Cognitive Science

Isbell Conjugacy for Developing Cognitive Science

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Abstract:	What is cognition? Equivalently, what is cognition good for? Or, what is it that would not be but for human cognition? But for human cognition, there would not be science. Based on this kinship between individual cognition and collective science, here we put forward Isbell conjugacy--the adjointness between objective geometry and subjective algebra--as a scientific method for developing cognitive science. We begin with the correspondence between categorical perception and category theory. Next, we show how the Gestalt maxim is subsumed by the mathematical construct of colimit, a generalization of summation. The universal mapping property definitions of mathematical constructs, by virtue of being the best with respect to the universe of discourse, can be learned using reinforcement learning algorithms, which raises the possibility of abstracting the architecture of mathematics by artificial intelligence. Subsequently, we present naturality (to be contrasted with miracles), understood as 'Becoming consistent with Being', which governs the transformations of both things and their theories, as the zeroth law of change. Furthermore, the contrast--physical [mechanism] vs. biological [organism]--is smoothed via natural transformation, wherein transformations are respectful of the cohesion of the objects transformed. In closing, upon recognizing the scientific value of learning difficult-to-master differential calculus by physicists, of learning a strange four-letter language by biologists, and of learning the grammar of our respective mother tongues, we make a case for learning the theory of naturality/category theory for developing cognitive science.

To,

Professor Rick Dale

Executive Editor

Cognitive Science

Dear Professor Dale,

I am herewith submitting our Letter, coauthored with Professor Sisir Roy, and entitled 'Isbell Conjugacy for Developing Cognitive Science' to be considered for publication as a 'Letter to the Editorial Board' in your journal: Cognitive Science. Our submission is in response to your call: CfL: "Progress and Puzzles of Cognitive Science".

Our Letter was motivated by a profound progress in cognitive science understood as the science of knowing. First, let us recall the main objective of cognitive science: How do we know? Recognizing the limitations of the Fregean definition of CONCEPT as a SET of properties (or features), the definition of CONCEPT was refined into a CATEGORY, with properties and their mutual determinations as objects and morphisms, respectively (of the category). Notwithstanding the realization that concepts is where cognitive science went wrong, this category theoretic advance in our understanding of how we go from particulars to generals (theory and models) has been largely ignored, presumably under the pretext: mathematical knowing is too special to inform knowing in general. However, reflecting on the development of science readily brings to mind that it is the too special motion of dropped objects that led to the development of the science of motion.

The kinship between mathematics (in particular and science in general) and cognition has been brought into clear focus by way of spelling out how 'representation', figuring in the foundational tenet of cognitive science: "cognition is computation of representations" (Nat. Hum. Behav. 3, 782, 2019), is calculated (Mind & Matter 15, 161-184, 2017). Representation (or model), according to functorial semantics, is a contravariant functor interpreting a theory (into a background category, with the theory abstracted from the given category of particulars; Reprints in Theory and Applications of Categories 5, 8-12, 2004).

Returning to the main objective of cognitive science--how we know--we find that how we know depends on what we are trying to know, i.e. ontology determines epistemology (cf. sense organs along with telescope for distant objects vs. microscope for tiny objects). As such cognitive science can develop only as a complex: ontology vis-à-vis epistemology. Comparing ontology and epistemology to geometry and algebra, respectively, in our Letter, we introduce Isbell conjugacy--the adjointness between geometry and algebra--as a method to develop cognitive science.

We discuss these mathematical insights into knowing in a manner readily accessible to the multidisciplinary audience of your journal. In closing, our Letter brings out the reach of Isbell conjugacy between objective geometry and its subjective reflections in algebra into focus so as to facilitate ready recognition of the relevance of Isbell conjugacy for the development of cognitive science.

If I may, the following may be considered for reviewing our Letter since they are experts in both cognitive science and category theory.

Professor Michael A. Arbib (arbib@usc.edu)

Professor Andrée C. Ehresmann (ehres@u-picardie.fr)

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We'd also like to request the following members of your esteemed Editorial Board to be our Letter's Handling Editors.

