Linnebo on Analyticity and Thin Existence

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Abstract. In his groundbreaking book, *Thin Objects*, Linnebo (2018) argues for an account of neo-Fregean abstraction principles and thin existence that does not rely on analyticity or conceptual rules. It instead relies on a metaphysical notion he calls "sufficiency". In this short discussion, I defend the analytic or conceptual rule account of thin existence.

1. Introduction.[1]

In his groundbreaking book, *Thin Objects*, Linnebo (2018) argues for an account of neo-Fregean abstraction principles and thin existence that does not rely on analyticity or conceptual rules. It instead relies on a metaphysical notion he calls "sufficiency". Analyticity has traditionally been used to explicate the sense in which some claims, such as abstraction principles and existence claims for abstract objects, place no demands on the world. In other words, some objects have a "thin" existence, and analyticity has historically been a popular way of explicating what thin existence amounts to. One of Linnebo's central tasks in the book is to explicate the idea of thin existence without appeal to analyticity. That is what his metaphysical notion of sufficiency is for. Linnebo rejects using analyticity to explicate thin existence because he is concerned about 1) analytic existence claims, 2) *de re* analyticity, and 3) the fact that the notion of analyticity that is required is the debunked metaphysical one (2018, 13-4). In this short discussion, I defend analyticity or conceptual rules and show that they can meet his criteria for

[1] Thanks to Øystein Linnebo for discussion of his objections to *de re* analyticity, which helped me to avoid some elementary blunders.

explicating thin existence.

In Section 2, I address concern 3) above. I discuss possible non-epistemic notions of analyticity that can be used to explicate thin existence. I think all are compatible with the present discussion (though they may not be compatible with each other). I would like not to commit myself to any one of them here. In Section 3, I discuss Linnebo's abstractionism and address concerns 1) and 2). I think the analytic or conceptual rule conception of thin existence comes out unscathed.

2. Analyticity[2]

In this discussion, I will be defending the claim that abstraction principles and existence claims for the abstract objects posited therefrom are analytic or expressions of conceptual rules. In this section, I want briefly to canvass some ways of in which analyticity can explicate thin existence. I discuss this in greater detail in Author (2024). Claims about which sentences are analytic cannot (only) be understood as epistemic claims, that abstraction principles or certain existence claims are knowable a priori on the basis of knowledge of their meanings, if we want to use analyticity to explicate thin existence. This is usually explicated by appeal to *metaphysical analyticity*,[3] and it is precisely because Linnebo rejects metaphysical analyticity that he develops his own metaphysical notion of sufficiency, which I explain below, to explicate thin existence. According to the metaphysical notion of analyticity, analyticities *owe* their truth-values to the

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[2] I thank a reviewer for pushing me to clarify the sense of analyticity relevant for this discussion.

[3] See Boghossian (1996) for the classic distinction between epistemic and metaphysical analyticity, including a critique of the latter and a defense of the former. I discuss his critique below.

meanings of their constituent terms alone; their truth-values *depend* on or hold *because* of the meanings of their constituent terms alone. I don't think metaphysical analyticity is in as poor a position as many think. It has had several respectable defenses recently (e.g., Rabinowicz 2010, Russell 2008, Warren 2015). However, it is only one way of using analyticity to explicate thin existence. (If you think of the next three views as simply ways of *defending* metaphysical analyticity and not as *rivals* to it, that's fine). Let me briefly indicate several more.

According to Thomasson's (2020) modal normativism, analytic truths place no substantial demands on the world because they are not descriptions at all, but prescriptions for how to use concepts, or expressions of conceptual rules (or consequences[4] of such rules). She accommodates their truth-aptness by appeal to a deflationary conception of truth, according to which, roughly, the truth concept is governed solely by the 'equivalence schema': '*p*' is true if and only if *p*.[5] Of course, any non-descriptivist proposal needs to answer the Frege-Geach problem (among other problems). I don't think it is necessary to explain here how Thomasson addresses such problems. See Thomasson (2020) for her responses to the Frege-Geach problem, the necessary a posteriori, the contingent a priori, *de re* necessity, and more.[6] The only point I want to make here is that if modal normativism is correct, that would explicate thin existence in

[4] Obviously, the normativist needs an account of logical consequence. See Thomasson (2020) for discussion of this.

[5] Or: the proposition that *p* is true if and only *p*. There are many ways of cashing out semantic deflationism (see, e.g., Horwich 1998).

[6] Warren (2022b) also addresses the necessary a posteriori and the contingent a priori (2022a).

terms of analyticity. For example, the existence of the number 2 is thin because "2 exists"[7] is analytic – an expression of a conceptual rule – and expressions of conceptual rules are not descriptions. I will explain in much more detail below how this account can be fleshed out.

According to Nyseth (2021), analyticities place no substantial demands on the world because the application conditions of the concepts involved are "fulfilled no matter what the world is actually like" (280).[8] Nyseth doesn't put his view this way, but it seems plausible that being composed of concepts whose application conditions are fulfilled no matter what the world is actually like is a way of explicating a sense in which analyticities make no demands on the world. He writes that, "'ewe' can be correctly applied to whatever 'adult female sheep' can be correctly applied to (roughly: the criteria laid down interact so that establishing that 'adult female sheep' applies to some entity immediately establishes that 'ewe' also applies). This then guarantees that, in the case of 'all adult female sheep are ewes' the truth-condition…will be fulfilled no matter what the world is actually like" (Ibid.). Note that this is not an epistemic point

[7] Throughout the paper I follow Thomasson (2020) and Ludwig (in progress) in taking analyticity to be predicated of meaningful strings, what they call *statements* or *claims*. For the purposes of this paper, sentences (and mutatis mutandis for terms) in quotation marks should be thought of as naming claims, not strings, and not restricted to English. I could appeal instead to concepts rather than terms to make my arguments even less language-dependent. The substance of my arguments shouldn't be affected by instead talking of concepts and rules governing them.

[8] Nyseth claims that, according to normativism, analyticities are true *because* they express rules (275). But that is dangerously close to saying analyticities describe or are made true by rules, which normativists deny.

about what we *know* or *can know* applies to what, but a point about our application conditions and how they can conspire to produce a truth that places no demands on the world. Other defenders of analyticity make this point too. Cf. Thomasson (2007, 70): "The relevant rule of use is: 'apply 'bachelor' only where 'male' may be applied' so the truth-conditions for 'x is a bachelor' include that x is male. … This also makes sense of the idea that the truth of analytic claims such as 'All bachelors are men' is independent of all empirical facts—even of there being bachelors or men, or indeed anything at all." Cf. Sidelle (2009, 229): "the rule tells us that whatever might be postulated about some situation, 'bachelor' applies only if the item in question is male. As 'male' also applies only to males, then, 'bachelors are male' will be true with respect to every possible world, and so, 'necessarily, bachelors are male' is true as well." These ideas can help explicate thin existence in terms of analyticity. The existence of the number 2 is thin because "2 exists" is analytic and analytic truths are those whose terms' application conditions are fulfilled no matter what the world is like.

Finally, according to many conventionalistic philosophers, including Carnap (1950), Wittgenstein (1974), Sidelle (2009), and Warren (2020), analyticities place no demands on the world because they are derivable from (and *explained by*, Warren would add) conventions which are not theoretically (as opposed to practically) evaluable at all; they are completely theoretically (as opposed to practically) unconstrained. In other words, we wouldn't be getting anything metaphysically *wrong* if we had different conventions, for there is no such transconventional sense of getting things wrong. This point is one stressed often by Wittgenstein and Carnap when discussing language[9] choice or framework choice. To choose a linguistic framework is to choose

[9] Wittgenstein would often say "grammar" choice – grammar cannot be true or false, cannot get

a set of analyticities, and framework choice is pragmatic. There is no such thing as getting things wrong in choosing a framework. The very possibility of getting things wrong – i.e., of saying anything at all – requires that one already possess a framework with its set of analyticities. Much the same can be said by Warren to explicate the sense in which analytic truths make no demands on the world. For him, similarly to Carnap, a sentence is analytic in a language just in case it is derivable from, and explained by,[10] the basic inferential rules of the language. This is not an overtly epistemic characterization of analyticity, but I also don't think it alone quite explicates why analytic truths make no demands on the world. I think this is achieved by adding Warren's unrestricted inferentialism, the thesis – similar, I think, to Carnap's principle of tolerance – that *any* basic inferential rules are automatically epistemically permissible and automatically valid. Carnap's frameworks, Wittgenstein's grammars, and Warren's basic inferential rules are completely theoretically unconstrained – there is no *sense* in their being theoretically (as opposed to practically) right or wrong. This complete unconstrainedness by the world helps to explicate why analyticities make no demands on said world.[11] The existence of the number 2 is thin because "2 exists" is analytic and analytic truths are derivable from (and, for Warren, explained by) conventions that are theoretically unconstrained in the above sense.

