May 2013, Dortmund

On Heisenberg's Notion of a Closed Theory

François-Igor PRIS

(frigpr@gmail.com , Dortmund)

Abstract

I claim that Heisenberg's notion of a closed theory and its analysis by Erhard Scheibe fit well with the philosophy of later Wittgenstein or its generalization. The notion of a closed theory corresponds to the notions of a form of life and rule/concept. I suggest the possibility of reconciling the views of Heisenberg, Dirac, and Bohr about inter-theoretical relations within a rational naturalistic pragmatism à la Wittgenstein and Robert Brandom's analytic interpretation of Kantian synthetic unity of apperception. In particular, I explain why a "closed theory" is "closed", "accurate" (and even "perfect"), and "final" (as Heisenberg claims), and why it is also "open" and "approximate" in Dirac's sense. That being said, unlike Alisa Bokulich (2004, 2006, 2008), I rather favour Heisenberg's philosophical position.

1. *Quantum mechanics as a generalization of classical mechanics. Mechanics as a synthetic super-unity of apperception*

Bohr saw quantum theory as a "rational generalization" of classical theory.[1] Dirac, too, saw quantum theory as a generalization of classical theory, but in a more concrete, more

[1]Bohr understands "the natural principle of generalization" as follows: "It is the combination of features which are united in the classical mode of description but appear separated in the quantum theory that ultimately allows us to consider the latter as a natural generalization of the classical physical theories" (Bohr 1929; quoted by Bokulich 2008, p. 97). In fact, it is the Wittgensteinian family resemblance between the two mechanics that allows us to talk about generalization.

Bohr also uses the term "rational generalization". At first glance, it might look surprising, since his old quantum mechanics was contradictory. New (quantum) laws/commitments contradicted the old (classical) ones. This contradiction had been resolved by rejecting the classical laws. In terms of a synthetic unity of apperception (see below) this corresponds to the critical step of eliminating of the material incompatibilities.

mathematical sense, namely, as a move from a commutative algebra of variables to a non-commutative one. (In 1926 Dirac even said that only one basic assumption of classical theory was false – the assumption of commutativity. For details, see, for example, Bokulich (2008).) For Heisenberg, too, the move from classical theory to quantum theory was a generalization (see, for example, Pris 2013). But this move, for him, was rather an intuitive revolutionary jump into unknown, supposing a certain "mental act". Heisenberg used Bohr's principle of correspondence (see, for example, Bohr 1920, 1922) as the principle of analogy.[2]

In this paper, I will put Heisenberg's, Dirac's, and Bohr's views on relation between classical mechanics and quantum mechanics in correspondence with different steps in the development of science, viewed as construing a Kantian synthetic unity of apperception (see also footnote 3 and below).[3]

To begin with, let me mention three steps in the development of science according to Heisenberg.

He distinguishes the following steps: (1) expansion of the domain of applicability of a theory, (2) new laws occur, though concepts are left untouched (new laws contradict to the old ones, says Heisenberg), (3) the whole conceptual structure of the theory changes. (See, for example, Scheibe 2001, p. 138)

Hence, for example, Bohr's old quantum mechanics corresponds to the step 2. It was contradictory because its new (quantum) laws cohabitated with the old (classical) conceptual apparatus. Heisenberg's own matrix mechanics corresponds to the final step 3.

Now, the birth of a new theory generalizing (extending) an old one can be considered, I think, as the birth of a new super-judgment made in a radically new experimental context, that is, a judgment made within a (implicit) super-paradigm, not within the old theory. So, I briefly turn to the theory of judgment.

Very briefly, according to Robert Brandom's (2011b) analytic interpretation of Kant's normative concept of judgment, judging is integrating a new commitment into a constellation of prior commitments, so as to maintain the rational normative unity of apperception.

---

[2] About the correspondence principle Heisenberg says, for example, the following: «In its most general version, Bohr's correspondence principle states a qualitative analogy (which can be carried out in detail) between the quantum theory and the classical theory belonging to the respective picture employed. This analogy does not only serve as a guide for finding formal laws, rather, its special value is that it furnishes at the same time the physical interpretation of the laws that are found» (1930a, p. 78; *translated by* Falkenburg 2007).

[3] Bokulich (2008) distinguishes four approaches to intertheoretic relations: reductionism (classical mechanics is reducible to quantum mechanics), Heisenberg's theoretical pluralism (different theories describe different domains of reality), Dirac's view that there is a structural continuity between the two mechanics, Bohr's view on quantum mechanics as a rational generalization of classical mechanics. Bokulich's "interstructuralism" favors Dirac's view (Bokulich 2008). My suggestion is that these views can be reconciled. Reductionism is the final - logical - step in the development of science and its self-understanding.

In Brandom terms, synthesizing a Kantian unity of apperception is accomplishing three normative task-responsibilities: *critical* (rejecting commitments that are materially incompatible with other commitments one has acknowledged), *ampliative* (acknowledging commitments that are material consequences of other commitments), and *justificatory* (providing reasons for the commitments one has acknowledged, by citing other commitments one acknowledges of which they are material consequences). (Brandom 2011b, *München Lectures*, May 2011) (Hegel's term for the process of integration of a new commitment is experience (*Erfahrung*). According to him, the first step is acknowledging the material incompatibility of some of one's commitments (the experience of error); the second step is revising one's commitments so as to repair the discordance; the third step is providing reasons for rejecting bad commitments (see Brandom's *München Lectures* (Brandom 2011b, lecture 2)).

Wittgenstein's account of judgment can be understood in connection with his rule-following problem or the problem of an application of a concept. Explicitly, I think, it is a naturalized version of Brandom's account of the evolving Kantian synthetic unity of apperception. (I do not justify here this claim.)

And this is also an account of projection of a concept into a new context, that is, an account of the process of natural generalization.

The move from classical mechanics to quantum mechanics is just a more radical "judgment", made within a certain super-unity of apperception – called "Mechanics" - involving (in its final form) classical mechanics as well as quantum mechanics.

Roughly, Heisenberg's step (3) in the development of science corresponds to the task of justification of the commitments (because without an appropriate conceptual apparatus such a justification is not possible). Though Heisenberg's quantum mechanics (as well as classical mechanics) can be treated as a unity of apperception (Heisenberg has accomplished all three tasks in construing it), from the point of view of a more general, super-unity of apperception, involving both classical and quantum mechanics, Heisenberg's generalization of classical mechanics should be treated as accomplishing only the first – critical – task in integrating the new quantum mechanics into Mechanics as a super-unity of apperception. I suggest that Dirac is rather situated on the level of the ampliative task, and Bohr – justificatory task (for some details see below).

That is why Dirac's philosophy of science is almost the antithesis of Heisenberg's. Dirac's level of investigation is more mathematical (Dirac himself emphasized his education in engineering and applied mathematics). As for Bohr, he is situated on the conceptual level of investigation. Heisenberg was closer to the real experience and its connection with theory than Dirac and Bohr.

The principle of generalization (and the principle of correspondence which should be understood as the principle of generalization (Pris 2013)) is manifested a bit differently on different levels: it is manifested (1) as the principle of natural extension (projection, or generalization) in Wittgenstein's sense (this is Heisenberg's level), (2) as the principle of mathematical generalization (this is Dirac's level), (3) as the principle of conceptual generalization (this is Bohr's level).

It is not surprising that for Heisenberg, the development of science is discontinuous, and for Dirac and Bohr it is rather continuous. The discovery of a new theory is always a jump. However, retrospectively, the development of science can be represented as a continuous rational process. Dirac's continuism (theory changes are piecemeal modifications) and Heisenberg's discontinuism (theory changes are wholesale ones) are reconcilable.

So, I claim that the move from classical physics to quantum physics can be understood as a dynamical process of creating a super-unity of apperception. Heisenberg's creation of quantum mechanics preceded the subsequent developing of logical/mathematical consequences of the new theory and establishing connections between classical and quantum theories, and, finally, effacing the sharp borders between the two paradigms (Dirac's "engineering of science". Dirac's attempts to construct a Lagrangian quantum mechanics in addition to the Hamiltonian mechanics are a good example (Dirac 1932). Finally, this led to Feynman's path integral. The latter uses the classical notion of a path.[4])

A good (true) physical theory is born as "closed", "perfect" and "incommensurable" with its predecessor. Then it gradually becomes "open", "approximate" and "commensurable". For example, Dirac was concerned with getting relativistic quantum mechanics, that is, with a mathematical generalization of non-relativistic quantum mechanics. (For him, "quantum mechanics" also included quantum field theory.) He predicted the positron which would be discovered by Anderson independently from Dirac's theory. So, it is understandable why quantum mechanics was not a closed theory for him (it was developing), and why he accentuated the unity of science.

Using Wittgenstein's terminology (for more details see below), I would say that for Heisenberg physical theories are language games anchored in experience; for Dirac they are rather mathematical constructions (or mathematical language games), for Bohr - conceptual systems (conceptual language games).[5]

Heisenberg himself characterized his own scientific style as follows (Heisenberg 1970):

«I always looked as a final aim to the mathematical scheme. And that was not perhaps what Bohr did. Bohr looked to a scheme of concepts, a number of concepts, and not a mathematical scheme — if one wants to make such a strong distinction. (…)

I compared my three teachers which I have had, that were Sommerfeld, and Max Born, and Niels Bohr. And I would say: Sommerfeld he was most interested in solving problems which you could compare with experiments. He did not mind the concepts — I mean, the concepts somehow had to be all right, but he was not too much interested. And also he was not too much upset about contradictions, as in old quantum theory, there were contradictions. He wanted to do calculations and to get all correct results. And then there was Max Born who believed strongly in the existence of rigorous mathematical schemes. He was very much a mathematician, but he

[4] Dirac's attempts to extend classical concepts in a more or less straightforward way were not always successful. To give an example, Dirac unsuccessfully tried to extend the electrodynamics by introducing the concepts of ether and absolute time without spoiling the Lorentz-invariance of the theory.

[5] Heisenberg's own explicit goal was to abandon « rules » (concepts) of classical mechanics and to find new « rules » (concepts) allowing one to describe quantum phenomena. (Heisenberg 1924, 1925)

was not a one philosopher. He was interested...he always suggested that I must find the theory in quantum mechanics which replaces Newton's mechanics, but he did not see that in order to do that you have to derive or to apply new concepts.

And, finally, Bohr, he was a philosopher who looked for the concepts. And he said, well, first we must have our concepts right — that means actually we have to make our mind clear, and before we can do that we have no chance really to solve the problems."

All three physicists, Heisenberg, Dirac and Bohr, also used the principle of correspondence. But they did it a bit differently. For Heisenberg, the principle of correspondence was the intuitive principle of generalization (analogy). For Dirac and Bohr it was much more rational. Dirac makes appeal to analogies, correspondence, and structural continuity between the two mechanics. He also uses the reciprocal correspondence principle.[6] Bohr speaks of the principle of correspondence as the principle of rational generalization (see also footnote 35).

2. *Heisenberg's closed theory versus Dirac's open theory*

I suggest that Heisenberg's notion of a closed theory[7] and also its analysis by Erhard Scheibe (see for example 2001) can be interpreted in terms of the philosophy of later Wittgenstein or its natural generalization. (Notice that Heisenberg himself explicitly acknowledged the role of the problems of language in science (see, for example, Heisenberg, W. 1960).) The notion, I claim, corresponds to the Wittgensteinian notions of a form of life and rule/concept.[8] (Kuhn's notion of a scientific paradigm can also be interpreted as a "form of life".) So, I do not completely agree with Erhard Scheibe (1993, see an English translation of his paper in Scheibe 2001) that we are far from the understanding of the concept of a closed theory. We are not far more from its understanding than from the understanding of Wittgenstein's philosophy (alas, Pseudo-Wittgenstein is still popular). At the same time, I agree with him that the understanding of the notion of a closed theory requires understanding of the notion of an application of a concept (or the notion of a concept), and that this problem is not yet completely resolved by philosophers.[9]

[6]The reciprocal principle of correspondence says that the dynamical structures of classical mechanics can be used to build up the dynamical structures of quantum mechanics, and *vice versa*. (The inverse principle of correspondence consists in using quantum mechanics to guide the further developments of classical mechanics.)

[7]Heisenberg's German terms are *abgeschlossene Theorie* and *geschlossene Theorie*.

[8]A "form of life" contains its own (implicit or explicit) rules. And *vice versa*: a system of rules determines a "form of life". A theory taken in its numerous applications is a "form of life". A theory taken as a system of abstract propositions is a rule. The two notions are not identical, but equivalent and not separable from each other.

There is nothing mysterious about the notion of a closed theory which can also be identified with that of a law (system of laws) of nature. For example, Heisenberg says that the Archimedes law of the lever, that was formulated more than 2000 years ago, will be always correct. ("Abschluß der Physik?", 1970, In Collected Works, C, III, 1985, p. 386) The Archimedes "theory" of the level is closed in the Heisenberg sense.

[9] Scheibe says: "Are we that stupid that we don't even know what it means for concepts to be applicable? To this I say: This is stupidity of philosophers, and we owe it to the physicist Heisenberg that he was not afraid to share it with us" (Scheibe 2001, p. 141).

For Heisenberg, well established physical theories, such as classical mechanics, electrodynamics (including special relativity), thermodynamics (including statistical physics) and quantum mechanics (including quantum field theory), are *closed*, *accurate* (and even perfect), and *final*. A closed theory describes (corresponds to) a restricted domain of reality. Heisenberg is a realist, not a positivist. As concerning quantum theory, Heisenberg believed that there are real quantum entities (ontological thesis), and that wave and particle are two aspects of them.[10]

At first sight, Dirac's approach to physical theories contradicts Heisenberg's one. Dirac treats all physical theories as "open" and "approximate". For him, physics is much closer to engineering, and even the most well-established theories are open to revision.[11] (About philosophical views of Heisenberg, Dirac and Bohr see, for example, the book of Bokulich 2008, and also her papers 2004, 2006. It seems to me that Bokulich exaggerates the so-called Heisenberg's pluralism. Heisenberg's world is not dappled. See below.)

For Heisenberg, the move from classical mechanics to quantum mechanics is a revolutionary jump, a change of a paradigm. The process of theoretical generalization is intuitive. The systems of classical and quantum concepts are incommensurable.