Professor Kinga Morsanyi

Professor Iris van Rooij

Professor Rick Dale

Professor Dr. Max Louwerse

We earnestly hope that you will find our Letter suitable for publication in your journal Cognitive Science. We sincerely thank you for your kind consideration of our Letter and we eagerly look forward to hearing from you.

Thanking you,

Yours truly,

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Abstract

What is cognition? Equivalently, what is cognition good for? Or, what is it that would not be but for human cognition? But for human cognition, there would not be science. Based on this kinship between individual cognition and collective science, here we put forward Isbell conjugacy--the adjointness between objective geometry and subjective algebra--as a scientific method for developing cognitive science. We begin with the correspondence between categorical perception and category theory. Next, we show how the Gestalt maxim is subsumed by the mathematical construct of colimit, a generalization of summation. The universal mapping property definitions of mathematical constructs, by virtue of being the best with respect to the universe of discourse, can be learned using reinforcement learning algorithms, which raises the possibility of abstracting the architecture of mathematics by artificial intelligence. Subsequently, we present naturality (to be contrasted with miracles), understood as 'Becoming consistent with Being', which governs the transformations of both things and their theories, as the zeroth law of change. Furthermore, the contrast--physical [mechanism] vs. biological [organism]--is smoothed via natural transformation, wherein transformations are respectful of the cohesion of the objects transformed. In closing, upon recognizing the scientific value of learning difficult-to-master differential calculus by physicists, of learning a strange four-letter language by biologists, and of learning the grammar of our respective mother tongues, we make a case for learning the theory of naturality/category theory for developing cognitive science.

1. Categories and Naturality

The most advanced scientific method for defining objects and operations is in terms of 'what they are good for' (Lawvere & Rosebrugh, 2003, pp. 26-29), which is a refinement of functional definitions. Human cognition is good for developing science. Not surprisingly, individual cognition and collective science have much in common: cognition is science writ small (see Einstein, 1936, p. 349; Fodor, 2006, p. 93; Schapira, 2016). Based on this propinquity (Ehresmann & Vanbremeersch, 2007, p. 12; Lawvere, 1994; Posina, Ghista, & Roy, 2017), here we show how to develop cognitive science.

Our conscious experiences are categorical (Albright, 2013); so is mathematics (Lawvere, 1972, p. 10; Lawvere & Schanuel, 2009). But, what exactly is meant by 'categorical'? The cat that I see sitting on the wall across my window partakes in the essence--catness--that is characteristic of the category of cats. The essence--catness--specifies the way in which parts (eyes, whiskers, nose, mouth, tail, etc.) of a whole (cat) stick together. Equally importantly, this essence is preserved as one object of a category is transformed into another object of the category (e.g. catness is preserved in the transformation: playful cat --> watchful cat). Not unlike objects of different categories--textbook, chair, and table--populating our perceptual experience, mathematics also consists of various categories such as sets, dynamical systems, functions, and graphs (ibid. pp. 11-21, 133-151). The abstract essence/theory of a category of objects is adequate for completely characterizing every object of the category and to tell apart any two transformations of objects (Lawvere & Schanuel, 2009, pp. 23, 177-181, 213-215, 245-250).

The Gestalt maxim--the whole is different from the sum of its parts--figures prominently in cognitive neuroscience (Albright et al., 2000). In order to see the 'difference' that the Gestalt maxim highlights, we ask: what does it mean to say 'a whole is the sum of its parts' (e.g. **1** + **2** = **3**, where **1** = {*}, **2** = {*, *}, **3** = {*, *, *})? A whole (**3**) is the sum of its parts (**1** and **2**), if what every part does determines what the whole does. Just as in the case of concepts, where constituent features can be related to one another (Smith & Medin, 1981, p. 83), the summands can also be related (say, a function between **1** and **2**). Colimit, a generalization of sum, takes into account [any] morphisms relating objects, and, as such, spells out the "different" in the Gestalt maxim. The mathematical construct of colimit has been brought to bear on cognitive science (Ehresmann & Vanbremeersch, 2007, 2019). The aforementioned mathematical definition of sum as a whole that is determined by its parts is a universal mapping property definition, wherein the sum (**3**) is unique with respect to the summands (**1** and **2**). The universal mapping property definition of mathematical objects and operations, by virtue of being the best in the given universe of discourse, can be abstracted using reinforcement learning algorithms (Posina, 2022a), which, in turn, raises the possibility of abstracting the architecture of mathematics by artificial intelligence.