It would be impossible here to address every objection to (metaphysical) analyticity, but,

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things right or wrong.

[10] See Warren (2020) for what he means by "explanation" in this context.

[11] Obviously, proponents of views like those just discussed need to address things like the necessary a posteriori and the contingent a priori, and they have (see, e.g., Thomasson 2020, Sidelle 1989, Warren 2022a,b).

before moving on, let me briefly address two of Boghossian's (1996) most influential: the contingency objection and what Warren (2015, 2020) calls "the master argument". These are objections specifically to any account of necessity that appeals to analyticity. There are many slightly different ways of formulating each objection. Let us understand the contingency objection to be that any account of necessity that appeals to analyticity makes necessities contingent, since our semantic conventions are contingent. It is simply false that had our conventions been different, so would have the necessities. According to the master argument, any account of necessity that appeals to analyticity implies that our conventions have the power to make certain sentences (i.e., the necessary, analytic ones) true. And since necessarily, *S* is true if and only if *S* means that *p*, and *p*, that implies that our conventions have the power to make it the case that *p*. Thomasson (2020a) has argued that her normativism avoids both objections because both assume that any account of necessity that appeals to analyticity must claim that conventions are truthmakers of necessary truths or that necessary truths describe conventions. But according to normativism, necessary truths aren't *made true* by conventions, they *express* conventions. Necessary truths don't describe anything[12], let alone conventions; they're prescriptions. She also adds vis-à-vis contingency, as do many others (e.g., Sidelle 2009, Topey 2019, Warren 2020, Wright 1985), that conventionalists have a conventional explanation for why necessities don't counterfactually depend on convention – because it is one of our conventions that in counterfactual reasoning, we use our actual conventions (a point often emphasized by Kripke 1980 as well).

[12] At least not in any substantive sense. Thomasson allows a weak, syntactic sense in which necessary truths are descriptions – statements of them are declarative sentences.

Warren (2015, 2020) argues 1) that the master argument misses explanatory versions of conventionalism according to which our conventions don't make *p* the case, but instead explain why *p*; 2) that even if it's not the case that our conventions explain why *p*, it wouldn't follow that they don't explain why *S* is true, even if necessarily, *S* is true if and only if *S* means that *p*, and *p*, because explanatory contexts are hyperintensional; and 3) that if one got around this by accepting a special principle according to which if *C* explains why *S* is true, then *C* explains why *p*, then this would be just to accept a view of propositions that every conventionalist would reject on meta-semantic grounds. See Asay (2020), Donaldson (2020), Hale and Wright (2015), and Topey (2019) for more criticisms of the master argument.

I presented three ways of using analyticity to explicate thin existence. They may not be consistent with each other, but I think all are consistent with what I say below. As my argument progresses in the next section, I will take the normativist perspective, but I will make clear what a conventionalist like Warren could say. It seems to me that Thomasson's and Warren's views are roughly intertranslatable: what expresses conceptual rules for one is explained by conceptual rules for the other. Next, I introduce Linnebo's abstractionism and respond to his other two concerns about analyticity: analytic existence claims and *de re* analyticity.

3. Linnebo's Abstractionism

To understand Linnebo's abstractionism, it will be helpful to take a brief look at neo-Fregeanism. Neo-Fregeanism is modern revival of Frege's logicist platonism – the theses that mathematics is about independently existing objects and that mathematical truths are analytic in virtue of being logical truths derivable from suitable definitions (Hale and Wright 2001, 1).[13] It

[13] I'm ignoring the fact that often neo-Fregeans are (as Frege himself was) only concerned with

may seem odd to combine platonism with the claim that mathematical truths are analytic, but neo-Fregeans bill themselves as platonist in quite a lightweight sense. In fact, normativists and conventionalists like Thomasson (2014) and Warren (2020) are both generally sympathetic to neo-Fregeanism, with Warren (2020, 198, 203) calling it "conventionalist-adjacent," and it is often grouped with metaontological deflationisms or minimalisms (e.g., in Thomasson 2014).

Neo-Fregeanism is sometimes called (a version of) "abstractionism" because of its reliance on so-called abstraction principles (Ebert and Rossberg 2016, Linnebo 2018). The general form of an abstraction principle is:

$$\S\alpha = \S\beta \leftrightarrow \alpha \sim \beta$$

where "§" is a term-forming operator, and ~ is an equivalence relation. Here is one of Frege's famous non-mathematical examples of an abstraction principle:

> (d): The direction of line *a* = the direction of line *b* if and only if lines *a* and *b* are parallel.

Here "the direction of" is "§," "the direction of line *a*" being a term, and the relation of being parallel is the equivalence relation. The relation of being parallel is an equivalence relation because it is reflexive, symmetric, and transitive: a line *a* is parallel to itself; if line *a* is parallel to line *b*, then line *b* is parallel to line *a*; and if line *a* is parallel to line *b*, and line *b* parallel to line *c*, then line *a* is parallel to line *c*.

What is the philosophical significance of (d)? For Frege, terms that figure in abstraction principles must refer to objects. So, "the direction of line *a*" refers to an object, a *direction*. The principle tells us when any two such objects are the same – it offers criteria of identity for

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arithmetic and real analysis.

directions. Criteria of identity determine when objects are identical or distinct, and many philosophers hold that objects must have criteria of identity, in agreement with Quine's (1958) slogan, "no entity without identity". Furthermore, neo-Fregeans hold that it is only by grasping an object's identity conditions that we are able to grasp the concept of that object and possess thoughts about it. So, that (d) provides criteria of identity for directions is what allows its terms to refer to directions, conceived as objects – clearly *abstract* objects, not physical, concrete objects, nor mental objects. Objecthood, reference, and criteria of identity are linked, in what Linnebo (2018, 21) calls the Fregean triangle.

What makes this so radical is this. Imagine that we had no concept of direction in Frege's sense. Then Frege comes along and gives us principle (d). With (d), we can grasp a new concept and refer to a "new" object – new in the sense that we couldn't refer to it before, not new in the sense that Frege brought it into existence. Furthermore, all it takes, metaphysically speaking, for directions to exist is for lines to exist – the truth of the left-hand side of (d) requires no more, metaphysically speaking, than the truth of the right-hand side. Thus, we seem to have an unmysterious picture of the metaphysics and epistemology of at least some abstract object, *directions*.

This picture is transferred into the philosophy of mathematics with abstraction principles like Hume's principle:

(HP) The number of Fs = the number of Gs if and only if F and G are equinumerous (i.e., can be one-to-one correlated)

Let us engage in a similar thought experiment. Imagine that we had no concept of number. Then Hume comes along and gives us principle (HP). With it, we can now grasp the concept of number and use that concept to refer to new abstract objects: *numbers*. Furthermore, all it takes, metaphysically speaking, for numbers to exist is for extensions of concepts to exist – the truth of the left-hand side of (d) requires no more, metaphysically speaking, than the truth of the right-hand side. Neo-Fregeans like Hale and Wright (2001) take (HP) to be analytic, implicitly defining the number concept. This gives us an unmysterious metaphysics and epistemology of numbers, for, we gain access to facts about numbers via facts about one-to-one correlation of concepts.

