This is the authors' version of the following article:
Podlaskowski, A. and J. Smith. (2014). "Probabilistic Regresses And The Availability Problem For Infinitism." *Metaphilosophy*, April, 45 (2), pp. 211–220, which has been published in final form at:

http://onlinelibrary.wiley.com/doi/10.1111/meta.12077/abstract

Probabilistic regresses and the availability problem for infinitism

Adam C. Podlaskowski and Joshua A. Smith

Abstract

Recent work by Peijnenburg, Atkinson, and Herzberg suggests that the infinitist who accepts a probabilistic construal of justification can overcome significant challenges to their position by attending to mathematical treatments of infinite probabilistic regresses. In this essay, we argue that care must be taken when assessing the significance of these formal results. Though valuable lessons can be drawn from these mathematical exercises (many of which we do not dispute here), as we will argue, it is entirely unclear that the form of infinitism that results meets a basic requirement: namely, providing an account of infinite chains of propositions *qua* reasons made available to agents.

Keywords: available reason, infinitism, probabilistic justification, regress problem

Introduction

The original *regress problem* for epistemic justification arises because any belief is justified only if it is based on good reasons, and the beliefs serving as reasons also stand in need of justification, as do those beliefs serving as reasons, and so on; as such, we face an infinite regress. Because many epistemologists now accept a probabilistic construal of justification, the regress problem deserves an update: because any belief is probabilistically supported by another belief, which in turn is probabilistically supported by another one, and so one, we face an infinite regress.[1]

According to the infinitist response advanced by Peter Klein (1998, 1999, 2003, 2005, 2007), we should embrace the infinite regress, since the most we can hope for is provisional, progressive justification amidst an infinite, non-repeating chain of reasons for belief. Debates over the success of infinitism continue to rage on in the literature. Recent work (e.g., Peijnenburg, 2007; Peijnenburg and Atkinson, 2008, 2009, 2013; Herzberg, 2010, 2013) suggests that the infinitist who accepts a probabilistic construal of justification can overcome significant challenges to their position by attending to mathematical treatments of infinite probabilistic regresses.

In this essay, we argue that care must be taken when assessing the significance of these formal results. Though valuable lessons can be drawn from

[1]It has been variously suggested that infinitists should divest themselves of interest in *justification* in favor of a less demanding notion of *warrant*. While some discussions require attending to this distinction, the current one does not. As such, though the term 'justification' will be used, here it can be replaced with 'warrant' at no significant cost.

these mathematical exercises (many of which we do not dispute here), as we will argue, it is entirely unclear that the form of infinitism that results meets a basic requirement: namely, providing an account of infinite chains of propositions *qua* reasons made available to agents.

Infinitism and progressive justification

As Klein puts it, the regress of reasons arises for epistemically responsible agents, and further asks: "what kind of reasoning can satisfy the norms of epistemic responsibility" (2007, 5)? This involves a picture of epistemic agents according to which they "examine their beliefs in order to determine which, if any, are worthy of being kept" (2007, 6). Moreover, Klein holds "that being able to produce reasons for beliefs is a distinctive characteristic of adult human knowledge" (1999, 201). So, an epistemically responsible agent, reflecting on which of her beliefs are justified and which are not, must consider each of them, their supporting evidence, weigh competing options, and so on. And, especially given the emphasis on probabilistic justification, these practices lead to establishing the probability of various propositions being true, based on one's findings. The infinitist, of course, insists that this is a never-ending process.

Klein's version of infinitism rejects an assumption that appears to be driving the regress problem: that a belief can be justified only if one accumulates decisive reasons in support of it. Instead, he contends that the best one can

hope for is being provisionally or progressively justified, where justification increases as one acquires additional reasons and where there is no limit to the number of reasons to be so collected (Klein, 2007, 10). The infinitist suggests we adjust our expectations, becoming content with a picture according to which responsible epistemic agents cite the appropriate propositions as reasons as they *progressively* justify any given belief. Accordingly:

> Infinitism is committed to an account of *propositional justification* such that a proposition, p, is justified for S *iff* there is an endless series of non-repeating propositions available to S such that beginning with p, each succeeding member is a reason for the immediately preceding one. It is committed to an account of *doxastic justification* such that a belief is doxastically justified for S *iff* S has engaged in tracing the reason in virtue of which the proposition p is justified far forward enough to satisfy the contextually determined requirements. (Klein, 2007, 11, emphases in original)

Among other things, the distinction between propositional and doxastic justification is used by Klein to combat the *finite minds objection*, which proceeds from the fact that human minds and lifespans are finite. Because of these limits, one cannot ever be justified in holding a belief since one cannot actually hold an infinite number of beliefs—something that appears to be required on the infinitist account. Klein's response, roughly, is that the objection is based on a confusion between different sorts of justification.

The infinitist flavor of the account comes from propositional justification, where an infinite chain of reasons is made *available* to an agent; and the provisional, progressive nature of doxastic justification is what dispenses with the expectations driving the finite minds objection.

The progressive feature of justification has been given an altogether more precise treatment by Peijnenburg (2007), Peijnenburg and Atkinson (2008, 2009, 2013), and Herzberg (2010, 2013). By attending to the formal details of various infinite probabilistic regresses, they have provided several mathematical illustrations of progressive justification. Suppose that a proposition E_0 is made probable by another proposition E_1, E_1 is made probable by E_2, and so on *ad infinitum*. It has been variously proven that the probability of E_0 can be precisely calculated even if the number of steps of probabilistic justification n is infinite.[2] Furthermore, Peijnenburg, Atkinson, and Herzberg

[2]Consider briefly one such example, found in Peijnenburg and Atkinson (2009) and Herzberg (2013). The probability of a proposition E_0 being true after n steps of probabilistic justification can be decided *via* the following equation:

$$P(E_0) = P(E_{n+1}) \prod_{i=0}^{n} \gamma_i + \sum_{\ell=0}^{n} \beta_\ell \prod_{i=0}^{\ell-1} \gamma_i$$

where N denotes the set of non-negative integers, and where all $k \in \mathrm{N}$:

$$\alpha_k = P(E_k|E_{k+1}), \quad \beta_k = P(E_k|\neg E_{k+1}), \quad \gamma_k = \alpha_k - \beta_k$$

After n steps of justification, a lower estimate for the probability of $P(E_0)$ might obtain; and given that $P(E_{n+1})$ is unknown and that $P(E_{n+1}) \prod_{i=0}^{n} \gamma_i$ might be very small:

$$P(E_0) \geq \sum_{\ell=0}^{n} \beta_\ell \prod_{i=0}^{\ell-1} \gamma_i$$

In the formal limit that n goes to infinity, we arrive at:

$$P(E_0) = \lim_{n\to\infty} P(E_{n+1}) \prod_{i=0}^{n} \gamma_i + \sum_{\ell=0}^{\infty} \beta_\ell \prod_{i=0}^{\ell-1} \gamma_i$$

have shown that the probability for any given proposition can actually be decided by a finite agent, even if that proposition belongs to an infinite series of propositions; not only is this probability computable, but it is completable by such an agent.[3]

Key to their case is the finding that deciding the probability of any given proposition being true *emerges* from the conditional probabilities making up an infinite chain of propositions, rather than some grounding proposition with an unconditional probability *transferring* justification up the chain (Peijnenburg and Atkinson, 2013). Even if there were some proposition grounding such a chain (as foundationalists insist), its contribution to deciding the conditional probability of another proposition much farther down the chain would be diminished by the number of links between them; and in the limit of an infinite chain, any such grounding proposition makes no contribution whatsoever. So, when wielding any of the equations of interest to decide the precise probability of E_0, where E_0 belongs to a chain of reasons with n steps of probabilistic justification with n being infinite in order:

> The standard way to investigate the convergence of an infinite series is first to look at a finite series of, say, $n+1$ terms only, with a remainder term, and then to investigate what happens as n tends to infinity.... In the formal limit that n goes to infinity, we find that the series has an infinite number of terms, and

[3]See Gwiazda (2010) for the criticism that something being computable does not imply that it is completable; and see Peijnenburg (2010) for a defense against that criticism.

