COMPARATIVE ANALYSIS OF MODELS FOR ADJUSTMENT PROCEDURE IN ASSETS VALUE INDEPENDENT EVALUATION PERFORMED BY COMPARATIVE APPROACH

This paper addresses the field of economic measurements of the value of assets, carried out by the methods of independent expert evaluation. The mathematical principles of application, within a comparative methodical approach, of additive and multiplicative models for correcting the cost of single indicator of compared objects have been considered. The differences of mathematical basis of the compared models were analyzed. It has been shown that the ambiguity in the methodology of correction procedure requires studying the advantages and disadvantages of known models, as well as the justification and elaboration of recommendations for their application.

Possible forms of correction representation using several alternative units of measurement have been defined; formulas for their interconnection have been built. Analytic expressions have been derived that mathematically describe the algorithms for performing the evaluation correction procedure using various forms of correction representation. The influence of the correction execution model on the characteristics of uncertainty in the independent evaluation result has been analyzed. The scope of two possible types of correction introduction models has been determined. A specific numerical example was used to demonstrate the methodological advantages of using a multiplicative model when summarizing percentage corrections. The independence of the correction result on the sequence of correction introduction has been confirmed. It is proposed to use the selected measure of partial corrections as a criterion for the adequacy of the correction introduction model. It is proved that the result of the independent expert evaluation depends on the chosen model and does not depend on the sequence of correction introduction.

The reported study results are important in terms of theory and practice since they make it possible to improve the accuracy and reliability of the result of independent expert evaluation

Keywords: independent evaluation, market value, comparative approach, additive model, multiplicative model 4

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1. Introduction

When performing economic measurements by the methods of independent expert evaluation based on the current regulatory framework, in particular [1], it is envisaged to use the cost, revenue, and comparative methodological approaches. The latter is now one of the most widely used one, given its high degree of marketability. This predetermines the increased attention to the methodological base of the comparative approach, which is not yet fully worked out – in particular, in terms of the evaluation adjustment procedure. One of the main evaluation procedures when using it is the

introduction of corrective adjustments to the cost of a single indicator of the selected compared objects. The corrections take into consideration the differences between the characteristics of the valuation object and the compared objects in individual pricing factors, which makes it possible to improve the accuracy and reliability of the result of determining the cost.

When performing evaluation operations using a comparative approach in a certain sequence, corrections to the price characteristics of compared objects should be introduced that correspond to the detected discrepancies with the corresponding parameters of the valuation object. It is important to choose an adequate mathematical model for correction introduction. National valuation standards do not establish a priority method for introducing adjustments in a comparative approach, declaring only the general requirement for their implementation. Adjustments should be made by adding or deducting a monetary amount, either using the coefficient (percentage) to the sale price (offer) of the specified property, or by combining them [1].

Depending on the chosen mathematical model for the method of correction introduction, the results could be completely different. That is, the mathematical model of correction introduction directly affects the result of economic measurements. Therefore, it can be argued that the justification of an adequate method of introducing adjustments is one of the most pressing problems of independent expert evaluation, solving which affects the degree of uncertainty of the result. The choice of the model to account for corrections during adjustment directly affects the results of the assessment, and an in-depth study of the model's impact on the reliability of the cost determining result is certainly expedient both from a theoretical and practical point of view.

2. Literature review and problem statement

In econometrics, extensive experience in applied modeling of characteristics of economic systems has been accumulated. The world practice of numerical analysis of the effectiveness of foreign trade policy involves the use of models of partial and general equilibrium [2]. It is noted that a characteristic feature of applied models of general equilibrium is their ability to assess the economic consequences of the implementation of measures of state trade and economic policy at the macroeconomic level. In addition, these models are used for investigating economic activity of individual enterprises and clusters at the microeconomic or regional levels. However, this class of models is not provided for use in valuation practice. The study of quantitative ratios of characteristics of economic processes and phenomena is based on economic dimensions, the accuracy of which greatly affects the effectiveness of asset management. The use of mathematical modeling in measurement processes made it possible to quantify various aspects and phenomena of socio-economic development and increased the completeness of the data obtained, protecting them from deliberate and technical distortions. From the point of view of the interests of reliable modeling of the economy, the most pressing problems of improving economic measurements are to increase the reliability of valuation of real estate and intangible assets [3]. One of the possible directions of achieving this goal is an in-depth study of mathematical models used in economic measurements. The mathematical principles of such models

are not sufficiently studied, and in practice the justification of the feasibility of using models of one or another type does not rely on the study of the adequacy of the performed evaluation procedures to a mathematically correct algorithm.

In econometrics, the additive and multiplicative mathematical models are widely used for analysis and forecasting of time series [4], in particular – for forecasting seasonal fluctuations in production volumes and demand for goods, export-import supplies, transport loads. However, their use in the field of independent expert evaluation has not been sufficiently worked out, and the question of the impact of model selection on the uncertainty of the assessment result remains unexplored. Studies have confirmed that additive models are able to reflect relatively constant seasonal fluctuations while multiplicative models more adequately take into consideration fluctuations that dynamically change depending on the trend [5]. At the same time, the researchers ignored the question of choosing an adequate model for generalization of partial corrections. The peculiarities of the implementation of the evaluation adjustment procedure have not yet been considered in terms of the analysis of methodological principles for taking into consideration partial corrections and their impact on the accuracy and reliability of the result of the independent expert evaluation.

In the practice of valuation activity, when performing the adjustment procedure, the most widespread are the two mathematical models – additive and multiplicative, named after the method of generalization of corrections in the indicator of general adjustment. The difference between these types of models is defined in [6]. In the cited work, the additive model is defined as a model to which individual factors are included in the form of an algebraic sum; accordingly, the multiplicative model is defined as a model to which these factors are included in the form of a product. However, in the cited work there is no analysis of the impact of the type of model on the indicators of uncertainty of the evaluation result. In addition, the issue of justifying the choice of the appropriate model has not been considered and the criterion for the implementation of this choice has not been formulated. The possibilities of economic and mathematical modeling depend on the degree to which the model constructed reflects objective conceptual patterns. The effectiveness of modeling also depends on the availability, completeness, and quality of data in the formation of the scorecard, methods of their evaluation and processing, and the quality of the analysis carried out in the interpretation of modeling results [7]. The author's conclusions can be appreciated but the actual issue of modeling efficiency and, in particular, the reliability of the result of the generalization of partial corrections in the implementation of economic measurements by the methods of independent evaluation is not considered.

The economic and mathematical model should formally describe financial and economic phenomena and processes with the degree of adequacy corresponding to the task. In general, the assessment of the adequacy of the model is carried out on the basis of comparing the results accumulated during the pilot operation of the model. Data received about the real object under investigation are also taken into consideration. In the process of model verification, the discrepancies of mathematical description and actual characteristics are detected and analyzed. If necessary, corrections are made to the model [3]. The described sequence of the study of model adequacy is not in doubt but, to this day, such a study has not been performed for models of generalization

of partial corrections. The degree of adequacy of the models used in the adjustment in the comparative approach is not fully investigated. This often leads to their misuse, in particular the additive model for generalization of percentage corrections.

