Towards a General Definition of Modeling

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Abstract. What is a model? Surprisingly, in philosophical texts, this question is asked (sometimes), but almost never – answered. Instead of a general answer, usually, some classification of models is considered. The broadest possible definition of modeling could sound as follows: *a model is anything that is (or could be) used, for some purpose, in place of something else*. If the purpose is "answering questions", then one has a *cognitive model*. Could such a broad definition be useful? Isn't it empty? Can one derive useful consequences from it? I'm trying to show that there is a lot of them.

Keywords: model, theory, model template, model-based picture of cognition, Dappled World, limits to modeling.

This article is written from an anti-Platonist point of view, according to which, mental structures and activities are considered to be parts of the physical Universe. It follows the program proposed in my paper Podnieks (2009).

What is a model?

Surprisingly, in philosophical texts, this question is asked (sometimes), but almost never – answered. Instead of a general answer, usually, some classification of models is considered. A general answer can't be found even in the prominent account given by Roman Frigg and Stephan Hartmann (2006). Should this mean that a sufficiently general definition of modeling is impossible?

Computer scientists are proposing very broad unified definitions of modeling since many years.

For example, A* is a model of A, if one can use A* in place of A to answer questions about A. This very general definition of the term "model" was proposed by Marvin L. Minsky (1965, p. 45):

"We use the term "model" in the following sense: To an observer B, an object A* is a model of an object A to the extent that B can use A* to answer questions that interest him about A. The model relation is inherently ternary. Any attempt to suppress the role of the intentions of the investigator B leads to circular definitions or to ambiguities about "essential features" and the like."

A more compact definition was proposed by Jeff Rothenberg (1989, p. 75): "Modeling in its broadest sense is the cost-effective use of something in place of something else for some purpose."

It seems, philosophers are approaching a comparable level of generality only recently. For example:

Paul Teller (2001, p. 397): "... in principle, anything can be a model, and that what makes the thing a model is the fact that it is regarded or used as a representation of something by the model users. ... it would be a mistake for the general account of the use of models in science to specify more narrowly what can function as a model."

Mauricio Suarez (2004, p. 774): "[inf]: A represents B only if (i) the representational force of A points towards B, and (ii) A allows competent and informed agents to draw specific inferences regarding B."

Ronald N. Giere (2010, p. 269): "... since just about anything can be used to represent anything else, there can be no unified ontology of models”.

Thus, the broadest possible definition of modeling could sound as follows: *a model is anything that is (or could be) used, for some purpose, in place of something else*. If the purpose is a kind of “answering questions”, then one has a *cognitive model*.

Could such a broad definition be useful? Isn't it empty? Can one derive useful consequences from it?

According to the above definition, a model is a *single concrete system* replacing, for some purpose, another *single concrete* system (*target system*).

For example, a model of the Solar system might include: a specified number of planets (8), a specified diameter and mass of Sun (1.4 x 10^6 km, 2.0 x 10^{20} kg), and for each planet: specified diameter, mass, initial location and velocity. No satellites, no asteroids, no Milky Way, no Universe.

To enable computer simulation, the Newtonian mechanics with the Gravitation law also should be included in the model as means of reasoning. *Without means of reasoning included, cognitive models would be useless semi-defined structures.* Most of information contained in the model, is not represented explicitly. For example, at which moments, the 3rd, 4th and 5th planet will be located “on line”? Thus, one must use some means of reasoning to answer non-trivial questions about the model. And, only by answering questions about the model, one can try answering questions about the target system.

The “competent and informed agents” mentioned in the above definition by Suarez are allowed to use means of reasoning without formulating them explicitly. Thus, different agents might apply different means of reasoning to the same model structure, obtaining possibly different results. To avoid this kind of confusion, I would propose to specify the *allowed means of reasoning* explicitly and always consider them as *part of the model*.

The model of an “arbitrary” system of *n* planets, is *not* a model in the above sense (it's a *model template*, see below).

According to the above definition, models exist as *independent systems*. Hence, some models may exist *before* the modeling relation with the target system is established. For example, the design of a building is created before the building is built. And it is possible even that some designs will not be implemented at all. But, in principle, non-implemented designs don't differ from implemented ones. Thus, one can create and explore models that “aren't modeling” (currently, or at all), but are similar to some “modeling models”. “Non-modeling models” can be obtained also by modifying the “modeling” ones. Thus, virtual realities used in computer games, dreams and even hallucinations can be regarded (and best understood!) as models. And thus, the addition of “or could be” in the above definition of modeling is fully justified.

