The Directionality of Distinctively Mathematical Explanations[1]

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Abstract: In "What Makes a Scientific Explanation Distinctively Mathematical?" (2013b), Lange uses several compelling examples to argue that certain explanations for natural phenomena appeal primarily to mathematical, rather than natural, facts. In such explanations, the core explanatory facts are modally stronger than facts about causation, regularity, and other natural relations. We show that Lange's account of distinctively mathematical explanation is flawed in that it fails to account for the implicit directionality in each of his examples. This inadequacy is remediable in each case by appeal to ontic facts that account for why the explanation is acceptable in one direction and unacceptable in the other direction. The mathematics involved in these examples cannot play this crucial normative role. While Lange's examples fail to demonstrate the existence of distinctively mathematical explanations, they help to emphasize that many superficially natural scientific explanations rely for their explanatory force on relations of stronger-than-natural necessity. These are not opposing kinds of scientific explanations; they are different aspects of scientific explanation.

Keywords: distinctively mathematical explanation, modal conception, ontic conception

1. Introduction.

In "What Makes a Scientific Explanation Distinctively Mathematical?" (2013b), Lange uses several compelling examples to argue that certain natural phenomena are best explained by appeal to mathematical, rather than natural, facts. In distinctively mathematical explanations, the core explanatory facts are modally stronger than facts about, e.g., statistical

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relevance, causation, or natural law. A distinctively mathematical explanation might describe causes, Lange allows, but its explanatory force derives ultimately from appeal to facts that are 'more necessary' than causal laws. Lange advances this thesis to argue for the importance of a purely modal view of explanation (a view that emphasizes necessities, possibilities, and impossibilities, showing that an event had to or could not have happened) in contrast to the widely discussed ontic view (a view that associates explanation with describing the relevant natural facts, e.g., about how the event was caused or how its underlying mechanisms work).[2]

Lange operates with a narrower understanding of the ontic conception. He describes it as the view that all explanations are causal. He cites Salmon, who claimed that, "To give scientific explanations is to show how events and statistical regularities fit into the causal structure of the world" (Salmon 1984)[3] and "To understand why certain things happen, we

2 There is a growing body of literature on mathematical explanation (Baker 2005; Baker and Colyvan 2011; Huneman 2010; Pincock 2011). We focus on Lange because his examples have become canonical and because his commitments are so explicitly formulated. We suspect that the directionality problem will arise in these other papers as well, but these authors are mostly concerned with indispensability and the ontology of mathematics, a topic that we (like Lange) hope to sidestep to focus on explanation alone. See Craver (forthcoming) for a discussion of directionality problems in network explanation. Andersen's (forthcoming) response to Lange is complementary to ours, fleshing out a point about explananda at which we only gesture in the conclusion. Our main focus is directionality.

3 See the passages quoted in Povich (forthcoming) for evidence that Salmon did not think

need to see how they are produced by these mechanisms [processes, interactions, laws]" (Salmon 1984). He also cites Lewis ("Here is my main thesis: to explain an event is to provide some information about its causal history"; 1986) and Sober ("The explanation of an event describes the 'causal structure' in which it is embedded"; 1984).[4] In contrast to Lange, we adopt a more inclusive understanding of the ontic that embraces any natural regularity (Salmon 1989; Craver 2014; Povich forthcoming), e.g., statistical relevance (Salmon 1977), natural laws (Hempel 1965), or contingent compositional relations might also figure fundamentally in explanation. This point will become crucial below, given that the ontic relations that explain the directionality of some explanations are not specifically causal relations; but they are ontic in this wider sense.[5] Lange's arguments should, however, work

the ontic conception was strictly causal. As we note, Lange's conception of the ontic conception is narrower than one might allow. The primary aim of the ontic conception is to insist that whether X explains Y is an objective matter of (natural) fact.

4 One can believe that mechanistic explanation is important without believing that all explanations are causal or mechanical. We show why $C = 2\pi r$ without describing mechanisms. We explain why Obama can sign treaties without describing causes. Explanations in epistemology, logic, and metaphysics often work without describing causes. The question here is not whether one should be a pluralist about explanation but about whether Lange's account of distinctively mathematical explanation is complete and whether his contrast with the ontic conception is substantiated by his examples.

5 For purposes of focus, we leave aside the question of whether the existence of distinctively mathematical explanations in fact commits one to the denial of the ontic

equally well against this broader understanding of the ontic conception, given that he uses the examples to show that some explanations of natural facts depend fundamentally on relations of necessity that are stronger than mere natural necessity.