With regard to the field of economic measurements carried out by expert evaluation methods, the degree of adequacy of the model of making adjustments would be determined, first of all, by the achieved reliability of the results after the implementation of the chain of transformations according to a certain algorithm [8]. However, the authors of the cited work ignored questions about justifying the choice of the algorithm for processing the source data. The choice of algorithm and model of generalization of partial corrections has a great influence on the result of the assessment and the degree of its uncertainty. The high degree of uncertainty is typical of the real estate market, which is an open system with a large number of factors of influence. Uncertainty, therefore, is an integral part of the process of formation of market prices. Numerical values of asset value indicators, determining which is the goal of economic measurements, are also the result of the influence of many pricing factors. The analysis carried out in [9] gives reason to believe that uncertainty is a factor that forms the real estate market. The authors argue that the real estate market is subject to constant changes caused by the influences of various pricing factors and changes in the internal and outer environment. Consequently, all information related to the real estate market is probabilistic. This important conclusion also concerns the uncertainty of models and methods as one of the components of the general uncertainty of the evaluation result. The authors of [9] do not focus on considering the contribution to this indicator of the uncertainty of the models used in economic measurements and do not analyze their mathematical principles. This also applies to [10], where these important problems remained beyond the topic of research.

It is worth noting that the regulatory framework for the evaluation of some post-Soviet countries reflects the uncertainty of its result [11]. However, they also contain only a statement of the fact that the uncertainty of the models and methods used in economic measurements has a great impact on the outcome of the assessment. At the same time, the principles of choosing an adequate model in the implementation of the evaluation adjustment procedure in a comparative approach are not specified and the criterion for its compliance is not determined. When applying expert methods, the degree of uncertainty of the result of economic measurements is influenced by various sources and factors of uncertainty, in particular: market uncertainty; uncertainty of models and methods; uncertainty of the source data. Inadequate application of models in valuation practice could critically increase the degree of uncertainty of the result, up to obtaining completely unacceptable results in terms of accuracy. The indisputable fact that different models of corrections lead to different evaluation results requires a more reliable mathematical justification for the adequacy of the applied model. After all, the choice of model always remains within the competence of the appraiser, and the appropriate methodology for each case of evaluation or expert study should be focused on minimal uncertainty of the result. Experience in solving problems related to the presence of

uncertainty in an independent assessment is summarized and systematized in monograph [12]. In particular, the author considers the issues of influencing the reliability of the result of models used in economic measurements. At the same time, the difference between the mathematical basis of additive and multiplicative models of performing adjustments of the values of the single indicator of compared objects is not sufficiently analyzed. In addition, the criterion of adequate model selection was not formulated and no relationship between the units of measurement of partial corrections and the corresponding type of model of their generalization was found. In addition to objective errors, which are made by the uncertainty of information about the market environment and the object under study, subjective errors also take place. The latter are due to bias, lack of attentiveness or incompetence of experts and their possible interest in exerting impact on the result.

The characteristic features of the methods of expert evaluations and models of their implementation as a tool for the scientific solution to complex poorly formalized problems are scientifically justified organization of all stages of expertise. It is noted that the use of quantitative methods both in the process of examination and in assessing the reliability of expert opinions contributes to the improvement of the degree of objectivity of expert evaluations [3]. Actually, the evaluation procedure of adjustment in a comparative approach is a typical example of the use of quantitative methods. However, its mathematical principles are not deeply researched enough, and there are no recommendations for a justified choice of the type of model for generalization of partial corrections. This leads to a completely unacceptable situation when the adjustment procedure is carried out by different specialists according to different models chosen at their own discretion and without sufficient justification. At the same time, in the same evaluation situations, completely incomparable results of the assessment can be obtained. Obviously, such ambiguity cannot be considered a normal phenomenon, and this large gap in the methodological provision of evaluation works should finally be eliminated right on.

The degree of adequacy of the model used is one of the main sources of uncertainty of the results of economic measurements. It is proposed to choose the accuracy of the assessment result as a criterion for the effectiveness of the model, the degree of uncertainty expressed through the assessment of error (with a point representation of the result) or the value of the confidence probability and the limits of the confidence interval (in the interval representation of the result). This corresponds to the classical formulation of the method for solving multicriteria optimization problems when the process cannot be described as a single-criterion dependence. From a mathematical point of view, there is no perfect way to solve such problems since each of the alternatives has its advantages and disadvantages. The optimization task is reduced to the task of maximizing (minimizing) the objective function, in accordance with the selected criterion, taking into consideration the specified restrictions. For example, in the considered case, a variety of criteria may be selected as the objective function. For example, the minimum absolute error of a point result; the minimum of its relative error; the maximum confidence probability, etc. If there are several criteria, it is recommended to choose an additive criterion – if the absolute values of the criteria have

the highest value when selecting a vector of parameters. If the main role belongs to changing the absolute values of individual criteria when varying a vector, it is proposed to choose a multiplicative criterion [13]. These recommendations are quite general in nature and are not adapted to the studied field of application of the additive and multiplicative models for correcting the costs of a single indicator. However, when performing their comparative analysis, it is advisable to consider the relationship between different forms of representation of partial corrections in different units of measurement. As well as analyze the indicators of the general correction obtained according to alternative models for their adequacy to the results of adjustment, calculated using a sequentially distributed method of partial corrections.

Our review of the professional literature demonstrates the importance of the problem of optimal choice of the mathematical model when performing economic measurements. It follows from the analysis of the above sources [2–13] that the adequacy of the model and the method of making adjustments are of great importance as regards the uncertainty of the result, and directly affect its effectiveness. However, none of those studies [2–13] proposed or provided justification for the application of models of corrections in the comparative approach. Mathematical principles of this evaluation procedure are also not considered. The task of adequately choosing a model, if there are certain requirements of the current regulatory framework for independent evaluation [1], ultimately comes down to a motivated choice between additive and multiplicative models. Actually, the choice of one of them is mathematically justified and should determine the correct methodology for calculating adjusted values and summarizing partial corrections. Thus, it is a relevant task to perform a comparative analysis of possible models of the adjustment procedure and justify the choice of the most adequate one.

3. The aim and objectives of the study

The purpose of this study is a comparative analysis of the mathematical bases of the evaluation adjustment procedure, with correction introduction using additive and multiplicative models when using a comparative approach of an independent expert assessment of asset value. This would make it possible to determine the scope of each model and ensure its adequacy to each specific evaluation event. Based on the study, practical recommendations aimed at increasing the accuracy of the assessment result can be worked out. In this way, the uncertainty that the methodical error of the adjustment procedure, introduced in the result of the assessment, could be minimized.

To accomplish the aim, the following tasks have been set:
– to consider mathematical principles for the implementation of the evaluation procedure for making corrections;
– to derive analytical expressions for a mathematical description of mutual relations of their parameters;
– to identify the dependence of indicators of general adjustment and their increase on the number of price-forming factors taken into consideration (adjustments) and demonstrate the peculiarities of the implementation of the evaluation adjustment procedure according to two compared models using a specific numerical example;

– to substantiate the principles of choosing an adequate model based on the selected criterion; determine the areas of correct application of the compared models;
– to theoretically justify the independence of the evaluation result on the sequence of consideration of the impact of corrections of certain pricing factors (adjustments); to work out recommendations for the use of compared models in valuation practice.

4. The study materials and methods

The choice of methodological approaches is due to the specificity of the field of economic measurements carried out by the methods of independent expert evaluation. This research is based on methods of mathematical analysis and mathematical modeling. Our study revealed which models in evaluation practice generalize partial corrections, and how it affects the result of evaluation work. The comparative analysis of mathematical principles and algorithms of implementation of the evaluation procedure of corrections has been performed. It has been shown that the procedures of generalization of percentage partial corrections in the comparative approach are possible according to the linear additive and nonlinear multiplicative models of their accumulation. We have proposed a criterion for choosing an adequate model and substantiated the limits of the applicability of each of the models according to this criterion.