I won't object to anything I've said so far. From the roughly conventionalist point of view I will be defending, abstraction principles are devices for introducing new concepts into a framework. One might take issue with the nature of such devices, whether they are really suitable for introducing a concept (e.g., debates about impredicativity and bad company), and so on. Some abstraction principles are impredicative, meaning that they quantify over objects some of which fall under the concept being defined, and there is debate over whether this seeming circularity is vicious. Frege's abstraction principle Basic Law V is impredicative and (combined with other plausible principles) famously leads to contradiction in Russell's Paradox. According to Basic Law V:

The extension of F = the extension of G if and only if F and G are coextensive

Think of extensions as sets, and ask whether the set of all sets not members of themselves is a member of itself. Contradiction quickly follows. The bad company problem (sometimes put in the form of an objection) is the problem of distinguishing "good" abstraction principles (i.e., those that successfully introduce or illuminate a concept[14]) from "bad" (i.e., those that fail, like

[14] I thank a reviewer for the distinction between abstraction principles that introduce new concepts and those that illuminate existing ones.

Basic Law V). Thomasson (2014, 138-9), who is otherwise sympathetic to neo-Fregeanism, also raises concerns about the need for criteria of identity when introducing a new concept. She's not certain that neo-Fregeans take criteria of identity to be necessary when introducing a new concept though, and Linnebo (2018, 33) is explicit that he takes criteria of identity only to be sufficient for introducing a new concept. Unlike Hale and Wright, Linnebo rejects the analytic/synthetic distinction.

Linnebo's (2018, 2023a,b) abstractionism is a deflationary metaontology similar to neo-Fregeanism in that it relies on abstraction principles, but it eschews analyticity. Thus, Linnebo needs some other way of cashing out the thin, lightweight, insubstantial existence of abstract objects. The way he does this is with his concept of *sufficiency*. This is his analyticity-replacement. Linnebo uses his notion of sufficiency to explicate abstraction principles and to account for the kinds of inference that Thomasson (2014) takes to be trivial analytic entailments, such as "if there are exactly three exams this semester, then the number of exams this semester is three". For Linnebo, the truth of the antecedent *suffices*, in his special metaphysical sense, for the truth of the consequent. This notion of metaphysical sufficiency is used to explicate similar kinds of claim, such as "all it takes for the number of exams this semester to be three is for there to be exactly three exams this semester" and thin existence generally. Linnebo rejects using analyticity to analyze these kinds of inference and kinds of claim because he is concerned about 1) analytic existence claims, 2) *de re* analyticity, and 3) the fact that the notion of analyticity that is required is the debunked metaphysical one. We addressed 3) in Section 2, where I discussed several ways for the defender of analyticity (henceforth "analyticist") to explicate thin existence using analyticity. Now I address objections 1) and 2).

Regarding 2), I think Warren (2020) has adequately rebutted the usual objections to analytic existence claims. The most common is this. If "God exists" were derivable from our basic conventional rules, it would be analytic, but such a being wouldn't come to exist, so something is amiss with the very idea that there are analytic existence claims. Warren's response is as simple as it is convincing: in a language in which "God exists" is derivable from its basic rules, so, analytic in that language, "God exists" wouldn't mean what it means in our language. This follows directly from Warren's inferentialism, as well as from plausible criteria governing translation, such as "[w]hen translating language L into English, we should reject any translation that maps a provable (via basic rules) sentence of L to a non-provable sentence of English, or vice versa." (Warren 2020, 129). "God exists" is provable from the basic rules in the hypothetical language, but it isn't in English, so we should reject the homophonic translation. Field (2022) raises the following objection to this idea. Suppose "there is an all-powerful creator" were derivable from the basic rules of some language. The analyticist would say that "there is an all-powerful creator" in that language wouldn't mean that there is an all-powerful creator. But Field (2022, 8, fn. 13) responds, "But if their use of 'powerful', 'create' etc. was otherwise like ours?" That, however, still wouldn't justify translating their "there is an all-powerful creator" to mean that there is an all-powerful creator. As before, "There is an all-powerful creator" is provable from the basic rules in their hypothetical language, but it isn't in English, so we should reject the translation. It would seem, in the language where "all," "powerful," and "creator" are *otherwise* used exactly like ours, that the meaning of "there is an all-powerful creator" is either not compositionally determined – its meaning is not a function of the meanings of "all," "powerful," and "creator" – or that (at least) one of the subsentential expressions has a different meaning. In that language, "all" would mean *all*, "powerful" would mean *powerful*, and "creator" would mean *creator*, since those words are used as they are in English, but "all-powerful creator" wouldn't mean *all-powerful creator*. There's nothing mysterious about that. There are many phrases in English whose meanings aren't compositionally determined, such as "red herring". The meaning of "red herring" is not a function of the standard meanings of "red" and "herring". We can easily imagine a language that uses "red" and "herring" in the ways that we do, so that we should homophonically translate those, but where they do not use "red herring" the way we do, so we should *not* homophonically translate that. They don't mean *red herring* by "red herring". We can imagine them not even possessing the concept of a red herring. Thus, in that language, "red" means *red* and "herring" means *herring*, but "red herring" doesn't mean *red herring*. If instead we imagine that, say, "there exists an x such that for every action y, x can perform y and x is a creator" were derivable from the basic rules of some language, and that the use of "action," "perform," "creator," etc. is otherwise like ours, then it seems the analyticist must say that (at least) one of these expressions has a different meaning, which accounts for the fact that "there exists an x such that for every action y, x can perform y and x is a creator" does not mean in that language what it means in ours.[15]

The problem of *de re* analyticity in 3) is this. In *de dicto* sufficiency statements, only formulas with no free variables flank the sufficiency operator; in *de re* sufficiency statements, formulas with free variables flank the sufficiency operator. Take the claim that there are thin objects, which Linnebo takes to equivalent to the sufficiency claim $\exists x(\top \Rightarrow Ex)$, where Ex is an existence predicate, and $\top$ is a tautology. The problem is not that this is an existence claim or that it uses an existence *predicate*, but that a formula with a free variable, Ex, flanks the sufficiency operator. However, Linnebo writes, "it is only *sentences* that are analytic, not open formulas

[15] I thank a reviewer for this point.

relative to variable assignments. Analyticity is meant to be an entirely linguistic phenomenon, whereas variable assignments typically involve non-linguistic objects" (2018, 13, original emphasis).

Linnebo thus assumes (as does Burgess 1997) that the analyticist must interpret the claim that there are thin objects to mean or at least be logically equivalent to $\exists x\mathrm{A}(\top \rightarrow Ex)$, or more simply $\exists x\mathrm{A}Ex$, where "A" is the analyticity operator, "it is analytic that". The central problem for the normativist is how to explain *de re* necessity, or the satisfaction by an object of open modal formulas like "x is necessarily F," without attributing analyticity to non-linguistic objects, quantifying into quotational contexts, or other confusions. I will now present what I consider the basics of such an account.

It is important to note that while *de re* necessity is often taken to be independent semantics, it is closely related to rigidity (Sidelle 1989, 73-4). For example, "The number of my eyes is necessarily even" is false when "The number of my eyes" is read non-rigidly but true when read rigidly. This rigidity, of course, gives the sentence a *de re* reading: it is true, of the number of my eyes, that it is necessarily even; the open sentence "*x* is necessarily even" is true of or satisfied by the number of my eyes. When an assignment assigns an object to a variable, the variable rigidly designates the object (Stanley 2017, 926).[16] Here, then, is my proposal for

[16] I think it is worth quoting in full: "Since, in each possible situation, we are considering whether or not the object o satisfies the formula, we need to ensure that the variable 'x' denotes o in all of the possible situations. That is, on the objectual interpretation of QML [quantified modal logic], when taken with respect to an assignment s, variables are rigid designators of the objects which s assigns to them. The reason that variables are de jure rigid designators is because there is

normativist truth/satisfaction conditions for open modal formulas:

(*Normativist Satisfaction*) "$\Box Fx$" is true of or satisfied by *o* just in case the claim that *n* is *G* is analytic (i.e., expresses an actual semantic rule or rule consequence).[17]

Two immediate comments on *Normativist Satisfaction*: 1) per footnote 6, I follow Thomasson and Ludwig in taking analyticity to be a property of claims. Hence the "claim" talk. 2) This should be read as an open-ended schema. Permissible substituends are restricted as follows: "*F*" may be replaced with any predicate of the object language, "*G*" with a translation[18] of "*F*" in the metalanguage, "*o*" with any singular term of the metalanguage, and "*n*" with any rigid designator of *o* (i.e., any rigid designator of what the singular term "*o*" refers to) in the metalanguage. I discuss rigidity from a normativist perspective below. You might worry that the two points I've just made conflict, for the phrase "the claim that *n* is *G*" is senseless because no claim is being made there, since "*n*" and "*G*" are schematic letters. This is not a problem. A schema is not a

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nothing else to the semantics of variables besides the stipulation that, when taken with respect to an assignment s which assigns the object o to a variable, it designates o in every possible situation." (Ibid.)