In some situations, the modeling relation may be *reflexive* (“target system is the best model of itself”), but not for all purposes. For example, direct experimenting with the target system may be too expensive, too dangerous, or impossible at all – as put by Jeff Rothenberg (1989, p. 78). Similarly, in some situations, the modeling relation may be *symmetric* and *transitive*, but again – not for all purposes.

Role of models in cognition

For an elaborated model-based picture of cognition see Carlos Hernandez et al. (2008, Section 4).

The ultimate purpose of cognition is knowing how to handle concrete systems surrounding us. Even when trying to reconstruct the history of the Universe we intend obtaining a useful knowledge for everyday practice. Thus, models as concrete replacements of “concrete systems surrounding us” are the central element of cognition. Building and exploring of models is the *ultimate phase* of cognition.

This simplifies greatly the picture of cognition. *All we need are models, means allowing to build and explore models, means allowing to create means allowing to build and explore models, etc.* Various other products of cognition (languages, logics, ontologies, theories, laws, methods, paradigms, ideologies, etc.) are useful only to the extent they allow for better building and exploring of models.

Any activities of human mind can be best understood as modeling.

Role of theories

What should be the role of theories in this picture of cognition? None of theories is modeling a single concrete system. Thus, theories are not models. Newtonian mechanics is not a model, but it allows for building of models of various concrete mechanical systems. Could you imagine a good theory that merely "explains the world", but does not help in building of good models and exploring them?

Thus, in this (model-based) picture of cognition, *theories are useful only as means of building and exploring models.* Theories are best understood as "The Toolbox of Science" – as put by Mauricio Suarez and Nancy Cartwright (2008).

Model templates

Between models and theories, there is an important intermediate concept that is called *model template* by computer scientists. The term "parametric model" also would be appropriate.

A *model template* is a structure containing parameters. By setting these parameters to concrete values, we obtain models (*instances* of the template). The above-mentioned "arbitrary" system of *n* planets is a model template having as parameters: *n* – the number of planets, D, M – diameter and mass of the central body, d_i, m_i – diameter and mass of i-th planet, etc. Newtonian mechanics with the Gravitation law are included in the template as means of reasoning. By setting parameters appropriately (n=8 etc.) we can obtain, for example, a computer simulation of Solar system.

Some theories can be represented almost entirely as model templates. For example, the Hamiltonian formulation of Newtonian mechanics can be regarded as a model template of an arbitrary mechanical system having *6n+2* parameters: *n* – the number of components, *H* – Hamiltonian function of the system, and *6n* initial coordinates and impulses. Quantum mechanics allows a similar formulation in terms of parametric quantum systems. Thus, some theories are "models" indeed – but parametric models.

The so-called London model of super-conductivity analyzed by Mauricio Suarez and Nancy Cartwright (2008) also is a model template, and not a model in the above sense.

The idea that some theories are, in fact, model templates can be found already in Ronald N. Giere (1979). See also Ronald N. Giere (1985), where on p. 78 model templates are called *general models*.

Less organized modeling...

Model templates generated by a single theory represent, in a sense, the "homogeneous" or "strictly organized" extreme of model-building.

Less organized cases of model-building are discussed, for example, by Mauricio Suarez and Nancy Cartwright (2008). Namely, when building models, people are applying not only accepted theories in a regular way. As put by Suarez and Cartwright, sometimes they introduce *ad hoc* assumptions that "do not follow from theory either by de-idealisation or by introducing otherwise acceptable descriptions of the facts" (p. 70). One may use assumptions with no theory behind, or, what Suarez and Cartwright are calling “piecemeal borrowing” (p. 73) – applying the results of theories out of their regular context.

Piecemeal borrowing, and even applying of *ad hoc* combinations of entire theories is, in a sense, the “heterogeneous” extreme of model-building. The Millennium Simulation Project performed by The Virgo Consortium represents a striking example of this kind – as argued by Stephanie Ruphy (2008).

Precision of modeling

Since models are independent systems, their potential of answering questions about target systems (“precision of modeling”) may be limited.