In contrast, Dirac is a continuist. He focuses on structural similarity between classical and quantum theories. For him, classical and quantum concepts are commensurable.

Heisenberg started with the empirical quantum phenomenon of dispersion. And he developed a new general quantum theory by constructing the quantum formalism "as close as possible" to that of classical formalism (paraphrasing Einstein, as close as possible but not closer.[12] Bohr's old quantum mechanics was « too close » to classical mechanics. Notice that "as close as possible" is a condition for a correct, not arbitrary generalization of a physical theory.)

Jürgen Renn (Renn 2010) explains how it happened that independently and practically simultaneously, from different experimental data, two different in form but « equivalent » in content theories – Heisenberg's matrix mechanics and Schrödinger's wave mechanics (he calls them dizygotic twins) – were created. Initially they were destined to describe different phenomena: the former – the phenomenon of dispersion, the latter – the hydrogen atom.

Renn proposes a genetic point of view, which, as I see it, is an analysis of the process of generalization (or the process of integration of a new judgement into the Kantian unity of apperception).

According to Renn, both theories are transformations of a common ancestor – the old quantum theory. Both retain the formal structure of the Hamiltonian mechanics, extending it only to the necessary degree. Both include

[10] Heisenberg defines a *theory* as follows: "Die Theorie ist also nicht ein von der Erfahrung unabhängiges Gedanken-gebäude, sondern der Versuch, viele Erfahrungen durch einige einfache Grundgesetze zusammenzufassen" (a theory is not an independent from experience construction of thought, but an attempt to bring together a lot of experiences by means of simple fundamental laws. *Translation mine*). (Heisenberg 1943, p. 202) A closed theory is not a special theory, but just a well established theory. And it is "not a dogma".

[11] For Dirac, for example, special relativity is a modification (improvement) of classical mechanics, and both together with classical electrodynamics constitute the same theory. He makes a more sharp distinction between classical and quantum theories.

[12] Einstein said that a theory must be as simple as possible, but not simpler.

the translation procedure connecting classical and quantum concepts: Heisenberg reinterprets the canonical variables as non-commuting matrices; Schrödinger, in his turn, uses an optico-mechanical analogy and introduces the wave function (see, for example, Joas 2009). Both theories integrate the new knowledge about the connection between energy and frequency.

Notice that Heisenberg explains his requirement that a new revolutionary theory must be a minimal modification of an old one from the point of view of realism: a new theory must not depend on the authority, but must be forced by the nature itself ("Änderungen der Denkstruktur im Fortschritt der Wissenschaft", 1970, In Collected Works, 1985. C, III, p. 357). Notice also that the Heisenberg requirement supposes that the concepts of the new theory are generalizations of the concepts of the old one (so, I do not agree with Schiemann (2008, p. 76) who claims that according to Heisenberg there must be no consistent relations between the concepts of the old and the new theories). I claim that this revolutionary modification of a theory (jump to another theory) can be treated in terms of the Wittgensteinian rule-following problem (or integration of a judgement into a naturalized Kantian unity of apperception). A new application of a rule/concept must be both rational and natural.

Dirac started with already created by Heisenberg and Schrödinger two forms of quantum mechanics. His work was more mathematical (though he was not a rigorous mathematician, but a theoretical physicist). At this level of investigation the question of theoretical understanding the connections between classical and quantum physics, and, respectively, the view that there is some continuity between them, were unavoidable.[13]

Dirac understands the move from classical mechanics to quantum mechanics as a process of mathematical generalization, as a move from a commutative algebra to a non-commutative one. He also speaks of "structural continuity of physics".

Let us in this connection mention that in (Born, Heisenberg & Jordan 1967, p. 322; quoted in Camilleri 2009, p. 90) Heisenberg's matrix quantum mechanics was characterized as a move from «classical visualised geometry» to «symbolic quantum geometry». (A general theory of non-commutative geometry has been developed only at the end of 20 and the beginning of 21 century.) As we can see, sometimes Dirac's and Heisenberg's views are almost identical.

My suggestion is that Heisenberg's "pluralism" of closed theories and Dirac's understanding of "unity" of physics correspond to different stages of the same research process – the process of natural extension (generalization) of science. And this process can be explained in different but related terms.

I claim that Heisenberg's "closed theories" are "closed", or self-contained in the same sense in which Wittgenstein's language games are. However, a language game is not only "closed". It is also open in the sense that it can be extended, or projected into a new domain.

From a metaphysical point of view, I think, roughly, a language game is Heidegger's *Dasein* (in this paper, I take it without justification). *Dasein* is thrown in a world, and is disclosed in its throwness (this is the first basic mode of disclosedness of *Dasein*). In its second basic

[13] For example, in 1925 Dirac discovered that the quantum commutator corresponds to the classical Poisson bracket, that is, that there is a formal correspondence (analogy) between the two mechanics. This discovery was purely mathematical. (Dirac 1925)

mode of disclosedness *Dasein* is projecting itself onto the possibilities of existence. (Gorner 2007, Mulhall 2005)

To speculate a little bit: classical mechanics, for instance, was born as a throwness of *Dasein* in a world. And it disclosed itself according to the second basic mode of disclosedness by projecting onto the possibility of quantum mechanics (see footnote 35).

The disclosedness of *Dasein* is the truth. This is the primary, the deepest sense of truth corresponding to the old Greek notion of truth as *aletheia*.[14] For Wittgenstein, as I understand him, the disclosedness corresponds to the authenticity (spontaneity) of a language game, or, to speak in pragmatic terms, to its "successfulness".

This is also the primary sense of truthfulness of Heisenberg's closed theories: first of all, the closed theories are *successful* (see § 7). Heisenberg (1974) says:

> "The closed-off theory contains no perfectly certain statement about the world of experiences. For how far one may be able to grasp phenomena by means of the concepts of this theory remains in the strict sense uncertain, and can be seen only by success."

In particular, quantum mechanics is a successful generalization of classical mechanics. And they are also true in another, more metaphysical sense: a closed theory does correspond to the domain of reality that it describes. Its primary truthfulness as disclosedness transforms into its truthfulness as correspondence (by the very process of its establishing and repetitive use).

Ontology of *Dasein* or language games is, I think, consistent with Heisenberg's ontological view on quantum states as collections of potentialities or possibilities of actualization in the act of measurement (Heisenberg's notion of *potentia* was inspired by Aristotle). (See, for example, Shimony 1983) For Heisenberg, "the elementary particles themselves are not as real; they form a world of potentialities or possibilities rather than one of things or facts" (Heisenberg 1958, p. 160). Heisenberg means that the elementary particles are not real in the sense of classical ontology of things/objects/particles. They are not classical particles.

3. *A "closed" theory is not falsifiable*

In later Wittgenstein, as I understand him, the notion of a language game is a key notion. A Wittgensteinian language game is a use, or application, of a Wittgensteinian rule; it is a normative practice.

The notion of a rule is equivalent to the notion of a form of life, or a language game of the second order.

A form of life involves a number of language games of the first order, which are concrete realizations of it. The corresponding rule is implicit or more or less explicit in them. The

[14] According to Heidegger, the original meaning of « truth » is « being unconcealed » (see, for example, Pattison 2000, p. 49). Compare with Wittgenstein's view that « nothing is hidden » in language.

explicit rule, or a "hinge proposition", gives the "philosophical grammar" (explicit concept) of the form of life and its language games.[15]

That is why I take it that a closed theory can also be identified with a (established) Wittgensteinian rule (I will use the abbreviation W-rule), or concept (in a broad sense). (For both Kant and Wittgenstein, concepts are rules.)

A "physical system" *in abstracto*, can be identified with a closed theory as an abstract W-rule. A physical system (or a physical phenomenon) *in concreto*, is a (correct) application of this rule. The application connects theory and reality so that a concrete real system is naturally identified. (In this sense one can interpret Einstein's words (explicitly accepted by Heisenberg) that the theory says what is measurable. It says this not a *priori*, but in its application.)

In a correct application of a theory, the epistemological gap between it and reality is closed (the real is known). The correct application of a theory amounts to it "anchoring" in reality (and *vice versa*, the reality "aliments" the theory, in particular, in all its new applications), that is, to the condition of *adequacy*. The *appropriateness* is another, more general condition. A theory is appropriate if and only if there is a formal correspondence between it and a domain of reality to which it is being applied. A theory/rule, which is appropriate to describe a domain of reality (this domain of reality will be the true domain of its application), is inappropriate to describe other domains of reality (where it is not applicable).

Now it is clear why and in which sense a closed theory cannot be false, is not falsifiable. The falsity could appear in the following three cases.

(1) A theory is inappropriate, or bad. This amounts to a bad choice of a theory/rule (of a method of acquiring knowledge) for a given domain of reality or *vice versa*. That is, in this case there is no correspondence between a theory and a domain of reality.

An inappropriate theory is false, but it is not "closed" (though it could be closed mathematically, that is as a mathematical, not physical theory). That is, it is not an established scientific theory, but a formal schema which does not fit a domain of its presumed applications.

A closed theory is not a formal mathematical schema; it always supposes a domain if its applications which is not separable from the theory. This domain is more or less implicit in the theory (reality is more or less implicit in a closed theory), and, conversely, the theory makes explicit the domain of its application (a closed theory is more or less implicit in a reality).

For a purely mathematical theory, the condition of closedness is purely mathematical (notice that Heisenberg himself talked only about closed physical theories). Roughly, it seems to be the condition of consistency. The Euclidean geometry and the Riemannian geometry, for

[15] I do not explain here in details Wittgenstein's notions. I refer to the philosophy of "later Wittgenstein".

instance, are closed theories. In another sense, however, the former can also be seen as an open theory, since the latter is an extension of the former. From the point of view of non-commutative geometry, in its turn the classical (commutative) Riemannian geometry can be seen as an open theory, too. The closedness/openness of a theory depends on the context.

A physical theory is "bi-dimensional" in the sense that physical concepts have mathematical meaning as well as physical meaning. Hence, a closed physical theory must also be closed empirically in the sense that it must not contradict empirical facts in the domain of its application (because these facts are explicitly or implicitly involved in the determination of the corresponding W-rule).

The Riemannian geometry is closed in virtue of the meanings of its concepts. The corresponding physical theory - General Relativity – is not yet sufficiently empirically confirmed to be considered as a closed theory.

Though such, for example, objects predicted by General Relativity as black holes (yet recently considered as highly hypothetical objects) are now more and more considered as real and (indirectly) observable objects of our Universe, other problems, such as the so-called dark matter problem or the dark energy problem remain mysterious.

In his time Heisenberg had all reasons to be undecided about the status of General Relativity as a closed theory. (Einstein himself considered for a short period of time the possibility of quantum "modification" of General Relativity (Rickles 2012, p. 170).) The theory of elementary particles was not closed for Heisenberg either. (In fact, in 1963, Heisenberg thought that the theory of elementary particles, that is, physics of high energies (but not other branches of physics, for example, physics of low energies) was almost closed: "I certainly do not mean that physics will be closed. (...) What I mean here is only that it looks as if at very high energies nothing very new and exciting will happen", (Heisenberg 1984, p. 627) He was mistaken.) Is it closed now, with the discovering of the Higgs like boson? The plausible answer is "no". The Standard Model has numerous theoretical problems. Physicists try to solve them, in particular, by trying to go beyond the Standard Model.

(2) A theory is appropriate but the condition of its adequacy is violated. This amounts to a wrong application of an appropriate (good) theory. In this case, an epistemic gap between a theory and its application - a concrete physical phenomenon - appears. In principle, it can be closed by a correct application of a theory.

In this case, the theory is not wrong. What is wrong is its application.

(3) One tries to apply a good theory in a domain where it is not applicable (by consequence, the theory becomes inappropriate). Here the problem of the applicability of a concept arises: what is a concept and what is a (correct) application of a concept? What are the limits of the application of a concept? (The problem of the application of a concept is related to Wittgenstein's rule-following problem).

To apply a theory outside the domain of its application, strictly speaking, does not make any sense. By consequence, it does not work well or does not work at all. However, this is not a problem with the theory. The problem is dissolved when one notices that some applications of a theory are meaningless.

There are of course borderline cases, where it is not clear if a closed theory is applicable or not. But even in these cases a closed theory is either applicable or not (I agree with Timothy Williamson that the classical principle of bivalence holds, though I do not share his epistemic conception of vague concepts[16]).

In substance, by definition, a closed theory viewed as a genuine theoretical normative (rule governing) practice (making explicit a more fundamental normative practice - experience) cannot be false. It is as it is. And by definition, a closed theory viewed as a rule/concept cannot be false either.

Conventionally, one can speak of a closed theory as being approximate in a domain outside the domain if its application.[17] Obviously, it can be "approximate" only in the same degree in which the concepts of theory are applicable. Now it is clear why one of Heisenberg's formulations of the notion of a closed theory says: "The laws are valid with the same degree of accuracy with which the appearances are describable using the concepts" (quoted by Scheibe 2001, p. 136). This claim is tautological.

If concepts of a theory are considered as perfectly applicable, and nevertheless the theory is approximate, such theory will be inexact; it will be an approximate theory. In Heisenberg's terminology it will not be closed. What Heisenberg calls "phenomenological theories" are approximate in this sense; they are "Vorstufen zum endgültigen Verständnis" (preliminary stages to the final understanding. *Translation mine*) (Collected Works, C, II, p. 336; quoted in (Schiemann 2008, p. 73)). For Heisenberg, unlike a phenomenological theory, a closed theory involves a consistent mathematical theory; it is a tight connection of axioms, definitions, and laws. And it is flexible in its applications (hence has a certain depth). For example, for Heisenberg, the Ptolemaic system is a "phenomenological" theory. (For Kuhn, the Ptolemaic system was a paradigm like others.) Heisenberg accepted Einstein's thesis that theory determines what is observable (to translate it to my Wittgensteinian terms, rule determines the domain of its applications; and rule determines its concrete application, not purely *a priori*, of course, but in a context. So, Einstein's thesis is not idealistic[18]). On the contrary, a

[16] If I understand correctly Timothy Williamson's epistemic conception of vague concepts, for him the answer to the question of applicability of a vague concept is predetermined, though might be unknown or even unknowable.

