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The Paradigm Shift in the 19th-century Polish Philosophy of Mathematics

Abstract

The Polish philosophy of mathematics in the 19th century had its origins in the Romantic period under the influence of the then-predominant idealist philosophies. The decline of Romantic philosophy precipitated changes in general philosophy, but what is less well known is how it triggered changes in the philosophy of mathematics. In this paper, we discuss how the Polish philosophy of mathematics evolved from the metaphysical approach that had been formed during the Romantic era to the more modern positivistic paradigm. These changes are attributed to the philosophers Henryk Struve, Antoni Molicki and Julian Ochorowicz, and mathematicians Karol Hertz and Samuel Dickstein. We also show how implicit ideas (i.e., those not declared openly) from the area between the philosophy

of science and general philosophy played a crucial role in the paradigm shift in the Polish philosophy of mathematics.

Keywords: *Polish philosophy, philosophy of mathematics, Romantic philosophy, Positivism, Samuel Dickstein, Karol Hertz, Antoni Molicki, Julian Ochorowicz, Henryk Struve*

Zmiana paradygmatu w polskiej filozofii matematyki dziewiętnastego wieku

Abstrakt

Polska dziewiętnastowieczna filozofia matematyki ma swe źródła głównie w okresie romantyzmu, co ma związek z wpływem dominującej wówczas filozofii idealistycznej. Upadek filozofii romantycznej wyzwolił proces przemian w obrębie filozofii polskiej, ale dotychczas nie analizowano, jak przełożyło się to na przemiany w obrębie polskiej filozofii matematyki. W niniejszym artykule przedstawiono, w jaki sposób dokonało się przejście od metafizycznego podejścia do filozofii matematyki ukształtowanego w dobie romantyzmu do nowszego paradygmatu pozytywistycznego. Te przemiany związane są z działalnością filozofów: Henryka Struvego, Antoniego Molickiego i Juliana Ochorowicza oraz matematyków: Karola Hertza i Samuela Dicksteina. W artykule ukazano również jak pewne ukryte (tj. nie wymieniane wprost) idee z pogranicza filozofii nauki i szeroko rozumianej filozofii odegrały kluczową rolę w zmianie paradygmatu w polskiej filozofii matematyki.

Słowa kluczowe: *historia filozofii polskiej, filozofia matematyki, filozofia romantyczna, pozytywizm, Samuel Dickstein, Karol Hertz, Antoni Molicki, Julian Ochorowicz, Henryk Struve*

1. Introduction

The history of the 19th-century Polish philosophy of mathematics, compared to its counterpart in the 20th century, is still rather poorly understood. From this era, only five personalities of great importance are usually mentioned (see e.g. Murawski 2014), namely Jan Śniadecki (1756–1830), Józef Maria Hoene-Wroński (1776–1853), Henryk Struve (1840–1912), Samuel Dickstein (1851–1939), and Edward Stamm (1886––1940). Unfortunately, several key issues have often been omitted

from such discussions: First, the contributions of these five researchers are discussed in isolation from other developments in philosophy. Second, the specificity of individual research centers is not accounted for or elucidated upon. Third, several important issues, such as the reasons for the growing interest in the philosophy of mathematics in Poland and the main philosophical currents of this era, are not explored. Fourth, it is never clarified as to how the 19th-century philosophy laid the foundations for the modern Polish philosophy of mathematics in the 20th century. Even if the last issue is beyond the scope of this paper, it is worth giving some comments to initiate discussion of the issue.

In the recent study "Mathematics and metaphysics: Romantic period of Polish philosophy of mathematics heritage" (Polak 2021), the author shows how philosophical reflection on mathematics emerged in Polish philosophy. Indeed, this process originated from two traditions, namely a short-lived Enlightenment tradition initiated by Jan Śniadecki and the more-enduring Romantic philosophy that started with Józef Maria Hoene-Wroński.

Polish Romantic philosophers[1], in contrast to the perceived view of Romantic philosophy as being anti-scientific, were very interested in the philosophical problems of mathematics. In fact, these philosophers founded a lasting tradition of metaphysical interpretations for mathematics (for details see Polak 2021). This tradition survived the Romantic period, and after some adjustments, it found some continuation in a revival of Catholic philosophy, namely Neo-Scholasticism. Yet, despite its initial successes, the metaphysical interpretation of mathematics was summarily rejected and subsequently forgotten by the 20th-century thinkers. The history for the development of this metaphysical perspective in the philosophy of mathematics in Poland, from its rise to its eventual dismissal and the emergence of a new paradigm in the philosophy of mathematics, is the topic of this paper.