> that the terms [in the equations of interest] that contain the unknown $P(E_{n+1})$ tend to zero, and hence disappear completely. (Peijnenburg, 2007, 600)

Given these demonstrations, it is not surprising that Peijnenburg and Atkinson suggest that Klein's own response to the finite minds objection "concedes too much" (2008, 336-337).

Infinitism and available reasons

Given the distinctive emphasis that Peijnenburg, Atkinson, and Herzberg place on calculability, we have doubts about the extent to which (on their account) an infinite chain of propositions can serve as *reasons* that are *available* to an agent. (This is what shall be called the *availability problem* facing the distinctive brand of infinitism under consideration.)

To tease out the availability problem, consider first an variation of the finite minds objection, that we have raised elsewhere (i.e., Podlaskowski and Smith, 2011) against Klein's infinitism. Our original objection turns, in part, on Klein's reaction to the traditional finite minds objection: namely, that agents need not actually possess each reason in a chain of reasons; rather, each such reason need only be *available* to the agent. A reason is available to an agent so long as she either possesses a second-order disposition to form a disposition to take on the reason (Klein, 1999), or possesses an epistemically credible disposition to cite that reason (Klein, 2007). We have denied that

appealing to any such disposition improves Klein's position (Podlaskowski and Smith, 2011, 521-522). More specifically, we have argued that finite agents are not appropriately disposed for every given belief within an infinite chain of reasons. In order to possess the requisite second-order dispositions to form beliefs to cite as reasons—as Klein suggests—those dispositions must be sensitive to the *order* of *every* given reason in the pertinent chain of reasons. But because we are *finite* agents (with limited abilities and lifespans), we lack the requisite dispositions to attend to the particular order in which reasons occur in an *infinite* chain of reasons. In short, our original objection is that Klein's infinitist fails to provide an account of *availability* that can be used to help avoid the finite minds objection.

In a recent article, Herzberg (2013, 373-374) brings the apparatus introduced by Peijnenburg to bear on this objection. Herzberg suggests that the formal treatment of infinite probabilistic regresses demonstrates (*contra* Podlaskowski and Smith, 2011) the genuine possibility that, as the infinitist maintains, justification is progressive.[4] Conspicuously absent from Herzberg's own response, however, is mention of *available reasons*. More specifically, though Herzberg mentions available reasons in his initial presentation of our charge (raised in Podlaskowski and Smith, 2011), his reply to that charge does not address our challenge to availability for Klein's brand

[4]Peijnenburg and Atkinson do not distinguish between propositional and doxastic justification. In contrast, Herzberg (2013, 371) suggests that their work pertains primarily to propositional justification and to doxastic justification derivatively (since the latter is parasitic on the former).

of infinitism. Clearly, this presents a problem, since the charge we originally raised is aimed at precisely this notion, as characterized by Klein. The importance of this notion for infinitism is made vivid by Klein's comment that:

> We don't have to traverse infinitely many steps on the endless path. There just must be such a path and we have to traverse as many as contextually required. (2007, 13)

Insofar as justificatory relationships holding between propositions serve as this endless path, to say that any given reason is available to an agent amounts to an agent (as a matter of principle) being able to traverse any given step along the endless path. It appears that, without some notion of an *available reason* in place, the infinitist appears without their characteristic claim that there are infinitely long chains of propositions that can serve as *reasons*. This brings us to our principal worry about the infinitist position championed by Peijnenburg, Atkinson, and Herzberg (the aforementioned *availability problem*). Using Herzberg's defense of infinitism as our jumping-off point, we will argue that it is hard to see, more generally, how the emphasis on calculability yields a notion of *available reason* (or *availability*) that can serve the infinitist's purposes.