Comparing the apparently incongruent physical mechanism vs. biological organism with change vs. unity, we find that the mechanistic transformations underlying the growth and development of a biological organism are respectful of the cohesion of the organism (e.g. aging did not tear me apart). This 'Becoming consistent with Being' (Lawvere & Schanuel, 2009, p. 152) or 'naturality' is what called for the abstraction of category theory from the mathematical practice (Eilenberg & MacLane, 1945; see also Lawvere, 2017, p. 12). Given that every morphism transforming one object into another of a category is respectful of the structural essence of the category (Lawvere and Schanuel, 2009, p. 210), we now put forward 'Becoming consistent with Being' or, equivalently, 'all changes are natural' (with Set-valued contravariant functors as objects; Lawvere & Schanuel, 2009, p. 378; see also Posina, 2022b) as the zeroth law of motion. This scientifically advanced understanding of 'natural' subsumes not only physical, biological, and cognitive sciences, but also cultural, political, and social sciences (cf. societies do not change willy-nilly; see Lawvere, 1999; Posina, 2020). Also note that not only are the transformations of things natural, but also that of their theories, all of which asserts that science--understood as a reflective part of reality (Lawvere and Schanuel, 2009, pp. 84-85; see also Clementino & Picado, 2008, p. 6)--is not a miracle machine (cf. Sarewitz, 2017). In accordance with commonsense, miracles (and prophetic revelations) are unnatural.

2. Compounding Epistemology and Ontology

The most basic question of cognitive science is: How do we know? How we know depends, not surprisingly, on what we are trying to know (cf. sense organs along with a telescope for distant objects vs. microscope for tiny objects). One immediate implication of the just stated realization is that the development of cognitive science can only take place as a complex: epistemology vis-à-vis ontology. Inspired by eminently useful analogies such as the Bohr atom, here we put forward Isbell conjugacy (Lawvere, 2005, pp. 16-20; 2016, pp. 1-3; see also Lawvere & Rosebrugh, 2003, pp. 171-176) as a method to compound epistemology and ontology into which reality is resolved.

What is Isbell conjugacy? How does it help in the much-desired maturation of cognitive science? In comparing algebra and geometry to epistemology and ontology, respectively, we find that Isbell conjugacy, which spells out the adjointness between subjective algebra and objective geometry, can inform the synthesis of epistemology and ontology into which reality is analyzed (see appendix A4 in Posina, Ghista, & Roy, 2017 for an accessible discussion of adjointness). In doing so, Isbell conjugacy constitutes the scientific foundation solid enough to build cognitive science.

To facilitate the scientific program of bringing Isbell conjugacy to bear on cognitive science, we begin with a familiar mathematical construct: function. A function f: A --> B, geometrically speaking, is an A-shaped figure in B; the very same function f: A --> B, algebraically, is a B-valued property of A (Lawvere & Schanuel, 2009, pp. 81-85, 370-371). As an illustration of resourcefulness of the aforementioned function, we note that implicit in it--in the function mapping elements to elements--is the most basic principle of Becoming (or change) consistent with Being (unity) or naturality (see Exercise 7.22a in Lawvere & Rosebrugh, 2003, p. 135). But for the naturality, science would not be possible (e.g. reconstruction of objects from observed changes, as in characterizing the receptive field of a neuron from observed changes in firing rate in response to stimulus changes; see Lawvere & Schanuel, 2009, pp. 360-361 for the mathematics of reconstruction as a category of right actions of a monoid objectifying given changes).

Summing it all, since reality consists, as noted above, of parts--individual cognition and collective science--reflective of the reality, we need a mathematical category of Reflecting in addition to the categories of Being and Becoming in order to bridge the two categories of objective reality on the one hand and its subjective reflections on the other. The mathematical category of Reflecting, with conjugate adjoints dualizing subjective algebra into objective geometry as objects, makes room for the basis of science--human cognition--in the scientific representation of reality.

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