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[1] I refer in this paper to Polish Romantic philosophy. It was a specific philosophy strongly influenced by Hegel and partially by Fichte or Schelling. Because of this specific character, each of the terms currently used in the historiography of Polish philosophy: Romanticism, Idealism, Messianism, does not fully describe this philosophy unambiguously. Therefore, the term Polish Romantic philosophy is most often used, which, although imprecise, allows for the best collective description of this historical current.

We trace the characteristic elements of this transformation by following the works of five Polish philosophers, namely Struve, Ochorowicz, Molicki, Dickstein, and Hertz. Their ideas seem to be crucial to the evolution of the 19th-century philosophy of mathematics. To place this discussion within the larger context, however, we begin by briefly discussing the background for the Polish philosophy of mathematics, presenting its early incarnations, and describing how Romantic philosophy became responsible for the growing interest in mathematical sciences. This wider perspective will help us to understand how changes in the philosophy of mathematics were galvanized by changes in the broader philosophical context and new mathematical discoveries, and it will also help us to understand the process by which the modern Polish philosophy of mathematics emerged in the 1920s and 1930s.

2. Background to the development of the 19th-century Polish philosophy of mathematics

Polish thought in the 19th century was strongly influenced by a very unfavorable geopolitical situation. Poland had been partitioned between Russia, Prussia, and Austria (later Austria-Hungary). The loss of political independence and the subsequent persecution and suppression of the Polish language and culture strongly influenced the 19th-century Polish philosophy. The philosophy of mathematics was also subject to the same limitations and pressures.

Polish Romantic philosophy dominated Polish thought for much of the 19th century, which was mainly occupied with ideas of independence and national liberation. The failed national uprisings in 1830, 1848, and 1863 somewhat undermined the confidence in Romantic ideals, however, and something new was needed. Not surprisingly, beginning in the 1870s, the idea of positivism, with Warsaw acting as its main center, became dominant. The initial center of positivist thought was the short-lived (1862–1869) Polish university in Warsaw, the Szkoła Główna Warszawska. Unfortunately, after a few years, this school was transformed into a Russian institution with Russian being the official language. Outside the university confines, however, Polish philosophical thought continued to flourish. It was within such circles that the most important transformations in the Polish philosophy of mathematics took place.

Going back to the Romantic period, Polish Romantic philosophers, who opposed the Enlightenment ideas of Jan Śniadecki and his followers, focused on the metaphysical aspects of mathematics. This metaphysical approach to mathematics was partially inspired by Hegelian logic and Platonic ontology, thus accepting the Platonic ontology of mathematical objects (i.e., all mathematical objects exist in the absolute mind).

Unfortunately, most Romantic philosophers, aside from Hoene--Wroński, perceived mathematics narrowly as a formal science for the quantitative aspects of reality. They were uninterested in the practice of mathematics, so their philosophy of mathematics was merely a philosophical enterprise developed exclusively for the consumption of fellow philosophers. Thus, the changes that began in the 1870s included not just changes in the philosophical framework but also meta-philosophical changes. In other words, there were changes in the methods of philosophical reflection.

This transformation began with the two philosophers Henryk Struve and Julian Ochorowicz, but it was not until the 1920s that the modern paradigm for the Polish philosophy of mathematics eventually emerged.[2] This came only after the new mathematical ideas of Gauss, Łobaczewski, Bolyai, Riemann, and Helmholtz had been assimilated into philosophy by philosophers and mathematicians Samuel Dickstein and Karol Hertz.

3. Henryk Struve: At the border of paradigms

Henryk Struve (1840–1912) was one of the most important Polish philosophers of the 19th century. He taught at Szkoła Główna Warszawska in Warsaw and later at the Russian Imperial University of Warsaw, and he was certainly a man of strong convictions. He opposed Romantic ideas, which he dismissed as "romantic dreaming" (Jadacki 1997, p. 147). He also opposed a new generation of Polish positivists and their anti-metaphysical philosophy that had been inspired by Comte (Borzym 1974 chap. 5), despite the fact that many of them were his students. His own original philosophy, called "ideo-realism", was also known as "scientific metaphysics" (Skarga 1983).

