area of increasing (decreasing) the probability of the bell curve of probability density distribution, or of the linear change of the positive alternatives. In areas, in which such an approximation is significantly different, forces, fields, and their energies should appear to compensate for the inaccuracy of approximation.

The principle of least action as a principle of least changing the probability actually includes also the case of its amendment in time corresponding to the above introduced concept of logical difference. In terms of the alternatives accepted to be favorable of a choice, ontological difference is the difference between the amount of favorable and unfavorable alternatives and all the alternatives while logical difference is that between all the alternatives (e.g. at different points in time). Logical difference, however, should be understood sufficiently generally: since all alternatives form a totality, it means that we can assume for the difference as preservation as change in an equal degree. Therefore, *the logical difference* seeks to quantify the relationship of the totality to itself.

Finally, the context of the work can be used to address a few – seemingly disparate and unrelated – exceptions for the dimensionality of **2** Gleason's (Gleason 1957: 132), in Kochen – Specker's theorem (Kochen, Specker 1967: 70; § 6; Specker 1975: 139-140), and at last … Fermat's last theorem proven at the end of last century by Andrew Wiles. Kochen and Specker directly indicated that the displayed result can be obtained from the theorem of Gleason. (Kochen, Specker 1967: 70). However our work has grounded on the isomorphism of qubit and three-dimensional sphere (which in fact is similar to Kochen and Specker's counter-example of the § 6 of their article), on the representation of Hilbert space by qubits and hence, by Minkowski space. Further a surprise occurs: as three-dimensional sphere is obtained from the complex Hilbert space of dimension **2**, and Minkowski space is an additive combination of qubits, at that representable as Hilbert space, then there is a direct pathway the exception about dimensionality two to be transferred to *infinite dimensions*. It should however again be noted that this is not a way to carry over to any finite dimensions. Further the axiom of choice carries the outcome to any transfinite power provided it to be valid for that power. We have statements – all of those that appear by the exception of dimensionality **2**, – which are not true for any finite integer, but they are true for an infinite number. The only way out of the situation, if we are to preserve the principle of induction, moreover it is included in the Peano axiomatic of integers, is to accept that there is a number that we cannot point out, for which that type of statements are not valid.The argued could immediately refer to Fermat's last theorem if we have taken a sufficiently powerful axiom of choice as long as higher dimensions are obtained multiplicatively. Is the axiom of choice or weaker version of it (the theorem of prime Boolean ideals) used in Wiles' proof (McLarty 2010) to be displayed that the field of rational numbers has an algebraic closure or all fields have algebraic closures Wiles (1995; Taylor, Wiles 1995)? On the base of just made considering as well as the whole context of the article we tend to insist on a negative answer: rather Fermat's last theorem as in Wiles' proof as at all is equivalent to the negation of the axiom of choice or even of a stronger version of it. It could display by its eventual deduction from the Kochen – Specker or Gleason theorem.

Instead, in the spirit of the philosophical nature of this discussion, we may propose the following problem: whether the propositions valid for any instant of time are valid for the eternity, i.e. whether they are tautologies. In statements on eternity, we find ourselves in an analogue of well-known difficulties on the set of all sets; in the case: the alleged validity / invalidity of the claim on all valid claims. Obviously, the proposed isomorphism between Minkowski space and Hilbert space and their physical interpretation because of the exception of alleged impossibility usual Boolean logic to be embedded (Kochen, Specker 1967: 70; Specker 1975: 139-140) as to dimension **1** or **2** transfers its statements which are valid, i.e. tautological, for any moment of time into „valid at all", i.e. valid as to eternity, however even if they are not valid as to any future moment (but not present one!). The statements about invalidity towards an arbitrary future, but not present instant of time turn out to be self-contradictory on the base of the proposed argument. So „the problem of unobservables" (Feyerabend 1975: 110) after interpreting quantum mechanics by means of three-values logic (Reichenbach 1975) reveals its self-contradiction. They are statements about unobservables which turn out to be contradictory as such ones about future moments in our approach.

It turns out that the usual binary logic has an unusual privilege with regard to eternity: in a sense it makes or brings an equivalence between present and eternity, interpreting eternity by generality quantifier as „for all presents".

Finally we will apply the used argument also to the theorem that a field (i.e. multiplication is commutative) of dimensionality higher than two does not exist: the principle of induction requires such to exists anyway: Of course, it comes to a clean and unconstructive proof of its existence.

But later, according to the conventional interpretation of the theorem known as the „paradox Banach – Traski" (Banach, Tarski 1924: 244), a qubit is equivalent to two ones hence ultimately to Hilbert space as a whole, as a model of the Universum. According to the theorem of Kochen and Specker (more precisely, as its direct consequence) it cannot be represented as a bit. However being valid the axiom of choice, it should be able, as well as according the very example (as a counter-example given by them in § 6 of their article). In last analysis, the starting point for their consideration may be reduced to a kind of a „bit": wave-particle duality, or according to the discussion made in this study, the Skolemain or Einsteinian type of relativity between discreteness and continuity. Choice within a relativity is guaranteed, but immaterial. Therefore, the primal philosophical choice we have made is between the importance of the very choice and the relativity of its alternatives, and hence their immateriality, ultimately, of the very choice. In gnomic words, the being of the world is reduced to a single choice (even of the simplest kind, between two equal possibilities, i.e. to a single bit). On the other hand, this is trivial because the question might be: is there a world? That problem turns out most surprisingly (of course, not to the successors of the antimetaphysic trend à la Wittgenstein in the contemporary philosophy) to be immaterial: in a Skolemian way, even being and non-being are relative.

If we pass the route in reverse order, we can create the universe, including gravity, the ensemble of all the possible states of it: each of them is simultaneously the actual state of some part of it. In other words, we can create the universe as „consisting only of itself", but not limited to acting as one, i.e. a whole, and also of all of its own parts equivalent to the states of the whole.

Summarizing the entire current statement, we can highlight several major problems:

1. The fundamental importance of the axiom of choice in the discussion of issues around the theorem of Kochen and Specker.

2. The status of the theorem of Kochen and Specker: Is not it an axiom?

3. The relationship of the axiom of choice and the theorem of Kochen and Specker: whether and how the latter can be seen as a direct negation of the former?

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