The general methodological basis was the basic principles of independent expert evaluation, on which one of the three classic evaluation approaches is based – comparative sales approach. According to the principles of this approach, under certain preliminary conditions and restrictions, the value of the valuation object is based on the comparison of the valuation object with its analogs, for which there is information about the prices of transactions with them, or about offers of sale of such property. This approach is based on the basic principles of real estate valuation: supply and demand; substitution; balance; contribution.

To determine the total value of the property being evaluated, the necessary stage is to adjust the sales price data or offer the sale of such property. Calculation and making of adjustments are carried out based on the mathematical and logical analysis of the influence of price-forming factors, taking into consideration the significance of each indicator. The most important is perhaps more accurate determining of the size of corrections and an adequate algorithm for their introduction. Sufficient justification of the size of the correction is achieved when at the date of assessment according to the available market data, a pattern is clearly traced, which reflects the tendency to change the specific value of such property when price-forming factors change.

5. Results of studying adjustment procedure models when using a comparative approach in an independent assessment

5. 1. Comparative analysis of mathematical principles and algorithms of implementation of the evaluation procedure for making corrections using additive and multiplicative models

The analysis of valuation practice shows that in fact the implementation of adjustments is most often carried

out mainly in two ways. The first of them is to define corrections for each price-forming factor in the dimension of a single indicator, a dimensionless coefficient, or a percentage to the initial (previous) value. The additive model involves their subsequent algebraic summation to determine the general correction. Sometimes appraisers make a one-time introduction of such a general correction by applying it to its original value. According to the second method, correction is carried out by gradually determining corrections for each price-forming factor, with their distributed step-by-step calculation, application to the previous value, and fixation of adjusted indicators after each adjustment (multiplicative model). Then partial corrections are determined in the dimension of percentages or dimensionless correction coefficients.

In the first case, to calculate the general correction indicator, individual corrections in the dimension of a single indicator for each price-forming factor are algebraically summed up; in the second case, to this end, the corrective coefficients for each price-forming factor are multiplied. That is, the use of the multiplicative model differs from the additive one by the correction introduction algorithm. The correction to each subsequent adjustment can be expressed in any units of measurement – percentage, units of specific value of the corrected indicator, or dimensionless coefficient. However, when using a distributed algorithm, it should be calculated not from the same primary indicator as in the additive model but from each other base – the cost adjusted at the previous stage.

In this case, the indicator of the general correction is defined as the product of individual partial corrections for each price-forming factor, represented in the form of dimensionless coefficients – multipliers to the previous adjusted value. The two models of adjustment procedure reviewed above with a large number of price-forming factors taken into consideration demonstrate sharply different results that directly affect the final result of the economic measurements performed.

In the regulatory framework of independent evaluation in Ukraine, the issue of choosing a model for making corrections during adjustments in a comparative approach is not covered. In international standards, this issue has also not been unambiguously resolved. Only general requirements for the implementation of adjustments are specified: it is indicated that statistical tools can be used to justify adjustments, in particular, regression analysis methods. It is noted that the presence of a weak data correlation directly affects the reliability of the appraiser's conclusions obtained on the basis of this information. When using such analytical tools, the appraiser must be sure that the data processing technology used is relevant, and the result of the assessment is mathematically correct [14]. For example, in the regulatory framework of Poland (p. 4. 2 of the interpretive note No. 1 "Application of a comparative approach in the evaluation of real estate") we also find a clear indication of the requirement for sequential corrections but without determining the model of making adjustments: "4.2.8. Consistently conducting comparisons of the object of evaluation and objects of comparison and determining the value of corrections arising from the difference in the prices of real estate of the object of evaluation and objects of comparison" [15].

Very limited information on two alternative models of correction introduction can be found in the literature in other countries [16]. Here, the additive model of making relative adjustments is defined as a model that assumes the calculation of the cumulative adjustment correction as the sum of all relative percentage adjustments made, in accordance with

$$\Delta C\% = \sum_i \Delta C\%_i, \qquad (1)$$

where $\Delta C\%$ is the cumulative relative (percentage) correction during adjustment,

$\Delta C\%_i$ is the relative (percentage) correction during adjustment introduced for a separate price-forming factor

The multiplicative model of corrections is defined in [16]:

$$\Delta C\% = 1 - \prod_i (1 - \Delta C\%_i), \qquad (2)$$

with the same components on the right side of the equation; the author refers to clause 22 of FSO-7 [17] where the model of making adjustments is also not defined. In accordance with p. 22 FSO-7, when using the method of adjustments, each analog object is compared with the object of evaluation according to certain pricing factors (elements of comparison). Next, the differences between objects on these factors are revealed. Accordingly, the price of the analog object (or its specific indicator) is adjusted according to the identified differences, in order to further determine the value of the valuation object.

In this case, the adjustment for each element of comparison is based on the principle of investing this element in the cost of the object. It should be noted that for both cases (1), (2), the indicators of corrections as a percentage were used – provided that percentage corrections during adjustments can be made and summarized both by additive and multiplicative models. However, in this case, expressions (1), (2) produce a completely false result, which is easy to verify in any numerical example. In order to correctly determine the cumulative relative correction when applying the multiplicative model, partial corrections made by individual price-forming factors should be represented not as a percentage but expressed by absolute dimensionless coefficients

$$\Delta C_i = \frac{C\%_i}{100}, \qquad (3)$$

and then the multiplicative model of making adjustments using the characters given in [16] is defined as

$$\Delta C = \prod_i (1 + \Delta C_i), \qquad (4)$$

where ΔC is a cumulative relative (percentage) correction, expressed, in contrast to $\Delta C\%$, by an absolute dimensionless coefficient that can easily be represented also in the form of a percentage correction:

$$\Delta C\% = (1 - \Delta C) \cdot 100 \%. \qquad (5)$$

Further analysis was based primarily on the methodological recommendations developed in Ukraine [18], which recorded important general conditions for the correctness of the evaluation adjustment procedure. First, its implementation should reduce the discrepancy between the specific prices of compared objects. Theoretically, if the corrections are introduced properly, and the prices of

sale/lease of analogs are set in an ideal market, then, after making corrections, they should get closer, and, ideally, become identical. Second, it is noted that the adjustment procedure is carried out in stages, at each stage the price-forming characteristics of the object of comparison are brought to the object of evaluation, while the cost of a single indicator of compared objects is reduced to the probable value of the valuation object. The requirement for the phased introduction of adjustments implicitly determines the recommendation for the use of a multiplicative model of corrections.

5. 2. Derivation of analytic expressions for a mathematical notation of the mutual relations among the additive and multiplicative models' parameters

For the completeness of the comparative analysis of the advantages and disadvantages of the methods of implementation of the evaluation adjustment procedure, it is necessary to clearly determine the mathematical basis of the model of correction introduction. Below are the mathematical principles of correction introduction based on both methods compared. Since the evaluation adjustment procedure is performed for each of the compared objects used independently of the others, the methods for performing adjustments for only one of them are further analyzed. For all other objects of comparison, the execution of this procedure is described by similar mathematical expressions but in practice the numerical indicators of corrections for different analogs, as a rule, differ.

When applying an additive model to obtain the value of the adjusted cost v_a, the initial specific cost of compared objects v_0 is summed up with the general correction Δv

$$v_a = v_o + \Delta v, \tag{6}$$

or adjusted for the total adjustment percentage δv

$$v_a = v_o + v_o \frac{\delta v}{100}, \tag{7}$$

which can also be represented in the form

$$v_a = v_o \left(1 + \frac{\delta v}{100} \right). \tag{8}$$

In the case of the application of the additive model to absolute corrections, the general correction Δv is an algebraic amount of partial corrections $\Delta v_1 ... \Delta v_i$ for each price-forming factor, represented in the dimension of a single indicator (absolute correction)

$$\Delta v = \sum_{i=1}^{n} \Delta v_i. \tag{9}$$

In the second case, the application of the additive model to the relative percentage of corrections, the total percentage of adjustment δv is the algebraic amount of partial percentage corrections $\delta v_1 ... \delta v_i$ for each price-forming factor, represented in the dimension of percentage to the initial value

$$\delta v = \sum_{i=1}^{n} \delta v_i, \tag{10}$$

where n is the total number of price-forming factors taken into consideration (adjustments).