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[2] This list would not be complete without mentioning Stefan Pawlicki, who in the 1870s became a priest and contributed significantly to modern Catholic philosophy. Pawlicki's philosophy of mathematics has been described by Polak (2021).

Struve was primarily a logician and a historian of logic. His approach to logic was discussed extensively by Murawski (Murawski 2018, 2016, 2014), but little attention has been paid to Stuve's reflection on mathematics, mainly because "Struve's aversion towards mathematics and mathematical methods in logic was connected with his views on the function of language in logic and cognition as well as with his conception of truth" (Murawski 2016, p. 188).

Struve's ideas about the philosophy of mathematics, specifically for the relationship between logic and mathematics, were presented in his book *Wykład systematyczny logiki, czyli nauka dochodzenia i poznania prawdy. Tom I. Część wstępna* (Struve 1870).[3] Struve recognized logic as being at the foundation of mathematics and something closely interlinked. For him, mathematics was "the expression of the rules of existence", while logic was "the expression of the rules of thinking", and "In this connection between the two sciences, […], *logic* is the basis of *mathematics* as well, because mathematical law can be known and developed only by means of and on the basis of the laws of thought, logical principles of reasoning and proof, etc." (Struve 1870, p. 23).[4]

Struve's ideas about logic and mathematics can be traced back to two German thinkers, namely Dobisch (1836) and Lentzen (1861), who he quoted directly, as well as the Polish philosophers Trentowski (1842) and Cieszkowski (1838). However, Struve's approach was different, as was his idea of logic.

Struve stressed that mathematics must be based on philosophy because it needs to use notions like space and time, movement and

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[3] We could trace similarities in his and Trentowski's remarks about the difference between mathematics and philosophy (see Polak 2021). It is also very probable that Struve was also familiar with Kremer's view on mathematics, because he later wrote a very good analysis of Kremer's philosophy (Struve 1881).

[4] „*Matematyka* jest wyrazem prawidłowości całego istnienia; *Logika* wyrazem prawidłowości myśli. A ponieważ myśl jest cząstką istnienia, więc nie dziw, że się logika często z matematyką spotyka; jak się to później, przy danej sposobności nieraz wykaże. W tym zaś związku tych dwóch nauk, *logika* jest jednak podstawą i *matematyki*, bo prawidłowość matematyczną poznać i rozwinąć można tylko przy pomocy i na podstawie prawidłowości myśli, zasad logicznych rozumowania i dowodzenia itd."

Struve later changed some aspects in his view on logic (for more on this topic see Murawski (2014, p. 7–12)). Struve distinguished three types: "(1) formal, (2) metaphysical, and (3) logic treated as the theory of knowledge" (Murawski 2014, p. 10), and he accepted the last one.

force, and quantity and number, notions that are (meta)physical in nature. These concepts have a dogmatic character, so in mathematics, there is no need to analyze them. However, they are not established in mathematics but rather a product of logic and metaphysics, so their analysis is a subject for philosophy (Struve 1875, p. 25, 1896, p. 128; see also Borzym 1974, p. 151–152). (Here, Struve is evidently drawing on the tradition of taking a metaphysical approach to the foundations of mathematics.)

Struve, like his predecessors in philosophy, accepted the distinction of form and content, so for him mathematics was the science of form (i.e., a formal science), so it could describe relationships between observations, but it could not explain their content (Struve 1890, 1896, pp. 352–353).

Unfortunately, Struve's views on mathematics were still tainted by ideas from the Romantic era, demonstrating how strong this tradition still was in Polish philosophy. Struve accepted a very narrow concept of mathematics as a science for the quantitative aspects of reality (Struve 1890), and this perspective was grounded in the traditional (Romantic) concept of mathematics and conflicted with some of his own ideas.

Struve (1896, p. 352) described mathematical methods as "a deduction, or a logical finding, of a specific sentence from general sentences." For him, such methods were not specific to mathematics but also used in logic and philosophy, and as such, they should be critically examined through logic and philosophy from the point of view of metaphysics and epistemology.