Given the gap just identified in Herzberg's defense of infinitism from our original objection (i.e., Podlaskowski and Smith, 2011), an expansion of that defense seems to be in order, one that incorporates the notion of an *available reason*. The natural expansion seems to be that, since mathematical means

exist with which a finite agent can decide the probability of any given proposition being true (even if it belongs to an infinite series), all of the members of an infinite chain of propositions must thereby be *available* (as reasons) to an epistemic agent. Even so expanded, though, nothing in Herzberg's defense undermines the case made in Podlaskowski and Smith, 2011, that finite agents fail to possess the dispositions that Klein suggests, and in the required order. To be fair, Herzberg et al. have shown that deciding the probability of any given proposition is possible, even if there are infinite chains of propositions. But this is still a far cry from showing that, as a matter of principle, each proposition in a chain of propositions is one that can serve as a *reason* for another proposition in that chain, and do so in the right order. It appears that two kinds of dispositions have been conflated: those to make a certain sort of calculation, and those to accept any given proposition *qua* reason for another proposition. The proposed expansion of Herzberg's defense is akin to claiming that, just because we can use a universal generalization (e.g., Everything William says is true) to do the same work as using an infinitely long conjunction (e.g., William says p which is true, and William says q which is true, and so on, *ad infinitum*), and one is disposed to use the former, that one is also disposed to use the latter. But, quite plausibly (owing to our finite limits), we are not disposed to assert infinitely long conjunctions; and, similarly, we are not disposed to form dispositions to accept each of the dispositions belonging to an infinitely long chain (in

the right order).[5] So, again, a demonstration that finite agents can actually calculate the probability of a proposition's truth—even if it belongs to an infinite chain of reasons—does not thereby show that each reason is equally *available* to a finite agent.

To show how deep the availability problem goes, imagine Carl, whose impressive talent in calculating conditional probabilities is strangely at odds with his ability to grasp various concepts. Carl has no problem solving all manner of complex equations, including those involving conditional probabilities (such as Peijnenburg, Atkinson, and Herzberg provide). Yet, there are various concepts which he is entirely incapable of grasping, some of which might feature in reasons whose probabilities of being true are conditional on other reasons. Suppose that Carl is given two lists, an infinite list of conditional probability assignments and an infinite list of reasons. Unbeknownst to Carl, the two lists correspond perfectly: the list of probabilities is meant to capture the probability of each reason being true, conditional on its predecessor. Moreover, some of the members of the list of reasons are comprised of those concepts that Carl is incapable of grasping. Even if Carl were capable of working through some infinite list of reasons, at some point on the list at hand, Carl would fail to comprehend the concepts deployed. But he would have *no problem* doing the corresponding calculations. Does merely

[5]Turri (2013) has argued (*contra* Podlaskowski and Smith, 2011) that agents do, in fact, possess dispositions pertaining to an infinitely long chain of reasons (and in the right order). See Smith and Podlaskowski, 2013, for a rejoinder to Turri, where it is argued that the requisite dispositions are indeed lacking in agents like us, at least in any sense that the infinitist should accept.

calculating the probability of the chain make Carl justified in holding any of those beliefs, when Carl is *incapable* of understanding the concepts on which those beliefs depend? Surely not. If an agent *cannot* understand some of the reasons in the infinite chain, it is difficult to see how those reasons can do any justificatory work for him.

The case of Carl puts infinitists such as Peijnenburg et al. in familiar—and perhaps unwelcome—company, for the problem illustrated by the Carl case closely parallels a problem commonly raised against reliabilism. Reliabilists (whose aim is to explain knowledge and justification in terms of reliable processes) are often faced with cases where a person is perfectly reliable (in the relevant respect), but has *no idea whatsoever* about her reliability.[6] In such cases, the typical intuition—*contra* reliabilism—is that the non-reflective yet reliable individual has unjustified beliefs. Similarly, the Carl case shows how Peijnenburg et al.'s brand of infinitism faces a serious problem if calculability plays an important role in making propositions available as reasons.