[17] In this sense of "approximate" one can understand the following Dirac (1934) words: "As we know, no physical theory should be regarded as absolutely correct; rather, it must have definite limits of validity, within which it can be regarded as a sufficiently good approximation to reality. In particular, quantum theory itself possesses such limits of applicability (...)."

[18] Einstein is a « tamed metaphysician » : "I believe that every true theorist is a tamed metaphysicist (…). The metaphysicist believes that the logically simple is also the real. The tamed metaphysicist believes that not all that is logically simple is also real, but that the totality of all sensory experience can be 'comprehended' on the basis of a conceptual system built on premises of great simplicity." (Einstein 1950. Quoted from Scheibe 2001, p. 155) I think this is a form of realism, but not a metaphysical one.

phenomenological/positivistic theory supposes that there is an atheoretical and uninterpreted observation which the theory describes/interpretes. Such a weakening of the connection between theory and observation/experience/experiment could entail a strong version of underdetermination thesis.

One can also understand “approximate” in another sense - as something that makes part of an “approximate” language game. For Wittgenstein an “approximate” phenomenological description is not inferior in comparison with a more exact description; it is a genuine phenomenological language game. (To give an example, a good impressionistic picture is not less “perfect” than a good realistic picture.) In the same sense one can understand the approximate character of a closed theory. For example, the “approximate” character of Newtonian mechanics (for example, in comparison with relativistic mechanics) as a form of life (or a language game – when it is said about a concrete use of Newtonian mechanics) is consistent with its “perfectness” (rules/concepts are “perfect”, and a genuine form of life (or a genuine language game) is “perfect”, too).

For Heisenberg, all theories are perfect and closed. Heisenberg is rather situated at the level of a logically not-predetermined intuitive pragmatic-theoretical jump into a new domain of reality. This kind of jump corresponds to the Kuhnian change of a paradigm. However, unlike Kuhn, Heisenberg does not reject the old theories. And for Heisenberg, the intuitive jump is not irrational.[19]

In particular, Heisenberg says: “(…) Kann wirkliches Neuland in einer Wissenschaft wohl nur gewonnen werden, wenn man an einer entscheidenden Stelle bereit ist, den Grund zu verlassen, auf dem die bisherige Wissenschaft ruht, und gewissermaßen ins Leere zu springen” (in science something really new can be obtained only if in a decisive moment one is ready to leave the foundations on which the previous science is based and so to speak to jump into a void. *Translation mine*). (“Aufbruch in das neue Land”, 1926-1927, In Collected Works. C, III, 1985, p. 101) I claim that this “jump into a void” is the Wittgensteinian spontaneous/instinctive and, at the same time, rational/natural “jump”.

In contradistinction, Dirac is rather situated at the more mathematical level of explicit intertheoretic relations[20]. For Dirac, all theories are approximate and open (notice that

[19] Notice also that the key word in the title of Heisenberg’s revolutionary paper (Heisenberg 1925) is *Umdeutung* («interpretation»). Heisenberg generalized (extended) classical mechanics by re-interpreting its formalism. This move can be viewed as a *Gestalt switch* - a change of aspect.

A change of aspect supposes the existence of an underlying invariant which plays the role of a rule. An aspect is the result of a use of the rule in the corresponding context. Hence, *Gestalt Switch* - a move between contexts (aspects, respectively) - cannot be considered as irrational or arbitrary.

It is remarkable that Kuhn used the expression *Gestalt Switch* to characterize scientific revolutions. In his later work he also drew attention to similarities between evolution and scientific change. (Kindi 2012) In this connection, let me emphasize that the process of natural generalization can be interpreted as a process of natural evolution of science.

[20] Dirac (1971) tells how he began to work on quantum mechanics. He read Heisenberg’s revolutionary paper and understood the importance of the idea of noncommutation (Heisenberg himself was not happy with the noncommutativity of physical quantities appearing in his work). In his view, this idea was more important than

Heisenberg also called his closed theories “idealizations”). This is because, I explain, on the one hand, all theories have a restricted domain of applicability, and on the other hand, this domain can be extended by the process of generalization. All theories are “approximate” because they are “exact” only within the domain of their application.

I would say that Dirac’s and Heisenberg’s positions are two sides of the same coin.

Notice, that Heisenberg, too, considers the question of relations between closed theories. For him, this is the relations between sets of concepts. For Heisenberg, some concepts are more *a priori* than others. For example, for him, the set of classical mechanics concepts is contained in the set of concepts of the special relativity and also in the set of quantum mechanics as their limiting cases.

Hence, this is not true that Heisenberg is an absolute pluralist (for instance, for Bokulich, Heisenberg is a pluralistic realist).[21] He envisages the existence of relations and subordination between different closed theories. I rather agree with Erhard Scheibe for whom Heisenberg’s understanding of unity and plurality of physics is a “compromise between the extremes of unity and plurality” (Scheibe 2001, p. 83).

4. *A “closed” theory is a true theory*

In this paper, I take it without proof that *Dasein* – the kind of Being we have – is the metaphysical equivalent of the notion of the language game. (To corroborate my view, let me only mention that both philosophers, Wittgenstein and Heidegger, are normative pragmatists (see for example Brandom 2002, 2009, 2011). See also, for example (Rentsch 2003)). Hence, as I already said in § 2, a closed theory as a genuine form of life or a genuine (reflective) language game can be also seen as rooted in authentic *Dasein* (roughly, a closed physical theory is a kind of (theoretical/reflective) Being the physicists have).

For Heidegger, there is a close relation between being and truth. In particular, a truth *is* if and only if *is* an authentic *Dasein*. Hence, a closed theory is a true theory. Moreover, a closed theory is knowledge (since for Heidegger "knowing has the phenomenal character of a Being which is in and towards the world" (Heidegger 1962, p. 87), it is also true that knowledge *is* if and only if an authentic Dasein *is*).

To explain in more detail, in Heideggerian terms, I interpret a closed theory as a propositional (predicative) *uncovering* accomplished by statements or judgments. It is founded in a more basic pre-predicative uncovering. The most primordial sense of truth is the disclosedness of

---

Heisenberg’s idea of building up the theory in terms of quantities closely connected with experimental results. (I note that the Heisenberg idea has a very important broad philosophical significance. I connect his idea with a non-metaphysical Wittgensteinian realism (see § 7). The idea of noncommutativity has rather mathematical significance.) Dirac combined the idea of noncommutation with the Hamiltonian approach used in the old quantum mechanics.

[21] This is also not true that Wittgenstein is an absolute language game pluralist.

world. The latter is an aspect of the disclosedness of *Dasein* (corresponds to the old Greek *aletheia*) (for more on terminology, see, for example, Gorner 2007).

The truth as correspondence is the propositional truth. This is the *ontic* notion of truth. It is secondary and depends on the more fundamental – *ontological* - notion of truth as *disclosedness* of *Dasein*.

In other terms, the truth of a scientific theory, I suggest, is a combination of its appropriateness and its adequateness. That is, this is a combination of the truth as correspondence and the truth as *disclosedness*. (And ultimately truth is one.) A closed theory satisfies both conditions by definition. It is a well established propositional uncovering.

One can agree with Heisenberg, that a closed theory reflects reality, or, more precisely, the domain of reality where it is determined. (For Heisenberg, a closed theory is "perfectly accurate within its domain", and it is "correct for all time".) And, in this sense, it is an absolute truth.

However, this kind of realism is not metaphysical (see also § 7 below). "Before there was *Dasein* there was no truth and when there is no longer *Dasein* there will be no truth", - says Heidegger (quoted by Gorner 2007, p. 102). For instance, before their discovery the Newton laws, that is, the propositions in which they are expressed, were neither true nor false (because, according to Heidegger, true propositions uncover entities). (Gorner 2007, p. 103)

Since a closed theory is a well established one (not just a new-born one), it can be detached from experience and put in a "metaphysical" correspondence with it. According to Heisenberg, the corresponding system of concepts becomes "rigid" (this is close to Einstein`s notion of a rigid theory). This explains, I think, the persistence of the idea of a metaphysical reality.

Indeed, Heisenberg himself interprets the notion of the "thin-in-itself" as follows: "The 'thing-in-itself' is for the atomic physicist, if he uses this concept at all, finally a mathematical structure: but this structure is - contrary to Kant - indirectly deduced from experience." (Heisenberg 1958)

Let me also quote, in connection with this, Thomas J. Hickey's description/translation of Heisenberg's view on development of physics:

"The historical processes that have given rise to the whole of modern physics since the conclusion of the Middle Ages, is a developmental process consisting of a succession of intellectual constructs, which take shape as if from a "crystal nucleus", out of individual queries raised out of experience, and which eventually once the complete crystal has developed, again detach themselves from experience as purely intellectual structures that forever illuminate the world for us as closed-off theories.» (Hickey 1995/2005, p. 26 ; see also the original in German : (Heisenberg 1948, pp. 335-336))

Such an intellectual construct (in original : *geistige Struktur*) is taken as an *a priori* for a new experience allowing one to deduce a new intellectual construct.

Normative practices (*Dasein*, *language games*) are complete in themselves. And Heisenberg's closed theories viewed as reflective normative practices are "complete in themselves", too. The mathematical consistency is the tip of the iceberg. However, if a physical theory were not complete, its mathematical structure would not be a logically consistent axiomatic system. That is why Heisenberg emphasized the latter condition.

Moreover it seems that the mathematical structure of a closed theory can be identified with the characteristic one for the theory and the corresponding domain of reality (even though this structure may also have different physical uses/interpretations). For example, Lagrangian and Hamiltonian formalisms characterize mechanics (in the first instance, classical mechanics, and then its generalizations: quantum mechanics and field theory), Minkowski space – special relativity, (pseudo-)Riemannian geometry – general relativity, Hilbert space – quantum mechanics, theory of connections on fiber bundles – gauge field theory. And so on.

This suggests that there is a positive answer to the following question posed by Erhard Scheibe: "Can we then point to kind of mathematical structure and claim that this kind of structures is characteristic for the given theory in the sense that another physical theory would have another kind of structures non-equivalent to the first as being characteristic for it (…)" (Scheibe 2001, p. 163)

Classical mechanics is a complete theory. Quantum mechanics is a complete theory, too. If the quantum mechanics were "completed" it would transform into a theory with an essentially different mathematical structure and, by consequence, would become a different physical theory. The famous discussion about completeness/incompleteness of quantum mechanics can, I think, be understood in these terms.

Born and Heisenberg, for example, say: "We regard quantum mechanics as a complete theory for which the fundamental physical and mathematical hypotheses are no longer susceptible of modification ». (Born 1927) This is completeness in the sense of Heisenberg's closedness.

For Born and Heisenberg (1927), quantum mechanics is intuitive (*anschaulich*) and *complete* (see also Bacciagaluppi 2006/2009, p. 408). They explain *Anschaulichkeit* as follows (Born & Heisenberg 1927, p. 172):

„Eine physikalische Theorie glauben wir dann anschaulich zu verstehen, wenn wir uns in allen einfachen Fällen die experimentellen Konsequenzen dieser Theorie qualitativ denken können, und wenn wir gleichzeitig erkannt haben, daß die Anwendung der Theorie niemals innere Widersprüche enthält" (we believe that we intuitively understand a physical theory if in all simple cases we can qualitatively think about experimental consequences of the theory, and if, at the same time, we recognize that the theory is internally consistent. *Translation mine*)

Roughly, in my interpretation, « all simples cases » correspond to the paradigmatic cases of the application of a rule/concept/theory.

In Wittgensteinian terms, a closed theory gives the "grammar" of the domain of its application. The corresponding "laws of nature" are part of such a theory/concept/rule. So they cannot be violated. That is why Heisenberg can say about a closed theory that "wherever the concepts can be used for the description of natural processes, the laws are exactly correct" (quoted by Scheibe 2000, p. 137). This is a *tautology*, because within a closed theory laws are on a par with concepts.

This may look like a form of rigid holism. However, it is a holism of basic principles, or conceptual holism. Concepts in application are not rigid. There is a family resemblance relation between different applications of a concept. The rigid theoretical holism is compatible with flexibility of a closed theory in applications. This allows one to describe the multiplicity and complexity of real phenomena which obey the same implicit ("rigid") rules.

A change of a closed theory cannot be small. If a change of a rule or a form of life is small, it is still the same rule or a form of life. This is why, as emphasizes Weizsäcker, a closed theory cannot be corrected by means of small (or: only by means of large) change (see Scheibe 2001, p. 137). (A theory can be corrected by means of small changes only if it is not highly coherent (or "rigid" – to use Einstein's term.) (For Heisenberg, in a closed theory "die Verbindung zwischen den verschiedenen Begriffen des Systems ist so eng, daß man im allgemeinen nicht irgendeinen dieser Begriffe ändern könnte, ohne gleichzeitig das ganze System zu zerstören" (the connection between different concepts of the system is so close that in general one cannot change these concepts without at the same time destroying the whole system. *Translation mine*). (Heisenberg, W. 1984. Collected Works, C, II, p. 81)

The truth of a closed theory/rule can be "broken" by a new, "deeper" scientific practice (rules/concepts). (For example, in a sense, classical mechanics is more "superficial" than quantum mechanics. Its general structure plays the role of a kind of *a priori* for quantum mechanics.) This process is not logical, not deductive. However, it is not irrational either, though its logic can be seen only *post factum*.

The understanding of the world and action in it suppose the use of (natural) rules/concepts – "physical laws" and theories. The extension of the domain of the understanding and action is at the same time the extension of the domain of the application of rules/concepts, that is, the process of natural/rational generalization. In this consists, in the first approximation, the meaning of the so-called unity of physics and world (we do not separate them from each other), which supposes the existence of connections, but also of gaps between theories and between things. The notion of an absolute unity does not make any sense.[22]

In particular, the principle of correspondence in quantum physics is not a positive principle, but the principle of natural/rational generalization (Pris 2013).

5. *A closed theory as a "closed unity"*

Let me begin with a citation from Werner Heisenberg:

[22] Analysing the so-called principle of simplicity in physics, Dirac says that we have to change the principle of simplicity into a principle of mathematical beauty (Dirac 1939, pp. 908-909). In the same paper he talks about the unity of nature (p. 911). I would continue in the same spirit and would say that we have to change the principle of beauty into a principle of unity, and the latter into a principle of generalization. "Unity", "beauty" or "simplicity" means the existence of an (in general implicit) W-rule, or truthfulness. A closed theory is a true theory. Hence, it is also "simple", "beautiful" and "unified".