We argue that Lange's account of distinctively mathematical explanation is flawed. Specifically, it fails to account for the directionality implicit in his examples of distinctively mathematical explanation. This failure threatens Lange's argument because it shows that his examples do not, in fact, derive their explanatory force from mathematical relations alone (independent of ontic considerations). The inadequacy is in each case easily remediable by appeal to ontic facts that account for why the explanation is acceptable in one direction and unacceptable in the other. That is, Lange's exemplars of distinctively mathematical explanation appear to require for their adequacy appeal to natural, ontic facts about, e.g., causation, constitution, and regularity. More positively, we suggest that all mechanistic explanations are constrained, and so partly constituted, by both ontic and modal facts. Rather than seeing an opposition between distinctively mathematical explanations and causal (or more broadly ontic) explanations, Lange's examples, as we reinterpret them, direct us to understand how these distinct aspects of explanation, these distinct sources of explanatory

conception or even to the idea that there is a modal form of explanation independent of ontic considerations. The fact that mathematics is important to explanation doesn't necessarily commit one to the idea that the modal conception has a role to play independently of ontic considerations absent further commitments about the relationship between mathematics and ontology. Like Lange, we remain silent on the ontology of mathematics (492).

power, intermingle and interact with one another in most scientific explanations.

2. Lange's Account of Distinctively Mathematical Explanation.

Lange's goal is to show “how distinctively mathematical explanations work” by revealing the “source of their explanatory power” (486). He accepts as a basic constraint on his account that it should “fit scientific practice,” that is, that it should judge as “explanatory only hypotheses that would (if true) constitute genuine scientific explanations” (486). In short, the account should not contradict too many scientific common-sense judgments about whether an explanation is good or bad. Lange's goal and his guiding constraint are conceptually related: to identify the source of an explanation's power requires identifying the key features that sort acceptable explanations from unacceptable explanations of that type. In causal explanations, for example, much of the explanatory power comes from knowledge of the causal relations among components in a mechanism. Bad causal explanations of this kind fail when they misrepresent the relevant causal structure (in ways that matter). In mathematical explanations, on Lange's view, the explanatory force comes from mathematical relations that are 'more necessary' than mere causal or correlational regularities.

Given this set-up, Lange's account of the explanatory force of distinctively mathematical explanations can be undermined by examples that fit Lange's account but that would be rejected as bad explanations as a matter of scientific common-sense. The account would fail to identify fully the explanatory force in such explanations and so would fail to account for the norms governing such explanations.

Lange does not address the canonical form of mathematical explanations. However, his examples are readily reconstructed as arguments in which a description of an explanandum phenomenon follows from an empirical premise (EP) describing the relevant natural facts, and a mathematical premise (MP) describing one or more more-than-merely-naturally-necessary facts. To begin with Lange's simplest example:

Strawberries: Why can't Mary divide her strawberries among her three kids?[6]

Because she has 23 strawberries, and 23 is not divisible by three.

This explanation can be reconstructed as an argument:

1. Mary has 23 strawberries (EP)

<u>2. 23 is indivisible by 3 (MP)</u>

C. Mary can't divide the strawberries equally among her three kids.[7]

We would have to tighten the bolts to make the argument valid (e.g., no cutting of strawberries is allowed), but the general idea is clear enough. The empirical premise works by describing the natural features of a system. They specify, for example, the relevant magnitudes (Mary starts with 23 strawberries), and the causal or otherwise relevant dependencies among them. All mathematical explanations of natural phenomena require at least some empirical premises to show how the mathematics will be applied and to specify

6 Or "Why didn't she on some particular occasion?" or "Why didn't or couldn't anyone ever?" Lange intends all these explananda to be explained by the same explanans; a similar multiplicity of explananda can be generated for the examples below.

7 This example is reconstructed as a sketch of a deductive argument, but distinctively mathematical explanations might be inductive. For example, one might explain why fair dice will most likely not roll a string of ten consecutive double-sixes on mathematical grounds, using logical probability and some math.

the natural (empirically discovered) constraints under which the mathematical premises do their work. The question is whether those mathematical premises are supplying the bulk of the 'force' of the explanation, as appears to be the case in Strawberries.[8]

Lange's other examples can similarly be reconstructed as arguments mixing empirical and mathematical premises.

> **Trefoil Knot**: Why can't Terry untie his shoes? Because Terry has a trefoil knot in his shoelace (EP). The trefoil knot is not isotopic to the unknot in three dimensions (EP), and only knots isotopic to the unknot in three dimensions can be untied (MP) (489).
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> **Königsberg**: Why did Marta fail to walk a path through Königsberg in 1735, crossing each of its bridges exactly once (an Eulerian walk)? Because, that year, Königsberg's bridges formed a connected network with four nodes (landmasses); three nodes had three edges (bridges); one had five (EP). But only networks that contain either zero or two nodes with an odd number of edges permit an Eulerian walk (MP) (489).
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> **Chopsticks**: Why is it likely that more tossed chopsticks will be oriented horizontally rather than vertically? Because the they were tossed randomly (EP) and there are

8 Lange might object to the inclusion of the empirical premise in this formulation. Instead, he might treat the empirical premise as a presupposition of the why question: "Why can't Mary divide her 23 strawberries among her three kids?" Answer: "Because 23 is indivisible by 3." In what follows, all of our examples can be so translated without affecting the principled incompleteness in the cases, but this reformulation comes at considerable cost to the clarity with which the incompleteness can be displayed (see Section 4).

more ways for a chopstick to be horizontal than to be vertical (MP). If we focus on the sphere produced by rotating the chopstick through three dimensions, a chopstick can be horizontal anywhere near the equator; it is vertical only near the poles (490).