[17] I thank a reviewer for pointing out flaws in several abortive attempts to formulate and explain *Normativist Satisfaction.*

[18] Philosophical issues regarding translation are beyond the scope of this paper. However, conventionalists like Thomasson and Warren are inferentialists, so, for them, "*G*" will be a predicate that plays the same inferential role (or is governed by the same inferential rules) in the metalanguage that "*F*" plays (or is governed by) in the object language. I interpret roles and rules broadly to include any empirical application conditions.

claim or a claimable – its instances are. And any instance of *Normativist Satisfaction* will itself be a claimable, and so will the instances of "the claim that *n* is *G*" therein. A related point will come up below. This doesn't mean that we can't believe *Normativist Satisfaction*, since it is a schema; it just means that to believe it is to believe that its instances are true (cf. Lavine 2006, 118).

I intend the schema to possess more or less what Lavine (2006) calls "fully schematic generality". It is very important to note that Lavine (among others, e.g., Dieveney 2013, Warren 2020) argues that fully schematic generality is distinct from quantificational generality of either the objectual/referential or substitutional type.[19] Although instances of the schema are generated by substitution, we must not equate this with substitutional quantification with a fixed substitution class of expressions. If we did this, open-ended schemas could not play the role they play in the foundations of mathematics, such as in axiomatizing set theory or in securing the categoricity and determinacy of arithmetic and of mathematics generally (see, e.g., McGee 2000, Warren 2020).

Treating *Normativist Satisfaction* as an open-ended schema allows me to avoid worries regarding, e.g., the fact that there are more real numbers than expressions in our language that can denote them. This would seem to imply that some real numbers aren't necessarily real numbers, since for some real numbers there isn't a name "*n*" such that "*n* is a real number" expresses a rule, thus those real numbers don't satisfy "□(x is a real number)". However, open-

[19] He also argues that fully schematic generality does not require absolutely unrestricted or absolutely general quantification, if one is concerned about that.

ended rules hold in any coherent expansion of the language.[20] It seems obvious that many rules relevant to *de re* necessity are open-ended, and this justifies the open-endedness of *Normativist Satisfaction*.[21] When a person is born and a new person-name is introduced[22] into the language, it too is governed by the open-ended rule according to which any person-name must be applied to a person. That person is thus necessarily a person, this being an object-level expression of the rule according to which their name must be applied to a person. The open-ended rules remain in effect as long as they remain in effect, i.e., as long as expansions of the language don't change them, directly or indirectly, and clearly there is no reason to think that adding names to our language would change the rules governing names. The normativist might also plausibly suggest that there is also an actual open-ended rule according to which any name for a real number must be applied to a real number. Since the rule is open-ended, it governs any coherent expansion of the language. If schemas can be truly open-ended in the way that some (e.g., Dieveney 2013,

[20] Warren also appeals to open-endedness for this purpose: "For when we move beyond arithmetic to mathematical truth quite generally, the need for open-ended schematic but non-infinitary rules becomes apparent. Many branches of mathematics, such as set theory, deal with more objects than can possibly be put into a canonical infinite list. If our language is countable, there will be real numbers and sets that are unnamed in our language" (2020, 271).

[21] Thomasson (2020) and Warren (2020) emphasize the open-endedness of many semantic rules, and Sidelle (1989, 44) appeals to a schema in his conventionalist explanation of *a posteriori* necessity. Schemas also appear throughout Ludwig's (in progress) account of necessity in terms of analyticity.

[22] Or when one simply introduces a new name for an already named person.

Lavine 1994, 2006, McGee 2000, Warren 2020) claim, then there is no cardinality problem for *Normativist Satisfaction*. Whether the antecedent of the previous sentence is true is beyond the scope of this paper. An area of future research for conventionalists/normativists is clarifying the requisite notion of open-endedness in a normativistically friendly way.

However, this open-endedness seems to open *Normativist Satisfaction* to another objection. Suppose that the claim that 2 exists expresses a rule or rule-consequence. Then, by *Normativist Satisfaction*, so should the claim that Tarski's actual favorite number exists, supposing "Tarski's actual favorite number" rigidly designates 2. But it is implausible that this claim expresses a rule or rule-consequence. I think such worries are easily handled, because rule-consequences include consequences that can only be derived with empirical auxiliaries. This is the key to the normativist/conventionalist account of *a posteriori* necessity (elaborated in Sidelle 1989, Thomasson 2020, Warren 2022), of which the claim that Tarski's actual favorite number exists seems to be an example. Here is an example of how *a posteriori* necessities are handled. According to Sidelle, it is an analytic truth that water necessarily has whatever microstructure it actually has. It is of course an empirical truth that water has the microstructure H_2O.[23] From these we derive that water necessarily has the microstructure H_2O. Warren's (2022) explanation runs similarly. Thomasson's (2020) explanation of *a posteriori* necessity is also quite similar, though it puts more emphasis on rules/normative talk. In the objector's case, the Sidellean explanation starts with the analytic truth is that 2 exists. The empirical auxiliary is that Tarski's

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[23] If you worry that the conventionalist cannot appeal to this truth because it is necessary, Sidelle says that "the real empirical import of, say, 'Water is H_2O' can be found in 'Most (enough) of the samples that we call 'water' are composed of H_2O'" (1989, 44).

actual favorite number = 2.[24] From these we derive that Tarski's actual favorite number exists. A more Thomassonian, rule-based explanation might go something like: "2" applies in every possible scenario. (Something like this is the rule [consequence] that is conveyed by the claim that 2 exists, as I explain below.) "Tarski's actual favorite number" and "2" must be applied to the same thing. (Something like this is the rule that is conveyed by the claim that Tarski's actual favorite number = 2.) So, "Tarski's actual favorite number" applies in every possible scenario. (Something like this is the rule that is conveyed by the claim that Tarksi's actual favorite number exists.) This is basically Povich's [forthcoming] normativist explanation of the substitutivity of identicals into modal contexts. It shows how, if the claim that *n* exists expresses a rule (consequence), where "*n*" is *some* rigid designator of *o*, then that claim expresses a rule (consequence), where "*n*" is *any* rigid designator of *o*, since flanking an identity sign with rigid designators of the same object results in a necessary identity statement.[25][26] Regardless of the

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[24] Similar to the previous footnote, here we could say something like: the real empirical import of "Tarski's actual favorite number = 2" can be found in "'Tarski's actual favorite number' and '2' apply to the same thing", which is obviously contingent.

[25] If you worry that only a schema's *instances* can be claimed (asserted), take the conditional in the footnoted sentence as shorthand for: if the claim expressed by *some* instance of the schema [*n* exists] expresses a rule (consequence), where "*n*" is a rigid designator of *o*, then the claim expressed by *any* instance of the schema [*n* exists] expresses a rule (consequence), where "*n*" is a rigid designator of *o*. Ditto for similar "schema-claims" throughout the paper.

[26] From a normativist perspective, the phenomenon of the substitutivity of identicals into modal contexts seems to show – what should be unsurprising – that any names of the same object are

details, if one of the normativist accounts of *a posteriori* necessity works, I think it can handle these and any similar examples, though here is not the place for a defense of any of those account specifically.

Now, one might worry about the appeal to rigidity in *Normativist Satisfaction*. It might seem that the normativist is not allowed the notion of rigidity or that the notion is useless for the normativist since rigid designators are conceptually contentless, and, so, there can be no analyticities involving them. However, normativists/conventionalists have argued for a semantic construal of rigidity (see Author 2024, Povich forthcoming, Sidelle 1992, 1995). It requires, however, the apparently controversial claim that objects/expressions have transworld identity criteria, which determine when two individuals in different worlds are identical.[27] As the

governed by the same rules.