Absolute corrections $\Delta v_1 ... \Delta v_i$ for each price-forming factor, represented in the dimension of a single indicator, as well as percentage corrections $\delta v_1 ... \delta v_i$, can be positive or negative – depending on the ratio of characteristics of the objects being compared.

Completely different is the procedure for correction introduction when using a multiplicative model. Partial corrections for each price-forming factor are determined in the form of dimensionless coefficients – multipliers to the previous adjusted value. Calculation of adjusted values is carried out step-by-step, with the obligatory fixation of intermediate adjusted indicators after each adjustment. This is a fundamentally important difference between the multiplicative model and the additive one since the definition of corrections for each price-forming factor in the form of dimensionless coefficients is calculated from each other base – each previous adjusted value of the single indicator. This completely changes the appearance of the adjusted metrics dependences on the number of adjustments. In this case, the value of the adjusted value v_a can also be obtained by multiplying the initial cost v_0 of the single indicator of the objects of comparison by the total corrective coefficient C,

$$v_a = v_o \cdot C, \tag{11}$$

which is the product of the partial coefficients $C_1 ... C_i$ for each price-forming factor, represented by dimensionless positive numbers (total relative correction)

$$C = \prod_{i=1}^{n} C_i. \tag{12}$$

In this case, partial adjustment coefficients $C_1 ... C_i$ for each price-forming factor are always represented by positive numbers – regardless of whether the absolute correction to the previous value is positive or negative. For positive absolute corrections, the partial adjustment coefficients would be greater than one, and for negative absolute corrections, respectively, these coefficients would be less than one.

Note that all the forms of corrections discussed above are quite equivalent and interconnected by simple ratios

$$\delta v_i = \frac{\Delta v_i}{v_{(i-1)}} \times 100\%, \tag{13}$$

$$\Delta v_i = \frac{\delta v_i \cdot v_{(i-1)}}{100\%}, \tag{14}$$

$$\Delta v_i = v_i - v_{(i-1)}, \tag{15}$$

$$\delta v_i = \frac{v_i}{v_{(i-1)}} - 1, \tag{16}$$

where $v_{(i-1)}$ is the previous cost value (initial v_0 for the first adjustment or the previous adjusted value according to the last price factors taken into consideration). It is easy to prove that the total correction coefficient C_i, which is defined as a dimensionless coefficient, reflecting the degree of change in the primary indicator after the i-th number of adjustments, is associated with the following equations considered above:

$$C_i = 1 + \frac{\delta v_i}{100\%}, \tag{17}$$

$$C_i = 1 + \frac{\Delta v_i}{v_{(i-1)}}, \qquad (18)$$

$$\Delta v_i = (C_i - 1) v_{(i-1)}. \qquad (19)$$

In the case of practical implementation of evaluation operations in the table of adjustments, it is possible to submit corrections either in any one form or at the same time in several. For example, absolute – in units of measurement of the specific indicator of the cost of compared objects; percentage – as a percentage of its change to the previous value; and coefficient – dimensionless corrections can be shown side by side. The result of the adjustment will be the same – only methodically correct implementation of the adjustment procedure is important, with adequate consideration of the forms, units of measurement and values of these corrections. It should be emphasized that these forms of correction representation express the same correction in a very different way. The first two, absolute and percentage, indicate the value by which the previous adjusted metric changes. The latter – coefficient – indicates what the value of this indicator is after adjustment, in relation to the previous value.

5. 3. Studying the dependences of indicators of general adjustment and their increase on the number of price-forming factors

In order to study the impact of the choice of the model for the introduction of corrections when applying a comparative approach to the uncertainty of the assessment result, we shall perform a comparative analysis of the additive and multiplicative models of correction introduction using specific numerical examples close to valuation practice. Below, we analyze two cases of introducing the same adjustments using two models defined above. The comparative analysis was carried out assuming that the number of pricing factors is 10, corrective adjustments for all factors are the same, and each percentage correction is −10 % (adjustment coefficient is 0.9). The chosen quantitative data were selected solely for the purpose of further analysis visibility. After all, in fact, in each evaluation operation, the number of price-forming factors taken into consideration may be different, and the size of individual corrections may be different, including opposite signs of percentage and absolute partial corrections.

To compare the results obtained when using additive and multiplier methods, comparable indicators were determined – the total adjustment percentage and the total adjustment coefficient, in line with (10) and (12), respectively. Both indicators represent the same degree of change in the primary (previous) indicator but in different representations of the dimensions: as a percentage and in absolute units, respectively. Mathematically, the total percentage of adjustment is determined from expression (10) as the percentage of change in the primary indicator of the specific value of compared objects after all the adjustments made (price-forming factors are taken into consideration). Accordingly, the total adjustment coefficient is determined from expression (12) as a dimensionless coefficient, which also defines the degree of change in the primary indicator after all the adjustments made.

The greatest interest is the dependence of these two indicators on the number of adjustments made. The following is a graphical interpretation of the resulting dependences. For the

above-formulated conditions of the example, the total percentage and the total adjustment coefficient when applying additive and multiplicative models are illustrated by plots in Fig. 1.
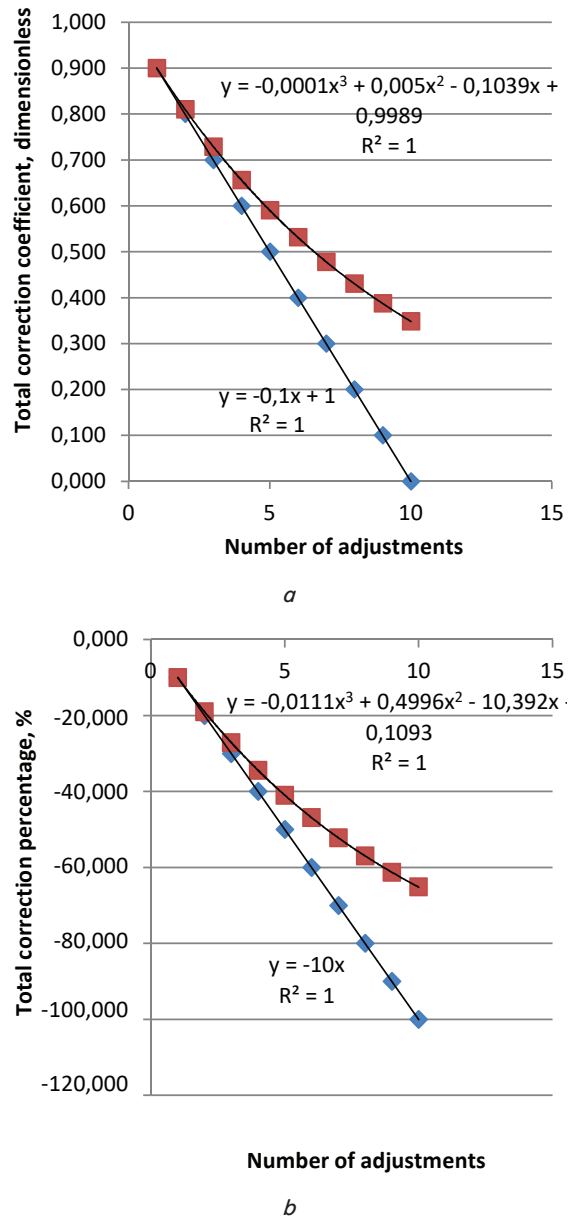

a

b

Fig. 1. Dependence plots of the total coefficient and the total percentage of adjustment on the number of corrections: *a* – plot of change in the total correction coefficient in accordance with (12); *b* – plot of change in the total correction percentage depending on the number of price-forming factors taken into consideration (adjustments) in accordance with (10). Both plots demonstrate the use of the additive (diamond markers) and multiplicative (square markers) correction introduction models