Cicadas: Why do cicadas with prime life-cycle periods tend to suffer less from predation by predators with periodic life cycles than do cicadas with composite periods? Because it minimizes predation to have a life cycle that intersects only infrequently with that of your periodic predators (EP) and because prime periods minimize the frequency of intersection (MP) (498).

Honeycombs: Why do honeybees use at least the amount of wax they would use to divide their combs into hexagons of equal area? Because honeybees divide their combs (which are planar regions with dividing walls of negligible thickness) into regions of equal area (EP) and a hexagonal grid uses the least total perimeter in dividing a planar region into regions of equal area (MP) (499).[9]

Pendulum: Why does Patty's pendulum have at least four equilibrium configurations? Because Patty's pendulum is a double pendulum (EP) and any double pendulum's configuration space is a torus with at least four stationary points (MP) (501).

Central to Lange's broader purposes is the claim that these distinctively mathematical

9 Lange 'narrows' the explananda in these last two cases. Note that the explananda are *not*, respectively, that cicadas have prime periods and that honeycombs are hexagonal. Those explananda have causal (etiological and constitutive) explanations. The narrower explananda, Lange argues, have distinctively mathematical explanations.

explanations gain their explanatory force from non-causal, and more broadly, non-ontic sources: i.e., stronger-than-naturally-necessary relations. Explanatory priority flows downward from the more necessary to the less necessary:

> In my view, the order of causal priority is not responsible for the order of explanatory priority in distinctively mathematical explanations in science. Rather, the facts doing the explaining are eligible to explain by virtue of being modally more necessary even than ordinary causal laws (as both mathematical facts and Newton's second law are) or being understood in the why question's context as constitutive of the physical task or arrangement at issue. (506)

For Lange, distinctively mathematical explanations gain their explanatory force from the fact that they rely fundamentally on mathematical relations that are more necessary than are relations of causation and natural law. The norms by which good mathematical explanations are sorted from bad mathematical explanations would, according to this account, turn on the relevant mathematics and facts about how that mathematics is being applied. In the following section we argue that Lange's analysis is inadequate.

3. The Inadequacy of Lange's Model.

Lange's account currently leaves unspecified a crucial feature for sorting the mathematical arguments that have explanatory power from those that fail as explanations. Our argument for this thesis is inspired by Bromberger's example of the flagpole and the shadow (1966). At least according to scientific common-sense, one can explain the shadow's length by appealing to the flagpole's height, the sun's angle of elevation, and the natural fact that light propagates in straight lines. One cannot, in accord with scientific common-sense, explain the flagpole's height in terms of its shadow's length, the angle of the sun's elevation, and the natural fact that light propagates in straight lines (in non-intentional contexts; cf. van Fraassen 1980, 132–4). In complete accordance with the norms of the once-received, covering-law model of explanation (Hempel 1965), one can write a deductive argument relating law statements and true descriptions of 'initial' conditions to either conclusion. For Bromberger (and Salmon 1984), the example demonstrates an *asymmetry* in *natural* explanations that the covering-law model could not accommodate. The covering-law model is thereby shown to be an inadequate account of the norms of scientific explanation.

More generally, the example demonstrates that at least some (and in fact, many) explanations have a preferred direction. Salmon, for example, used this example (among others) to argue that scientific explanations work by tracing the antecedent causal structure of an event: Light leaves the sun, passes the flagpole, and lands on the ground. Causation enforces this temporal direction. No such causal sequence proceeds from the shadow to the height of the flagpole (outside intentional contexts). Considerations of just this sort underlie both Lewis' and Sober's emphasis on causation as the fundament of scientific explanation. In what follows, we emphasize the *directionality* of explanations, not their asymmetry. It does not matter for our purposes whether all the same statements in one explanation are reordered in the other. In some cases this is possible; in others it is not. What matters, instead, is that one can generate an explanation that fits the form of a distinctively mathematical explanation that appears to violate our common-sense norms about the acceptable and unacceptable directions of scientific explanation.

If one is committed to the existence of distinctively mathematical explanations of natural phenomena, then one must find a way to reconcile the directionlessness of many applications of mathematics with the directionality of natural explanations. The kinds of relation described in algebra, geometry, and calculus are directionless; with addition or division, a variable on one side of the equation can be moved to the other side. They have no intrinsic left-right directions; rather, these must be imposed from the outside. This is why Lange's examples of putative distinctively mathematical explanation face a directionality challenge. Each of Lange's examples can be 'reversed' to yield an argument that appeals to the same mathematical premise and that has the same form as Lange's examples but that would not be counted as an acceptable explanation (absent considerable revision in scientific common-sense). Consider, for example:

> **Reversed Strawberries.** Why doesn't Mary have 23 strawberries? Because she divided her strawberries equally among her three kids (EP) and 23 is indivisible by 3 (MP).