Struve (1890) was the first Polish academic philosopher to realize the important contribution to the philosophy of mathematics that was made by the discoveries of Gauss, Lobachevsky, Bolyai, Riemann, and Helmholtz (Struve quoted their research in his works) for n-dimensional Euclidean spaces and non-Euclidean geometries. (This stream of research was called "pan-geometry" or "general geometry.")

Struve rejected the idea of the absoluteness of mathematical theories, which would also include Mill's empiricist concept of mathematics. A critical examination of mathematics led Struve to accept a new branch of science called "meta-mathematics."[5] This concept was inspired by the

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[5] In the 20th century, meta-mathematics changed meaning, but in the 19th century, because of the Kantian tradition, it was typically conceived as a part of philosophy.

works of the mathematician Otto Schmitz-Dumont (1878). (Of course, Kant's philosophy was also an obvious source of inspiration, because Kant was seen as a forerunner to the critical reflection on mathematics.) According to Struve, meta-mathematics belonged to philosophy, and it had an impact on the philosophy of science, as well as on the "general philosophical worldview." Struve stressed that as meta-mathematical ideas were important to the metaphysics of science, philosophers of science must also be familiar with the foundations of mathematics (Struve 1896, p. 364), so he posited that scientific metaphysics should be grounded in concrete mathematical results.[6]

The meta-mathematics promoted by Struve represented an important step in the development of philosophical reflection on mathematics. However, by considering meta-mathematics to be a part of philosophical reflection that was independent of mathematical methods, Struve was following the traditional metaphysical approach of the Romantic era. Later conceptions of meta-mathematics in the Warsaw school were based on different approaches, such as Tarski's, where mathematical methods were used in meta-mathematics (see e.g. Blok; Pigozzi 1988).

Struve's ideas about philosophy and mathematics qualify him as a representative of the metaphysical tradition for the philosophy of mathematics. Struve's views on logic and mathematics can be positioned between the old (Romantic) and the new (early 20th-century) schools of mathematical thought. At the beginning of the 20th century, Kazimierz Twardowski accurately characterized his philosophy "as if a link connecting this new period with the previous one. Between the generations of the Cieszkowskis, the Gołuchowskis, the Kremers, the Libelts, the Trentowskis and the contemporary generation, there appears the distinguished figure of this thinker, writer, who saved from the past what was of lasting value [...]" (Twardowski 1912, p. 102; English translation from: Murawski 2014, p. 7–8). The next step toward the paradigm shift in the Polish philosophy of mathematics was undertaken by a generation of thinkers younger than Struve.

We may speculate that Struve's impact on the generation of positivist philosophers from Warsaw determined, to some degree, the metaphysical style of their thinking. We can see these influences in the many

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[6] This approach resembles Michael Heller's concept of "philosophy in science" (Heller 2019; Polak 2019).

similarities in Struve's and Ochorowicz's styles (the latter is discussed next) of thinking about mathematics, as well as some evident influences in the concept of metaphysics that may have been inspired by science in Ochorowicz's studies (e.g. Ochorowicz 1872, p. 77–79).

4. Mathematics as philosophy: The case of Julian Ochorowicz

Julian Ochorowicz (1850–1917) was Struve's student and one of the most prominent Polish positivist philosophers (Gawor 2009). He is now mainly remembered in Poland as a pioneer of empirical psychology. Ochorowicz's positivist philosophy was built on rejecting the old (Romantic) metaphysics as being erroneous and meaningless, although he did not condemn every form of metaphysics. For him, a good example of metaphysical thinking that was compatible with positivist philosophy was the philosophy of Jan Śniadecki, which in reality was a kind of inductive metaphysics.[7] Moreover, Ochorowicz, following the example of Śniadecki, added that for specific sciences (e.g., physics, chemistry, biology), a critical evaluation of their methodology and an establishment of their fundamental notions would benefit from employing philosophical methodology (Ochorowicz 1872, p. 73).

For Ochorowicz, who was under Comte's inspiration, mathematics was fundamental to other sciences, including philosophy, and he claimed that significant philosophy could only be created by people who were well-versed in mathematics and recognized its fundamental value (Ochorowicz 1872, p. 16–36). Both these sciences, in Ochorowicz's view, have many similarities, although they also differ significantly. For example, they are both general, abstract, and purely rational, but the manner in which they use symbols to express their ideas differs: Philosophy uses words more or less from the common vocabulary, while mathematics uses special symbols, syntax, and semantics. These distinctions led Ochorowicz to the concept of mathematical philosophy:[8]

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[7] Śniadecki's view was also criticized by Libelt as being anti-metaphysical. For more about Śniadecki's and Libelt's philosophy of mathematics, see the work of Polak (2021).