Fig. 1 demonstrates that when using the additive correction introduction model, the total percentage and the overall adjustment ratio change linearly. And this is a big drawback of this model – after all, as we see in Fig. 1, when performing 10 djustments of −10 % each, the total adjusted indicator takes a zero value. Moreover, when making the next adjustments, if there are more than 10, it would accept negative values. In a given context, this does not make any economic sense – because in

practice, a measurable single indicator of the value of a valuation object in general cannot be either zero or negative. In the case of correct resolution of the evaluation problem, the total percentage and the total adjustment coefficient should take numerical values limited by the requirements of economic realities and common sense. Too high dimensions of partial adjustments indicate unacceptably large discrepancies in the characteristics of the compared objects and the object of evaluation, that is, the unjustified choice of compared objects. It should be noted that in the considered example, the abnormally large size of the total adjustment of –100 % at 10 price-forming factors is due, first of all, to the incorrect use of the additive model of correction introduction.

Instead, when using the multiplicative model of correction introduction, the total percentage and the overall adjustment ratio change nonlinearly. This fundamentally eliminates the possibility of obtaining incorrect zero or negative cost values and too large values of the total percentage and adjustment coefficient. Then the curve describing the dynamics of the overall adjustment indicators may asymptotically approach the abscissa axis but it will never reach zero cost values. Fig. 1 also shows that in the considered example, when using the multiplicative model of correction introduction, the pattern of changing the general indicators of adjustment without loss of accuracy is described by the 3rd power polynomial.

This pattern, under the condition of equality of individual partial adjustments, is inherently an indicator (power) function. In this case, the value of the total adjustment coefficient is defined as the value of one adjustment raised to the power, which is the number of adjustments [19].

For a better understanding of the differences between the two compared models, it is even more interesting to consider the dependences of absolute percentage gains and the adjustment coefficient on the number of adjustments. Taking into consideration the fact that for the analyzed case, all corrections are the same and are –10 % (adjustment coefficient is 0.9), it is worth tracking how the indicators of the general adjustment would change when using both compared models. Below, Fig. 2 shows a graphical interpretation of the obtained indicators when representing on the axis of abscissa the number of accounted-for price-forming factors (adjustments), and on the axis of verticality – the absolute values of the growth of the parameters under study.

Fig. 2 demonstrates that the nature of the dependences studied for both alternative models is dramatically different. When applying the additive model of correction introduction, the value of absolute increases in the total percentage and the adjustment coefficient remain stable and do not depend on the number of price-forming factors (adjustments) taken into consideration. Instead, when using the multiplicative model of correction introduction, these gains reduce non-linearly. The weight factor, that is the contribution to the indicator of the general adjustment, of each subsequent partial correction is less than the previous one. However, this does not mean that the overall adjustment indicators would depend on the sequence of adjustments introduced. Absolute increases on individual pricing factors, in this case, would vary but the indicators of the total percentage and adjustment coefficient would remain the same, regardless of the chosen procedure for making corrections. Thus, when using the multiplicative model, the plots of the dependences of the module of absolute increase of the total percentage of

adjustment and the module of increase of the total adjustment coefficient show the nonlinearly-descending nature of functions with negative first and second derivatives. Due to this pattern, the use of the multiplicative model avoids the possibility of obtaining inadequate results devoid of economic content (zero and negative values of the adjusted value of compared objects).
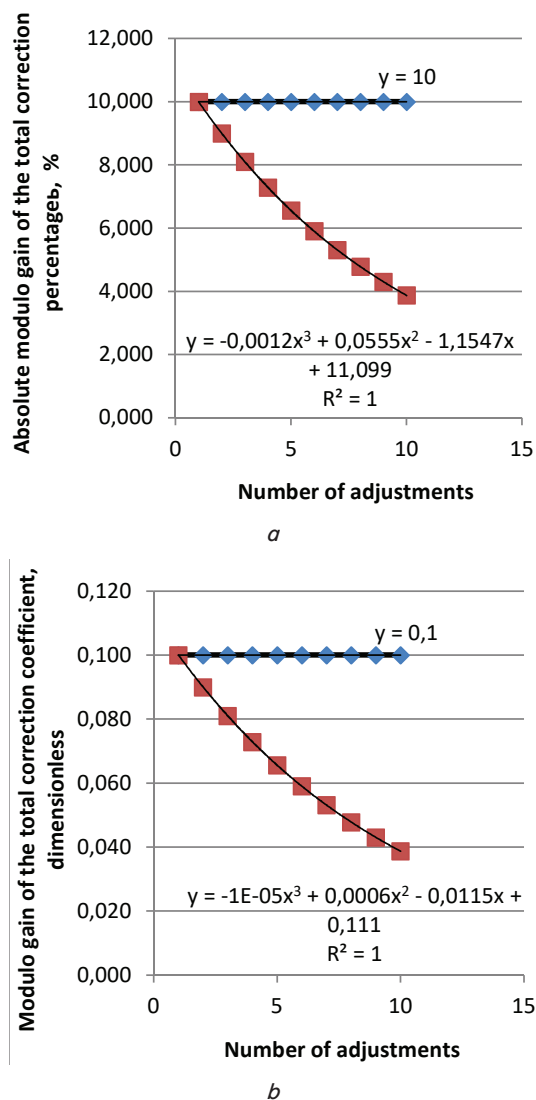

Fig. 2. Dependence plots of the absolute modulo gain of the total correction percentage on the number of adjustments and the dependence of the modulo gain of the total correction coefficient on the number of adjustments: *a* — plot of change in the absolute modulo gain of the total percentage; *b* — plot of change in the modulo gain of the total correction coefficient depending on the number of considered pricing factors (adjustments) when applying additive (diamond markers) and multiplicative (square markers) model of correction introduction

Our comparative analysis allows us to conclude that applying an additive model for adjustments expressed as a percentage is incorrect. Their proper generalization is made possible only when using a multiplicative model – after converting the corrections expressed as a percentage to the form of representation by dimensionless coefficients. Instead, the use of an additive model is fully justified for absolute

corrections expressed in units of measurement for a single indicator. Then the results of the application of both models would be identical. Thus, each of the compared models has its own exclusive area of proper application. And this area of application is determined by the selected measurer of partial corrections, which is expedient to recognize as the criterion of adequacy for the correction introduction model.

5. 4. Substantiating the principles of choosing an adequate model based on the selected criterion

In the additive model, the algebraic amount of partial corrections is applied to one base – the primary single indicator of the cost of the offer of the comparison object. Instead, in the multiplicative model, each partial correction is made from a different base – the result of the adjusted cost after the previous adjustment. It is this option that meets the requirements of methodological recommendations [18]. Our results of the generalization of corrections and the analysis of increments in a general adjustment confirm the undoubted methodological advantage of the multiplicative model for percentage/coefficient corrections, compared to the widely used additive one. The identified features of the multiplicative model, thus, make it possible to carry out a more correct, balanced accumulation of partial corrections in the indicators of the total percentage and the total adjustment coefficient.