One might, at this point, suspect that we have crafted the Carl case too narrowly, and that it misses some important aspect of what mathematical analyses of probabilistic regresses are supposed to be doing. Perhaps there is a notion of *available reason* that can supplement the project of Peijnenburg et al. that avoids the problems raised by the Carl case. The problem with successfully developing such a response, however, is that it is entirely unclear

[6]See, for example, BonJour (1985).

what sort of notion they could use, given their emphasis on calculability. To see this, consider the spectrum of possible views. On one end, the notion of *availability* drops out. This end of the spectrum has the unfortunate consequence that the view collapses into maintaining that a belief is justified for a person when there merely exists an infinite, non-repeating chain of reasons that makes the belief probable. But this dramatically externalist picture would run face-first into many well-known problems facing externalism, including clairvoyance cases and, potentially (depending on how the notion of a reason is understood), the new evil demon problem (Cohen, 1984). On the other end of the spectrum, one might hold a very strong notion of *availability*, according to which it is required that one *actually believe* a reason for it to be available. But this is far too strong, as it runs face-first into the original finite minds objection to infinitism. Framed in this way, Klein's view is an attempt to find a middle ground on this spectrum, one designed to cast infinitism as a reasonable position. One lesson to draw from the Carl case is that moving a brand of infinitism beyond Klein's middle ground on the notion of *availability* proves seriously problematic; this case illustrates the fragility of Klein's position on availability, and how attempts to abandon it in either direction are open to familiar problems.

The upshot of the Carl case is that one must be sensitive to reasons *as reasons* in order for them to do justificatory work. Infinitist positions invoking any notion of availability that does not make this point clear will face serious difficulties motivating infinitism. After all, Klein's motivation

for infinitism stemmed from the thought that an epistemically responsible agent will always seek another reason for her belief. However, as argued above, there is quite a difference between being able to calculate a probability (on the one hand) and recognizing something *as a* reason for another belief (on the other). But it is exactly *that*—the ability to recognize a reason *as a reason* and the need to continue doing so—which motivates infinitism. Peijnenburg, Atkinson, and Herzberg owe an account of availability before their impressive work can vindicate infinitism; and seeing how they might go about successfully articulating the notion of *availability* is difficult indeed.

Having illustrated the availability problem in a general way, we are now positioned to consider a more subtle expansion of this defense of infinitism. Instead of thinking of the notion of *available reason* as being fixed somewhere along a spectrum, one might think of the availability of a reason as tied to the contribution that a reason makes to the justificatory status of a proposition. Indeed, it might appear that such a conception comes from appreciating the impact of a finding central to the brand of infinitism espoused by Peijnenburg, Atkinson, and Herzberg: namely, the farther down a chain of a reasons some proposition is from another proposition, the less the former proposition contributes to the justification of the latter. Accordingly, because some distant proposition makes virtually no justificatory contribution, that proposition need not be available whatsoever to an agent.[7]

This might, at first blush, seem attractive as a solution to the availability

[7]This response has been suggested to us by Peijnenburg and Atkinson.

problem. But closer consideration reveals that this solution is incompatible with the infinitist's signature view that being justified requires (among other things) belonging to an *infinite* chain of *reasons*. To see this, recall the point (made earlier) that it is hard to see how a proposition can qualify as a *reason* in the first place unless it is *available* to an agent. By conceding (as the proposed solution does) that some of the propositions belonging to an infinite chain are not available to an agent (nor need they be), we reach a curious result: those propositions do not count as reasons whatsoever. So, the proposed solution (if fused with the stance of Peijnenburg et al.) results *not* in an infinitist position that *infinite* chains of *reasons* support propositions; rather, it yields merely an anti-foundationalist view that deciding the probability of a proposition does not require a grounding reason. Of course, this would still be an important anti-foundationalist result. But Peijnenburg et al. take their mathematical insights also to be relevant to discussions of infinitism and its defense.[8] If the proposed solution were embraced, their results would not really have a proper place in those discussions. As such, it is recommended that they not champion this solution to the availability problem. And so we are left with our original concern: pending further arguments, focusing on the ability to calculate probabilities (at least in the fashion of Peijnenburg, Atkinson, and Herzberg) fails to get infinitism off the ground.

[8]Though Peijnenburg and Atkinson take their results to be especially relevant to discussions of infinitism, they have also shown their willingness to entertain the possibility that some chains of justification are only finite in length (2013, p. 559).