“Das Gebäude der exakten Naturwissenschaft kann also kaum in dem früher erhoffen, naiven Sinn eine zusammenhängende Einheit werden, so, daß man von einem Punkte in ihm einfach durch die Verfolgung des vorgeschriebenes Weges in alle anderen Räume des Gebäudes kommen kann. Vielmehr besteht es aus einzelnen Teilen, von denen jeder, obwohl er zu den anderen in den mannigfachsten Beziehungen steht und manche andere umschließt und von manchen umschlossen wird, doch eine in sich abgeschlossene Einheit darstellt“ (“Wandlungen...”, p. 702. In Collected Works, C, I, 1984, p. 101).

The building of the exact natural science can hardly become a coherent unity in the previously expected naive sense so that following prescribed rules on can move from a place in it to any other place. Rather it consists of individual parts each of which, though it has diverse relations with others, includes some others and is included in some others, represents a self-contained (closed) unity. *Translation mine*.

This is not a rejection of unity of science, but a more accurate view on it (in this connection, see, for example, Scheibe 1997, 1999). Obviously, any closed theory is a more coherent unity than the whole science. However, because there are connections between closed theories, their pluralism should not be understood in the absolute sense. In (Pris 2013) I argued that the meaning of the unity of science and, in particular, the meaning of the correspondence principle in quantum physics, is given by the principle of rational/natural generalization.

Moreover, Heisenberg himself talks about the unification tendency in science, the overcoming of the borders between different disciplines, such as physics, mathematics, informatics, biology and philosophy. In this sense these disciplines cannot be closed. (“Abschluß der Physik?” 1970. In *Collected Works*, C, III, 1985. pp. 385-392) Notice that this observation as it stands is also applicable to Heisenberg’s “closed theories”. For example, in semi-classical mechanics the borders between classical and quantum mechanics are vague and one can even talk about semi-classical mechanics as a new theory (see also § 7 below).

6. *Erhard Scheibe about the notion of a closed theory*

Scheibe poses the question about the epistemological status of a proposition with which we ascribe closedness to a theory. His conclusion is that the familiar dichotomy of the logical and the empirical is inadequate to describe the situation (Scheibe 2001, p. 141).

This is in agreement with my interpretation of a closed theory as a Wittgenstein form of life (rule or concept).[23] Moreover, from a contemporary point of view, the dichotomy of the logical (*a priori*) and the empirical (*a posteriori*) is almost never inadequate.[24] Timothy Williamson (2011), for instance, claims that the distinction between *a priori* and *a posteriori*

[23] A theory is a theoretical “form of life”, which is connected with the corresponding practical “form of life” – experience. The notions “form of life” and “rule” are not identical, but equivalent, because a form of life determines a rule (of this form of life) and *vice versa*.

[24] For Scheibe, too, “the path of physics (…) passes *in between* rationalism and empiricism – somewhat like the path between Scylla and Charybdis” (Scheibe 2001, p. 71). He looks for a “synthesis” (apparently beyond Kant) of rationalism and empiricism. This intermediary way or synthesis, I think, can be understood from the point of view of pragmatism, or more precisely a synthesis of rationalism, empiricism and pragmatism inspired by later Wittgenstein.

is superficial. Any experience plays a role which is more than simply enabling but less than simply evidential. I think this is also Wittgenstein's position.[25]

Notice that already in his 1928 lecture on 'The Epistemological Problems of Modern Physics' Heisenberg declared that the theories of relativity and quantum mechanics had rendered problematic Kant's distinction between a priori and a posteriori knowledge.

A form of life involves (explicitly or implicitly) the so-called hinge propositions. They are a "philosophical grammar", or natural rules, of the form of life in which they are anchored. They are neither purely a priori ("logical") nor purely a posteriori ("empirical"), but natural as well as Wittgenstein's language games are.[26]

The epistemological status of the theory itself is not really different. In general the propositions of a theory (the theory itself, or theoretical knowledge) are neither purely *a priori* nor purely *a posteriori*. A true ("correct", "genuine", "beautiful", "simple", "best" are synonymous) theory is a synthesis of *a priori* reflection (including theoretical and mathematical investigation) and empirical investigation.

For example, usually, "I have two hands" is a hinge proposition.[27] The laws of Newton, the Maxwell equations, the axioms of quantum mechanics are the hinge propositions of the corresponding scientific forms of life.

A closed theory is born as an intimately connected with experience "meaningful" reflective language game, and when it becomes what it is - a closed theory, that is, a system of "meaningless" hinge propositions.

The "laws of nature" (hinge propositions) can be stronger (more necessary) or weaker. For example, not all nomological (natural) necessities are metaphysical ones (Fine 2005). Notice also that natural laws may have larger or smaller domains of applicability. For example, the laws of Newton are more contingent than the general Hamiltonian structure of mechanics. The latter also holds in quantum mechanics.

In this connection, let me mention that Heisenberg talks about "practical realism" and introduces the notion of a relative *practical a priori* by transforming the Kantian *synthetic a priori* from a metaphysical notion to a practical one. (Heisenberg 1958. See also *Post Scriptum* below) The Kantian synthetic *a priori* has absolute character and, according to Kant, an *a priori* truth is necessary and *vice versa*. In Heisenberg's "reinterpretation the Kantian 'a priori' is indirectly connected with experience in so far as it has been formed through the development of the human mind in a very distant past." (Heisenberg 1958)

---

[25] Recently Williamson (2012) treated logics as scientific theories. According to him, the principle of the best explanation is applicable to them. So, even logical propositions are not purely *a priori*.

[26] In general, making explicit deeply implicit rules requires both *a priori* reflection and empirical/theoretical investigation.

[27] For someone who has only one hand "I have one hand" will be a hinge proposition of his "form of life". On the contrary, the proposition "I have two hands" will be false for him. Both, *a priori* reflection and empirical investigation are required to establish a hinge proposition and also to ascribe the status "hinge" to a hinge proposition.

In the spirit of Heisenberg (I hope) and in my Wittgensteinian terms, I would say that the propositions of a closed theory are absolutely necessary only within the corresponding form of life (hence, the propositions can be more or less necessary), and some of them are *a priori* relatively to a generalization of the theory (these relative *a priori* truths are, obviously, more necessary than others, since they also hold within the new form of life, associated with the new theory). For example, the laws of Newton are necessary, but only within the Newtonian form of life (in terms of the pragmatist C. S. Peirce, they are “habits”). The Hamiltonian structure of classical mechanics is necessary, but also *a priori* relatively to the quantum mechanical form of life (so its necessity is stronger than the necessity of Newton’s laws). The classical deterministic causal law is *a priori* in classical mechanics, but it is violated in quantum mechanics. The Kantian view on absolute space and time as *a priori* forms of pure intuition is not valid in theory of relativity. And so on.

A closed theory is a well established true theory, a system of “hinge propositions”. The question about which theories are closed and which are not is the question about which theories are true and well established, and which are not. Science is the search for truth, absolute truth, and even certainty. And closed theories provide certain knowledge.

Erhard Scheibe’s analysis of the notion of a closed theory leads him to the conclusion that the understanding of the notion of a closed theory requires the understanding of what we mean when we say that concepts are applicable. This is because, I claim, a closed theory is a W-concept.

According to Scheibe, in the notion of a closed theory the applicability of concepts and validity of laws reverse their roles: the former become the premise and the latter the conclusion. This is the case when, I claim, the new (not paradigmatic) uses of a closed theory are being considered. For such uses, it looks as though the question of applicability of concepts comes first. In reality a closed theory concepts and laws are on a par. Concepts are implicit in laws and *vice versa*. Classical mechanics is an easy example.

Scheibe remarks that the validity of certain laws follows from applicability of certain concepts in case when the applicability of the latter presupposes the validity of the former. This is, for example, the case with the second law of Newton. There is a *presuppositional* relation in this case. The laws of Newton are *presupposed* by the *elementary* concepts of force and mass. He poses the question about the status of Heisenberg’s implication “concepts are applicable, then the respective laws are valid”. This status is unclear for him.

I would say that the laws of Newton and the corresponding concepts can be obtained by making explicit the corresponding normative practice, or language game. First of all we make explicit some pieces of the game - the concepts of force and masse (leaving the connections between these pieces implicit) - and then we make explicit the connections. Applicability of concepts (materially) implies validity of laws. This is a “presuppositional” relation in a deep sense. The laws are not analytically deducible from concepts (they are not definitions; in particular, the second law of Newton is not a definition of the concept of a force, nor is it a purely “empirical” relation). Both, laws and the concepts are on a par.

A closed theory can be more or less closed. For example, it seems that classical logic is “more real”, “more true”, more “closed” than so called non-standard logics. It looks as though there were a mathematical reality in virtue of which it holds. The principles of non-standard logics are true rather in virtue of the meanings of their terms.

For Timothy Williamson, who defends the classical principle of bivalence, "(…) trivalent and fuzzy logics are undoubtedly less convenient than bivalent logic" (Williamson 2007, p. 39) I agree with him.

For Scheibe, a non-classical logic is suited to give an account of quantum situation and is closed in the sense of the Heisenberg's notion (Scheibe 2001, p. 140). I would say that non-classical logics are less closed than the classical bivalent logic. And I would rather agree with Bohr (1948, 2007, p. 317) that

> « the recourse to three-valued logic, sometimes proposed as means for dealing with the paradoxical features of quantum theory, is not suited to give a clearer account of the situation, since all well-defined experimental evidence, even if it cannot be analysed in terms of classical physics, must be expressed in ordinary language making use of common logic. »

According to Scheibe, Heisenberg favors view of empirical ascription of closedness to theories (notice that this is because Heisenberg deals with physical, not purely mathematical theories. A physical theory whose empirical confirmation is not sufficient cannot be considered as definitely established). According to Scheibe, this would mean that some theories should be more and some should be less likely candidates for closedness. One can agree with this conclusion. For example, General Relativity and the Standard Model of fundamental interactions are less likely candidates for closedness, or less closed (or may be even not closed at all), than other established physical theories. Obviously, string theory, non-commutative geometry, quantum gravitation theories are examples of not closed and even highly speculative theories of the real world.

Scheibe notes that Heisenberg sometimes goes to empirical direction, sometimes to logical direction. This oscillation is perfectly understandable because of the very status of a closed theory.

7. *A contextualist non-metaphysical realism and quantum physics*

The realistic "success of science argument" and the anti-realistic "pessimistic meta-induction argument", both share the same metaphysical presupposition about the nature of reality. The fact that the conclusions of these arguments contradict each other suggests that the presupposition is false.

The arguments can be dissolved within a Wittgensteinian contextualist realism of language games (language games are contextual by their very nature).

Very briefly, different scientific theories in their use are different "language games" or "forms of life" (in this context, a form of life is a set of interconnected uses of a theory – a set of language games). After having been detached from its uses a theory becomes a rule/concept or a "hinge proposition" (a closed theory in Heisenberg's sense). (Notice that a theory as such is a theoretical "language game" that reflects, or makes explicit, an experience - a certain practical "language game".) Different theories are related with each other within more general forms of life.

The development of science is not a progressive discovery of a pre-determined reality, but a dynamical normative/natural process of forming more general scientific forms of life, extending human activity to new domains. This process is not always more or less continuous, though it resembles the process of natural evolution/selection.

In this connection, let me mention an interesting position of Hasok Chang (2012) who argues in pragmatic Wittgensteinian terms ("system of practice" is a unit of his analysis) in favour of "active normative epistemic pluralism", or "active scientific realism", allowing multiple systems simultaneously ("more windows on nature, more enriched understanding of it", he says).

In particular, according to Chang, there was no convincing reason to kill off phlogiston. The phlogistonist system of chemistry was not inferior to Lavoisier's oxygenist system of chemistry. And, actually, the phlogiston had been revived under different names ("chemical potential energy" and others). (Chang 2012, p. 64)

Another conclusion of Chang's analysis is that we should not regard "Water is H2O" as an inevitable piece of scientific knowledge. "Water is H2O in all systems of practice familiar to modern science. But it has been, and will be, other things in some other systems." (Chang 2012, p. 214)

A new-born theory/language game "gets" reality as it is, in its very nature. The notion of a (pseudo-) "metaphysical reality", situated in front of the subject, is secondary; it corresponds to a closed theory as an *established* language game.

A closed theory as a "tightly knit system of axioms, definitions, and laws that provide a perfectly accurate and final description of a certain limited domain of phenomena"[28] (Heisenberg) fits in the notion of a descriptive language game. Heisenberg is a realist. However, he is not a metaphysical realist. That is why sometimes he was mistakenly taken to be a positivist.[29]

Let me also notice that Heisenberg often talks about "connections between the observables quantities". At first sight, this is a positivistic attitude. But in reality it is not, because for Heisenberg the observable quantities are expressed with the help of non-phenomenological physical/mathematical concepts (example: coordinates and momentum of a particle are operators/matrices, not real numbers); and the role of these concepts is not purely instrumental. This makes difference between positivism or instrumentalism and Heisenberg's non-metaphysical realism which he himself calls Platonism.

As I already said above, Heisenberg distinguishes between "phenomenological" (hence positivistic) theories and "closed" theories. A phenomenological theory is more superficial, and by consequence less flexible. It does not reflect the genuine laws of nature.

Talking about true nomological connections in the world, or laws of nature, Heisenberg uses the term "Platonic" (notice that a lot of mathematicians are Platonists). He says, for example, the following: "I think that modern physics has definitely decided in favor of Plato. These smallest units of matter are not in fact physical objects in

[28] Given a closed theory, every new notion which could be introduced within it would be secondary and reducible to the fundamental notions of the theory. A closed theory cannot be ameliorated or « repaired ».