Note that belonging to one or another of the two models in question is determined only by the selected algorithm for making corrections. According to the requirements set out in [18], corrections must be taken into consideration sequentially, with the registration of intermediate results. That is, each subsequent correction must be made to the previous adjusted value. Then, for relative corrections, the indicators of general adjustment would be determined according to the multiplicative model, regardless of which metric each correction is expressed by (as a percentage or a dimensionless coefficient). If a single general correction is calculated in units of the specific cost of the corrected indicator, as an algebraic sum of partial corrections for all pricing factors, and then it is once applied to the primary unit indicator of compared objects – obviously, this indicator of general adjustment corresponds to the additive model. In both cases, the result of the adjustment would be identically the same if the model of generalization of corrections is adequate to their metrics.

When applying a distributed algorithm, when each subsequent correction is made to the previous adjusted value, the size of the same percentage corrections would correspond to the different sizes of the corrections expressed in units of measurement of the unit indicator. Because the same percentage corrections apply to each time other base values of the adjusted value. Then there is the dependence of the adjusted value on the location of the correction at a certain price-forming factor in the chain of consistently performed adjustments. If the percentage corrections for different pricing factors were referred to as the same base value, then the same percentage corrections would correspond to the same size of absolute corrections expressed in units of unit of measurement of a unit indicator. However, with chain sequential adjustment, it is not, and this feature should be taken into consideration when determining the sequence of corrections in the additive model. In general, in the additive model, the summing up of corrections, expressed as a percentage, is a rather silly procedure. The result of its implementation gives a very vague idea of the total amount of adjustments made – regardless of whether the percentage corrections are summed up by absolute value, or they are com-

bined taking into consideration the signs. The total percentage correction, algebraically defined according to the additive model, cannot be applied at all to calculate the adjusted cost of a single indicator. There is no valid base value for it to which it can be attributed. After all, its components – partial percentage corrections received for different pricing factors – were determined for different basic values of the single indicator. Even more absurd and inappropriate is to calculate the total percentage correction determined without taking into consideration the sign – it does not say anything but the amount of modules of these partial corrections. There is no method for interpreting this metric that can be solved by an evaluation task. The practical value of both indicators seems highly questionable, and their submission in a report only overshadows the content of the adjustment procedure.

If one feels an urgent desire to get generalized indicators of the adjustments made, then there are the following indicators:

1) a product of partial corrective coefficients determined for different price-forming factors (the general coefficient correction obtained from the multiplicative model);

2) summation taking into consideration the signs of partial absolute corrections expressed in units of measurement of a unit indicator (the general absolute correction obtained from the additive model).

It is easy for us to mathematically prove that both above indicators give identical and methodically correct values of the adjusted single indicator of the sale offer. These values fully meet the requirements set out in [18] for an adjusted single metric correctly obtained by a distributed method of performing a chain of sequential adjustments and registering intermediate adjusted values.

Instead, the widespread application of the additive model to partial percentage corrections is completely incorrect. This is a rather typical example of the erroneous application of the model of generalization of corrections for indicators of partial corrections that are not suitable for it – in this case, an additive model for generalization of percentage partial corrections. In fact, in this case, the sum of percentage corrections does not reflect any objective characteristics of the adjustment procedure. It should be understood that the numerical values of partial percentage corrections (as well as their sum) do not make sense unless their place in the adjustment chain is determined. In this case, it is not known what base indicator and what size of the correction expressed in units of single indicator they correspond to.

Therefore, it seems completely impractical to sum up the percentage of adjustments – as a result, a certain fictitious indicator would be obtained, which is not informative enough and can only mislead both the appraiser himself and the users of the report. The application of an additive model makes sense only when summing up absolute corrections expressed in units of measurement of a unit of indicator. The so-called "general percentage correction", obtained by the algebraic summation of partial percentage corrections, is an informative and methodically erroneous indicator. It is artificially obtained by inadequate application of the additive model to indicators that should be summarized exclusively according to the multiplicative model. Correct resulting values of the general adjustment cannot be obtained by summing up partial percentage corrections (and even more so, partial correction coefficients expressed in dimensionless units). Therefore, you should not mix elements of additive and multiplicative models – on the contrary, they must be clearly delineated and should be applied only to corrections expressed in the appropriate format of dimensions. Professional appraisers should realize that the only

criterion for the adequacy of model selection is their chosen unit of partial corrections.

5. 5. Theoretical substantiation of the independence of the evaluation result on the sequence of partial corrections consideration

An important general property of the multiplicative and additive models described above is the independence of the indicator of general adjustment (total coefficient or general absolute correction) on the order (sequence) of corrections. In the multiplicative model, the order of multipliers – partial coefficient corrections – can be arbitrarily changed, with the unchanging value of the product. This is also true for the products in an additive model where the resulting values of the general adjustment are determined by the algebraic sum of partial absolute corrections expressed in units of measurement of the corrected unit indicator – the specific cost (the total absolute correction). Changing the order (sequence) of multipliers or products does not change their sum. This is quite obvious as a consequence of the switching law of multiplication and addition operations.

Consequently, an important general property of additive and multiplicative models is the independence of the general adjustment on the order (sequence) of corrections. After all, in the additive model, the resulting values of the general correction are determined by the algebraic sum of partial corrections expressed in absolute units of measurement of the specific value of the corrected indicator. Accordingly, in the multiplicative model – the product of partial dimensionless adjustment coefficients according to certain pricing factors. As a result of the switching, or permutation, law of mathematical operations of addition and multiplication, the algebraic sum and product of several rational numbers would not change if one swaps them [20–22]. That is why, when determining the amount and product, the order of products and multipliers can be arbitrarily changed – with the immutability of the final result.

The resulting values of the general corrections during the adjustment, and, accordingly, the adjusted value of the single indicator, are determined by the choice of the model and its adequate algorithm for the implementation of the adjustment but not by the sequence of taking into consideration individual pricing factors. For the multiplicative model, the problem of consideration of correction signs is also eliminated – all adjustment coefficients have the same signs; they are always positive.

6. Discussion of results of investigating the differences between the compared mathematical models

Unlike [16, 23–29], where the mathematical principles of the evaluation procedure of adjustment were not analyzed and the criteria of adequacy and scope of models were not formulated, our results from their comparative analysis make it possible to achieve this. Although the priority of applying the multiplicative method of corrections is confirmed by the research data reported by some authors. Thus, we can find the statement that when several types of independent adjustments are taken into consideration at the same time, a multiplicative model should be used, and not an additive model [28] – without analyzing the mathematical basis of models. However, one can also find sources where preference is given to the additive model [29], again – without analyzing its mathematical principles. Our study has made it possible to identify advantages, disadvantages, and areas of proper application of compared mathematical models. That became possible due to the establishment of differences in

the dependences of absolute increases in the total percentage and the adjustment coefficient on the number of price-forming factors taken into consideration (partial corrections).

Our results of the comparison analysis are explained by the fundamental difference in the weight of individual partial corrections in their generalization. Fig. 2 shows that, unlike the additive model, when using a multiplicative one, the total percentage and the overall adjustment coefficient change nonlinearly. This corresponds to a better distributed algorithm of corrections, the values of which are calculated from each other base – a unit value indicator adjusted at the previous stage. This fundamentally eliminates the possibility of obtaining incorrect zero or negative cost values and too large values of the total percentage and adjustment coefficient. Then the curve describing the dynamics of accumulation of general indicators of adjustment may asymptotically approach the axis of abscissa but it would never reach zero cost values.