[8] This program of mathematical philosophy was partially formulated by mathematician Stanisław Zaremba in his philosophy of science (see Polak 2015), but Ochorowicz impact on Zaremba seems improbable.

> […] if the ultimate aim of theoretical philosophy is
> discovering the laws explaining phenomena in the Universe,
> this aim will be reached when the laws discovered by
> the philosophical analysis are presented by us explicitly
> (unambiguously) in mathematical formulas. For this
> reason, we consider philosophy a kind of mathematics,
> or mathematics a kind of a philosophy. […] In a word,
> in our belief, philosophical knowledge and mathematical
> knowledge are only different explications of the same
> general, abstract knowledge (Ochorowicz 1872, p. 82).

Ochorowicz's rejection of the "classical" concepts for the relationships
that mathematics has with philosophy and logic was a revolutionary
step. Nevertheless, his ideas did not have much impact on the Polish
philosophy of mathematics, because they were mostly viewed as being
vague declarations of a possible future philosophy and therefore
confined to the meta-philosophical discourse. In reality, Ochorowicz
himself did not explicitly use mathematics in his philosophical analyses,
possibly due to his ignorance of higher mathematics. (He indicated this
as the main obstacle to developing this type of philosophy.)

Similar attempts to formulate a strict philosophy can be found in the
reflections on mathematics in the Kraków milieu. Antoni Molicki, who
"was connected with the positivist movement by significant postulates,"
proposed redefining mathematics and its relation to philosophy
(Głombik 1978, p. 252). By redefining philosophy, and even calling
it "tagmonlogia," Molicki broke from the concept of mathematics as
a science of quantities, instead considering it to be a science of the
general principles that serve to order (our learned) reality. According
to him, the aims of mathematics and his new philosophy were the
same, differing only in terms of their subject matter (Molicki 1875, p.
183nn, 203, 1914, p. 17–18). Thus, Molicki's ideas were close to those
of Ochorowicz, which is unsurprising seeing as he was well acquainted
with the writings of the Warsaw positivists. Within the philosophy
of mathematics, Molicki was inspired by the views of Peacock
and Henkel, as well as Dickstein (described below). Molicki also
mentioned some of Jan Śniadecki's ideas. Despite the clear changes
in the understanding of mathematics and the great hope presented
by Molicki's new "tagmonlogia" philosophy, his reflections on the

nature of mathematics did not influence the development of the philosophy of mathematics in Poland.

The real changes in the philosophy of mathematics in 20th century were inspired by two late 19th-century mathematicians whose work is discussed below.

5. Forerunners to a new philosophy of mathematics: Karol Hertz and Samuel Dickstein

A revolutionary shift in the Polish philosophy of mathematics was brought about by mathematicians Karol Hertz (1843–1904) and Samuel Dickstein (1851–1939). They rejected the teachings of the old masters and instead ushered in a new modern perspective for the Polish philosophy of mathematics.

Karol Hertz was a student of the Warsaw Main School (1862–1866), and in 1871, he obtained his PhD from the University of Halle. He was subsequently appointed to a teaching post at II Gimnazjum Męskie in Warsaw (see Maligranda 2014), where he taught for the rest of his life. His two most significant publications were *Symbolic reasoning* [*Rozumowanie symboliczne*] (Hertz 1880) and *Recent research on space* [*Najnowsze badania nad przestrzenią*] (Hertz 1897), both written in Polish.

The Polish philosophy of mathematics reached a turning point when Hertz rejected the typical, in Polish philosophy at least, distinction between mathematics and logic. This development was somewhat precipitated by the arrival of the modern symbolic logic of Boole and McCole (Hertz 1880). Thus, Hertz became an early adopter of symbolic logic in Poland.

Hertz also rejected the traditional concept of mathematics that was widespread in Polish philosophy, and he significantly changed the (again) traditional view of the relationship between metaphysics and mathematics (Hertz 1887, pp. III–IV). According to Hertz, the concept of mathematics as the science of quantity was "too narrow" to apply to general relationships between real or formal objects (Hertz 1887, p. IV).