[27] While controversial, this claim seems consistent with the idea that transworld identity can be stipulated (Kripke 1980, Salmon 1996, Fiocco 2007). *This* stipulation is not of the identity of individuals between *possible* worlds, but between the worlds simpliciter under consideration. Accounts of modal epistemology that accept that transworld identity can be stipulated must answer the question of what distinguishes possible worlds from impossible worlds, for, the latter are just as stipulable as the former. So, I can stipulate that I'm considering a world where a poached egg is identical to Hubert Humphrey. This does not make the world I'm considering a possible one. Such limits of stipulation are widely recognized (e.g., Fiocco 2007 and references therein). The normativist has a straightforward solution: what is stipulated is some content. That content is possible if and only if it is consistent with actual semantic rules – including identity criteria or other criteria that play the same essential-property-entailing role – and their

previous sentence should make clear, by “transworld identity criteria” I intend the metaphysical, not the epistemic, sense of that phrase. A full defense of this is beyond the scope of the present paper, but the basic idea is that if an expression is rigid, then it is governed by a rule according to which it must be applied[28] in every possible world to the individual that satisfies or fulfills the transworld identity criteria associated with it. This is Sidelle’s (1992, 1995) account of rigidity put in explicitly rule form; it ties the criteria of counterfactual application of an expression, i.e., criteria for application in merely possible worlds, to satisfaction or fulfillment of its transworld identity criteria.[29]

The appeal to “possible worlds” shouldn’t be taken to imply anything metaphysically significant that is inconsistent with normativism. Sidelle often talks instead of “actual or counterfactual scenarios” or “real or imagined scenarios,” and those phrases would do just as

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consequences. Since we can be ignorant or incorrect about the actual semantic rules and their consequences, we can be mistaken about whether what we’ve stipulated is possible. Normativism thus seems consistent with haecceitism, or at least what Salmon (1996) means by “haecceitism”.

[28] By “must be applied” here, I mean that it may be applied to a certain individual and may not be applied to anything else.

[29] Note that if a term’s criteria of counterfactual application are fulfilled in every world, yet it does not have transworld identity criteria, it seems that we cannot construe this as giving rise to any *de re* necessity, for there are no criteria that determine whether the thing that fulfills the counterfactual application criteria in world w_1 is identical to the thing that fulfills them in world w_2.

well here. What I – following Sidelle – mean is that we, in the actual world, are constrained by the relevant rules to apply the expression to one and the same object in any, possibly counterfactual, scenario that we happen to consider.[30]

So, for "*n*" to be a rigid designator of *o* is for "*n*" to be governed by a rule according to which it must apply in every possible scenario to the individual there that satisfies *o*'s transworld identity criteria. However, it seems that the normativist must say that transworld identity criteria are associated with terms rather than objects.[31] We should instead say that for "*n*" to be a rigid designator of *o* is for "*n*" to be governed by a rule according to which it must apply in every possible scenario to the individual there that satisfies the transworld identity criteria associated with a sortal under which *o* falls. This is not *ad hoc* but motivated by considering what assignment of an object to a variable requires. An open modal formula is simply not truth/satisfaction evaluable unless the object assigned to the variable is conceived under some sortal. Trying to evaluate whether an open formula is satisfied by an object without conceiving of it under a sortal is like trying to evaluate whether "$\Box Fx$" is true of *that* (pointing at, say, the dog). Well, what am I assigning to "*x*"? The dog? The collection of the dog's cells? A time-slice of the dog? What's necessarily true of the dog is not necessarily true of the collection of its cells. This is not an epistemic point about how we can know which object is being assigned. To secure

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[30] I thank a reviewer for pressing me to clarify this and for this way of putting the point.

[31] Unless, perhaps, one thinks objects themselves are sortally individuated. Then one can simply talk of the transworld identity criteria of an object, rather than those of a sortal under which it falls. I think some conventionalists (e.g., Sidelle 1989) hold this view, but I won't assume it here.

determinate reference at all, I must intend my ostension to be associated with a certain sortal.[32] It is already part of the normativist position that there cannot be direct reference, that reference requires a disambiguating sortal with transworld identity conditions (Sidelle 1992, 1995, Thomasson 2007; Warren presumably must say similar things, though I don't think he has in print). I am simply pointing out that insofar as assignment of an object to a variable is the determination of a variable's referent, there similarly cannot be "direct assignment," i.e., assignment without a disambiguating sortal. In logic a disambiguating sortal is not made explicit, but we do it in practice.

Put another way, transworld identity criteria are sortal-relative and can be specified in general as follows: $\forall x \forall y$ (if x is an S in w_1 and y is an S in w_2, then $x=y$ iff $R_S xy$). For example, $\forall x \forall y$ (if x is a person in w_1 and y is a person in w_2, then $x=y$ iff x and y have the same biological origin) (for example). So, when I say that "*n*" must be applied in every possible world to the individual that satisfies the transworld identity criteria of a sortal under which *o* falls, I mean that "*n*" must be applied in every possible world to the individual that bears R_S to *o*, for a sortal *S* under which *o* falls. We look at examples below.

One might worry that reference to "a sortal" in the normativist account of rigidity could

[32] Or perhaps only a categorial term (Lowe 2007). If that is the correct view, perhaps we should replace all instances of "sortal" with "categorial term" in my arguments below. However, Lowe's categorialism seems to be distinguished from sortalism only in what the view says is required to individuate an object *cognitively* – to single it out in thought, as Lowe says – rather than metaphysically. So, perhaps the categorialism/sortalism distinction doesn't matter so much when it comes to the metaphysical issues I'm concerned with here.

cause problems. Is satisfaction relative to which sortal is chosen? This is not a problem. All sortals that apply to a given object share their transworld identity criteria. An object can only fall under one category, and every sortal within a category shares its identity criteria (Dummett 1973, Lowe 1989, 2007).[33] Suppose that *o* falls under sortals with different identity criteria. Things falling under sortals with different identity criteria cannot be identical. Therefore, *o* cannot be identical with itself. That's absurd. So, *o* cannot fall under sortals with different identity criteria. What distinguishes sortals falling under a category are their criteria of application, not their identity criteria (Ibid.).

Let's look at an example of *Normativist Satisfaction* in action. "$\Box$(*x* is a person)" is true of or satisfied by Socrates just in case the claim that Socrates is a person expresses a rule or rule-consequence – as does any other claim of an instance of the schema [*n* is a person], where "*n*" must be applied in every possible world to the person that satisfies the transworld identity criteria of a sortal under which Socrates falls. The relevant sortal is, of course, "person," so "$\Box$(*x* is a person)" is true of or satisfied by Socrates just in case the claim[34] that *n* is a person expresses a rule or rule-consequence, for any "*n*" that must be applied in every possible world to the person who shares Socrates's biological origin (assuming sameness of biological origin is the transworld identity criterion of "person"). Is it true that the claim that *n* is a person expresses a rule or rule-

[33] Dummett and Lowe were talking about *intra*world identity criteria, but the arguments clearly generalize to transworld identity criteria.