The simplest kind of models is obtained from target systems by applying various kinds of simplification (omission, abstraction, linearization, other kinds of idealization, etc.). When compared to target systems, such models may contain deliberate *deformations*.

But as independent systems, models may possess also – when compared to target systems – *“excessive” properties*. Consider, for example, rats as model organisms used to test human drugs.

But, in a sense, the most advanced kind of models is produced by physicists. Most of modern physical models – starting already with Ptolemy, Copernicus and Kepler – are not “derived” from experimental data, they are almost *pure inventions* of human mind. As put by Albert Einstein (1930) on the occasion of 300th death anniversary of Kepler:

"It seems that the human mind has first to construct forms independently, before we can find them in things. Kepler’s marvelous achievement is a particularly fine example of the truth that knowledge cannot spring from experience alone, but only from the comparison of the inventions of the intellect with observed fact." – an English translation from Bruce Director (2006).

In physics, the distance between theoretical constructs (such as, for example, quarks, or Big Bang) and their “points of confirmation” may be considerable.

Limits to modeling

Now, the main point: there are systems that *can't be modeled in full detail even in principle*.

For example, how much of the Universe can be captured in a *single* model? How detailed can be a *simulatable computer model* of the Universe? Could it simulate *every* photon and *every* neutrino traveling across the Universe? Could this model include the model of the simulating computer itself?

Laplace was aware of this problem: in 1812 he concluded that despite having a “Theory of Everything” (Newtonian mechanics with the Gravitation law), we will never be able to predict the future in every detail: “... our efforts will always fall infinitely short of this mark” – an English translation from Kevin D. Hoover (2001, p. 101).

But, contrary to Laplace, the impossibility of such detailed models is caused not by the (actual, or future) limitations of human minds or technical resources. The *limitation is built into the very principle of modeling*: we are trying to *replace some system by another one*. In full detail, this may be impossible. And *this* limitation is “built” into the very structure of the physical Universe. Very detailed models of the Universe simply can't exist... in the Universe – at least, as we know it today. Of course, this conviction represents a metaphysical hypothesis about “how the world really is”. How much of the Universe can be captured by a tiny fragment of it?

Are there systems, that are much smaller than the Universe, but for which detailed models also can't exist in the Universe? Which of the systems are “too unique” to be modeled in full detail?

Thus, it makes sense trying to determine the *limiting conditions*, behind which a detailed modeling becomes impossible. Let's ask the following question: for which values of N, two physical systems each consisting of N “separable components” can't be “isomorphic enough” to represent each other in every detail (whatever all that means)?

Let's denote by L the least such N, and let's call it *Laplace's constant*. My hypothesis:

$$10^{19} \leq L \leq 10^{22}.$$

Indeed, could we build 10^6 computers each having a terabyte (i.e. 10^{13} bit) hard disk, store identical terabytes of data, and start identical programs on them. Hence, it seems, $L \geq 10^{19}$?

To “prove” $L \leq 10^{22}$, let's consider an isolated container containing one liter of air. How detailed can be a *simulatable computer model* of such a system? If we believe that air consists of molecules, then our container includes about 10^{22} molecules. Thus, to represent the state of this system at a particular moment of time, we need to store at least 6×10^{22} numbers (coordinates and velocities). And to simulate the evolution of this state in real time, we need to compute – at a very high speed – the solutions of 6×10^{22} Hamiltonian equations. (If, instead of a classical model, we will try building a quantum mechanical one, this won't decrease the number of details necessary to represent precisely “this particular one liter of air”.)

We can't build such a computer. Moreover – such a computer can't exist in the Universe! Because (my metaphysical hypothesis), in the Universe, *two systems each consisting of 10^{22} “separable components” simply can't be “isomorphic enough” to represent each other in every detail*! No two identical liters of air can exist in the Universe! Hence, it seems, $L \leq 10^{22}$?

So, let's establish “Laplace Prize” for determining the exact value of L?

Of course, too large a number of “separable components” is not the only obstacle to a detailed modeling. The impossibility of determining precisely enough of various parameters (masses, coordinates, velocities etc.) is the next one. And, when trying to model complex social systems, the network of different “laws” to be implemented in the model, becomes too complicated, making model-based prediction impossible, as argued by Michael Batty and Paul M. Torrens (2001).