Like Strawberries, Reversed Strawberries can be represented as a deductive argument with both an empirical and a mathematical premise:

> 1. Mary evenly distributed her strawberries among her three kids (EP).
>
> 2. 23 is indivisible by 3 (MP).
>
> C. Mary doesn't have 23 strawberries.

From a common-sense perspective, at least, Mary's even-numbered pile of strawberries explains but is not explained by her dividing the pile equally among the children.[10] (And

10 Catherine Stinson (personal communication) emphasizes that this claim must be bracketed to nonintentional contexts. Mary might decide, for example, to bake a certain number of

surely the number of children Mary had is not explained by her distribution of strawberries today, though a mathematical argument of that sort could be constructed as well.) Note further that the implicit directionality in this explanation is plausibly accounted for by ontic assumptions about the kinds of relations that properly carry explanatory force: i.e., that Mary's pile is the cause (the source) of the portions each kid gets. In contrast, the portions do not cause the number of strawberries or the number of children. The trefoil knot example faces a similar reversal:

> **Reversed Trefoil Knot**: Why doesn't Terry have a trefoil knot in his shoelace? Because Terry untied the knot (EP) and the trefoil knot is not isotopic to the unknot in three dimensions, and only knots isotopic to the unknot in three dimensions can be untied (MP).

But it would seem more in line with scientific common-sense to explain why Terry has a particular kind of knot by describing how he tied it and not by describing his ability or inability to untie it.

> **Reversed Königsberg**: Why did either zero or two of Königsberg's landmasses have an odd number of bridges in 1756? Because Marta walked through town, hitting each bridge exactly once (EP) and only networks containing zero or two nodes with an odd degree contain an Eulerian path (MP).

As in the other examples, Königsberg's layout is arguably better explained by the decisions of

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cookies knowing they will have to be evenly divided among her kids, or she might decide to have three kids because she decides that three is the maximum number of children she can support on her income. These are intentional, causal explanations.

the Burgermeister than by Marta's walk, yet facts about Königsberg's layout follow reliably from descriptions of either.

> **Reversed Chopsticks**: Why were the chopsticks tossed non-randomly? Because more of the tossed chopsticks were oriented vertically than horizontally (EP) and there are more ways for a chopstick to be horizontal than to be vertical (MP).

In this 'reversal,' the unexpected number of vertically oriented chopsticks provides evidence that some biasing force must be acting upon them (much as deviations from the Hardy-Weinberg equilibrium detect selective forces). As in Lange's forward-directed version of the example, the argument here is inductive. But while we are apt to count Lange's original example as explanatory, it seems more fitting with scientific common-sense to describe Reversed Chopsticks as describing an evidential, not explanatory, relation. In the case of Cicadas, suppose that a field scientist discovers a species of Cicadas that thrives despite the fact that its life cycle overlaps considerably with that of its periodic predators:

> **Reversed Cicadas**: Why doesn't it minimize predation in these Cicadas to have a life cycle that intersects only infrequently with that of your periodic predators? Because cicadas with prime life-cycle periods don't tend to suffer less from predation by predators with periodic life cycles than do cicadas with composite periods (EP) and because prime periods minimize the frequency of intersection (MP).

To modify the example and give a more intuitive appeal, suppose that the life-cycles of a species of Cicada and its periodic predator overlap only every 21 years. This places constraints on the space of possible periods for the life-cycles in these species: 1, 3, 7, and 21 years are the available options. If we know on empirical grounds that 1 and 21 are not live options and that the life-cycle of the cicada is 7 years, and we package that into the request for explanation, we can infer with mathematical certainty that the predator cycle is 3 years. But it would seem that the frequency of intersection is explained by the life-cycles, not that the life-cycles are explained by the frequency of their intersection.

> **Reversed Honeycombs**: Why does this species of honeybees divide their combs into regions of unequal area? Because honeybees use less than the amount of wax they would use to divide their combs into hexagons of equal area (EP) and a hexagonal grid uses the least total perimeter in dividing a planar region into regions of equal area (MP).

But it is a stretch from common-sense to think of the bee's hive-construction as explained by the fact that it uses less wax than a hexagonal grid. (If there were such an explanation, it would be a selectionist, and so causal, explanation on Lange's view [498].) The mathematical premise is directionless, but the explanatory force runs in a preferred direction. And finally:

> **Reversed Pendulum**: Why isn't Patty's pendulum a double pendulum? Because Patty's pendulum doesn't have at least four equilibrium configurations (EP) and any double pendulum's configuration space is a torus with at least four stationary points (MP).

But surely Patty's engineering explains the kind of pendulum she has or does not have better than does fact that the pendulum has more or fewer than four equilibrium points (again, outside intentional contexts).