[34] Recall that this is shorthand for: the claim expressed by any instance of the schema [*n* is a person] expresses a rule or rule-consequence, for any "*n*" that must be applied in every possible world to the person who shares Socrates's biological origin. Keep this in mind below.

consequence, for any "*n*" that must be applied in every possible world to the person who shares Socrates's biological origin? Yes, for, it is a rule that "*n*" must be applied to the person in every possible world who shares Socrates's biological origin, which implies that "*n*" must be applied to a person in every world. This is the rule expressed by the *de re* necessary claim that Socrates is a person. This comports well with Thomasson's (2020) own account of *de re* necessities such as this, according to which a name must be associated with a sortal, which gives rise to the rule that the name must be applied to something that satisfies the sortal. "Socrates" is a person-name, so there is a rule that "Socrates" must be applied to a person in every actual or counterfactual scenario, which is expressed by the *de re* necessary claim that Socrates is a person. My account fleshes out in more detail why it is that the fact that "Socrates" is a person-name gives rise to the rule that "Socrates" must be applied to a person – this follows from the rule that "Socrates" must be applied to the person in every possible world who shares Socrates's biological origin and the fact that this implies being a person. For the normativist/conventionalist, there just can't be any other source of *de re* necessity than transworld identity criteria and what they imply.[35] Metaphorically, as we go from world to world "seeing" which properties Socrates has in all of them, the only thing that *makes* a person in each world identical to the actual Socrates is that he shares Socrates's biological origin (Sidelle 1989, 73-4). Thus, the properties Socrates himself has necessarily can only be those that are implied by his biological origin. Socrates could've been a

[35] For this reason, you might want to state *Normativist Satisfaction* as: "*Fx*" is true of or satisfied by an object *o*, just in case the transworld identity criteria of a sortal under which *o* falls imply that *o* is *F*. However, we will see below when we discuss the thinness of 2's existence that it may be a bit more complicated than this.

senator because being a person having actual Socrates's biological origin doesn't rule that out (Ibid.). Thomasson's account of *de re* necessity, however, does not mention this crucial role for transworld identity criteria. Her account seems to explain the necessity of *de re* necessities but not their *de re*-ness. My account does both. The claim that Socrates is necessarily a person doesn't just express the rule that "Socrates" must be applied to a person; it expresses the rule that *any* name of Socrates must be applied to a person.[36] This is what gives rise to the central intuition behind *de re* necessities, that they are true of the individuals themselves, independently of how the individuals are picked out.

One might object as follows. Socrates also falls under the sortal "collection of cells". Since the claim that *n* is a person does *not* express a rule or rule-consequence, where "*n*" must be applied in every possible world to the individual who satisfies the transworld identity criteria of associated with the sortal "collection of cells," "$\Box$(x is a person)" is *not* true of or satisfied by Socrates. Contrary to what I said above, choice of the sortal under which an object falls matters! This is confused from the beginning. Socrates is a person, *not* a collection of cells. The collection of cells composing Socrates at a time is an object not identical to Socrates, precisely because they have different identity criteria (see Lowe 1989, especially chapter 7). If "Schmocrates" names a collection of cells composing Socrates at *t*, then "$\Box$(x is a person)" isn't true of or satisfied by Schmocrates, but Schmocrates is not identical to Socrates.

Now – getting back to thin objects – what, then, according to the normativist, is

[36] Thomasson would not, of course, deny that any name of Socrates must be applied to a person. But it seems that she denies – or at least does not affirm – that this is the rule expressed by the *de re* necessary claim that Socrates is a person.

expressed by the claim that there are thin objects or objects whose existence makes no demands on the world? (The claim that there are thin objects is itself necessary, so it should express a rule or rule-consequence.) I think the normativist should take the claim that there are thin objects to express that there is an *x* such that the claim that *n* exists expresses a rule or rule-consequence, for any "*n*" that must be applied in every possible world to the individual that satisfies the transworld identity criteria of a sortal under which *x* falls. This, of course, is the normativist precisification of the claim that there is an *x* such that the claim that *n* exists is analytic, where "*n*" rigidly designates *x*. We look at an existential instantiation of this below. The rule or rule-consequence that the claim that there are thin objects expresses can then be thought of as an existential generalization of the instantiation we examine below.

What about claims of *relative* thinness? Linnebo writes that, "An object can also be thin *relative to some other objects* if, given the existence of these other objects, the existence of the object in question makes no substantial *further* demand" (2018, 4, original emphasis). For the normativist, the claim that there are relatively thin objects should be understood to mean that there are objects *x* and *y* such that "if *m* exists, then *n* exists" expresses a rule or rule-consequence, for any "*m*" that must be applied in every possible world to the individual that satisfies the transworld identity criteria of a sortal under which *x* falls and for any "*n*" that must be applied in every possible world to the individual that satisfies the transworld identity criteria of a sortal under which *y* falls.

Let us look at an existentially instantiated thinness claim. The natural number 2 is thin; its existence makes no demands on the world. For the normativist, this should be understood to mean that the claim that 2 exists expresses a rule or rule-consequence. The rule (consequence) it expresses is that any name of 2 applies[37] in every world. By what reasoning? What explains this rule-consequence? Any name of 2 must apply in every possible world to the individual that satisfies the transworld identity criteria of a sortal under which 2 falls, the relevant sortal being "natural number". Now, I have no account of the transworld identity criteria associated with the sortal "natural number". One might need a specific account of the nature of natural numbers before such criteria can be suggested,[38] or perhaps they are given by the Peano axioms (see below for more on this thought). Regardless, 1) the above considerations on determinate reference suggest that *some* such criteria are required, and 2) I think I can say enough here to make plausible that the normativist has an account of *de re* necessary existence claims like this even without a specific account of the transworld identity criteria for natural numbers.

Why, for the normativist, isn't the existence of Socrates necessary? Because fulfilling the relevant transworld identity criteria in a world isn't guaranteed – there are worlds where nothing shares actual Socrates's biological origin, so in those worlds Socrates doesn't exist. A *de re* necessary existence claim is true when something fulfills the relevant transworld identity criteria

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[37] To ward off a confusion, I must emphasize that when I say "apply" in this context, I do not mean what is often called the application of mathematics. So, when I talk of the application of "2" or any name of 2, I mean its use to pick out the number 2; I don't mean its use in, for example, counting.

[38] So, e.g., a structuralist might say that the nature of a natural number is determined by its place in the natural number structure, so that the identity conditions for natural numbers involve occupying the same place in the natural number structure. Perhaps Hume's Principle could be of help too (Hale and Wright 2001).

in every world. In such a case, there is an individual in every world that is identical to the actual individual, so the individual exists necessarily. So, the normativist can say that the necessary existence of 2 reflects the fact that fulfillment of the relevant transworld identity criteria – whatever they are – is guaranteed in every world.

It would be nice, though, if the normativist can say more about *why* the transworld identity criteria are guaranteed to be fulfilled in every world. Here is a suggestion. Some conventionalists (e.g., Friederich 2011, Warren 2020) have argued that the Peano axioms can be thought of as the rules governing our natural number concepts. Perhaps it is these rules that determine the transworld identity criteria for "natural number". And the Peano axioms simultaneously imply the existence of things that fulfill the transworld identity criteria in any world in which they are true (i.e., in any world in which the rules are in force). Several relevant axioms are: 0 is a natural number (which clearly implies the existence of a number); the successor of any natural number is a natural number; if the successor of a natural number m = the successor of a natural number n, then $m = n$. Now, it is already part of the normativist/conventionalist position that in counterfactual reasoning we hold fixed our actual rules (see e.g., Sidelle 2009, Thomasson 2020, Wright 1985).[39] So, we go from world to world "looking" for something that fulfills the relevant transworld identity criteria. As we go from world to world, we are holding fixed the Peano axioms, our rules governing our natural number

[39] It allows them to respond to the contingency objection, according to which normativism makes necessities contingent, since our rules are contingent. The response is that we hold our actual rules fixed in all counterfactual reasoning, so the normativist can accept the intuition that necessities don't counterfactually depend on our contingent rules.

concepts, which simultaneously supply the relevant transworld identity criteria and imply the existence of things that fulfill them. So, as we go from world to world "looking" for something that fulfills the relevant transworld identity criteria, we are guaranteed to find it. The *de re* necessary claim that 2 exists thus expresses a rule-consequence – a consequence of the rule that any name of 2 must be applied in every possible world to the individual that satisfies the transworld identity criteria of a sortal under which 2 falls and the fact, just explained, that in every world it is guaranteed that there exists something that fulfills the criteria. That names of 2 apply in every world is, I suggest, the rule-consequence expressed by the *de re* necessary claim that 2 exists.[40]

How about a specific *relative* thinness claim, such as that singleton Socrates is thin relative to Socrates? For the normativist, this should be understood to mean that the claim that if m exists, then n exists expresses a rule or rule consequence, for any "*m*" that must be applied in every possible world to the individual that satisfies the transworld identity criteria of a sortal under which Socrates falls, and for any "*n*" that must be applied in every possible world to the individual that satisfies the transworld identity criteria of a sortal under which singleton Socrates falls. The application criteria for "set" require only the existence of things to act as members – wherever there are some things, there is a set of those things – and the transworld identity

[40] What I've argued for the natural number 2 obviously applies to every natural number, and, presumably, to other kinds of number (e.g., real numbers), mutatis mutandis (e.g., by substituting in the axioms of real analysis for the Peano axioms). A similar story might be told for *de re* necessary existence claims for other kinds of abstract object, though the details await future work.

criterion for sets is identity of members. Thus, any world where there's an individual that satisfies the transworld identity criteria of a sortal under which Socrates falls – i.e., any world where Socrates exists – is a world where there's an individual that satisfies the transworld identity criteria of a sortal under which singleton Socrates falls – i.e., is a world where singleton Socrates exists. The claim that if Socrates exists, then singleton Socrates exists thus expresses a rule such as that in any actual or counterfactual scenario in which any name of Socrates must be applied, any name of singleton Socrates must be applied.[41] (This would be an example of what Thomasson [2007] calls an analytic entailment, explained below.)