Our consideration of the mathematical principles of the evaluation procedure for making corrections has made it possible to obtain analytical expressions for a mathematical description of the mutual ratios of parameters (6) to (19) when applying various dimensional corrections. This is extremely useful for the practice of economic measurement of the value of assets carried out by the methods of independent expert evaluation. After all, this realizes the possibility of formalizing the mutual changes of various dimensions of partial corrections that can be used in the table of adjustments to the valuation report. This simplifies work of the appraiser, increases the productivity, and eliminates the possibility of accidental errors in calculations. A once-developed and proven template for calculating the adjusted cost when submitting partial corrections in several metrics, with a well-founded correct choice of models of their generalization, avoids errors with its subsequent repeated use.

The dependences of indicators of general adjustment and their growth on the number of price-forming factors taken into consideration have made it possible to formulate and substantiate the criterion for choosing an adequate model of generalization of corrections. That has made it possible to exclude possible gross errors with inadequate application of the model of generalization of corrections. Such errors occur from inexperienced appraisers and lead to a critically unacceptable distortion of the results of economic measurements. A specific numerical example was used to clearly demonstrate the consequences of such gross errors when performing an evaluation adjustment procedure for two compared models. Our study has confirmed that the fairly common practice of summing up percentage partial corrections without justifying the choice of an adequate model inevitably leads to unreliable results of the assessment. Unlike [16], which gives expressions (1), (2) producing a completely false result, our work reports a ratio (12) for the correct generalization of percentage partial corrections. Only the justified choice of the model based on the criterion formulated in our work makes it possible to correctly determine the areas of correct use of the compared models.

The above theoretical justification of the independence of the assessment result on the sequence of taking into consideration the impact of partial corrections of individual price-forming factors (adjustments) with the correct choice of the model has made it possible to remove this issue for good. Previously, it was given an undeserved lot of attention while, when choosing the correct model, the sequence of adjustments does not matter. The above results of this study have made it possible to work out recommendations for the use of compared models in evaluation practice.

Taking into consideration the comparative analysis of the mathematical principles of the evaluation procedure of adjustment and the results of the application of both models in question, the additive model of corrections should be considered suitable for use in valuation practice only for absolute corrections. As demonstrated in the above example, when it is incorrectly applied to percentage corrections, completely absurd results can be obtained. The additive model of corrections does not formally contradict the requirements set out in p. 49 of the National Standard No. 1 [1] but the study showed that its scope is limited solely to the form of representation of absolute corrections expressed in units of single indicator. The results of our study confirm that when it is applied to percentage corrections, the result obtained is incorrect.

This study was carried out under certain preliminary assumptions and limitations. Namely, the considered examples used an idealized case of using 10 identical lowering partial corrections. In practice, the number, the absolute value, and the correction sign are variable parameters of the evaluation adjustment procedure. In fact, the number of partial corrections may be less and greater than 10, depending on the number of price-forming factors taken into consideration. The size and sign of partial corrections depend on the degree and nature of the discrepancies between the object of evaluation and the object of comparison. All these indicators are determined by the appraiser and belong solely to his/her competence and responsibility.

Regardless of the above assumptions and limitations applied in the examples under review, expressions (3) to (19), reported in this work, are universal in nature. They are provided to quantify the parameters of models for the implementation of the evaluation adjustment procedure for any quantities and sizes of partial corrections. A certain disadvantage of this study is the lack of a real example of the use of compared models in valuation practice, in the typical volume assessment report. In the future, this disadvantage can be eliminated in the preparation of a separate monograph on this topic.

Of practical and theoretical interest for the further research in this area is to derive analytical expressions for absolute and relative errors with an inadequate model selection and their quantitative analysis using examples from valuation practice. A promising direction is also the development and verification of quantitative methods for determining the size of corrections in the implementation of the evaluation adjustment procedure in a comparative approach. Most of the corrections are determined by the so-called "expert way", that is, based on own beliefs, ideas, experience of each appraiser. It is clear that at the same time there is a strong influence of the subjective factor. The introduction into practice of valuation activities of objective estimation methods for determining the size of corrections in the implementation of the evaluation adjustment procedure would make it possible to reduce the influence of the subjectivity of appraisers. In this way, it would be possible to reduce the degree of uncertainty of the results of the independent assessment of the value of assets [12] and a corresponding increase in accuracy and reliability. However, in this case, the procedure of objective digitization of various qualitative characteristics of the objects of evaluation and comparison is associated with certain difficulties of a methodical and mathematical nature.

Based on the comparative analysis of the results of the application of both models in question, the following recommendations for their application in valuation practice were formulated. The additive model of corrections is proposed to be considered suitable for use only for absolute corrections. Absolute corrections in units of unit of measure of a unit indicator, as practice shows, are used relatively rarely by appraisers. However, the additive model is provided only for this form of partial corrections. When it is incorrectly applied to partial percentage corrections, quite absurd results can be obtained, in the form of zero or negative adjusted value of a single indicator. It follows from our study that each subsequent adjustment should be made from the base obtained after the previous adjustment. Therefore, the scope of the correct application of the additive model is limited to summation, taking into consideration the signs of partial absolute corrections expressed in units of measurement of a unit indicator (general absolute correction).

Taking into consideration the general unpopularity of the use of partial absolute corrections expressed in units of unit of measurement of a unit indicator, the scope of the additive model is very narrow. The generally accepted priority of using partial percentage (as well as relevant coefficient) corrections gives reason to consider a much more common area of application of the multiplicative model, which is adequate for this case. At the same time, it should be used exclusively to obtain the product of dimensionless coefficients (coefficient partial corrections). Therefore, the most convenient measurements of partial corrections are their own dimensionless corrective coefficients. And the submission in the table of partial correction adjustments in three dimensions for each price-forming factor (coefficient correction, dimensionless; percentage correction, %; absolute correction, in units of measurement of a unit indicator) most fully and transparently reveals the essence of the adjustment procedure. This form of representation usually removes all users' questions about the report.

Based on the results of the above analysis, it is possible to formulate a recommendation for the priority use of the multiplicative model in an arbitrarily wide range, taking into consideration its undoubted methodological advantages. It should be emphasized that the methodically correct result, corresponding to the multiplicative method of making corrections, can be obtained in any form of submission of corrections. They can be filed in the form of a dimensionless coefficient, or a percentage, or an absolute correction with the dimension of the total cost of the offer (for example, USD) or a single indicator (for example, USD/sq. m for premises). However, each form of representation must meet an adequate model for generalization of partial corrections.

Important is the algorithm of correction introduction during adjustments and the model of their generalization. As recommended in [18], the procedure for applying adjustments should be carried out in stages, that is, with step-by-step calculation and registration of intermediate adjusted indicators after the introduction of each partial adjustment. In this case, each subsequent adjustment would be made from each other base – the result of the previous adjustment. At the same time, it is completely optional to determine the indicator of the general adjustment (a generalized correction) since the correction introduction procedure is carried out in stages and distributed, and not once. However, if this generalized indicator is to be determined by the appraiser, it can be established by the correct application of the multiplicative model to partial corrective coefficients. Or, again, the correct application of the additive model to partial absolute corrections expressed in units of measurement of a unit. Both indicators obtained in this way are methodically correct, identically mutually relevant, and exactly equal the value of the generalized correction, which corresponds to its correct analytical description. Such an indicator of general adjustment can be used as a generalized correction for a one-time adjustment – with the same result, which was obtained during a phased distributed adjustment.