Conclusion

In closing, while the defense of infinitism pioneered by Peijnenburg introduced important new materials to the discussion, we must take care when evaluating the reach of those materials. The mathematical illustrations of interest reveal some important features of probabilistic justification. For instance, Peijnenburg et al. take themselves to be dispelling an objection according to which there cannot be an infinite chain of reasons, owing to the fact that a chain of reasons transfers justification up the chain *via* its links: without an initial source of justification (e.g., supplied by a foundation), there is nothing to be transferred up the chain. They regard their mathematical illustrations to serve as counterexamples to this charge. We take no issue with this response. Rather, our concern is that, though these illustrations serve one important purpose for the infinitist, they do not help fashion a notion of an *available reason* required by the infinitist. It remains to be seen if a rigorous treatment of probabilistic regresses can be provided that adequately captures this notion which remains important to infinitism.

College of Liberal Arts
Fairmont State University
1201 Locust Avenue
Fairmont, WV 26554, USA
apodlaskowski@fairmontstate.edu

Department of Philosophy and Religion
Central Michigan University
288 Anspach Hall
Mt. Pleasant, MI 48858, USA
smith45j@cmich.edu

References

BonJour, Laurence. 1985. *The Structure of Empirical Knowledge*. Cambridge, Mass.: Harvard University Press.

Cohen, Stewart. 1984. "Justification and Truth." *Philosophical Studies*, 46: 279–96.

Gwiazda, Jeremy. 2010. "Infinitism, Completability, and Computability: Reply to Peijnenburg." *Mind*, 119, no. 476: 1123–1124.

Herzberg, Frederik. 2010. "The Consistency of Probabilistic Regresses. A Reply to Jeanne Peijnenburg and David Atkinson." *Studia Logica*, 94: 331–345.

Herzberg, Frederik. 2013. "The Consistency of Probabilistic Regresses: Some Implications for Epistemological Infinitism." *Erkenntnis*, 78, no. 2: 371–382.

Klein, Peter. 1998. "Foundationalism and the infinite regress of reasons." *Philosophy and Phenomenological Research*, 58, no. 4: 919–925.

Klein, Peter. 1999. "Human knowledge and the infinite regress of reasons." *Philosophical Perspectives*, 13: 297–325.

Klein, Peter. 2003. "When infinite regresses are *not* vicious." *Philosophy and Phenomenological Research*, 66, no. 3: 718–729.

Klein, Peter. 2005. "Is infinitism the solution to the regress problem?" In *Contemporary Debates in Epistemology*, edited by Matthias Steup and Ernest Sosa, 131–140. Malden, Mass.: Blackwell.

Klein, Peter. 2007. "Human knowledge and the infinite progress of reasoning." *Philosophical Studies*, 134: 1–17.

Peijnenburg, Jeanne. 2007. "Infinitism Regained." *Mind*, 116, no. 463: 597–602.

Peijnenburg, Jeanne. 2010. "Ineffectual Foundations: Reply to Gwiazda." *Mind*, 119, no. 476: 1125–1133.

Peijnenburg, Jeanne and David Atkinson. 2008. "Probabilistic Justification and the Regress Problem." *Studia Logica*, 89: 333–341.

Peijnenburg, Jeanne and David Atkinson. 2009. "The Solvability of Probabilistic Regresses. A Reply to Frederik Herzberg." *Studia Logica*, 94: 347–353.

Peijnenburg, Jeanne and David Atkinson. 2013. "The Emergence of Justification." *The Philosophical Quarterly*, 63, no. 252: 546-564.

Podlaskowski, Adam and Joshua Smith. 2011. "Infinitism and Epistemic Normativity." *Synthese*, 178: 515–527.

Smith, Joshua and Adam Podlaskowski. 2013. "Infinitism And Agents Like Us: Reply To Turri." *Logos & Episteme*, IV, no. 1: 125–128.

Turri, John. 2013. "Infinitism, Finitude, and Normativity." *Philosophical Studies*, 163, no. 3: 791–795.