The story I've told has many moving parts that all need much more defense, so it is by no means conclusive. But I think we can conclude that it is at least not obvious that Linnebo, following Quine and Burgess, has ruled out understanding abstraction principles and thin existence in terms of analyticity or conceptual rules. Let's now examine how Linnebo conceives sufficiency and the criteria he thinks should be used to explicate thin existence. He argues that sufficiency meets these criteria. I argue that analyticity or conceptual rules can meet these criteria as well.

Linnebo argues that if sufficiency is to explicate thin existence, it should be a relation that meets the following criteria: 1) to be less demanding than analytic entailment,[42] 2) to be more demanding than strict (i.e., necessary) implication, 3) to imply metaphysical explanation (so that if φ suffices for ψ, φ metaphysically explains ψ), 4) to be ontologically ampliative (so that if φ

[41] Warren could argue that all the rules I've appealed to throughout this section, or similar ones, are what explain why thinness claims are true.

[42] P analytically entails Q if and only if "if P, then Q" is analytic.

suffices for ψ, the ontological commitments of ψ exceed those of φ), and 5) to imply that it is possible to know that φ implies ψ (Linnebo 2018, 14-7).[43] What relation could possibly do all these things? With some finessing, analytic entailment can be made a plausible candidate (ignoring the first criterion, of course). Let's examine each of these criteria in turn and see how analyticity can meet them.

The second criterion means that we shouldn't define $\varphi \Rightarrow \psi$ as the strict conditional $\Box(\varphi \rightarrow \psi)$. The reasons are 1) that it would count God, if They exist, as a thin object since $\Box(\top \rightarrow \text{God exists})$, 2) that it would run afoul of the third criterion, because no tautology metaphysically explains God's existence, and 3) that it would run afoul of the fifth criterion, because it is not possible to move from knowledge of a tautology to knowledge of God's existence.[44] This is a problem, since analyticists usually view necessitation (i.e., strict implication) as analytic entailment. I will address the second objection below when I discuss metaphysical explanation. The solution to the first and third objections is to recognize that only *metaphysical* necessitation is analytic entailment. Thomasson (2020, 115) considers whether the claim that God exists is a counterexample to her normativism. The thought is that if it's true, it's metaphysically necessary, but it isn't analytic, i.e., it doesn't express any conceptual rule. So, it seems like a counterexample to her view that all metaphysical necessity is analytic or expresses

[43] This is the simplified criterion he uses in Linnebo (2023a). The full criterion is: "If $\varphi \Rightarrow \psi$, then it is possible to know $\varphi \rightarrow \psi$; and if additionally φ is known, then this possible knowledge is compatible with continued knowledge of φ" (2018, 16). All this criterion is supposed to do is ensure that if $\varphi \Rightarrow \psi$, it is possible to move from knowledge of φ to knowledge of ψ.

[44] I'm sure there are philosophers of religion who think otherwise.

conceptual rules (or their consequences). Thomasson suspects that some other kind of necessity is at play here than metaphysical necessity – perhaps nomological necessity – and I agree. Lange (2009) suggests that there are various strengths of nomological necessity. Perhaps the necessity at play is the strongest kind of nomological necessity. But what if there *were* a metaphysically necessary God? Again, the only way I can make sense of this question is by imagining a being with the strongest kind of nomological necessity. If you stipulated that it is metaphysically necessary that God exists, you'd be making the claim that God exists analytic, i.e., an expression of conceptual rules, and, thus, changing its meaning.

What about objects that *are* metaphysically necessary? Consider $\Box(\top \rightarrow 2$ exists). Is it possible to move from knowledge of a tautology to knowledge of 2's existence? Yes, you can move from knowledge of anything to knowledge of 2's existence, because 2's existence is analytic and a priori. Linnebo includes the epistemic criterion to account for knowledge of abstract objects. How do we acquire knowledge of directions? Somehow, knowledge of lines must suffice for knowledge of directions. Supporters of analyticity have an easy answer: knowledge of lines suffices for knowledge of directions because claims about lines analytically entail claims about directions. Thus, Linnebo's second criterion, that sufficiency be more demanding than strict implication, is unnecessary if you have analyticity and believe that God's existence wouldn't be *metaphysically* necessary.

The third criterion requires that that if φ suffices for ψ, φ metaphysically explains ψ. It certainly does seem like there being some things arranged tablewise metaphysically explains there being a table, rather than vice versa. And it is not generally the case that if X analytically entails Y, then X metaphysically explains Y. For example, in some cases, analytic entailment is symmetric where metaphysical explanation isn't. That Jeremy is a bachelor analytically entails that he is an unmarried man, and vice versa, but it doesn't seem like metaphysical explanation similarly goes in both directions. So, I agree with Linnebo that metaphysical explanations are usually asymmetric – one side of a sufficiency claim usually does have explanatory priority – but what is the motivation for requiring that sufficiency imply metaphysical explanation? About this requirement, Linnebo writes (16-7):

> A second promised benefit of thin objects is a response to the worry about the seeming ontological extravagance of modern mathematics and certain other bodies of knowledge, such as classical mereology. How can these sciences get away with postulating such an abundance of objects when ontological economy is otherwise regarded as a virtue? [1] Again, the minimalist has an answer, namely that the generous ontologies in question either make no substantial demand on the world (in the case of pure abstract objects such as numbers and sets), or their demands on the world do not substantially exceed demands that have already been met (in the case of impure sets or mereological sums). [2] This answer motivates another constraint on $\Rightarrow$. Assume that $\varphi \Rightarrow \psi$. Then any metaphysical explanation of ϕ must also explain ψ, or at least give rise to such an explanation.

But for the analyticist, the claim of insubstantial demand is justified by appeal to analyticity or conceptual rules. The formal sciences get away with postulating an abundance of objects because their demands are insubstantial, which means their existence is analytic. So, for the analyticist, the requirement that sufficiency claims imply metaphysical explanation is unnecessary. Recall that the appeal of abstraction principles is that they explain how we can grasp a new concept and use that concept to refer to a "new" object. From a conventionalist point of view, they answer the question of how legitimately to introduce new concepts to our scheme. Metaphysical explanation is irrelevant to that question. If we had the concept of a table but not the concept of things arranged tablewise, we could use an abstraction principle to introduce the concept of things arranged tablewise, even though tables do not metaphysically explain things arranged tablewise. Explanatory priority is irrelevant when introducing a new concept. Linnebo might say that this introduction of the concept of things arranged tablewise is illegitimate because it doesn't track the direction of metaphysical explanation, but remember that he says that only because he eschews analyticity; he thinks appeal to metaphysical explanation is required to explicate the notion of insubstantial demand. Since tables do not metaphysically explain things arranged tablewise, we can't be sure that our introduction of this concept hasn't substantially increased the demands we make on the world. This is not a worry when you have analyticity and accept that there being tables analytically entails there being things arranged tablewise. So, although analytic entailment does not imply metaphysical explanation, that isn't a problem.[45]

Linnebo's fourth criterion for sufficiency is that it be ontologically ampliative, so that if φ suffices for ψ, the ontological commitments of ψ exceed those of φ. For the analyticist, an inference's being ontologically ampliative just means it is ampliative with respect to the linguistic framework – i.e., when its conclusion contains a noun term not contained in the premise (Carnap 1950, Thomasson 2017). Thus, the fourth criterion is met so long as we stipulate that the consequent analytically entailed contain a noun term not contained in the antecedent. Let us say that X analytically entails* Y if and only if X analytically entails Y, and "Y" contains a noun term not contained in "X". Thus, it seems that analytic entailment* misses

[45] Also, while I can't argue for this here, I think metaphysical explanation is best understood *in terms of* analyticity (see Locke 2020), although, as I have already argued, the concept of metaphysical explanation is not required to explicate thin existence once you have analyticity.

no feature of Linnebo's "sufficiency" that is well-motivated.[46]

4. Conclusion

There are many fascinating arguments in Linnebo's (2018) book that I could not address. Here, I was only concerned to defend an analytic or conceptual rule conception of thin existence from Linnebo's objections. I have argued that analyticity can meet Linnebo's criteria for explicating thin existence, and I have defended analyticity or conceptual rules against objections regarding analytic existence claims and *de re* analyticity. While I don't claim to have settled the matter, I hope to have shown that analyticity or conceptual rules can still plausibly be used to explicate thin existence.