[29] Heisenberg himself criticized positivism and even said that the Copenhagen interpretation of quantum mechanics was not at all positivistic. (*Physik und Philosophie* 1959) (*Gesammelte Werke, Collected Works*, Ableitung C, Band II, 1984, p. 135)

the ordinary sense; they are forms; ideas which can be expressed unambiguously only in mathematical language." („Das Naturgesetz und die Struktur der Materie", p. 37 in Gesammelte Werke. Collected Works. C, II, 1984. p. 373)

He also says that the connections in the nature are not made by humans (so, they are objective), and they are visible (*sichtbar*) in the Platonic ideas (so, they are not identical with the Platonic ideas) ("(...) die ganz grossen Zusammenhänge werden in den Grundstrukturen, in den so sich manifestierenden platonischen Ideen sichtbar" (the very large connections are visible in the basic structures, in the Platonic ideas manifesting themselves in this way. *Translation mine*). The Platonic ideas, for him, may be taken not only from the rational domain of human Psyche, but also from other domains which have immediate relation to the world ("(...) diese Ideen können, da sie von der dahinterliegenden Gesamtordnung Kunde geben, vielleicht auch von anderen Bereichen der menschlichen Psyche als nur von der Ratio aufgenommen werden, von Bereichen, die eben selbst wieder in unmittelbarer Beziehung zu jener Gesamtordnung und damit auch zur Welt der Werte stehen" (pp. 407-408)). (Heisenberg, W. 1967. „Das Naturbild Goethes und die technisch-naturwissenschaftliche Welt", p. 40. In *Gesammelte Werke*. *Collected Works*. Vol. II. Physik und Erkenntnis. 1984, 407) (See also Schiemann 2008, p. 79)

Bokulich (2008) says that Heisenberg uses this term for realistic position; and, for example, Schiemann (2008) says that Heisenberg's Platonism can be designated as Platonism only in a broad sense. Indeed, Heisenberg rather uses his *method of abstraction* and Goethe's idea of capacity of intuitive knowledge (remind of Wittgenstein's reference to Goethe). I agree that Heisenberg is a realist (I emphasize: a Wittgensteinian kind of realist. In particular, Heisenberg rejected the Democritus atomism (however, he went too far by rejecting on philosophical grounds the quark model in the sixties)). However, I think the term "Platonic" is not accidental by Heisenberg. Natural laws or real nomological connections are being made explicit as (naturalized) platonic ideas (I say that an idea is naturalized if it is not detached from reality, but determined by it (see also on Jocelyn Benoist's intentional realism below)).

I would talk about Wittgensteinian Platonism (though Wittgenstein is considered as criticizing Platonism, his philosophy, I think, can be extended in a way to be able to properly correct the abstract Platonism, to make it compatible with rational naturalistic pragmatism.)

Heisenberg uses the word "pragmatism" in the pejorative sense (as a successful phenomenological description). I use this word in a very different sense. In this latter sense Heisenberg is also a pragmatist. (Thomas J. Hickey (1995, 2005), for example, too, thinks that Heisenberg contributed to pragmatism in physics.) (I also notice that Heisenberg himself explicitly used the pragmatic slogan: "Success (not the "goal") justifies the means" *(Der Erfolg heiligt die Mittel*) to pass from classical mechanics to quantum mechanics (Heisenberg 1921))

According to Schiemann (2008), Heisenberg has not noticed any relations between his notion of a closed theory and Platon's ideas, in particular, the similarity between plurality of closed theories and plurality of Platonic ideas. I think that this relation is implicit in Heisenberg's writings. A closed theory is a Platonic idea in the Heisenberg sense. In particular, for Heisenberg, elementary particles are "nicht Stoff, sondern mathematische Form" (not substance, but mathematical form. *Translation mine*) of a very complicated and abstract kind. (Heisenberg, W. 1984. In *Gesammelte Werke*. *Collected Works*. Vol. II. Physik und Erkenntnis. 1984, 407)

For Heisenberg, progress in physics consists in extending the domain of experience and the respective theoretical description. This is an anti-Kuhnian view.

He says, for example (Heisenberg 1928. In (Heisenberg, W. 1984. *Gesammelte Werke. Collected Works*. Abteilung C. Band I. Physik und Erkenntnis. Piper München Zürich.: p. 23)

„Die Geschichte der Physik zeigt, dass alle großen Disziplinen der Physik: wie die Newtonsche oder klassische Mechanik, die Elektrodynamik unverändert richtig geblieben sind in dem Erfahrungsbereich, für den sie aufgestellt waren, und daß sämtliche Physiker über die Richtigkeit dieser Disziplinen einig sind. Daß diese Theorien in der modernen Physik nicht mehr verwendbar oder gültig sind, beruht darauf, daß der Erfahrungsbereich der modernen Physik weiter ist, als der der früheren sogenannten klassischen Physik" (the history of physics shows that all major disciplines of physics, such as Newton's (classical) mechanics, or electrodynamics, remain valid in the domain of experience for which they were established, and that all physicists agree on the correctness of these disciplines. These theories are not more used in modern physics. This is due to the fact that the domain of experience of the modern physics is larger than that of the so-called classical physics. *Translation mine*).

Here is Einstein and Infeld's anti-Kuhnian view on the development of physics (Einstein 1938, p. 159):

"To use a comparison, we could say that creating a new theory is not like destroying an old barn and erecting a skyscraper in its place. It is rather like climbing a mountain, gaining new and wider views, discovering unexpected connections between our starting point and its rich environment. But the point from which we started out still exists and can be seen, although it appears smaller and forms a tiny part of our broad view gained by the mastery of the obstacles on our adventurous way up."

For both, Heisenberg (see, for example, "Über die Grundprinzipien der Quantenmechanik") and Einstein (1938), a new theory supposes new concepts which correspond to a new domain of experience (I even argued that quantum concepts are not simply new; they are a new kind of concepts (Pris 2010)). An old theory is not eliminated or replaced by a new one; it continues to be used in its domain of application.

Some philosophers suggested that there is a "structural continuity" between different (at least close) physical theories. Structuralism tries to answer the "success of science argument" and the "pessimistic meta-induction argument" by rejecting the realistic assumption. It is taken as an alternative to realism and anti-realism.

I think that the structuralism can be transformed into a non-metaphysical realism by reinterpreting the "structural continuity" as the conceptual continuity, that is, as the principle of conceptual generalization. "Structural continuity" would mean that there is a common, in general implicit, W-rule/concept. For example, the fact that classical and quantum Hamiltonian mechanics share a general Hamiltonian structure is a manifestation of the existence of a common for them W-rule.[30]

So, the bold idea of "structural similarity" is not enough. It is necessary to understand the nature of natural similarity, that is, in my Wittgensteinian terms, the nature of language games.

Let me make here the following remark concerning the notion of a structure. For example, Einstein and Infeld say that Maxwell's equations are laws representing (describing) the structure of the field. (Einstein 1938) Their claim can be generalized: physical equations represent/describe the structure of reality. These claims, I think, should rather be understood in the sense of "making explicit" the implicit structure of the field, which does not exist in itself, but only as a correct mathematical description (that is, only as a structure made explicit. Think of rules/structure of most of games). I would say that Maxwell's laws are W-rules, or "philoso-

[30] This is, I think, in agreement with Dirac's view that "the correspondence between the quantum and classical theories lies not so much in the limiting agreement $h \to 0$ as in the fact that the mathematical operators in the two theories obey in many cases the same laws" (Dirac 1925; quoted by Bokulich 2004, p. 16). I interpret "the same laws" as the same W-rule.

phical grammar" of the field. To talk about the "structure-in-itself" as if it were something ontologically substantial (metaphysical structural realism) would be a doubling the entities, that is, introducing a pre-determined ontological structure literally reflected by the mathematical Maxwell's laws. (In this sense, for Heisenberg, the mathematical structures are not material.)

Einstein and Infeld's own words seem to corroborate my non-metaphysical interpretation of them. They say:

"In the beginning, the field concept was no more than a means of facilitating the understanding of phenomena from the mechanical point of view. In the new field language it is the description of the field between the two charges, and not the charges themselves, which is essential for an understanding of their action. The recognition of the new concepts grew steadily, until substance was overshadowed by the field. It was realized that something of great importance had happened in physics. A *new reality was created, a new concept* for which there was no place in the mechanical description. Slowly and by a struggle the field concept established for itself a leading place in physics and has remained one of the basic physical concepts. *The electromagnetic field is, for the modern physicist, as real as the chair on which he sits*." (*Emphasis mine*) (Einstein 1938, pp 157-158)

(I note that such a transformation of a conception into a fullblood concept and a discovered and established reality is common in physics. For a recent example see the history of the concept and discovery of the Higgs boson (see, for example, Iliopoulos 2013).

Another eloquent example: the quantum thought-experiments, as conceived by the founding fathers of quantum physics, have become reality. (See, for example, Haroch 2006. 2007) As known, Heisenberg himself did not believe that thought experiments with single atoms could be realized.)

It seems to me that this is close to the Wittgensteinian contextualist non-metaphysical realism (see also below Jocelyn Benoist's intentional realism).

Sure, Einstein and Infeld do not want to say that the electromagnetic field is of the same kind of reality as chairs, but rather that there are different kinds of reality. And the objects of one kind of reality are not more or less "real" than those of another one. The notion of the essence of reality as such does not make any sense.

They do not want to say that *reality is our*, in the sense that we can easily manipulate/create it (they say: "a new reality was created"!).

From now on I refer to a Wittgensteinian contextualist realism, or *intentional realism*, of Benoist (2010/2011, 2011. See also my presentation of his books: Pris 2012b, 2013) which is, in my view, one of the best interpretations/extensions of later Wittgenstein. According to Benoist the real is *what one has*. However, this metaphor must not be understood in the literal sense, that is, in the sense of possession. It must be understood in the sense in which one says "let us see what we have!"[31] That being said, what is important is *what we do with what we have* (it may happen that we do not know what to do with what we have).

---

[31] « Le monde n'a rien de « notre », et nous le dit encore moins » (the world has nothing of « ours », and tells us this even less. *Translation mine*) (Benoist 2011, p. 92).

Further, consciously or unconsciously Einstein and Infeld also emphasize the intimate connection between concepts and reality (they say: "a new reality was created, a new concept").

For Benoist, contextually, there is no sense in speaking of the essence of reality in general or in justifying it. A theory of reality in general is an absurd project, but one can construct theories of different forms of the real.

As for Heisenberg, he writes:

« But definitely one must now and always realize that the reality of which we can speak is never reality 'in itself', but is just a reality of which we can have knowledge, to wit in many cases a reality to which we ourselves have given form. If one objects to this last formulation that there is, nevertheless, definitively an objective world, completely independent of us and our thought … which is what we truly aim for in the case of scientific research, then one must counter this objection … with the fact that the expression 'there is' is nevertheless already derived from human language and cannot therefore really mean something which would not be connected, in one way or another, with our capacity for knowledge. For us, the only world 'there is', is precisely the one in which the expression 'there is' has a meaning ». (Manuscript 1942. In Heisenberg 1984, p. 236)

Benoist's *intentional realism* can be understood in two closely related senses (to pass from one to another it is sufficient to move the emphasis in the expression *intentional realism* from one word to the other): every reality is given to us within a point of view, an intention, a context (these notions are not separable from each other). In this sense, reality is intentional. On the other hand, genuine intention, not a pseudo-intention, is anchored in reality which is a condition of its existence (that is, there is a realist "constraint" on intentionality). In this sense and only in this sense, the intention is real. Intentional realism rejects the metaphysical approach to intentionality, which considers intentionality as something *sui generis*, as something which is added to the matter from outside.

A point of view is not separable from the thing viewed from this point of view. It must not be imposed from outside, as something independent from the thing itself. Within a context, a point a view, a thing is given to us as it is.[32] In one context we are dealing, for example, with a book; in another context, with a parallelepiped. This does not mean that there is a third object considered sometimes as a book and sometimes as a parallelepiped (though such a point of view is possible). In our case there are only two "things-in-themselves" which are at the same time "things-for-us": a book and a parallelepiped.

I think that both Heisenberg and Einstein are non-metaphysical realists, though the latter, but not the former, is a classical determinist. Einstein talks about "the creation of new concepts, forming a new picture of reality", but also about "the creation of reality". He says: "The electromagnetic field once created exists, acts, and changes according to Maxwell's laws." (Einstein 1938) It does not look like a talk about discovering of a pre-determined reality.

[32] « Il n'y a de contexte que là où on rentre dans ce jeu normatif dans le réel que, en un sens ou en un autre, on appelle « pensée ». Mais le contexte lui-même (…) est ce qui reste dans le silence » (context only exists where one enters into this normative game within the real which, in one sense or another, is called "thought". But the context itself remains in silence. *Translation mine*) (Benoist 2011, p. 88).

Heisenberg, in his turn, wrote that he was « fabricating quantum mechanics ». (Letter to Pauli dated June 21, 1925) The title of his revolutionary paper (1925) contains the key word "Umdeutung" (reinterpretation).

Heisenberg explicitly rejects the classical materialist ontology of particles (Heisenberg, *Physik und Philosophie* 1959 in (Heisenberg, W. 1984. *Gesammelte Werke. Collected Works,* Ableitung C. Band II), p. 136; see also Falkenburg 2007, 2010):

"Die Ontologie des Materialismus beruhte auf der Illusion, daß die Art der Existenz, das unmittelbar Faktische der uns umgebenden Welt, auf die Verhältnisse im atomaren Bereich extrapolieren könne. Aber diese Extrapolation ist unmöglich" (the materialistic ontology is based on the illusion that the immediate world of facts around us can be extrapolated into the atomic domain. But this extrapolation is not possible. *Translation mine*).

Indeed, according to Heisenberg's uncertainty relations (Heisenberg also called them "law of nature") the nature of quantum particles is such that it does not make any sense to talk about their exact simultaneous values of position and velocity. Hence, they are not "real" in the sense of the classical ontology, that is, they are not "things". (Heisenberg reminds that the word "real" comes from the Latin word "res", which means "Thing". Things are located in ordinary three-dimensional space, not in a mathematical/physical space. (See Heisenberg, W. 1984. *Gesammelte Werke. Collected Works,* Ableitung C. Band II, 1984, p. 121))

So, as already said above, the question about the nature of reality as such is meaningless.[33] The deepest and unique ontological essence does not exist. The ontology is variable, and it depends on the context. Classical ontology is different from quantum ontology. And it seems that at least in some cases, the semi-classical phenomena could have their own specific ontology.[34] The classical referents of some semi-classical descriptions using classical as well as quantum concepts are not always purely fictional.