Each of Lange's examples can be used to generate a putative distinctively mathematical explanation, with the same mathematical premise and the same form, that few scientists would accept as a genuine explanation. Given that Lange is not aiming to revise radically our scientific common-sense ideas about the nature of scientific explanation, it would appear that Lange's model of distinctively mathematical explanation is inadequate.

To amplify this point, note that each example of reversal seems to confuse justification and explanation (see Hempel's [1965] distinction between reason-seeking and explanation-seeking why-questions). An argument justifies believing thesis P (at least partially) when it provides evidence that P. The pristine form of the covering-law model, i.e., one conjoined to the strongest form of the explanation-prediction symmetry thesis, can be seen as attempting to erase this boundary. The goal was to cast explanation as fundamentally an epistemic achievement: explanation is reduced to rational expectation. The problem, of course, is that one can have reason to believe P without explaining P. An Archaeopteryx fossil gives one reason to believe that Archaeopteryx once existed, but it does not explain Archaeopteryx's existence. The same point has been made time and again: with barometers and storms, spots and measles, yellow fingers and lung cancer, and roosters and sunrises. Indicators are not always explainers. It was in recognition of this problem that defenders of the covering-law model quickly backed away from strong forms of the explanation-prediction symmetry thesis and sought other means to account for the directionality of scientific explanations. It was in the face of these challenges that Salmon raised his flag in favor of the ontic conception.

Yet precisely the same problem appears to arise for Lange's examples: We learn something about Terry's knot when we learn he's untied it; we learn something about Königsberg from Marta's stroll; we learn something about our chopsticks when we observe their contra-normal behavior; we learn something about the structure of honeycombs from the amount of wax used; we learn something about the life-cycles of cicadas when we observe predation patterns; and we learn something about a pendulum from how many equilibrium configurations it has. But learning something about the system is not in all cases tantamount to explaining that feature of the system.[11]

11 We leave aside whether our critique also applies to Lange's (2013a) account of 'really statistical explanation,' though we note that similar 'reversals' seem possible. Consider two of Lange's examples of 'really statistical explanation': regression toward the mean and Rutherford and Geiger's explanation of certain behavior of alpha particles.

> **Regression**: Why do students with the lowest scores on the first exam tend not to be the students with the lowest scores on the second exam? Because there is a statistical relation rather than a perfect correlation between the outcomes of two tests (i.e., insofar as one student scored lower than another on the first test, the former student likely – but not certainly – scored lower on the second test) (EP) and when there is a statistical relation rather than a perfect correlation between two variables, extreme scores in one variable tend to be associated with less extreme scores in the other variable (i.e., "regression toward the mean" from the extremes) (MP) (170).
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> **Alpha Particles**: Why is the average number of alpha particles emitted from a steady source nearly constant when a large number is counted, but subject to wide fluctuations at shorter intervals? Because particles are emitted at random (EP) and "the laws of probability" (MP) (174).

Now 'reversed':

Lange argues that the order of explanatory priority in his examples follows the degree of modal necessity, with more necessary things explaining less necessary things. Yet this restriction on distinctively mathematical explanations cannot block the above examples. After all, the same mathematical laws are involved in the forward and reversed cases. We simply have changed the empirical facts. The problem appears to be that the mathematics in these examples is sufficiently flexible about what goes on the right and left hand side of an equation that it doesn't seem to have the resources internal to it to account for the directionality enforced in scientific common-sense. Some extra ingredient is required to sort genuine mathematical explanations from pretenders and, specifically, to sort explanation from justification. In other words, these putative cases of distinctively modal, mathematical

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> **Reversed Regression**: Why isn't there is a statistical relation rather than a perfect correlation between the outcomes of the two tests? Because the students with the lowest scores on the first exam tend to be the students with the lowest scores on the second exam (EP) and when there is a statistical relation rather than a perfect correlation between two variables, extreme scores in one variable tend to be associated with less extreme scores in the other variable (MP).
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> **Reversed Alpha Particles**: Why aren't alpha particles emitted from a steady source at random? Because it is not the case that the average number of particles emitted from a steady source is nearly constant when a large number is counted, but subject to wide fluctuations at shorter intervals (EP) and “the laws of probability” (MP).

Like Reversed Chopsticks, these ’reversals’ are statistical in flavor and similarly seem to conflate evidence and explanation.