The Dappled World perspective refined

The Dappled World perspective was introduced by Nancy Cartwright (1999, p. 1): “... we live in a world rich in different things, with different natures, behaving in different ways. The laws that describe this world are a patchwork, not a pyramid.”

For me, it isn't a metaphysical hypothesis about “how the world really is”. Because there is no such thing as “laws of nature” – again, as put by Nancy Cartwright (1983, p. 129): “My basic view is that fundamental equations do not govern objects in reality; they only govern objects in models.”

A similar formulation is attributed to Niels Bohr: "There is no quantum world. There is only an abstract quantum physical description. It is wrong to think the task of physics is to find out how nature is. Physics concerns what we can say about nature." – quoted after David Favrholdt (1994, p. 92).

Thus, there are no “laws of nature”, there are only laws implemented in models, and hence, it is only our world picture, that may be “dappled”, or not. For me, the Dappled World perspective is a hypothesis about how our world picture is and always will be.

The above-mentioned limitations to modeling represent an argument in favor of the Dappled World perspective: neither robots, nor humans can hope to create a single model for extensive parts of its/his/her environment. *At the model level, we will always have only a patchwork of models each very restricted in its application scope.*

But at the level of theories, i.e. at the level of means of model-building-exploring? Will we have, some time, a *single* complete Theory of Everything (or, at least, a complete pyramid of theories) not only "explaining everything in the world", but also allowing, *without any additional postulates*, to generate *all* the models we may need? According to the Dappled World perspective, the answer is negative.

The above two negative theses represent a refinement of the Dappled World perspective.

A model template does exist, but its instances don't?

Now, a possible Platonist objection: the above-mentioned detailed models of systems containing 10^{22} separable components do not exist in the physical Universe, but we can *imagine* them! There are "imagined systems" that "would be physical things if they were real" – as put by Roman Frigg (2010, p. 253).

But how could one imagine a computer simulation of 10^{22} molecules, when such computers can't exist in the Universe even in principle? For an anti-Platonist, this means that, in fact, one is using some *mathematical axioms* to derive the "existence" of such an impossibly huge computer. Indeed, even the axioms of first order arithmetic would suffice to prove, for any N, the "existence" of a computer having a 10^N bit storage device. But almost all of that is far beyond what can exist in the physical Universe!

What really exist in the physical Universe, are *definitions of model templates* – because they can be written down on a paper. For example, one can write down a definition of the following model template: an isolated container containing the so-called *hard-ball-gas* having as parameters: *n* – the number of spherical molecules; dimensions of the container; and for each molecule: diameter, mass, initial location and velocity. The Hamiltonian function of this system is defined as a simple sum of kinetic and potential energies of molecules, assuming elastic collisions. This *model template* written down on a paper exists in the physical Universe in the most solid sense possible.

But, if we set $n=10^{22}$, then there is no real way of assigning particular values even to the initial coordinates and velocities of each hard-ball molecule. Thus, while the model template itself does exist in the physical Universe, for $n=10^{22}$, its *instances* don't – they "exist" only according to the axioms used for theoretical analysis of the template.

What could be done in such a situation?

At the model level, we can try computer simulation only of very small models of hard-ball gas containers: for $n=10^3$, 10^4 etc. For impressive educational programs of this kind see Paul Falstad (2009: Gas Molecules Simulation) and John I. Gelder et al. (2000: Chemistry Web Server). For a research application involving such simulations – see Yakov G. Sinai et al. (2008). It appears that even for such small amounts of molecules, for example, the observed (model!) distribution of velocities is similar to the well-known Maxwell-Boltzmann distribution.

But, perhaps, the most striking "small" simulation experiment of this kind – The Millennium Simulation Project was performed by The Virgo Consortium. The simulated Universe of this experiment consists of about 10^{10} big gravitating "particles" (each about 10^9 solar masses in the first simulation, and about 10^7 – in the second one). Trillion times less details – when compared to one liter of air!

And, at the level of theory?

For realistic numbers of molecules, most *particular* instances of the hard-ball gas model templates are not accessible to analysis. But one can try proving *general* theorems about *all* instances, or about large (but simply definable) sets of them. One can try calculating various *average properties* over these sets of instances. This is exactly what is going on in statistical mechanics since Gibbs introduced the so-called *microcanonical ensemble*!

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