Quantum ontology and classical ontology are connected because there are connections between quantum concepts and classical concepts. Indeed, the former presupposes the latter. Quantum mechanics is a generalization of classical mechanics. In fact, this is the true meaning of the most general principle of correspondence: it is the principle of natural generalization (Pris 2013; see also footnote 35).

As a presupposition of quantum mechanics, classical mechanics is implicit in it. That is why the classical limit makes sense.

---

[33] Even the question about the nature of space or time as such is meanigless: „Der Physiker kann nicht mehr hoffen, das allgemeine Wesen des Raums oder der Zeit zu ergründen. Er findet nur Sätze über einen kleinen Teil der Welt" (the physicist cannot more hope to discover the general essence of space and time. He finds the sentences only about a small part of the world. *Translation mine*) (Heisenberg, W. 1984. *Gesammelte Werke. Collected Works*. Abteilung C. Band I. Physik und Erkenntnis. Piper München Zürich)

[34] In her book, Falkenburg (1994/1995, 2007/2010) shows how the classical metaphysical notion of reality and classical ontology of particles are more and more broken when one goes to the smaller and smaller scales of the high energy physics. For her, "the correspondence principle prevents classical realism from breaking down at once in the quantum domain" (Falkenburg 2007, 2010, c. 193).

In another sense, it is quantum mechanics which is deeply implicit in classical mechanics; the former is implicit in the latter as a tendency, that is, as a possible extension of classical mechanics.[35]

Yet in another sense, classical and quantum mechanics are two different explicit rational paradigms within a super-paradigm "Mechanics" (notice that according to Erhard Scheibe, unlike, for example, classical and relativistic mechanics, classical and quantum mechanics do not have a "common home" (Scheibe 1999, p. 174)). When these paradigms are established in the course of the development of physics, the next step consists in establishing explicit connections between them, blending them. Thus, in particular, the semi-classical mechanics occurs.

This is the same process of natural/rational (at this level – more "rational" than "natural") generalization. However, it is already based on the two previously established paradigms. At this level, it makes sense to talk about semi-classical mechanics and quantum mechanics as the improvements of classical mechanics or about relativistic quantum mechanics as an improvement of non-relativistic quantum mechanics. When Dirac talks about the improvement of theories, he is situated at this level. For him, all theories are open; all theories can be modified and improved.

Dirac did not exclude even the return to determinism. However, he realistically emphasized that it could be possible only at a high conceptual prize:

„I think it might turn out that ultimately Einstein will prove to be right, because the present form of quantum mechanics should not be considered as the final form. (...) And I think it is quite likely that at some future time we may get an improved form quantum mechanics in which there will be a return to determinism and which will, therefore, justify the Einstein point of view. But such a return to determinism could only be made at the expense of giving up some other basic idea which we now assume without question. We would have to pay for it in some way which we cannot yet guess at, if we are to re-introduce determinism." (Dirac 1978, p. 10)

In this connection, think, for example, of David Wallace (2012) attempts to give a deterministic version of the Everett interpretation of quantum mechanics. I argued that this interpretation of quantum mechanics can and should be demystified within a Wittgensteinian non-metaphysical realism (Pris 2008).

[35]Bohr refers to the correspondence principle as a "law of quantum theory", a "principle of rational generalization", or "the principle of natural generalization". For him "the correspondence principle expresses the tendency to utilize in the systematic development of the quantum theory every feature of the classical theories in a rational transcription appropriate to the fundamental contrast between the [quantum] postulates and the classical theories" (Bohr 1925; quoted by Bokulich 2008, p. 90).

In a sense quantum mechanics is "deeply implicit" in classical mechanics, because "the whole apparatus of quantum mechanics can be regarded as a precise formulation of the tendencies embodied in the correspondence principle" (Bohr 1925; quoted by Bokulich 2008, p. 90). Bohr talks about the tendencies of natural extension of classical mechanics, of its projection in a new context. In a very broad sense quantum mechanics and classical mechanics are part of one and the same Mechanics.

De Broglie-Bohm theory is also deterministic. At the same time it is a new (non-classical) form of dynamics. In particular, the theory is dualistic and non-local. The ontology of the theory contains the classical trajectories which are hidden variables and the quantum wave function. The original de Broglie pilot-wave theory (first order dynamics) was an incomplete (dualistic) synthesis of waves and particles. De Broglie himself emphasized in the last paragraph of his thesis (Broglie 1924) that the physical content of his theory was not entirely specified. Bohm's (1952) theory (second order dynamics – development, transformation and completion of de Broglie's theory) is "complete" in the sense that it solves the measurement problem by taking into account both the quantum system and the measuring apparatus. Nevertheless, philosophically, it seems to me it has the same problems as de Broglie theory. That being said, recently the Bohmian trajectories have been "observed" (a set of "average trajectories" is defined operationally for an ensemble of quantum particles) by using the so-called weak measurements. (Kocsis 2011, Coffey 2011) They are less real than (or real not in the same sense as) classical trajectories, but may be not less than the wave function itself. The trajectories are not *actual*, but *potential*. One could also talk about a kind of "implicit reality". This kind of reality is similar to that of virtual particles.

By contrary, for Heisenberg quantum mechanics is neither a falsification of classical mechanics nor an improvement of it. As a closed theory, it has its own domain of applicability whose boundaries are not given *a priori*, but discovered empirically. In this and only this context, Heisenberg rejects Bohr's dualism of complementary co-existence of classical and quantum concepts.[36]

Let me refer here again to Jocelyn Benoist's interpretation of later Wittgenstein which is close to mine own (Pris 2008/2009).

> Concepts possess some flexibility, plasticity; they have an "open texture". The domain of their applications is restricted, that is, it is not without limits (there are *real possibilities* of applications of concepts), but at the same time it is "vague". In general it cannot be known *a priori*. *A posteriori* it often happens that a concept has an application which one has not been able to predict. In many cases, it does not make any sense to say whether a concept is applicable or not in an imaginary situation until a real practice permits us to make a justified decision. For example (Benoist borrows this example from Charles Travis (2000)), given the state of our current knowledge it is not possible to answer the question of whether a genetically created animal which resembled a pig but which could fly should be identified by the same concept of a pig or by another concept (maybe a concept of a different kind of a pig). In other words, one cannot answer *a priori* the question of its identification with another kind of a pig or the same kind of a pig. (Benoist 2010/2011, pp. 180-182)

At the same time, Heisenberg (like Bohr) says: "Newtonian mechanics is a kind of *a priori* for quantum theory. It is *a priori* in the sense that it is the language which enables us to say what we observe." (Heisenberg 1963; quoted by Bokulich 2004, p. 13) This is very close to Bohr's principle of complementary.

[36]Bohr's principle of complementarity (that is, indispensability of classical concepts) is easily understandable: quantum concepts presuppose classical ones. Hence, the classical concepts cannot be eliminated. In particular, the process of measurement demands both classical and quantum concepts. (The experimental macro-situation and the result of measurement are described by classical concepts.)

For Bohr, quantum mechanics is a universal theory. That is why, unlike Heisenberg, he explicitly proclaims the principle of complementarity (classical concepts are indispensable to be able to cover the whole physical world). However, neither Bohr nor Heisenberg is an eliminativist with respect to classical mechanics.[37]

According to Bokulich, for Dirac and Bohr quantum concepts are not incommensurable with classical concepts. Indeed quantum concepts are generalizations of classical concepts. However, Bohr's principle of complementarity supposes that classical and quantum concepts are very different: they complement each other, but they cannot be related with each other "smoothly". In this sense they are incommensurable.

Notice that the incommensurability or commensurability of two sets of scientific concepts is not a matter of all or nothing. And it depends on the stage in the development of science. Quantum concepts had been created as incommensurable with classical ones. Their (relative) commensurability was established later (about the precise mathematical/physical meaning of incommensurability of classical and quantum theories (concepts) see (Scheibe 2009, ch. 10)).

For example, within a quantum mechanics viewed as a non-commutative generalization of classical mechanics the latter is a limit (commutative) case of the former. This means that the relations between classical and quantum concepts become rationally justified; the two sets of concepts become commensurable.

It seems that Kuhn underestimated the rational phase in the development of science. He focused too much on the initial phase of the "irrational" jump from one paradigm to another one. For Heisenberg, the "jump" between paradigms is "intuitive", but not irrational. This makes an essential difference between him and Kuhn.[38]

Within the contextual realism the incommensurability/commensurability of classical and quantum concepts entails the respective ontological incommensurability/commensurability. To apply this to semi-classical quantum mechanics, some classical trajectories in it must be more than just calculational devices; they should not be considered as mere fictions.[39] They could really exist in the same sense in which they exist in classical mechanics. Sometimes this

[37]In 1931, for example, Bohr proposed to imagine that the experimental discoveries of electron diffraction and photonic effects (…) were made before the work of Faraday and Maxwell. He says that the state of science would then be farther away from a consistent view of the properties of matter and light than Newton and Huygens were (see the corresponding quotation in Bokulich 2008, p. 99). That is because quantum physics presupposes classical physics: the former cannot be understood without the latter, but not *vice versa.*

[38] Kuhn's later position is more rationalistic. He says : « The claim that two theories are incommensurable is (…) the claim that there is no language, neutral or otherwise, into which both theories (…) can be translated without residue or loss. » (Kuhn 1983 ; quoted in Scheibe 1999, p. 174)

[39]Semi-classical methods blend classical and quantum ideas and have their own domain of applicability. For example, recently the semi-classical mechanics turned out to be able to solve the helium atom problem in the spirit of Bohr's old quantum mechanics. Classical mechanics gives new insights into quantum dynamical structure of the Helium atom. (Bokulich 2008) It looks as if the status of the applicability or usefulness of classical concepts were stronger than purely instrumental.

is the question of a point of view or context. (See the example with book/ parallelepiped above.)

Here is another example. One can distinguish between the following two objects: a table as a set of atoms, and “the same” table as an ordinary macroscopic object. It does not make any sense to ask what the table is in reality.

Analogously, one can suppose that sometimes a quantum object which looks like a (fictional) classical trajectory in another context could be also considered as a (non-fictional) classical trajectory.

In this vein, Heisenberg himself says, for example, the following (see Heisenberg 1934; quoted in Scheibe 2001, p. 138):

“(…) Completely dissimilar schemata of laws of nature can be applied to the same physical events without contradicting one another. This is because of the fact that in a certain system of laws, due to the basic concepts on which it is built, only certain types of questions have a sense, and that through this it closes itself off against other systems in which other questions are posed.”

Hence, in a sense theories are underdetermined by empirical data However, we must remember that Heisenberg accepts Einstein’s dictum that it is the theory who tells us what is observable. Then we must conclude that the rough empirical data (« physical events ») is ambiguous and, strictly speaking, different theories describe different empirical data (in Heisenberg’s terms, different theories pose different types of questions). The underdetermination is dissolved.

That is why I do not agree with Schiemann (2008, p. 76-77) who thinks that a Heisenberg’s closed theory can be empirically undetermined. And that is why, I think, Heisenberg himself does not discuss the question of empirical underdeterminacy

Only in an approximate sense, I think, one can give reason to Schiemann (2008, p. 76-77), who talks about the possibility of describing empirically equivalent states of affaires by means of conceptually incompatible theories.

For Heisenberg our concepts, including the concept of *existence*, are vague. And this precludes establishing an absolute truth alone by rational thinking. (Heisenberg, W. 1984. *Gesammelte Werke. Collected Works*. C, II, 1984, p. 78)

In understanding ontology, the nature of physical phenomena mathematics plays a fundamental role. When it is correctly applied to a phenomenon, that is, when it is “the best explanation” of it, it discovers its nature, allows one to say what really exists.[40] Mathematical models are more than merely calculational devices. If “fictions can explain” (Bokulich 2008), they are more than simply fictions.

[40] For example, Heisenberg says : « one could not describe at the same time the exact position and the exact velocity of an electron ; one had this uncertainty relation » (In *Gesammelte Werke. Collected Works*. C, II, 1984, p. 433) And this means that the nature of the electron differs from that of the classical particle.

The idea of incompleteness of quantum mechanics is based on the classical metaphysical presupposition about the nature of reality.

For example, "hidden variables theories" have classical ontology. However, according to a non-metaphysical contextual realism unobservable hidden variables do not describe reality. For Heisenberg, it is "unattractive to try to describe more than just the 'connection between experiments'" "(…) Heisenberg is unhappy with a theory containing unobservable causal connections". (Bacciagaluppi 2006/2009, p. 265)

In his article, Bohr (1935) concludes that quantum mechanics is complete, but the dependence on the reference system in relativity theory and quantum mechanics (supposing the principle of complementarity of classical and quantum descriptions/concepts[41]) demand a radical revision of our attitude as regards physical reality. This is, I think, a form of contextualist realism.

8. *Cartwright's anti-realistic view on ("closed") theories*

I would like briefly to contrast Heisenberg's realistic understanding of (closed) theories exposed above with Nancy Cartwright's anti-realistic one.

Cartwright characterises a "closed" theory (without referring to Heisenberg) as an unambiguous, precise, mathematically represented, non-modal, and expressed in equations theory. (Cartwright 1999, p. 7) And she writes:

"Exactly what kind of closure do the concepts of our best theories in physics have? (...) The kind of closure that is supported by the powerful empirical successes of these theories, I shall argue, is of a narrowly restricted kind: so long as no factors relevant to the effect in question operate except ones that can be appropriately represented by the concepts of the theory, the theory can tell us, to a very high degree of approximation, what the effect will be." (p. 7)

To better understand what she means, let us look at her view on laws of nature.