explanations of natural phenomena appear to retain an ineliminable ontic component, perhaps working implicitly in the background, but required to account for the preferred direction to the explanation. Mary's pile explains the kids' allotment, and not vice versa, because the allotment is produced from pile. The trefoil knot explains the failure to untie it, and not vice versa, perhaps because structures constrain functions and not vice versa. Similarly, the structure of Königsberg explains which walks are possible around town, but the walks do not explain the structure of the town. Perhaps the movement of the sticks does not explain the forces acting on the sticks because the pattern in the sticks is not causally relevant to the forces acting upon them. Perhaps life-cycle periods explain predation patterns, and not vice versa, because the length of a life-cycle period is causally relevant to the amount of predation. Perhaps the structure of honeycombs explains the amount of wax used, but not vice versa, because the structure of a honeycomb determines the amount of wax needed to build it. And perhaps the shape of Patty's pendulum is explained by her desires in choosing it and not by the fact that it does or does not have four stable equilibrium points precisely because Patty's desires are causally relevant and (in most non-intentional contexts) the four equilibrium points are not. In other words, in each case, it would appear that various ontic assumptions about what can explain what are called upon to sort out the appropriate direction of the explanation and to weed out inappropriate applications of the same argumentative forms appealing to the same mathematical laws.[12]

12 Aggregative explanations apply to constitutive relations but exhibit a preferred direction. The mass of the pile of sand is explained by summing the masses of the individual grains. But one can infer the mass of an individual grain from the mass of the whole and the mass

The dialectical situation might be put expressed as a tension between three propositions: first, that there are distinctively mathematical explanations of natural phenomena; second; that mathematical explanations are directionless; and third, that explanations of natural phenomena are not directionless.

To resolve this tension, one might deny the first of these propositions, holding that all distinctively mathematical explanations of natural phenomena have at least implicit within them a set of ontic commitments that account for the directionality of the explanations and so for the norms that sort good from bad mathematical explanations. Perhaps once the explanandum has been narrowed to the point that it is susceptible of a distinctively mathematical explanation, the explanandum has been transformed into a mathematical rather than a natural fact. Our above discussion is consistent with this view but in no way forces it upon us. One might also deny that mathematics is directionless. Perhaps some areas of mathematics enforce a direction that corresponds to the explanatory norms in a given domain. This appears not to be the case in Lange's examples, but it does not follow that there are no such cases. Perhaps, that is, there are distinctively mathematical explanations of natural phenomena that do not face a directionality problem (Philippe Huneman, personal communication).

Finally, one might reject the third proposition and allow that explanations of natural

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of the other grains. This aggregative explanation appears to have the same simple mathematical structure as Strawberries. In this case, it is a constitutive (not causal) relation that apparently accounts for the preferred direction. Perhaps parts explain wholes and not vice versa: an ontic commitment.

phenomena are directionless. This is the extreme caricature of the covering-law model we mentioned above, one that holds to the strong form of the prediction-explanation symmetry thesis. This option involves biting the bullet and accepting that shadows explain flagpoles, that spots explain measles, and that yellow fingers explain lung cancer. (Railton [1981], for example, includes such things in his 'ideal explanatory text'.)

Even if one is tempted to give up on the first proposition and to deny that there truly are distinctively mathematical explanations of natural phenomena, Lange's discussion highlights an important feature of causal and mechanistic explanation that has thus far received very little attention: namely, that all mechanisms are constrained to work within the space of logical and mathematical possibility. If how something works is explained by revealing constraints on its operation (as Craver and Darden [2013], for example, suggest), then one cannot neglect these modal constraints in a complete understanding of mechanistic explanation. In our view, that thesis is interesting enough even if there are not distinctively mathematical explanations of natural phenomena.

4. Presuppositions and Constitutive Contexts.

Although we have modeled our reconstructions on Lange's discussion, in which he explicitly states that contingent, empirical facts are part of the explanantia (506), he may object to the form of our examples. He considers and rejects the following pseudo-explanation:

> **Elliptical Orbits**: Why are all planetary orbits elliptical (approximately)? Because each planetary orbit is (approximately) the locus of points for which the sum of the distances from two fixed points is a constant [EP], and that locus is (as a matter of

mathematical fact) an ellipse [MP]. (508)

Like the previous examples, this one has an empirical premise and a mathematical premise. This is not a distinctively mathematical explanation, according to Lange, because “the first fact to which it appeals [i.e., EP] is neither modally more necessary than ordinary causal laws nor understood in the why question's context to be constitutive of being a planetary orbit (the physical arrangement in question)” (508). However, if we presuppose that the planetary orbits in question are just those that are loci of points for which the sum of the distances from two fixed points is a constant, then that fact is understood in the why question's context to be constitutive of being a planetary orbit. The why-question then becomes: Why are all planetary orbits that are loci of points for which the sum of the distances from two fixed points is a constant, elliptical? It is constitutive of the planetary orbits in question that they are loci of points for which the sum of the distances from two fixed points is a constant. The distinctively mathematical explanation is that those loci are necessarily ellipses. Should Lange object to our “reversed” examples on similar grounds, their empirical premises can also be presupposed and shifted into their associated why-questions. For example, in Reversed Trefoil Knot, instead of asking, “Why doesn't Terry have a trefoil knot in his shoelace?” and stating as an empirical premise that Terry untied the knot, we could instead ask, “Why doesn't Terry have a trefoil knot in the shoelace he untied?” Now the former empirical premise is part of the constitutive context of the why-question. We presuppose that Terry untied his shoelace, rather than stating it as an empirical premise. This seems to fit Lange's criteria for distinctively mathematical explanation.