It is worth remembering that if it is necessary to apply partial corrections expressed by different measurements, for their correct generalization, all of them should be brought to the same metric. That is, for all objects of comparison, the condition of comparability of partial corrections and adjusted indicators should be met – regardless of in which units of measurement individual partial corrections are expressed. The form of representation of consistently received adjusted indicators must be the same and compatible, in terms of the dimension of the selected single metric. This makes it possible the report user to observe a consistent change in the size of the single metric, with the consistent introduction of corrections from the beginning to the end of the adjustment procedure.

Mathematical principles of mutual transformation of various forms of correction representation, defined above by expressions (13) to (19), make it possible to easily algorithmize the implementation of all evaluation procedures of a comparative approach using the Microsoft Excel platform. It would be rational to represent, in the table of adjustments, several forms of representation of partial corrections at the same time, in all possible and appropriate units of measurement. This corresponds to the maximum transparency and informativeness of the report and makes the representation of the implementation of the evaluation adjustment procedure in the report clear, visual, and completely transparent. Meeting this condition makes understanding the report easier for the customer and greatly facilitates work of the reviewer who, at the same time, gets the opportunity to analyze and verify interim results.

Within the framework of the concept of information and metrological approach to the evaluation of the use of research results in valuation practice, it makes it possible to reduce the degree of uncertainty of the results of evaluation operations. Our study is the next step in the development of the information and metrological paradigm of independent expert evaluation [30], which is a promising way to further improve its methodological base. The above approaches to the mathematical description of the patterns of making corrective adjustments would undoubtedly contribute to increasing the accuracy and improving the reliability of the assessment results. In this way, it is possible to increase the level of methodological support for evaluation activities, which is positively reflected at the level of uncertainty of their results [31–35].

7. Conclusions

1. We have analyzed the mathematical principles of implementation of the evaluation procedure of corrections with the use of two comparable models in determining the cost according to the comparative approach – additive and multiplicative. This has made it possible to build a mathematical description of the evaluation adjustment procedures and determine the areas of correct application of each model. Unlike the widespread erroneous practice of applying an additive model to generalize percentage partial corrections, its scope is limited to absolute corrections expressed in units of unit of measurement of a unit indicator. This study has proven the incorrectness of its application to percentage partial corrections. It is shown that each of the models is characterized by its inherent scope: for the additive model – this is a generalization of absolute partial corrections; for the multiplicative model, respectively, the generalization of relative partial corrections, represented in the form of dimensionless coefficients. This realizes the possibility of qualitative improvement of the degree of validity of the

methodological base of evaluation operations in terms of using a comparative methodological approach.

2. Based on the comparative analysis, we have derived analytical expressions (6) to (19) for the mathematical description of mutual ratios of parameters of these models and indicators of general correction. Equations (6) to (10) make it possible to determine the value of adjusted value when using an additive model. Accordingly, expressions (10) to (12) mathematically describe the procedure for correction introduction when using a multiplicative model. In the latter case, partial corrections for each price-forming factor are determined in the form of dimensionless coefficients – multipliers to the previous adjusted value. Formulas (13) to (19) represent ratios that allow for the equivalent mutual transformations of different forms of correction representation in all possible dimensions. All analytical expressions have been constructed on the basis of our comparative analysis of additive and multiplicative models.

The derived formulas for the interrelation of parameters of the additive and multiplicative models and indicators of the generalized correction made it possible to obtain appropriate numerical indicators when performing evaluation activities. In terms of their structure, equations (6) to (19) are formulas of functional interrelations among the basic parameters of the mathematical description of mathematical principles of the evaluation procedure of adjustment, with correction introduction when using the additive and multiplicative models. These parameters include: the value of the adjusted cost v_a; the initial cost of compared objects v_0; general absolute correction Δv; the total percentage of adjustment δv; n is the total number of price-forming factors taken into consideration; it is also the number of partial corrections during adjustments. The general absolute correction Δv is an algebraic sum of partial corrections $\Delta v_1 ... \Delta v_i$ for each price-forming factor, represented in the dimension of a single indicator. The total adjustment percentage δv is the algebraic sum of partial percentage corrections $\delta v_1 ... \delta v_i$ for each pricing factor represented in the dimension of percentage to the initial value v_0. In the multiplicative model, the value of the adjusted cost v_a can also be obtained by multiplying the initial cost v_0 of the single indicator of the objects of comparison by the total corrective coefficient K, which is the product of partial coefficients $C_1 ... C_i$ for each price-forming factor, represented by dimensionless positive numbers.

The formulas obtained in this paper for the relationship between the above parameters of the additive and multiplicative models and the indicators of generalized correction provided for the opportunity to mathematically formalize the parameters of corrections in all possible measures. This, of course, increases the transparency and informativeness of the evaluation report and makes it more understandable to users. This also provides a more reliable rationale for the results of the independent evaluation

3. A specific numerical example was used to show the peculiarities of performing an evaluation adjustment procedure based on two compared models. Discrepancies in the dependences of indicators of general adjustment and their increase in the number of price-forming factors (adjustments) were found. It has been clearly demonstrated that the result of the assessment strongly depends on the correctness of the model selection. It has been shown that the erroneous choice of the model leads to distortion of the assessment result, when its uncertainty increases sharply due to an increase in the influence of methodical error. The numerical examples considered demonstrated the inappropriate use of the additive model in the generalization of percentage corrections. They also con-

firmed the correctness of the use of the multiplicative model for this case, with the representation of partial corrections in the form of dimensionless coefficients. The limited amount of this work does not make it possible to include an example of the use of comparable models in valuation practice in the typical volume assessment report, given its large volume. In addition, the specific numerical indicators of any example represent only one partial implementation of the application of the considered methods and algorithms of adjustment. The performance of each specific evaluation activity is associated with obtaining several other numerical indicators that do not make sense to summarize.

4. It is shown that each of the models is characterized by its inherent area of correct application. For the multiplicative model, this is the implementation of the adjustment procedure when representing partial corrections in the form of dimensionless coefficients – multipliers to the previous adjusted value. Accordingly, for the additive model – the implementation of the adjustment procedure when representing partial corrections in units of measurement of a unit indicator. It has been proven that it is completely unacceptable to violate the boundaries of the above-mentioned areas of correct application of models. In particular, it is incorrect to use an additive model when representing partial corrections in the form of percentages to the previous adjusted value. Algebraic or arithmetic summation of partial percentage corrections leads to false results, sometimes completely unacceptable and devoid of meaning (zero and negative adjusted value).

The principles of choosing an adequate adjustment model based on the criterion determined in the study – the unit of partial corrections chosen by the appraiser – have been formulated. This paper proved that partial corrections for each price-forming factor can be equally informatively represented in any of the three possible forms. Namely: in percentage measurers (percentage correction, %); dimensionless coefficients (coefficient correction); in units of single indicator (absolute correction).

However, at the same time, the algorithm for introducing and summarizing corrections should strictly correspond to the selected unit of measurement of partial corrections. With the correct application of the model, adequate for the used unit of corrections, the final result of the adjustment would be the same. Actually, the applied form of the representation of partial corrections in the selected unit of measurement determines the choice of the corresponding model. This result is extremely important for valuation practice since, due to the lack of research on the mathematical basis of the adjustment procedure, examples of mistaken application of models can often be found – with obvious consequences of obtaining a false assessment result.

5. The theoretical justification of the formulated restrictions of the scope of correct application of justified models of corrections is given. It is shown that in chain adjustment, when each subsequent partial correction is made to the previous adjusted value, the size of the same percentage corrections would correspond to the different sizes of absolute corrections expressed in units of unit of measurement of the unit indicator. The difference between these absolute corrections is due to the fact that the same percentage corrections refer to each time other basic values of the adjusted value of a single indicator.

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