Cartwright (1999) defends the following three theses about laws. 1. Our best physics theories are not universal. 2. All physics laws are *ceteris paribus* laws. They hold as a consequence of the repeated, successful operation of a *nomological machine*. 3. Our most wide-ranging knowledge is not knowledge of laws but knowledge of the *natures* of things (this is a realistic trend). This knowledge allows us to build new *nomological machines* giving rise to new laws. (Cartwright 1999, p. 4)

A nomological machine "is a fixed (enough) arrangement of components, or factors, with stable (enough) capacities that in the right sort of stable (enough) environment will, with repeated operations, give rise to the kind of regular behaviour that we represent in our scientific laws." (Cartwright 1999, p. 50) "Nomological machines (...) require the conditions to be just right for a system to exercise its capacities in a repeatable way, and the empirical indications suggest that this kinds of conditions are rare." (Cartwright 1999, p. 73)

I translate these theses into my Wittgenstenian realistic terms as follows:

---

[41] At first sight, the principle of complementarity contradicts the idea of completeness of quantum mechanics. How can quantum mechanics be complete, of one needs classical concepts, for example, to describe the results of measurements? In fact, from a certain non-standard point of view, quantum mechanics as a generalisation of classical mechanics, that is, as a super-paradigm "Mechanics", includes classical mechanics. From this very broad point of view, classical concepts are quantum ones. To describe physical phenomena one needs all concepts of Mechanics. In particular, the description of semi-classical phenomena needs not only quantum concepts in the narrowly sense, but also classical concepts, that is, quantum concepts in the broad sense.

1. Laws are (natural) rules/concepts that have a limited domain of applicability (in this sense, and only in this sense they are not universal).

2. The correct application of a law is not "secured". In the end the "criterion" of a correct application of a law/rule/concept is pragmatic (this is the rule-following problem). The *ceteris paribus* condition reflects the condition of pragmatic successfulness.

A "nomological machine" is a set of paradigmatic cases of application of a law/rule/concept, or a "form of life".

3. Most of our knowledge is implicit pragmatic knowledge (this is Cartwright's "knowledge of natures of things"). The "laws of nature" are the result of making explicit the implicit rules/concepts of our developed practices (their implicitness means that they cannot be identified with properties of the nature – on this point, I agree with Cartwright). Building a "nomological machine" is "building" a normative practice ("a form of life").

For example, for Cartwright, the laws of physics like Newton's second law, f=ma, obtain only in special circumstances, that is, only "when a nomological machine is at work" (Cartwright 1999, p. 25).

Unlike "phenomenological" laws (in the philosophical sense, that is, laws describing observable entities, what one has in hand[42]), which can be true or false, the fundamental principles/laws of physics are not regularity laws; they do not state the facts, but describe the "capacities", that is, not what things do or what they can do, but what they tend to do (in my terms, they are rules which make explicit the corresponding implicit norms). At the same time, she anti-realistically says: "Fundamental laws do not govern objects in reality; they govern only objects in models" (Cartwright 1983, p. 18). "The falsehood of fundamental laws is a consequence of their great explanatory power." (Cartwright 1983, p. 4)

For Cartwright, the "capacities can be assembled and reassembled in different nomological machines, unending in their variety, to give rise to different laws" (Cartwright 1999, p. 52). As one can see, Cartwright combines a realistic/naturalistic talk about capacities (and natures) with a rather anti-realistic one about nomological machines (and laws). (Though she is an anti-realist about fundamental laws, she is a realist about (observable) entities.)

Cartwright also rejects the inference to best explanation. "No inference to best explanation, only inference to most likely cause", she says (Cartwright 1983, p. 6). As for me, I take it that an explanation is the best if, and

[42] It seems to me that within a Wittgensteinian kind of realism the philosophical distinction between the observables and the unobservables (theoretical entities) is false. The distinction of physicists between phenomenological laws (which are not about the "observables" in the philosophical sense) and fundamental laws is true, but rather superficial.

Strictly speaking, one can distinguish between the observables and the unobservable hypothetical entities only in theories which are not yet closed.

In this connection, let me mention that Heisenberg too says that the boundary between "observable" and "unobservable" quantities is indeterminate. He denies the necessity of the distinction. (See Bacciagaluppi 2006/2009, p. 407)) According to him, one must take care only of what is "observable in principle", that is, also indirectly (by inference) with the help of an appropriate conceptual/mathematical scheme/theory. If the conceptual scheme is unknown, one cannot draw the conclusions about "unobservables" (that is, observable only indirectly) from what is directly observable. (I note that in this latter case the rather philosophical and temporarily distinction between observables and unobservables acquires a certain sense. Example: in the 19th century atoms were not observable. The "energeticists" regarded them as metaphysical fictions. I the 20th century they became observable, first indirectly and then directly. Recently (Stodolna 2013) the first direct observation of the orbital structure of an excited hydrogen atom has been made using a quantum microscope. The "photoionization microscopy" allows one to visualize the structure of the hydrogen atom.)

only if it makes explicit the implicit rules. So, I do not reject the inference to best explanation, but reinterpret it so that it becomes able to take into account Cartwright's critique of it.

On Cartwright's *simulacrum account* - an alternative to the conventional picture of explanation – "the route from theory to reality is from theory to model, and then from model to phenomenological law." (1983, p. 4) On a realistic account one proceeds in the opposite order.

In substance, the best way to appreciate Cartwright's insights is to see them as a critique of meta-physical realism. However, if we want to dig deeper we must accept a Wittgensteinian (or Jocelyn Benoist's contextualist) version of realism. My claim is that Heisenberg's philosophy was on the right track.

9. *Conclusion*

In this paper, I tried to answer the following questions posed by Erhard Scheibe (2001): "What precisely is a closed theory?" "What is the relation between concepts and laws as conceived in the concept of a closed theory?" "What is the relation between two successive close theories?"

Very briefly, my suggestion is that a closed physical theory has the status of the Wittgensteinian rule, concept (or, equivalently, a form of life). Its truthfulness is logical in the sense of the Wittgensteinian "philosophical grammar". It can be either applicable or not, but cannot be false.

At the same time, a closed theory does "reflect" reality (even a "metaphysical reality", that is, "things-in-themselves", when the theory is detached from experience). And it does it in a non-metaphysical way, by means of its Wittgensteinian uses/applications. Its correct applications are anchored into reality. A closed theory is both appropriate and adequate in the sense used by Jocelyn Benoist (2010/2011). It is a well established true theory.

My interpretation agrees with Heisenberg's methodological approach (see footnote 5) and Scheibe's view on "a physical theory as being a *concept* of physical systems" (Scheibe 2001, p. 354).

10. *Post scriptum*

I must confess that I had written this paper when I read the original paper of Heisenberg "Der Begriff "Abgeschlossene Theorie..." (Heisenberg 1948, 1974) In this *post scriptum* I briefly compare my analysis with the original Heisenberg's one (Heisenberg 1948). It seems to me that my interpretation of the Heisenberg notion is correct.

Heisenberg characterizes a closed theory by means of the following four criterions which, as we will see, are logically connected and cannot be separated from each other.

Firstly, a closed theory must be consistent and axiomatized. Its notions are deduced from experience and precised by axioms and definitions. (Thereafter, I remind, the theory can be detached from immediate experience.) The possible solutions of a system of equations of a closed theory correspond to the possible natural events.

In my terms, a closed theory is a W-rule/concept. I also take it to be a Wittgensteinian view that the domain of possible solutions of a system of equations (hence, the corresponding natural events) is not pre-determined or pre-determiened only partially.

Secondly, a closed theory, its concepts must « represent » something in the world of experiences (natural events). (In other words, let me say, a physical theory must have, besides a purely mathematical meaning, a physical one.) However, the connection between theory and experience is not immediate. Otherwise, the concepts of a closed theory would depend on changing experience. The concepts do not refer to experiences (events) in a « secured » way ; their correct application is a matter of success.

In my terms, Heisenberg raises the problem of a correct application of a concept/rule. This is also Wittgenstein's rule-following problem. In general, a correct application of a concept/rule is not pre-determined ; it must be successful in the sense of Wittgenstein's pragmatism. (So, in this respect, Heisenberg is a pragmatist, in spite of his critique of « pragmatism » in (Heisenberg 1929).)

Thirdly, Heisenberg says that one cannot know (*a priori*) the borders of the domain of applicability of a closed theory. He mentions Newton's mechanics. The domain of its applicability was first delimited by relativity at the beginning of the 20th century.

In my Wittgensteinian terms, the domain of applicability of a rule/concept can not be known *a priori* and is not pre-determined. It can be established only in practice.

Fourthly, a new system of concepts describing new experiences could appear when one crosses the borders of a closed theory. The old theory remains a part of scientific language. It is presupposed by a new theory. So, it plays the role of an *a priori* for the new theory. Heisenberg suggests that there are degrees of *a priori* (hence, I note, degrees of closedness of a closed theory).

In my terms, Heisenberg talks about the extension/generalization of a rule/concept (closed theory). A rule/concept can be generalized. A new more general rule/concept can be formed. In this process of extension/generalization the old rule/concept plays the role of an *a priori*. This is a kind of a relative *a priori*, because the old rule/concept is not applicable in the domain of applicability of the new rule/concept (it is applicable only relative to a restricted domain).

Notice that in (Heisenberg 1958) Heisenberg speaks of « practical realism »[43] and introduces the notion of a *practically a priori* (that is, indirectly deduced from experience and hence relative to the experience, valid only in a restricted domain). The *practically a priori* statements are neither necessary nor universal. They are historically contingent, yet indispensable because they make part of our language (think of Wittgensteinian hinge

[43] In (Heisenberg 1989), he claims that the point of view of classical physics is "dogmatic realism". According to him, Einstein criticized quantum mechanics from the point of view of dogmatic realism (Heisenberg 1989, 43 - 45).

propositions). This notion must replace the Kantian notion of a *synthetic a priori*. For example, as it turned out the classical deterministic law of causality and classical views on space and time - the result of a long process of evolution of human knowledge (hence, undirectly related to experience) - have a restricted domain of applicability. (On Heisenberg's transformation of Kantian philosophy and, in particular, the notion of a *practically a priori*, see, for example, Camilleri 2005.)

Like Camilleri (2005) I distinguishe two senses in which Heisenberg uses the *a priori* in a non-Kantian way. First, any "a priori" has a limited range of applicability; it is only a "practically a priori". Second, classical concepts (or theory) are *a priori* in the sense that they are presupposed (but not valid) in quantum mechanics.

Camilleri also writes that Heisenberg's view "should not be confused with the kind of 'pragmatic Kantianism' espoused by such thinkers as C. I. Lewis, Quine, and the later Wittgenstein, usually associated with a constructivist understanding of science". (Camilleri 2005, p. 272)

First of all, I do not agree that later Wittgenstein is associated with a constructivist understanding of science. Secondly, my thesis is indeed that Heisenberg's "practical transformation" of Kant can be understood in terms of later Wittgenstein pragmatic philosophy.

Heisenberg mentions "the fundamental paradox in quantum mechanics": though the classical concepts are not more valid, they are indispensable because their use is "the condition for observing atomic events and is, in this sense of the word, 'a priori'. (Heisenberg, W. 1958. Physics and philosophy: The revolution in modern science. London: George Allen & Unwin. p. 82)

I notice that there is no paradox. Quantum concepts are simply generalizations of classical concepts. It is a very confused (but very suggestive!) way to talk about the latter as invalid (in the quantum domain) and at the same time indispensable or *a priori*.

We do not arbitrarily invent concepts/rules.[44] If a concept/rule (or a system of concepts/rules) is not *appropriate*, that is if, in one sense or another, it does not "correspond" to a domain of reality it is supposed to do, it can be called "false" or "empty". In this sense, the concepts/rules can be true (or genuine) or false. A true concept/rule can have true (or successful) or false uses/applications which are judgements. (Hence, the talk about *a priori* concepts is also the talk about *a priori* judgements (or knowledge), and *vice versa*.)

The Kantian *a priori* is universal and necessary. Heisenberg's practically *a priori* is "universal" and "necessary" only in a limited sense. It is universal within the domain of its validity and it also has a degree of necessity. In terms of Peircian pragmatism, I would say that a *practically a priori* is a natural "habit" (for Peirce laws of natures are "habits"); and the more strong the "habit" is, the more necessary the corresponding *a priori*.

Historically, a *practically a priori* is a result of human experience. In particular, he avoids the view that the language has an *a priori* structure. (See Catherine Chevalley (1998, p. 163); quoted in Camilleri 2005, p. 280) However, Heisenberg is not a pure empiricist. He allows the contribution of rationality, for example, of mathematics, into constitution of human (in particular, scientific) experience. In particular, in his manuscript 1942, Heisenberg (see also 1984) claims that his conception of space and time is an intermediary one between the two extremes of apriorism and empiricism. In general,

„Die Annahme, daß die *a priori* gegebenen Anschauungsformen und Kategorien im biologischen Sinne angeborene Schemata seien, hält also gerade die richtige Mitte zwischen den extremen Auffassungen, die sie also als unabhängig von aller Erfahrung für unbedingt gültig oder im Gegenteil als reines Erfahrungsgut erklären wollen" (the assumption that the *a priori* forms of intuition and categories are biologically innate schemata is the happy medium between the two extreme points of view which want to explain the necessary valid *a priori* as

[44] Camilleri (2005) mentions that regarding the meaning of the *a priori* Heisenberg's work offers an alternative to both Husserl's phenomenology, on the one hand, and Poincaré's conventionalism, on the other hand.

independent of any experience or, on the contrary, as a pure product of experience. *Translation mine*). (Heisenberg 1984, C, I, p. 284 "Ordnung der Wirklichkeit" (Manuscript 1942))

The origin of « biologically innate schemata » is not purely natural. They are being formed in fight for survival (*Kampf ums Dasein*). (Manuscript 1942. In Heisenberg 1984, p. 284)

Heisenberg also resumes his analysis of the notion of a closed theory in three points (Heisenberg 1948, p. 335): (1) a closed theory holds true for all time, (2) it contains no perfectly certain statements about the world of experiences. The domain of its applicability is uncertain. A correct application of the theory is a matter of success, (3) inspite of its experiential uncertainty, it constitutes an integral part of scientific language.

In my terms, these three points concern: (1) the rule/concept, (2) the use/application of the rule (the domain of applicability), (3) the generalization/extension of the rule. And (1) - (3) are inseparable from each other : any of the three notions presupposes the two others.

For Heisenberg, what mathematics (of a closed theory) allows us to determine is an « objective fact » to a small degree only, but to a large degree it is an overview of possibilities (Heisenberg 1948, p. 333).