Lange could respond to this move by distinguishing between what is *understood* to be constitutive of the physical task or arrangement at issue and what is *actually* constitutive of the physical task or arrangement at issue.[13] Lange could then argue that, for example, in Trefoil Knot it is actually constitutive of the physical task or arrangement at issue that Terry's shoelace is a trefoil knot. However, Lange could continue, in the version of Reversed Trefoil Knot where we presuppose that Terry untied his shoelace, that fact is not actually constitutive of the physical task or arrangement at issue. This seems to be the kind of move Lange has in mind when he distinguishes between what is and isn't constitutive of the arrangement or physical task at issue in a given why-question's context (2017, 43). We are unsure how this distinction between what is "understood" to be and "actually" constitutive could be drawn. Granted, in Trefoil Knot there is a single structure (i.e., the knot) the constitutive properties of which are determined by context (i.e., that it is trefoil) and there is not a similar single structure in Reversed Trefoil Knot (the same point applies, mutatis mutandis, to Königsberg). We do not see how this is a relevant difference though. When we request an explanation for the fact that Terry failed to untie his shoelace, we grant that context determines that it is actually (and not merely understood to be) constitutive of that fact that his shoelace is a trefoil knot. However, when we request an explanation for the fact that Terry doesn't have a trefoil knot in the shoelace he untied, it seems to us constitutive of that very fact that Terry untied his shoelace. It wouldn't be the same explanandum had Terry not untied his shoelace. We do not see how one can claim that Terry's untying the knot is merely understood to be

13 We thank an anonymous referee for this suggestion and careful discussion of the points in this section. Note that Lange (2013b) always speaks of what is "understood" to be constitutive in the context of the why-question (e.g., 491, 497, 506, 507, 508).

constitutive of this explanandum, while claiming that the shoelace's being a trefoil knot is actually constitutive of the former explanandum.

We don't think there's anything objectionable about so restricting the range of our explananda/why-question (e.g., to just those planetary orbits that are loci of points for which the sum of the distances from two fixed points is a constant). Notice that such a restriction is required of Lange's examples as well. For example, it is not constitutive of all shoelaces that they contain trefoil knots; it is constitutive only of the shoelace under consideration, which actually contains a trefoil knot. Nor is it constitutive of all pendula that they are double pendula; nor of all arrangements of strawberries and children that there are 23 of the former and 3 of the latter; nor of all bridges that they have a non-Eulerian structure. This response to our challenge, in other words, requires an account of how context determines what is constitutive of the physical task or arrangement in question[14], especially if it relies on a distinction between what is actually and merely understood to be constitutive in a given context.

5. Conclusion: Modal and Ontic Aspects of Mechanistic Explanations.

Return again to the flagpole and the shadow. As discussed above, Bromberger and Salmon used this example to demonstrate the directionality of scientific explanations. They enlist this point to argue for an ineliminable causal (or more broadly, ontic) component in our normative analysis of scientific explanation. We have used the same strategy to argue for an ineliminable ontic component in Lange's examples of distinctively mathematical explanation.

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14 This worry is raised by Pincock (2015: 875). We thank an anonymous reviewer for bringing this to our attention.

But the example can be yoked for another duty.

One might, in fact, describe the flagpole example as a distinctively mathematical (or at least trigonometric) explanation of a natural phenomenon, one that calls out for a distinctively modal interpretation. *Presupposing* that the angle of elevation of the sun is θ and that the height of the flagpole is *h* (and the flagpole and ground are straight and form a right angle, and that the system is Euclidean, etc.; EP), why is the length of the flagpole's shadow *l*? Once the contingent causal facts are presupposed in our empirical premise, the only relevant fact left to do the explaining seems to be the trigonometric fact that $\tan \theta = h/l$ (MP). Moreover, once these natural facts are presupposed, the length of the flagpole's shadow seems to follow by trigonometric necessity. So if we package all the natural facts into an empirical premise and highlight the relation $\tan \theta = h/l,$ which is crucial for the argument to work, then we might see this as a case in which the bulk of the explanatory force is carried by a trigonometric function. The example thus seems to provide a recipe for turning at least some mechanistic explanations into distinctively mathematical explanations: simply package all of the empirical conditions, such as the rectilinear propagation of light, or the Euclidean nature of spacetime, into the empirical premise or the context of the request for explanation, and leave a mathematical remainder or a tautology to serve as the premise with stronger-than-natural necessity.[15]

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15 This could presumably be done with any kind of necessity. For example, take an explanation one of whose premises is a *conceptual* necessity. Fix or presuppose all the premises other than the conceptual necessity. You then have a distinctively *conceptual* explanation. Lange appears to recognize this possibility (504).

