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On Horwich's Way Out

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The minimalist view of truth endorsed by Paul Horwich denies that truth has any underlying nature. According to minimalism, the truth predicate 'exists solely for the sake of a certain logical need'; 'the function of the truth predicate is to enable the explicit formulation of schematic generalisations'. Horwich proposes that all there really is to truth follows from the equivalence schema:

> The proposition *that p* is true iff *p*,

or, using Horwich's notation, $\langle p \rangle$ is true $\leftrightarrow p$. The (unproblematic) instances of the schema form 'the minimal theory of truth'. Horwich claims that all the facts involving truth can be explained on the basis of the minimal theory.

However, it has been pointed out, e.g. by Gupta (1993), that the minimal theory is too weak to entail any *general* facts about truth, e.g. the fact that

> Every proposition of the form '$p \to p$' is true.

The minimal theory only implies every particular instance of such generalisations. This observation actually goes back to Tarski (1935) (cf. Ketland 1999). Nevertheless, it was, according to the minimalist conception of truth, a key function of the truth predicate to enable such generalisations!

In his 'Postscript' to the second revised edition of his book *Truth*, Horwich grants the problem, but proposes a way out:

> … it is plausible to suppose that there is a truth-preserving rule of inference that can take us from a set of premises attributing to each proposition some property, *F*, to the conclusion that all propositions have *F*. No doubt this rule is not logically valid …. But it is a principle we find plausible. (Horwich 1998: 137)

There is indeed such a rule. What Horwich is demanding here amounts to what is called the ω-rule, also known as the rule of infinite induction (see e.g. Hazen 1998). In its simplest form, it allows the inference of $\forall x\ \varphi(x)$ from the infinitely many premises $\varphi(\mathbf{0})$, $\varphi(\mathbf{1})$, $\varphi(\mathbf{2})$, … that result from replacing the variable x in $\varphi(x)$ with the numeral for each natural number. Obviously, Horwich considers propositions, not numbers, but let us ignore this difference for a moment. How successful is Horwich's move in saving minimalism? In what follows, I aim to argue that it fails badly.

One problem derives from the fact that the ω-rule is intimately connected to the substitutional interpretation of quantifiers. Namely**,** the ω-rule is valid only if the relevant quantifier can be interpreted substitutionally (see Dunn & Belnap 1968, Hazen 1998). Substitutional quantification is standardly explained in terms of the *truth* of the substitution instances of quantified sentences. Although this is perfectly unproblematic in many contexts, it is questionable whether minimalism, which aims to give an explanation of truth, can make use of the notion of truth in this way, and thus lean on substitutional quantification (as Horwich himself admits; see e.g. Horwich 1998: 25).

Even if one would manage to circumvent this problem, Horwich's strategy meets unbearable problems. The ω-rule has its uses in theoretical contexts, but because of its infinitary nature, it is not a rule of inference in the ordinary sense. That is, the usual rules of inference are *decidable* relations between (conclusion) formulas and *finite* sets of (premise) formulas. This is not so with the ω-rule. It requires that one can, so to say, have in mind and check infinitely many premises, and then draw a conclusion.

Consequently, we finite human beings are never in a position to apply the ω-rule. That is, even if the rule would in theory entail the desired generalisations about truth, we human beings would never reach any of these generalisations. It would only be possible for an idealised infinite mathematical super-being. But certainly we want ourselves to be able to attain, and in real life we do attain, such generalisations.

It is indeed important to understand just how ideal, abstract and highly undecidable the ω-rule is. If it is added to the elementary first-order theory of arithmetic, the resulting system proves all the truths of the first-order arithmetic. Consequently, the set of theorems of the system with this rule not only fails to be recursively enumerable (i.e. they cannot be effectively generated), it also fails to be decidable in the limit (trial-and-error decidable), and in general, it goes beyond the whole arithmetical hierarchy. It follows from Tarski's theorem of the undefinability of truth that the ω-rule itself cannot even be defined in the language of arithmetic.

Wouldn't it be much more reasonable to accept the full Tarskian theory of truth? This may mean giving up the grand hopes of minimalism, but the Tarskian theory is at least effectively axiomatisable (that is, its set of axioms is decidable, and it uses only the standard rules of inference, which are decidable relations) and can nevertheless prove all sorts of generalizations about truth which minimalism fails to entail (see e.g. Ketland 1999).

Certain features of Horwich's theory make the situation even worse. Namely, as Horwich explicitly notes (Horwich 1998: 20, fn 4), the minimal theory cannot be regarded as *the set* of propositions of the form $\langle\langle p\rangle$ is true $\leftrightarrow p\rangle$. What this actually means is that it is a proper class – larger than any, however highly infinite, set. Assuming the standard set theory, the totality of the axioms of the minimal theory has thus the same extreme level of infinity as the whole set theoretic universe (at least the power of the first inaccessible cardinal). Certainly this is nobody's theory of truth.

This may be worrying enough, but in the present context the main issue is that neither the ω-rule nor anything analogous to it makes sense if the intended universe is uncountable. For such rules require that there is a canonical name for every element of the universe, and this is totally incredible if the universe is uncountable. But this is the case with Horwich's totality of propositions.

Under closer scrutiny, Horwich's rescue strategy emerges as desperately implausible. Minimalism is still in deep trouble as it stands with the problem of generalizations.

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