References

Asay, Jamin. "Truth(making) by convention." *American Philosophical Quarterly* 57.2 (2020): 117-128.

Author. [Redacted for complete anonymity]. 2024. Oxford University Press.

Boghossian, Paul. "Analyticity reconsidered." *Noûs* 30.3 (1996): 360-391.

Burgess, John P. "Quinus ab Omni Nævo Vindicatus." *Canadian Journal of Philosophy Supplementary Volume* 23 (1997): 25-65.

Davidson, Donald, 1967, "Truth and Meaning," *Synthese*, 17(3): 304–323; reprinted in *Inquiries into Truth and Interpretation*, Oxford: Clarendon Press, 2001, pp. 17–36.

Dieveney, Patrick. "Anything and Everything." *Erkenntnis* 78 (2013): 119-140.

Donaldson, Thomas. "Analyticity." In Michael J. Raven (ed.), *The Routledge Handbook of Metaphysical Grounding*, pp. 288-299. 2020.

[46] I thank an anonymous reviewer for this point.

Dummett, M., *Frege: Philosophy of Language*, London: Duckworth, 1973.

Ebert, Philip A., and Marcus Rossberg, eds. *Abstractionism: Essays in philosophy of mathematics*. Oxford University Press, 2016.

Field, Hartry. “Conventionalism about mathematics and logic.” *Noûs* (2022): 1-17.

Fiocco, M. Oreste. "Conceivability, imagination and modal knowledge." *Philosophy and Phenomenological Research* 74.2 (2007): 364-380.

Friederich, Simon. “Motivating Wittgenstein’s perspective on mathematical sentences as norms.” *Philosophia Mathematica* 19.1 (2011): 1-19.

Hale, Bob, and Crispin Wright. *The reason's proper study: Essays towards a neo-Fregean philosophy of mathematics*. Oxford University Press, 2001.

Hale, Bob, and Crispin Wright. "Bolzano's Definition of Analytic Propositions." *Grazer Philosophische Studien* 91.1 (2015): 325-364.

Horwich, Paul 1998, *Truth*, Oxford: Blackwell, 2nd edition.

Kripke, Saul. 1980. *Naming and Necessity*, Cambridge, MA: Harvard University Press.

Lange, Marc. *Laws and Lawmakers*. Oxford University Press, 2009.

Lavine, Shaughan. *Understanding the infinite*. Harvard University Press, 1994.

Lavine, S. “Something About Everything: Universal Quantification in the Universal Sense of Universal Quantification”. In Rayo and Uzquiano (eds.), *Absolute Generality*. Oxford: Oxford University Press, 2006.

Linnebo, Øystein. *Thin Objects: An Abstractionist Account*. Oxford University Press, 2018.

Linnebo, Øystein. “Précis.” *Theoria* 89, 3 Special Issue: Thin Objects (2023a): 247-255.

Linnebo, Øystein. “Replies.” *Theoria* 89, 3 Special Issue: Thin Objects (2023b): 393-406.

Locke, Theodore. 2020. Metaphysical Explanations for Modal Normativists. *Metaphysics*, 3(1), pp. 33–54.

Lowe, E. J., 1989, *Kinds of Being: A Study of Individuation, Identity and the Logic of Sortal Terms*, Oxford: Blackwell.

Lowe, E. J. "Sortals and the individuation of objects." *Mind & Language* 22.5 (2007): 514-33.

Ludwig, Kirk. In progress. "De re necessities." https://socrates.sitehost.iu.edu/papers/De%20Re%20Necessities.doc

McGee, Vann. (2000). "Everything." In G. Sher and R. Tieszen (eds.), *Between Logic and Intuition*. Cambridge: Cambridge University Press. pp. 54-78.

Nyseth, Fredrik. "Linguistic Conventionalism and the Truth-Contrast Thesis." *The Philosophical Quarterly* 71.2 (2021): 264-285.

Povich, M. "(A Little) Quantified Modal Logic for Normativists". *Analysis*, forthcoming.

Quine, W.V.O., 1951, "Two Dogmas of Empiricism", *Philosophical Review*, 60: 20–43.

Quine W.V.O., "Reference and modality." In *From a logical point of view*, by Willard Van Orman Quine, Harvard University Press, Cambridge, Mass., 1953, pp. 139–159.

Quine, W.V. (1958). "Speaking of Objects." *Proceedings and Addresses of the American Philosophical Association* 31: 5–22.

Rabinowicz, Wlodek. "Analyticity and possible-world semantics." *Erkenntnis* 72 (2010): 295-314.

Russell, Gillian. *Truth in virtue of meaning: A defence of the analytic/synthetic distinction*. OUP: Oxford, 2008.

Salmon, Nathan. "Trans-world identification and stipulation." *Philosophical Studies: An International Journal for Philosophy in the Analytic Tradition* 84.2/3 (1996): 203-223.

Sidelle, Alan. 1989. *Necessity, Essence and Individuation: A Defense of Conventionalism*. Ithaca:

Cornell University Press.

Sidelle, Alan. "Rigidity, ontology, and semantic structure." *The Journal of Philosophy* 89.8 (1992): 410-430.

Sidelle, Alan. "A semantic account of rigidity." *Philosophical Studies: An International Journal for Philosophy in the Analytic Tradition* 80.1 (1995): 69-105.

Sidelle, Alan. “Conventionalism and the Contingency of Conventions.” *Noûs* 43.2 (2009): 224-241.

Stanley, Jason. “Names and Rigid Designation”. In *A Companion to the Philosophy of Language*, Hale, Wright, and Miller, eds. (Oxford, Blackwell Press, 2017): 920-947.

Thomasson, Amie L. 2007. *Ordinary Objects*. New York: Oxford University Press.

Thomasson, Amie L. *Ontology made easy*. Oxford University Press, 2014.

Thomasson, Amie L. “Why we should still take it easy.” *Mind* 126.503 (2017): 769-779.

Thomasson, Amie L. *Norms and Necessity*. New York: Oxford University Press, 2020.

Thomasson, Amie L. “How it All Hangs Together.” In Miguel Garcia-Godinez, ed. *Thomasson on Ontology*. Palgrave Macmillan, 2023. 9-38.

Topey, Brett. “Linguistic convention and worldly fact: Prospects for a naturalist theory of the a priori.” *Philosophical Studies* 176 (2019): 1725-1752.

Warren, Jared. “The possibility of truth by convention.” *The Philosophical Quarterly* 65.258 (2015): 84-93.

Warren, Jared. *Shadows of syntax: Revitalizing logical and mathematical conventionalism*. Oxford University Press, 2020.

Warren, Jared. *The a priori without magic*. Cambridge University Press, 2022a.

Warren, Jared. “Inferentialism, Conventionalism, and A Posteriori Necessity.” *The Journal of*

Philosophy 119.10 (2022b): 517-541.

Wittgenstein, Ludwig. *Philosophical Grammar*, 1974, R. Rhees (ed.), A. Kenny (trans.), Oxford: Blackwell.

Wright, Crispin. 1985. "In Defence of the Conventional Wisdom." In *Exercises in Analysis*, ed. Ian Hacking, 171–97. Cambridge University Press.