The importance of geometry to mechanistic explanation is readily apparent in artifacts, such as the coupling between an engine and the drive crank shaft of a car. Machamer, Darden and Craver (2000) describe the organization of such mechanisms as geometrico-mechanical in nature. Vertical motion produced by explosions in the piston chambers drive the pistons out. The center of each piston is connected via a rod to the crankshaft at some distance (r) from the center of the crankshaft so that when the piston is driven out, the crankshaft is rotated in a circle. This mechanism very efficiently transfers the vertical force of the pistons into a circular motion that drives the car forward. These engine parts are organized geometrically in circles and triangles. The angle of the connecting rod, for example, determines the position of the piston, though the explanation would appear to work the other way around. Yet these mathematical facts surely are relevant to why the car accelerates as it does and not faster or slower.[16]

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16 Baron, Colyvan, and Ripley (forthcoming; see also Chirimuuta forthcoming and Reutlinger 2016) propose assimilating this mathematical dependence to a "counterfactualist" account of explanation (i.e., an account according to which explanatory power consists in the ability to answer what-if-things-had-been-different questions or w-questions) and they show how to assess the relevant counterpossible counterfactuals within a structural equation modeling framework. We find this assimilation plausible but as yet inadequate, because Baron et al. (and Chirimuuta and Reutlinger) do not address the question of which true counterfactuals are explanatorily relevant and which are not. For example, there are contexts in which it is true that had the flagpole's shadow been length *l* then the flagpole's height would have been *h*. There are

But as Lange's examples aptly illustrate, mathematics appears to play an essential role in mechanistic explanations in at least many areas of science. After all, the space of possible mechanisms is constrained by the space of mathematical (and logical) possibility. If one considers the mechanisms of sound transduction in the inner ear, one finds an arrangement most similar to the engine and the crankshaft, except in this case the mechanism converts

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also contexts in which it is true that had Mary divided her strawberries evenly among her children, then 23 would have been divisible by 3. Thus there is a similar problem of directionality with respect to counterfactuals: in one direction, a counterfactual can seem explanatory; in the other direction, it does not seem explanatory. We think that the distinction between explanatorily relevant and irrelevant counterfactuals must be made by appeal to ontic considerations (Salmon 1984; Povich forthcoming).

Note that if the counterfactualists are right, this will go some way to dissolving the distinction between ontic and modal conceptions of explanation. According to counterfactualists, causal, mechanistic, distinctively mathematical, and all other kinds of explanation derive their explanatory power from their ability to answer w-questions about their explananda. No one, as far as we know, takes the distinction between causal and mechanistic explanation to be significant enough to warrant relegating each to a different conception of explanation. The distinction between them is real and there is disagreement about how to make it, but, even noting the real differences between causal and constitutive relevance, no one takes the distinction to mark two wholly different conceptions of what it means to explain. If the counterfactualists are right, the distinction between distinctively mathematical explanations and causal/mechanistic explanations seems as insignificant for vibrations in the air into vibrations in fluid. Still, parts are arranged geometrically. Likewise, when we look into the intricate mechanisms gating ion channels, we seem to find structures that are understood geometrically, in terms of sheets and helices, which structures allow or prohibit certain activities (Kandel *et al*. 2013). Structural information has been essential to understanding the mechanisms of protein synthesis and inheritance and to understanding features of macro evolution (Craver and Darden 2013). Perhaps not all of these explanations are distinctively mathematical, but the mathematics does ineliminable work in revealing how the mechanism operates, how it can operate, and how it cannot.

This blend of the mathematical and the mechanical (or more broadly, the ontic) is, in fact, precisely what one would expect based on the history of the mechanical philosophy. Aristotle's (or pseudo-Aristotle's) mechanics works fundamentally by reducing practical problems to facts about circles (1936). Hero of Alexandria and Archimedes, though celebrated for the practical utility of their simple machines, viewed those machines equally as geometrical puzzles to be solved. Descartes' conception of the mechanistic structure of the world was directly connected with his planar representation of geometrical space, in which extended things interact through contact. Galileo demonstrated his results with thought

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the theory of explanation as the distinction between causal and mechanistic explanation. There is no philosophically significant reason to lump a few kinds of explanation together and say that they explain in accordance with an "ontic conception" and the others in accordance with a "modal conception". For the counterfactualist, all are simply species of a genus, and all explain by providing answers to w-questions.

experiments, such as the Tower of Pisa, that rely on basic mathematical truths (i.e., an object cannot both accelerate and decelerate at the same time). Newton wrote the *Principia*, like the great physicists before him, in the language of geometry. Dijksterhuis (1986) closes his masterly *Mechanization of the Scientific World Picture* with the cautionary note that, "serious misconceptions would be created if mechanization and mathematization were presented as antitheses" (500). It is a misconception because the mathematization of nature and the search for basic mechanistic explanatory principles have been treated historically as distinct aspects of the same explanatory enterprise. The very idea of mechanism, and the idea of the world as a causal nexus, has always been expressed in tandem, rather than in opposition, to the idea that the book of nature is written in the language of mathematics and the belief that a primary aim of science is to leave nothing in words.

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