Hertz revealed some deep analogies between the symbolism in mathematics and logic, but reducing logic to mathematics was not so easy to accept because of the reaction of logicians in defending classical logic. Hertz therefore did not reject the connection between logic and metaphysics but rather redefined it. Pure logic, which seems to be formal

and symbolic like mathematics, deals only with mechanical operations on symbols. Metaphysics, meanwhile, was characterized as "research on principles (*początki*) and source of knowledge in a human's mind," so it was in fact a kind of epistemology.

His view on the ontology of mathematics was close to formalism, because he rejected the existence of non-Euclidean and non-three-dimensional spaces,[9] seeing them merely as formal properties of a set of axioms. However, in an earlier publication (Hertz; Dickstein 1875, p. 1), he and Dickstein suggested a kind of mathematical Platonism for describing the objective and independent existence of relationships between objects and notions, with these being later discovered by mathematicians. This was probably Dickstein's concept (see below), or Hertz may have later changed the ontological assumptions due to the arrival of works on non-Euclidean geometries.

Hertz also rejected the widespread concept of space as given to mathematics from "external" metaphysics or epistemology. He argued that the research on non-Euclidean geometry, mainly undertaken by Gauss, showed that notions connected with the notion of space (e.g., curvature) were obtained using pure analytical methods without any philosophical consideration (Hertz 1897, p. 29). He showed that this research voided Kant's concept of Euclidean space as an *a priori* (i.e., necessary) form of thought (Hertz 1897, p. 37–38). This breakaway from the broadly conceived post-Kantian tradition of the philosophy of mathematics appears to have been Hertz's most important innovation in philosophy.

Hertz proposed approach to the metaphysics of science was far more extreme than that of other Polish positivists. Hertz's works certainly laid down the foundations for the modern 20th-century philosophy of mathematics. However, while the Polish 19th-century philosophers often quoted from Hertz's book on non-Euclidean geometries and recognized the philosophical significance of his work, they were generally not influenced by his ideas. Hertz had simply come too early for most of them.[10]

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[9] Hertz critiqued para-psychologist attempts to research four-dimensional spaces. It is very probable that this was a veiled criticism of Ochorowicz's controversial research in this field (Hertz 1897, pp. 40–41).

[10] Hertz's views on the relationship between logic and mathematics were echoed only by those of Stanisław Piątkiewicz (1888), a mathematician teaching at Lwów's

Another important step in the modernization of the Polish philosophy of mathematics was taken by Samuel Dickstein (1851–1939), a mathematician who was a generation younger than Hertz (see also Murawski 2011, p. 21–23). Dickstein was one of the key figures in Warsaw positivism, and he was not just a mathematician but also a historian and philosopher of science (Woleński 2010).

Dickstein, similar to Hertz, rejected the narrow concept of mathematics as the science of quantity (Dickstein 1891, p. 1). His views on the relationship between mathematics and logic were strongly influenced by Boole's works, as indeed Hertz's were. He conceived mathematics as a formal logic that applies to specific mathematical forms, with logic being more general than mathematics and connected with the theory of knowledge (i.e., epistemology). Dickstein carefully identified similarities between mathematics and formal logic (i.e., the algebra of logic) based on the distinction of the "different meaning of their operations" (Dickstein 1891, p. 21–22). He also established a "logic of mathematics," a branch that examines logical relationships between the notions and methods of mathematics (Dickstein 1891, p. 23). This kind of meta-mathematics was conceptually much closer to the 20th-century view than that of the 19th-century Struve.

Dickstein formulated an unusual opinion, at least for his time, about the independence of mathematics from direct philosophical influences:

> Mathematical truths are, and have to be, independent of ideas concerning the essence of their products, which [ideas] are important mainly for epistemology (*teoria poznania*), or for the philosophy of knowledge (*filozofia wiedzy*) [...] (Dickstein 1893a, p. 187).

This evidently new concept announced a new attempt at the philosophy of mathematics, one that would be developed later in Poland in the early decades of the 20th century.

Dickstein was a positivist philosopher (Woleński 2010), but he did not reject all forms of metaphysical thinking. For him, philosophy and

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high school. Piątkiewicz also rejected the simple definition of mathematics as a science of quantity and deepened the philosophical reflection on possibly using algebra in logic. It is very probable that he did not know about Hertz's work, however, so we may speculate that any similarity derives from having similar sources of inspiration.

metaphysics played important heuristic roles in mathematics. (He gave the examples of Cantor, Dedekind, and Kronecker.) Dickstein was aware that mathematical objects are constructions inspired by external objects, but the active mind and creative imagination played a crucial role in constructing these objects.