As I said above, these possibilities are real (that is, those in accordance with rules, not arbitrarily imagined) and, in general, they are not pre-determined. These are possibilities of the correct application of the rule. These are also Dasein's possibilities of projecting into the world.

References

Bacciagaluppi, B. & Valentini, A. 2006/2009. *Quantum Theory at the Crossroads: Reconsidering the 1927 Solvay Conference*. arXiv:quant-ph/0609184

Brandom, R. 2002. *Tales of the Mighty Dead*. Harvard UP

Brandom, R. 2008. *Between Saying and Doing*. Oxford UP.

Brandom, R. 2009. *Reason in Philosophy. Animating Ideas*. Harvard UP

Brandom, R. 2011. *Perspectives on Pragmatism*. Harvard UP

Brandom, R. 2011b. *Brandom's München Lectures* (May 2011)

Brandom, R. 2012. Brandom's lecture at the Cambridge University on 31 May 2012

Benoist, J. 2010/2011. *Concepts*. Paris: Les éditions CERF.

Benoist, J. 2011. *Eléments de philosophie réaliste*. Paris: Vrin.

Berry, Michael, 2010. “Review of Bokulich’s book”. *Brit. J. Phil. Sci*. 0, 1-7.

Bohr, N. 1920. „Uber die Serienspektren der Elemente“, *Z. Physik* **2**, 423–469. Engl. transl. in: Bohr (1922)

Bohr, N. 1922. *The Theory of Spectra and Atomic Constitution*. Cambridge UP

Bohr, N. 1935. „Can Quantum-Mechanical Description of Physical Reality be Considered Complete?” *Physical Review*. Vol. 48. October 15, pp. 696-702.

Bohr, N. 1948. “On the Notions of Causality and Complementarity”. *Dialectica,* 2, issue 3-4, pp. 312–319 (published online: 31 May 2007)

Bokulich, Alisa, 2004. “Open or Closed? Dirac, Heisenberg, and the Relation between Classical and Quantum Mechanics.” *Studies in History and Philosophy of Modern Physics*. 35 (3).

Bokulich, Alisa, 2006. „Heisenberg Meets Kuhn: Closed Theories and Paradigms”. *Philosophy of Science,* Vol. 73, No. 1 (January 2006), p. 9.

Bokulich, Alisa, 2008. *Re-Examining the Quantum-Classical Relation: Beyond Reductionism and Pluralism*. Cambridge UP.

Born, M. and Heisenberg, W. 1927. "Quantum mechanics", paper delivered to the Fifth Solvay Congress of 1927

Broglie, L. de. 1924. *Recherches sur la théorie des quanta. Ph.D.Thesis,* University of Paris.

Camilleri, K. 2005. “Heisenberg and the Transformation of Kantian Philosophy. »

International Studies in the Philosophy of Science. Volume 19, Number 3/October 2005 , pp. 271-287(17)

Camilleri, K. 2009. *Heisenberg and the interpretation of quantum mechanics.* CUP.

Cartwright, N. 1983. *How The Laws of Physics Lie*. Clarendon Press

Cartwright, N. 1999. *Dappled World*. Cambridge UP

Cartwright, N. 2007. *Hunting Causes and Using Them*. Cambridge UP

Chang, Hasok, 2012. *Is Water H2O? Evidence, Realism and Pluralism*. Springer

Coffey, T. M. et al. 2011. “Comment on ‘Observing the Average Trajectories of Single

Photons in a Two-Slit Interferometer’ “, arXiv:1109.4436v1 [quant-ph] 20 Sep 2011

Dirac, P.A.M. 1925. “The Fundamental Equations of Quantum Mechanics.” In The Collected Works of P.A.M. Dirac., p. 67)

Dirac, P. A. M. 1925. “The Fundamental Equations of Quantum Mechanics.” *Proc. R. Soc. Lond. A*. December 1, 109 752, 642-653

Dirac, P. A. M. 1931. “The Relation Betweem Mathematics And Physics” Communicated to the Royal Society of Edinburg , February 6, 1939 In Collected Works, p. 907

Dirac, P.A.M. 1932. “The Lagrangian in Quantum Mechanics”. In (Dirac, P.A.M. 1995. Collected Works. 1924-1948. Edited by R.H. Dalitz , p. 661)

Dirac, P.A.M. 1966. *Lectures On Quantum Field Theory*. Academic Press Inc.

Dirac, P.A.M. 1971. *The Development of Quantum Theory*. Science Publishers.

Dirac, P.A.M. 1978. *Directions in Physics*. A Wiley-Interscience Publication

Dirac, P.A.M. 1934. “Theory of the positron”, in (Dirac, P.A.M. 1995. *Collected Works*. 1924-1948. Edited by R.H. Dalitz, p. 725)

Dirac. 1939. “The Relation Betweem Mathematics And Physics” Communicated to the Royal Society of Edinburg , February 6, 1939. In (Dirac, P.A.M. 1995. Collected Works. 1924-1948. Edited by R.H. Dalitz, p. 907)

Dirac, P.A.M. 1978. *Directions in Physics*. A Wiley-Interscience Publication

Einstein, A., Podolsky, B., and Rosen, N. 1935. ‘Can quantum-mechanical description of physical reality be considered complete?’, Physical Review 47, 777–780.

Einstein, A., Infeld, L. *The Evolution of Physics*. The scientific book club in Charing cross road. London W.C. (Available on Internet) (Originally published by Cambridge UP)

Falkenburg, B. 1994. 1995. *Teilchenmetaphysik*. Heidelberg: Spektrum

Falkenburg, B. 2007. 2010. *Particle Metaphysics*. Springer

Falkenburg, B. 2013. “On the Contributions of Astroparticle Physics to Cosmology.” *Talk given* on 29 May 2013 (Universität Dortmund)

Fine, K. 2005. « The Varieties of Necessity. » in *Modality and Tense: Philosophical Papers*. Oxford UP

Gorner, P. 2007. *Heidegger's Being and Time. An Introduction*. Cambridge UP.

Haroch, R. 2006. 2007. *Exploring the Quantum*. Oxford University Press: Oxford

Heidegger, M. 1962. *Being and Time*, Trans. by John Macquarie and Edward Robinson. New York: Harper and Row Press

Heidegger, M. 1996. *Being and Time.* (Trans. Joan Stambaugh) State University of New York: Albany

Heidegger, M. 2007. *The Essence of Truth*. Continuum.

Heisenberg, W. 1921. Letter to Pauli. November 19, 1921.

Heisenberg, W. 1924. "Über eine Abänderung der formalin Regeln der Quantentheorie beim Problem der anomalen Zeeman-Effekte". *Z. Phys.* **26**: 291–307.

Heisenberg, W. 1925. "Über quantentheoretische Umdeutung kinematischer und mechanischer Beziehungen". *Zeitschrift für Physik* **33**: 879–893.

Heisenberg, W. 1927. ¨Uber den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik, Z. Physik **43**, 172–198

Heisenberg, W. 1929. Atomic Physics and Pragmatism

Heisenberg, W. (1930a): *Physikalische Prinzipien der Quantentheorie*, Hirzel, Leipzig. Repr.: Bibliographisches Institut, Mannheim (1958)

Heisenberg, W. 1930b. *Physical Principles of Quantum Theory*, transl. by C. Eckart and F.C. Hoyt, University of Chicago Press, Chicago

Heisenberg, W. 1942. "Über das Weltbild der Naturwissenschaft" (Unveröffentlichter Vortrag) In (*Gesammelte Werke. Collected Works. Band 1. Physik und Erkenntnis*. Piper, München, 1984)

Heisenberg, W. 1943a. Die Bewertung „der modernen theoretischen Physik". (Unveröffentlichter Vortrag) In (*Gesammelte Werke. Collected Works. Band 1. Physik und Erkenntnis*. Piper, München, 1984)

Heisenberg, W. 1943b. Discussion after "The Present Situation in the Theory of Elementary Particles" in (Heisenberg, W. 1984. Gesammelte Werke, Series B. Springer)

Heisenberg, W. 1948. Der Begriff „Abgeschlossene Theorie" in der Modernen Naturwissenschaft". *Dialectica*, 2, 331-336

Heisenberg, W. 1958. *Physics and Philosophy*. Published by George Allen and Unwin Edition, 1959

Heisenberg, 1960. "Sprache and Wirklichkeit in der modernen Physik"

Heisenberg, W. 1967. *Einführung in die einheitliche Feldtheorie der Elementarteilchen*, Hirzel, Stuttgart

Heisenberg, W. 1969. *Der Teil und das Ganze. Gespräche im Umkreis der Atomphysik*, Piper, München. Repr. in: Heisenberg (1985) pp. 3–334

Heisenberg, Werner. 1970. *Interview with Werner Heisenberg by Joan Bromberg*. June 16, 1970.

Heisenberg, W. 1971. Der Begriff der kleinsten Teilchen in der Entwicklung der Naturwissenschaft. In: *Meyers Enzyklopädisches Lexikon*, Vol. 2, Bibliographisches Institut, Mannheim, pp. 870–879. Repr. in: Heisenberg (1985) pp. 395–

404.
Heisenberg, W. 1976. Was ist ein Elementarteilchen? Die Naturwissenschaften **63**, 1–7. Repr. in: Heisenberg (1985) pp. 507–513
Heisenberg, W. 1974. *Across the Frontiers*. New York: Harper & Row, Publishers, Inc.
Heisenberg, W. 1948. « The Notion of a « Closed Theory » in Modern Science. » In (Heisenberg, W. 1974. Across the Frontiers. New York: Harper & Row, Publishers, Inc. pp. 39-46)
Heisenberg, W. 1984. *Gesammelte Werke. Collected Works*, Series B, Berlin, Heidelberg : Springer Verlag
Heisenberg, W. 1984b. *Gesammelte Werke, Collected Works,* Ableitung C, Band II
Heisenberg, W. 1985. *Collected Works*, Part C III, ed. by W. Blum, H.-P. Dürr and H. Rechenberg, Piper, München, Zürich
Heisenberg, W. 1989. *Физика и философия. Часть и Целое*. Москва: Наука
Hickey, J. Thomas, 1995/2005. "Werner Heisenberg and the Quantum Mechanics."
Iliopoulos, J. 2013. "De la conception a la découverte." Talk given at Colloque *Un boson nommé Higgs*. Collège de France. Paris. Le 24 Mai 2013
Joas, C., Lehner, C. 2009. „The classical roots of wave mechanics: Schroedinger's transformations of the optical-mechanical analogy." *Studies in History and Philosophy of Modern Physics*. 40, p. 338-351

Kindi, Vasso and Arabatzis, Theodore (eds.) 2012. *Kuhn's The Structure of Scientific Revolutions Revisited*. Routledge

Koscis, S. et al. 2011. "Observing the Average Trajectories of Single Photons in a Two-Slit Interferometer". *Science*. Vol. 322, 3 June

Kuhn, T. 1983. "Commensurability, Comparability, Communicability." In: PSA 1982, vol. 2, Ed. By P.D. Asquith and Th. Nickles. East Lansing, Mich., 1983. Pp. 669-688.

Mulhall, S. 2005. *Heidegger and Being and Time*. Second Edition. Oxford: Routledge

Pattison, G. 2000. *The Later Heidegger*. Routledge

Pris, François-Igor. 2008. "On the Everett Interpretation of Quantum Mechanics". 39 pages. *Unpublished Draft*. [in English]

Pris, François-Igor. Le fossé explicatif dans la philosophie de l'esprit du point de vue de la deuxième philosophie de Wittgenstein vue comme un naturalisme normatif, 330 pages. Thèse

(PhD) 2008 (Diffusion ANRT 2009) [in French].

Pris, François-Igor. 2010. “Heidegger’s Quantum Phenomenology.” *PhilPapers Internet*. [in English]

Pris, François-Igor. 2012. « Réalité et concepts ». *Al-Mukhatabat Journal* (e-journal), 04, pp. 227-240 [in French].

Pris, François-Igor. 2013b. « Reality, Phenomenology, and Concept. » *PhilPapers Internet*. [in English]

Pris, François-Igor. 2013. “The Meaning of the Unity of Physics.” *PhilPapers Internet*. [in Russian]

Renn, Jürgen. 2010. Talk given at HQ-3 (Berlin).

Rentsch, T. 2003. *Heidegger und Wittgenstein*. Klett-Cotta

Rickles, D. 2012. “Quantum Gravity Meets & HPS”. In (Mauskopf, S. & Schmaltz, T. *Editors*. 2012. *Integrating History and Philosophy of Science*. Springer)

Scheibe, E. 1997. *Die Reduktion physikalischer Theorien. Ein Beitrag zur Einheit der Physik*. Teil I: *Grundlagen und elementare Theorie*. Berlin: Springer

Scheibe, E. 1999. *Die Reduktion physikalischer Theorien. Ein Beitrag zur Einheit der Physik*. Teil II: *Inkommensurabilität und Grenzfallreduktion*. Berlin: Springer

Scheibe, Erhard. 2001. (Ed. by Falkenburg, B.) *Between Rationalism and Empiricism. Selected Papers in the Philosophy of Physics*. Springer.

Schiemann, G. 2008. *Werner Heisenberg.* C.H. Beck

Shimony, A. 1983. “Reflections on the philosophy of Bohr, Heisenberg, and Schrödinger.” In Physics, philosophy and psychoanalysis, edited by R. S. Cohen and L. Laudan. Dordrecht: D. Reidel.

Stodolna, A. S., et al. 2013. “Hydrogen Atoms under Magnification: Direct Observation of the Nodal Structure of Stark States », *Phys. Rev. Lett*. 110, 213001

Travis, Charles. 2000. *Unshadowed Tthought*. Harvard UP

Wallace, D. 2012. *The Emergent Multiverse: Quantum Theory according to the Everett Interpretation*, Oxford UP

Williamson, T. 2007. *Philosophy of Philosophy*. Blackwell

Williamson, T. 2011. “How deep is the distinction between *a priori* and *a posteriori* knowledge?”

Williamson, T. 2012. A talk given at the University of Hamburg.

Wittgenstein, L. 1953. 2001. *Philosophical Investigations.* Second Edition. Oxford: Blackwell.

Wittgenstein, L. 1991. *On Certainty*. Tr. G. E. M. Anscombe and George Henrik von Wright. Oxford: Blackwell.