Dickstein also recognized that mathematics included pure theoretical speculations that exist only in analyses of the formal properties of mathematical objects (Dickstein 1893b, p. 3–4). Using the example of imaginary numbers, he showed how mathematics could construct new objects that were not inspired by the empirical world and had their own form of reality. He implicitly tried to demarcate mathematical reality from physical reality (referred to simply as "reality"). Dickstein therefore rejected the value of using "external" philosophy to solve the internal problems of mathematics:

> Metaphysical question, if the infinite numbers, or infinite forms, "exist" in a metaphysical sense, it is not a mathematical question; in mathematics, it is meaningless (*próżne*) (Dickstein 1893b, p. 10).

Dickstein pointed out that some mathematical notions could not be analyzed using philosophical methods but only with the help of formal, mathematical apparatus. For example, only mathematical analysis can provide clarity in defining abstract, mathematical concepts like continuity. In this way, mathematics could help philosophical speculations, but philosophy alone would be helpless in this analysis (Dickstein 1893b, p. 36).

Dickstein criticized the use of "external" (i.e., non-mathematical) philosophy in explaining mathematical concepts. However, he seemed to accept some kind of mathematical Platonism for the existence of mathematical objects. Of course, the source of this concept was different from that of the Romantic philosophers. He posited that a mathematician does not need to ask if some objects exist in nature, but

> for him, reality could serve as a logical connection between constructions and truths, or obtained system of truths. […] Reality in mathematics […] is a possibility, which is bestowed the dignity of reality by a mathematician [...] (Dickstein 1889, p. 267).

However, Dickstein also criticized extreme forms of mathematical Platonism, referring to them as "mathematical mysticism." For him, it was impossible to treat existence in mathematics and existence in physical reality as being equal (Dickstein 1893b, p. 35).

In the preface to the Polish translation of Riemann's famous lecture (Riemann 1877), Dickstein stressed the significance of philosophy in scientific research:

> […] philosophy finds in it [i.e., Riemann's work] guidelines as to how to examine the foundation of knowledge and how each exact science derives constructions from their fundamental notions using some system of hypothesis [...] (Dickstein's preface in: Riemann 1877, p. 6).

In this declaration, we can see concepts similar to those in the philosophy of science (Heller 2019; see also Polak 2019). These are not mentioned by accident, because his concept of philosophy was based directly on science (Dickstein 1889, pp. 259–261; 1893b, p. 36, endnote 5).

In the philosophy of mathematics, Dickstein was an important forerunner to a new style of thinking that would be typical of 20th-century philosophy. His philosophy was derived from mathematics and enriched with references to the history of science.

6. Conclusions

While the changes in the Polish philosophy of mathematics had been brewing since the 1870s in the works of Henryk Struve and Julian Ochorowicz, the real change in this tradition was precipitated by the mathematicians Hertz and Dickstein. The arrival of the newest mathematical works, particularly non-Euclidean geometry and Boolean logic, led them to redefine the widespread concept of mathematics and its relationships with other sciences. They tried to think in the spirit of what we now call the philosophy of science. Thus, they rejected the main philosophical assumptions that were typical of the metaphysical tradition, and their works became precursors to the modern Polish philosophy of mathematics.

The intellectual inertia of the 19th-century Polish philosophers and their uncritical adherence to philosophical tradition meant that any

real change had to wait for the advent of new mathematics. These developments eventually led to mathematics flourishing in Poland in the early 20th century, accompanied by rapid development in the Polish philosophy of mathematics (see e.g. Woleński 2015; Murawski 2010).

Examinations of the long tradition for the philosophy of mathematics in Polish thought, which lasted over a century, show that instead of perpetuating the Romantic philosophers' erroneous philosophical claims, the 19th-century Polish philosophers created a sound fundamental base for the later development of this branch. This study also shows how important the change in the philosophical attitude toward sciences was to the evolution of philosophical thought on mathematics. Indeed, this shift in attitude marked a pivotal movement for the development of the modern Polish philosophy of mathematics.

7. Acknowledgments

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